The operations defined on sets satisfy the following properties:

- Commutative properties:

1. $A \cup B = B \cup C$;
2. $A \cap B = B \cap C$;

- Associative properties.

3. $A \cup (B \cup C) = (A \cup B) \cup C$
4. $A \cap (B \cap C) = (A \cap B) \cap C$

- Distributive properties.

5. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- Idempotent properties:

7. $A \cup A = A$
8. $A \cap A = A$

- Properties of the complement.

9. $\overline{(\bar{A})} = A$
10. $A \cup \bar{A} = U$
11. $A \cap \bar{A} = \varnothing$
12. $\overline{\varnothing} = U$
13. $\overline{U} = \varnothing$
14. $\overline{A \cup B} = \bar{A} \cap \bar{B}$
15. $\overline{A \cap B} = \bar{A} \cup \bar{B}$

- Properties of a universal set

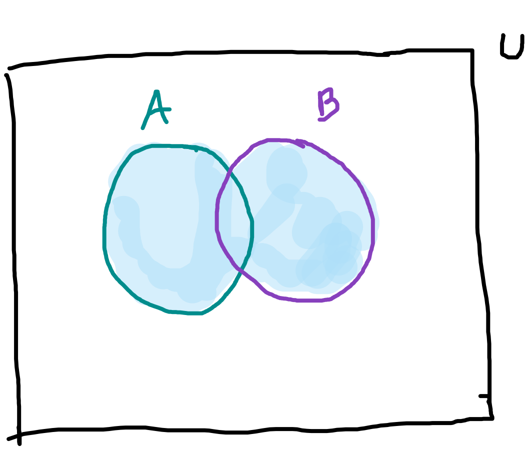
16. $A \cup U = U$
17. $A \cap U = A$

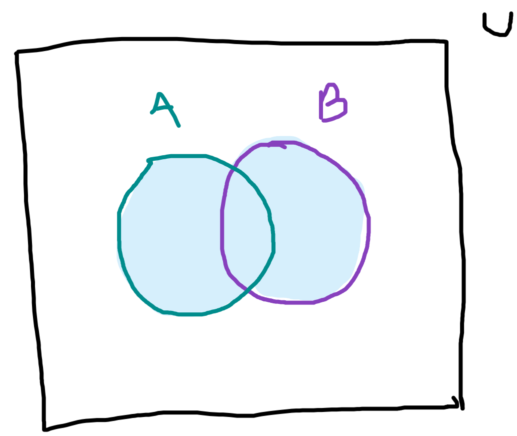
- Properties of the empty set

18. $A \cup \varnothing = A$
19. $A \cap \varnothing = \varnothing$

☆ Venn diagrams of the properties:

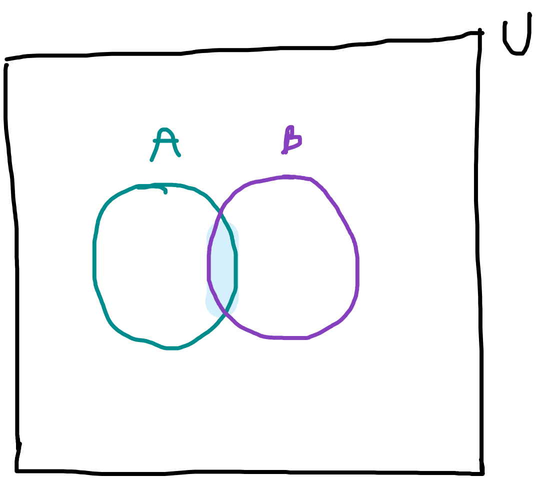
1.

