Introduction to Magnetic Bearings

Lecture presented in Quality Improvement Program (QIP'08) at Indian Institute of Technology Guwahati

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Overview of the Presentation

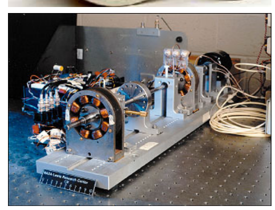
- Introduction
- Design of Active Magnetic Bearings
- Control Engineering of Magnetic Bearings
- Control of Rotor by using Magnetic Bearings
- Conclusions

Introduction

- **An active magnetic bearing (AMB) system supports a rotating shaft, without any physical contact by suspending the rotor in the air, with an electrically controlled (or/and permanent magnet) magnetic force.**
- **It is a mechatronic product which involves different fields of engineering such as Mechanical, Electrical, Control Systems, and Computer Science etc.**

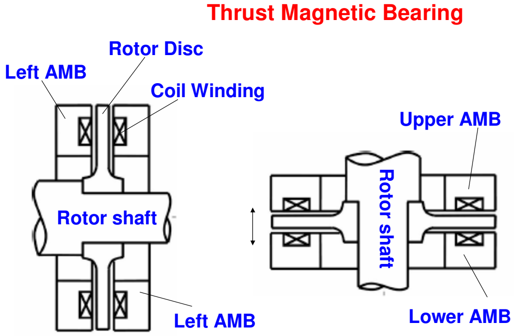
Radial Magnetic Bearing

Eight-Pole Radial Magnetic-Bearing



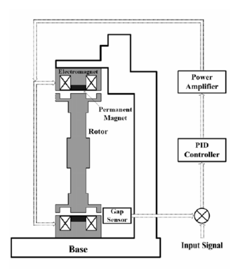
Test Apparatus for rotor control

Thrust Magnetic Bearing



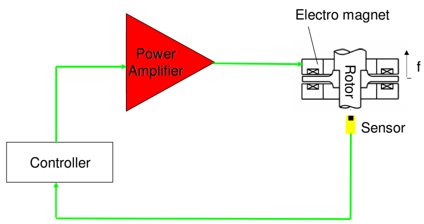
Horizontal shaft **Vertical shaft**

Introduction to Active Magnetic Bearings



Typical Actuator – Controller unit of an AMB

Introduction to Active Magnetic Bearings



Working principle of magnetic bearing

Advantages of Magnetic Bearings

- Magnetic Bearings are free of contact and can be utilized in vacuum techniques, clean and sterile rooms, transportation of aggressive media or pure media
- Highest speeds are possible even till the ultimate strength of the rotor
- Absence of lubrication seals allows the larger and stiffer rotor shafts
- Absence of mechanical wear results in lower maintenance costs and longer life of the system
- Adaptable stiffness can be used in vibration isolation, passing critical speeds, robust to external disturbances

Classification of Magnetic Bearings

According to
- control action
  - Active
  - Passive
  - Hybrid
- Forcing action
  - Repulsive
  - Attractive
- Sensing action
  - Sensor sensing
  - Self sensing
- Load supported
  - Axial or Thrust
  - Radial or Journal
  - Conical
- Magnetic effect
  - Electro magnetic
  - Electro dynamic
- Application
  - Precision flotors
  - Linear motors
  - Levitated rotors
  - Bearingless motors
  - Contactless Geartrains

Applications of Magnetic Bearings

- Turbo molecular pumps
- Blood pumps
- Molecular beam choppers
- Epitaxy centrifuges
- Contact free linear guides
- Variable speed spindles
- Pipeline compressor
- Elastic rotor control
- Test rig for high speed tires
- Magnarails and maglev systems
- Gears, Chains, Conveyors, etc
- Energy Storage Flywheels
- High precision position stages
- Active magnetic dampers
- Smart Aero Engines
- Turbo machines

Fields of Applications of Magnetic Bearings

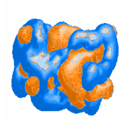
- Semiconductor Industry
- Bio-medical Engineering
- Vacuum Technology
- Structural Isolation
- Rotor Dynamics
- Maglev Transportation
- Precision Engineering
- Energy Storage
- Aero Space
- Turbo Machines

Electromagnetism

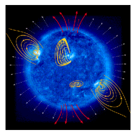
- **Electromagnetic field**
- **Lorenz force**

Electromagnetism

- When a charged particle is at rest it won't emit electromagnetic waves rather it is surrounded by electrostatic field
- When the charged particle is in uniform motion (i.e. the motion with uniform velocity in a direction) the electrostatic field is associated with magnetostatic field.



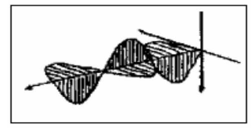
3d electrostatic field surrounding a charged particle



Magnetostatic field

Electromagnetism

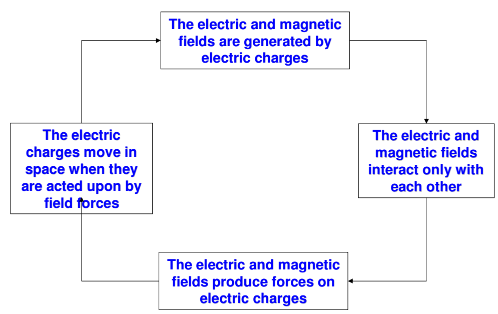
- When the particle is in accelerated motion then the magnetic field will be oscillating.
- In electromagnetic waves both the electric and magnetic fields are oscillating and harmonic.



Feed back loop of electromagnetism

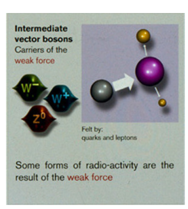
- The electric and magnetic fields are generated by electric charges
  - Charges generate electric fields
  - Movement of charges generate magnetic fields
- The electric and magnetic fields interact only with each other
  - Changing electric field acts like a current, generating vortex of magnetic field
  - Changing magnetic field induces (negative) vortex of electric field
- The electric and magnetic fields produce forces on electric charges
  - Electric force which is generated by the electric field and is in same direction as electric field
  - magnetic force which is generated by the magnetic field and is perpendicular both to magnetic field and to velocity of charge
- The electric charges move in space
  - The electric charges move in space when they are acted upon by field forces

Feed back loop of electromagnetism



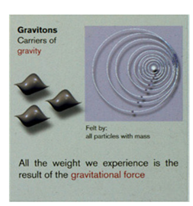
The four fundamental forces

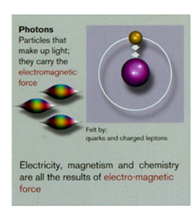
- Strong nuclear force
  - which holds atomic nuclei together
- Weak nuclear force
  - which causes certain forms of radioactive decay



