Universidade de São Paulo
Instituto de Física de São Carlos - IFSC

FCM 0410 Física para Engenharia Ambiental

Rotação, torque e momento angular

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Agradescimentos

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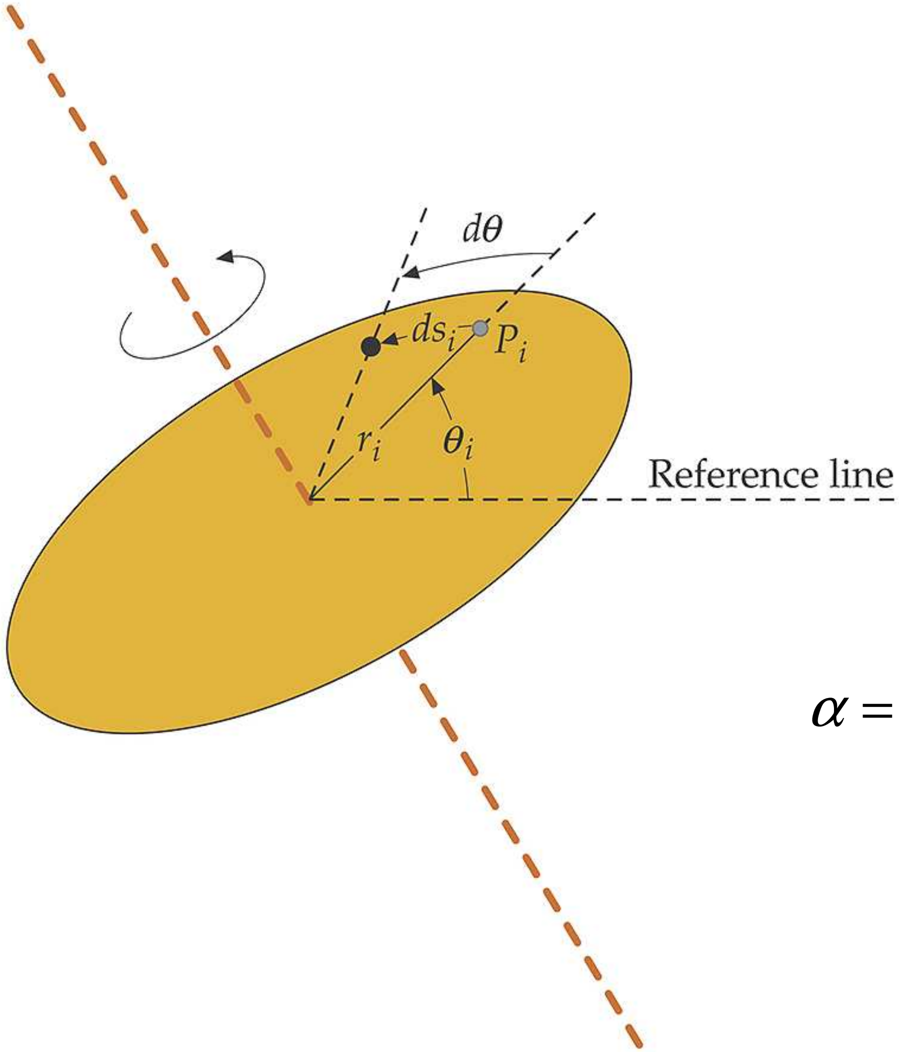
Disco girando em torno de um eixo: velocidade e aceleração angular



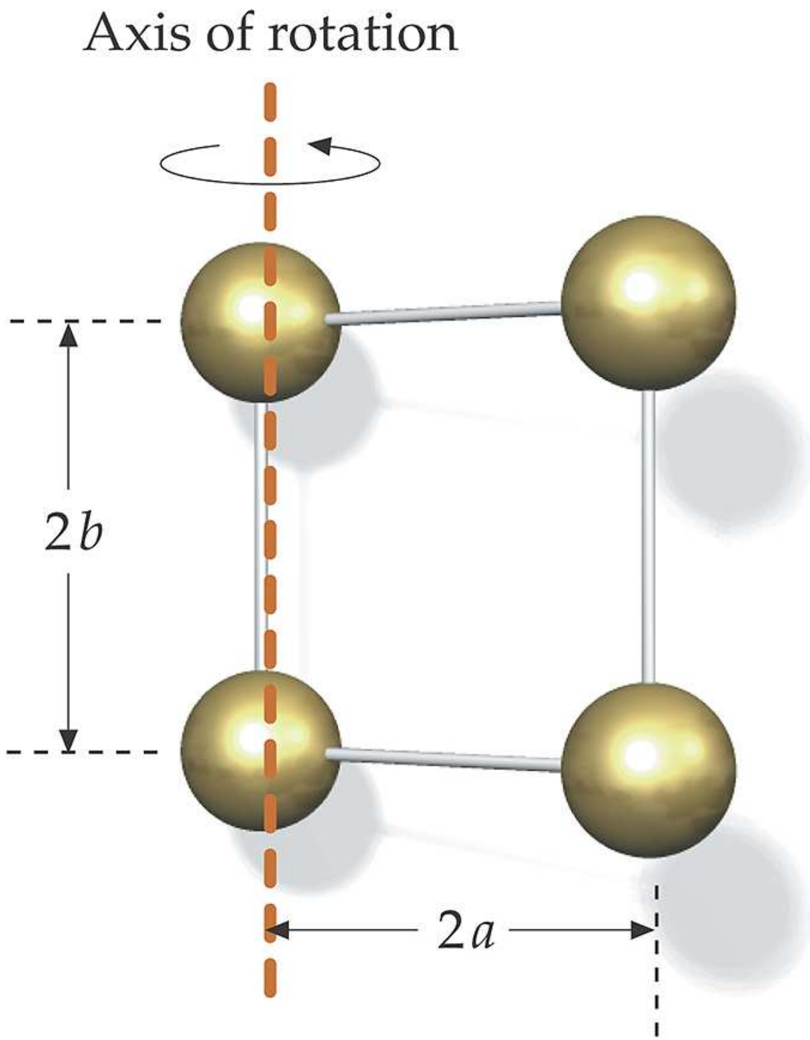
$$ds = r d\theta$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

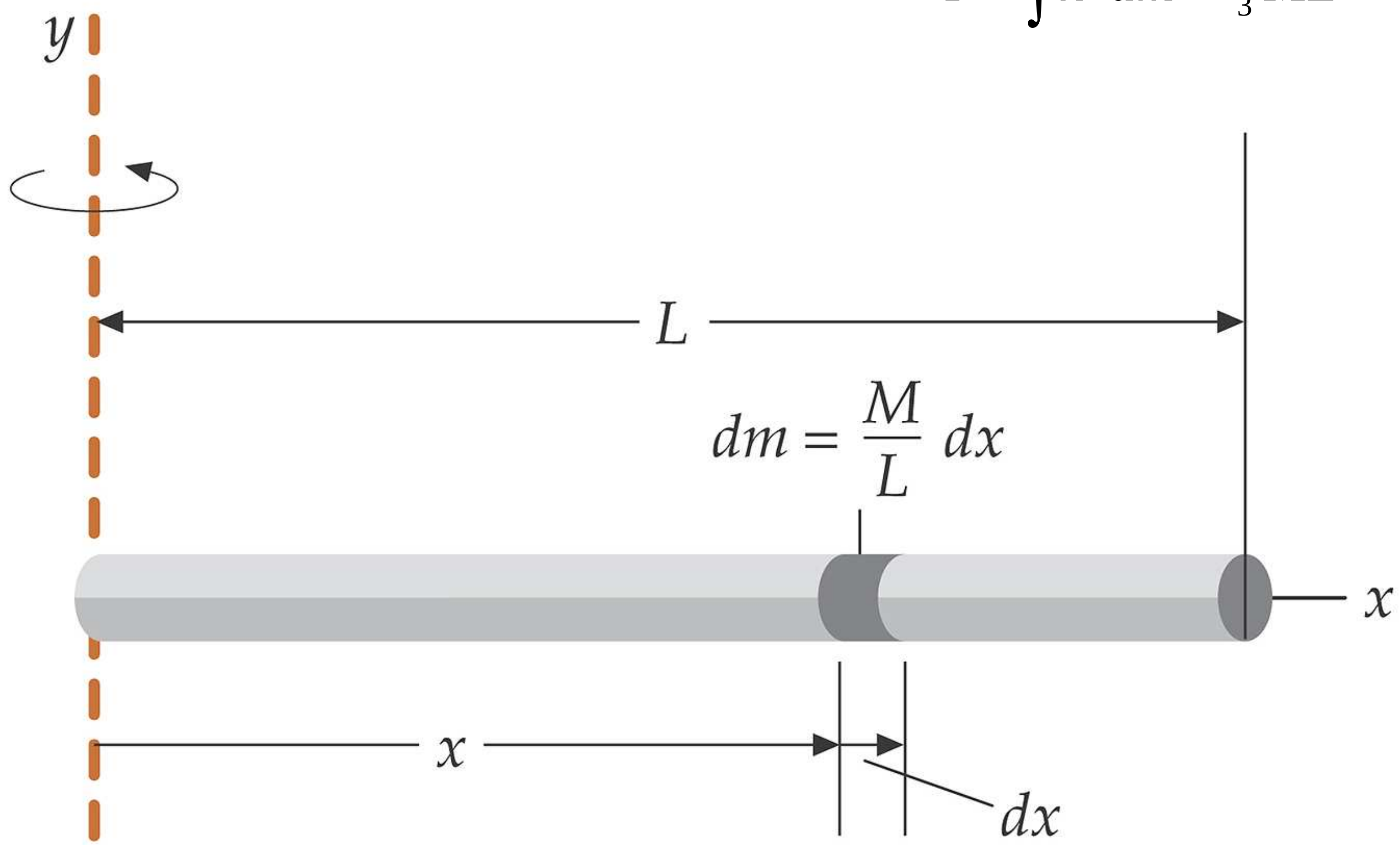
Sistema de partículas girando

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

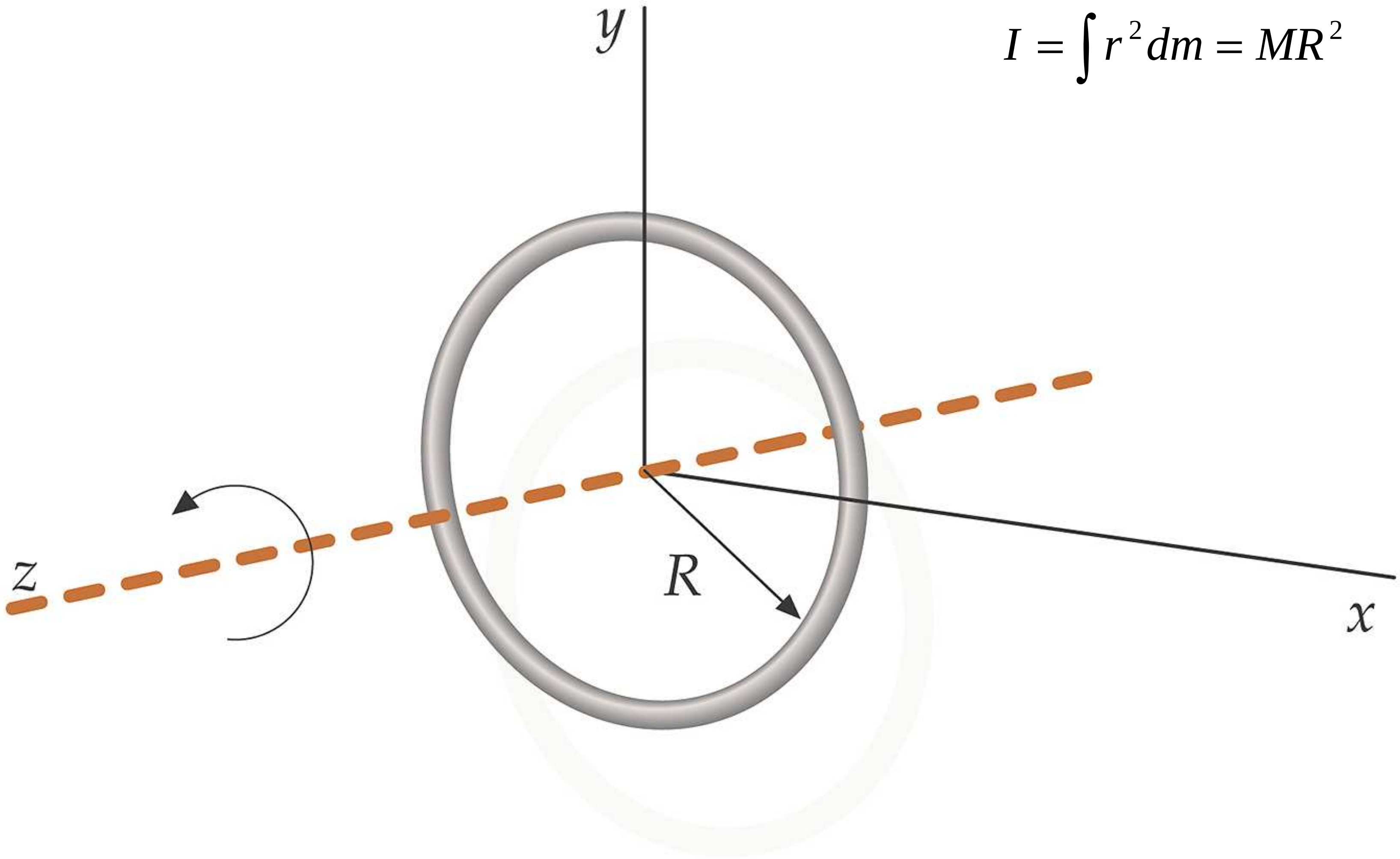


Momento de inercia de uma barra (eixo na extremidade)

