

# Heterogeneous Graphs and Knowledge Graph Embeddings

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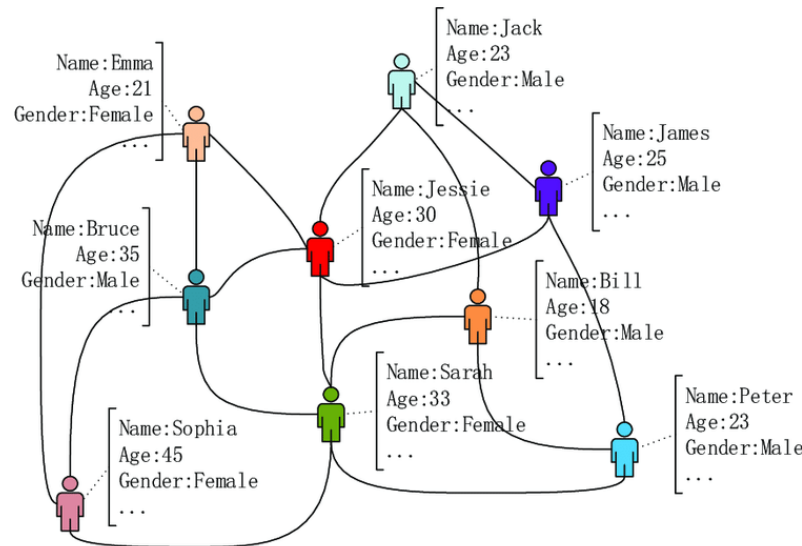
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# Contents



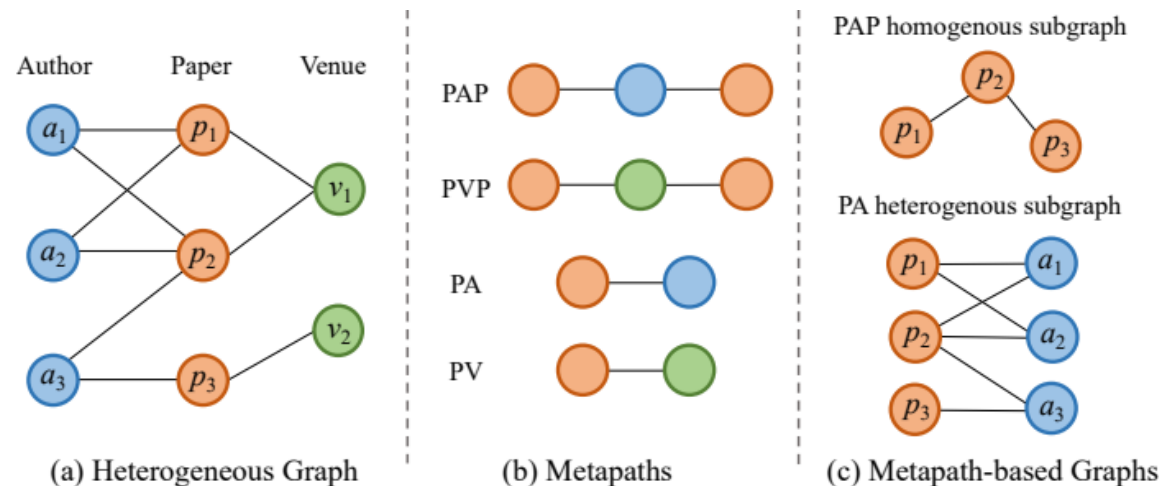
- From Homogeneous Graphs to Heterogeneous Graphs
- Knowledge Graphs
- Knowledge Graph Representation Learning
  - Translation-based Embedding Approaches
  - Graph Neural Network-based Approaches

- Many real-world systems, such as molecular graphs, social networks, and knowledge graphs, involve multiple entities and relations.
- For example: In Social Networks
  - Entities (Nodes): Users, posts, or hashtags.
  - Relations (Edges): Friendships, likes, shares, or mentions.
  - Reason for Heterogeneity: Interactions vary in type and significance, and users/posts exhibit diverse roles in the network.



## ➤ Objective:

- So far we only handle graphs with one edge type.
- Question: How to handle (directed) graphs with multiple edge types (heterogeneous graphs)?
- For example:
  - a) heterogeneous graph with three types of nodes (i.e., authors, papers, venues).
  - (b) Four metapaths: PAP, PVP, PA, and PV.
  - (c) The metapath-based homogeneous graph and heterogeneous graph

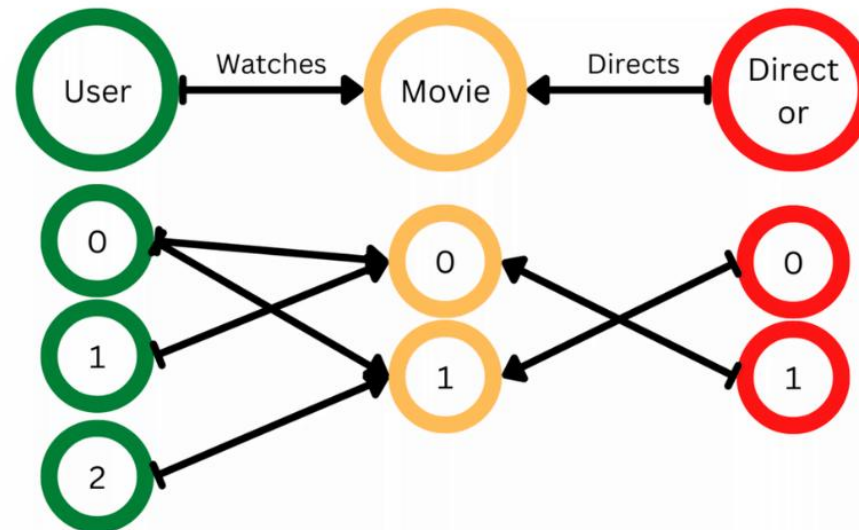




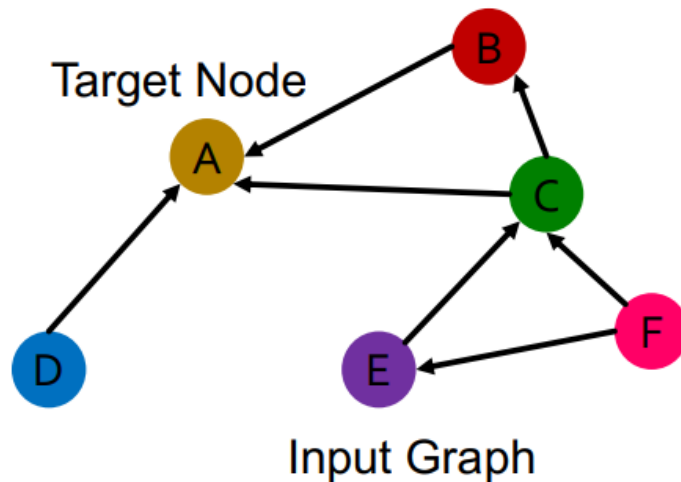
- A heterogeneous graph is defined as:

$$G = (V, E, R, T)$$

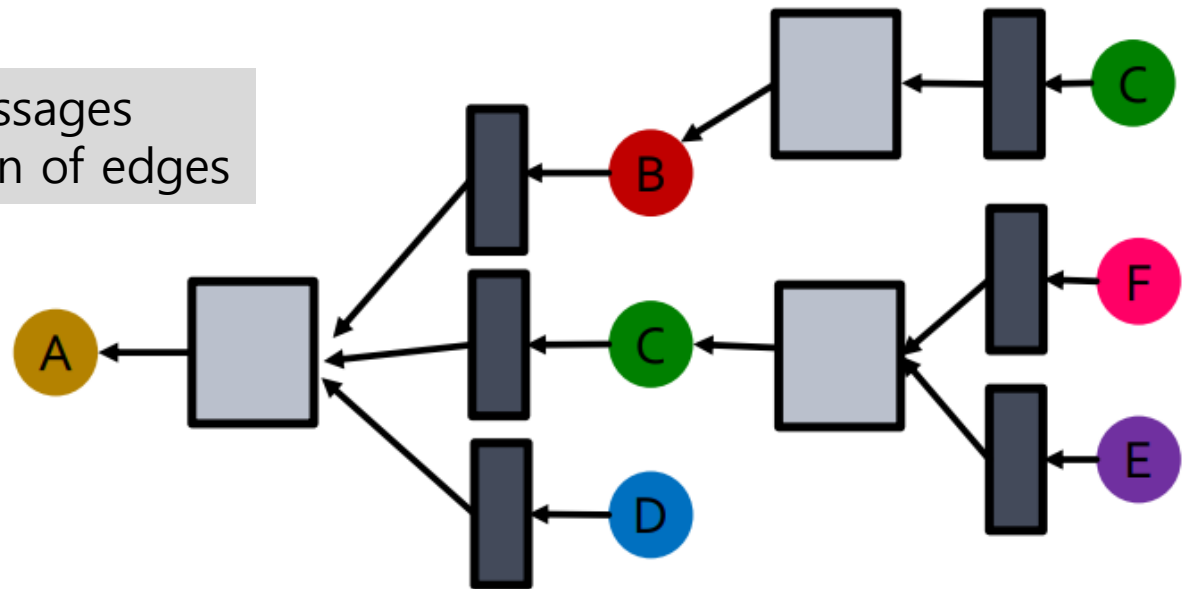
- Nodes with node types  $v_i \in V$
- Edges with relation types  $(v_i, r, v_j) \in E$
- Node type  $T(v_i)$
- Relation type  $r \in R$



