Distribution of Electromagnetic Force in Permanent Magnets

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Abstract - **Two dual formulations are proposed for the calculation of the electromagnetic forces in permanent magnets. The formulations are based on the virtual work principle with the use of nodal elements. Both methods allow the calculation of global force as well as local force densities. These densities depend on the expression of the magnetic energy or co-energy of the magnet. The energies of a permanent magnet are discussed on physical basis.**

Index terms : **local force densities, permanent magnet, virtual work, magnetic energy.**

INTRODUCTION

Two dual formulations are used to calculate electromagnetic forces by the Finite Element Method (FEM). One is based on the vector potential A and the other one is based on the scalar potential Ψ. By the use of the virtual work principle, the electromagnetic force on a magnet is obtained by deriving either the energy ($W=W(\boldsymbol{B})$) or co-energy ($W'=W'(\boldsymbol{H})$) with respect to a virtual displacement, according to the formulation used to solve the FEM problem [1][2][3]. The global force on a magnet is well known and is calculated currently [5][6]. Nevertheless, when mechanical calculation is done, a local distribution of forces is necessary. Though the local force densities obtained directly by the above expressions are not significant. They do not represent the actual interaction force distribution between the magnets. The results depend on the expression of the magnetic energy or co-energy. It is shown that the use of simplified expressions can lead to erroneous results. It is shown also that, with these expressions, the total energy on a magnet is composed of two terms : the intrinsic and the interaction energy. An analysis of the calculated "local force densities" from the above energies is given. The adequacy of the methods is shown as the same results are obtained by the two different approaches.

DESCRIPTION OF THE METHOD

For a permanent magnet, one can define the magnetic induction, in terms of the remanent induction $\boldsymbol{B}_r$ as $\boldsymbol{B}=\mu_0\boldsymbol{H}+\boldsymbol{B}_r$. Considering the relation $\boldsymbol{B}_r=\mu_0\boldsymbol{M}$, where $\boldsymbol{M}$ is the magnetization, the magnetic induction is given by $B=\mu_0(H+M)$ [4][6][7]. The electromagnetic force on the magnet can be calculated either with $\boldsymbol{B}_r$ or $\boldsymbol{M}$, according to the formulation used. The formulations to be used are described below.

A - Vector Potential Formulation

The vector potential as used, is defined from the magnetic induction as $\boldsymbol{B}=\boldsymbol{curl}\,\boldsymbol{A}$. By the virtual work principle, the electromagnetic force is calculated in terms of the magnetic energy, which on a permanent magnet is [4][6][7] :

$$W = \frac{1}{2\mu_0}\int_\Omega (B-B_r)(B-B_r)d\Omega \tag{1}$$

In this basis, the electromagnetic force on the magnet is given by $F=-\partial W/\partial s$ at constant flux, which in the direction i is :

$$\begin{aligned} F_i &= -\frac{\partial}{\partial s_i}\left[\frac{1}{2\mu_0}\int_\Omega (B-B_r)(B-B_r)d\Omega\right] \\ &= -\frac{1}{2\mu_0}\sum_e \frac{\partial}{\partial s_i}\int_{\Omega_e}(B-B_r)(B-B_r)|G|d\Omega_e \end{aligned} \tag{2}$$

where e stands for the elements of the magnet, μ_0 is the permeability of the air, s_i is the virtual displacement in the direction i, $|G|$ is the determinant of the jacobian matrix and $d\Omega_e = du\,dv\,dw$.

The derivation of (2) for a node k of the magnet results in :

$$\begin{aligned} F_{ik} = -\frac{1}{2\mu_0}\sum_{ek}\int_{\Omega_{ek}}\Bigg[&\frac{\partial B}{\partial s_i}(B-B_r)+(B-B_r)\frac{\partial B}{\partial s_i}+ \\ &(B-B_r)(B-B_r)|G|^{-1}\frac{\partial |G|}{\partial s_i}\Bigg]d\Omega_{ek} \end{aligned} \tag{3}$$

where $|G|^{-1}$ is the inverse of the jacobian matrix and ek concerns the elements which have in common the node k. The virtual displacement is applied with constant vector potential $\boldsymbol{A}$ which is equivalent to constant flux.

