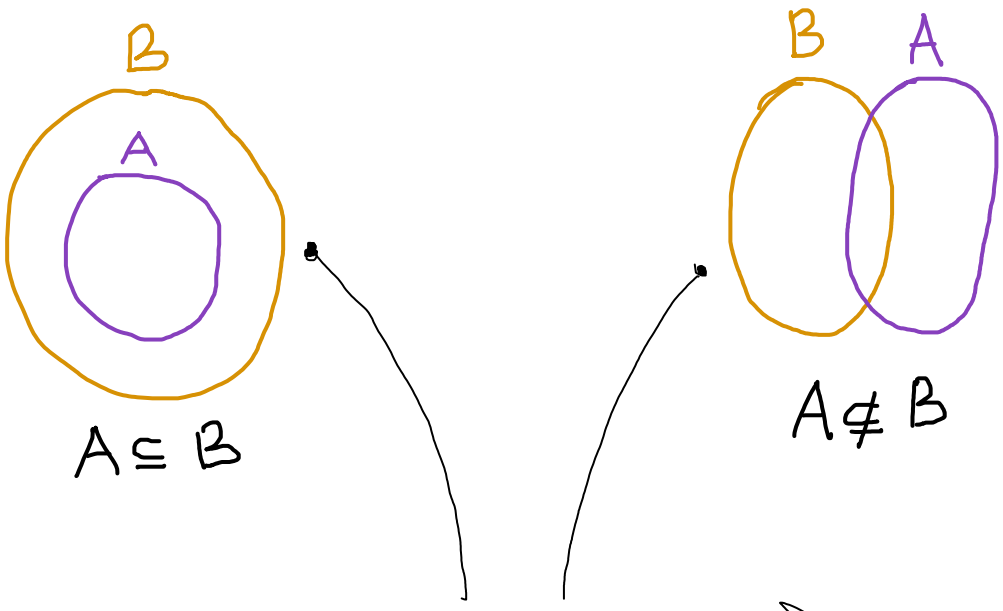
Subsets

If every element of A is also an element of B, that is, if whenever $x \in A$ then $x \in B$, we say that A is a subset of B, or that A is contained in B, and we write $A \subseteq B$.

If A is not a subset of B, we write $A \nsubseteq B$.



Venn diagrams

(used to show relationships between sets)

Example 1:

$Z^+ \subseteq Z$

$Z \subseteq Q = \{x \mid x \text{ is a rational number}\}$

Example 2:

Let $A = \{1,2,3,4,5,6\}$, $B = \{2,4,5\}$ and $C = \{1,2,3,4,5\}$.

Then $B \subseteq A$, $B \subseteq C$, and $C \subseteq A$

However, $A \not\subseteq B$, $A \not\subseteq C$, and $C \not\subseteq B$

Example 3:

If A is any set, then $A \subseteq A$. ✡

Example 4:

Let A be a set and let $B = \{A, \{A\}\}$. Then, since A and $\{A\}$ are elements of B, we have $A \in B$ and $\{A\} \in B$

It follows that $\{A\} \subseteq B$ and $\{\{A\}\} \subseteq B$

However $A \not\subseteq B$ as long $A \notin A$ and $\{A\} \notin A$

Empty set

For every set A, since there are no elements of $\varnothing$ that are not in A, we have $\varnothing \subseteq A$.

The collection of everything

This collection that contains everything, it turns out, cannot be considered a set without destroying the logical structure of mathematics. To avoid this, we will assume that for each discussion there is an universal set U (which will vary with the discussion) containing all objects for which the discussion is meaningful Any other set mentioned in the discussion will automatically be assumed to be a subset of U

Thus, if we are discussing real numbers and we mention sets A and B, then A and B must (we assume) be sets of real numbers, not matrices, electronic cincuits, or animals So $U = \mathbb{R}$ for that case.

In most problems, a universal set will be apparent from the setting of the problem

★ In Venn diagrams, the universal set U will be denoted by a rectangle, while sets within U will be denoted by circles