MTD2 Research Aircraft Flight Dynamic Model

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Abstract

This document outlines the constant parameters and aerodynamic model of the MTD2 research aircraft used by the Nonlinear System Laboratory (NSL) at Virginia Tech. The airframe, sensors, and flight computer are purchased separately but are assembled and instrumented in-house by NSL members. Additionally, NSL members conducted ground tests to determine the aircraft's geometric and inertial parameters, as well as flight test to determine performance and aerodynamics parameters.

Nomenclature

α, β	=	Angle of attack and sideslip angle [rad]
b	=	Wing span [m]
$\bar{c}$	=	Mean aerodynamic chord [m]
D	=	Aircraft propeller diameter [m]
δ_{a}, δ_{e}, δ_{r}	=	Control surface deflection (aileron, elevator, roll) [rad]
δ_{rps}	=	Propeller rotation speed [rad/s]
η_{n}	=	Number of propellers
η_{e}	=	Propeller efficiency
I_{xx}, I_{yy}, I_{zz}, I_{xz}	=	Aircraft moment of inertia [$\mathrm{kg} \cdot \mathrm{m}^2$]
m	=	Mass of aircraft [kg]
p, q, r	=	Body-fixed angular rates [rad/s]
S	=	Wing reference area [m^2]
σ	=	Standard deviation
θ	=	Parameter
V_{r}	=	True airspeed [m/s]

1 Research Aircraft

The research aircraft used, the My Twin Dream (MTD), is a radio-controlled (RC) twin-engine, electrically powered, foam-structured, fixed-wing aircraft manufactured by My Fly Dream; see Figure 1. The aircraft is powered by a four-cell 14.8 V, 6750 mAh Lithium-polymer battery with a counter-rotating twin propeller configuration. The MTD was chosen because of its simple construction, benign flight dynamics, endurance greater than 20 minutes, and ample payload capacity for the desired instrumentation package. Mass and geometric properties are shown in Table 1.

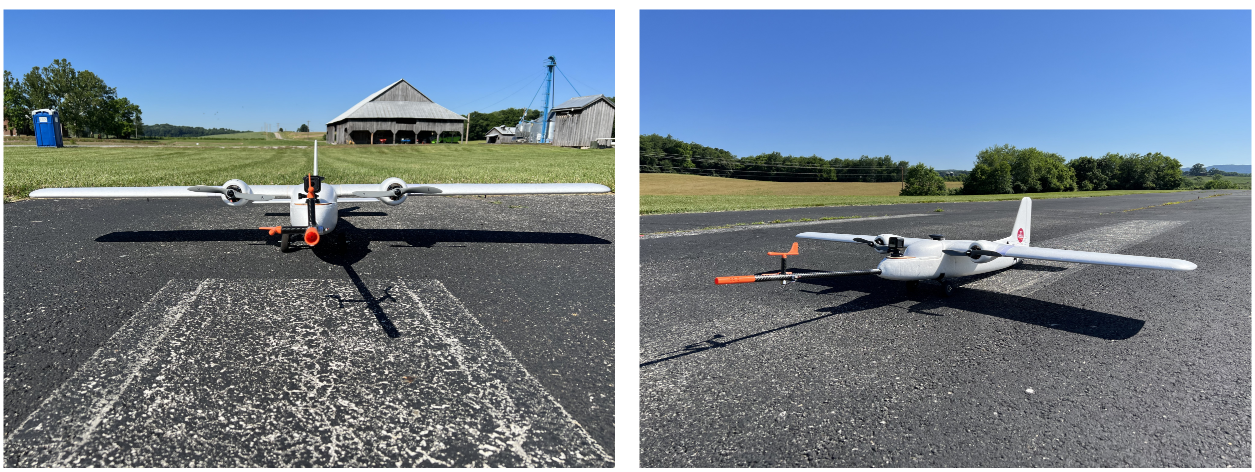
2 Flight Computer

The MTD aircraft is instrumented with a Pixhawk Cubepilot Cube Orange flight computer running PX4 firmware for data collection and its inertial navigation solution (position, attitude, velocity, and angular veloc-