The derivation of $|G|$ with respect to the virtual displacement is given by [1]. To calculate the derivation of B with respect to the displacement s_i, the finite element approximation of nodal unknowns is defined as :

$$A(u,v,w)=\sum_k \alpha_k(u,v,w)A_k \tag{4}$$

where α_k are the nodal shape functions. The magnetic induction B is given by :

$$B = curl\sum_k \alpha_k A_k = \sum_k grad\,\alpha_k \times A_k \tag{5}$$

so, $$B=\sum_k G^{-1}\partial_{uj}\alpha_k \times A_k \tag{6}$$

which gives $$\frac{\partial B}{\partial s_i}=\sum_k \frac{\partial G^{-1}}{\partial s_i}\partial_{uj}\alpha_k\times A_k \tag{7}$$

for the derivation of B. With the identity

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$\frac{\partial G^{-1}}{\partial s_i}=-G^{-1}\frac{\partial G}{\partial s_i}G^{-1}$, the derivation of the magnetic induction is [3] :

$$\frac{\partial B}{\partial s_i}=-\sum_k G^{-1}\frac{\partial G}{\partial s_i}\,grad\,\alpha_k \times A_k \qquad (8)$$

With (8), all the variables of (3) are defined and forces can be obtained by the vector potential formulation.

Therefore, when the FEM is solved with the scalar potential, to minimize the errors due to the derivation of the field variable, a formulation based on the scalar potential can be used.

A second formulation relying on the scalar potential ψ allows to obtain another expression of force. It is described below.

B - Scalar Potential Formulation

In the scalar potential formulation, the magnetic field is defined as $\boldsymbol{H}=-\boldsymbol{grad}\ \Psi$. One calculates the electromagnetic force, by the virtual work principle, with the use of the magnetic co-energy, which on a permanent magnet is [4][6][7] :

$$W' = \frac{\mu_0}{2}\int_\Omega (H+M)(H+M)d\Omega \qquad (9)$$

The electromagnetic force on the magnet ($F=\partial W'/\partial s$) at constant current, which on a direction i is given by :

$$F_i = \frac{\partial}{\partial s_i}\left[\frac{\mu_0}{2}\int_\Omega (H+M)(H+M)d\Omega\right]$$
$$= \frac{\mu_0}{2}\sum_e \frac{\partial}{\partial s_i}\int_{\Omega_e}(H+M)(H+M)|G|d\Omega_e \qquad (10)$$

therefore, the electromagnetic force is :

$$F_i = \frac{\mu_0}{2}\sum_e\int_\Omega\left[\frac{\partial H}{\partial s_i}(H+M)+(H+M)\frac{\partial H}{\partial s_i}+\right.$$
$$\left.(H+M)(H+M)|G|^{-1}\frac{\partial|G|}{\partial s_i}\right]d\Omega \qquad (11)$$

Interpolating the scalar potential Ψ with nodal shape functions, such that :

$$\psi(u,v,w)=\sum_k \alpha_k(u,v,w)\,\psi_k \qquad (12)$$

the magnetic field H is given by :

$$H=-grad\ \psi=-G^{-1}\left[\frac{\partial\psi}{\partial u}\right] \qquad (13)$$

The derivation of the magnetic field with respect to the virtual displacement s_i is given by [1] :

$$\frac{\partial H}{\partial s_i}=-\frac{\partial G^{-1}}{\partial s_i}\left[\frac{\partial\psi}{\partial u}\right] \qquad (14)$$

so, $$\frac{\partial H}{\partial s_i}=G^{-1}\frac{\partial G}{\partial s_i}G^{-1}\left[\frac{\partial\psi}{\partial u}\right]=-G^{-1}\frac{\partial G}{\partial s_i}H \qquad (15)$$