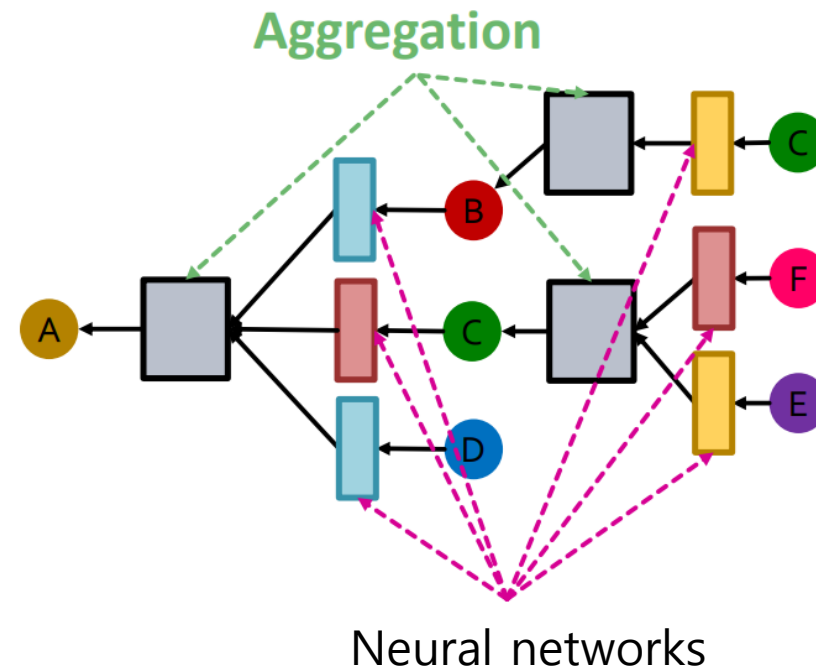
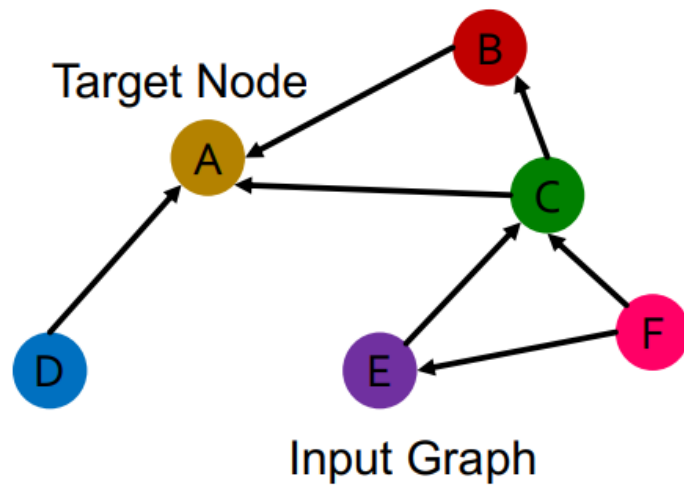
- We will extend GCN to handle heterogeneous graphs with multiple edge/relation types
- We start with a directed graph with one relation
  - How do we run GCN and update the representation of the target node A on this graph?



Only pass messages  
along direction of edges



- What if the graph has multiple relation types?
- Use different neural network weights for different relation types.



- Relational GCN (RGCN):

$$\mathbf{h}_v^{(l+1)} = \sigma \left( \sum_{r \in R} \sum_{u \in N_v^r} \frac{1}{c_{v,r}} \mathbf{W}_r^{(l)} \mathbf{h}_u^{(l)} + \mathbf{W}_0^{(l)} \mathbf{h}_v^{(l)} \right)$$

- How to write this as Message + Aggregation?

- **Message:** Each neighbor of a given relation & Self-loop::

$$\mathbf{m}_{u,r}^{(l)} = \frac{1}{c_{v,r}} \mathbf{W}_r^{(l)} \mathbf{h}_u^{(l)} \qquad \mathbf{m}_v^{(l)} = \mathbf{W}_0^{(l)} \mathbf{h}_v^{(l)}$$

- **Aggregation:** Sum over messages from neighbors and self-loop, then apply activation

$$\mathbf{h}_v^{(l+1)} = \sigma \left( \text{Sum} \left( \left\{ \mathbf{m}_{u,r}^{(l)}, u \in N(v) \right\} \cup \left\{ \mathbf{m}_v^{(l)} \right\} \right) \right)$$



- How to define Message + Aggregation?

$$\mathbf{h}_v^{(l+1)} = \sigma \left( \sum_{r \in R} \sum_{u \in N_v^r} \frac{1}{c_{v,r}} \mathbf{w}_r^{(l)} \mathbf{h}_u^{(l)} + \mathbf{w}_0^{(l)} \mathbf{h}_v^{(l)} \right)$$

- Aggregation:

$$\mathbf{h}_v^{(l+1)} = \sigma \left( \text{Sum} \left( \left\{ \mathbf{m}_{u,r}^{(l)}, u \in N(v) \right\} \cup \left\{ \mathbf{m}_v^{(l)} \right\} \right) \right)$$

Relational GCN

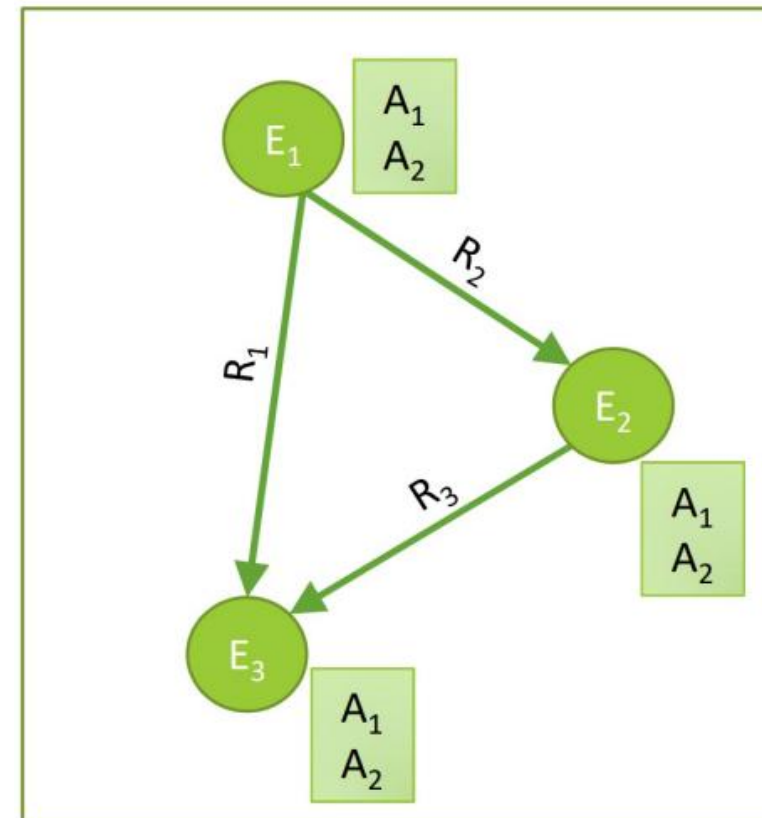
$$\mathbf{h}_v^{(l)} = \sigma \left( \mathbf{w}^{(l)} \sum_{u \in N(v)} \frac{\mathbf{h}_u^{(l-1)}}{|N(v)|} \right)$$

$$\mathbf{h}_v^{(l)} = \sigma \left( \underbrace{\sum_{u \in N(v)} \mathbf{w}^{(l)} \frac{\mathbf{h}_u^{(l-1)}}{|N(v)|}}_{\text{Aggregation}} \right)$$

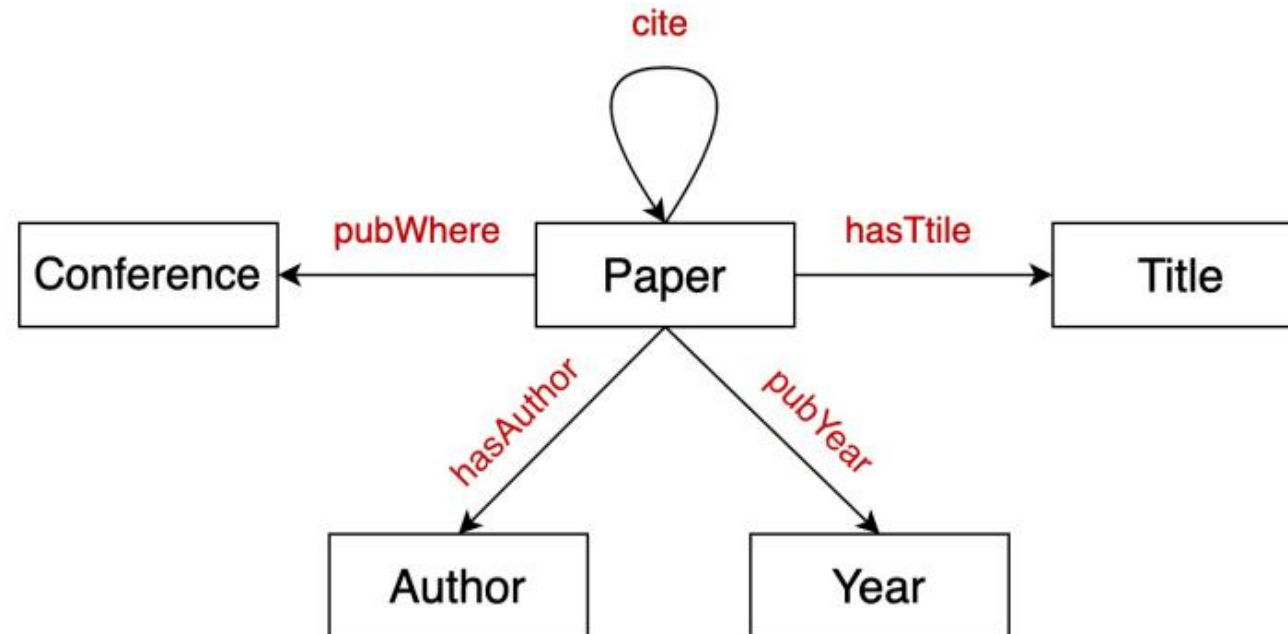
Message

GCN

- Knowledge in graph form:
  - Capture entities, types, and relationships
  - Nodes are entities
  - Nodes are labeled with their types
  - Edges between two nodes capture relationships between entities
- A Knowledge Graph (KG) is indeed a prime example of a heterogeneous graph.