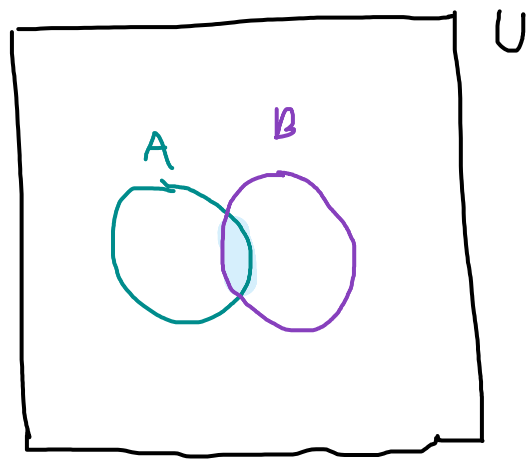




$A \cup B = B \cup A$

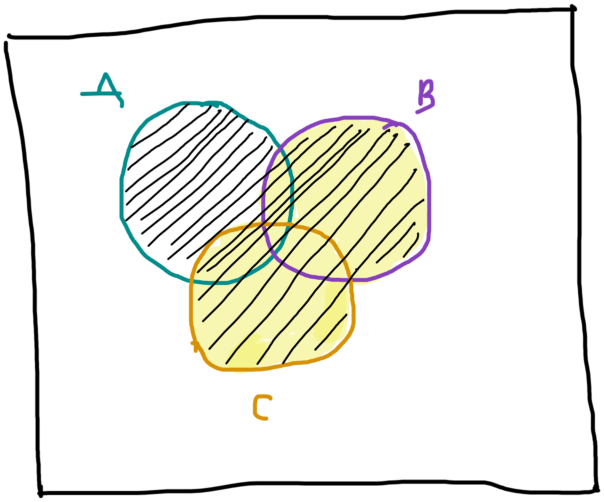
2.

