In Exercises 1 *through* 4, *let* $U = \{a, b, c, d, e, f, g, h, k\}$, $A = \{a, b, c, g\}$, $B = \{d, e, f, g\}$, $C = \{a, c, f\}$, *and* $D = \{f, h, k\}$.

1. Compute

(a) $A \cup B$ (b) $B \cup C$ (c) $A \cap C$

(d) $B \cap D$ (e) $A - B$ (f) $\overline{A}$

(g) $A \oplus B$ (h) $A \oplus C$

(a) $\{a, b, c, d, e, f, g\}$

(b) $\{a, c, d, e, f, g\}$

(c) $\{a, c\}$

(d) $\{f\}$

(e) $\{a, b, c\}$

(f) $\{d, e, f, h, k\}$

(g) $\{a, b, c, d, e, f\}$

(h) $\{b, f, g\}$

In Exercises 1 *through* 4, *let* $U = \{a, b, c, d, e, f, g, h, k\}$, $A = \{a, b, c, g\}$, $B = \{d, e, f, g\}$, $C = \{a, c, f\}$, *and* $D = \{f, h, k\}$.

2. Compute

(a) $A \cup D$ (b) $B \cup D$ (c) $C \cap D$
(d) $A \cap D$ (e) $B - C$ (f) $\overline{B}$
(g) $C - B$ (h) $C \oplus D$

(a) $\{a, b, c, f, g, h, k\}$
(b) $\{d, e, f, g, h, k\}$
(c) $\{f\}$
(d) $\varnothing$
(e) $\{d, e, g\}$
(f) $\{a, b, c, h, k\}$
(g) $\{a, c\}$
(h) $\{a, c, d, e, g\}$

In Exercises 1 *through* 4, *let* $U = \{a, b, c, d, e, f, g, h, k\}$, $A = \{a, b, c, g\}$, $B = \{d, e, f, g\}$, $C = \{a, c, f\}$, *and* $D = \{f, h, k\}$.

3. Compute

(a) $A \cup B \cup C$ (b) $A \cap B \cap C$

(c) $A \cap (B \cup C)$ (d) $(A \cup B) \cap C$

(e) $\overline{A \cup B}$ (f) $\overline{A \cap B}$

(a) $\{a, b, c, d, e, f, g\}$

(b) $\emptyset$

(c) $\{a, c, g\}$

(d) $\{a, c, f\}$

(e) $\{h, k\}$

(f) $\{a, b, c, d, e, f, h, k\}$

In Exercises 1 *through* 4, *let* $U = \{a, b, c, d, e, f, g, h, k\}$, $A = \{a, b, c, g\}$, $B = \{d, e, f, g\}$, $C = \{a, c, f\}$, *and* $D = \{f, h, k\}$.

4. Compute

(a) $A \cup \emptyset$ (b) $A \cup U$ (c) $B \cup B$

(d) $C \cap \{\,\}$ (e) $\overline{C \cup D}$ (f) $\overline{C \cap D}$

(a) A

(b) U

(c) B

(d) $\emptyset$

(e) $\{b, d, e, g\}$

(f) $\{a, b, c, d, e, g, h, k\}$

In Exercises 5 and 6, let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 4, 5, 9\}$, $C = \{x \mid x$ *is a positive integer and* $x^2 \leq 16\}$, *and* $D = \{7, 8\}$.

$C = \{1, 2, 3, 4\}$

5. Compute

(a) $A \cup B$ (b) $A \cup C$ (c) $A \cup D$
(d) $B \cup C$ (e) $A \cap C$ (f) $A \cap D$
(g) $B \cap C$ (h) $C \cap D$ (i) $A - B$
(j) $B - A$ (k) $C - D$ (l) $\overline{C}$
(m) $\overline{A}$ (n) $A \oplus B$ (o) $C \oplus D$
(p) $B \oplus C$

(a) $\{1, 2, 4, 5, 6, 8, 9\}$
(b) $\{1, 2, 3, 4, 6, 8\}$
(c) $\{1, 2, 4, 6, 7, 8\}$
(d) $\{1, 2, 3, 4, 5, 9\}$
(e) $\{1, 2, 4\}$
(f) $\{8\}$
(g) $\{2, 4\}$
(h) $\emptyset$
(i) $\{1, 6, 8\}$
(j) $\{5, 9\}$
(k) C
(l) $\{5, 6, 7, 8, 9\}$

(m) $\{3, 5, 7, 9\}$
(n) $\{1, 5, 6, 8, 9\}$
(o) $\{1, 2, 3, 4, 7, 8\}$
(p) $\{1, 3, 5, 9\}$

In Exercises 5 and 6, let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 6, 8\}$, $B = \{2, 4, 5, 9\}$, $C = \{x \mid x$ *is a positive integer and* $x^2 \leq 16\}$, *and* $D = \{7, 8\}$.

