In Exercises 1 through 4, give the set corresponding to the sequence.

1. 2, 1, 2, 1, 2, 1, 2, 1

2. 0, 2, 4, 6, 8, 10, . . .

3. *aabbccddee* · · · *zz*

4. *abbcccdddd*

1. $\{1, 2\}$
2. $\{x \mid x \in N \wedge (x = 0$ or x is even$)\}$
3. $\{x \mid x$ is a symbol from english alphabete$\}$
4. $\{a, b, c, d\}$

5. Give three different sequences that have $\{x, y, z\}$ as a corresponding set.

x, y, z ; x, z, y ; y, x, z ;

6. Give three different sequences that have $\{1, 2, 3, \ldots\}$ as a corresponding set.

1. $S: s_n = n, \; 1 \leq n \leq \infty$
2. $S: s_1 = 2, s_2 = 1, s_n = s_{n-2} + 2, \; 3 \leq n \leq \infty$
3. S $1, 1, 2, 2, 3, 3, 4, 4, 5, 5, \ldots <=>$

$s_n = [n/2] + (n \% 2)$

In Exercises 7 through 10, write out the first four terms (begin with $n = 1$) of the sequence whose general term is given.

7. $a_n = 5^n$

8. $b_n = 3n^2 + 2n - 6$

9. $c_1 = 2.5, c_n = c_{n-1} + 1.5$

10. $d_1 = -3, d_n = -2d_{n-1} + 1$

7. A. 5, 25, 125, 625

8. B: −1, 10, 27, 50, 79, 114

9 C: 2.5, 4, 5.5, 7

10. D. −3, 7, −13, 27

In Exercises 11 through 16, write a formula for the nth term of the sequence. Identify your formula as recursive or explicit.

11. 1, 3, 5, 7, . . .

12. $\underbrace{0, 3}_{3}, \underbrace{3, 8}_{5}, \underbrace{8, 15}_{7}, \underbrace{15, 24}_{9}, \underbrace{24, 35}_{11}, \ldots$

13. 1, −1, 1, −1, 1, −1, . . .

14. 0, 2, 0, 2, 0, 2, . . .

15. 1, 4, 7, 10, 13, 16

16. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

11 $s_n = 1 + 2(n-1)$ – explicit, $1 \le n \le \infty$

12. $s_1 = 0$, $s_n = s_{n-1} + (1 + 2(n-1))$, $2 \le n \le \infty$ – recursive

13. $s_n = (-1)^{n\%2+1}$, $1 \le n \le \infty$ – explicit

14. $s_n = (1 - n\%2) \cdot 2$, $1 \le n \le \infty$ – explicit

15. $s_n = 1 + 3(n-1)$ – explicit, $1 \le n < \infty$

16 $s_n = 2^{1-n}$, $1 \le n \le \infty$ – explicit

17. Write an explicit formula for the sequence 2, 5, 8, 11, 14, 17, ...

$S_n = 2 + 3(n-1), \quad 1 \leq n \leq \infty$

18. Write a recursive formula for the sequence 2, 5, 7, 12, 19, 31, ...

$s_1 = 2, \quad s_2 = 5, \quad s_n = s_{n-1} + s_{n-2}, \quad 3 \leq n \leq \infty$

19. Let $A = \{x \mid x$ is a real number and $0 < x < 1\}$, $B = \{x \mid x$ is a real number and $x^2 + 1 = 0\}$, $C = \{x \mid x = 4m, m \in Z\}$, $D = \{(x, 3) \mid x$ is an English word whose length is 3$\}$, and $E = \{x \mid x \in Z$ and $x^2 \leq 100\}$. Identify each set as finite, countable, or uncountable.