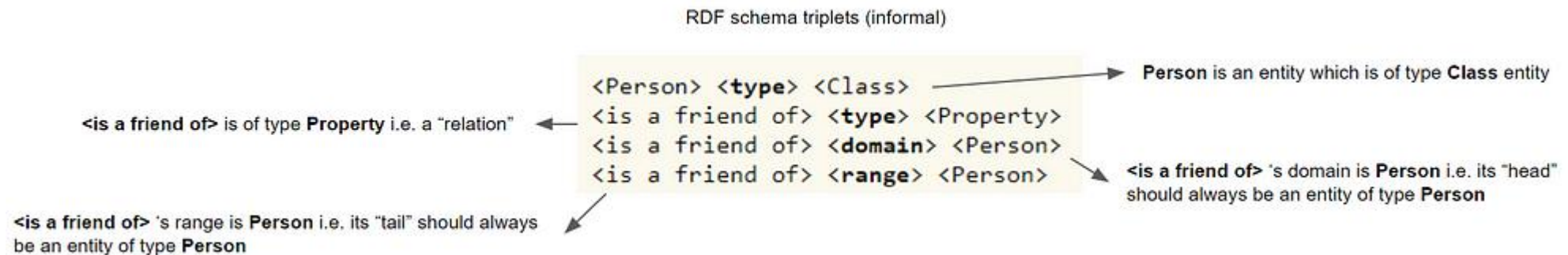


- An example of Bibliographic Networks
  - Node types: paper, title, author, conference, year
  - Relation types: pubWhere, pubYear, hasTitle, hasAuthor, cite



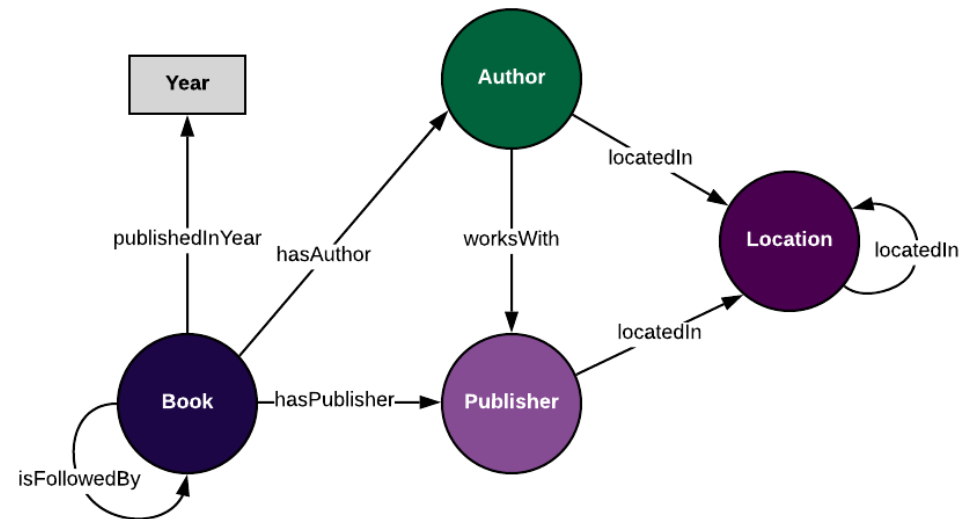
## ➤ Knowledge graph Ontology

- An ontology is a model of the world (practically only a subset), listing the types of entities, the relationships that connect them, and constraints on the ways that entities and relationships can be combined.
- Resource Description Framework (RDF) and Web Ontology Language (OWL) are some of the vocabulary frameworks used to model ontology.



## Why we need Ontologies?

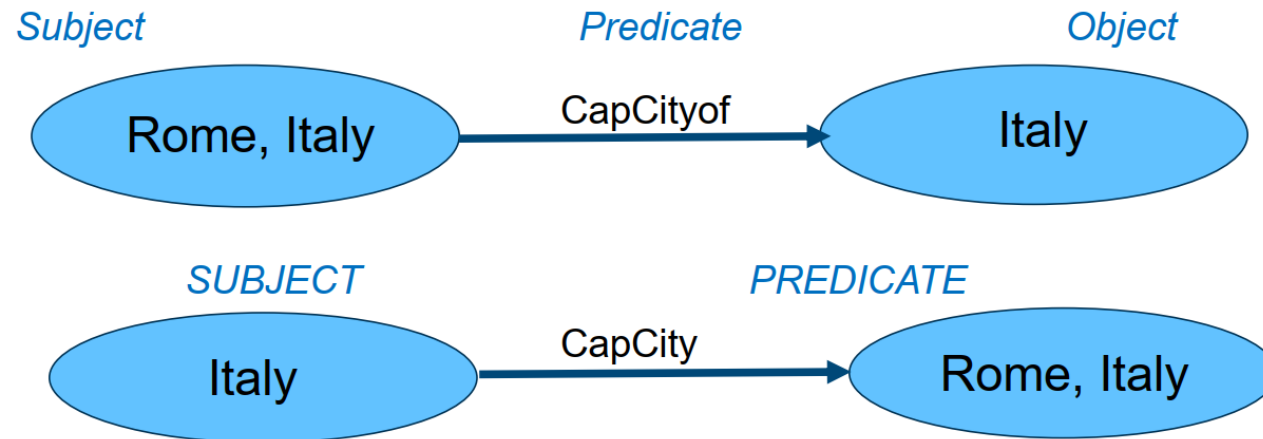
- To share common understanding of the structure of information among people or objects
- To enable reuse of domain knowledge
- To make domain assumptions explicit
- To separate domain knowledge from the operational knowledge
- To analyze domain knowledge



## Knowledge Graphs and Ontologies are based on RDF

- RDF, a standard model for data interchange on the Web, uses URIs to name things and the relationship between things, which are referred to as triples:

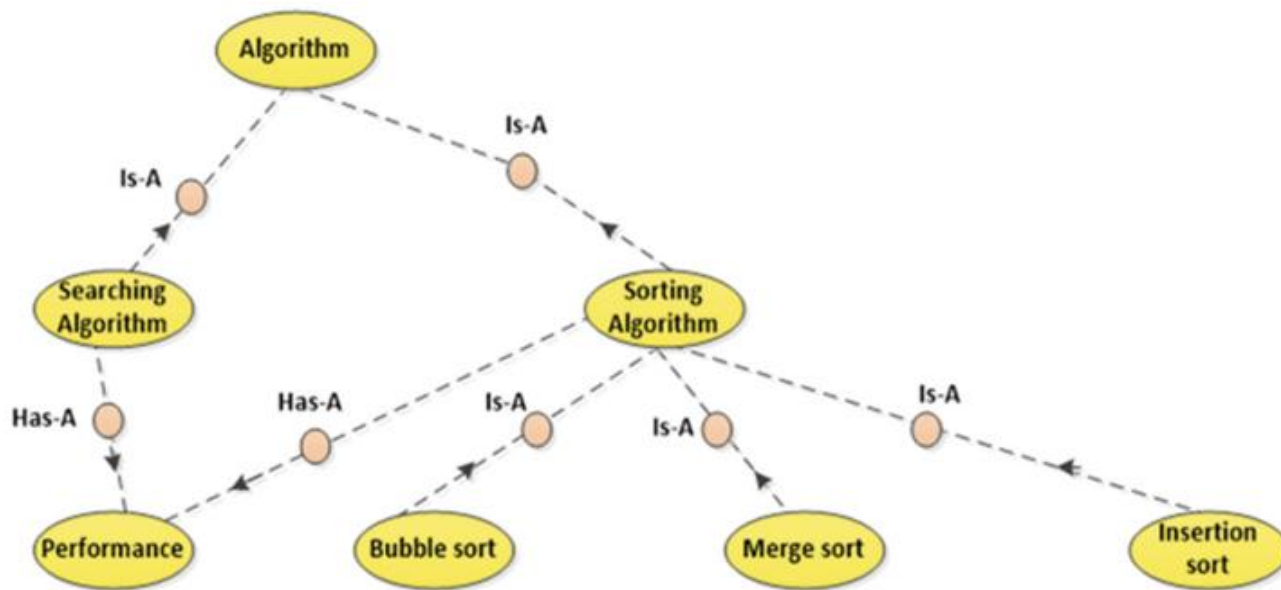
(1) Subject – (2) Predicate – (3) Object





## ➤ Ontology as Foundation Layer for KG

- Ontology: extract taxonomic relations and attributes, plus some semantic relations.
- Knowledge Graph focuses on extracting relationships in all forms with the same priority.