$C = \{1, 2, 3, 4\}$

6. Compute

(a) $A \cup B \cup C$ (b) $A \cap B \cap C$

(c) $A \cap (B \cup C)$ (d) $(A \cup B) \cap D$

(e) $\overline{A \cup B}$ (f) $\overline{A \cap B}$

(g) $B \cup C \cup D$ (h) $B \cap C \cap D$

(i) $A \cup A$ (j) $A \cap \overline{A}$

(k) $A \cup \overline{A}$ (l) $A \cap (\overline{C} \cup D)$

(a) $\{1, 2, 3, 4, 5, 6, 8, 9\}$

(b) $\{2, 4\}$

(c) $\{1, 2, 4\}$

(d) $\{8\}$

(e) $\{3, 7\}$

(f) $\{1, 3, 5, 6, 7, 8, 9\}$

(g) $\{1, 2, 3, 4, 5, 7, 8, 9\}$

(h) $\varnothing$

(i) A

(j) $\varnothing$

(k) U

(l) $\{6, 8\}$

In Exercises 7 and 8, let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, c, f, g\}$, $B = \{a, e\}$, $B = \{a, e\}$, *and* $C = \{b, h\}$.

7. Compute

(a) $\overline{A}$ (b) $\overline{B}$ (c) $\overline{A \cup B}$

(d) $\overline{A \cap B}$ (e) $\overline{U}$ (f) $A - B$

(a) $\{b, d, e, h\}$

(b) $\{b, c, d, f, g, h\}$

(c) $\{b, d, h\}$

(d) $\{b, c, d, e, f, g, h\}$

(e) $\varnothing$

(f) $\{c, f, g\}$

In Exercises 7 and 8, let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, c, f, g\}$, $B = \{a, e\}$, $B = \{a, e\}$, *and* $C = \{b, h\}$.

8. Compute

(a) $\overline{A} \cap \overline{B}$ (b) $\overline{B} \cup \overline{C}$ (c) $\overline{A \cup A}$

(d) $\overline{C} \cap \overline{C}$ (e) $A \oplus B$ (f) $B \oplus C$

(a) $\overline{A \cup B} \overset{(7c)}{=} \{b, d, h\}$

(b) $\overline{B \cap C} = \overline{\emptyset} = U$

(c) $\overline{A} \overset{(7a)}{=} \{b, d, e, h\}$

(d) $\{a, c, d, e, f, g\}$

(e) $\{c, e, f, g\}$

(f) $\{a, b, e, h\}$

9. Let U be the set of real numbers, $A = \{x \mid x \text{ is a solution of } x^2 - 1 = 0\}$, and $B = \{-1, 4\}$. Compute

(a) $\overline{A}$ (b) $\overline{B}$ (c) $\overline{A \cup B}$

(d) $\overline{A \cap B}$

$x^2 - 1 = 0 \Leftrightarrow (x+1)(x-1) = 0 \Rightarrow A = \{-1, 1\}$

(a) $\{x \mid x \in \mathbb{R} \text{ and } x \neq -1 \text{ and } x \neq 1\}$

(b) $\{x \mid x \in \mathbb{R} \text{ and } x \neq -1 \text{ and } x \neq 4\}$

(c) $\{x \mid x \in \mathbb{R} \text{ and } x \neq -1 \text{ and } x \neq 1 \text{ and } x \neq 4\}$

(d) $\{x \mid x \in \mathbb{R} \text{ and } x \neq -1\}$

In Exercises 10 *and* 11, *refer to Figure* 1.13.