Table 1: My Twin Dream (MTD) aircraft properties

Property	**Symbol**	**Value**	**Units**
Mass	m	3.311	kg
Mean aerodynamic chord	$\bar{c}$	0.254	m
Projected wing span	b	1.80	m
Wing reference area	S	0.457	m^2
Number of propellers	η_{n}	2	—
Propeller efficiency	η_{e}	0.9	—
Propeller Diameter	D	0.254	m
Roll moment of inertia	I_{xx}	0.319	kg-m^2
Pitch moment of inertia	I_{yy}	0.267	kg-m^2
Yaw moment of inertia	I_{zz}	0.471	kg-m^2
Product of inertia	I_{xz}	0.024	kg-m^2

ity) [1]. The flight computer sensor suite includes three accelerometers and gyroscopes, two magnetometers, a barometer, and a global navigation satellite system (GNSS) receiver capable of real-time kinematic (RTK) positioning. The pulse-width modulated (PWM) signals sent from the Pixhawk to the control surface servos are also recorded. The control surface deflection angle is calculated post-flight from a servo actuator model developed in [2]. The MTD also carries a vaned ADU for measuring air-relative velocity. The ADU, visible in Figure 1, gives the aircraft's true airspeed (V_{r}), angle of attack (α), and flank angle (β_{f}).



(a) Front view of MTD2.

(b) Left view of MTD2.

Figure 1: My twin dream (MTD) 2 aircraft

2.1 Aircraft Aerodyanmic Model

The aerodynamic longitudinal and lateral-directional force and moment coefficient equations acting on the aircraft can be expressed as:

$$C_X = C_{X_o} + C_{X_{\delta_e}}\delta_e + C_{X_\alpha}\alpha \tag{1a}$$

$$C_Z = C_{Z_o} + C_{Z_q}\hat{q} + C_{Z_\alpha}\alpha + C_{Z_{\dot\alpha}}\hat{\dot\alpha} \tag{1b}$$

$$C_m = C_{m_o} + C_{m_{\delta_e}}\delta_e + C_{m_q}\hat{q} + C_{m_\alpha}\alpha + C_{m_{\dot\alpha}}\hat{\dot\alpha} \tag{1c}$$

$$C_Y = C_{Y_p}\hat{p} + C_{Y_r}\hat{r} + C_{Y_{\delta_a}}\delta_a + C_{Y_{\delta_r}}\delta_r + C_{Y_\beta}\beta + C_{Y_{\dot\beta}}\hat{\dot\beta} \tag{1d}$$

$$C_l = C_{l_p}\hat{p} + C_{l_{\delta_a}}\delta_a + C_{l_\beta}\beta + C_{l_{\dot\beta}}\hat{\dot\beta} \tag{1e}$$

$$C_n = C_{n_r}\hat{r} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r + C_{n_\beta}\beta \tag{1f}$$

where the nondimensional terms are

$$\hat{p} = \frac{pb}{2V_t} \quad \hat{q} = \frac{q\bar{c}}{2V_t} \quad \hat{r} = \frac{rb}{2V_t} \quad \hat{\dot\alpha} = \frac{\dot\alpha\bar{c}}{2V_t} \quad \hat{\dot\beta} = \frac{\dot\beta b}{2V_t}$$

The parameters in Equation 1 are shown in Table 2 and Table 3. In these tables, $\hat\theta$ represents an estimate for the value of a given parameter value and σ represents the standard deviation in that estimate. Parameter estimation was accomplished with the output error method [3]. Parameter uncertainty estimates in Table 2 and Table 3 account for colored residuals [3]. The quasi-steady model is valid in the normal operating range, just prior to stall; it does not capture propulsion effects, as discussed in [4]. The model validation

Table 2: Longitudinal aerodynamic parameter

Term	$\hat\theta \pm \sigma$	Term	$\hat\theta \pm \sigma$	Term	$\hat\theta \pm \sigma$
C_{X_0}	+0.009 ± 0.026	C_{Z_0}	-0.255 ± 0.074	C_{m_0}	+0.008 ± 0.006
C_{X_α}	+0.282 ± 0.000	C_{Z_α}	-4.436 ± 0.015	C_{m_α}	-0.444 ± 0.027
$C_{X_{\delta e}}$	+0.051 ± 0.025	C_{Z_q}	-12.540 ± 0.030	C_{m_q}	-14.019 ± 3.363
		$C_{Z_{\dot\alpha}}$	+4.614 ± 0.034	$C_{m_{\delta e}}$	-0.415 ± 0.058
				$C_{m_{\dot\alpha}}$	+0.514 ± 0.036

