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Fault tolerant control for satellites with four reaction wheels

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Abstract

The authors propose a simple and effective fault tolerant control method for satellites with four reaction wheels. The proposed method is based on dynamic inversion and time-delay control theory. Faults of reaction wheels are modeled as additive and multiplicative unknown dynamics, which are estimated by using one-step previous state information and canceled out by the estimated values. Therefore, this method can accommodate faults rapidly without any explicit reconfiguration. Numerical simulations demonstrate the performance of the proposed method by comparing with a conventional proportional-derivative control.

Keywords: Satellite attitude control; Reaction wheels; Redundant wheels; Fault tolerant control; Time-delay control

1. Introduction

Fault tolerance means a system's ability to maintain its stability and performance in spite of unknown faults within the components of the system. One typical method for fault tolerance is to provide a system with redundant components or modules. In satellite systems, for example, a fault in one reaction wheel among four reaction wheels, which are commonly used for attitude control (Bang, Tahk, & Choi, 2003; Wisniewski & Kulczycki, 2005), can be accommodated such that the remaining wheels maintain the controllability for the three axes motions. For this purpose, effective methods to manage redundant wheels and to deal with unknown faults are necessary. Such control methods are called fault tolerant controls (Blanke, Izadi-Zamanabadi, Bøgh, & Lunau, 1997; Bodson & Groszkiewicz, 1997; Edwards & Tan, 2005; Izadi-Zamanabadi & Blanke, 1999). This topic also has been researched in various engineering fields (Romanenko, Santos, & Afonso, 2007; Wu, Thavamani, Zhang, & Blanke, 2006).

In this paper, the authors propose a fault tolerant control method for a satellite with four reaction wheels, based on dynamics inversion and time-delay control (TDC). TDC has been developed for robust control and known to be effective for dealing with unknown dynamics or uncertainties (Chang & Lee, 1996; Youcef-Toumi & Reddy, 1992). Here, faults are considered as unknown dynamics and estimated by using one-step previous state information. Since the motions of satellites are generally slow, such estimation approximates unknown dynamics well. The proposed method is a kind of passive accommodation because it does not need any explicit reconfiguration based on the information of faults (Edwards & Tan, 2005).

The structure of this paper is as follows: in the next section, a dynamic model using the modified Rodrigues parameters (MRPs) are introduced. Then Section 3 reviews a conventional attitude control method using proportional-derivative (PD) control. A fault tolerant control method based on TDC is proposed in Section 4 and then simulation examples are presented in Section 5 to demonstrate effectiveness of the proposed approach. Finally, Section 6 concludes this study.

2. Satellite dynamics modeling

2.1. Attitude description

In this paper, matrices are denoted by boldface upper case (e.g. $\boldsymbol{J}$), vectors by boldface lower case (e.g. $\mathbf{f}$), and

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scalars by lower case (e.g. τ). The notation (t) indicating dependence of a variable $\mathbf{x}$ on time t as in $\mathbf{x}(t)$ is omitted for simple notation, unless it is required in the context. $\boldsymbol{I}_{n\times n}$ denotes the n-by-n identity matrix.

Among several methods for describing the attitude of a satellite such as the Euler angles, the direction cosines, the quaternion, and the MRPs, the MRPs are used in this paper. In this description, singularities do not occur within $\pm 360^\circ$ and three parameters (p_1, p_2, p_3) are updated. The definition of the MRPs and the update equations are given as (Myung & Bang, 2003)

$$p_i = \frac{q_i}{1+q_0}, \quad i = 1,2,3, \tag{1}$$

$$\mathbf{p} = \mathbf{n}\tan(\phi/4) = [p_1 \;\; p_2 \;\; p_3]^{\mathrm{T}}, \tag{2}$$

$$\dot{\mathbf{p}} = \boldsymbol{F}(\mathbf{p})\boldsymbol{\omega}, \tag{3}$$

$$\boldsymbol{F}(\mathbf{p}) = \tfrac{1}{4}\{(1-\mathbf{p}^{\mathrm{T}}\mathbf{p})I_{3\times 3} + 2[\mathbf{p}\times] + 2\mathbf{p}\mathbf{p}^{\mathrm{T}}\} = \tfrac{1}{4}\boldsymbol{A}, \tag{4}$$

$$\begin{aligned}\boldsymbol{F}^{-1}(\mathbf{p}) &= \frac{16}{(1+\mathbf{p}^{\mathrm{T}}\mathbf{p})^2}\boldsymbol{F}^{\mathrm{T}}(\mathbf{p}) \\ &= \{(1-\mathbf{p}^{\mathrm{T}}\mathbf{p})I_{3\times 3} - 2[\mathbf{p}\times] + 2\mathbf{p}\mathbf{p}^{\mathrm{T}}\} \\ &= \frac{4}{(1+\mathbf{p}^{\mathrm{T}}\mathbf{p})^2}\boldsymbol{A}^{\mathrm{T}},\end{aligned} \tag{5}$$

$$[\mathbf{p}\times] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}, \tag{6}$$

$$\boldsymbol{A} = \begin{bmatrix} 1+p_1^2-p_2^2-p_3^2 & 2(p_1p_2-p_3) & 2(p_3p_1+p_2) \\ 2(p_1p_2+p_3) & 1+p_2^2-p_3^2-p_1^2 & 2(p_2p_3-p_1) \\ 2(p_3p_1-p_2) & 2(p_2p_3+p_1) & 1+p_3^2-p_1^2-p_2^2 \end{bmatrix}. \tag{7}$$

Here q_0, q_1, q_2, and q_3 are the quaternion parameters, $\mathbf{n}$ is the unit vector representing the Euler principal axis, ϕ is the Euler principal angle, and $\boldsymbol{\omega} \in R^{3\times 1}$ is the angular velocity vector.