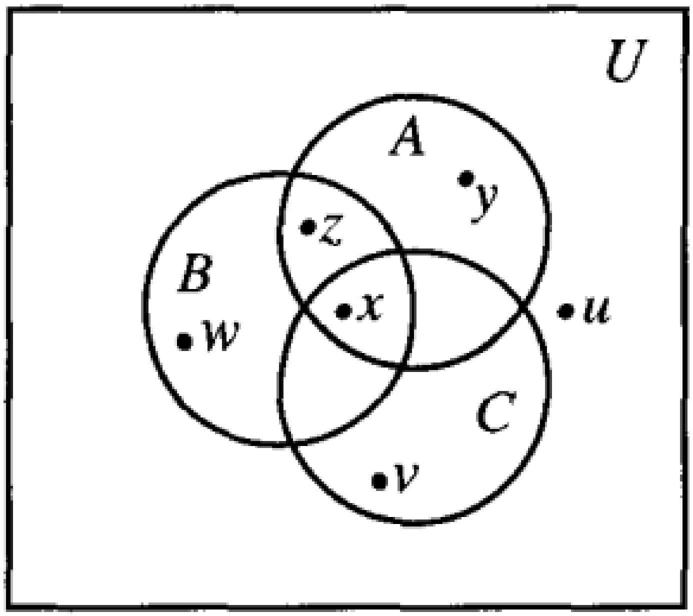


Figure 1.13

10. Identify the following as true or false.

(a) $y \in A \cap B$ (b) $x \in B \cup C$

(c) $w \in B \cap C$ (d) $u \notin C$

11. Identify the following as true or false.

(a) $x \in A \cap B \cap C$ (b) $y \in A \cup B \cup C$

(c) $z \in A \cap C$ (d) $v \in B \cap C$

12. Describe the shaded region shown in Figure 1.14 using unions and intersections of the sets A, B, and C. (Several descriptions are possible.)

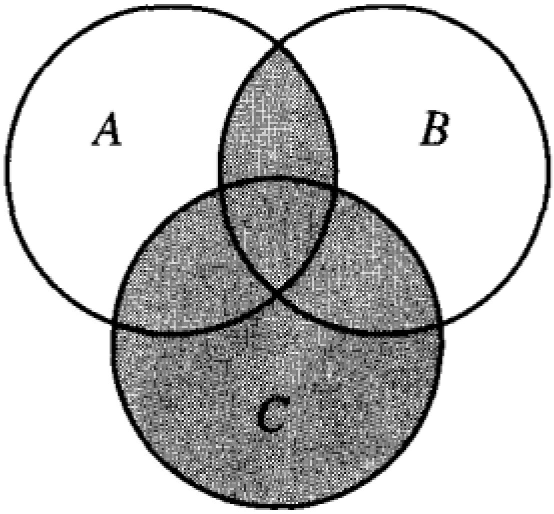


Figure 1.14

$C \cup (A \cap B)$

13. Let A, B, and C be finite sets with $|A| = 6$, $|B| = 8$, $|C| = 6$, $|A \cup B \cup C| = 11$, $|A \cap B| = 3$, $|A \cap C| = 2$, and $|B \cap C| = 5$. Find $|A \cap B \cap C|$.

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$\Leftrightarrow$

$11 = 6 + 8 + 6 - 3 - 2 - 5 + |A \cap B \cap C| \Leftrightarrow$

$11 = 20 - 10 + |A \cap B \cap C| \Leftrightarrow$

$11 = 10 + |A \cap B \cap C| \Rightarrow |A \cap B \cap C| = 1$

14. Verify Theorem 2 for the following sets.

(a) $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6, 8\}$; $|A| = 4$, $|B| = 5$
(b) $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7, 8, 9\}$; $|A| = 4$, $|B| = 5$
(c) $A = \{a, b, c, d, e, f\}$, $B = \{a, c, f, g, h, i, r\}$; $|A| = 6$, $|B| = 7$
(d) $A = \{a, b, c, d, e\}$, $B = \{f, g, r, s, t, u\}$, $|A| = 5$, $|B| = 6$
(e) $A = \{x \mid x \text{ is a positive integer} < 8\}$, $|A| = 7$
$B = \{x \mid x \text{ is an integer such that } 2 \le x \le 5\}$ $|B| = 4$
(f) $A = \{x \mid x \text{ is a positive integer and } x^2 \le 16\}$, $|A| = 4$
$B = \{x \mid x \text{ is a negative integer and } x^2 \le 25\}$, $|B| = 5$

(a) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\} \Rightarrow |A \cup B| = 7$
$A \cap B = \{2, 3\} \Rightarrow |A \cap B| = 2$

