PARAMETRIC EXCITATION OF A HIGH ALTITUDE GRAVITY GRADIENT SATELLITE

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(Received 18 December, 1981; accepted 27 January, 1982)

Abstract. Attitude dynamics of a gravity oriented satellite in the presence of solar radiation pressure is examined. It is shown that even with a small offset between the satellite center of mass and center of pressure, significant pointing errors may result from the parametric excitation of the attitude motion by the radiation pressure. The phenomenon is illustrated through the analysis of a simple configuration involving a spherical shaped satellite possessing a nonspherical but axisymmetric mass distribution.

1. Introduction

Attitude motion of satellites, in orbits beyond the sensible atmosphere of the earth, is essentially subject to the gravity gradient and solar radiation pressure torques. When the offset between the satellite center of pressure and center of mass is small, the gravitational torques dominate. The satellite then tends to attain an earth-pointing orientation with its minimum moment of inertia axis along the local vertical and the radiation torques represent the major disturbance. The relative importance of the solar torques has been pointed out by Roberson (1958) and Wiggins (1964). The modelling of the solar radiation torques acting on satellites of varying shapes was considered by Hall (1961), Evans (1962), and Tidwell (1970) and others. Flanagan and Modi (1970) evaluated the attitude response of a flat plate satellite due to the radiation torque. The possibility of forced resonance due to the solar torque at certain critical satellite parameter combinations was recognized by Modi and Pande (1973, 1975) during the dynamic analysis of slowly spinning satellites in presence of solar radiation pressure. Whisnant and Anand (1968) pointed out the possibility of roll resonance due to periodic discontinuities in the solar torque during the earth's shadow passage.

The existence of conditions under which significant attitude deviations may result even due to radiation torques of small magnitude, unfortunately, has not been fully explored. The purpose of this note is to show that, besides forced or external resonance, the behaviour may also result from parametric excitation of the attitude motion by the solar radiation pressure. The phenomenon is illustrated through the analysis of a simple configuration consisting of a spherical shaped satellite possessing a non-spherical but axisymmetric mass distribution.

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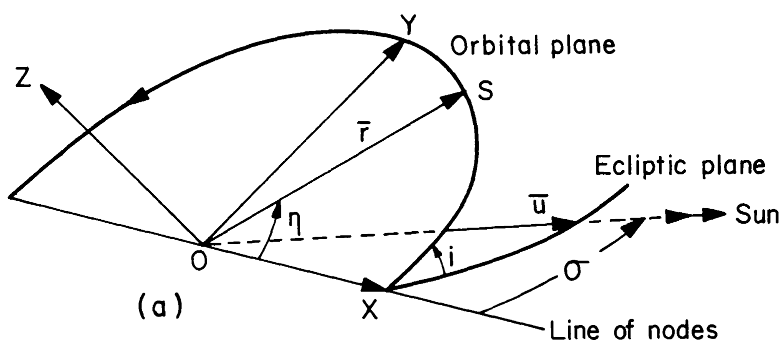
Celestial Mechanics **29** (1983) 101–106. 0008–8714/83/0292–0101 $00.90.

2. Formulation and Analysis

Figure 1a shows the satellite S moving in a circular orbit, with an inclination i from the ecliptic, about the earth's center 0. X, Y, Z represents a geocentric inertial frame of reference with X along the line of nodes, X, Y defining the orbit plane and Z perpendicular to the orbit plane. The apparent position of the sun in the ecliptic plane is indicated by the solar aspect angle σ measured from the line of nodes. The geometry of attitude motion is shown in Figure 1b. The system x_0, y_0, z_0 represents the rotating orbital coordinates with Sx_0, Sy_0, Sz_0 along the local vertical, local horizontal and the orbit normal, respectively. The spatial orientation of the symmetry axis x of the satellite is specified by two successive finite rotations, ψ about the z_0-axis and ϕ about the y_1-axis, which bring the orbital coordinates x_0, y_0, z_0 in coincidence with the satellite principal axes x, y, z. Relative to the principal axes, the satellite has an angular velocity $\mathring{\lambda}$ about the x-axis.

The center of pressure P of the vehicle is assumed to be at a distance l from the center of mass S along the symmetry axis (Figure 2). The solar radiation force acting



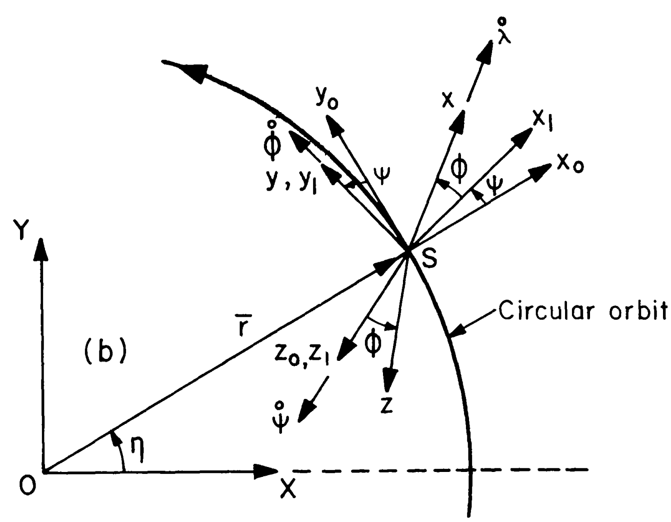


Fig. 1. Geometry of orbital and attitude motion.