The four fundamental forces

- Gravitational force
  - Which causes the masses to attract each other
- Electromagnetic force
  - Which is caused by electromagnetic fields on electrically charged particles





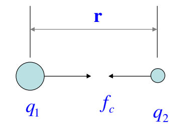
The four fundamental forces

- All the other forces are derived from these four fundamental forces
- Electro-magnetic force is one of these four fundamental forces

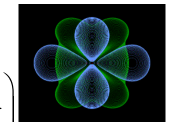
Force between two electrically charged particles

- Coulomb force (Static)

$$f_c = \frac{q_1 q_2}{4\pi\varepsilon_0 r^3}\mathbf{r}$$



- Lorenz force (Dynamic)



$$f_l = \left(\frac{\gamma q_1 q_2 \mathbf{r}}{4\pi\varepsilon_0 r^3}\right) + \mathbf{v}\times\left(\frac{\gamma q_1 q_2 \mathbf{v}\times\mathbf{r}}{4\pi\varepsilon_0 c^2 r^3}\right)$$

Electric and magnetic components of Lorenz force

- If $q_1 = q$ then

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

$\mathbf{E} = \frac{\gamma q_2 \mathbf{r}}{4\pi\varepsilon_0 r^3}$ **Electric flux;** $\mathbf{B} = \frac{\gamma q_2 \mathbf{v}\times\mathbf{r}}{4\pi\varepsilon_0 c^2 r^3}$ **Magnetic flux;**

$r = \|\mathbf{r}\|;$ $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ **Lorenz factor;**

$\varepsilon_0 = 8.854\times10^{-12}\ \mathrm{C^2/J\text{-}m}$ **Electric permeability of vacuum;**

$\frac{1}{\varepsilon_0 c^2} = \mu_0 = 4\pi\times10^{-7}\ \mathrm{N/A^2}$ **Magnetic permeability of vacuum;**

Comparison Electric and magnetic components of Lorenz force

$$\frac{|\mathbf{v}\times\mathbf{B}|}{|\mathbf{E}|} \le \frac{v^2}{c^2} \le \frac{1}{10^{23}}$$

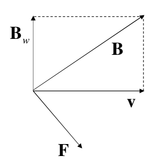
Three conclusions:

- Magnetic component of Lorenz force is at least smaller by a factor of 10^{23}! But we don't face the effect of electric field in conductors because *protons and electrons are equal in number and generate equal and opposite electric fields canceling each other*
- Protons have no motion with reference to conductor and there won't be magnetic component from them. Thus *the magnetic component observed is the relativistic effect of electrons only*
- When the conductor is moving with reference to another frame both the protons and electrons will move with the same velocity thus the *relativistic effects due to the velocity of conductor will be cancelled out*

Effective Lorenz force in macro calculations

- For macro calculations Lorenz force is reduced to the form

$$\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$$



- Lorenz force acts perpendicular to both velocity of charged particle and magnetic flux

Relations between **E** and **B**

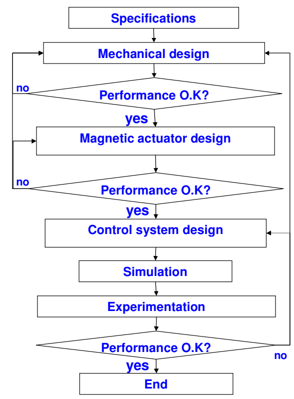
Gauss' Law for linear materials	$\nabla\cdot\mathbf{E} = \frac{q}{\varepsilon_0}$	$\int_S \mathbf{E}\cdot\mathbf{ds} = \frac{1}{\varepsilon_0}\int_V q\,dv$
Gauss' Law for magnetism	$\nabla\cdot\mathbf{B} = 0$	$\int_S \mathbf{B}\cdot\mathbf{ds} = 0$
Faraday's law of magnetic induction	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	$\int_L \mathbf{E}\cdot\mathbf{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B}\cdot\mathbf{ds}$
Ampere's law and Maxwell's extension	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$	$\int_L \mathbf{B}\cdot\mathbf{dl} = \mu_0\int_S\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)\cdot\mathbf{ds}$

These relations are called simplified Maxwell's relations who formulated the original relations from previous works

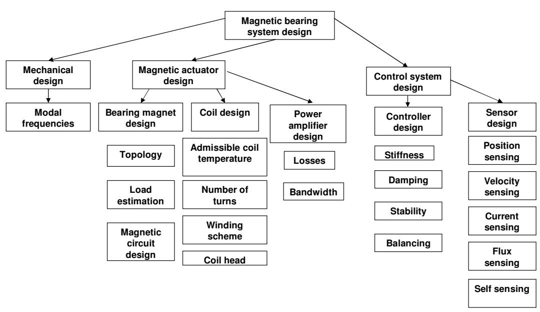
Design of magnetic actuator

- Bearing magnet
- Magnetic circuit
- Coil

Design methodology of magnetic bearing systems



Areas involved in the design of magnetic bearing systems

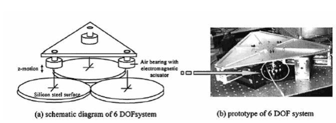


Magneto mechanical systems

According to the known technology till now, magnetic bearings can be classified for their design according to the purpose of the levitated object as

Magneto mechanical systems

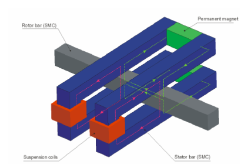
- **Precision flotors** (precision stages, isolation bases, isolation springs)
  - Levitation force
  - Propulsion force

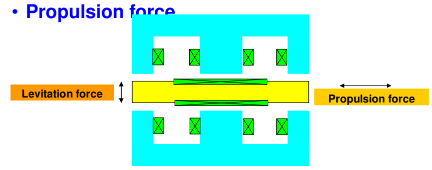


A magnetic Precision Stage

Magneto mechanical systems

Linear motors
(Contactless sliders, maglev trains and conveyors)
- Levitation force
- Propulsion force

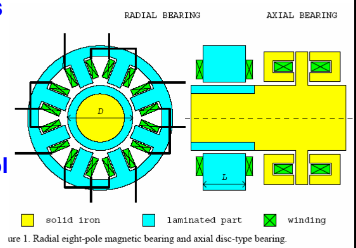




Principle of a linear motor

Magneto mechanical systems

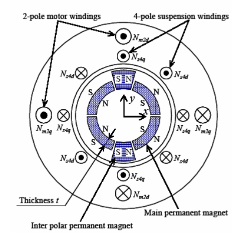
Levitated rotors
(gas turbines, energy storage flywheels, high speed spindles, balancing and vibration control of rotors)
- Radial load
- Thrust load