$|A \cup B| = |A| + |B| - |A \cap B| = 4 + 5 - 2 = 9 - 2 = 7$

(b) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \Rightarrow |A \cup B| = 9$
$A \cap B = \emptyset \Rightarrow |A \cap B| = 0$

$|A \cup B| = |A| + |B| - |A \cap B| = 4 + 5 - 0 = 9$

(c) $A \cup B = \{a, b, c, d, e, f, g, h, i, r\} \Rightarrow |A \cup B| = 10$
$A \cap B = \{a, c, f\} \Rightarrow |A \cap B| = 3$
$|A \cup B| = |A| + |B| - |A \cap B| = 6 + 7 - 3 = 13 - 3 = 10$

(d) $A \cup B = \{a, b, c, d, e, f, g, r, s, t, u\} \Rightarrow |A \cup B| = 11$
$A \cap B = \emptyset \Rightarrow |A \cap B| = 0$
$|A \cup B| = |A| + |B| - |A \cap B| = 5 + 6 - 0 = 11$

(e) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\} \Rightarrow |A \cup B| = 7$
$A \cap B = \{2, 3, 4, 5\} \Rightarrow |A \cap B| = 4$

$|A \cup B| = |A| + |B| - |A \cap B| = 7 + 4 - 4 = 7$

(f) $A \cup B = \{-5, -4, -3, -2, -1, 1, 2, 3, 4\} \Rightarrow |A \cup B| = 9$
$A \cap B = \emptyset \Rightarrow |A \cap B| = 0$
$|A \cup B| = |A| + |B| - |A \cap B| = 4 + 5 - 0 = 9$

15. If A and B are disjoint sets such that $|A \cup B| = |A|$, what must be true about B?

If: A & B are disjoint $\Rightarrow A \cap B = \emptyset$,

$|A \cup B| = |A| + |B| - |A \cap B| \Leftrightarrow$

$|A \cup B| = |A| + |B|$, becouse $A \cap B = \emptyset \Rightarrow$
$|A \cap B| = 0$.

And becouse of that

$|A \cup B| = |A| + |B| - 0 = |A| + |B|$
, but $|A \cup B| = |A|$ $\Rightarrow$

$\Rightarrow |A| + |B| = |A| \Rightarrow |B| = 0 \Rightarrow B = \emptyset$

16. Verify Theorem 3 for the following sets:

(a) $A = \{a, b, c, d, e\}$, $B = \{d, e, f, g, h, i, k\}$, $|A| = 5$, $|B| = 7$, $|C| = 8$
$C = \{a, c, d, e, k, r, s, t\}$

(b) $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 7, 8, 9\}$, $|A| = 6$, $|B| = 5$, $|C| = 6$
$C = \{1, 2, 4, 7, 10, 12\}$

(c) $A = \{x \mid x \text{ is a positive integer} < 8\}$,
$B = \{x \mid x \text{ is an integer such that } 2 \leq x \leq 4\}$,
$C = \{x \mid x \text{ is an integer such that } x^2 < 16\}$

(a) $A \cup B \cup C = \{a, b, c, d, e, f, g, h, i, k, r, s, t\} \Rightarrow |A \cup B \cup C| = 13$

$A \cap B = \{d, e\} \Rightarrow |A \cap B| = 2$

$A \cap C = \{a, c, d, e\} \Rightarrow |A \cap C| = 4$

$B \cap C = \{d, e, k\} \Rightarrow |B \cap C| = 3$

$A \cap B \cap C = \{d, e\} \Rightarrow |A \cap B \cap C| = 2$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| =$$
$$= 5 + 7 + 8 - 2 - 4 - 3 + 2$$
$$= 20 - 7 = 13$$

(b) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\} \Rightarrow |A \cup B \cup C| = 11$

$A \cap B = \{2, 4\} \Rightarrow |A \cap B| = 2$

$A \cap C = \{1, 2, 4\} \Rightarrow |A \cap C| = 3$

$B \cap C = \{2, 4, 7\} \Rightarrow |B \cap C| = 3$

$A \cap B \cap C = \{2, 4\} \Rightarrow |A \cap B \cap C| = 2$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap D| - |B \cap C| + |A \cap B \cap C|$$
$$= 6 + 5 + 6 - 2 - 3 - 3 + 2 = 17 - 6 = 11$$