2.2. Dynamic model

A dynamic model of a spacecraft whose main actuators are reaction wheels is given by (Bang et al., 2003; Myung & Bang, 2003; Won, 1999)

$$\boldsymbol{J}_s\dot{\boldsymbol{\omega}} = \begin{cases} -\boldsymbol{\omega}\times\mathbf{H} + \boldsymbol{L}\mathbf{u}_W & \text{without faults,} \\ -\boldsymbol{\omega}\times\mathbf{H} + \boldsymbol{L}\boldsymbol{E}(\mathbf{u}_W+\mathbf{f}) & \text{with faults,} \end{cases} \tag{8}$$

$$\dot{\boldsymbol{\Omega}} = -\boldsymbol{J}_W^{-1}\mathbf{u}_W - \boldsymbol{L}^{\mathrm{T}}\dot{\boldsymbol{\omega}}, \tag{9}$$

$$\boldsymbol{J}_s = (\boldsymbol{J} - \boldsymbol{L}\boldsymbol{J}_W\boldsymbol{L}^{\mathrm{T}}v), \quad \mathbf{H} = \boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{L}\boldsymbol{J}_W\boldsymbol{\Omega}. \tag{10}$$

This model includes provisions for modeling additive faults (e.g. bias fault) and multiplicative faults (e.g. reduced control torque) of the reaction wheels represented by a vector $\mathbf{f}$ and a diagonal matrix $\boldsymbol{E}$. Here, $\boldsymbol{E}$ is given by

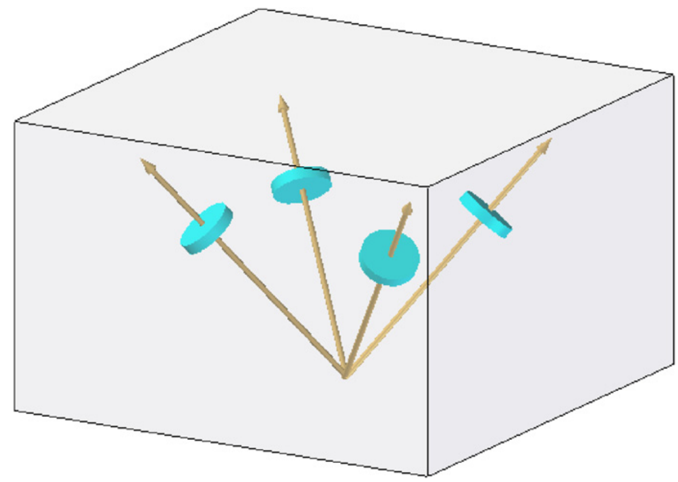

Fig. 1. Four reaction wheels for attitude control.

$$\boldsymbol{E} = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3, \alpha_4), \quad 0 \leqslant \alpha_i \leqslant 1, \;\; i = 1,\ldots,4. \tag{11}$$

Here $\alpha_i = 1$ means no multiplicative fault in the ith wheel and $\alpha_i = 0$ means the complete failure of the ith wheel from which no torque is generated. $\boldsymbol{J}$ is the inertia matrix of the satellite including wheels, $\boldsymbol{J}_W$ is the inertia matrix of reaction wheels, and $\boldsymbol{L} = [\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3,\mathbf{l}_4]$ is the input matrix representing the influence of each wheel on the angular acceleration of the satellite. A common configuration of four reaction wheels is a pyramid type shown in Fig. 1.

It is assumed that the angular velocity, $\boldsymbol{\Omega}$, of the reaction wheels are within the saturation limit and the input torque, $\mathbf{u}_W$, is unbounded. It is also assumed that the attitude and velocity information is available.

3. Conventional attitude control

Several control methods have been proposed for attitude control of satellites: linear control methods such as PD control (Wie, Weiss, & Arapostathis, 1989) and H_∞ control (Won, 1999), and nonlinear control methods such as sliding mode control (Crassidis & Markley, 1996) and predictive control (Myung & Bang, 2003).

PD control is one of the most popular methods, where the control input torque is formulated as

$$\mathbf{u}_W = \boldsymbol{L}^{+}[\boldsymbol{\omega}\times\mathbf{H} - \boldsymbol{D}\boldsymbol{\omega} - \boldsymbol{K}(\mathbf{p}-\mathbf{p}_d)], \tag{12}$$

where $\boldsymbol{L}^{+} = \boldsymbol{L}^{\mathrm{T}}(\boldsymbol{L}\boldsymbol{L}^{\mathrm{T}})^{-1}$ is the pseudo inverse of $\boldsymbol{L}$ and $\mathbf{p}_d$ is the desired attitude or angular position vector. The block diagram of this PD control is shown in Fig. 2.

Using (8) with no fault condition and (12) yield

$$\boldsymbol{J}_s\dot{\boldsymbol{\omega}} + \boldsymbol{D}\boldsymbol{\omega} + \boldsymbol{K}(\mathbf{p}-\mathbf{p}_d) = 0. \tag{13}$$

This is similar to a second order dynamic system and proper gain matrices $\boldsymbol{D}$ and $\boldsymbol{K}$ are chosen such that $\boldsymbol{\omega}$ and $\mathbf{p}$ approach to zero and $\mathbf{p}_d$, respectively, as time t tends to

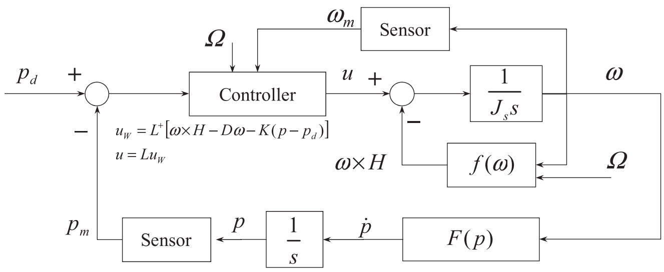

Fig. 2. Block diagram of PD control.

infinity. The gain matrices are chosen as

$$\boldsymbol{D} = 2\varsigma\omega_n \boldsymbol{J}_s, \quad \boldsymbol{K} = \omega_n^2 \boldsymbol{J}_s. \tag{14}$$

The parameters of the gain matrices which are the damping ratio ζ and the natural frequency ω_n are determined by considering dynamic properties of the target satellite and the actuators.