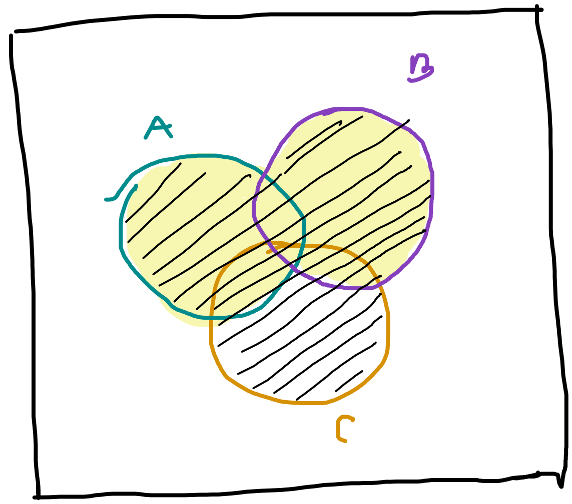




$A \cap B = B \cap A$

3

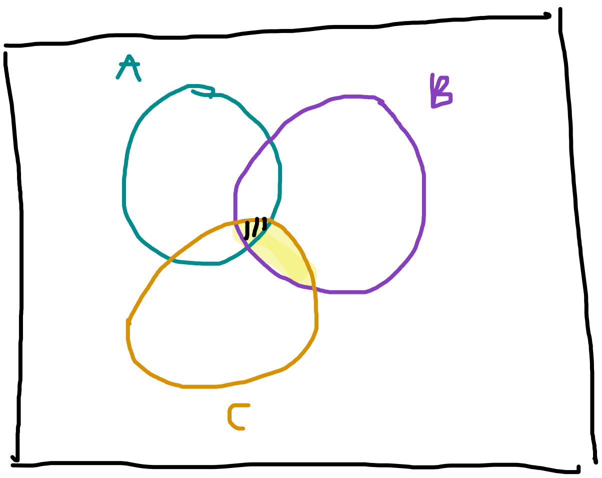


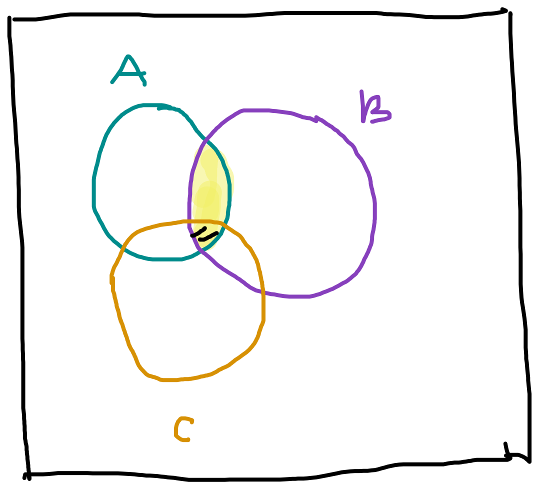


$B \cup C$ | $A \cup B$

$A \cup (B \cup C) = (A \cup B) \cup C$

4

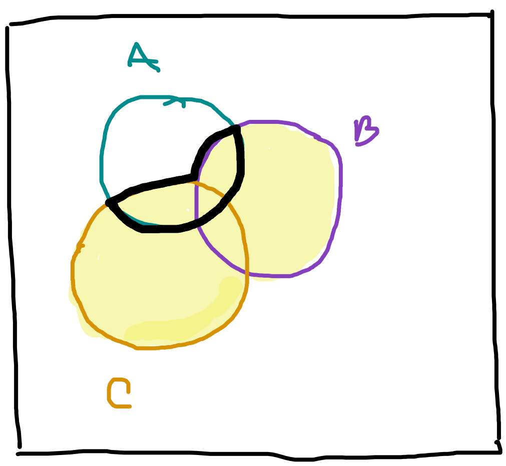


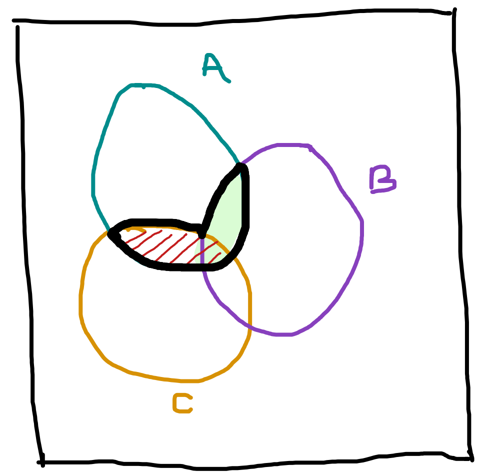


$B \cap C$ | $A \cap B$

$A \cap (B \cap C) = (A \cap B) \cap C$

5.





$B \cup C$

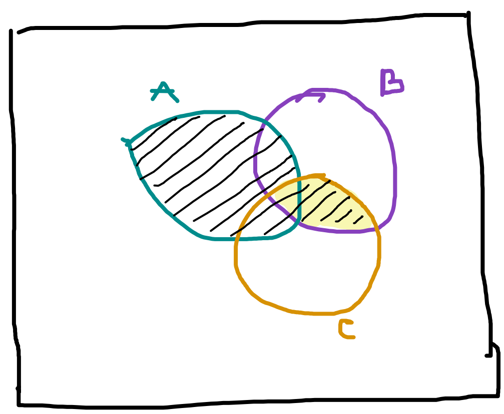
$A \cap (B \cup C)$

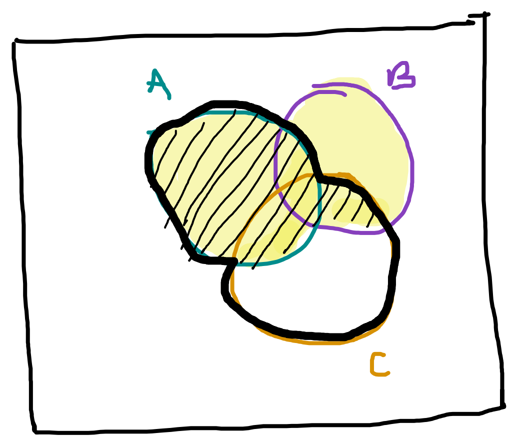
$A \cap B$

$A \cap C$

$= (A \cap B) \cup (A \cap C)$

6.





$B \cap C$

$A \cup (B \cap C)$

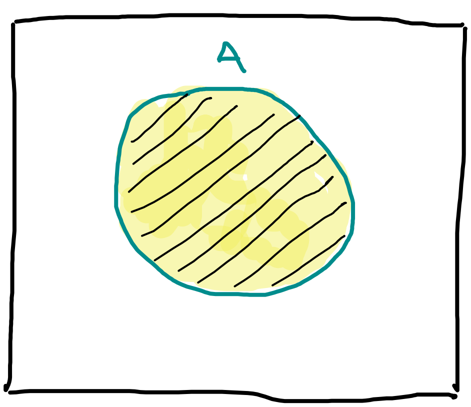
$A \cup B$

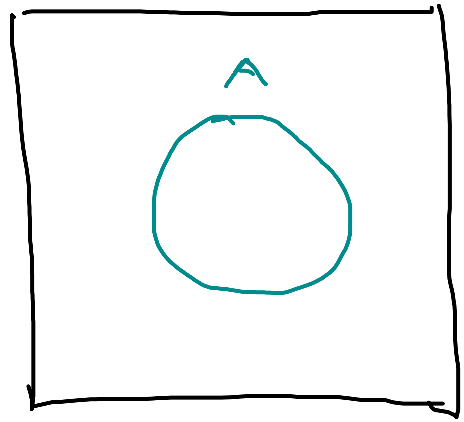
$A \cup C$

$= (A \cup B) \cap (A \cup C)$

7.

8.



