Set

A set is any well-defined collection of objects, also called elements of the set.

For exemple: the collection of all children of a class.

The "well-defined" from the definition of a set means that it is possible to decide if an object belongs to the set or not.

Describing a set I

One way of describing a set that has a finite number of elements is by listing the elements of a set between braces. Thus the set of all positive integers that are less than 4 can be written as:

$\{1, 2, 3\}$

The order in wich the elements of a set are listed is not important, this is just a diferent way of describing the same set. Thus $\{1,3,2\}$, $\{2,3,1\}$, $\{3,1,2\}$, $\{3,2,1\}$ and $\{2,1,3\}$ are all representation of the set $\{1,2,3\}$.

Moreover, repeted elements in the listing of the elements of a set can be ignored. Thus $\{1,1,1,1,3,2\}$ is just another representation for the set $\{1,2,3\}$.

Denote sets

Uppercase letters such as A, B, C are used to denote the sets.

And lowercase letters such as a, b, c are used to denote the elements of sets.

Belonging of an element to a set

We indicate the fact that an element x belongs to a set A by writing $x \in A$. We can also indicate the fact that x is not an element of A by writing $x \notin A$.

Example:

Let $A = \{1, 7, 8, 10\}$;

Then $1 \in A$, $2 \notin A$, $4 \notin A$, $7 \in A$, $10 \in A$, $12 \notin A$.

Describing a set II

Sometimes sets contains a large amount of elements, or even infinite number of elements, this means that it is inconvenient or imposible to describe a set by listing all its elements.

Another way to define a set is by specifying a property that the elements of the set have in common. We use the notation $P(x)$ to denote a sentence or statement P concerning the variable object x. The set defined by $P(x)$, writen $\{x \mid P(x)\}$ is the collection of all objects x for wich P is sensible and true.

Example 1:

$\{x \mid x$ is a positive integer less than $4\}$
is the set $\{1, 2, 3\}$

Example 2:

The set consisting of all letters in the word "byte" can be denoted by:

$\{b, y, t, e\}$

or

$\{x \mid x$ is a letter in the word "byte"$\}$

Example 3. ☆

Introducing several sets and their notations:

$\mathbb{Z}^+ = \{x \mid x \text{ is a positive integer}\}$

Thus $\mathbb{Z}^+$ consists of the numbers used for counting: 1, 2, 3, ..

$\mathbb{N} = \{x \mid x \text{ is a positive integer or zero}\}$

$\mathbb{Z} = \{x \mid x \text{ is an integer}\}$

$\mathbb{R} = \{x \mid x \text{ is a real number}\}$

$\emptyset = \{\}$

The set that has no elements, called "the empty set"

Example 4:

Since the square of any real number is positive,

$\{x \mid x \text{ is a real number and } x^2 = -1\} = \emptyset$

Equal sets

We say two sets A and B are equal if they have the same elements, and we write $A = B$.

If $A = \{1, 2, 3\}$ and $B = \{x \mid x \text{ is a positive integer and } x^2 < 12\}$, then $A = B$

If $A = \{\text{BASIC}, \text{PASCAL}, \text{ADA}\}$ and $B = \{\text{ADA}, \text{BASIC}, \text{PASCAL}\}$ then $A = B$.