However, note that since the angular velocity $\boldsymbol{\omega}$ is not the direct derivative of the MRPs, the parameters of (14) does not directly control the transient attitude response of the satellite.

4. Fault tolerant control

4.1. Baseline control method

For the fault-free case, an inversion-based control is used as a baseline control method. The torque input $\mathbf{u}_W$ controls the angular velocity $\boldsymbol{\omega}$, which, in turn, controls the attitude $\mathbf{p}$ (Crassidis & Markley, 1996). If the angular velocity is set as by choosing $\tau_1 > 0$

$$\boldsymbol{\omega} = -\boldsymbol{F}^{-1}(\mathbf{p})\frac{1}{\tau_1}(\mathbf{p} - \mathbf{p}_d), \tag{15}$$

the angular position is given by

$$\tau_1 \dot{\mathbf{p}} + \mathbf{p} = \mathbf{p}_d. \tag{16}$$

Then the attitude $\mathbf{p}$ converges to the desired attitude $\mathbf{p}_d$. This means that (15) can be used for the angular velocity command to (8) with the fault-free condition as

$$\boldsymbol{\omega}_d = -\boldsymbol{F}^{-1}(\mathbf{p})\frac{1}{\tau_1}(\mathbf{p} - \mathbf{p}_d). \tag{17}$$

Now the control input that makes $\boldsymbol{\omega}$ follow $\boldsymbol{\omega}_d$ is designed as by choosing $\tau_2 > 0$

$$\mathbf{u}_W = \boldsymbol{L}^+\left(\boldsymbol{\omega} \times \mathbf{H} - \boldsymbol{J}_s \frac{1}{\tau_2}(\boldsymbol{\omega} - \boldsymbol{\omega}_d)\right). \tag{18}$$

Then the angular velocity dynamics is given by

$$\tau_2 \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} = \boldsymbol{\omega}_d \tag{19}$$

and the desired angular velocity can be attained asymptotically.

Fig. 3 shows the block diagram of (17) and (18), which consists of an inner velocity loop and an outer attitude loop.

In the baseline control above, the design parameters are the time constants τ_1 and τ_2. In order to have a faster inner velocity loop than the outer attitude loop, τ_2 must be less than τ_1 ($\tau_2 < \tau_1$). For example, $\tau_2 = 0.1\tau_1$ means that the velocity loop is ten times faster than the attitude loop. Since those time constants well describe transient responses of the satellite, the proper values of the time constants can be easily chosen.

4.2. Time-delay control

Now the authors propose a fault tolerant control method to accommodate faults of reaction wheels by modifying the baseline control (18) and employing TDC. TDC has been successfully used to deal with unknown dynamics such as uncertainties and disturbances without any explicit estimation for unknown dynamics (Chang & Lee, 1996; Youcef-Toumi & Reddy, 1992). Therefore, by considering the faults as unknown dynamics, TDC can be used as fault tolerant control. This is the main contribution of this study.

Eq. (8) with a fault condition is rearranged by adding and subtracting the nominal control input $\boldsymbol{J}_s^{-1}\boldsymbol{L}\mathbf{u}_W$ as

$$\begin{aligned}\dot{\boldsymbol{\omega}} &= -\boldsymbol{J}_s^{-1}\boldsymbol{\omega} \times \mathbf{H} + \boldsymbol{J}_s^{-1}\boldsymbol{L}\boldsymbol{E}(\mathbf{u}_W + \mathbf{f}) \\ &= \boldsymbol{J}_s^{-1}\boldsymbol{L}\mathbf{u}_W + \mathbf{g},\end{aligned} \tag{20}$$

$$\mathbf{g} \equiv -\boldsymbol{J}_s^{-1}\boldsymbol{L}\mathbf{u}_W - \boldsymbol{J}_s^{-1}\boldsymbol{\omega} \times \mathbf{H} + \boldsymbol{J}_s^{-1}\boldsymbol{L}\boldsymbol{E}(\mathbf{u}_W + \mathbf{f}). \tag{21}$$

In order to cancel out $\mathbf{g}$ and thereby to have a desirable response, the input is chosen as

$$\mathbf{u}_W = \boldsymbol{L}^+\boldsymbol{J}_s\left(-\frac{1}{\tau_2}(\boldsymbol{\omega} - \boldsymbol{\omega}_d) - \mathbf{g}\right). \tag{22}$$

However, since the term $\mathbf{g}$ contains unknown fault information, it is approximately estimated by using one-step previous information as follows:

$$\hat{\mathbf{g}}(t) \equiv \mathbf{g}(t-T) = \dot{\boldsymbol{\omega}}(t-T) - \boldsymbol{J}_s^{-1}\boldsymbol{L}\mathbf{u}_W(t-T). \tag{23}$$

Here T is selected as the control update period. Since the dynamics of a satellite is slow enough, $\hat{\mathbf{g}}(t)$ is a good approximation of $\mathbf{g}(t)$ with this time-delay. The control input (22) is now rewritten by using the estimated one $\hat{\mathbf{g}}(t)$ as

$$\begin{aligned}\mathbf{u}_W(t) &= L^+J_s\Bigg(-\frac{1}{\tau_2}(\boldsymbol{\omega}(t) - \boldsymbol{\omega}_d(t)) - \dot{\boldsymbol{\omega}}(t-T) \\ &\qquad + J_s^{-1}L\mathbf{u}_W(t-T)\Bigg) \\ &= \mathbf{u}_W(t-T) + \boldsymbol{L}^+\boldsymbol{J}_s\Bigg(-\frac{1}{\tau_2}(\boldsymbol{\omega}(t) - \boldsymbol{\omega}_d(t)) \\ &\qquad - \dot{\boldsymbol{\omega}}(t-T)\Bigg).\end{aligned} \tag{24}$$

