All of the phases of the BLDC have the same resistance (H1)

All of the phases of the BLDC have the same inductance (H2)

The airgap length is constant (H3)

All of the components in the mechanical subsystem are rigid bodies (H4)

The friction torque acting on the shaft of the motor is composed of viscous friction (proportional to the angular velocity) and Coulomb friction (H5)

The mutual inductances are negligible improve this sentence... (H6)

The back emf in each phase is proportional to the angular velocity of the motor (H7)

0.1 Electrical subsystem analysis

A diagram of the electrical subsystem is shown in figure 1, assuming (H1) and (H2). From it, it's possible to obtain expressions for v_{ab}, v_{bc} and v_{ca} (defined as $v_a - v_b$, $v_b - v_c$ and $v_c - v_a$, respectively) [citation needed]. That, added to the fact that the currents in each phase must sum to zero (Kirchhoff's current law in the central node) and (H6), leads to the following equations:

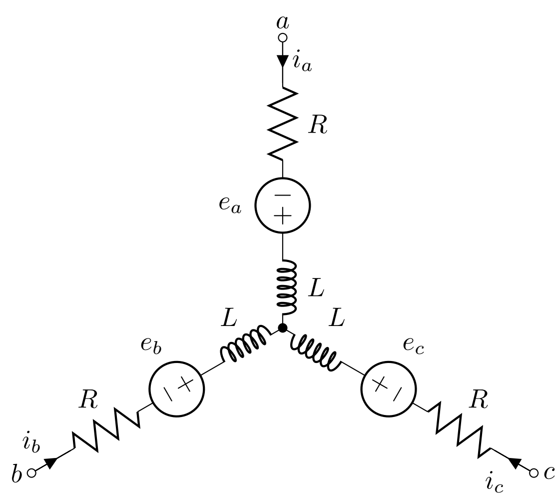

Figure 1: Diagram of the electrical circuit of a three-phase BLDC motor

$$v_{ab} = R(i_a - i_b) + L\frac{d}{dt}(i_a - i_b) + e_a - e_b \tag{1}$$

$$v_{bc} = R(i_a + 2i_b) + L\frac{d}{dt}(i_a + 2i_b) + e_b - e_c \tag{2}$$

$$i_a + i_b + i_c = 0. \tag{3}$$

Additionally, assuming (H7), we can write the back-emfs in each phase as

$$e_a = \frac{f_a(\theta_e).K_e}{2}.\frac{d\theta_m}{dt} \tag{4}$$

$$e_b = \frac{f_b(\theta_e).K_e}{2}.\frac{d\theta_m}{dt} \tag{5}$$

$$e_c = \frac{f_c(\theta_e).K_e}{2}.\frac{d\theta_m}{dt}. \tag{6}$$

Here, K_e is the motor's back-emf constant and $f_a(\theta_e)$, $f_b(\theta_e)$ and $f_c(\theta_e)$ are the functions that give the intended waveform for the back-emf; θ_m is the rotor's angular position in relation to the stator and θ_e is the electrical angular position, defined as

$$\theta_e = \theta_m.\frac{P}{2}, \tag{7}$$

where P is the motor's number of poles.

Through an analysis of the consumed power by the machine, it's possible to deduce the electromagnetic torque expression, assuming (H3) [citation needed]. This can be further simplified by substituting equations 4 through 6 in the result. The final expression is

$$T_e = \frac{K_t}{2}.\Big(f_a(\theta_e).i_a + f_b(\theta_e).i_b + f_c(\theta_e).i_c\Big). \tag{8}$$

Equations 1 through 8 model the dynamic relationship between the electromagnetic torque and the currents, back-emfs and voltages of each phase of the BLDC.

0.2 Mechanical subsystem analysis

A diagram for the mechanical subsystem is illustrated in figure 2 (to improve clarity, the load is not shown in this diagram).

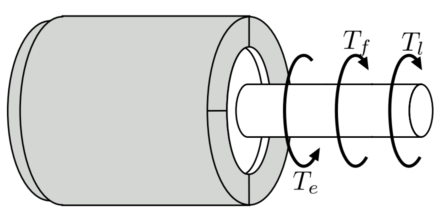

Figure 2: Mechanical subsystem

Applying Newton's second law to the motor's rotor, together with assumption (H4) we obtain the following equation

$$J.\frac{d^2\theta_m}{dt^2} = T_e - T_f - T_l, \tag{9}$$

where J is the motor's rotor inertia, T_f is the resistance torque due to friction and T_l is the torque applied by the load to the rotor.

Because of (H5), T_f can be decomposed as viscous friction (referred to as T_{f1}) and Coulomb friction (referred to as T_{f2})

$$T_f = T_{f1} + T_{f2}. \tag{10}$$

Viscous frictions can be calculated with

$$T_{f1} = K_d.\frac{d\theta_m}{dt}, \tag{11}$$

where K_d is the damping constant.

And Coulomb's model of friction [citation needed] is given by

$$T_{f2} = \begin{cases} -T_k.sign(\frac{d\theta}{dt}) & : \frac{d\theta_m}{dt} \neq 0 \\ -min(T_s, T_e - T_l).sign(T_e - T_l) & : \frac{d\theta_m}{dt} = 0 \end{cases} \tag{12}$$

where T_k is the static friction constant and T_s is the kinetic friction constant.

Analogously to section 0.1, equations 9 through 12 model the dynamic relationship between the electromagnetic torque and the angular position of the rotor. This equations, together with equations 1 through 8, constitute the full mathematical model used in the simulator.