(Domain, Data structure)  
 (Class, Algorithm)  
 (SubClass, Sorting algorithm, Algorithm)  
 (SubClass, Searching algorithm, Algorithm)  
 (Has/Property, Performance, Sorting algorithm)  
 (Has/Property, Performance, Searching algorithm)  
 (SubClass, Bubble sort, Sorting algorithm)  
 (SubClass, Merge sort, Sorting algorithm)  
 (SubClass, Insertion sort, Sorting algorithm)

// domain  
 //  $c_1$   
 // ( $r_1$ ,  $c_2$ ,  $c_1$ )  
 // ( $r_1$ ,  $c_3$ ,  $c_1$ )  
 // ( $r_2$ ,  $c_4$ ,  $c_2$ )  
 // ( $r_2$ ,  $c_5$ ,  $c_3$ )  
 // ( $r_1$ ,  $c_6$ ,  $c_2$ )  
 // ( $r_1$ ,  $c_7$ ,  $c_2$ )  
 // ( $r_1$ ,  $c_8$ ,  $c_2$ )

$c_i$ : concept  $r_i$ : relation

- **An example: From Ontologies to Knowledge Graphs**
  - We have three objects: books, authors, and publishers:

## Books

Title	Author	Publisher	Year Published	Followed By
To Kill a Mockingbird	Harper Lee	J. B. Lippincott Company	1960	Go Set a Watchman
Go Set a Watchman	Harper Lee	HarperCollins, LLC; Heinemann	2015	
The Picture of Dorian Gray	Oscar Wilde	J. B. Lippincott & Co.	1890	
2001: A Space Odyssey	Arthur C. Clarke	New American Library, Hutchinson	1968	

## Publishers

Name	City	Country
J. B. Lippincott & Company	Philadelphia	United States
HarperCollins, LLC	New York City	United States
Heinemann	Portsmouth	United States
New American Library	New York City	United States
Hutchinson	London	United Kingdom

## Authors

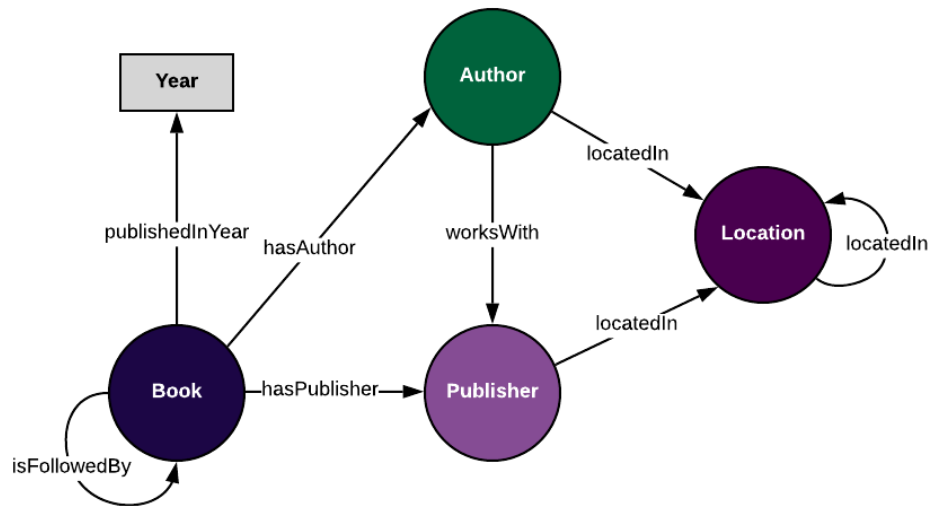
Name	Country of Birth
Harper Lee	United States
Oscar Wilde	Ireland
Arthur C. Clarke	United Kingdom

## We define the properties:

- Book → has author → Author
- Book → has publisher → Publisher
- Book → published on → Publication date
- Book → is followed by → Book
- Author → works with → Publisher
- Publisher → located in → Location
- Location → located in → Location

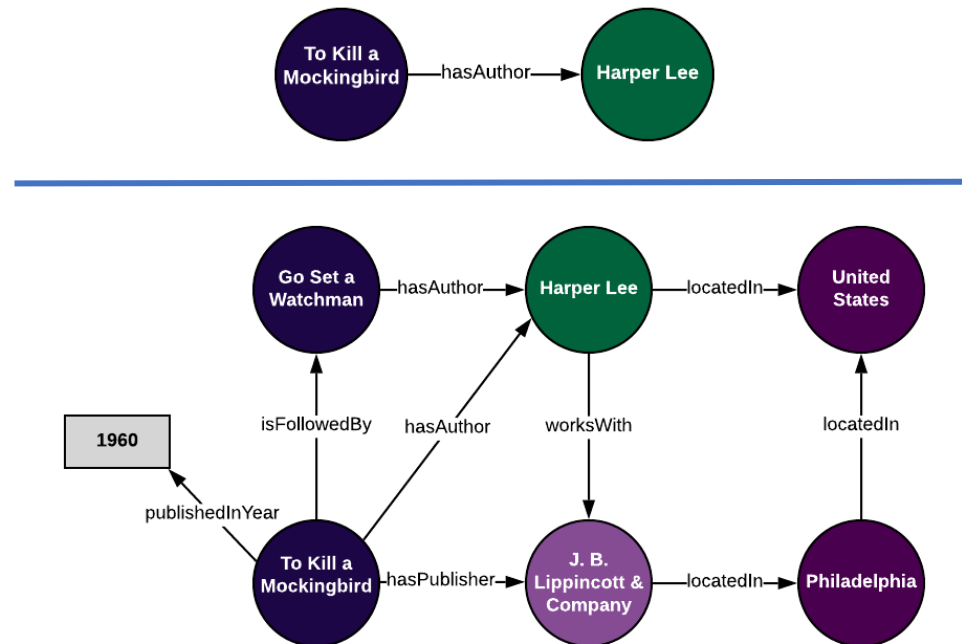
## ➤ From Ontologies to Knowledge Graphs: An example

### Ontology



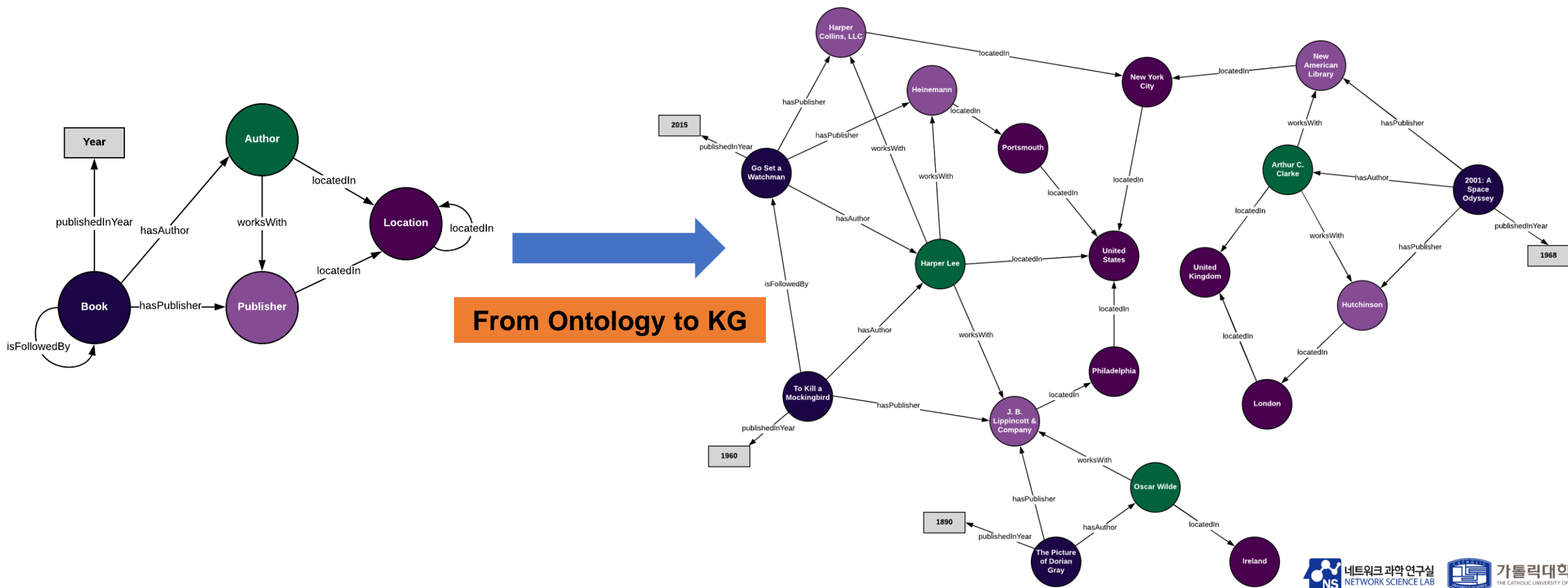
### Knowledge graph

Using ontology as a framework, we can add in real data about individual books, authors, publishers, and locations to create a **knowledge graph**

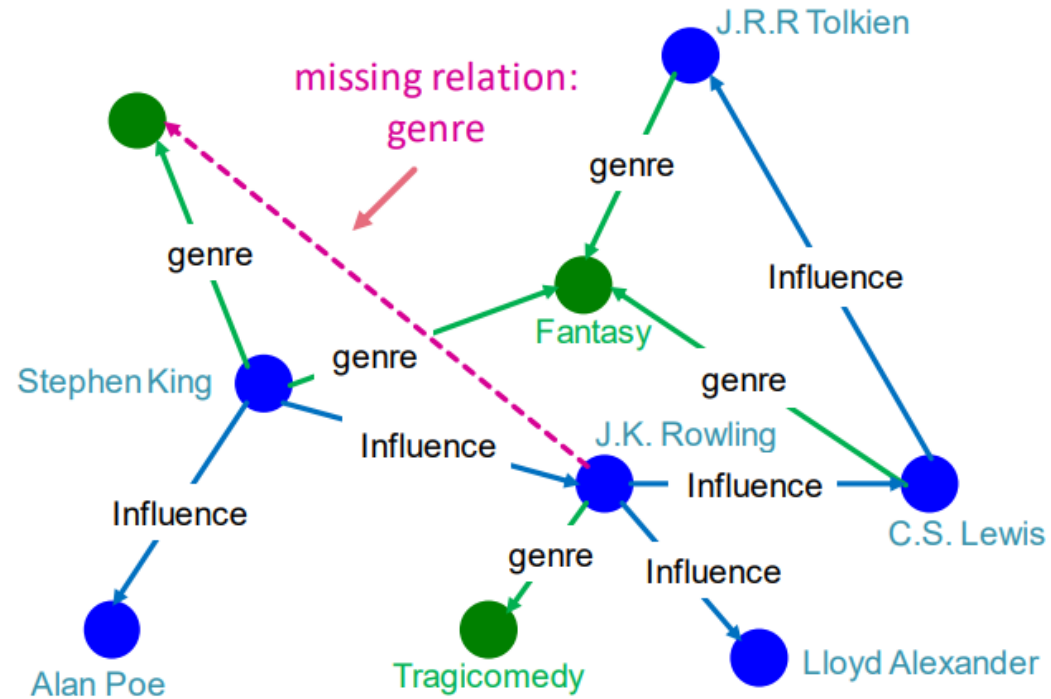


## ➤ Full knowledge graph representation: An example

Adding in real data about individual books, authors, publishers, and locations to create a complete KG.