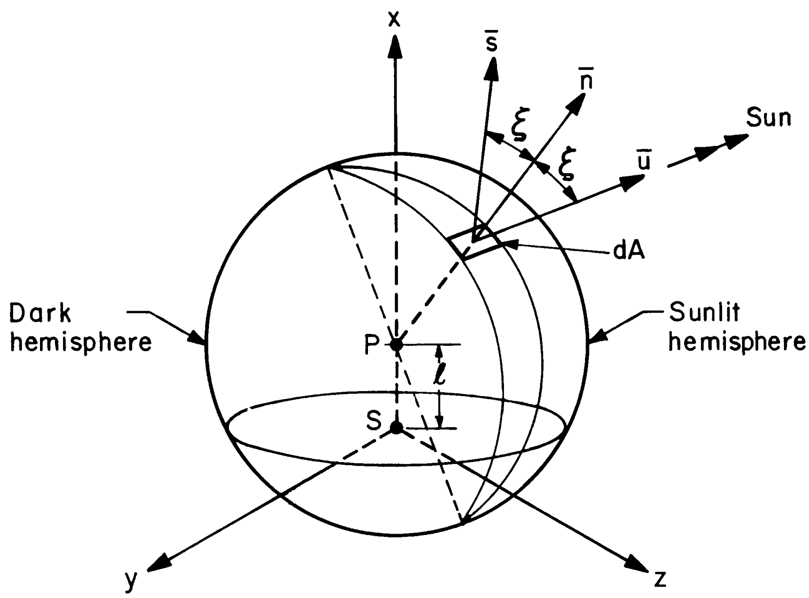


Fig. 2. Evaluation of solar radiation torque.

on the satellite depends on the optical properties of the surface material. The force acting on an area element dA may be written as

$$\mathrm{d}\bar{F} = -p\,\mathrm{d}A\cos\xi\{(1-\tau)\bar{u} + \rho\bar{s}\}, \tag{1}$$

where $p = 4.65 \times 10^{-6}\,\mathrm{N\,m^{-2}}$ is the solar pressure, ξ the local angle of incidence, ρ and τ the reflectivity and transmissibility of the surface, and $\bar{u}, \bar{s}$ denote the unit vectors along the satellite-sun line and the direction of reflection, respectively. Integrating the equation over the entire sunlit hemisphere, it turns out that the forces due to reflected light have no net contribution. The resultant force $\bar{F}$, acting through P, simply becomes

$$\bar{F} = -pA_0(1-\tau)\bar{u}, \tag{2}$$

where A_0 represents the projected area of the sphere. The moment about the center of mass is then given by

$$\bar{M} = l\bar{\imath}X\{-pA_0(1-\tau)\}\bar{u}. \tag{3}$$

Using the principle of virtual work, the generalized forces in the ψ, ϕ, and λ degrees of freedom are found to be:

$$\begin{aligned} Q_\psi &= -pA_0 l(1-\tau)\cos\phi(-u_1\sin\psi + u_2\cos\psi) \\ Q_\phi &= +pA_0 l(1-\tau)(u_1\cos\psi\sin\phi + u_2\sin\psi\sin\phi + u_3\cos\phi) \\ Q_\lambda &= 0, \end{aligned} \tag{4}$$

where u_1, u_2, u_3 denote the components of $\bar{u}$ along the orbital system x_0, y_0, z_0. Geometrical considerations lead to the following convenient form for these components:

$$u_1 = (1 - \sin^2\sigma \sin^2 i)^{1/2} \cos(\eta - \zeta)$$
$$u_2 = -(1 - \sin^2\sigma \sin^2 i)^{1/2} \sin(\eta - \zeta)$$
$$u_3 = -\sin\sigma \sin i \tag{5}$$

with $\zeta = \arctan(\tan\sigma \cos i)$, the quadrant of ζ being the same as that of σ.