Finally, substituting (15) in (11), and calculating the electromagnetic force in the direction i for a node k of the magnet, the following expression is obtained :

$$F_{ik}=\frac{\mu}{2}\sum_{ek}\int_{\Omega_{ek}}\left[-G^{-1}\frac{\partial G}{\partial s_i}H(H+M)+(H+M)\left(-G^{-1}\frac{\partial G}{\partial s_i}H\right)\right.$$
$$\left.+(H+M)(H+M)|G|^{-1}\frac{\partial|G|}{\partial s_i}\right]d\Omega_{ek} \qquad (16)$$

To assure the constant current, the derivation with respect to the virtual displacement is applied with constant scalar potential Ψ.

The local force is computed on the nodes of the permanent magnet. Only the elements surrounding a node have their energies modified by the virtual displacement of the node. Thus, the force on one node is given by the summation of all the force contributions from all the elements surrounding that node. The global force is calculated by the summation of the nodal forces on all the nodes of the magnet.

NUMERICAL RESULTS

The formulations have been tested on 2D and 3D magnetic problems. Expressions (3) and (16) are used to calculate the global and local force distribution in permanent magnets. The force calculation is investigated on a two cubic magnet model. Fig. 1 presents the geometry of the problem.

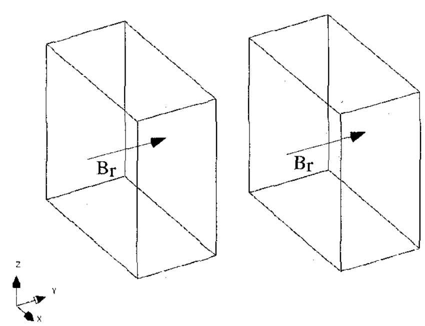

Fig 1. Geometry of the problem

The calculations of the global interaction force between the magnets obtained by the two methods are compared with analytical results [5]. The remanent induction of the magnets is 1T. The accuracy of the methods is presented in table I. Results correspond to a mesh of 256 elements per magnet.

Table I
Global interaction force between the magnets

	Vector Potential	Scalar Potential	Analytical Result
Fx	7,094	7,089	7,089
Fy	8,862	8,856	8,857
Fz	0,004	0,001	0,000

Thus, global force calculation are excellent for both formulations. The problem turns much more complex for the local force distribution. Discussion follows.

To set up clearly the problem, the total magnetic energy of a magnet is decomposed in two terms as in fig. 2. Consequently, different corresponding force densities appear.

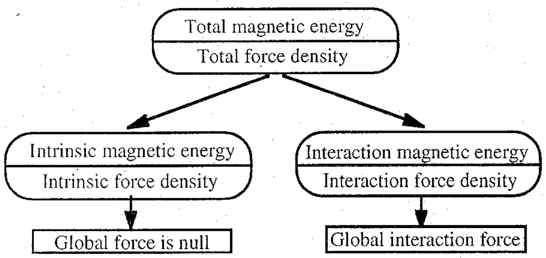

Fig. 2. Energies and force densities of a permanent magnet

Thus, three different quantities are to be considered, either for the energies implied as for the force : "total magnetic energy and total force density", "intrinsic magnetic energy and intrinsic force density", "interaction magnetic energy and interaction force density". The exact meaning of these quantities is explained below.

According to the expressions (3) and (9), the distribution of forces due to a single magnet in the air, calculated by the vector potential and scalar potential formulations are presented in fig. 3. These forces are called by convenience "intrinsic magnetic force densities", as they are supposed to represent the magnetic forces on the magnet due to its intrinsic magnetization. The summation of these forces for a single magnet is, of course, null.

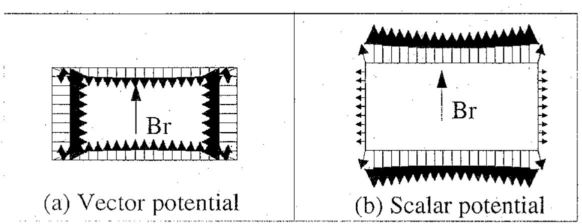

Fig. 3. Local force densities of a magnet alone (intrinsic forces)

These distributions completely differ. This can be explained by the fact that the expressions of the energy and co-energy from which the above electromagnetic forces derive are simplified models and do not account for the same quantities. Fig. 4 presents the energy and the co-energy in a magnet from expressions (1) and (9).