The stability condition for this control law is

$$\|\boldsymbol{I} - \boldsymbol{J}_s^{-1}\boldsymbol{L}\boldsymbol{E}\boldsymbol{L}^+\boldsymbol{J}_s\| < 1 \quad \text{or} \quad \|\boldsymbol{I} - \boldsymbol{L}\boldsymbol{E}\boldsymbol{L}^+\| < 1, \tag{25}$$

as proved in Appendix A. This condition was obtained by following a similar procedure of the reference (Youcef-Toumi & Reddy, 1992).

If more than two wheels stop, the matrix $\boldsymbol{LEL}^+$ becomes singular and the condition does not hold. In such cases, other actuators such as thrusters have to be used.

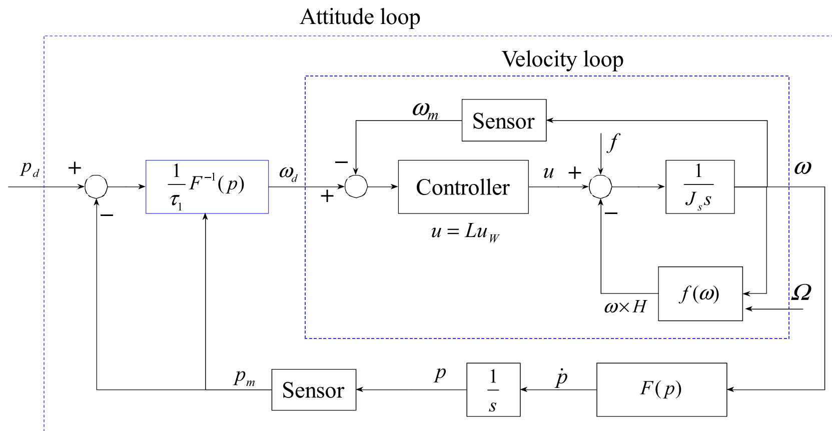

Fig. 3. Block diagram for attitude control using dynamic inversion.

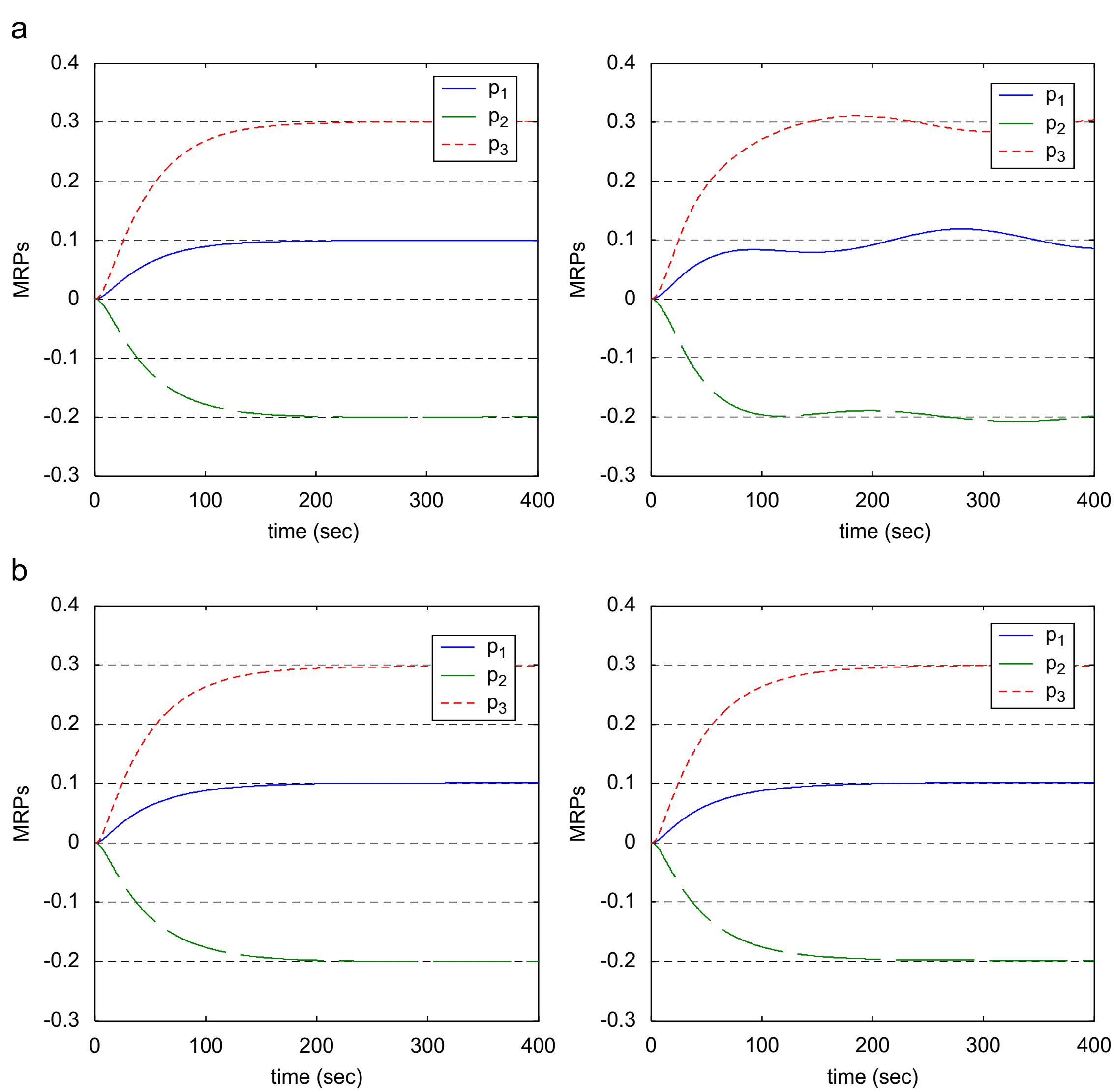

Fig. 4. MRP values: (a) PD control—without faults and with faults and (b) proposed method—without faults and with faults.

