The addition principle

Theorem:

$$|A \cup B| = |A| + |B| - |A \cap B|$$ ☆

Exemple:

$A = \{ \square, \bigcirc, \triangle \}$

$B = \{ -, \triangle, 1 \}$

$A \cup B = \{ -, 1, \triangle, \square, \bigcirc \}$

$A \cap B = \{ \triangle \}$

$|A| = 3, |B| = 3 ; |A \cup B| = 5 ; |A \cap B| = 1$

From the theorem:

$|A \cup B| = |A| + |B| - |A \cap B| = 3 + 3 - 1 = 6 - 1 = 5$

Theorem:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Example:

$A = \{a, b, c, x, y\} \Rightarrow |A| = 5$

$B = \{i, j, k, l, m, x, z\} \Rightarrow |B| = 7$

$C = \{a, c, i, k, m, y\} \Rightarrow |C| = 6$

$A \cup B \cup C = \{a, b, c, i, j, k, l, m, x, y, z\} \Rightarrow |A \cup B \cup C| = 11$

$A \cap B = \{x\} \Rightarrow |A \cap B| = 1$

$A \cap C = \{a, c, y\} \Rightarrow |A \cap C| = 3$

$B \cap C = \{i, k, m\} \Rightarrow |B \cap C| = 3$

$A \cap B \cap C = \{\} \Rightarrow |A \cap B \cap C| = 0$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 5 + 7 + 6 - 1 - 3 - 3 + 0$$

$$= 18 - 7 = 11 \quad (A)$$

Example 10. A survey has been taken on methods of commuter travel. Each respondent was asked to check BUS, TRAIN, or AUTOMOBILE as a major method of traveling to work. More than one answer was permitted. The results reported were as follows: BUS, 30 people; TRAIN, 35 people; AUTOMOBILE, 100 people; BUS and TRAIN, 15 people; BUS and AUTOMOBILE, 15 people; TRAIN and AUTOMOBILE, 20 people; and all three methods, 5 people. How many people completed a survey form?

A = people who choose BUS

B = people who choose TRAIN

C = people who choose AUTOMOBILE

$|A| = 30$

$|B| = 35$

$|C| = 100$

$|A \cap B| = 15$

$|A \cap C| = 15$

$|B \cap C| = 20$

$|A \cap B \cap C| = 5$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 30 + 35 + 100 - 15 - 15 - 20 + 5$$

$$= 120$$