Rotor levitated by Radial and Axial Active Magnetic Bearings

Magneto mechanical systems

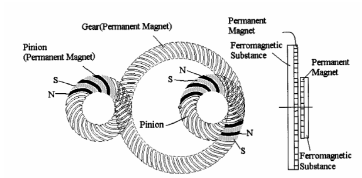
Bearingless motors
(canned pumps, compact pumps, blood pumps, spindle drives, semiconductor process)
- **Radial load**
- **Thrust load**
- **Torque**



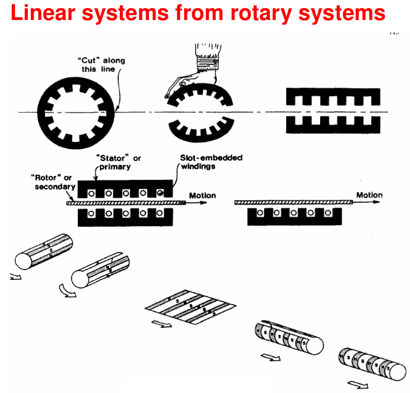
Bearingless Motor

Magneto mechanical systems

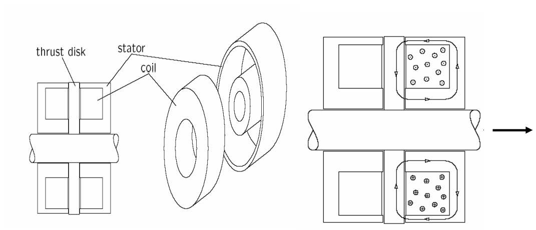
Contactless Gears and Couplers
- **Regulated torque transmission**



Linear systems from rotary systems



Design of a thrust magnetic bearing



Macro Geometry of Thrust Magnetic Bearing

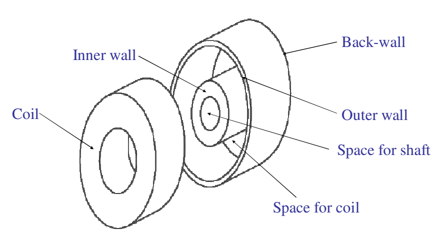
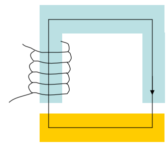


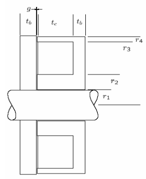
Figure 1: Parts of Thrust Magnetic Bearing

Optimal design

Optimal design is carried out in two steps

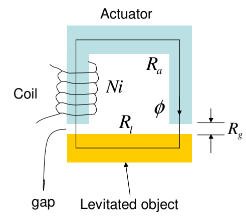
- **Modeling the magnetic circuit**
  - **Determines the accuracy of achieving the objective**



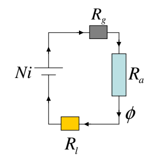


- **Optimization of the parameters**
  - **Determines the efficiency of the achieving the objective**

Magnetic circuit



Magnetic circuit



Equivalent electric (dc) circuit representation

Magnetic circuit analogy with electric circuit

Magnetic circuit	Electric circuit
Magneto Motive Force (MMF)	Electro Motive Force (EMF) or Voltage (V)
Magnetic Flux (ϕ)	Electric Current (i)
Reluctance (R)	Resistance (R)

Ideal magnetic circuit model

$$\oint_L H \cdot dl = \int_S J \cdot n da \quad \text{(Ampere's law)}$$

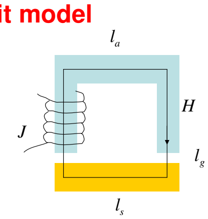
$$2H_g l_g + H_a l_a + H_s l_s = ni$$

$$B = \mu H \text{ or } H = B/\mu$$

$$2B_g l_g + \mu_0\left(\frac{B_a}{\mu_a} l_a + \frac{B_s}{\mu_s} l_s\right) = \mu_0 ni$$

if $\mu_0\left(\frac{B_a}{\mu_a} l_a + \frac{B_s}{\mu_s} l_s\right)$ is neglected

$$B_g = \frac{\mu_0 ni}{2 l_g}$$



Flux density is used to find the force exerted

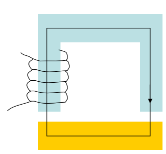
Extension of the ideal model

if K_a is added for $\mu_0\left(\frac{B_a}{\mu_a} l_a + \frac{B_s}{\mu_s} l_s\right)$ as core loss factor and K_i is added as coil loss factor, then

The model reduces to

$$2K_a B_g l_g = \mu_0 K_i ni$$

$$B_g = \frac{\mu_0 K_i ni}{2K_a l_g}$$

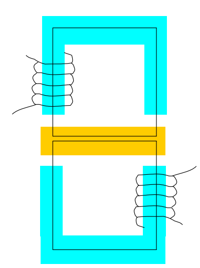


Force by using flux density

$$B = \frac{\phi}{A} = \frac{Ni\mu_0}{2(l_g - x)}$$

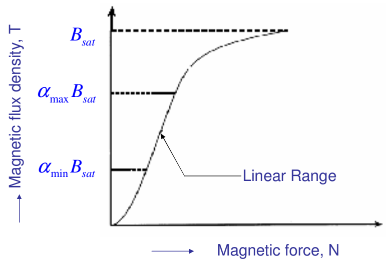
$$F = \frac{B_0^{\ 2}}{2\mu_0} A_g$$

$$\Delta F = \frac{2A_g}{\mu_0} B_0 \Delta B$$



Differential actuator

Linear range of flux density



Hysteresis is assumed to be negligible while setting the linear range

Terminology used in magnetic circuit

Quantity	Symbol	Formula	Units	Magnitude
Permeability of vacuum	μ_0	$\frac{1}{\varepsilon_0 c^2}$	Vs/Am	$4\pi\times10^{-7}$
Permeability	μ	$\mu_0\mu_r$	Vs/Am	0.026
Reluctance	R	$\frac{l_{fp}}{\mu A}=\frac{l_{fp}}{\mu_0\mu_r wl}$	Vs/A	7.95e5 for air 3.97e4 for Fe
Magneto motive force	ni	$n\times i$	A-turns	1600
Magnetic flux	ϕ	$\frac{Ni}{2R_g}=\frac{Ni\mu_0 wl}{2(g-x)}$	Wb	0.0010
Flux density	B	$\frac{\phi}{A}=\frac{Ni\mu_0}{2(g-x)}$	T	10.05
Magnetic flux linkage	λ	$N\phi$	Wb-turns	0.1005

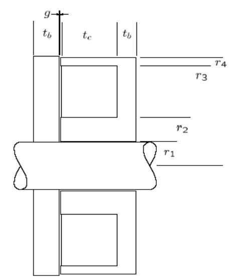
Different quantities used in magnetic circuit