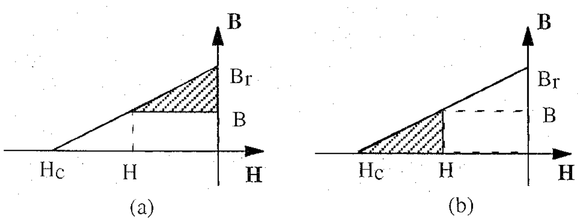

Fig. 4. (a) Energy and (b) Co-energy in a permanent magnet from expressions (1) and (9)

These expressions assume a linear-rigid model for the magnet which accounts for the variation of energy or co-energie from the remanent or coercitive point to the working point B. They obviously do not represent correctly the actual energy implied in the non linear magnetizing process. That energy will be called "intrinsic magnetic energy". The value of this intrinsic magnetic energy is unknown. At least, no clear value is given. Indeed, each permanent magnet material has his own behaviour when submitted to an exterior magnetic field. Fig. 5 presents the energy supplied during the magnetizing process.

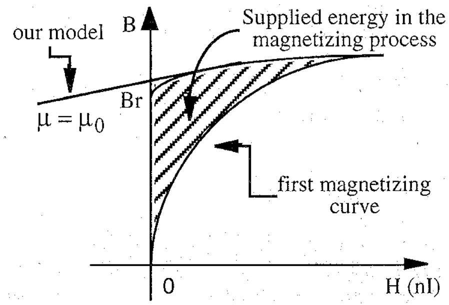

Fig. 5. Supplied energy during the magnetizing process

This supplied energy is the total energy supplied magnetically by the magnetizing circuit used to magnetize the magnet. This total energy is the energy that must be supplied to saturate the magnet. In any way that energy represents the stored magnetic energy. Indeed, only a part of this supplied energy is stored in the magnet under magnetic form. The necessary energy to magnetize the magnet is, in the most of the cases, over two to twenty times the energy of the single magnet [8]. The exceeding energy is lost in heat form, magneto mechanical form, ... [7][8][9][10].

We have seen that when a magnet is magnetized, during the magnetizing process, most of the supplied energy is lost in heat or magneto mechanical form while only a small part of this energy is stored in the magnet under magnetic form. Then, one should be able to affirm that in the remanent point B_r (in fig. 4), the magnet has a certain magnetic energy stored, which we shall call "intrinsic initial energy" which is the intrinsic magnetic energy at the remanent point. This energy is not taken into account by the expression (1), for which the magnetic energy is null if the magnet works on the remanent point ($B = B_r$).

Thus, one must take care that the classical expressions of forces in a permanent magnet obtained by the usual magnetic energy and co-energy expressions are so simplified that they do not represent the reality any more. Therefore, the intrinsic or magnetization energy, and so the real intrinsic force densities, cannot be obtained by such simple models. This explain that the distributions of force presented in fig. 3 are not the real intrinsic forces, obtained once and for all during the magnetizing process. The two different distributions are both wrong, which is the only point that can be ascertained. The intrinsic force densities will only be obtained when a correct expression of the intrinsic magnetic energy due to the magnetizing process is known.

Though, expressions (1) and (9) calculate a variation of energy or co-energy from the remanent point until the working point and account correctly for the variations of energy due to interactions of the exterior medium and the magnet, as long as the linear rigid model for the magnet is correct. Indeed, in such a rigid model, the magnetization $\boldsymbol{M}$ is kept constant which means that no domain rotates. Thus, there should be no losses. Hence, the supplied energy is actually equal to the variation of the magnetic energy. Therefore, interaction force densities can be correctly obtained.