$$I = \int x^2 dm = \tfrac{1}{3} ML^2$$



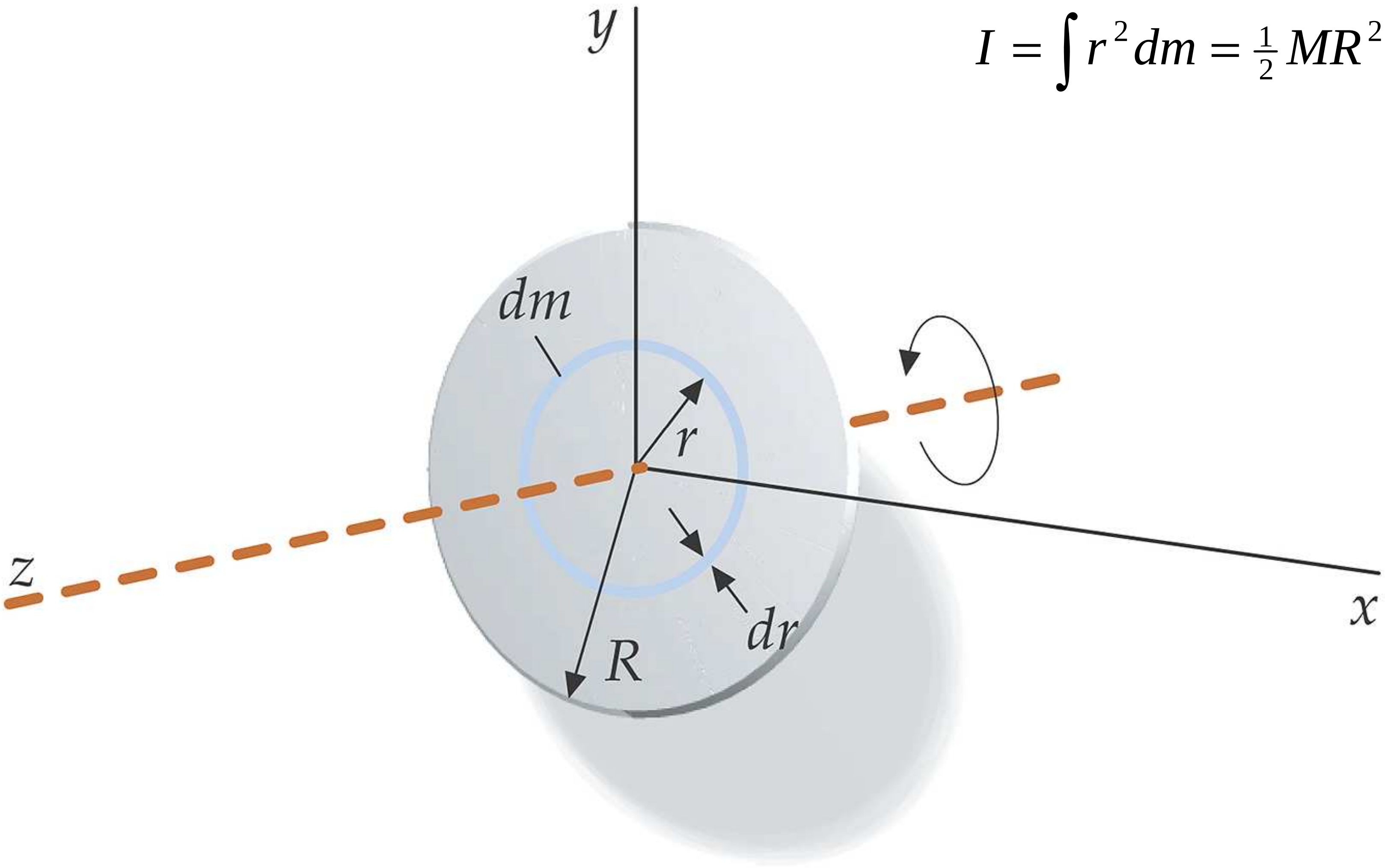
Momento de inercia de um aro (eixo pelo centro)



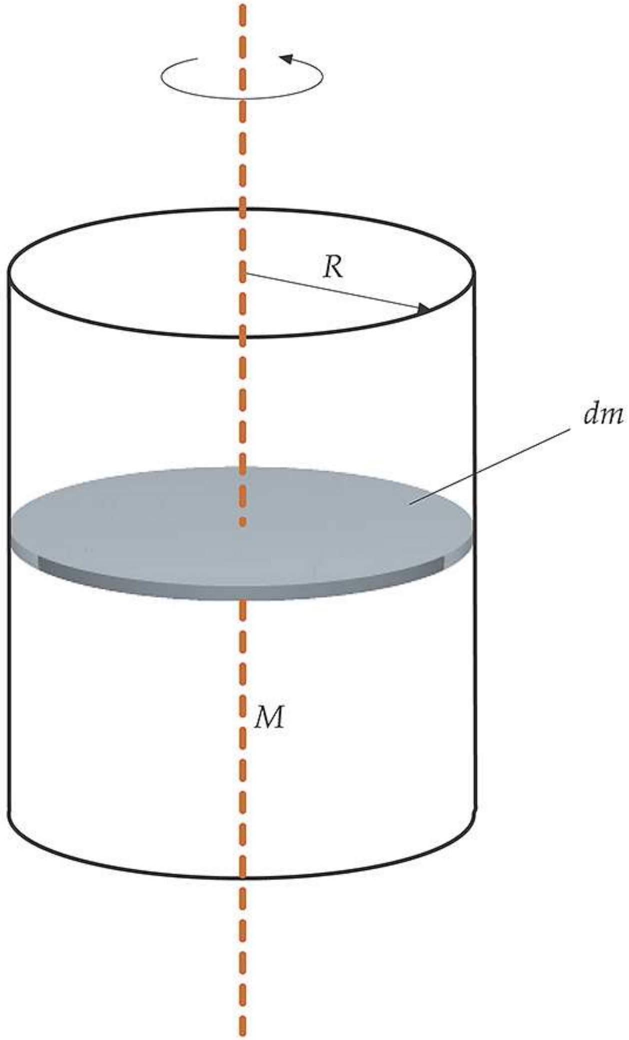
$$I = \int r^2 dm = MR^2$$

Momento de inercia de um disco (eixo pelo centro)

$$I = \int r^2 dm = \tfrac{1}{2} MR^2$$



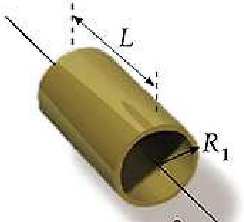
Momento de inercia de um cilindro (eixo pelo centro)



$$I = \int r^2 dm = \tfrac{1}{2} MR^2$$

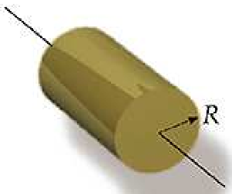
Table 9-1 Moments of Inertia of Uniform Bodies of Various Shapes

Thin cylindrical shell about axis



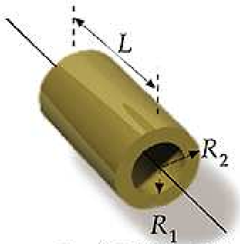
$I = MR^2$

Solid cylinder about axis



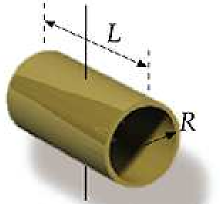
$I = \frac{1}{2}MR^2$

Hollow cylinder about axis



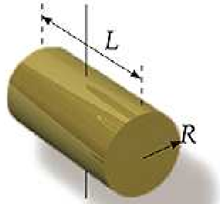
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Thin cylindrical shell about diameter through center



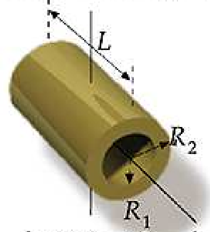
$I = \frac{1}{2}MR^2 + \frac{1}{12}ML^2$

Solid cylinder about diameter through center



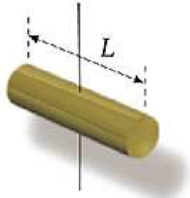
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$

Hollow cylinder about diameter through center



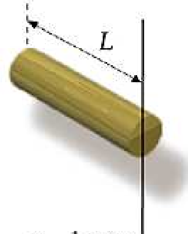
$I = \frac{1}{4}M(R_1^2 + R_2^2) + \frac{1}{12}ML^2$

Thin rod about perpendicular line through center



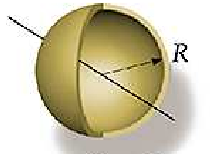
$I = \frac{1}{12}ML^2$

Thin rod about perpendicular line through one end



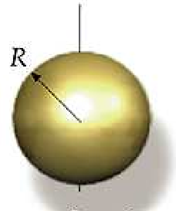
$I = \frac{1}{3}ML^2$

Thin spherical shell about diameter



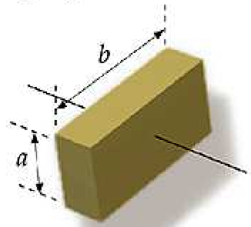
$I = \frac{2}{3}MR^2$

Solid sphere about diameter



$I = \frac{2}{5}MR^2$

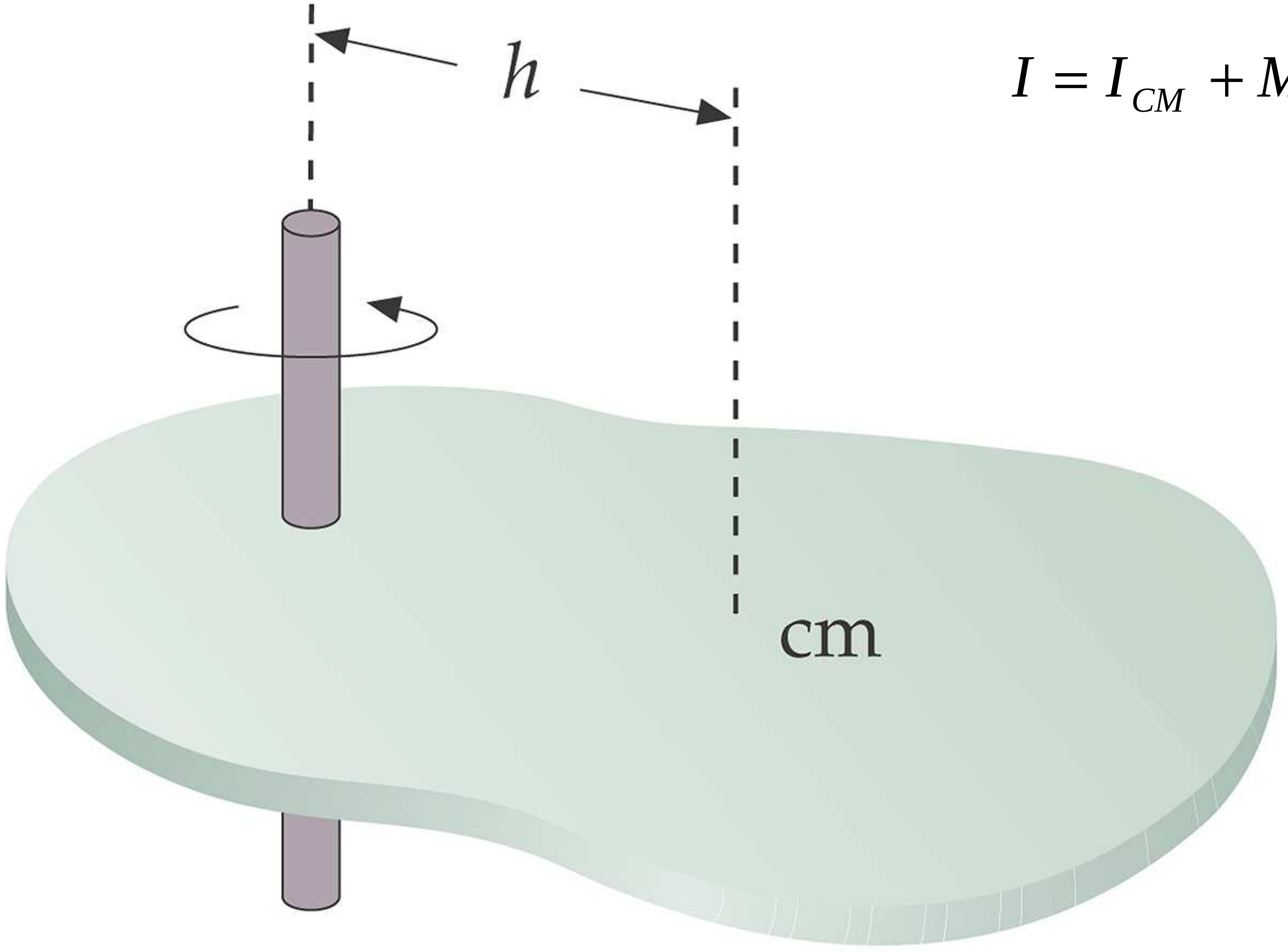
Solid rectangular parallelepiped about axis through center perpendicular to face