Quantity	Symbol	Formula	Units	Magnitude
Current density	J	$\frac{i}{A}=\frac{i}{wl}$	A/m²	16e4
Magnetic inductance	L	$\frac{\lambda}{i}=\frac{n^2\mu_0 wl}{2(l_g-x)}$	H=Wb/A	
Nominal inductance	L_0	$L\big\vert_{x=0}=\frac{n^2\mu_0 wl}{2l_g}$	H	0.0063
Magnetic force by inductance	F	$\frac{L_0 i^2}{2l_g}$	N	804.2
Magnetic force by flux density	F	$\frac{B_0^2}{2\mu_0}A_g$	N	804.2
Magnetic force for diff actuator	F	$\frac{A_g}{2\mu_0}\left(B_+^2-B_-^2\right)$	N	19.84

Design vector for optimal design

Known parameters are
- Gap
- Inner radius of the bearing
- Outer radius of the bearing

Free parameters
- Inner radius of the coil space
- Outer radius of the coil space
- Height of the coil space
- Current density supplied

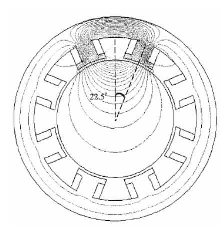


All the other parameters are dependant

Input parameters taken for the design of thrust magnetic bearing

Parameter	Value	Parameter	Value
Inner radius of the bearing	25.00mm	Specific gravity of the stator iron	7.77g/cm³
Operating air gap	4.00mm	Specific gravity of the copper	8.91g/cm³
Operating load	2025N	Specific gravity of permanent magnet material neodymium-iron-baron	7.5g/cm³
Variation in the gap	±5%	Coil mmf loss factor	1.394
Variation in the load	±10%	Actuator loss factor	1.072
Saturation flux density	1.00T	Flux leakage factor	0.840
Remnant flux density of bias magnets	1.2T	Packing factor	0.85
Saturation current density	4.0A/mm²	Maximum allowable coil volume	820mm³
Maximum outer radius of bearing	120mm	Maximum height of bearing	70mm

Eight pole radial magnetic bearing

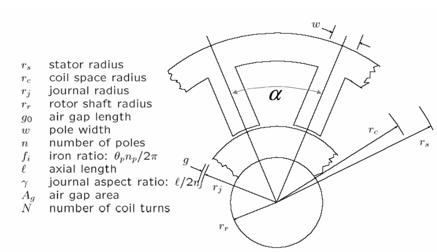


Eight Pole AMB

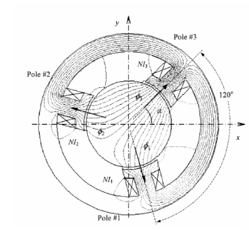
Radial magnetic bearing

The component of force will be at an angle of half of the angle between two poles

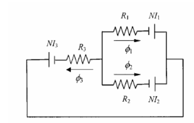
$$F = \frac{\mu_0 A_g (K_i n i)^2}{4(K_a l_g)^2} \cos\frac{\alpha}{2}$$



Three pole radial magnetic bearing



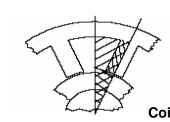
Three Pole AMB



Magnetic Circuit for three pole AMB

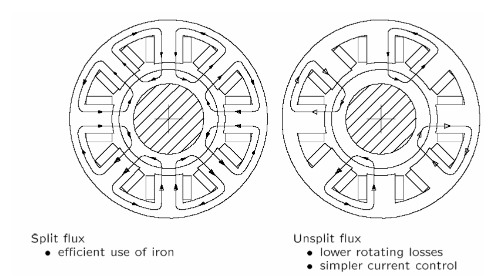
Coil design

- Admissible coil temperature is determined by the choice of insulation type
- Number of turns are chosen such that it generates maximum admissible magneto motive force at the maximum current supplied by the power amplifier

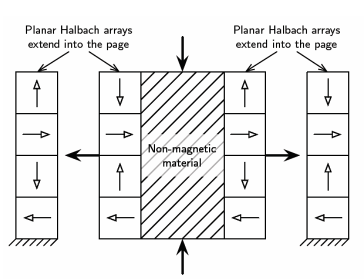


Coil

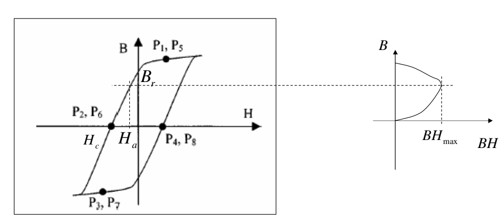
Winding scheme



Permanent magnetic bearings



Permanent magnetic bearings



MAGNETIC BEARINGS

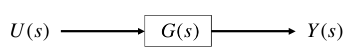
CONTROL

Introduction

- *Control is the process of bringing a system into desired path when it is going away from it*
- Earnshaw(1842) had shown that it is impossible to hover a body in all six degrees of freedom by using permanent magnets
- But it is possible to maintain the body in equilibrium condition by active control

Types of control systems

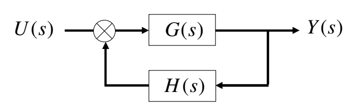
- *Open loop control systems*



  - The control in which the output of the system has no effect on input is called open loop control
  - Open loop control is used when the input is known and there are no external disturbances
  - An example of open loop control is washing machine which works on time basis rather than the cleanliness of clothes

Types of control systems

- *Closed loop control systems*



  - If the control maintains a *prescribed output and reference input relation by comparing them and uses their difference as controlling quantity*, it is called feedback or closed loop control
  - Temperature control of a room or a furnace is an example of closed loop system

Classification of controllers

- According to control action controllers are classified as:
  - Two-position or on-off controllers
  - Proportional controllers
  - Integral controllers
  - Proportional-integral controllers
  - Proportional-differential controllers
  - Proportional-differential-integral controllers

Classification of controllers

- Two-position or on-off controllers
  - The output of the controller $y(t)$ will be a maximum or minimum according to the state of error $e(t)$ as below:

$$\begin{aligned} y(t) &= y_0 \quad \text{for } e(t) < 0 \\ &= y_1 \quad \text{for } e(t) > 0 \end{aligned}$$

  - y_0 and y_1 are minimum and maximum values of output

Classification of controllers

- Proportional controllers:
  - The output of the controller $y(t)$ is proportional to the magnitude of the actuating error $e(t)$ signal as

$$y(t) = g_p e(t)$$

  - By Laplace transformation

$$\frac{Y(s)}{E(s)} = g_p$$

  - g_p is called proportional gain

Classification of controllers

- Integral controllers:
  - In integral control action, the value of the controller output $y(t)$ is changed at a rate proportional to the actuating error signal $e(t)$

$$\frac{dy(t)}{dt} = g_i e(t)$$

  - By Laplace transformation
  - $$\frac{Y(s)}{E(s)} = \frac{g_i}{s}$$
  - g_i is called integral gain