- **Knowledge Graph completion task**
  - Given an enormous KG, can we complete the KG?
  - For a given (head, relation), we predict missing tails



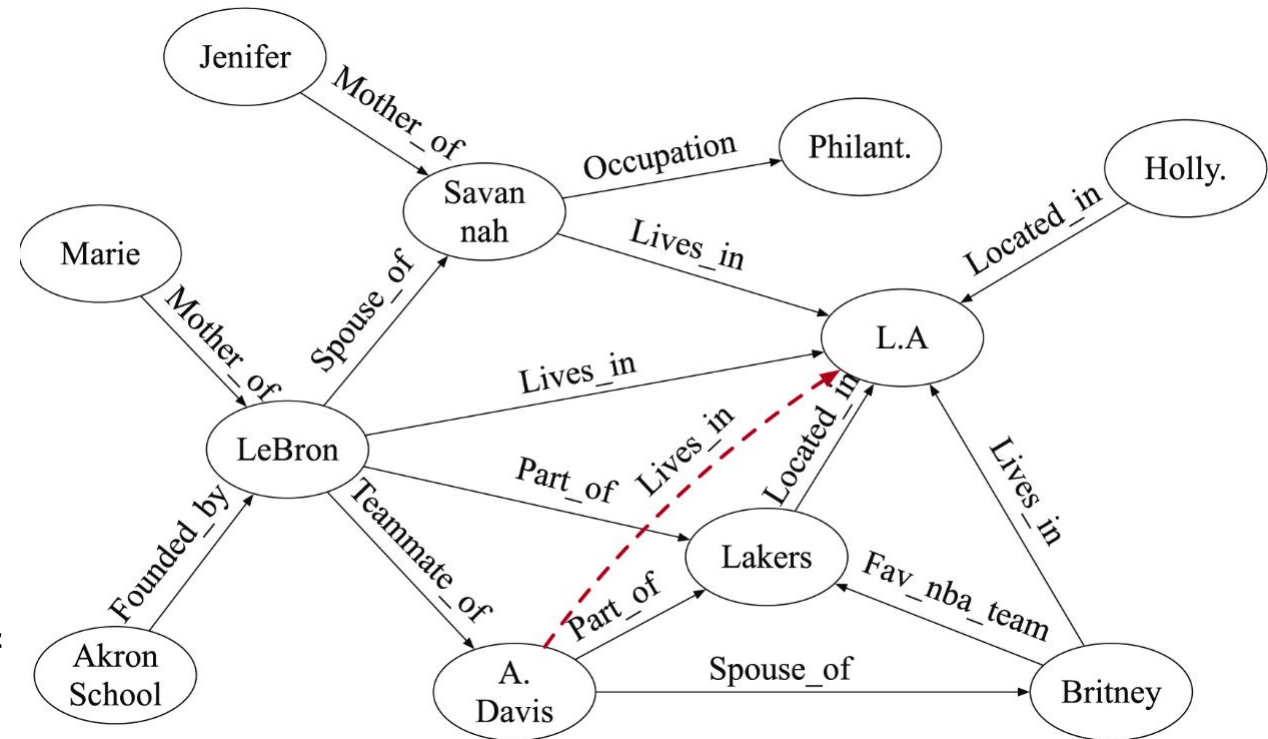
Example task:  
predict the tail “Science Fiction” for (“J.K. Rowling”, “genre”)

## ➤ Semantic Information

- Knowledge graph completion:
- Query relations:
  - Lives\_in
  - Head entity: A.Davis
- Reasoning result:
  - L.A

## ➤ KG question answering:

- Questions:
  - Where do the spouses of teammates of Lakers usually live?
- Reasoning result:
  - L.A



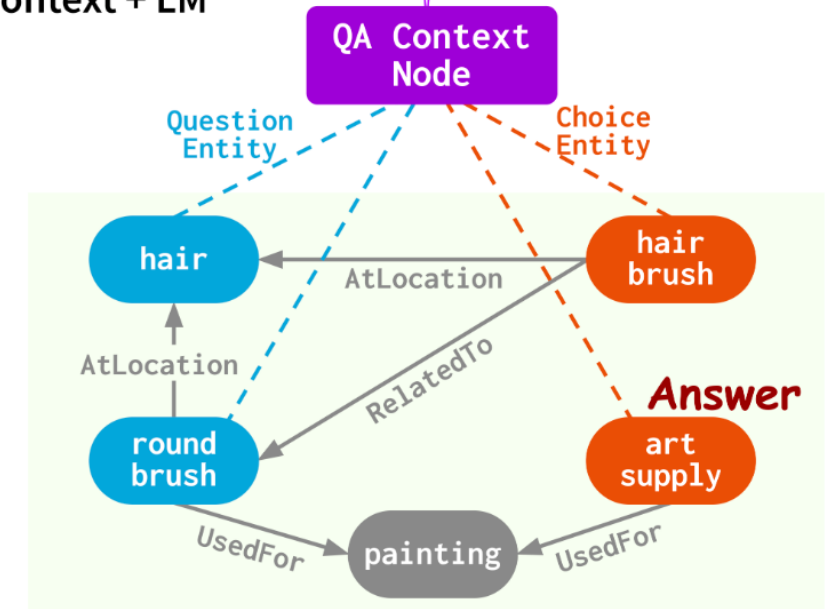


## ➤ Question Answering over Knowledge Graphs

- View the QA context as a node (Purple node) and connect it to each topic entity in the KG (blue and red nodes).
- Each node is associated with one of 4 types:
  - Purple is the QA context node
  - Blue is an entity in the question
  - Orange is an entity in the answer choices
  - Gray is any other entity.
- The representation is initialized as the LM representation of the QA context or entity name.

If it is not used for **hair**, a **round brush** is an example of what?  
 A. **hair brush** B. **bathroom** C. **art supplies\*** D. **shower**

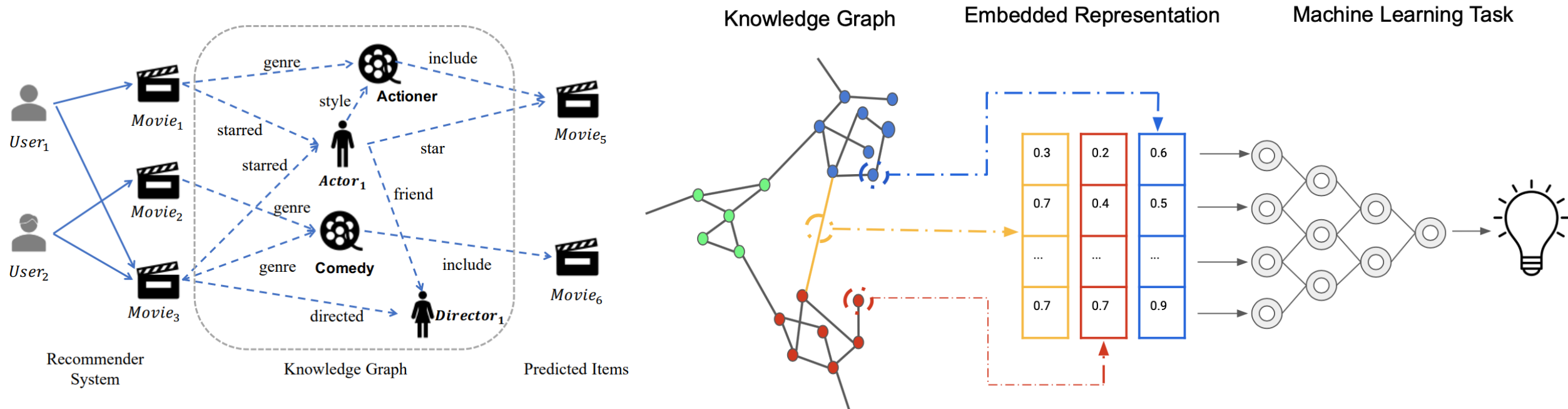
QA Context + LM



Knowledge Graph

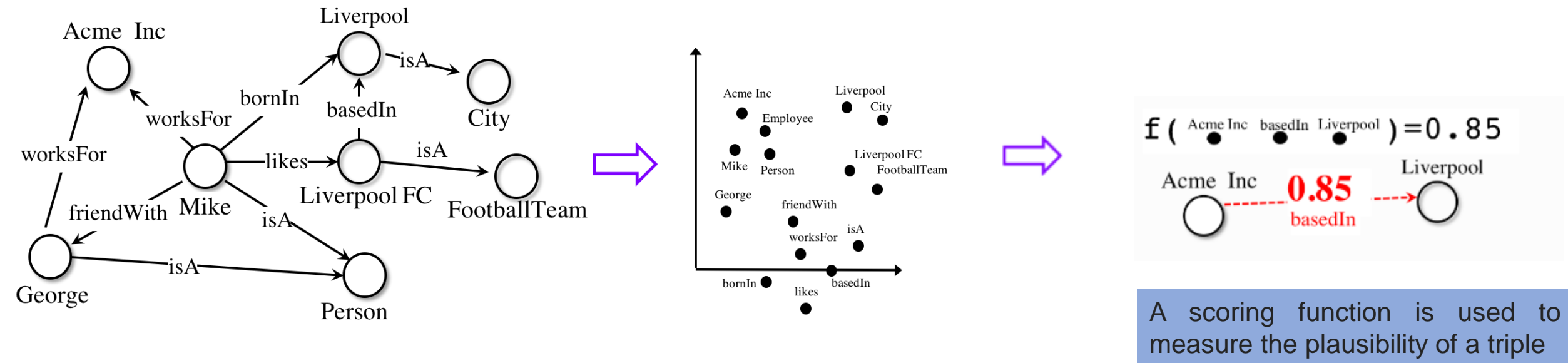
## ➤ Recommender system

- KGs have been used in recommender systems in order to overcome the problem of user-item interactions sparsity and the cold start problem.
- The vector representation of the entities and relations can be used for different machine learning applications.