$I = \frac{1}{12}M(a^2 + b^2)$

*A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.

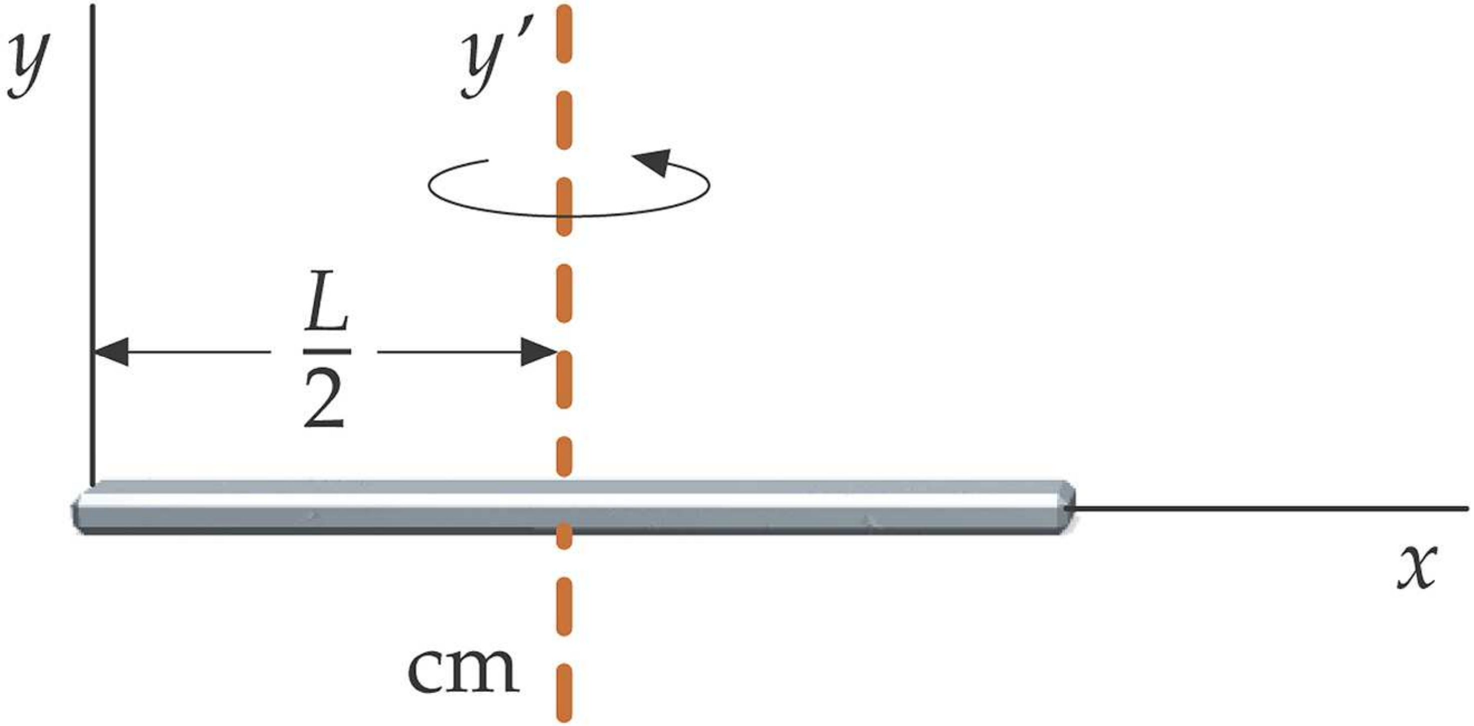
Momento de inercia: teorema dos eixos paralelos



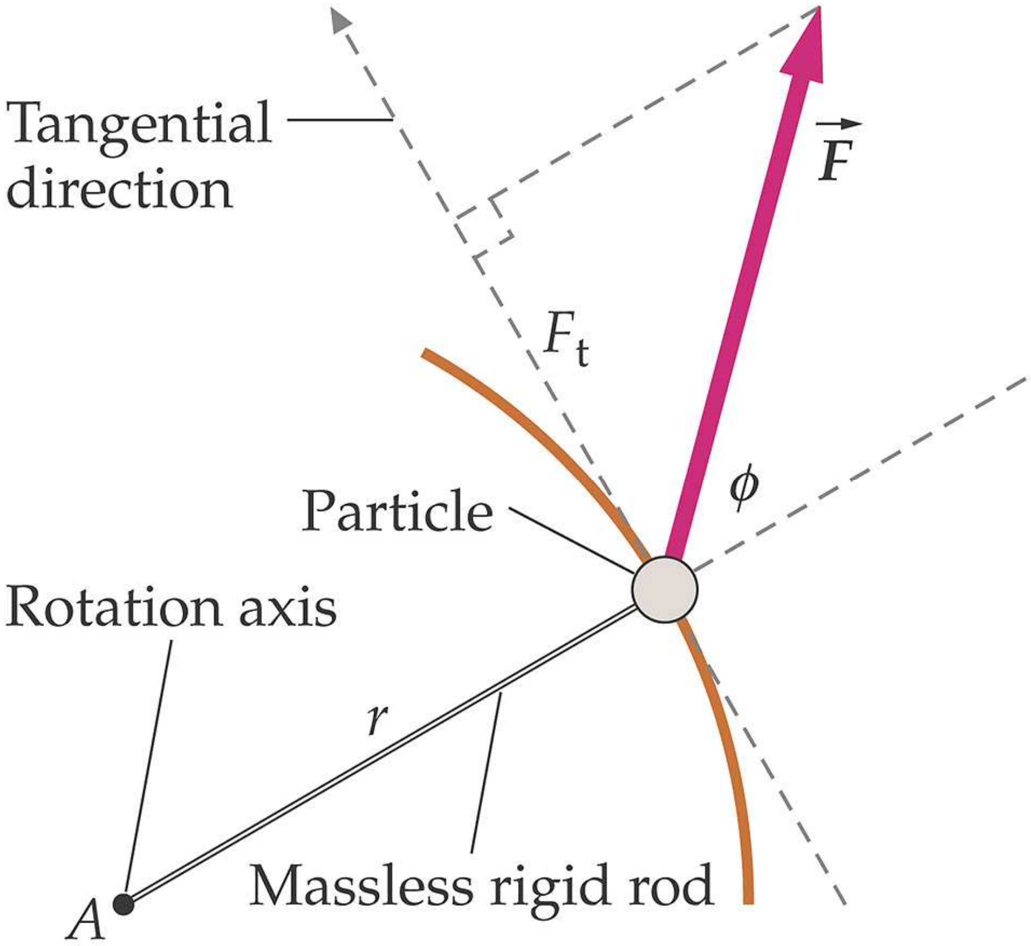
$$I = I_{CM} + Mh^2$$

Aplicando o teorema dos eixos paralelos: momento de inercia de uma barra

$$I_{CM} = I_y - Mh^2 = \tfrac{1}{3}ML^2 - M\left(\tfrac{1}{2}L\right)^2 = \tfrac{1}{12}ML^2$$