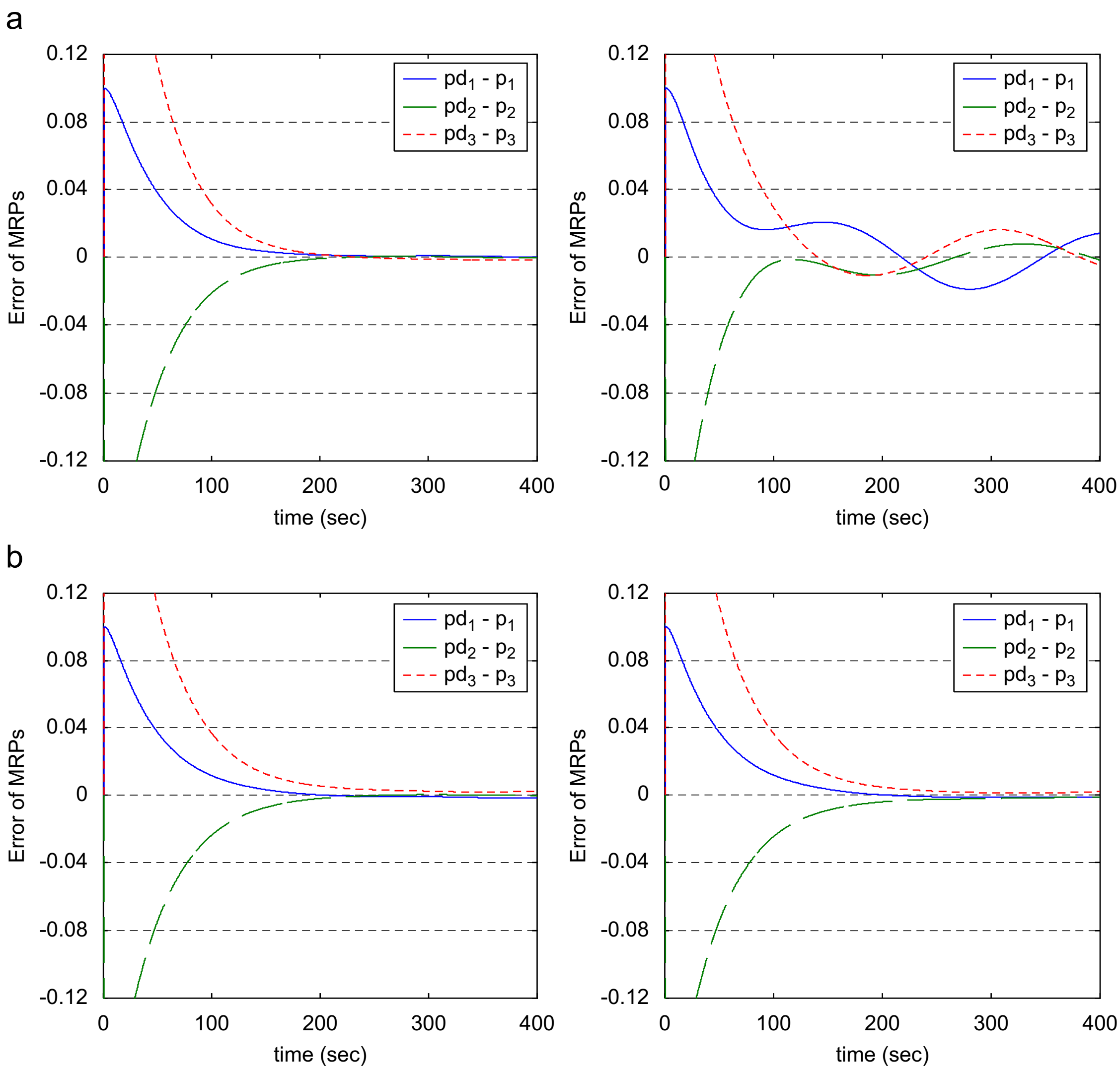

Fig. 5. Enlarged plots of errors: (a) PD control—without faults and with faults and (b) proposed method—without faults and with faults.

The last term in the parenthesis of (24) can be obtained simply as

$$\dot{\boldsymbol{\omega}}(t-T) = \frac{\boldsymbol{\omega}(t-T) - \boldsymbol{\omega}(t-2T)}{T}. \tag{26}$$

However, in order to cope with the problems such as noise amplification due to sensor noises, an observer scheme proposed by Chang and Lee (1996) can be used to estimate $\hat{\boldsymbol{\omega}}$ and $\dot{\hat{\boldsymbol{\omega}}}$ as

$$\dot{\hat{\boldsymbol{\omega}}} = \frac{1}{\tau_2}(\boldsymbol{\omega}_d - \hat{\boldsymbol{\omega}}) + \mathrm{G}(\boldsymbol{\omega}_m - \hat{\boldsymbol{\omega}}), \tag{27}$$

where $\boldsymbol{\omega}_m$ is the measured angular velocity and $\hat{\boldsymbol{\omega}}$ is the estimated value. $\boldsymbol{G}$ is a gain matrix to determine the convergence rate of the observer.

5. Numerical simulations

The performance of the proposed fault tolerant control is demonstrated by comparing with the PD control through numerical simulations. Both control strategies are compared for a fault-free condition first and then for a faulty condition.