Classification of controllers

- Proportional-Integral (PI) controllers:
  - Control action is a combination of both proportional and integral action

$$y(t) = g_p e(t) + \frac{g_p}{T_i}\int_0^t e(t)dt$$

  - By Laplace transformation

$$\frac{Y(s)}{E(s)} = g_p\left(1 + \frac{1}{T_i s}\right)$$

Classification of controllers

- proportional-differential (PD) controllers:
  - The control action is defined by

$$y(t) = g_p e(t) + g_p T_d \frac{de(t)}{dt}$$

  - By Laplace transformation

$$\frac{Y(s)}{E(s)} = g_p(1 + T_d s)$$

Classification of controllers

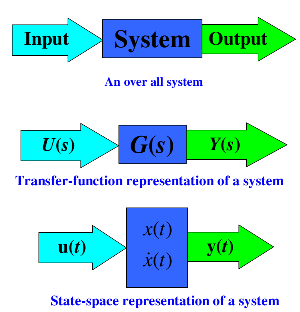
- proportional-Integral-differential (PID) controllers:
  - It has the advantages of all three actions. So this is the most common type of industrial controllers
  - Mathematical form of PID action is

$$y(t) = g_p e(t) + \frac{g_p}{T_i}\int_0^t e(t)dt + g_p T_d \frac{de(t)}{dt}$$

  - By Laplace transformation

$$\frac{Y(s)}{E(s)} = g_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

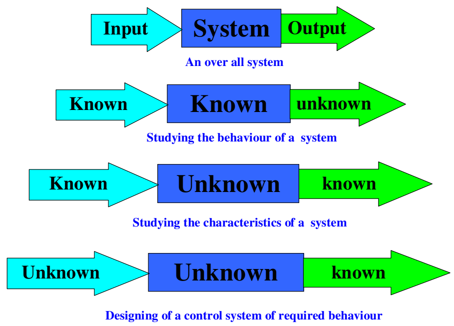
Control Design



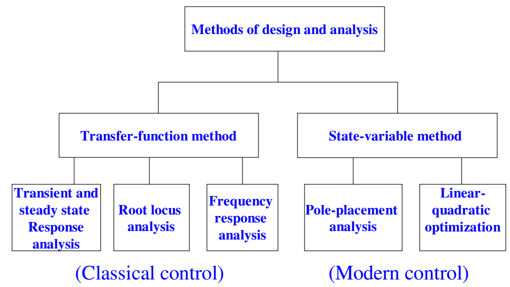
$$Y(s) = G(s)U(s)$$

$$\mathbf{y}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

Control Design

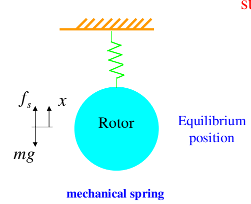


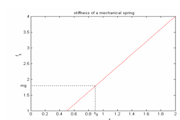
Methods of design and analysis of controllers



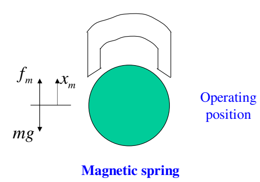
Transfer-function method	State-space method
❖Classical control method	❖Modern control method
❖Used for single input single output (SISO) systems	❖Used for multi input multi output (MIMO) systems can be used for SISO also
❖It is useful for linear and simple systems only	❖It is useful for nonlinear and complex systems also.
❖Frequency domain method	❖Time domain method
❖Analysis consists of single higher order differential equation	❖Analysis consists of system of *n* first order differential equations.
❖Steady state and transient response analysis, Root locus analysis and frequency response analysis are the main methods of design and analysis	❖Pole-placement method and Linear-quadratic optimization are the main methods of design and analysis.

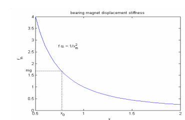
Mechanical and electro magnetic stiffness



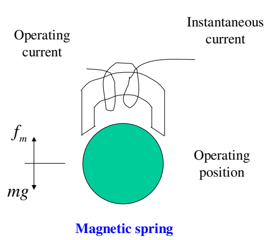


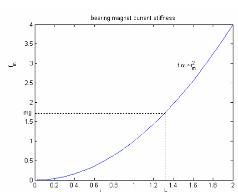
Mechanical spring stiffness





magnetic displacement stiffness

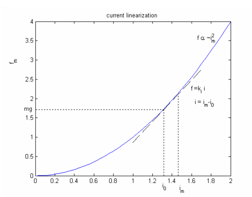




Magnetic Bearing Control

- Equilibrium and Operating points
  - For a mechanical spring there will be an *equilibrium point* where the force resisted by the spring is equal to the force applied on the spring
  - For electro magnets there will be a quantity of current corresponding to position of the object and force applied. At this point the gravity force and magnetic force will be equal. A slight movement form this point will cause indefinite movement of the body. This point is called *operating point*

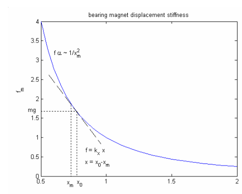
Linearization at operating point



Linearization of current

$$i = i_m - i_0$$
$$f = k_i i$$

i_m is the instantaneous current



Linearization of displacement

$$x = x_0 - x_m$$
$$f = k_x x$$

x_m is the instantaneous position

Linearized formula around the operating point will be

$$f(x,i) = k_x x + k_i i$$

where

x is the displacement from the operating position

i is the deviation of current from operating current

k_x is displacement stiffness

k_i is current stiffness

f is instantaneous force

- Linearized equation is suitable for most of the applications of magnetic bearings
- It is not valid in three occasions
  - When $x = x_0$ the rotor touches the bearing magnet
  - When there are strong currents such that magnetic saturation of the material occurs
  - When $i = -i_0$ or very small currents there won't be levitation of the rotor because of very small magnetic forces.

Magnetic Bearing Control

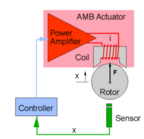
$$f = k_x x + k_i i$$

By Newton's law

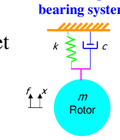
$$f = m\ddot{x}$$

Combing above two equations we get

$$m\ddot{x} - k_x x = k_i i$$



Active magnetic bearing system

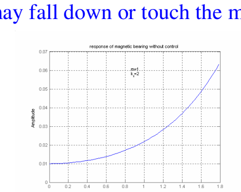


spring mass damper system

If controlling current i is zero then

$$m\ddot{x} - k_x x = 0$$

And the response grows exponentially thus the rotor may fall down or touch the magnet



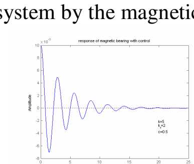
Response of magnetic bearing without control

If we supply controlling current i such that

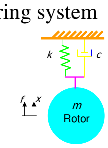
$$i(x) = \frac{k + k_x}{k_i} x + \frac{c}{k_i}\dot{x}$$

then it becomes $\quad m\ddot{x} + c\dot{x} + kx = 0$

And the response is imitated to a spring mass damper system by the magnetic bearing system