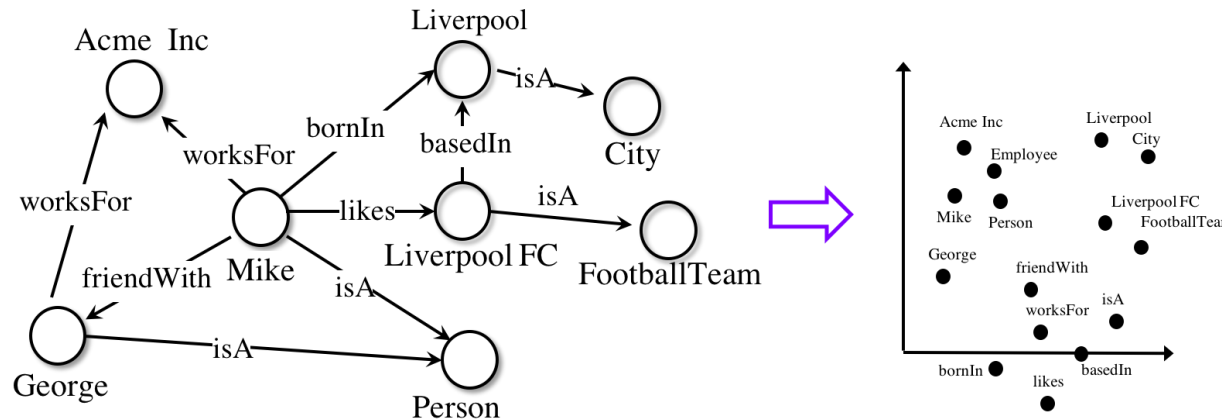


## ➤ Mapping from Graph domain to Space domain

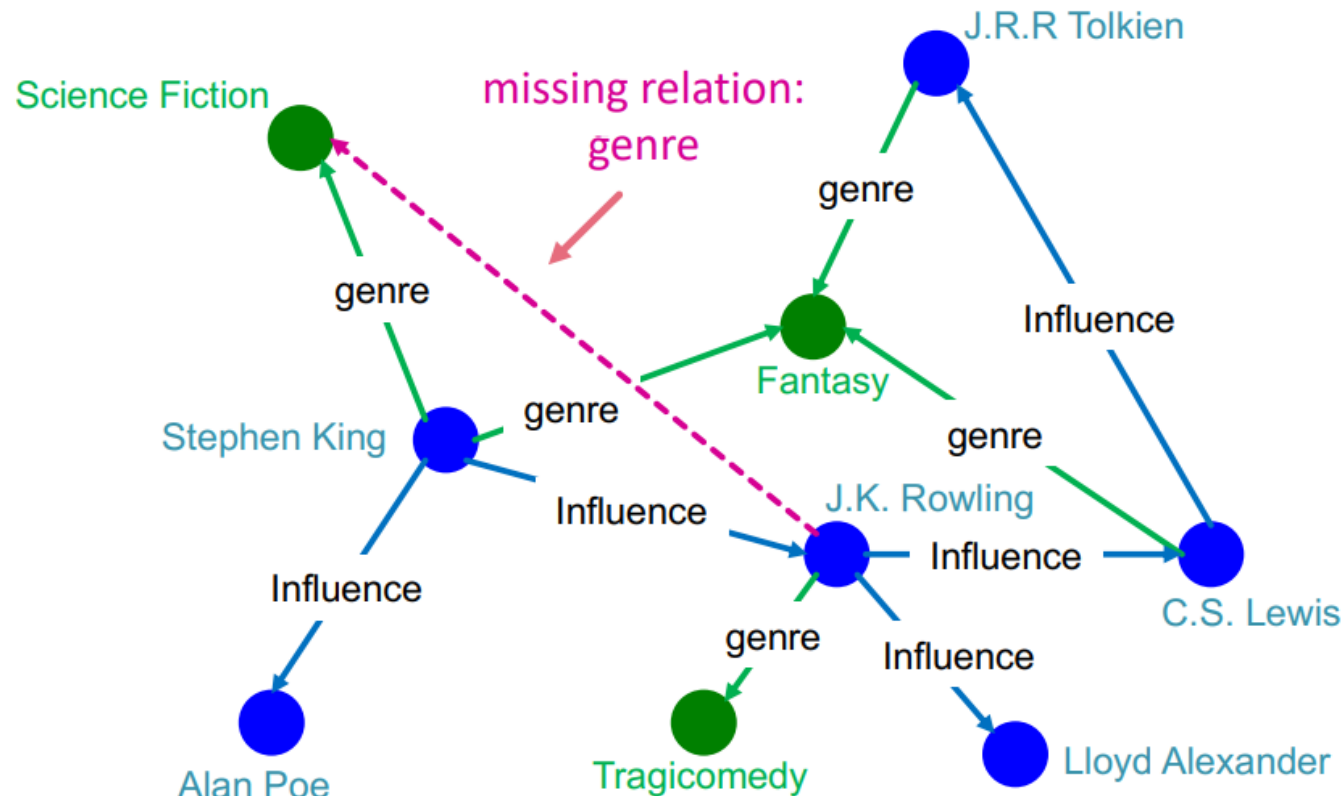
- Use Graph embeddings for a latent semantic representation of Knowledge Graphs
- Combining latent semantic representations of different (symbolic) representations



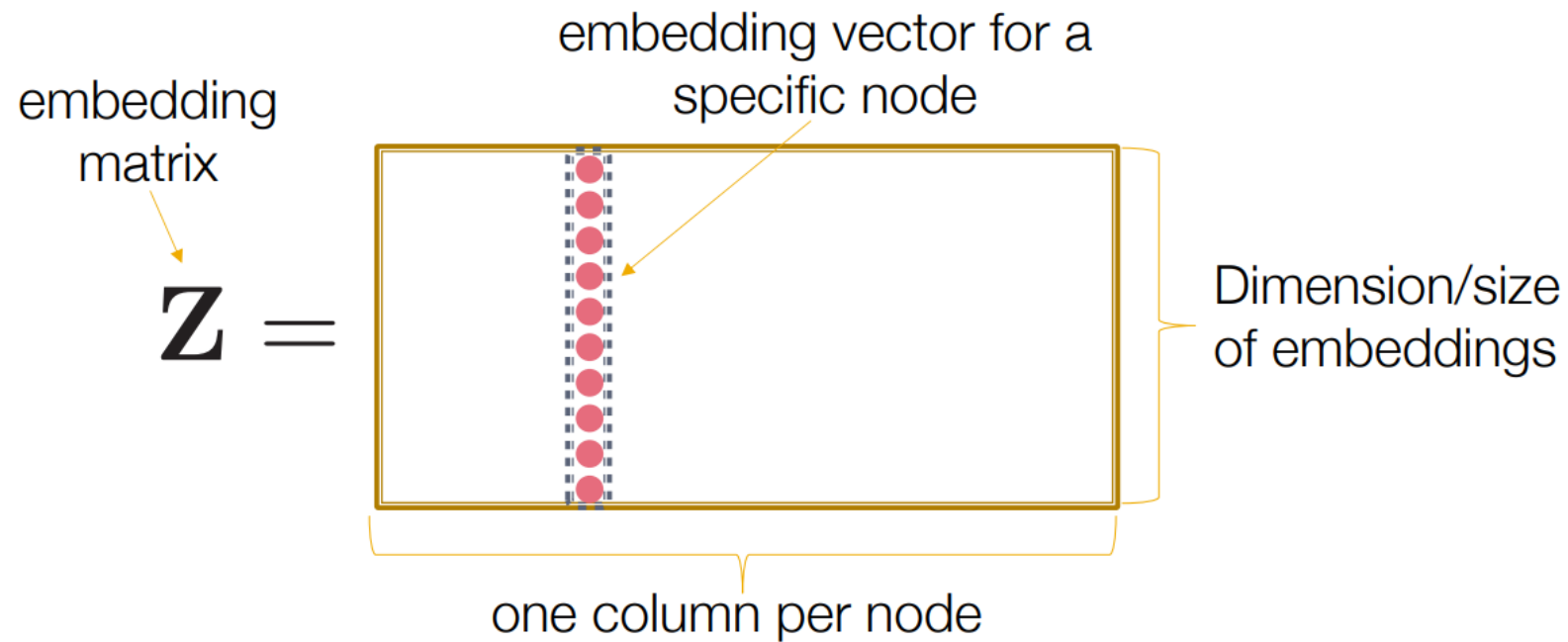
- **Why do we need vector embeddings?**
  - Embeddings make it easier to do machine learning on large inputs like sparse word vectors.
- **Where do the embeddings come from?**
  - arned from the knowledge base itself (e.g. KG completion)
  - Learned from text (e.g. word embeddings)
- **What is the underlying principle?**
  - Similarity-based reasoning is highly heuristic.
  - No strong reason to believe that something is true just because it is true for a similar predicate or individual.