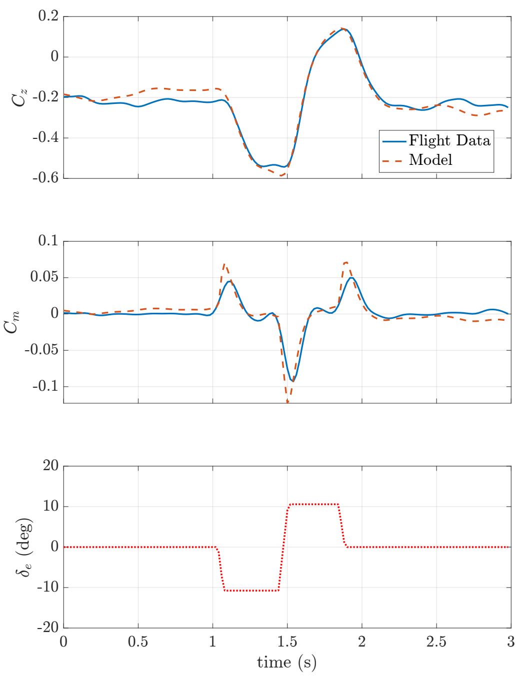
Table 3: Lateral-directional aerodynamic parameter

Term	$\hat\theta \pm \sigma$	Term	$\hat\theta \pm \sigma$	Term	$\hat\theta \pm \sigma$
C_{Y_β}	-0.410 ± 0.115	C_{l_β}	-0.035 ± 0.004	C_{n_β}	+0.083 ± 0.020
C_{Y_p}	+0.221 ± 0.133	C_{l_p}	-0.386 ± 0.041	C_{n_r}	-0.119 ± 0.029
C_{Y_r}	+0.230 ± 0.547	$C_{l_{\delta_a}}$	-0.137 ± 0.010	$C_{n_{\delta_a}}$	+0.013 ± 0.009
$C_{Y_{\delta_a}}$	+0.118 ± 0.011	$C_{l_{\dot\beta}}$	-0.079 ± 0.042	$C_{n_{\delta_r}}$	-0.068 ± 0.006
$C_{Y_{\delta_r}}$	+0.136 ± 0.030				
$C_{Y_{\dot\beta}}$	+0.041 ± 0.533				

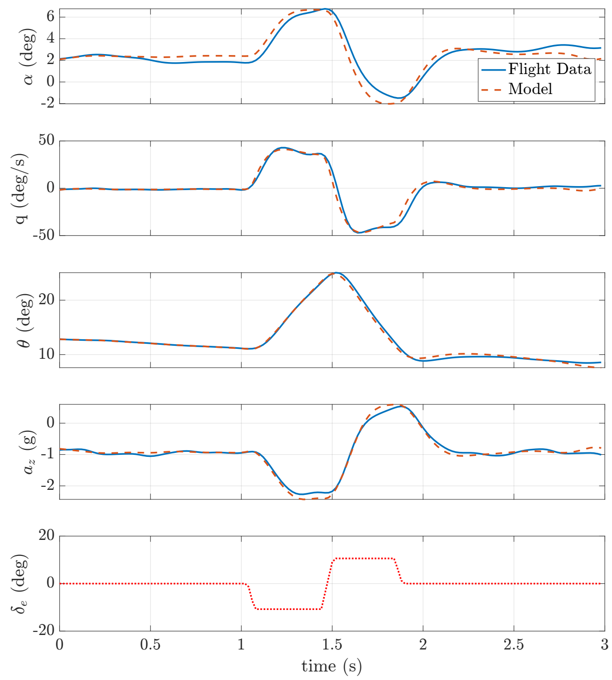
Table 4: Propeller model parameter

Term	$\hat\theta \pm \sigma$
C_{J_0}	$+0.116 \pm 2.212 \times 10^{-4}$
C_J	$-0.040 \pm 1.254 \times 10^{-3}$
C_{J^2}	$-0.131 \pm 1.488 \times 10^{-3}$

flight data were collected with an automated elevator doublet to characterize pitching motion and a rudder doublet followed by aileron 1-2-1 maneuver to characterize lateral-directional motion. A comparison of the quasi-steady model predictions to the validation flight data is shown in Figure 2 and Figure 3. The model predictions show good matching to the validation flight data.