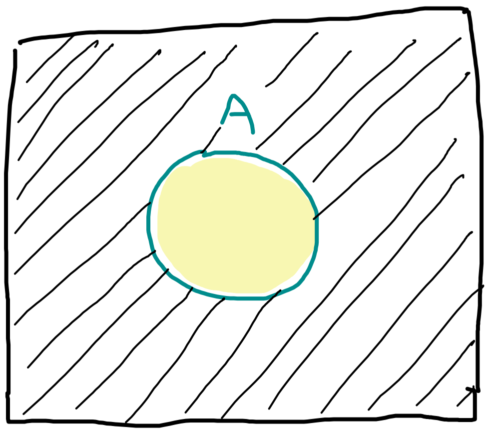


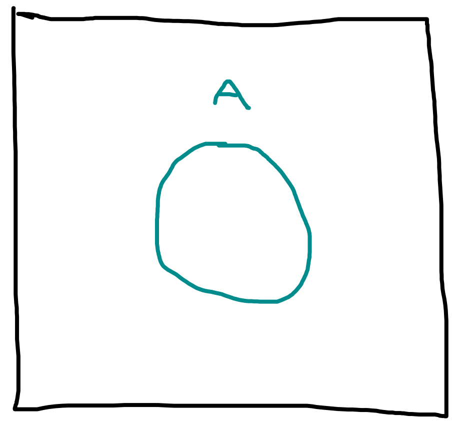
$A \cup A$

$A \cap A$

$=$ A

9.



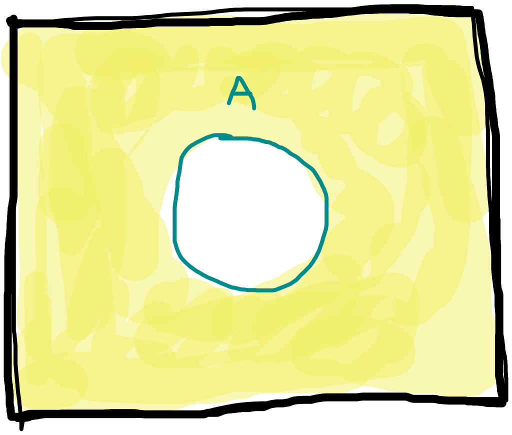


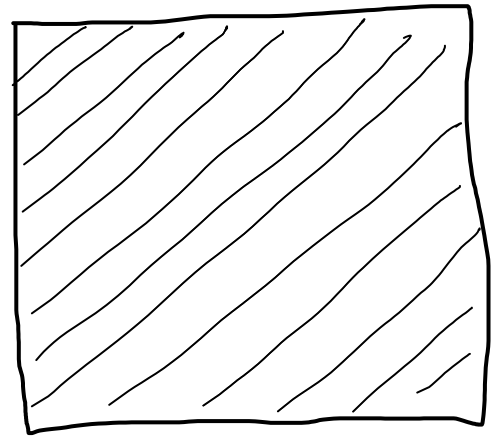
$\bar{A}$

$\overline{(\bar{A})}$

$=$ A

10.

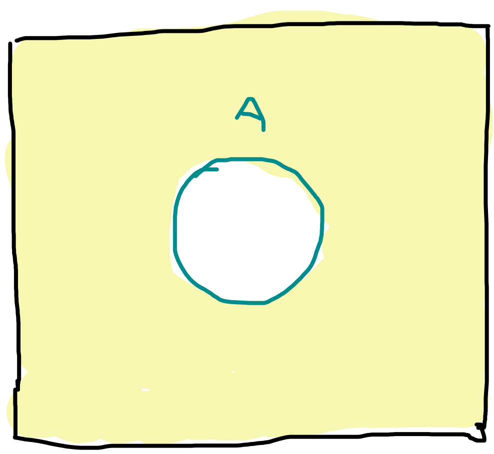


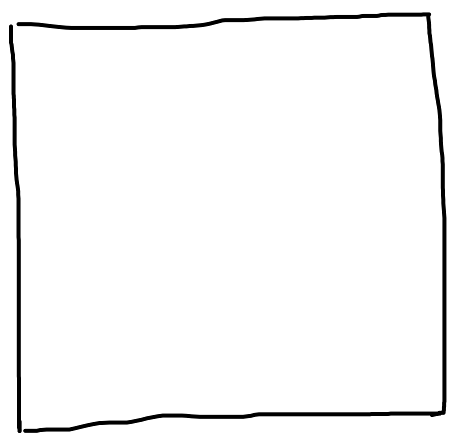


$\bar{A}$

$A \cup \bar{A}$ = U

11.