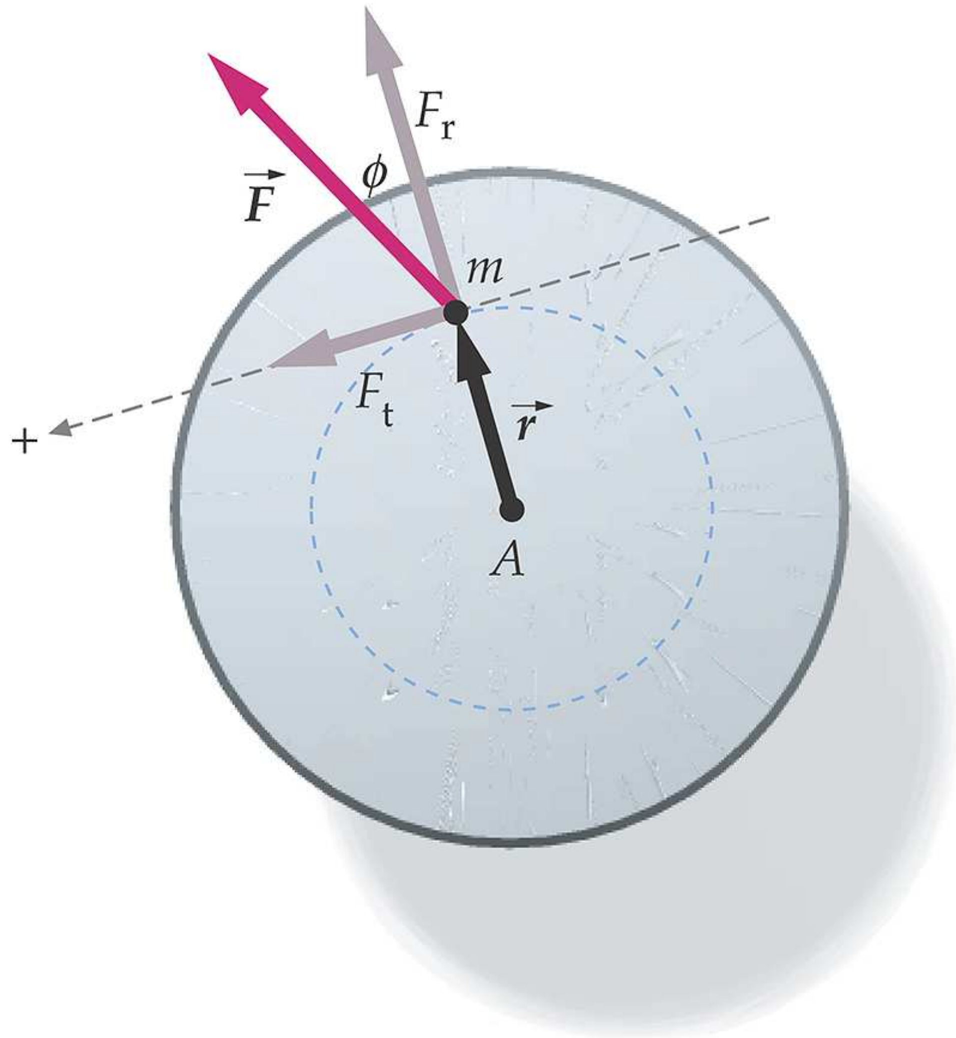


Partícula pressa a uma barra que pode girar livremente: torque



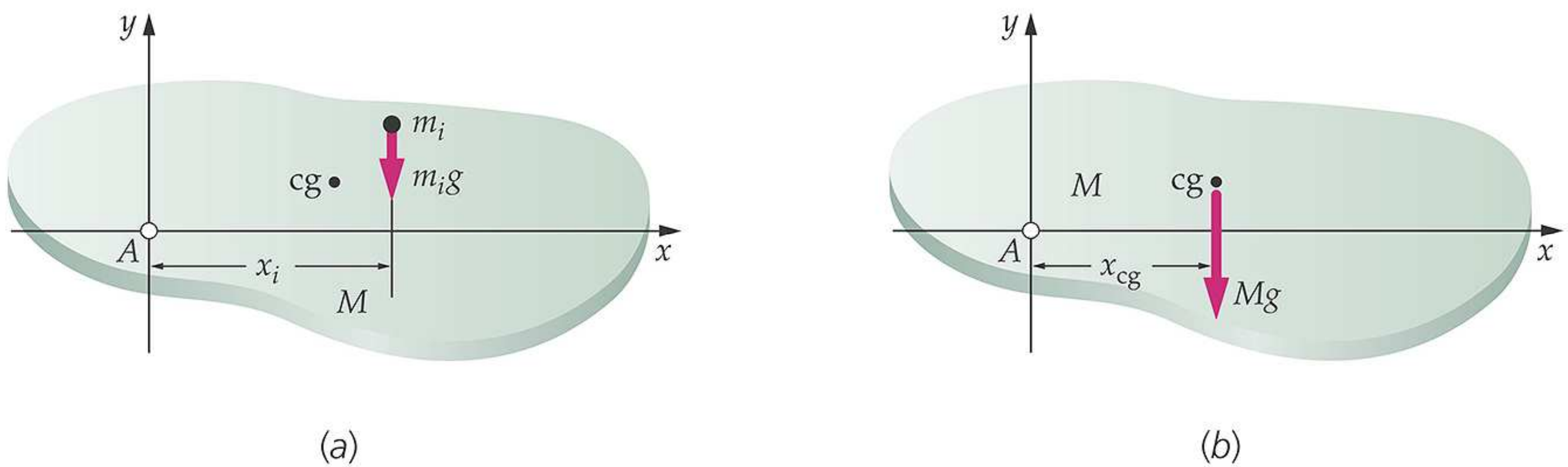
$$\tau = F_t r$$

A força F produz um torque $F_t r$ em relação ao centro



Torque devido à gravidade: ele é calculado como se toda a força gravitacional fosse aplicada no centro de massa

$$\tau = Mgx_{CM}$$



Barra pivotada: determine a aceleração angular

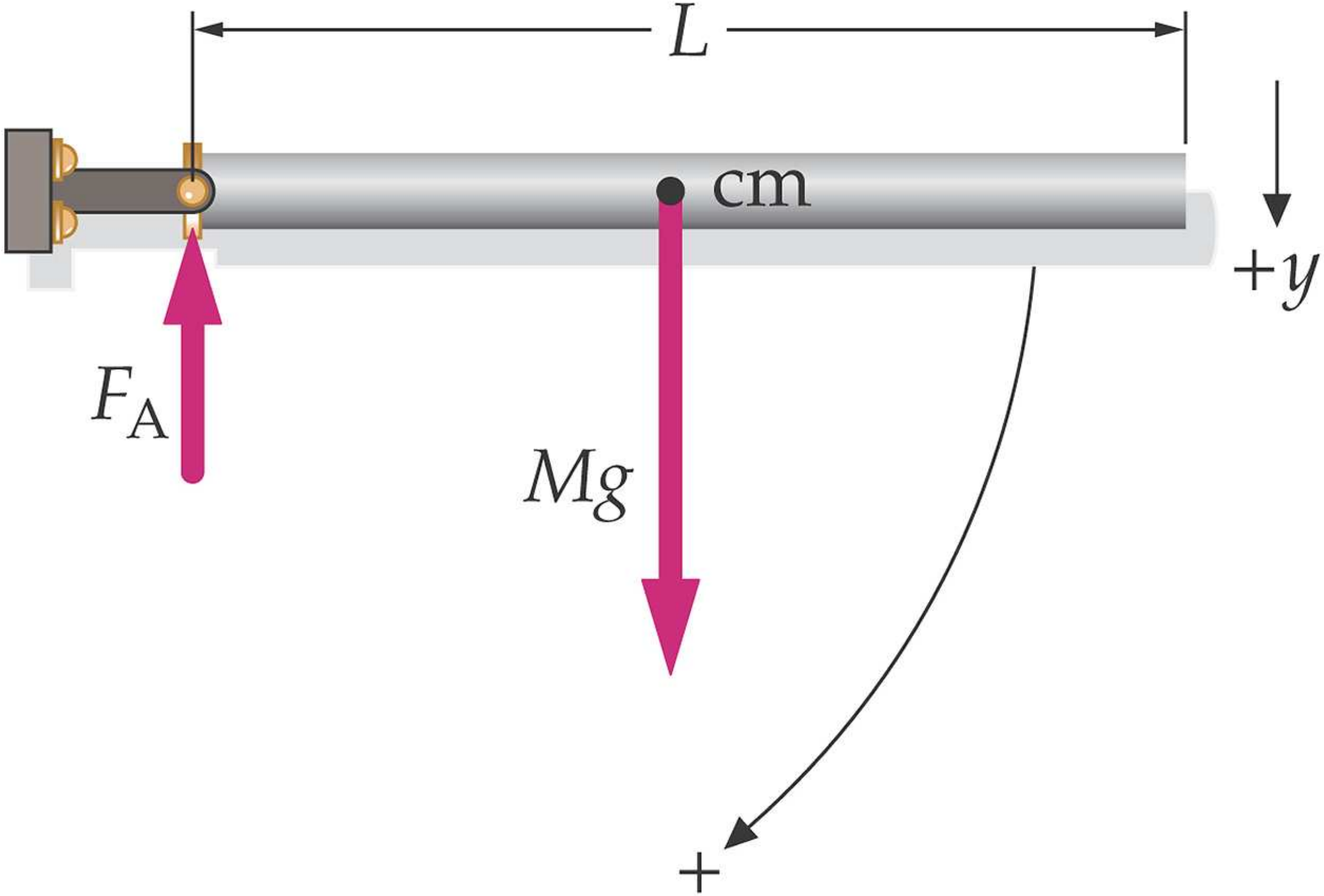


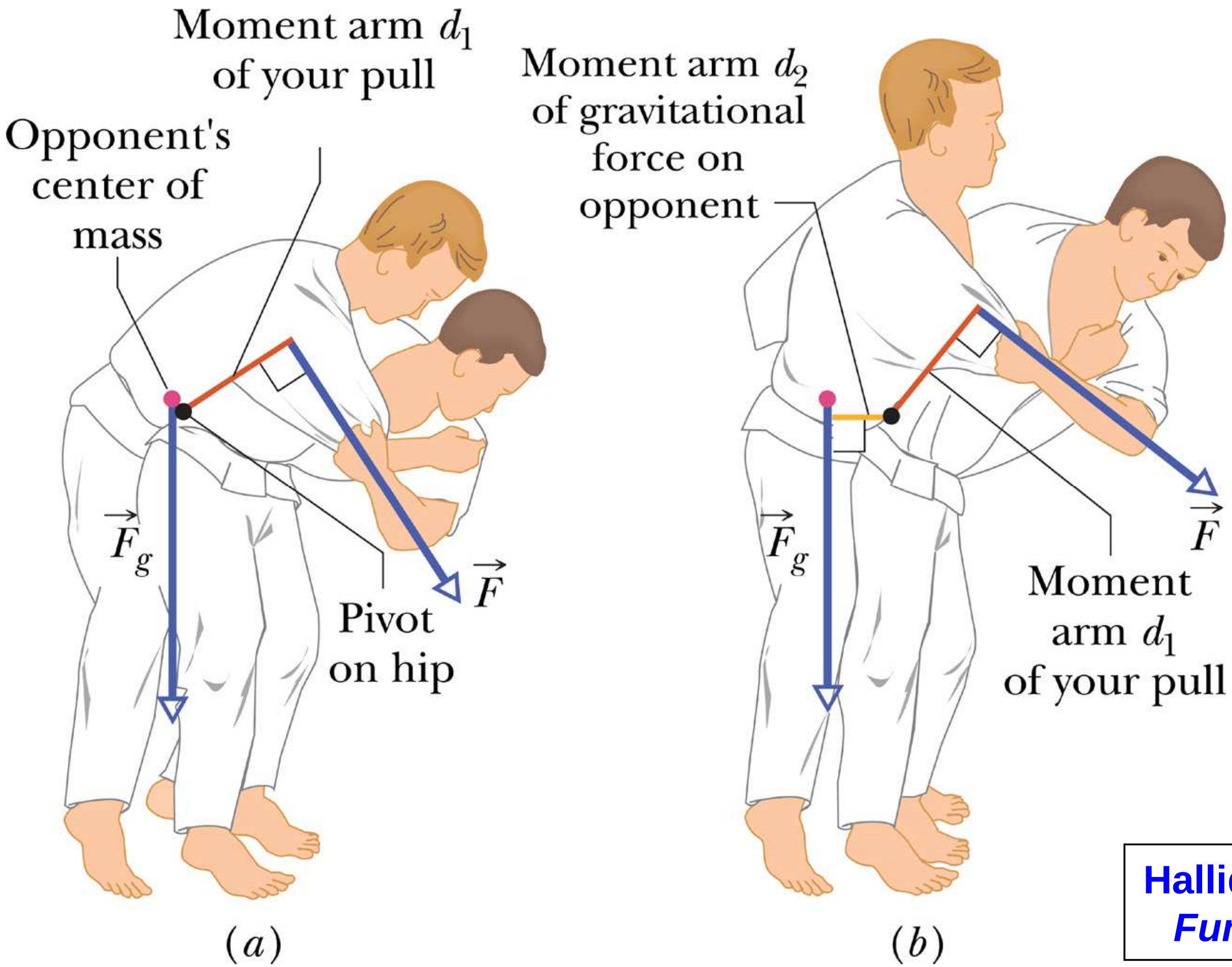
Table 9-2 Analogs in Fixed-Axis Rotational and One-Dimensional Linear Motion

Rotational Motion		Linear Motion	
Angular displacement	$\Delta\theta$	Displacement	Δx
Angular velocity	$\omega = \frac{d\theta}{dt}$	Velocity	$v_x = \frac{dx}{dt}$
Angular acceleration	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	Acceleration	$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$
Constant-angular-acceleration equations	$\omega = \omega_0 + \alpha t$	Constant-acceleration equations	$v_x = v_{0x} + a_x t$
	$\Delta\theta = \omega_{av}\,\Delta t$		$\Delta x = v_{av\,x}\,\Delta t$
	$\omega_{av} = \frac{1}{2}(\omega_0 + \omega)$		$v_{av\,x} = \frac{1}{2}(v_{0x} + v_x)$
	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$		$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$
	$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta$		$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$
Torque	τ	Force	F_x
Moment of inertia	I	Mass	m
Work	$dW = \tau\,d\theta$	Work	$dW = F_x\,dx$
Kinetic energy	$K = \frac{1}{2}I\omega^2$	Kinetic energy	$K = \frac{1}{2}mv^2$
Power	$P = \tau\omega$	Power	$P = F_x v_x$
Angular momentum*	$L = I\omega$	Momentum	$p_x = mv_x$
Newton's second law	$\tau_{net} = I\alpha = \frac{dL}{dt}$	Newton's second law	$F_{net\,x} = ma_x = \frac{dp_x}{dt}$

*Angular momentum is introduced in Chapter 10.

Judô: qual valor da força *F* para derrubar o adversário?