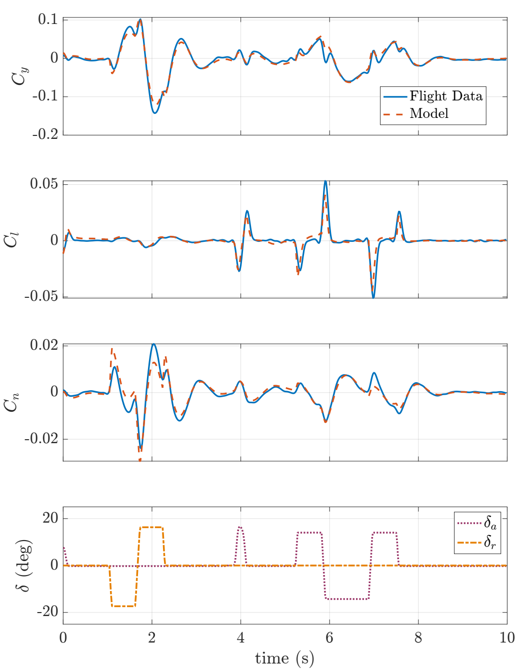


(a) Force and moment coefficient prediction

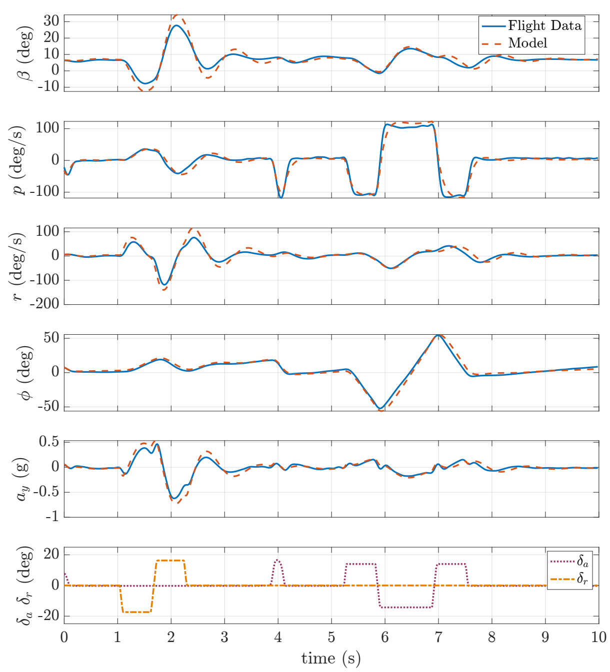


(b) Output prediction

Figure 2: Validation results for the quasi-steady model for an elevator doublet maneuver in calm air.



(a) Force and moment coefficient prediction



(b) Output prediction

Figure 3: Validation results for the quasi-steady model for a rudder doublet, then aileron 1-2-1 maneuver.

3 Propulsion Model

The propulsion model was developed using a least-squares polynomial fit in advance ratio from UIUC wind tunnel data[1]. The data for the APC 10x6E propellers is available at `https://www.apcprop.com/files/PER3_10x6E.dat`[2]). The propeller model is given below.

$$C_J = C_{J_0} + C_J J + C_{J^2} J^2 \tag{2}$$

where $J = \frac{V_{\mathrm{r}}}{\delta_{\mathrm{rps}} D}$ is the propeller advanced ratio, and the parameter values are given in Table 4.

References

[1] P. Autopilot, "Using the ECL EKF | PX4 User Guide," 2023.

[2] J. L. Gresham, J.-M. W. Fahmi, B. M. Simmons, J. W. Hopwood, W. Foster, and C. A. Woolsey, "Flight Test Approach for Modeling and Control Law Validation for Unmanned Aircraft," in *AIAA SCITECH 2022 Forum*, AIAA SciTech Forum, p. 2406, American Institute of Aeronautics and Astronautics, Jan. 2022.

[3] E. A. Morelli and V. Klein, *Aircraft System Identification: Theory and Practice*. Sunflyte Enterprises Williamsburg, VA, 2016.

[4] B. M. Simmons, J. L. Gresham, and C. A. Woolsey, "Aero-propulsive modeling for propeller aircraft using flight data," *Journal of Aircraft*, vol. 60, pp. 81–96, Jan. 2023. Publisher: American Institute of Aeronautics and Astronautics.

[1] See `https://m-selig.ae.illinois.edu/props/propDB.html`
[2] Accessed Feb. 21, 2024