For this, the following parameters of a satellite dynamics in Won (1999) are used:

$$\boldsymbol{J} = \mathrm{diag}(295, 130, 210)\ (\mathrm{kg\,m^2}),$$

$$\boldsymbol{J}_W = 0.01044\boldsymbol{I}_{4\times4}\ (\mathrm{kg\,m^2}),$$

$$\boldsymbol{L} = \frac{1}{2}\begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix}, \quad \mathbf{p}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{p}_d = \begin{bmatrix} 0.1 \\ -0.2 \\ 0.3 \end{bmatrix}. \tag{28}$$

For practical simulations, the following sensor noises modeled as zero-mean Gaussian random variables with

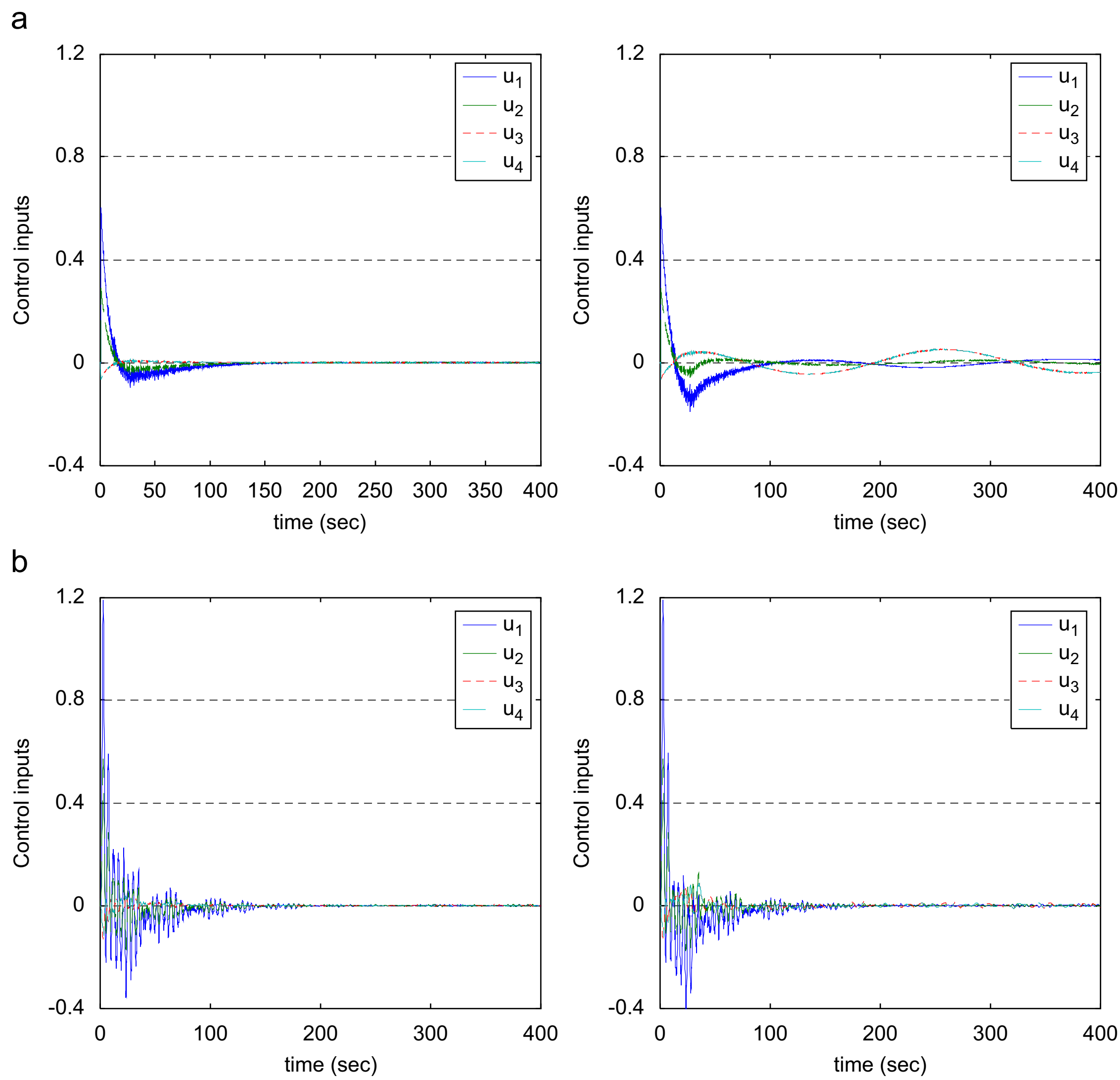

Fig. 6. Control inputs: (a) PD control—without faults and with faults and (b) proposed control—without faults and with faults.

variances of σ_{sf}^2, σ_b^2, and σ_p^2 are considered:

- gyro scale factor error (σ_{sf}^2): 0.1% (1σ)
- gyro bias (σ_b^2): 3°/h (1σ)
- attitude sensor noises (σ_p^2): 0.0001 (1σ)

in the measurement equations ($i = 1, 2, 3$)

$$
\begin{aligned}
&d(t+T) = d(t) + TN(0, \sigma_b), \\
&\omega_{m,i}(t) = (1 + N(0, \sigma_{sf}))(\omega_i(t) + d_t), \\
&p_{m,i}(t) = p_i(t) + N(0, \sigma_p).
\end{aligned} \tag{29}
$$

Here $N(0,\sigma)$ means a zero-mean Gaussian white noise with variance σ^2. Those noise levels were determined based on a reference (Sidi, 1997). The measurement noises of reaction wheels' angular velocity are not considered.