$\bar{A}$

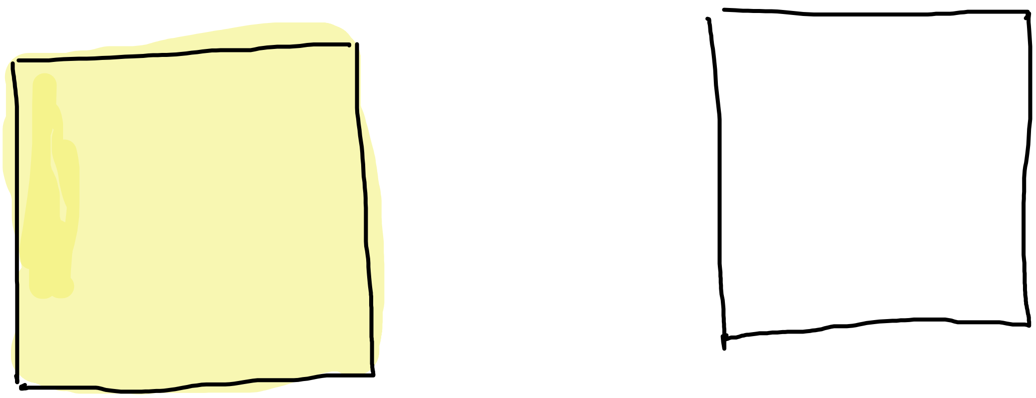
$A \cap \bar{A}$ = $\emptyset$

12

$\emptyset$

$\overline{\emptyset} = U$

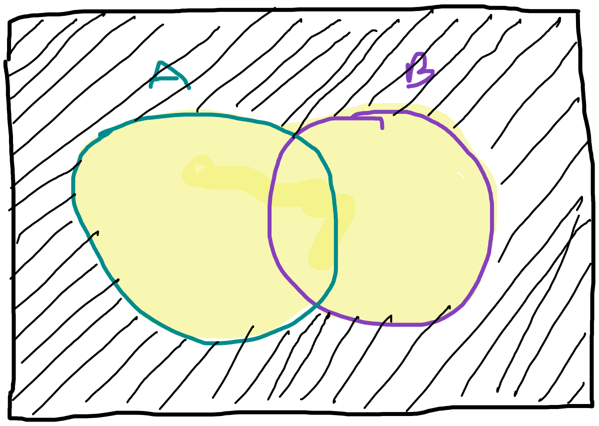
13.

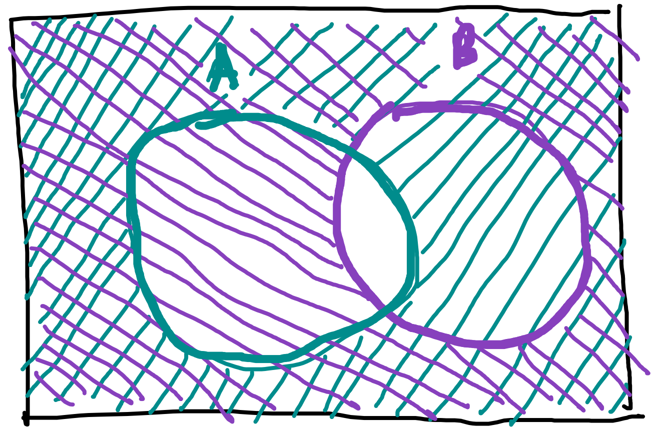


U

$\overline{U} = \emptyset$

14.





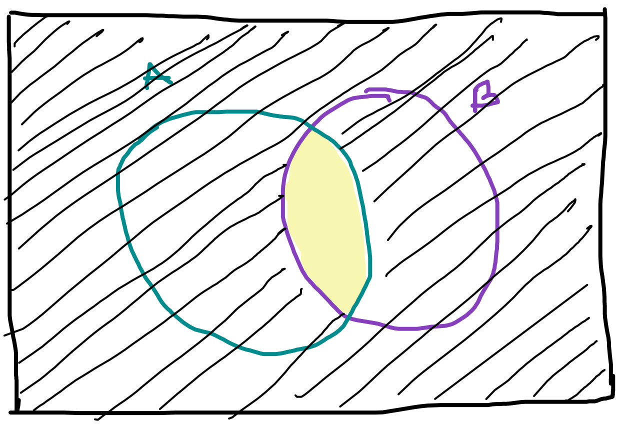
$A \cup B$

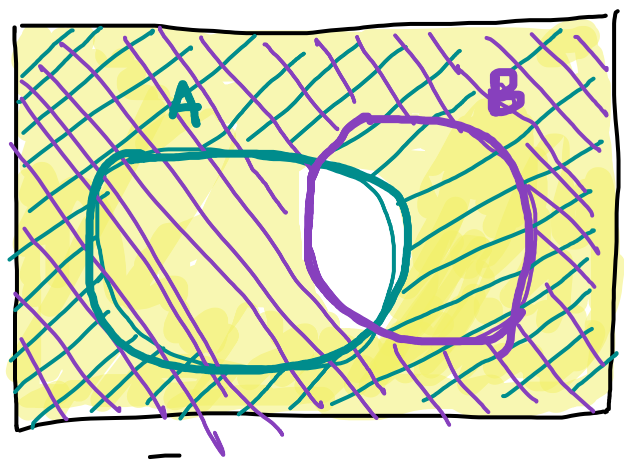
$\overline{A}$

$\overline{B}$

$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

15.





