Mei Hime Source Paper Notes

Study of cosmic ray muon flux and induced activity as a function of overburden - particular attention to muon-induced fast neutron activity for underground sites. developing a depth-sensitivity relation (DSR) in terms of total muon and muon-induced neutron flux.

Quick Overview

Sec II - reviews experimentally collected muon flux data and provides depth in terms of total muon flux, with differentiation between mountain and flat overburden.
Sec III - These experimental muon fluxes are then parameterized and used in **FLUKA** simulations to generate the **production rate of fast neutrons**. Then they quantify the agreement between simulation and experiment, while explaining some discrepancy specifically with the LVD.
Sec IV - Muon-induced cosmogenic radioactivity is discussed in terms of depth and the average muon energy.
Sec V - Apply results to based germanium-based experiments in search of neutrinoless double-beta decay and WIMP DM and demonstrate the utility of DSR in projecting the sensitivity and depth requirements of such experiments.

Detailed Notes

Depth-intensity-relation (DIR):

$$I(h) \ = \ (I_1 e^{-h/\lambda_1)} \ + \ I_2 e^{-h/\lambda_2})$$

I(h) is the differential muon intensity corresponding to the slant depth, h. The authors used the **experimental data in FIG 1 to find the free parameters** $I_1, \lambda_1, I_2, \lambda_2$
In FIG 2 the authors show that these free parameters reproduce the experimental data well with an error of around 5%.

Underground Lab with Flat Overburden
Flat-earth approximation, the muon intensity (I) for a specific slant depth (h) and in the direction of zenith angle (theta):

$$I_{th}(h, \ \theta) \ = \ I(h) G(h, \ \theta)$$

where $G(h, \ \theta) \ = \ sec(\theta)$ and $h \ = \ h_0 sec(\theta)$ and $I(h)$ is the DIR expressed in the previous equation. If you substitute in I, h, and G into this equation, you get:

$$I_{th}(h, \ \theta) \ = (I_1 e^{-h_0 \ sec(\theta)/\lambda_1)} \ + \ I_2 e^{-h_0 \ sec(\theta)/\lambda_2}) sec(\theta)$$

You can then **integrate over the upper hemisphere** (0 to 180 deg) to get the total muon intensity for an underground site with flat overburden positioned at vertical depth h_0.

- I am not taking the time to solve this integral rn but would be worthwhile to practice for my own sake lol
  After integration you can get a **differential muon intensity function** as a function of vertical depth h_0:

$$I_\mu(h_0) \ = \ 67.97 \ \times 10^{-6} \ e^{-h_0/0.285} \ + \ 2.071 \ \times \ 10^{-6} e^{-h_0/0.698}$$

Underground Lab with Mountain Overburden
Mountain shape or elevation map is required to determine the total muon flux - you essentially can either integrate over the shape of the mountain to find the total intensity, then put on left side of equation above and solve for the equivalent 'flat overburden' height, or you can compute the average vertical depth by integrating over the depth profile of the mountain. Either way - we don't need to factor this in at the moment.

Table I - lists depths (vertical or vertical-equivalent depths) of each underground lab in study in terms of km.w.e

Stopping muons - also a source of background. μ^- capture on a nucleus produces neutrons and radioactive isotopes. Total stopping-muon rate has contributions from cosmic-ray muons coming to the end of their range. Some secondary muons generated through these interactions as well.
Ratio of **stopping muons to throughgoing muons:**

$$R(h) \sim \gamma_\mu \frac{\Delta E e^{h/\xi}}{(e^{h/\xi} - 1)\epsilon_\mu}$$

The ratio is **less than 0.5% and is hereafter neglected for the underground sites considered in study**. Variable and coefficient values can be found in paper.