The control gain parameters are set as

$$\omega_n = 0.11, \quad \varsigma = 0.7, \tag{30}$$

$$\tau_1 = 50, \quad \tau_2 = 5, \tag{31}$$

$$\boldsymbol{G} = \boldsymbol{I}_{3\times 3}. \tag{32}$$

5.1. Simulations of a fault-free condition

For fault-free performance, 36.7%, 10%, and 5% error settling times to an initial error for a motion command $\mathbf{p}_d$ are compared. The dynamics update frequency for simulation is 10 Hz.

Figs. 4–7 show the time histories of the MRPs, attitude errors, control inputs, and reaction wheels' velocities obtained through the PD control and the proposed control. The settling times for the error ($\mathbf{p}_d - \mathbf{p}$) are indicated in Table 1. Two methods show a similar performance.

For the PD control, the design time constant for the second order system with the parameter values (30) is about 13.0 ($= 1/\varsigma\omega_n$) which is different from the values obtained from the simulation (see the 36.7% column in Table 1). This inconsistency results from the fact that the angular velocity is not the direct derivative of the MRPs (see Eq. (3)).

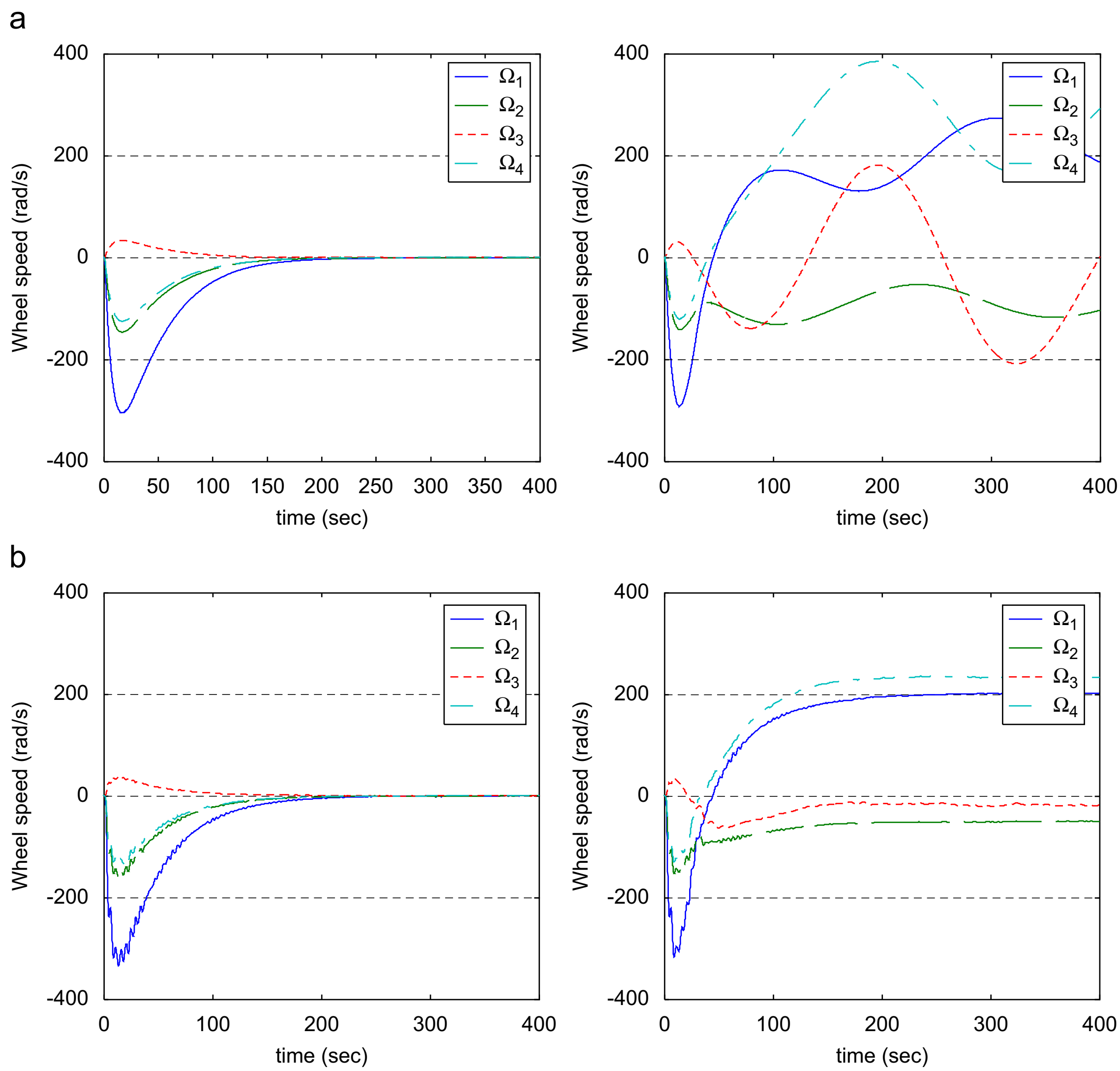

Fig. 7. Reaction wheels' angular speeds: (a) PD control—without faults and with faults and (b) proposed method—without faults and with faults.

Table 1
Attitude error settling times

MRP	Settling times (s)[a]		
	To 36.7% of initial error (= time constant)	To 10% of initial error	To 5% of initial error
PD control			
1	52	103	133
2	52	102	128
3	52	102	128
TDC			
1	51	107	136
2	51	108	137
3	51	110	142

[a]There may be small variations run-by-run due to random bias noises.

In contrast to the PD control, the result of the proposed method is consistent with the design time constant ($\tau_1 = 50$) of the attitude loop. The proposed method predicts the response very closely.

5.2. Simulations of a faulty condition

Now the following fault scenario is considered in the simulation:

$$\begin{cases} f_1 = 0.1, & 10 \leqslant t \leqslant 30\,\text{s}, \\ \alpha_3 = 0, & t \geqslant 30\,\text{s}, \\ \alpha_4 = 0.6, & t \geqslant 20\,\text{s}. \end{cases} \tag{33}$$

That is, a bias fault occurs at the first wheel for 20 s after 10 s after the simulation. The fourth wheel loses 40% of the control power after 20 s and the third wheel loses the complete control power after 30 s after the simulation.