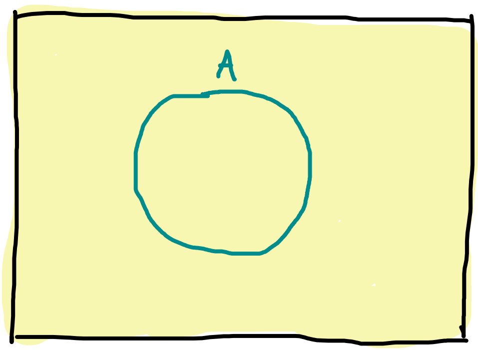
$A \cap B$

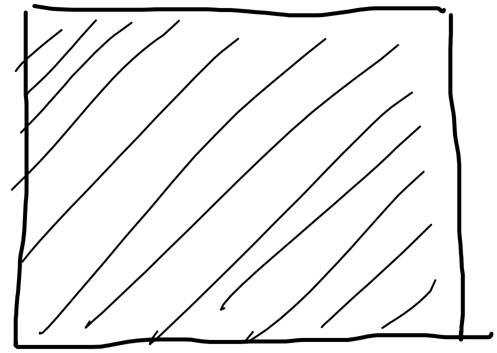
$\overline{A}$

$\overline{B}$

$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$

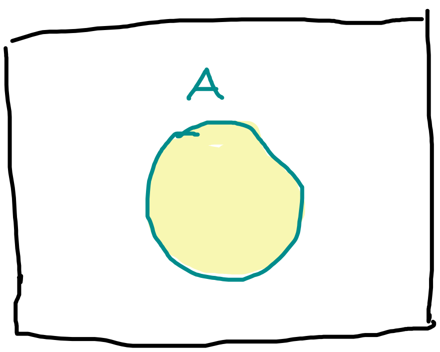
16.

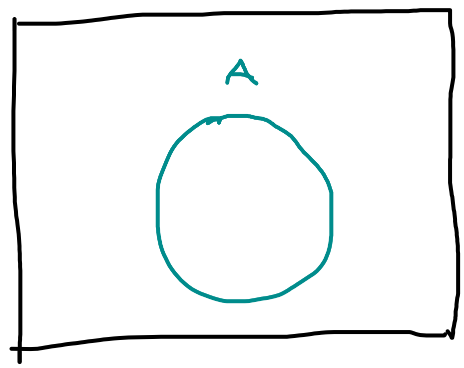




$A \cup U = U$

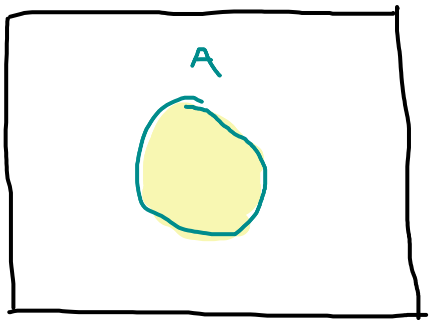
17.

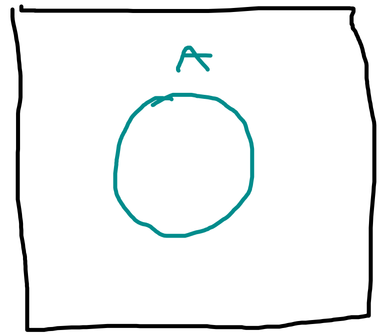




$A \cap U = A$

18

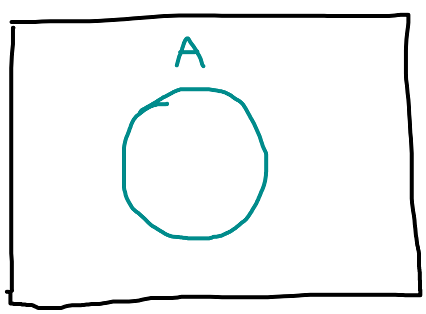


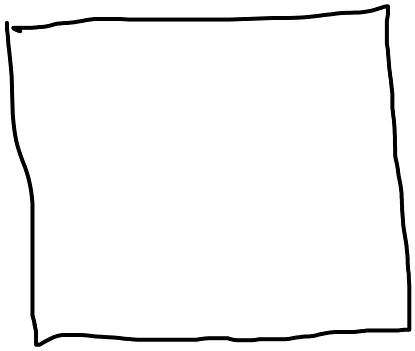


$\emptyset$

$A \cup \emptyset = A$

19.