Response of magnetic bearing with control



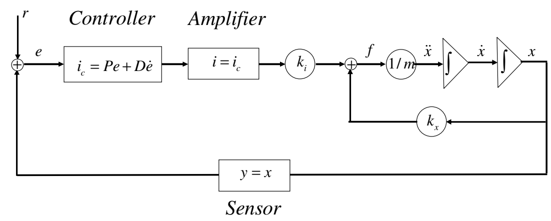
spring mass damper system

PD controller model

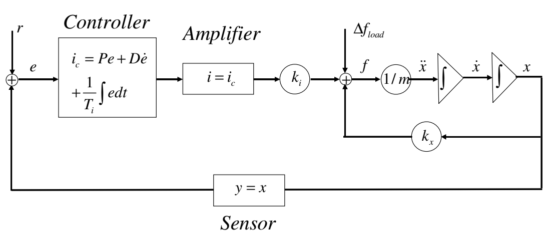
- The model is PD-controller with proportional and differential feed back

$$P = \frac{k + k_x}{k_i} \qquad D = \frac{c}{k_i}$$

- In design of controller we choose the stiffness and damping to ensure the system come to steady state in optimum time.
- The optimal stiffness suggested is
- The range of damping ratio for better systems suggested is 0.1 to 1



Block diagram of PD controller with current control



Block diagram of PID controller with current control

Control of rotors by using magnetic bearings

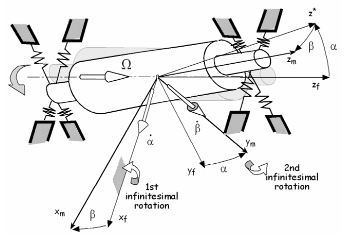
Topics to be covered

- Rigid rotor model
- Flexible rotor model

Differences between mechanical and magnetic bearing models

- Stiffness is very high thus the vibration of the rotor will be transmitted to foundation
- Stiffness is very low thus the rotor can rotate freely about the principal axes of inertia which results in a vibration isolation system.
- Damping is directly observed due to hydrodynamic effects
- As the rotor is free in the air there is no coulomb damping acting on the system. The control law will have damping term.

Rigid rotor model



Rotor mechanical bearing system

- Ω **Angular velocity of shaft**
- α **Infinitesimal rotation about x axis**
- β **Infinitesimal rotation about y axis**

$$\dot{\alpha} = \frac{d\alpha}{dt} \qquad \dot{\beta} = \frac{d\beta}{dt}$$

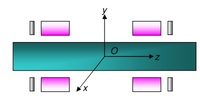
Rigid rotor model

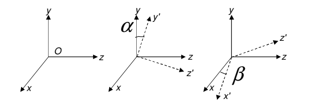
If the variable vector is chosen as

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \beta & x & -\alpha & y \end{bmatrix}^{\mathrm{T}}$$

Angular velocity vector can be expressed as

$$\omega = \begin{bmatrix} \omega_{x0} \\ \omega_{y0} \\ \omega_{z0} \end{bmatrix} = \begin{bmatrix} \dot{\alpha}\cos\Omega t + \dot{\beta}\sin\Omega t \\ -\dot{\alpha}\sin\Omega t + \dot{\beta}\cos\Omega t \\ \Omega \end{bmatrix}$$





Motion about x- axis Motion about y- axis

Rigid rotor model

Kinetic energy is expressed as

$$T = \frac{1}{2} m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) + \frac{1}{2}\left(J_{x0}\omega_{x0}^2 + J_{y0}\omega_{y0}^2 + J_{z0}\omega_{z0}^2\right)$$

Equations of equilibrium can be obtained as by using Lagrange's principle

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) + \frac{\partial T}{\partial x_i} = F_i$$

F_i is the generalized force corresponding to i^{th} variable

Rigid rotor model

Equations **(1)** can be expressed in matrix form by rearranging

$$M\ddot{\mathbf{x}} + (G + C)\dot{\mathbf{x}} = \mathbf{F}$$

M is the inertia matrix ($M = M^T$)

G is the gyroscopic matrix($G = -G^T$)

C is the damping matrix ($C = C^T$)

$\mathbf{F}$ can be expressed as

$$\mathbf{F} = -(K + N)\mathbf{x}$$

K is conservative force matrix ($K = K^T$)

N is non-conservative force matrix($N = -N^T$)

Rigid rotor model

- **Conservative forces include**
  - **forces due to stiffness**
- **Non-conservative or circulatory forces include**
  - **Internal or structural damping**
  - **Steam or gas whirl in turbines**
  - **Seal effects**
  - **Process forces such as in grinding**
  - **Unbalance, etc**
- **Damping include**
  - **Coulomb damping due to hydrodynamic effects**

Rigid rotor model

- From Eq. (2) and (3) we get

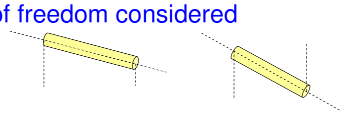
$$M\ddot{\mathbf{x}} + (G + C)\dot{\mathbf{x}} + (K + N)\mathbf{x} = 0$$

- If the non-conservative and gyroscopic forces neglected, we have

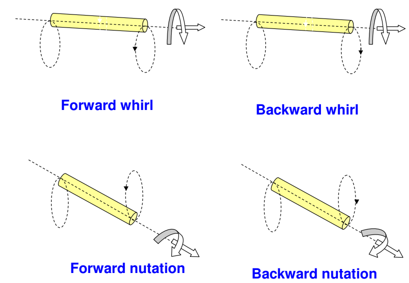
$$M\ddot{\mathbf{x}} + C\dot{\mathbf{x}} + K\mathbf{x} = 0$$

Natural modes

- The solution of the equations (5) gives four modes, for there are four degrees of freedom considered



Natural modes



Magnetic bearing model

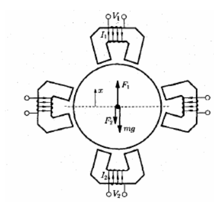
- In a magnetic bearing if we neglect the conservative, non-conservative, and damping effects, we will have

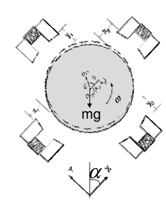
$$M\ddot{\mathbf{x}} + G\dot{\mathbf{x}} = \mathbf{F}$$