Figs. 4–7 show the results of the same motion command used in the previous subsection. The settling times are indicated in Table 2.

The results of two methods are obviously different. While PD control resulted in a poor performance, the proposed method did not show any big degradation of the

Table 2
Attitude error settling times

MRP	Settling times (s)[a]		
	To 36.7% of initial errors	To 10% of initial errors	To 5% of initial errors
PD control			
1	46	–[b]	–[b]
2	43	73	–[b]
3	49	100	–[b]
TDC			
1	51	109	138
2	51	110	144
3	51	110	141

[a]There may be small variations run-by-run due to random bias noises.
[b]Not settled down within the fixed simulation time.

desired performance in spite of unknown faults and maintained the desired time constants. (This is predictable since PD control strategy does not contain any fault tolerant property. So for the PD control it is desirable to reconfigure or redesign the gains.) These results show that the proposed method is robust to failures in the reaction wheels.

6. Conclusions

A fault tolerant control method based on dynamic inversion and TDC was proposed for the attitude control of satellites using four reaction wheels. The performance of the proposed method was compared with those of the conventional PD control using numerical simulations. For a fault-free condition, two methods yielded a similar performance. However, for a faulty condition, PD control resulted in a poor performance. The proposed maintained the desired performance. These simulation results show that the proposed method is robust to faults in the reaction wheels.

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Appendix A. Proof of the stability condition (25)

This proof for the stability condition followed a similar procedure used in a reference (Youcef-Toumi & Reddy, 1992). Since additive faults $\mathbf{f}$ do not affect the stability, $\mathbf{f}$ is set to zero in (20).

$$\dot{\boldsymbol{\omega}}(t) = -\boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t)\times\mathbf{H}(t)] + \boldsymbol{J}_s^{-1}\boldsymbol{LE}\mathbf{u}_W(t). \tag{34}$$

By substituting (24) into (34), the following is obtained.

$$\begin{aligned}\dot{\boldsymbol{\omega}}(t) = & -\boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t)\times\mathbf{H}(t)] + \boldsymbol{J}_s^{-1}\boldsymbol{LE}\mathbf{u}_W(t-T) \\ & + \boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s\left(-\frac{1}{\tau_2}\boldsymbol{\omega}(t) + \frac{1}{\tau_2}\boldsymbol{\omega}_d(t) - \dot{\boldsymbol{\omega}}(t-T)\right).\end{aligned} \tag{35}$$

From (34)

$$\boldsymbol{J}_s^{-1}\boldsymbol{LE}\mathbf{u}_W(t-T) = \dot{\boldsymbol{\omega}}(t-T) + \boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t-T)\times\mathbf{H}(t-T)]. \tag{36}$$

Then, (36) is substituted into (35):

$$\begin{aligned}\dot{\boldsymbol{\omega}}(t) = & -\{\boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t)\times\mathbf{H}(t)] - \boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t-T)\times\mathbf{H}(t-T)]\} \\ & + (\boldsymbol{I} - \boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s)\dot{\boldsymbol{\omega}}(t-T) \\ & - \frac{1}{\tau_2}\boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s(\boldsymbol{\omega}(t) - \boldsymbol{\omega}_d(t)).\end{aligned} \tag{37}$$

T is selected small enough for the following conditions to be satisfied.

$$\|\boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t)\times\mathbf{H}(t)] - \boldsymbol{J}_s^{-1}[\boldsymbol{\omega}(t-T)\times\mathbf{H}(t-T)]\| \ll 1, \tag{38}$$

$$\|\boldsymbol{\omega}(t) - \boldsymbol{\omega}(t-T)\| \ll 1. \tag{39}$$

And also if $\|\boldsymbol{\omega}_d(t) - \boldsymbol{\omega}_d(t-T)\| \ll 1$, the following is obtained.

$$\dot{\boldsymbol{\omega}}(t) - \dot{\boldsymbol{\omega}}(t-T) = (\boldsymbol{I} - \boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s)(\dot{\boldsymbol{\omega}}(t-T) - \dot{\boldsymbol{\omega}}(t-2T)). \tag{40}$$

If there is γ satisfying $\|\boldsymbol{I} - \boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s\| \leqslant \gamma < 1$, $\dot{\boldsymbol{\omega}}(t) - \dot{\boldsymbol{\omega}}(t-T)$ converges to zero by the stability theorem for discrete systems. Then (37) is rewritten by (38) as

$$\begin{aligned}\underbrace{\dot{\boldsymbol{\omega}}(t) - \dot{\boldsymbol{\omega}}(t-T)}_{\approx 0} = & -\boldsymbol{J}_s^{-1}\boldsymbol{LEL}^{+}\boldsymbol{J}_s \\ & \times\left(\underbrace{\dot{\boldsymbol{\omega}}(t-T)}_{\approx\dot{\omega}(t)} + \frac{1}{\tau_2}(\boldsymbol{\omega}(t) - \boldsymbol{\omega}_d(t))\right).\end{aligned} \tag{41}$$

Since $\dot{\boldsymbol{\omega}}(t) - \dot{\boldsymbol{\omega}}(t-T)$ converges to zero, it can be assumed that $\dot{\boldsymbol{\omega}}(t) \approx \dot{\boldsymbol{\omega}}(t-T)$. And if $\boldsymbol{LEL}^{+}$ is nonsingular, the following is established:

$$\dot{\boldsymbol{\omega}}(t) = -\frac{1}{\tau_2}(\boldsymbol{\omega}(t) - \boldsymbol{\omega}_d(t)) \tag{42}$$

and this completes the proof.

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