$\emptyset$

$A \cap \emptyset = \emptyset$

✩ Proofs.

14. Suppose $x \in \overline{A \cup B} \Rightarrow x \notin A$ and $x \notin B \Leftrightarrow x \in \bar{A}$ and $x \in \bar{B} \Leftrightarrow x \in \bar{A} \cap \bar{B} \Rightarrow \overline{A \cup B} \subseteq \bar{A} \cap \bar{B}$. In other order, if $x \in \bar{A} \cap \bar{B} \Rightarrow x \notin A$ and $x \notin B \Rightarrow x \notin A \cup B \Leftrightarrow x \in \overline{A \cup B} \Rightarrow \bar{A} \cap \bar{B} \subseteq \overline{A \cup B}$. And becouse the sets includes each other $\Rightarrow \overline{A \cup B} = \bar{A} \cap \bar{B}$ $\blacksquare$

15. Suppose $x \in \overline{A \cap B} \Rightarrow (x \notin A$ and $x \in (B \cup U))$ or $(x \notin B$ and $x \in (A \cup U)) \Leftrightarrow$ $(x \in U$ and $x \notin A)$ or $(x \in U$ and $x \notin B) \Leftrightarrow$ $(x \in \bar{A}$ and $x \in U)$ or $(x \in \bar{B}$ and $x \in U) \Leftrightarrow$ $x \in \bar{A} \cap U$ or $x \in \bar{B} \cap U \Leftrightarrow$ $x \in \bar{A}$ or $x \in \bar{B} \Leftrightarrow x \in \bar{A} \cup \bar{B} \Rightarrow \overline{A \cap B} \subseteq \bar{A} \cup \bar{B}$ In other order, if $x \in \bar{A} \cup \bar{B} \Rightarrow$ $x \notin A$ or $x \notin B \Leftrightarrow x \notin A \cap B \Leftrightarrow x \in \overline{A \cap B} \Rightarrow$ $\bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$, and becouse $\overline{A \cap B} \subseteq \bar{A} \cup \bar{B} \Rightarrow$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$. $\blacksquare$

1. $A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{x \mid x \in B \text{ or } x \in A\} = B \cup A$ ■

2. $A \cap B = \{x \mid x \in A \text{ and } x \in B\} = \{x \mid x \in B \text{ and } x \in A\} = B \cap A$ ■

3. $X = B \cup C$

$X' = A \cup X$

$Y = A \cup B$

$Y' = Y \cup C$

If $x \in X' \iff x \in A$ or $x \in X \iff x \in A$ or $x \in B$ or $x \in C \iff x \in Y$ or $x \in C \iff x \in Y' \Rightarrow X' \subseteq Y'$ (1). If $x \in Y' \iff x \in Y$ or $x \in C \iff x \in A$ or $x \in B$ or $x \in C \iff x \in A$ or $x \in X \iff x \in X' \Rightarrow Y' \subseteq X'$ (2).

From (1) and (2) $\Rightarrow X' = Y' \iff A \cup X = Y \cup C \iff A \cup (B \cup C) = (A \cup B) \cup C$ ■

4. $X = B \cap C$

$X' = A \cap X$

$Y = A \cap B$

$Y' = Y \cap C$

If $x \in X' \Leftrightarrow x \in A$ and $x \in X \Leftrightarrow x \in A$ and $x \in B$ and $x \in C \Leftrightarrow x \in Y$ and $x \in C \Leftrightarrow x \in Y' \Rightarrow X' \subseteq Y'$ (1). If $x \in Y' \Leftrightarrow x \in Y$ and $x \in C \Leftrightarrow x \in A$ and $x \in B$ and $x \in C \Leftrightarrow x \in A$ and $x \in X \Leftrightarrow x \in X'$ $\Rightarrow Y' \subseteq X'$ (2) From (1) and (2) $\Rightarrow X' = Y' \Leftrightarrow A \cap X = Y \cap C \Leftrightarrow A \cap (B \cap C) = (A \cap B) \cap C$ $\blacksquare$

5 $X = B \cup C$

$Z = A \cap X$

$Y = A \cap B$

$Y' = A \cap C$

$Z' = Y \cup Y'$

If $x \in Z \Leftrightarrow x \in A$ and $x \in X \Leftrightarrow x \in A$ and $(x \in B$ or $x \in C) \Leftrightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C) \Leftrightarrow x \in Y$ or $x \in Y' \Leftrightarrow x \in Z' \Rightarrow Z \subseteq Z'$ (1). If $x \in Z' \Leftrightarrow x \in Y$ or $x \in Y' \Leftrightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$ $\Leftrightarrow x \in A$ and $(x \in B$ or $x \in C) \Leftrightarrow x \in A$ and $x \in X \Leftrightarrow x \in Z \Rightarrow Z' \subseteq Z$ (2). From (1) and (2) $\Rightarrow Z = Z' \Leftrightarrow A \cap X = Y \cup Y' \Leftrightarrow$

