# The definition of sequences

A sequence is simply a list of objects in order. a first element, a second element, a third element, and so on.. This list may stop after $n$ elements, $n \in \mathbb{N}$, or it may go on forever. In the first case, we say that the sequence is finite, and in the second case we say that is infinite.

Example 1. The sequence 1, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1 is a finite sequence with repeted items. The digit zero, for example, occurs as the second, third, fifth, seventh, and eight elements of the sequence.

# Describing a sequence

Example 2. The list 3, 8, 13, 18, 23, . . . is an infinite sequence. The three dots in the expression mean "and so on"; that is, continue the pattern established by the first few elements. ◆

Example 3. Another infinite sequence is 1, 4, 9, 16, 25, . . . , the list of the squares of all positive integers. ◆

It may happen that how a sequence is to continue is not clear from the first few terms. Also, it may be useful to have a compact notation to describe a sequence. Two kinds of formulas are commonly used to describe sequences. In Example 2, a natural description of the sequence is that successive terms are produced by adding 5 to the previous term. If we use a subscript to indicate a term's position in the sequence, we can describe the sequence in Example 2 as $a_1 = 3$, $a_n = a_{n-1} + 5$, $2 \leq n < \infty$. A formula like this, which refers to previous terms to define the next term, is called **recursive**. Every recursive formula must include a starting place.

On the other hand, in Example 3 it is easy to describe a term using only its position number. In the $n$th position is the square of $n$; $b_n = n^2$, $1 \leq n < \infty$. This type of formula is called **explicit**, because it tells us exactly what value any particular term has.

For the example 2, we can make an explicit formula as well for the sequence:
$a_n = 3 + 5(n-1)$; So, $a_1 = 3 + 5(1-1) = 3$ ✓, $a_2 = 3 + 5(2-1) = 8$ ✓, $a_3 = 3 + 5(3-1) = 13$ ✓, $a_4 = 3 + 5(4-1) = 18$ ✓, . . . , $a_6 = 3 + 5(6-1) =$
$= 3 + 5 \quad 5 = 3 + 25 = 28$, . .

Example 4: $c_1 = 5$; $c_n = 2 \cdot c_{n-1}$, $2 \leq n \leq 6 \Rightarrow$ Sequence: 5, 10, 20, 40, 80, 160;

Example 5: Sequence: 3, 7, 11, 15, 19, 23, ..., has the formula: $d_1 = 3$, $d_n = d_{n-1} + 4$, $n \geq 2$.

Example 6. The explicit formula $s_n = (-4)^n$, $1 \leq n \leq \infty$, describes the sequence: $-4, 16, -64, 256, -1024, 4096, \ldots$

Example 7: Sequence: 87, 82, 77, 72, 67, has the formula $t_n = 92 - 5 \cdot n$, $1 \leq n \leq 5$.

Example 8. An ordinary English word such as "sturdy" can be viewed as the finite sequence

$$s, t, u, r, d, y$$

composed of letters from the ordinary English alphabet. ◆

String

In examples such as Example 8, it is common to omit the commas and write the word in the usual way, if no confusion results. Similarly, even a meaningless word such as "abacabcd" may be regarded as a finite sequence of length 8. Sequences of letters or other symbols, written without the commas, are also referred to as **strings.**

Example 9. An infinite string such as *abababab*. . . may be regarded as the infinite sequence $a, b, a, b, a, b, \ldots$. ◆

Example 10. The sentence "now is the time for the test" can be regarded as a finite sequence of English words: now, is, the, time, for, the, test. Here the elements of the sequence are themselves words of varying length, so we would not be able simply to omit the commas. The custom is to use spaces instead of commas in this case. ◆

The set corresponding to a sequence

The **set corresponding to a sequence** is simply the set of all distinct elements in the sequence. Note that an essential feature of a sequence is the order in which the elements are listed. However, the order in which the elements of a set are listed is of no significance at all.

Example 11
(a) The set corresponding to the sequence in Example 3 is $\{1, 4, 9, 16, 25, \ldots\}$.
(b) The set corresponding to the sequence in Example 9 is simply $\{a, b\}$. ◆

# Arrays

If we have a sequence $S$: $s_1, s_2, \ldots$, and we change any $s_i$ in $S$, we will obtain another sequence, different from $S$, denoted by, let say, $S'$. Also, we see sequences with all the elements completely determined

An array, on the other hand, may be viewed as a sequence of positions, which we represent in Figure 1.15 as boxes. The positions form a finite or infinite list, depending on the desired size of the array. Elements from some set may be assigned to the positions of the array $S$. The element assigned to position $n$ will be denoted by $S[n]$, and the sequence $S[1], S[2], S[3], \ldots$ will be called the **sequence of values** of the array $S$. The point is that $S$ is considered to be a well-defined object, even if some of the positions have not been assigned values or if some values are changed during the discussion. The following shows one use of arrays.


**Figure 1.15**