(c) $A = \{x \mid x \text{ is a positive integer} < 8\}$, $|A| = 7$
$B = \{x \mid x \text{ is an integer such that } 2 \leq x \leq 4\}$, $|B| = 3$
$C = \{x \mid x \text{ is an integer such that } x^2 < 16\}$, $|C| = 7$

$A \cup B \cup C = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\} \Rightarrow |A \cup B \cup C| = 11$

$A \cap B = \{2, 3, 4\} \Rightarrow |A \cap B| = 3$

$A \cap C = \{1, 2, 3\} \Rightarrow |A \cap C| = 3$

$B \cap C = \{2, 3\} \Rightarrow |B \cap C| = 2$

$A \cap B \cap C = \{2, 3\} \Rightarrow |A \cap B \cap C| = 2$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$= 7 + 3 + 7 - 3 - 3 - 2 + 2$$
$$= 17 - 6 = 11$$

17. In a survey of 260 college students, the following data were obtained:

64 had taken a mathematics course,
94 had taken a computer science course,
58 had taken a business course,
28 had taken both a mathematics and a business course,
26 had taken both a mathematics and a computer science course,
22 had taken both a computer science and a business course, and
14 had taken all three types of courses.

(a) How many students were surveyed who had taken none of the three types of courses?

(b) Of the students surveyed, how many had taken only a computer science course?

$|M| = 64$

$|C| = 94$

$|B| = 58$

$|M \cap B| = 28$

$|M \cap C| = 26$

$|C \cap B| = 22$

$|M \cap C \cap B| = 14$

(a) $|M \cup C \cup B| = |M| + |C| + |B| - |M \cap C| - |M \cap B| - |C \cap B| + |M \cap C \cap B|$

$= 64 + 94 + 58 - 26 - 28 - 22 + 14 = 154$

$260 - 154 = 106$

(b) $94 - 26 - 22 - 14 = 32$

19. In a psychology experiment, the subjects under study were classified according to body type and gender as follows:

	A	B	C
	Endomorph	Ectomorph	Mesomorph
M Male	72	54	36
F Female	62	64	38

(a) How many male subjects were there?
(b) How many subjects were ectomorphs?

(a) $72+54+36 = 162$

(b) $54+64 = 118$

(d) $|M|+|F|-|M \cap C|$
$= 162+164-36$
$= 290$

(c) How many subjects were either female or endomorphs?
(d) How many subjects were not male mesomorphs?
(e) How many subjects were either male, ectomorph, or mesomorph?

(e) $|M|+|B|+|C|$
$-|M \cap B|-|M \cap C|$
$= 162+118+74$
$-54-36$
$= 264$

(c) $|F| = 62+64+38 = 164$
$|A| = 72+62 = 134$
$|F \cup A| = |F|+|A|-|F \cap A|$
$= 164+134-62$
$= 164+72$
$= 236$

20. Prove that $A \subseteq A \cup B$.

$(\forall)\ x \in A \Rightarrow x \in A \vee x \in B \Leftrightarrow x \in A \cup B \Rightarrow$

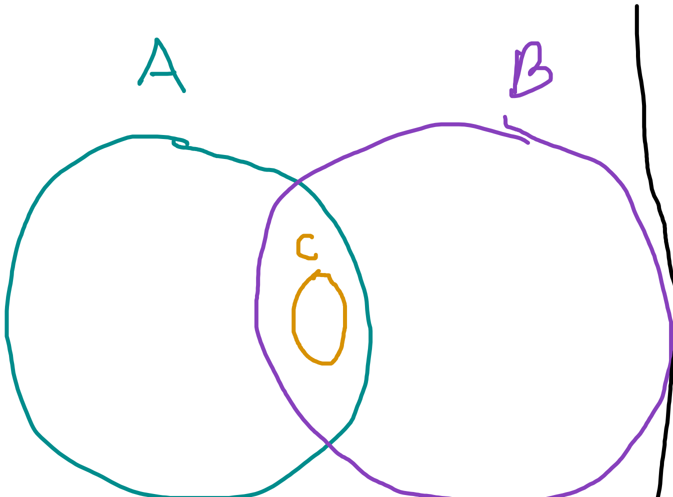
$\Rightarrow A \subseteq A \cup B$ $\blacksquare$

21. Prove that $A \cap B \subseteq A$.

$(\forall)\ x \in A \cap B \Rightarrow x \in A \wedge x \in B$, and because $x \in A$ and also B, for all element x in $A \cap B \Rightarrow A \cap B \subseteq A$ $\blacksquare$

22. (a) Draw a Venn diagram to represent the situation $C \subseteq A$ and $C \subseteq B$.
(b) Prove that if $C \subseteq A$ and $C \subseteq B$, then $C \subseteq A \cup B$.