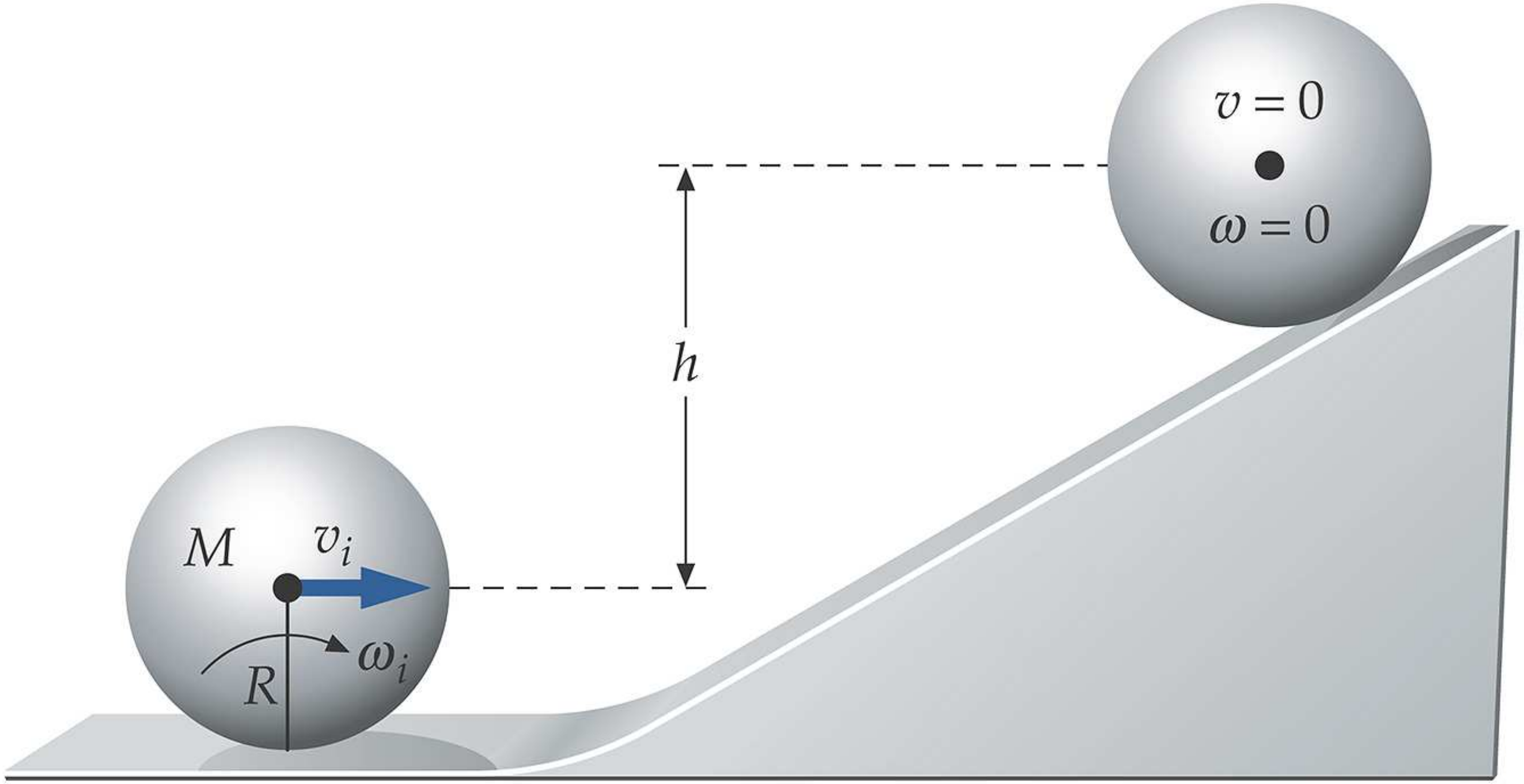
$$Torque : -d\vec{F} = I\alpha$$



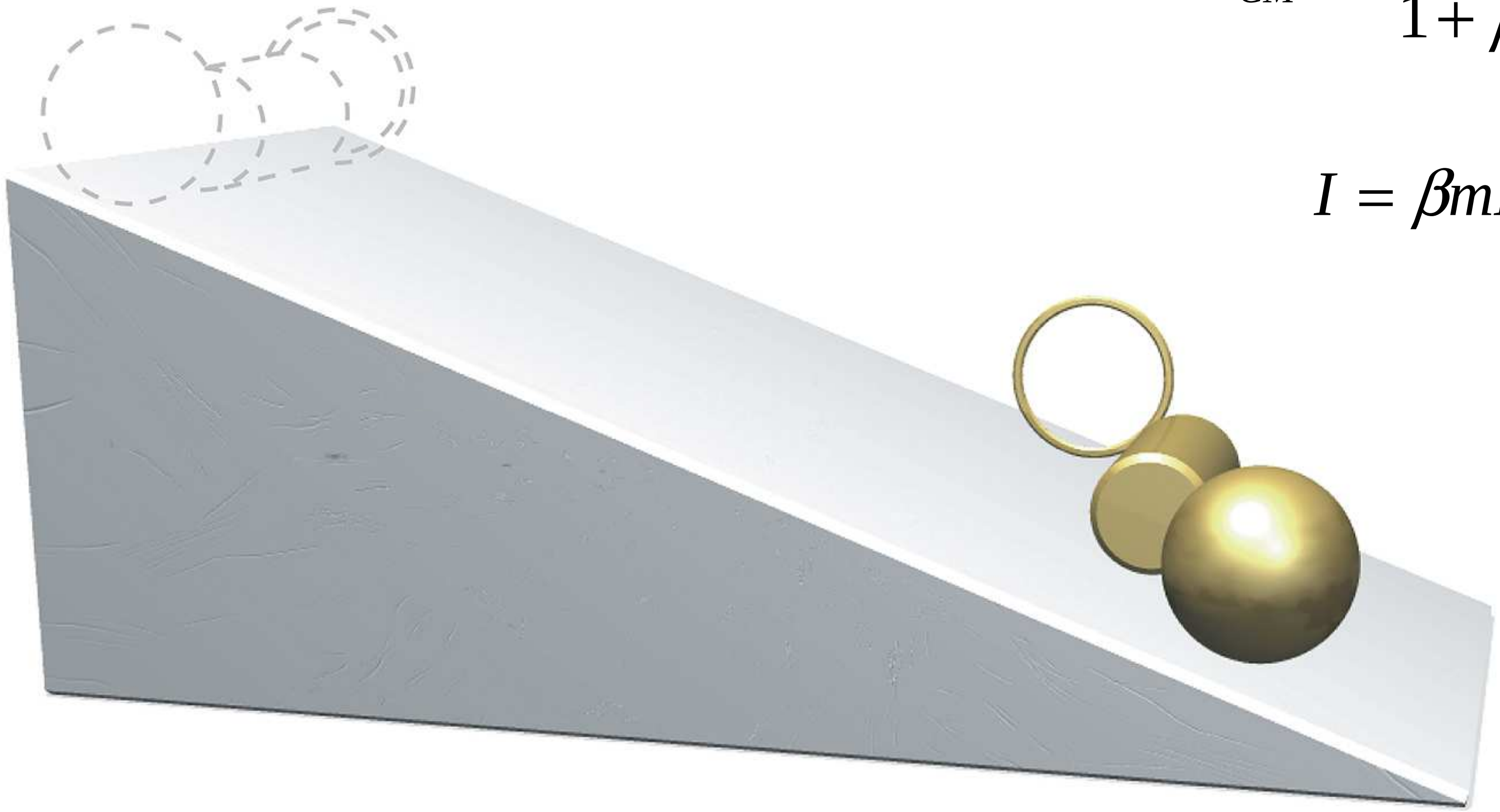
Halliday, Resnick, Walker, *Fundamentos da Física*

Conservação da energia

$$Mgh = \tfrac{1}{2} M v_{CM}^2 + \tfrac{1}{2} I_{CM}\, \omega^2$$



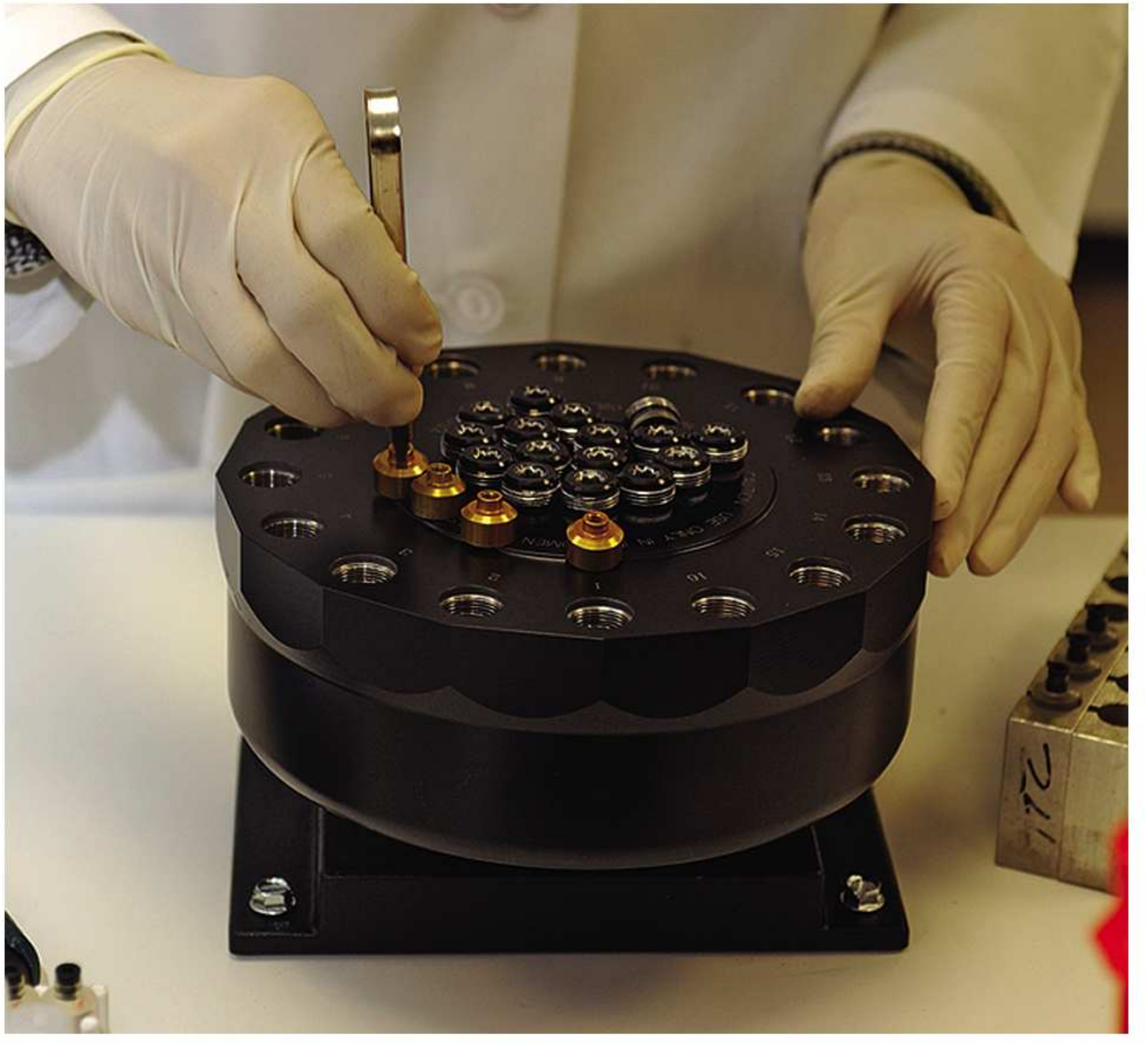
Uma esfera ($\beta = 2/5$), um cilindro ($\beta = 1/2$) e um aro ($\beta = 1$) num plano inclinado



$$a_{CM} = \frac{g sen\phi}{1+\beta}$$

$$I = \beta m R^2$$

Centrífugas de laboratório

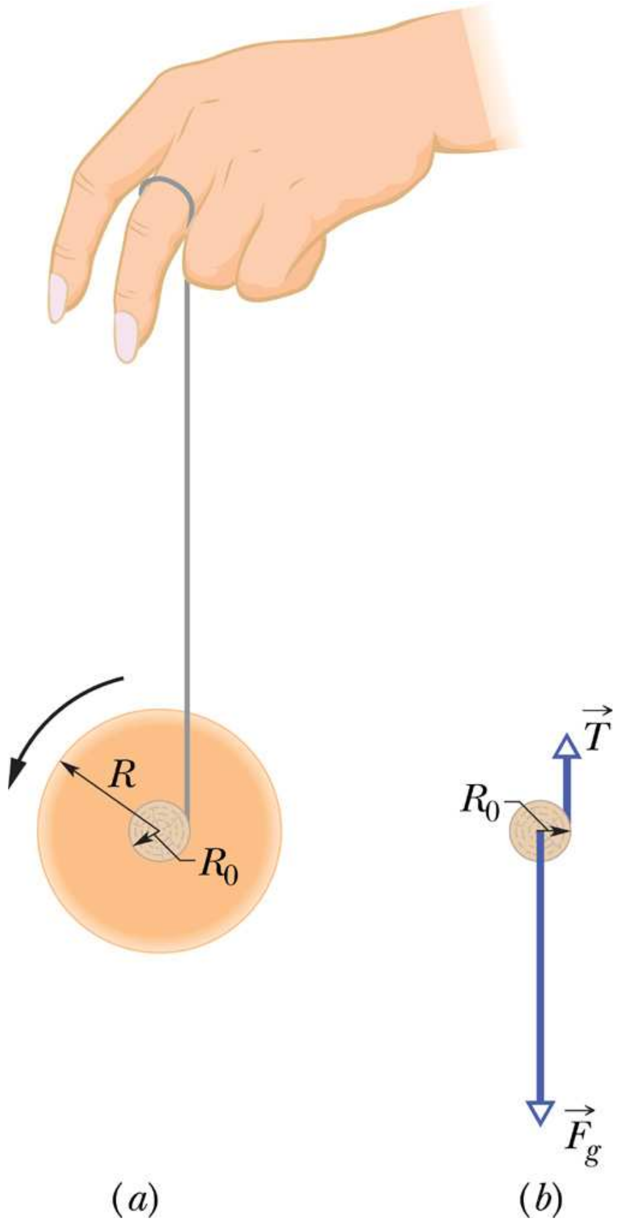


Explosão de um rotor de aço maciço em forma de disco

Empresa *Test Devices Inc.* (1985)