Muon Energy Spectrum and Angular Distribution
In addition to total muon intensity arriving at a given underground site, knowledge of the differential energy and angular distributions is required in order to generate the muon-induced activity within a **particular experimental cavern** (or target rock slab..)
Energy spectrum (from Refs 14, 15):

$$\frac{dN}{dE_\mu} \ = \ Ae^{bh(\gamma_\mu - 1)} \ \cdot \ (E_\mu \ + \ \epsilon_\mu(1 \ - \ e^{-bh}))^{-\gamma_\mu}$$

A is a normalization constant with respect to the differential muon intensity at a given depth and E_μ is the muon energy after crossing the rock slant depth h (km.w.e). The parameters the authors used (FIG 6):

$b = 0.4/k.w.e$
$\gamma_\mu = 3.77$
$\epsilon_\mu = 693\ GeV$

- these are correct in the PrimaryGeneratorAction.cc

FIG 7 - local angular distribution of muons valid for an excess of 1.5 km.w.e of depth
Overall angular distribution of muons at the surface is proportional to $cos^2(\theta)$ with an average muon energy of ~4 GeV.
this is very important for when we need to simulate background induced in our rocks after a given time at the surface

Average Muon Energy at Certain Depth

$$\langle E_\mu \rangle = \frac{\epsilon_\mu(1 - e^{-bh})}{\gamma_\mu - 2}$$

Single muon average energy for Sudbury: 327 GeV (Lipari) or 356 GeV (Groom).
Uncertainties in parameters are due to uncertainties in muon energy spectrum of atmosphere, details of energy loss in media, and local rock density/composition (in other words, I don't think we will every be able to know exactly. uncertainty increases dramatically when we consider minerals that have changed depth over age...)

Muon-Induced Neutrons
Two classes of fast neutrons - those produced by muons traversing the detector itself, and neutrons created in the external rock by muons missing the veto detector (this correlates to neutrons produced in surrounding rock, and then neutrons produced by spallation in the target minerals itself). **This paper assumes that muons inducing neutrons in the target/detector is negligible as you can have sufficient shielding - thus they focus only on neutrons produced in surrounding rock**.
I wonder if this is a pretty decent assumption - I mean our target are so incredibly small that I feel like the likelihood of muons producing neutrons within the target and THEN interacting within the same target is pretty low. Like negligibly low... maybe worth a quick question to josh? but fine for now running geant4 under same params
Production of neutrons depends strongly on the depth and composition of underground site.
Generally - the neutron production rate at large depths due to muons is 2-3 orders of magnitude smaller than that of neutrons arising from local radioactivity through (alpha, neutron) reactions.
However, radiogenic neutrons are low energy and easily to shield - the cosmogenic neutrons at this depth - although smaller in flux - have a very hard energy spectrum extending into several GeVs and can penetrate to significant depth in both surrounding rock and detector materials. (larger area to consider around target..)

TABLE III - rock composition for various sites. Sudbury: used in geant4, density ~2.894 g/cm3 with Z/A ~ 0.491.
Alex used norite in his simulations, this is a good approximation for surrounding rock of olivine gabbro, but to make this more reflective of our actual samples from SNOLAB, then we need to change this to match probably the stratigraphy of Kidd Mine or Creighton Mine (mostly granite and ore) and change the target to quartz....

Simualtion settings
rock thickness: 20 m x 20 m x 20 m
experimental cavern placed inside simulation - we don't want to do this so neglect
experimental boundary views 7 m on each side of rock
this ensures equilibrium between neutron and muon fluxes and the ratio of neutron to muon fluxes is constant
I honestly don't think the amount of boundary matters but we can check? idk maybe keep the extra boundary that Alex set

TABLE IV - mean neutron production rates from seven measurements using liquid scintillator covering a significant range in depth and mean muon energy. provides a global fit function in FIG 8 and compares to Monte Carlo calculations performed in a couple of refs and FLUKA simulation results
Overall good agreement - and they describe that the LVD (uncorrected) discrepancy is likely due to quenching of proton-recoil energy in the liquid scintillator

Data of neutron yield is well described by simple power law - but the FLUKA simulations do **underestimate the data by about 35%**.
Authors suggest this is due to neutron multiplicity in muon-induced nuclear cascades and electromagnetic cascades
Experimental results show higher multiplicity than FLUKA
Corrected the simulated data by using a correction function to the neutron production rate (so post-processing after muon simulation?)