- For small rotations gyroscopic effects can be neglected and the equations in x and y directions can be decoupled

$$M\ddot{\mathbf{x}} = \mathbf{F}$$

Weight considerations

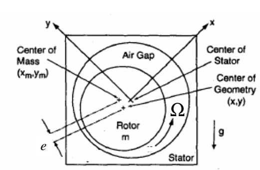




$$f_g = mg = k_i i_0$$

$$f_g = \frac{mg}{\cos\alpha} = k_i i_0$$

Imbalance considerations



$$f_\omega = \tilde{m}e\Omega^2\cos(\Omega t + \theta)$$

$\tilde{m}$ is the imbalance mass

e is the eccentricity of imbalance mass

θ is the angular position of imbalance mass

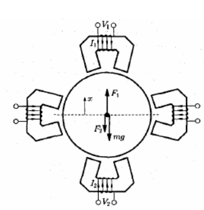
Magnetic bearing model

It can be written as

$$m\ddot{x} = f_k + f_c - f_g + f_\omega$$

where

$$f_k = k_x x$$
$$f_c = k_i i$$
$$f_g = mg = k_i i_0$$
$$f_\omega = \tilde{m}e\Omega^2\cos(\Omega t + \theta)$$



Magnetic bearing model

- It will be

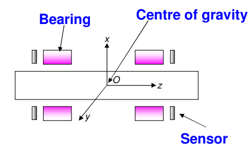
$$m\ddot{x} = k_x x + k_i (i - i_0) + \tilde{m} e \Omega^2 \cos(\Omega t + \theta)$$

- i at any instant will be

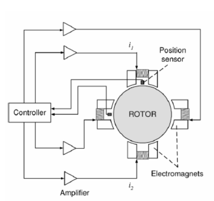
$$i = i_0 + \frac{m\ddot{x} - k_x x - \tilde{m} e \Omega^2 \cos(\Omega t + \theta)}{k_i}$$

Rigid rotor with magnetic bearing

- Three steps involved:
  - Formulation with respect to centre of gravity
  - Transformation with respect to the bearing coordinates
  - Transformation with respect to the sensor coordinates



Why with respect to sensor coordinates

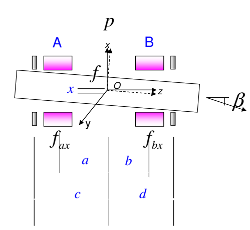


Structure of a two-DOF AMB system.

- Sensors cannot be arranged directly in the magnetic actuator.
- This requires certain gap between the magnet and the sensor.
- The displacements with respect to sensor coordinates will be transformed to bearing coordinates

With respect to centre of gravity

- In slow role x and y directions can be decoupled



$$m\ddot{x} = f$$

$$I_y \ddot{\beta} = p$$

In matrix form as

$$M\ddot{\mathbf{x}} = \mathbf{f}$$

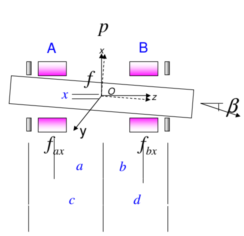
where

$$M = \begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x \\ \beta \end{bmatrix}$$

$$\mathbf{f} = \begin{bmatrix} f \\ p \end{bmatrix}$$

With respect to bearing coordinates

- **Forces are transformed as**



$$f = f_{ax} + f_{bx}$$

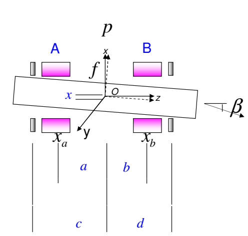
$$p = a f_{ax} + b f_{bx}$$

$$\mathbf{f} = T_f \mathbf{f}_B$$

$$T_f = \begin{bmatrix} 1 & 1 \\ a & b \end{bmatrix}, \mathbf{f}_B = \begin{bmatrix} f_{ax} \\ f_{bx} \end{bmatrix}$$

With respect to bearing coordinates

- Displacement vector can be transformed as



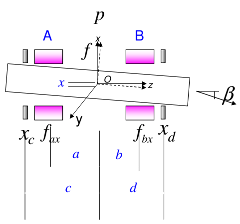
$$\mathbf{x} = T_B \mathbf{x}_B$$

$$\mathbf{x} = \begin{bmatrix} x \\ \beta \end{bmatrix}$$

$$T_B = \frac{1}{b-a} \begin{bmatrix} b & -a \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{x}_B = \begin{bmatrix} x_a \\ x_b \end{bmatrix}$$

With respect to sensor coordinates



$$\mathbf{x}_S = \begin{bmatrix} x_c \\ x_d \end{bmatrix}$$

$$T_S = \begin{bmatrix} 1 & c \\ 1 & d \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x \\ \beta \end{bmatrix}$$

$$\mathbf{x}_S = T_S \mathbf{x} = T_S T_B \mathbf{x}_B$$

$$\mathbf{x}_s = \begin{bmatrix} \mathbf{x}_S \\ \dot{\mathbf{x}}_S \end{bmatrix} = T_s \mathbf{x}_b$$

$$T_s = \begin{bmatrix} T_S T_B & 0 \\ 0 & T_S T_B \end{bmatrix}$$

State feed back

State space form with respect to sensor coordinates

$$\dot{\mathbf{x}}_s = A_s \mathbf{x}_s + B_s \mathbf{u} \qquad A_s = T_s A_b T_s^{-1} \quad B_s = B_b$$

- The control vector is found by using control law

$$\mathbf{u} = -F\mathbf{x}_s$$

- We do not know the velocity components directly from sensors. So a state observer is required to find the velocities

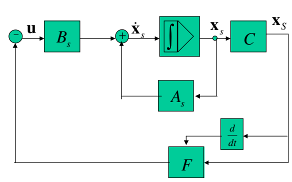
$$\mathbf{x}_s = C\mathbf{x}_S$$

$\mathbf{x}_s$ is the full state vector
$\mathbf{x}_S$ is the vector from the sensor

State feed back

- The whole closed loop system can be shown as block diagram

$$\dot{\mathbf{x}}_s = A_s \mathbf{x}_s + B_s \mathbf{u}$$
$$\mathbf{x}_S = C\mathbf{x}_s$$
$$\mathbf{u} = -F\mathbf{x}_s$$
$$\dot{\mathbf{x}}_s = (A_s - B_s F)\mathbf{x}_s$$



$|A_s - B_s F|$ decides the closed loop dynamics of the system

Model at high speeds

- At high speeds the gyroscopic effects cannot be neglected, thus the model becomes

$$M\ddot{\mathbf{x}} + G\dot{\mathbf{x}} = \mathbf{F}$$

- The displacements in x and y directions no longer decoupled, so four forces and four displacements should be taken into consideration simultaneously.
- The same procedure is to be followed as for the slow rotation

Model at high speeds

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_y & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & I_x \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

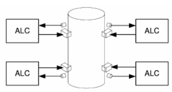
$$\mathbf{f} = \begin{bmatrix} f_x \\ p_y \\ f_y \\ -p_x \end{bmatrix} \qquad \mathbf{x}_B = \begin{bmatrix} x_a \\ x_b \\ y_a \\ y_b \end{bmatrix}$$