$\Leftrightarrow A \cap (B \cup C) = (A \wedge B) \cup (A \wedge C)$ $\blacksquare$

6 $X = B \cap C$

$Z = A \cup X$

$Y = A \cup B$

$Y' = A \cup C$

$Z' = Y \cap Y'$

If $x \in Z \Leftrightarrow x \in A$ or $x \in X \Leftrightarrow x \in A$ or $(x \in B$ and $x \in C) \Leftrightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C) \Leftrightarrow x \in Y$ and $x \in Y' \Leftrightarrow x \in Z' \Rightarrow Z \subseteq Z'$ (1). If $x \in Z' \Leftrightarrow x \in Y$ and $x \in Y' \Leftrightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C) \Leftrightarrow x \in A$ or $(x \in B$ and $x \in C) \Leftrightarrow x \in A$ or $x \in X \Leftrightarrow x \in Z \Rightarrow Z' \subseteq Z$ (2). From (1) and (2) $\Rightarrow Z = Z'$ $\Leftrightarrow A \cup X = Y \cap Y' \Leftrightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$\blacksquare$

7. $X = A \cup A$

If $x \in X \Leftrightarrow x \in A$ or $x \in A \Leftrightarrow x \in A \Rightarrow$ $X \subseteq A$ (1). If $x \in A \Leftrightarrow x \in A \Leftrightarrow x \in A$ or $x \in A \Leftrightarrow x \in A$ or $x \in A$ or $x \in A$ or ... $\Leftrightarrow$ $x \in A$ or $x \in A \Leftrightarrow x \in X \Rightarrow A \subseteq X$ (2). From (1) and (2) $\Rightarrow$ $X = A \Leftrightarrow A \cup A = A$ ■

8. $A \cap A = \{x \mid x \in A$ and $x \in A\} =$ $\{x \mid x \in A\} = A$ ■

9 $\overline{(\overline{A})} = \overline{\{x \mid x \notin A\}} = \{x \mid x \in A\} = A$ ■

10 $A \cup \overline{A} = A \cup (U - A) = \{x \mid x \in A$ or $(x \in U$ and $x \notin A)\} = \{x \mid (x \in A$ or $x \in U)$ and $(x \in A$ or $x \notin A)$ $\} = \{x \mid x \in A$ or $x \in U\} = A \cup U$, and because $A \subseteq U$ $\Rightarrow A \cup U = U$ ■

11. $A \cap \bar{A} = \{x \mid x \in A \text{ and } x \notin A\} = \emptyset$ $\blacksquare$

12. $\overline{\emptyset} = U - \emptyset = \{x \mid x \in U \text{ and } x \notin \emptyset\}$, and because $|\emptyset| = 0 \Rightarrow (\forall) x \in U, x \notin \emptyset \Rightarrow$ $\{x \mid x \in U \text{ and } x \notin \emptyset\} = \{x \mid x \in U\} = U$ $\blacksquare$

13. $\overline{U} = U - U = \{x \mid x \in U \text{ and } x \notin U\} = \emptyset$ $\blacksquare$

16. $A \cup U = \{x \mid x \in A \text{ or } x \in U\}$, and because $A \subseteq U \Rightarrow (\forall) x \in A, x \in U \Rightarrow \{x \mid x \in A \text{ or } x \in U\} =$ $\{x \mid x \in U\} = U$ $\blacksquare$

17. $A \cap U = \{x \mid x \in A \text{ and } x \in U\}$, and because $A \subseteq U \Rightarrow (\forall) x \in A, x \in U \Rightarrow \{x \mid x \in A \text{ and } x \in U\} =$ $\{x \mid x \in A\} = A$ $\blacksquare$

18. $A \cup \emptyset = \{x \mid x \in A \text{ or } x \in \emptyset\}$, and because $|\emptyset| = 0 \Rightarrow \{x \mid x \in A \text{ or } x \in \emptyset\} = \{x \mid x \in A\} = A$ $\blacksquare$

19. $A \cap \emptyset = \{x \mid x \in A \text{ and } x \in \emptyset\}$, and because $|\emptyset| = 0 \Rightarrow (\nexists) x \in A \mid x \in \emptyset \Rightarrow \{x \mid x \in A \text{ and } x \in \emptyset\} =$ $\emptyset$ $\blacksquare$