A - uncountable
B - finite
C - countable
D - finite
E - finite

20. Let $A = \{ab, bc, ba\}$. In each part, tell whether the string belongs to A^*.

(a) *ababab* ✓ (b) *abc* ✗ (c) *abba* ✓
(d) *abbcbaba* ✓ (e) *bcabbab* ✗ (f) *abbbcba* ✗

21. Let U = {FORTRAN, PASCAL, ADA, COBOL, LISP, BASIC, C^{++}, FORTH}, B = {C^{++}, BASIC, ADA}, C = {PASCAL, ADA, LISP, C^{++}}, D = {FORTRAN, PASCAL, ADA, BASIC, FORTH}, and E = {PASCAL, ADA, COBOL, LISP, C^{++}}. In each of the following, represent the given set by an array of zeros and ones.

(a) $B \cup C$ (b) $C \cap D$ (c) $B \cap (D \cap E)$
(d) $\overline{B} \cup E$ (e) $\overline{C} \cap (B \cup E)$

(a)
$B \cup C$ = { C^{++}, BASIC, ADA, PASCAL, LISP }
$B \cup C$: 0, 1, 1, 0, 1, 1, 1, 0
(b) $C \cap D$ = { PASCAL, ADA }
$C \cap D$: 0, 1, 1, 0, 0, 0, 0, 0
(c) $B \cap (D \cap E) = B \cap D \cap E$ = { ADA }
$B \cap (D \cap E)$ = 0, 0, 1, 0, 0, 0, 0, 0
(d) $\overline{B} \cup E$ = { PASCAL, ADA, COBOL, LISP, C++, FORTRAN, FORTH } ; $\overline{B} \cup E$: 1, 1, 1, 1, 1, 0, 1, 1
(e) $\overline{C} \cap (B \cup E)$ = { BASIC, COBOL }
$\overline{C} \cap (B \cup E)$: 0, 0, 0, 1, 0, 1, 0, 0

22. Let $U = \{b, d, e, g, h, k, m, n\}$, $B = \{b\}$, $C = \{d, g, m, n\}$, and $D = \{d, k, n\}$.

(a) What is $f_B(b)$? (b) What is $f_C(e)$?

(c) Find the sequences of length 8 that correspond to f_B, f_C, and f_D.

(d) Represent $B \cup C$, $C \cup D$, and $C \cap D$ by arrays of zeros and ones.

(a) 1 , (b) 0 ;

(c) f_B : 1, 0, 0, 0, 0, 0, 0, 0

f_C : 0, 1, 0, 1, 0, 0, 1, 1

f_D : 0, 1, 0, 0, 0, 1, 0, 1

(d) $B \cup C$: 1, 1, 0, 1, 0, 0, 1, 1

$C \cup D$: 0, 1, 0, 1, 0, 1, 1, 1

$C \cap D$: 0, 1, 0, 0, 0, 0, 0, 1

23. Prove Theorem 1(c).

✓ DONE

24. Using characteristic functions, prove that
$(A \oplus B) \oplus C = A \oplus (B \oplus C)$.

$$f_{(A \oplus B) \oplus C} = f_{A \oplus B} + f_C - 2 f_{A \oplus B} f_C$$

$$= f_A + f_B - 2 f_A f_B + f_C - 2 (f_A + f_B - 2 f_A f_B) f_C$$

$$= f_A + f_B + f_C - 2 f_A f_B - 2 f_A f_C - 2 f_B f_C + 4 f_A f_B f_C$$

$$f_{A \oplus (B \oplus C)} = f_A + f_{B \oplus C} - 2 f_A f_{B \oplus C} =$$

$$= f_A + f_B + f_C - 2 f_B f_C - 2 f_A (f_B + f_C - 2 f_B f_C) =$$

$$= f_A + f_B + f_C - 2 f_B f_C - 2 f_A f_B - 2 f_A f_C + 4 f_A f_B f_C$$

$$= f_{(A \oplus B) \oplus C} \Rightarrow (A \oplus B) \oplus C = A \oplus (B \oplus C) \quad \blacksquare$$