Conclusions on rigid rotor model

- There is an optimal design for each speed
- The optimal design at higher speed may not be stable at lower speeds, for the gyroscopic effects are reduced.
- The optimal design at zero speed may not be the optimal at higher speeds
- The gyroscopic effects will not destabilize the system which is stable at lower speeds.
- Further more the design at lower speeds is decoupled and easier to design. Decentralized designs for lower speeds can be implemented

Conclusions on rigid rotor model

– Thus for stability considerations and other advantages systems are designed for lower speeds and with decentralization



Decentralized control mode scheme

$$u_{xa} = -F_{xa} x_a$$

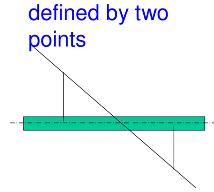
$$u_{xb} = -F_{xb} x_b$$

$$u_{ya} = -F_{ya} x_a$$

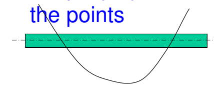
$$u_{yb} = -F_{yb} x_b$$

Flexible rotor model

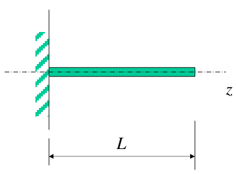
- Rigid rotor can be defined by two points



- Flexible rotor has infinite degrees of freedom. One cannot define uniquely by some of the points



Flexible rotor model



- Equation motion of an Euler-Bernoulli beam is given by

$$EI\frac{\partial^4 y}{\partial z^4} + m\frac{\partial^2 y}{\partial t^2} = 0$$

- The variable separable form is

$$y(z,t) = Y(z)q(t)$$

Flexible rotor model

- By substituting we get

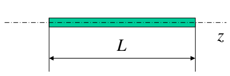
$$\frac{EI}{m}\frac{\left(\frac{d^4Y(z)}{dz^4}\right)}{Y(z)} = -\frac{\left(\frac{d^2q(t)}{dt^2}\right)}{q(t)} = \omega^2$$

- By rewriting we get

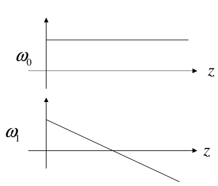
$$\frac{d^4Y(z)}{dz^4} - \beta^4 Y(z) = 0, \qquad \beta = \frac{\omega^2}{EI/m}$$

$$\frac{d^2q(t)}{dt^2} + \omega^2 q(t) = 0$$

Flexible rotor model

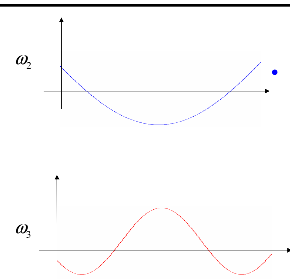


- By applying initial conditions and solving we get the natural frequencies ω



- By substituting the Eigen values in (29) we get the Eigen functions or model functions $Y(z)$

Rigid rotor modes

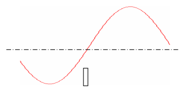


- The mode shapes or modal functions depend on the end conditions

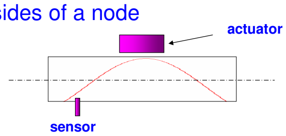
Flexible rotor modes

Actuator sensor location

- Sensor should not be set at nodes

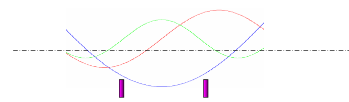


- Sensor and actuator should not lie on opposite sides of a node



Actuator sensor location

- We can conclude that the sensor can be set at a place where we can get information from each mode under consideration



Modal reduction

- While designing a flexible rotor system, we can not consider all the modes of the system for they are infinite
- Thus we consider first n number of modes corresponding to first n natural frequencies and neglect the remaining modes
- If we study the effect of the reduced modes we can find the number of modes which we can consider without destabilizing the system

Modal reduction (mathematical representation)

- Mathematical model of the
  - full system

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$\mathbf{y} = C\mathbf{x}$$

  - Divided system

$$\begin{bmatrix} \dot{\mathbf{x}}_M \\ \dot{\mathbf{x}}_R \end{bmatrix} = \begin{bmatrix} A_M & A_{MR} \\ A_{RM} & A_R \end{bmatrix} \begin{bmatrix} \mathbf{x}_M \\ \mathbf{x}_R \end{bmatrix} + \begin{bmatrix} B_M \\ B_R \end{bmatrix} \mathbf{u}$$
$$\mathbf{y} = \begin{bmatrix} C_M & C_R \end{bmatrix} \begin{bmatrix} \mathbf{x}_M \\ \mathbf{x}_R \end{bmatrix}$$

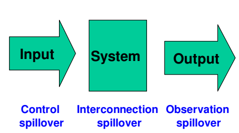
  - Reduced system

$$\dot{\mathbf{x}}_M = A_M\mathbf{x}_M + B_M\mathbf{u}$$
$$\mathbf{y} = C_M\mathbf{x}$$

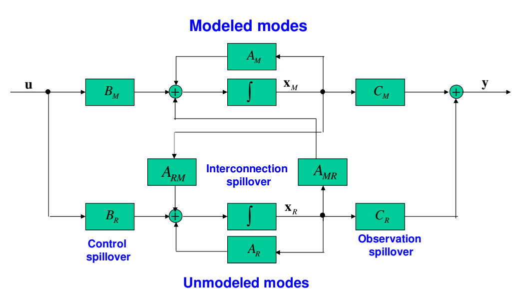
Modal reduction

- The reduced modes give three kinds of effects on the system called spillovers
  - Control spillover (By the input)
  - Interconnection spillover (By the parameters of the system)
  - Observation spillover (on the estimated output)

$$\begin{bmatrix} \dot{\mathbf{x}}_M \\ \dot{\mathbf{x}}_R \end{bmatrix} = \begin{bmatrix} A_M & A_{MR} \\ A_{RM} & A_R \end{bmatrix} \begin{bmatrix} \mathbf{x}_M \\ \mathbf{x}_R \end{bmatrix} + \begin{bmatrix} B_M \\ B_R \end{bmatrix} \mathbf{u}$$
$$\mathbf{y} = \begin{bmatrix} C_M & C_R \end{bmatrix} \begin{bmatrix} \mathbf{x}_M \\ \mathbf{x}_R \end{bmatrix}$$



Modal reduction



Block diagram of effect of model reduction

Conclusion on flexible rotor control

- Modal reduction is studied to consider the number modes to be taken into consideration for having stable control
- Mechanical design is studied for finding the sensor actuator locations

Conclusions

- Magnetic bearings advantages and applications have been discussed
- Electromagnetism and Control system technologies have been introduced
- Design of thrust and radial magnetic bearings have been studied
- Control of a rotor by rigid rotor and flexible rotor models have been studied

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Thank you