(a)



(b) $C \subseteq A$ and $C \subseteq B \Rightarrow$
$(\forall)\ c \in C,\ c \in A \wedge c \in B$
$\Leftrightarrow c \in A \cap B \Rightarrow$
$C \subseteq A \cap B \subseteq A \subseteq A \cup B$

$\blacksquare$

24. Prove that $A - A = \varnothing$.

If $x \in A - A \Rightarrow x \in A \land x \notin A$, there is no such elements $\Rightarrow A - A = \{x \mid x \in A \land x \notin A\} = \varnothing$ $\blacksquare$

25. Prove that $A - B = A \cap \overline{B}$.

$A - B = \{x \mid x \in A \land x \notin B\} = \{x \mid x \in A \land x \in \overline{B}\} = A \cap \overline{B}$

$\blacksquare$

26. Prove that $A - (A - B) \subseteq B$.

$A - (A - B) = A - (A \cap \overline{B}) = A \cap \overline{(A \cap \overline{B})} =$

$A \cap (\overline{A} \cup \overline{\overline{B}}) = A \cap (\overline{A} \cup B)$

$= (A \cap \overline{A}) \cup (A \cap B) = \varnothing \cup (A \cap B) =$

$= A \cap B \subseteq B$ $\blacksquare$

27. If $A \cup B = A \cup C$, must $B = C$? Explain.

for exemple.

$B = A \neq \emptyset$ and $C = \emptyset \Rightarrow$

$$\left.\begin{array}{l} A \cup B = A \cup A = A \\ A \cup C = A \cup \emptyset = A \end{array}\right\} \Rightarrow$$

$A \cup B = A \cup C$ and $B \neq C \Rightarrow$

If $A \cup B = A \cup C$, B is not mandatory $= C$ $\blacksquare$

28. If $A \cap B = A \cap C$, must $B = C$? Explain.

for example

$B = U$ and $C = A \neq U \Rightarrow$

$A \cap B = A \cap U = A$

$A \cap C = A \cap A = A$ $\not\Rightarrow$

$A \cap B = A \cap C$ and $B \neq C \Rightarrow$

If $A \cap B = A \cap C$, B must not $= C$ $\blacksquare$

29. Prove that if $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$ and $A \cap C \subseteq B \cap D$.

$A \subseteq A \cup C \subseteq B \cup D$ $\blacksquare$

$A \cap C \subseteq A \subseteq B$

$A \cap C \subseteq C \subseteq D$ $\not\Rightarrow$ $A \cap C \subseteq B \cap D$ $\blacksquare$

30. When is $A - B = B - A$? Explain.

If $A = B \Rightarrow$

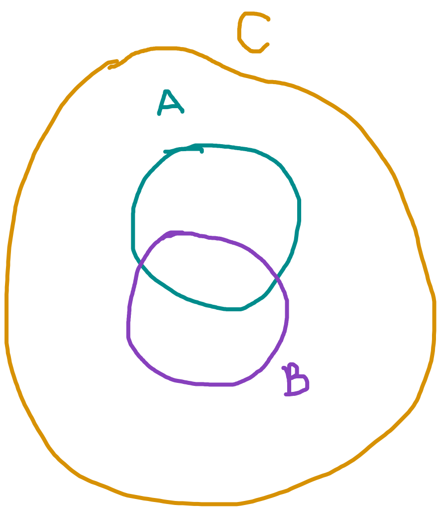
$$\left.\begin{array}{l} A - B = A - A = \emptyset \\ B - A = A - A = \emptyset \end{array}\right\} \Rightarrow A - B = B - A, \; (\forall) A, B,$$

$A = B$. $\blacksquare$

23. (a) Draw a Venn diagram to represent the situation $A \subseteq C$ and $B \subseteq C$.

(b) Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

(a)



(b) $A \subseteq C \wedge B \subseteq C \iff (\forall)\, a \in A \wedge b \in B,$

$a \in C \wedge b \in C \iff (\forall)\, x \in A \vee x \in B,$

$x \in C \iff A \cup B \subseteq C$ ∎