Halliday, Resnick, Walker, *Fundamentos da Física*

O Ioiô



Halliday, Resnick, Walker
Fundamentos da Física

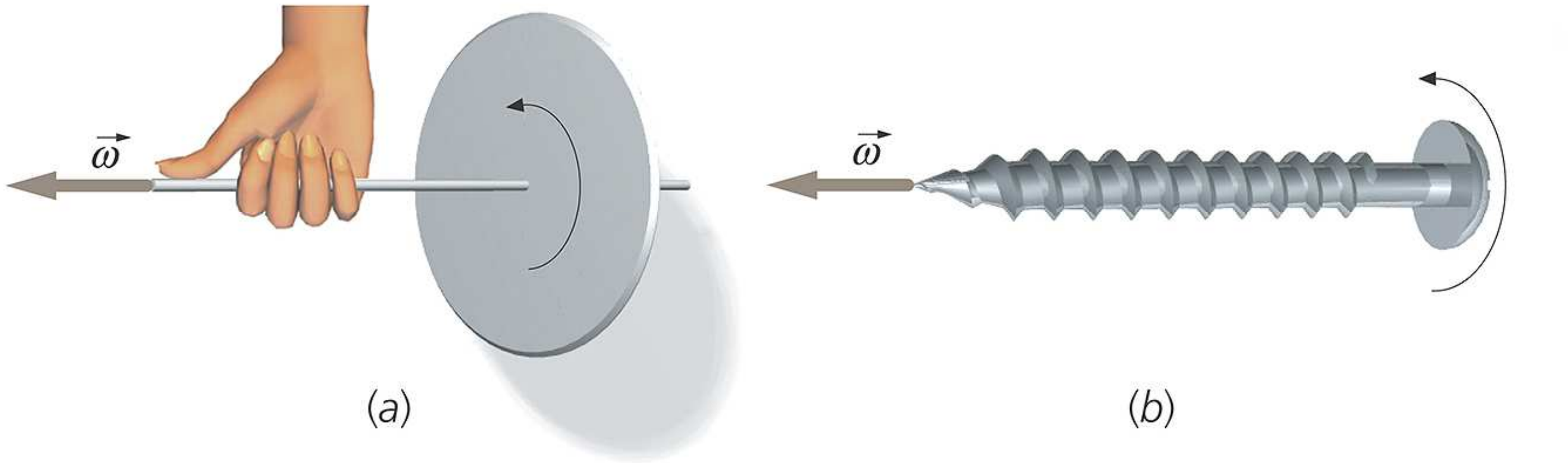
Correspondências entre movimentos de traslação e rotação

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dy$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{\text{net}} = ma$	Newton's second law	$\tau_{\text{net}} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Halliday, Resnick, Walker, *Fundamentos da Física*