- Given an enormous KG, can we complete the KG?
  - For a given (head, relation), we predict missing tails.
  - Note this is slightly different from link prediction task
- Example task: predict the tail “Science Fiction” for (“J.K. Rowling”, “genre”)



- Simplest encoding approach: encoder is just an embedding-lookup





- Edges in KG are represented as triples  $(h, r, t)$   
(head  $h$  has relation  $r$  with tail  $t$ )
- **Key Idea:**
  - Model entities and relations in the embedding/vector space  $R^d$
  - Associate entities and relations with shallow embeddings
- Given a true triple  $(h, r, t)$ , the goal is that the embedding of  $(h, r)$  should be close to the embedding of  $t$ .
- **Main questions:**
  - How to embed  $(h, r)$  ?
  - How to define the similarity between them?

- Focused on embedding monolingual triples  $(h, r, t)$ 
  - Exploit distance-based scoring functions
  - Measure the plausibility of a fact as the distance between the two entities

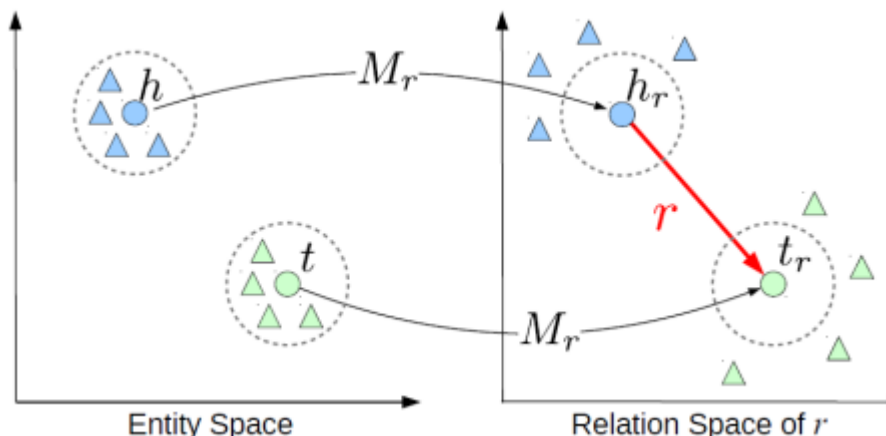
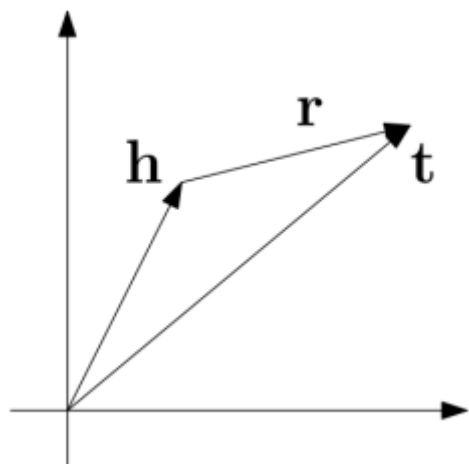
**TransE:  $h+r \approx t$**



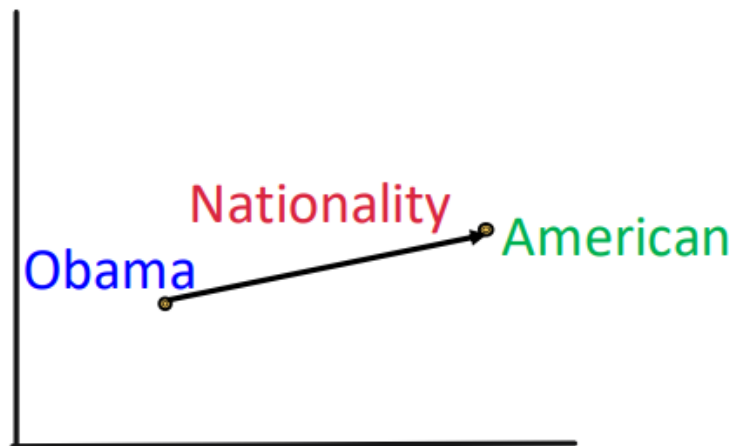
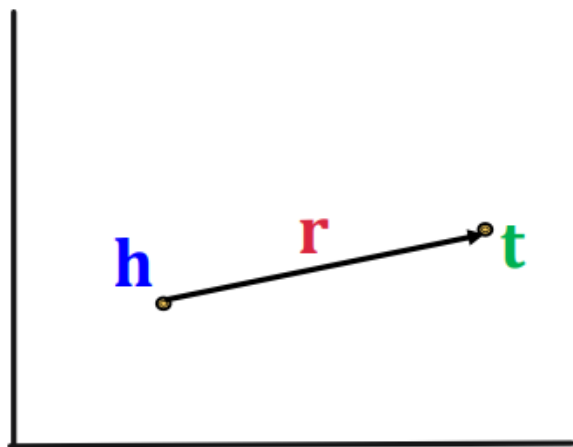
## Later approaches:

- TransH [Wang et al. 2014]
- TransR [Lin et al. 2015]
- TransD [Ji et al. 2015]
- HolE [Nickle et al. 2016]
- ComplEx [Trouillon et al. 2016]

**Embedding of monolingual knowledge seems to be well-addressed.**



- For a triple  $(h, r, t)$ :
  - $\mathbf{h} + \mathbf{r} \approx \mathbf{t}$  if the given fact is true
  - else  $\mathbf{h} + \mathbf{r} \neq \mathbf{t}$
- Scoring function:  $f_r(h, t) = -||\mathbf{h} + \mathbf{r} - \mathbf{t}||$



- **Symmetric** (Antisymmetric) Relations:

$$r(h, t) \Rightarrow r(t, h) \quad (r(h, t) \Rightarrow \neg r(t, h)) \quad \forall h, t$$

- Example:

- Symmetric: Family, Roommate
- Antisymmetric: Hypernym

- **Inverse** Relations:

- Symmetric (Antisymmetric) Relations:  $r_2(h, t) \Rightarrow r_1(t, h)$
- Example : (Advisor, Advisee)

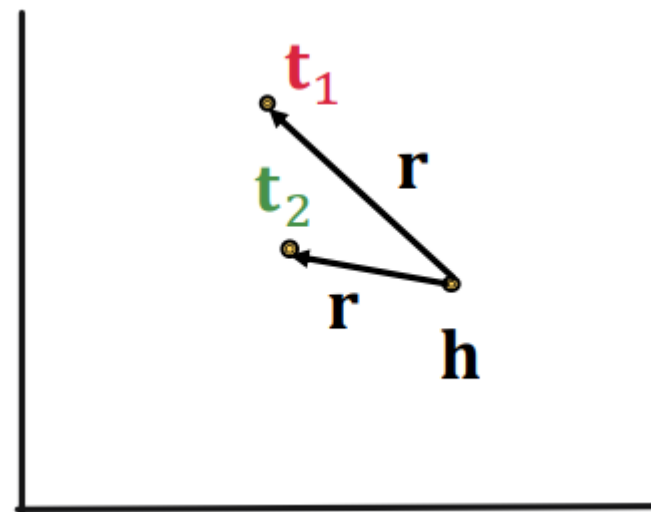
- **1-to-N** relations:

$$r(h, t_1), r(h, t_2), \dots, r(h, t_n) \text{ are all True.}$$

- Example:  $r$  is “StudentsOf”

- TransE Limitation: 1-to-N relations
  - 1-to-N Relations:  $(h, r, t_1)$  and  $(h, r, t_2)$  both exist in the knowledge graph.
  - TransE cannot model 1-to-N relations:  $t_1$  and  $t_2$  will map to the same vector, although they are different entities

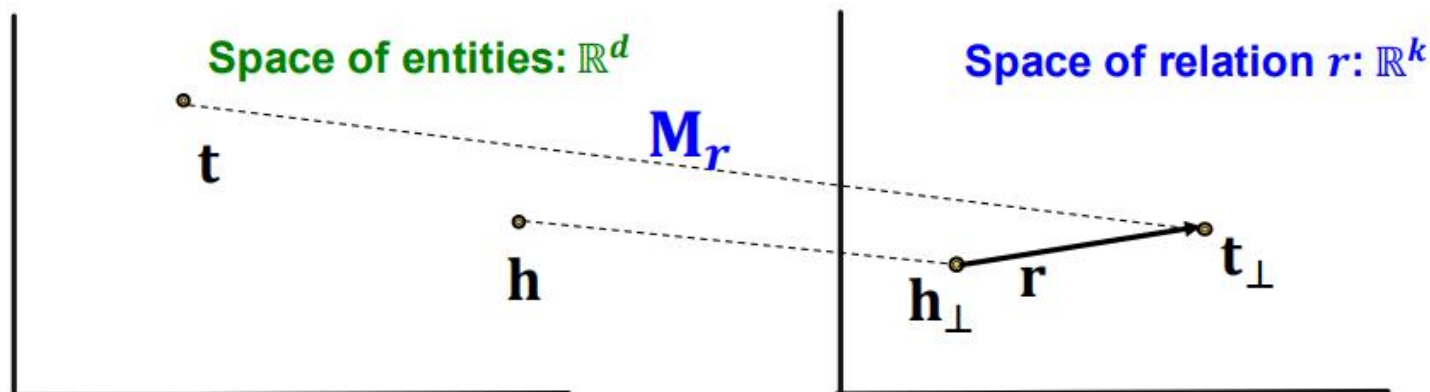
$$\begin{aligned} \mathbf{t}_1 &= \mathbf{h} + \mathbf{r} = \mathbf{t}_2 \\ \mathbf{t}_1 &\neq \mathbf{t}_2 \quad \text{contradictory!} \end{aligned}$$



- TransE models the translation of any relation in the same embedding space.