The classical Lagrangian formulation was employed to derive the complete equations of motion in presence of the gravity-gradient potential and the solar torques. With $Q_\lambda = 0$, the λ degree of freedom turns out to be cyclic. The associated constant generalized momentum must be zero for gravity-gradient stabilization to be possible. The cyclic generalized velocity $\mathring{\lambda}$ may then be eliminated from the equations governing the ψ and ϕ degrees of freedom. Finally, changing the independent variable from t to the orbital angle η, the pitch and roll equations of motion take the non-dimensional form:

$$\psi'' - 2(\psi' + 1)\phi' \tan\phi + 3K \sin\psi \cos\psi =$$
$$= \varepsilon \sin(\psi + \eta - \zeta)/\cos\phi \tag{6a}$$
$$\phi'' + \{(1 + \psi')^2 + 3K \cos^2\psi\} \sin\phi \cos\phi =$$
$$= \varepsilon\{\cos(\psi + \eta - \zeta)\sin\phi - \sin\sigma \sin i \cos\phi/(1 - \sin^2\sigma \sin^2 i)^{1/2}\}. \tag{6b}$$

where, the inertia parameter K and the solar parameter ε are defined as

$$K = 1 - I_x/I_y \tag{7a}$$
$$\varepsilon = pA_0 l(1 - \tau)(1 - \sin^2\sigma \sin^2 i)^{1/2}/I_x \mathring{\eta}^2 \tag{7b}$$

and $I_x, I_y (= I_z)$ denote the satellite principal moments of inertia about the symmetry and transverse axes, respectively.

The governing equations represent a rather complicated coupled, nonlinear, nonautonomous system of differential equations. The nonautonomous character of the problem arises due to the explicit presence of two time variables, namely, η associated with the orbital motion and σ associated with the apparent annual motion of the sun. The equations generally are not amenable to an analytical treatment. However, some information about the system behaviour may still be extracted analytically when the solar torque magnitude is small.

It is apparent that when P and S are coincident ($\varepsilon = 0$), $\psi = \phi = 0$ represents an identical solution of Equations (6). Perfect earth-pointing is thus achieved nominally. When the offset l between the centers of pressure and mass is small (ε small but nonzero), small amplitude pitch and roll motion would, in general, be expected. The governing equations of motion may then be linearized in ψ and ϕ. Furthermore, since $\mathring{\sigma} = 2\pi$ radians/year, σ represents an extremely slow variable compared to η. Hence, one may assume it to remain practically a constant over a number of satellite orbits. Shifting the independent variable from η to $(\eta - \zeta)$ for convenience, the linearized pitch and roll equations become

$$\psi'' + (3K - \varepsilon \cos \eta)\psi = \varepsilon \sin \eta \tag{8a}$$

$$\phi'' + (1 + 3K - \varepsilon \cos \eta)\phi = -\varepsilon \sin \sigma \sin i/(1 - \sin^2 \sigma \sin^2 i)^{1/2}. \tag{8b}$$

It is well known that the small amplitude in-plane (pitch) and out-of-plane (roll) motions decouple in absence of any environmental torques. The present formulation indicates that they remain uncoupled even in presence of the radiation torques. The radiation torque leads to a sinusoidal forcing function in the pitch equation. In the roll equation, it appears as a 'constant' input which, in general, is nonzero in orbits inclined to the ecliptic. The attitude dependent character of the solar torque manifests itself as periodic time-varying coefficients of orbital frequency in both degrees of freedom. It is the presence of these coefficients which may lead to parametric instability of the attitude motion under appropriate conditions.

As the driving terms in Equations (8) are bounded, the stability of the motion is governed by their homogeneous parts. In accordance with Floquet theory (Cessari, 1963), the stability depends on the parameters K and ε. The boundaries separating the stable and unstable regions in the $K - \varepsilon$ plane correspond to periodic solutions of period 2π and 4π. The boundaries may be determined analytically up to any order in ε using the perturbation method (Nayfeh, 1973). In fact, a change of variable from η to 2η converts the homogeneous parts of Equations (8) to the standard form of the Mathieu equation whose stability properties are well tabulated. For $\varepsilon \ll 3K$, which is the case under study, the parametric instability of a gravitationally stable ($K > 0$) satellite is found to occur in the neighbourhood of the inertia parameter values

$$K = 1/12,\ 1/3,\ 3/4 \qquad \text{for instability in } \psi \tag{9a}$$

$$K = 5/12,\ 1 \qquad \text{for instability in } \phi. \tag{9b}$$

The severity of the instability decreases as one proceeds from the lower to the higher K values. Under these conditions an amplitude build-up would occur even with small magnitudes of the disturbing solar torque. The avoidance of satellite inertia properties leading to an inertia parameter near the critical ones is thus desired for a safe design.

The nonresonant forced motions in pitch and roll, to a first approximation, are given by

$$\psi = \{\varepsilon/(3K - 1)\} \sin \eta \tag{10a}$$

$$\phi = -\varepsilon \sin \sigma \sin i/(1 + 3K)(1 - \sin^2 \sigma \sin^2 i)^{1/2}. \tag{10b}$$

In such a situation, of course, the attitude deviations may be reduced to any level by making the solar parameter ε sufficiently small through a reduction in the offset l. Again the occurrence of large amplitude motion due to external resonance in pitch for $K \simeq 1/3$ is observed.

In conclusion, the possibility of parametric excitation of a gravity-stabilized satellite by solar radiation pressure has been demonstrated. For a simple configuration involving an inertially axisymmetric spherical shaped satellite, the critical inertia parameter values have been identified. The more general stability analysis for a triaxial satellite in presence of solar torques is in progress. The results are expected to be useful during the configuration selection of gravity-oriented satellites.

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