25. Let $A = \{+, \times, a, b\}$. Show that the following expressions are regular over A.

(a) $a + b(ab)^*(a \times b \vee a)$

(b) $a + b \times (a^* \vee b)$

(c) $((a^*b \vee +)^* \vee \times ab^*)$

(a):

a, +, b are RE2, a+b are RE3

a, b are RE2, ab are RE3

$(ab)^*$ is RE5

$a+b(ab)^*$ are RE3

a, x, b are RE2, a x b are RE3

a is RE2

$(a x b \vee a)$ is RE4

$a + b(ab)^*(a x b \vee a)$ is RE3

25. Let $A = \{+, \times, a, b\}$. Show that the following expressions are regular over A.

(a) $a + b(ab)^*(a \times b \vee a)$

(b) $a + b \times (a^* \vee b)$

(c) $((a^*b \vee +)^* \vee \times ab^*)$

(b):

a, +, b, x with RE2 => regular

a + b x with RE3 => regular

a* with RE3 => regular

a* v b with RE4 => regular

a + b x (a* v b) with R3 => regular

25. Let $A = \{+, \times, a, b\}$. Show that the following expressions are regular over A.
(a) $a + b(ab)^*(a \times b \vee a)$
(b) $a + b \times (a^* \vee b)$
(c) $((a^*b \vee +)^* \vee \times ab^*)$

(c):

$a, b, +, \times \overset{RE2}{\Longrightarrow}$ regular

$a^* \overset{RE5}{\Longrightarrow}$ regular

$a^*b \overset{RE3}{\Longrightarrow}$ regular

$(a^*b \vee +) \overset{RE4}{\Longrightarrow}$ regular

$(a^*b \vee +)^* \overset{RE5}{\Longrightarrow}$ regular

$\times a \overset{RE3}{\Longrightarrow}$ regular

$b^* \overset{5}{\Longrightarrow}$ regular

$\times ab^* \overset{3}{\Longrightarrow}$ regular

$((a^*b \vee +)^* \vee \times ab^*) \overset{RE4}{\Longrightarrow}$ regular ■

26. Let $A = \{a, b, c\}$. In each part we list a string in A^* and a regular expression over A. In each case, tell whether or not the string on the left belongs to the regular set corresponding to the regular expression on the right.

(a) ac $\quad \cancel{a^*b^*}c$ ✓

(b) $abcc$ $\quad (abc \vee c)^*$ ✓

(c) $aaabc$ $\quad ((a \vee b) \vee c)^*$ ✓

(d) ac $\quad (a^*b \vee c)$ X

(e) $abab$ $\quad (ab)^*c$ X

27. We define T-numbers recursively as follows:

1. 0 is a T-number.
2. If X is a T-number, $X + 3$ is a T-number.

Write a description of the set of T-numbers.

$T = \{ x \mid x = 3 \cdot i , \; i \in \mathbb{N} \}$

28. Define an S-number by:

1. 8 is an S-number.
2. If X is an S-number and Y is a multiple of X, then Y is an S-number.
3. If X is an S-number and X is a multiple of Y, then Y is an S-number.

Describe the set of S-numbers.

2\. $\ldots, 8\cdot(-2), 8\cdot(-1), 8\cdot 0, 8\cdot 1, 8\cdot 2, \ldots$ are S numbers

3\. $\ldots$ $8\cdot(-2), \ldots$ are S numbers

$\Rightarrow \ldots -2, -1, 0, 1, 2, \ldots$ are S numbers

$\Rightarrow S = \mathbb{Z}$

29. Let F be a function defined for all nonnegative integers by the following recursive definition:

$$F(0) = 0, \qquad F(1) = 1,$$
$$F(N+2) = 2F(N) + F(N+1), \quad N \geq 0$$

Write the first six values of F; that is, write the values of $F(N)$ for $N = 0, 1, 2, 3, 4, 5$.

0, 1, 1, 3, 5, 11, 21, 43, 85, 171,

30. Let G be a function defined for all nonnegative integers by the following recursive definition:

$$G(0) = 1, \qquad G(1) = 2,$$
$$G(N+2) = G(N)^2 + G(N+1), \quad N \geq 0$$

Write the first five values of G.

1, 2, 3, 7, 16