Correction fuction:

$$\frac{M_d \; - \; M_{mc}}{M_d} \; = \; -0.64 E_\mu^{0.02} \; - \; 0.74 E_\mu^{-0.12}$$

M_d is measured neutron multiplicity, M_{mc} is the simulated neutron multiplicity in FLUKA and E_μ is this muon energy in GeV. After correcting with this equation, the simulated data matched pretty well.
Additional correction could be made by including deep inelastic scattering of muons and nucleons.
Still FLUKA is ~15% error on actual experimental data***

Media Dependence on Neutron Production Rates

The muon-induced production rate for neutrons depends critically on knowledge of chemical composition and density of medium in which muons propagate through. Authors say they have simulated this dependence on Z^2/A in FLUKA.

This will become important if surrounding rock is high density, or contains heavy elements - since neutron multiplicity from electromagnetic showers is more prominent since electromagnetic muon interaction cross section increases with atomic weight.

Neutron Flux

$$\Phi_n \ = \ P_0 \frac{P_1}{h_0} e^{-h_0/P_1}$$

where h_0 is the equivalent vertical depth in km.w.e relative to flat overburden.

$P_0 \ = \ (4.0 \pm 1.1) \ \times \ 10^{-7} \ cm^{-2} s^{-1}$

$P_1 \ = \ 0.86 \pm 0.0.5$ km.w.e

-- no notes on neutron production from shielding materials - unnecessary for this analysis but summary is that different shielding materials (copper, germanium, etc) have certain neutron production from muons passing through. Lead produces more, Polyethylene is least. (density dependent I assume)

Neutron Energy Spectrum

It is well known that energy spectrum of neutrons produced by muon spallation is uncertain and data are scarce.

Authors used muon energy spectrum as input into FLUKA simulations

$$\frac{dN}{dE_n} \ = \ A_\mu (\frac{e^{-a_0 E_n}}{E_n} \ + B_\mu(E_\mu) e^{-a_1 E_n}) \ + \ a_2 E_n^{-a_3}$$

where A_μ is a normalization constant, and $a_0 \rightarrow \ a_3$ are fitted parameters, E_n is the neutron energy, $B_\mu E_\mu$ is a function of muon energy, and E_μ is in GeV (FIND IN TABLE VII)

$$B_\mu(E_\mu) \ = \ 0.324 \ - \ 0.641 e^{-0.014 E_\mu}$$

valid for $E_n > 10 \ MeV$

Neutron Angular Distribution

FIG 18 - angular distribution of neutrons from simulation

good to reproduce this plot with our work and compare!

Parameterized angular distribution:

$$\frac{dN}{d \ cos(\theta)} \ = \ \frac{A_\theta}{(1 \ - \ cos(\theta))^{B_\theta(E_\mu)} \ + \ C_\theta(E_\mu)}$$

where A_θ is a constant, and $B_\theta(E_\mu)$ and $C_\theta(E_\mu)$ are weakly correlated to muon energy and E_μ is in GeV.
Corresponding functions:

$$B_\theta(E_\mu) \;=\; 0.482E_\mu^{0.045}$$

$$C_\theta(E_\mu) \;=\; 0.832E_\mu^{-0.152}$$

Neutron Multiplicity
The number of neutrons produced per muon interaction is the least known quantity in the production of neutrons induced by muons. Average multiplicity in FLUKA is smaller than that of the measurements. In this section - authors list the simulated multiplicity parameterization - but reference back up to correction function earlier.

Neutron Lateral Distribution
Neutron flux is attenuated by about 2 orders of magnitude at distances larger than 3.5 m from the muon track, however, as much as 10% remain as large as 2-2.5 m.
Not necessarily a think we would need to know... we will not have rock over 20 m3 in volume lmao

Muon-Induced Cosmogenic Activity
This section details the production of cosmogenic nuclides from muons at a certain depth. Neutrons can also interact with nuclei to produce long-lived radioactive isotopes and secondary neutrons. Long-lived radioactive isotopes are the most dominant product of muon-induced cosmogenic radioactivity near Earth's surface
I think this modeling will become very important for shallower rocks. Maybe keep this in mind - it would essentially account for a higher than predicted radiogenic background in our natural samples. Not something we would simulate...

Everything after this point talks about specific detectors, experiments, or future work. The above is what is relevant for our project.