For a magnet alone in the air, its working point is the induction B in the fig. 6. When the magnet is submitted to an exterior magnetic field, the working point changes to a new induction B'. This variation of (co)energy is the interaction (co)energy necessary to calculate correctly the interaction force distribution between the magnet and the exterior media. This interaction (co)energy is presented in Fig. 6.

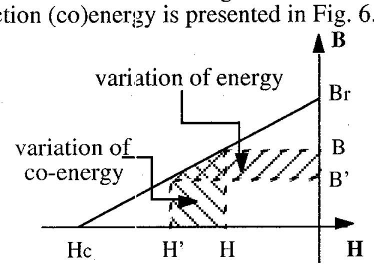

Fig. 6. Variation of (co)energy from a point B to B'

Therefore, if one calculates the distribution of the magnetic force densities in the new point B', with the energy or co-energy calculated by the expressions (1) and (9), the (co)energy that will be taken into account is the total variation from the remanent point B_r (or H_c) to B'. This total magnetic energy includes the interaction energy, that is the same no matter which formulation is used, and the variation of energy form B_r until B, which is the energy of the magnet alone and that depends on the formulation, and that is not correctly calculated.

Fig. 7 presents the total force densities between the two magnets of fig. 1. This total force distribution (calculated from the total magnetic energy) includes also two terms, one being the "intrinsic forces" (which is calculated for each magnet alone, and we know that they are wrong) and the other one being the interaction forces between the two magnets (which is correctly calculated).

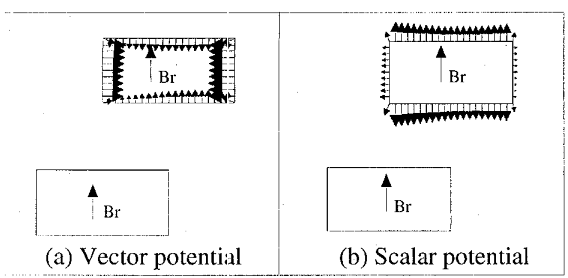

Fig. 7. Total force densities between two magnets

We are interested in the interaction force distribution between the magnets. Expressions (3) and (16) allow to calculate correctly the interaction forces between the magnet and the "rest of the world". As the intrinsic force densities are not correctly calculated, to obtain the interaction force densities, one must withdraw these intrinsic force densities from the total force densities. The resulting interaction force distribution between the two magnets is presented in fig. 8.

There is actually only one interaction force distribution between two objects. The coherence of the results and so of the methods is shown in fig. 8. The results do not depend on the formulation and the methods can be used to calculate the interaction force between a permanent magnet and any other exterior medium.

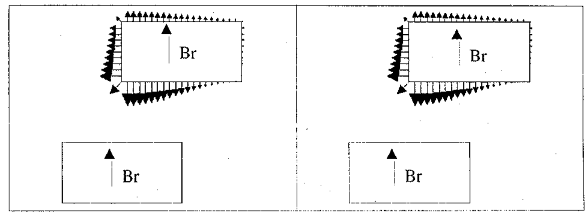

Fig. 8. Interaction force densities between two magnets obtained by both methods

CONCLUSION

Different physical considerations have led to decompose the total magnetic energy on a magnet in two terms : the intrinsic energy, which is the magnetic energy stored in a single magnet during the magnetization and the interaction energy corresponding to the modification of the total energy due to the surrounding media.

Then two formulations (vector potential and scalar potential) relying on the virtual work principle are used. They give the same interaction force distribution, while intrinsic forces sharply differ. This can be justified by the fact that the simple linear-rigid model used for the expressions of the energy and co-energy do not represent the real (co)energy implied on the non linear magnetizing process. These energies cannot be used to compute the intrinsic forces but are valid to determine the interaction forces (density and value) between the magnet and the exterior media.

As the methods are based on the virtual work principle, (physical principle), the interaction force distribution presented in fig. 8 seems to correctly represent the actual interaction force densities between the magnets.

Concerning the global interaction force, the methods agree well with the analytical result and present a good accuracy.

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