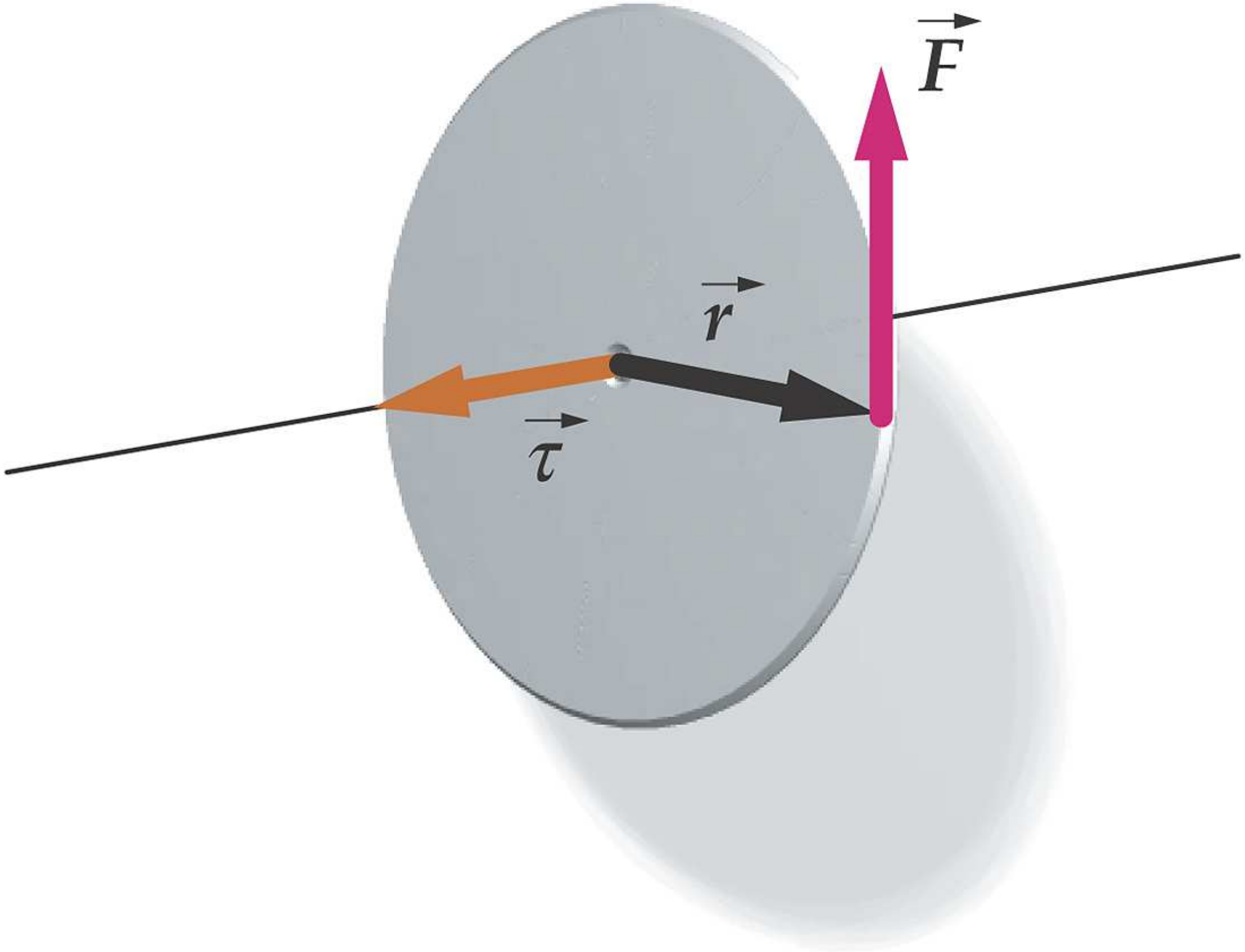
Momento angular

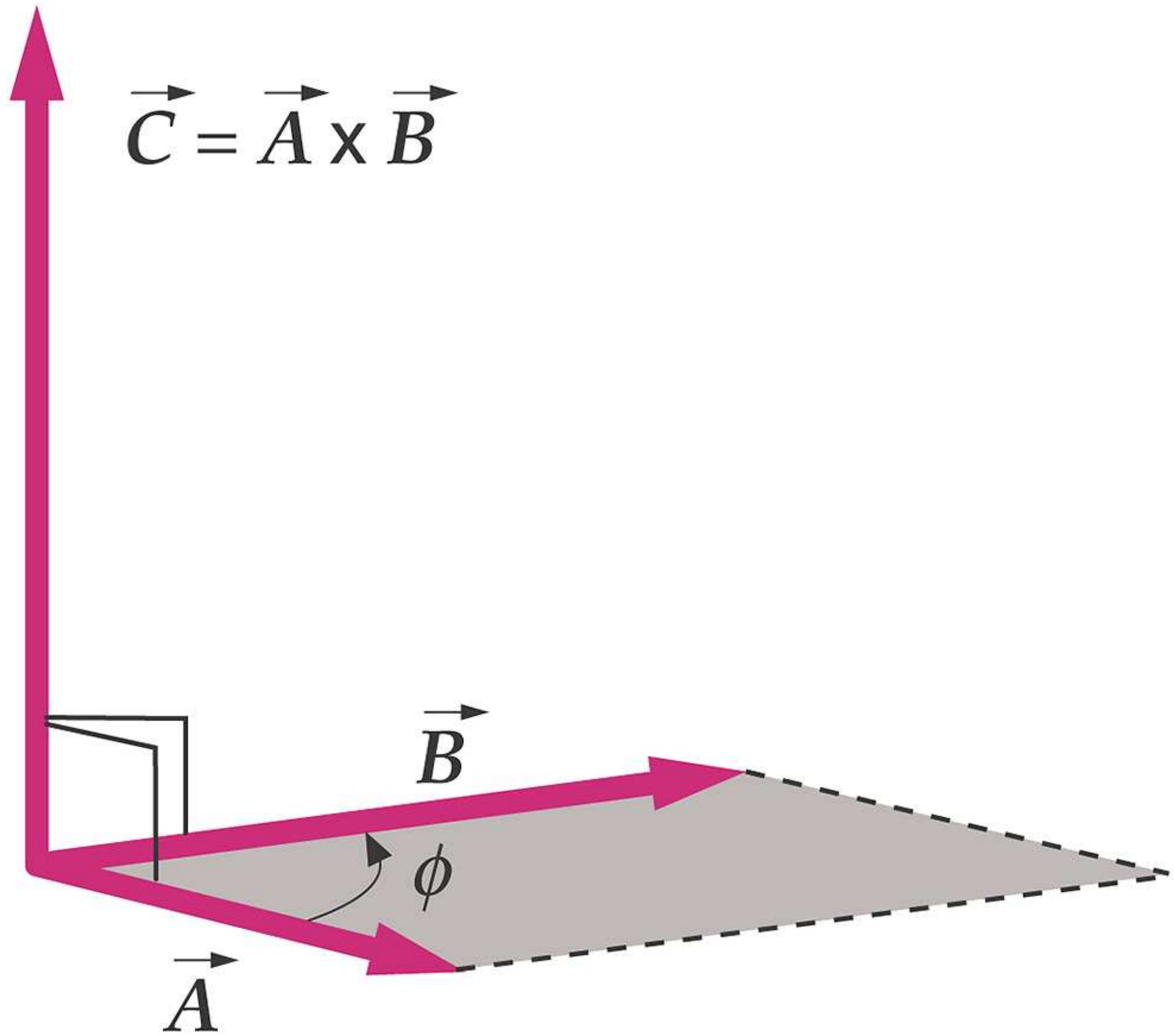
Natureza vetorial da rotação



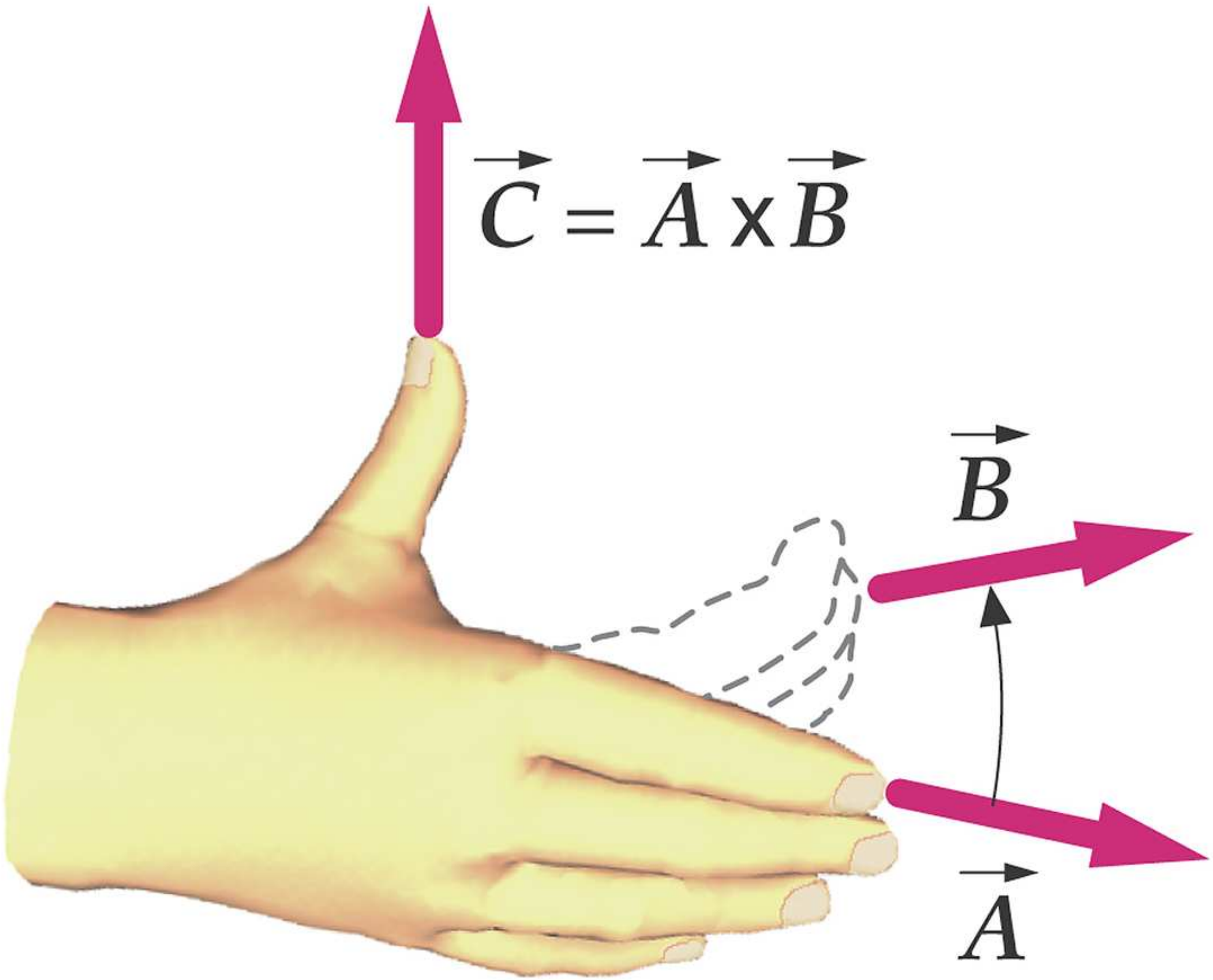
(a) (b)