Can we design a new space for each relation and do translation in relation-specific space?

- **TransR**: model entities as vectors in the entity space  $\mathbb{R}^d$  and model each relation as vector in relation space  $\mathbf{r} \in \mathbb{R}^k$  with  $M_r \in \mathbb{R}^{k \times d}$  as the projection matrix.





- **TransR**: model entities as vectors in the entity space  $\mathbb{R}^d$  and model each relation as vector in relation space  $\mathbf{r} \in \mathbb{R}^k$  with  $M_r \in \mathbb{R}^{k \times d}$  as the projection matrix:

$$\mathbf{h}_\perp = \mathbf{M}_r \mathbf{h}, \quad \mathbf{t}_\perp = \mathbf{M}_r \mathbf{t}$$

- Score function:

$$f_r(h, t) = -\|\mathbf{h}_\perp + \mathbf{r} - \mathbf{t}_\perp\|$$

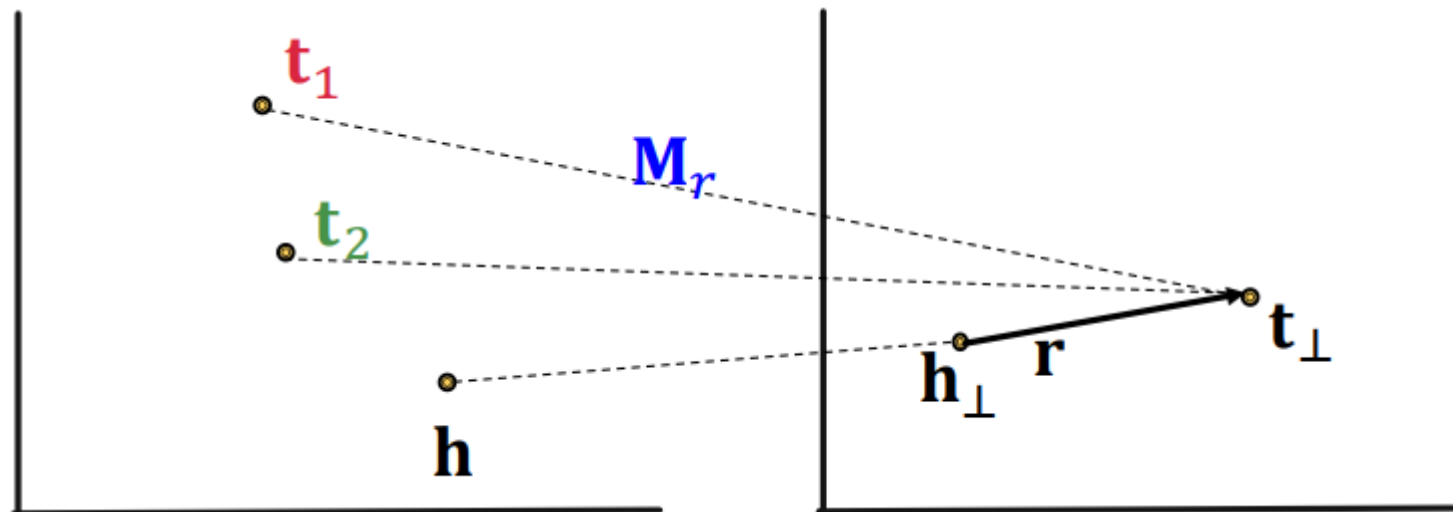
Use  $M_r$  to project from entity space  $\mathbb{R}^d$  to relation space  $\mathbb{R}^k$



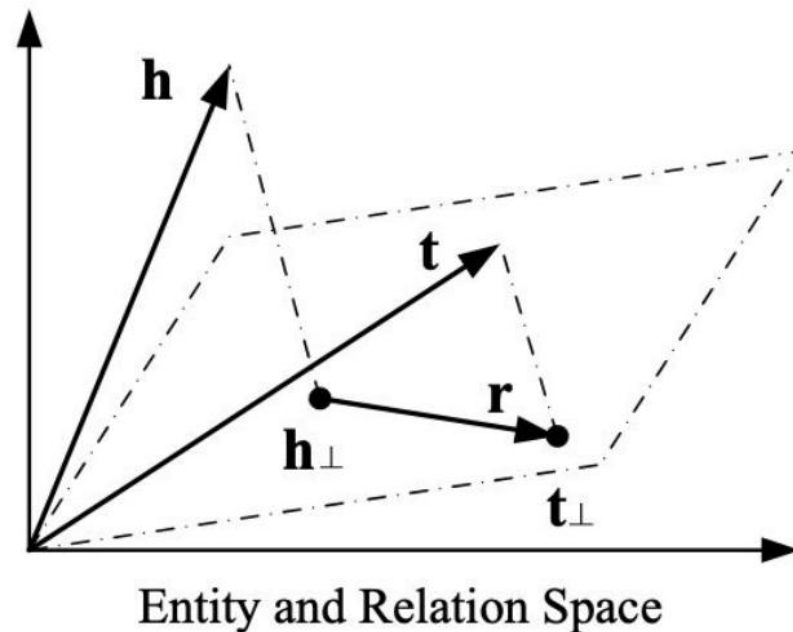
- TransR: 1-to-N relations in TransR
  - 1-to-N Relations:
    - Example: If  $(h, r, t_1)$  and  $(h, r, t_2)$  exist in the knowledge graph.
  - TransR can model 1-to-N relations
    - We can learn  $\mathbf{M}_r$  so that:

$$\mathbf{t}_\perp = \mathbf{M}_r \mathbf{t}_1 = \mathbf{M}_r \mathbf{t}_2$$

Use  $M_r$  to project from entity space  $\mathbb{R}^d$  to relation space  $\mathbb{R}^k$



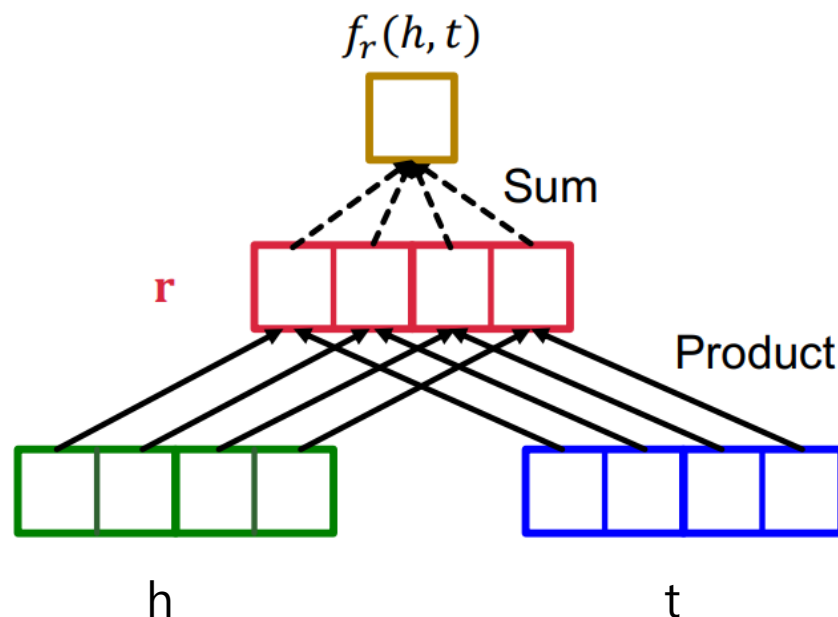
- From Original space to Hyperplane
- TransH enables different roles of an entity in different relations
- Entities  $h$  and  $t$  are projected into specific hyperplane of relation  $r$
- Then predict new links based on translation on hyperplane



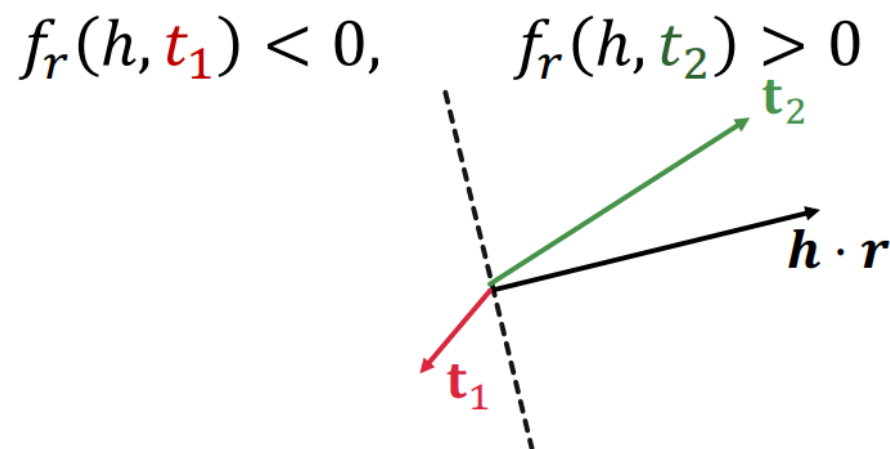
- So far: The scoring function  $f_r(h, t)$  is negative of L1 / L2 distance in TransE and TransR
- Another line of KG embeddings adopt bilinear modelling
- **DistMult**: Entities and relations using vectors in  $\mathbb{R}^k$
- Score function:

$$f_r(h, t) = \langle \mathbf{h}, \mathbf{r}, \mathbf{t} \rangle = \sum_i \mathbf{h}_i \cdot \mathbf{r}_i \cdot \mathbf{t}_i$$

$$\mathbf{h}, \mathbf{r}, \mathbf{t} \in \mathbb{R}^k$$



- **DistMult**: Entities and relations using vectors in  $R^k$
- Intuition of the score function: Can be viewed as a cosine similarity between  $h \cdot r$  and  $t$ 
  - where  $h \cdot r$  is defined as  $\sum_i h_i \cdot r_i$
  - Example:







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