Torque: $\tau = r \times F$

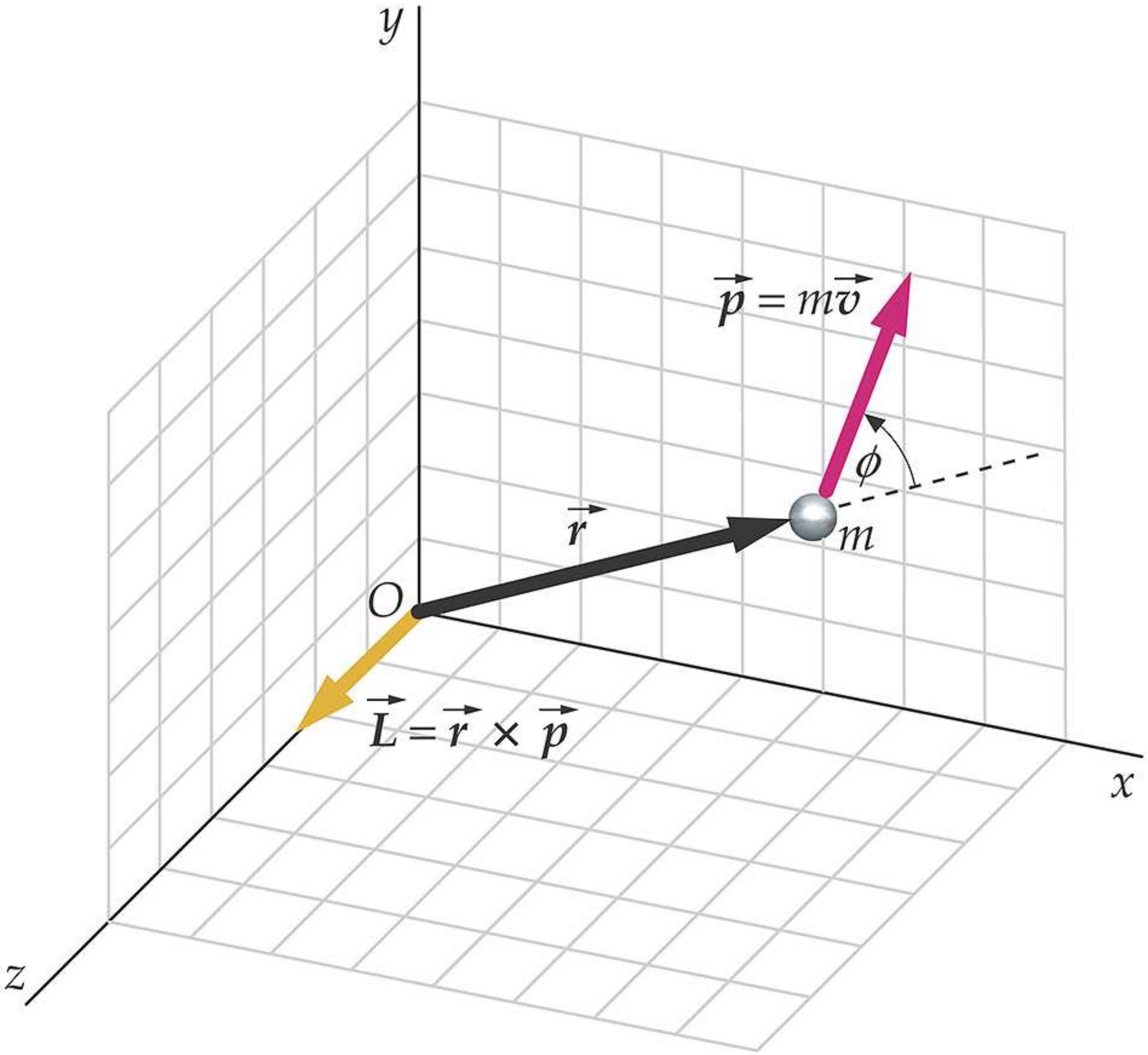



$\vec{C} = \vec{A} \times \vec{B}$
$\vec{B}$
ϕ
$\vec{A}$

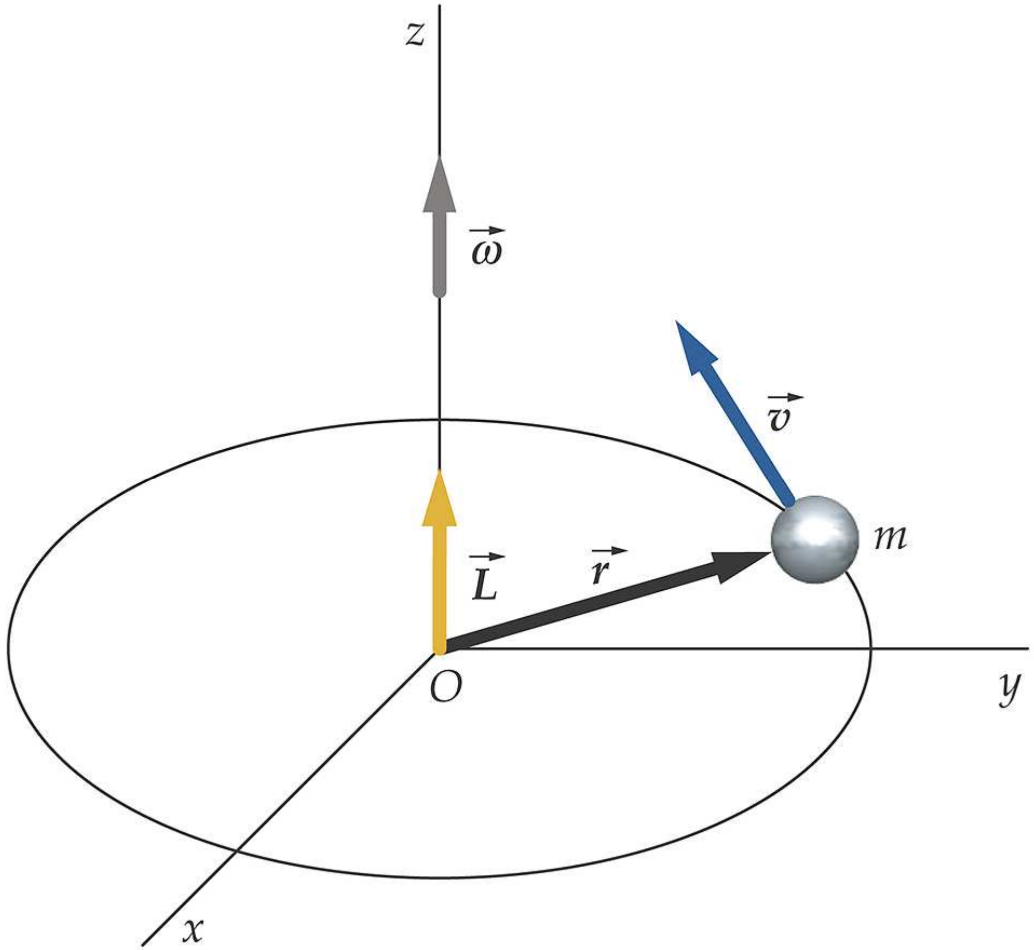
Regra da mão direita



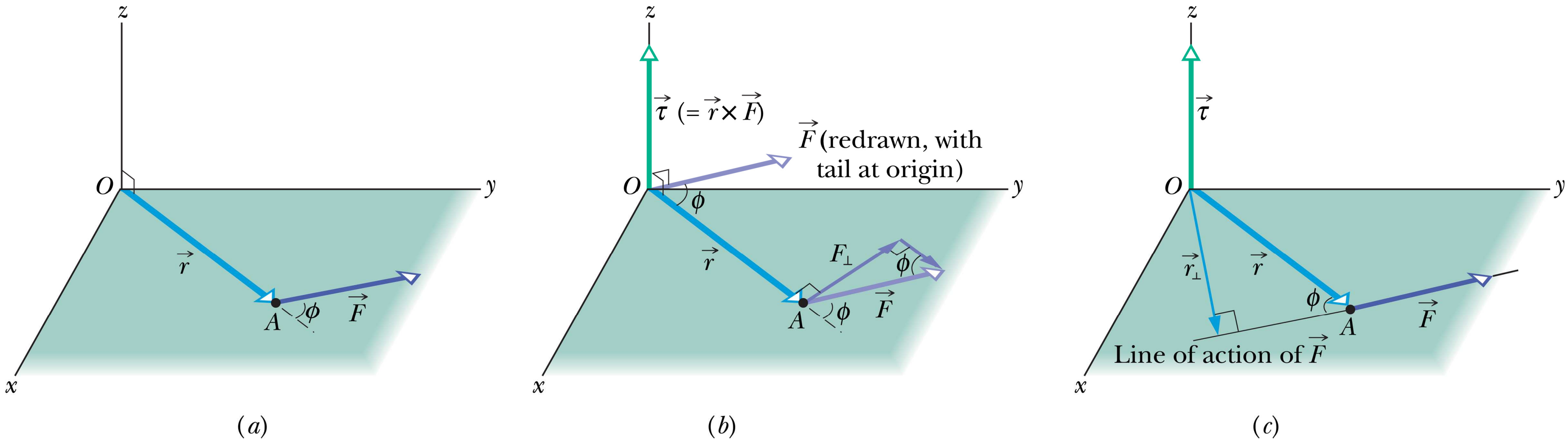
Torque e Momento angular



Partícula movendo-se num circulo no plano *xy*

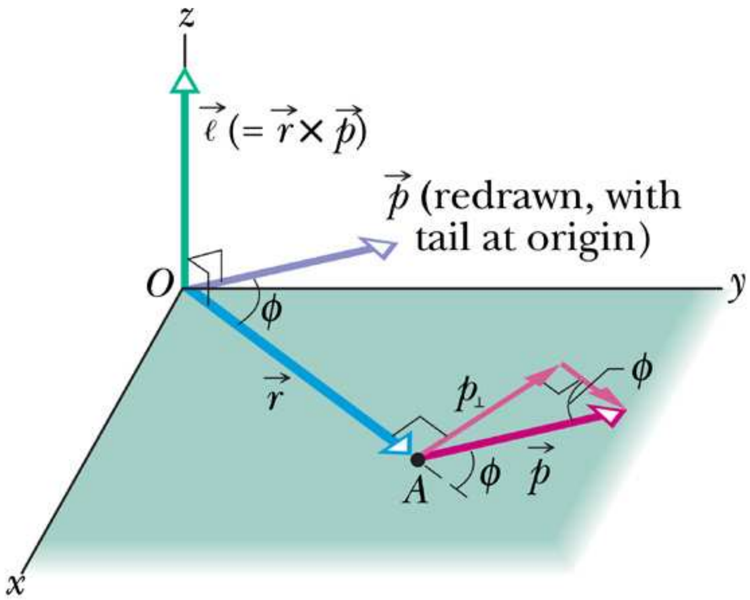


Revisão de torque

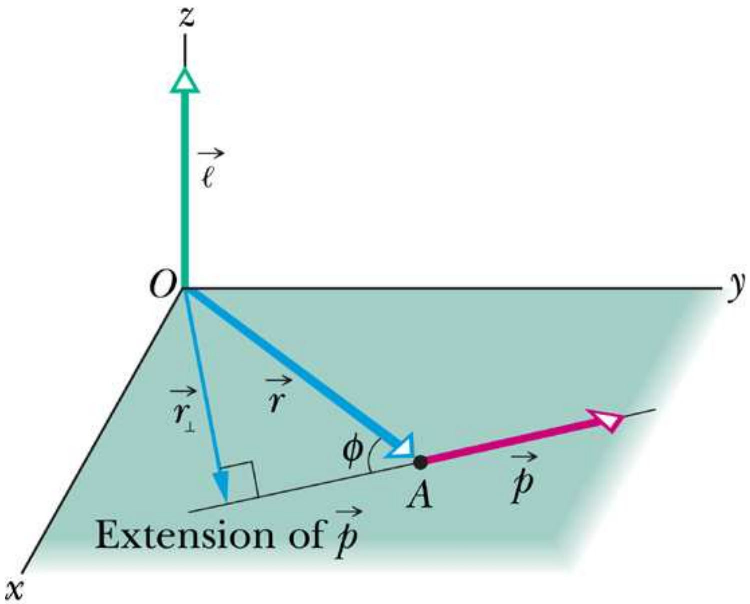


(a) Uma força age sobre uma partícula em A
(b) Essa força produz um torque sobre a partícula em relação à origem
(c) Regra da mão direita: o vetor torque aponta no sentido + Z

Halliday, Resnick, Walker
Fundamentos da Física



(a)



(b)

Momento angular

A partícula em *A* possui momento linear
P = mv com o vetor P no plano xy

A partícula possui momento angular
L = r×P em relação à origem O

Regra da mão direita: o vetor momento
angular aponta no sentido +Z

- O módulo de $L = rP_\perp = rmv_\perp$
- O módulo de L é dado também por
- $L = r_\perp P = r_\perp mv$

Halliday, Resnick, Walker
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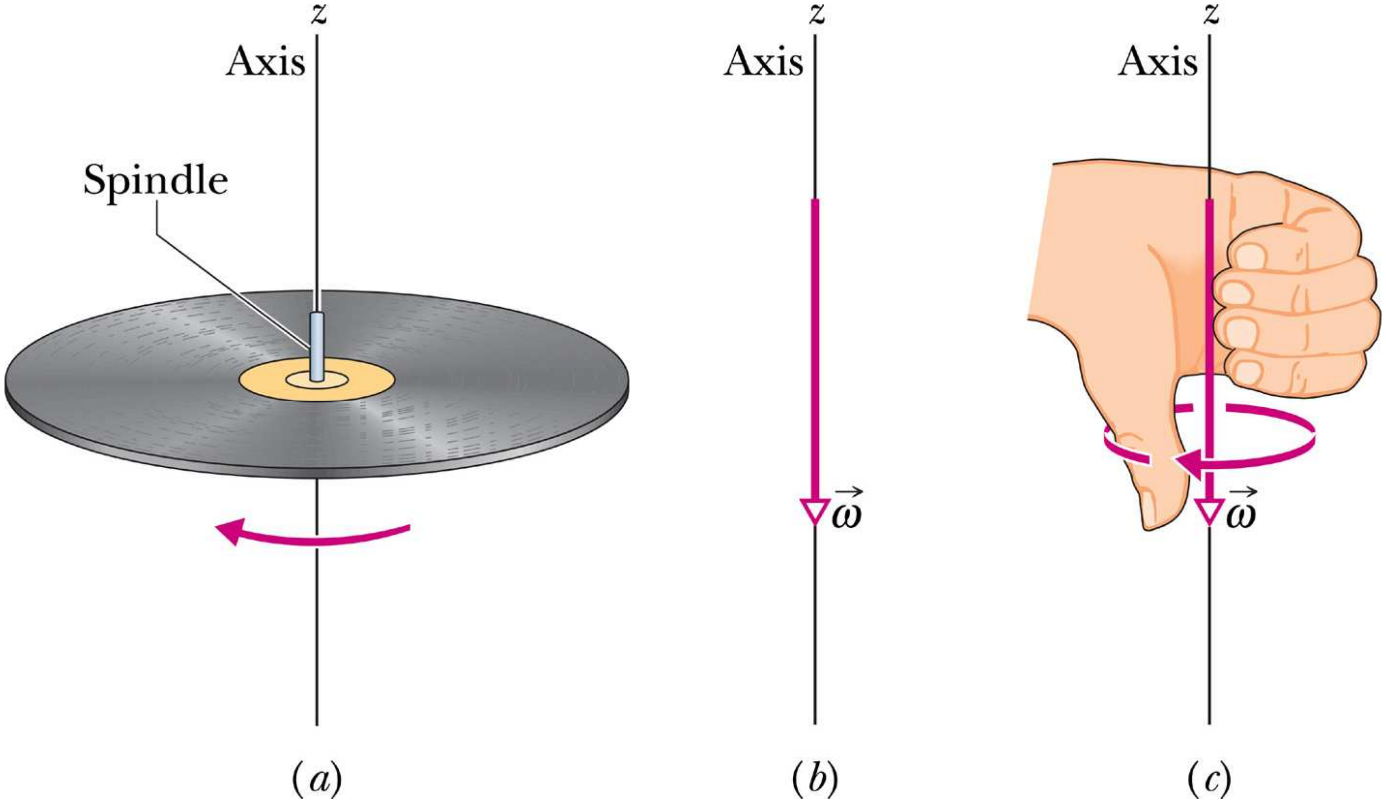
Exemplo: primeira roda gigante (1893)

Halliday, Resnick, Walker
Fundamentos da Física

(a)

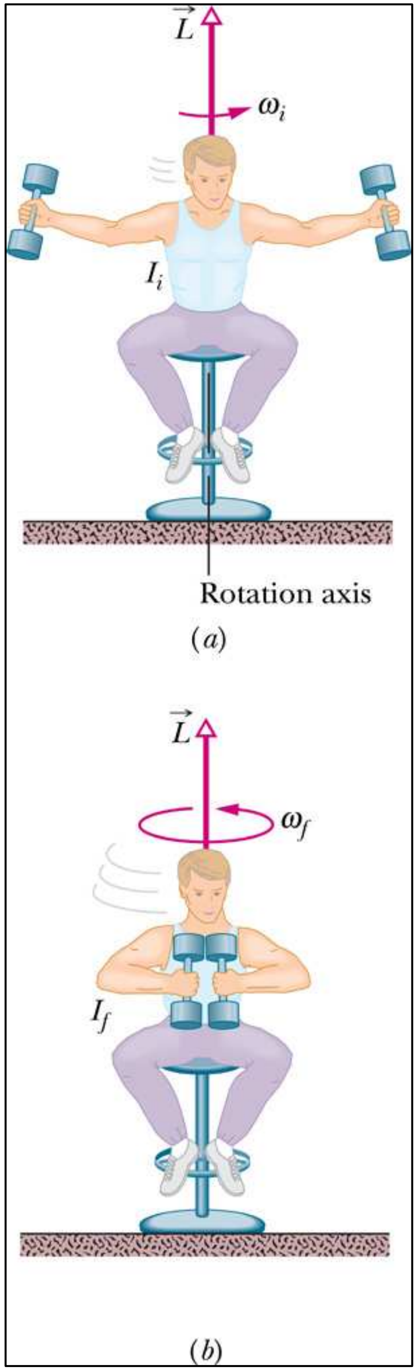
(b)

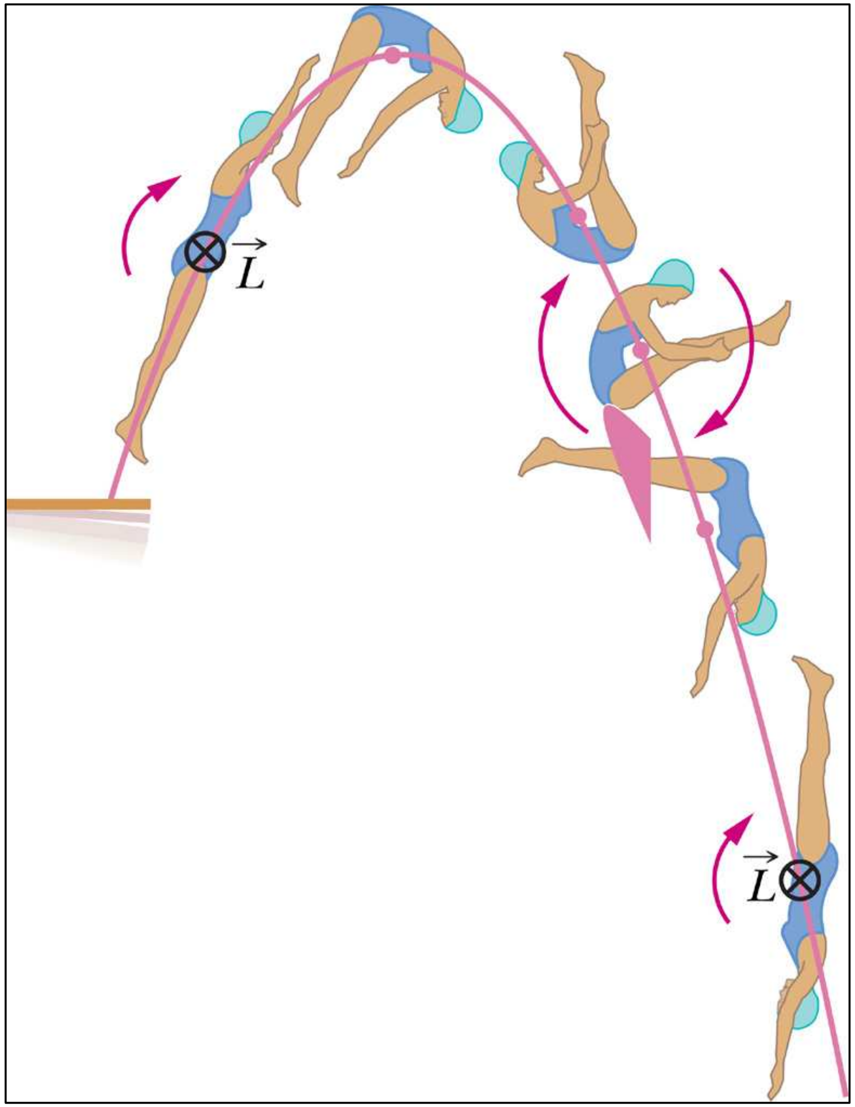


Patinadora Sasha Cohen em movimento de (a) traslação e (b) de rotação em torno de um eixo vertical

Ref: Halliday, Resnick, Walker, *Fundamentos da Física*

Conservação do Momento Angular





Halliday, Resnick, Walker, *Fundamentos da Física*


$\vec{L}_{\text{spin}}$