Operations on sets

If A and B are sets, we define their union as the set consisting of all elements that belong to A or B and denote it $A \cup B$. Thus,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Example 1:

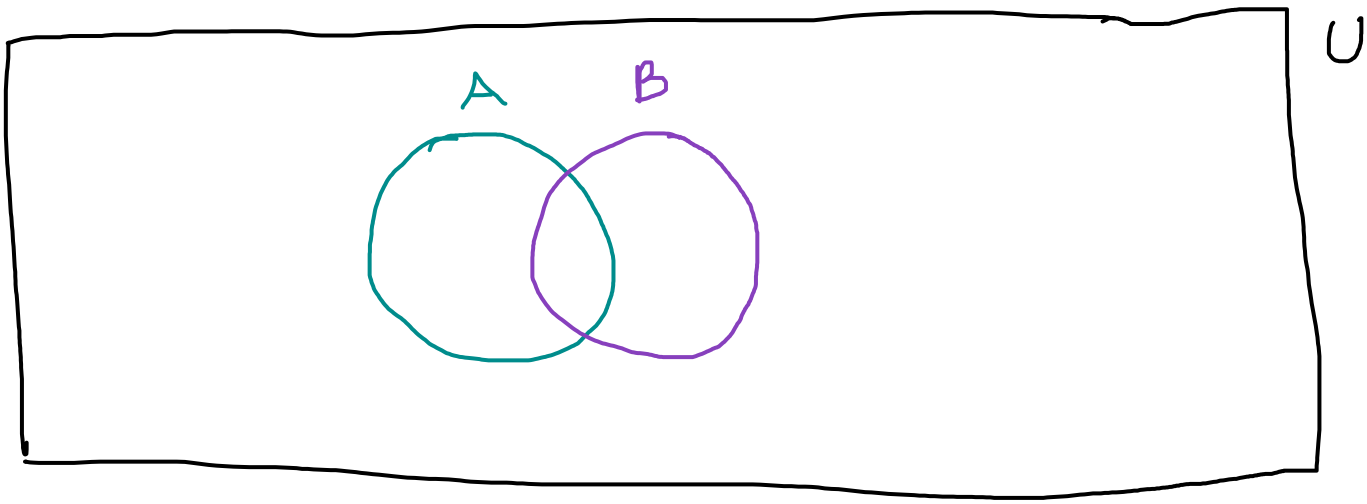
Let $A = \{a, b, c, e, f\}$ and $B = \{b, d, r, s\}$, then

$A \cup B = \{a, b, c, d, e, f, r, s\}$

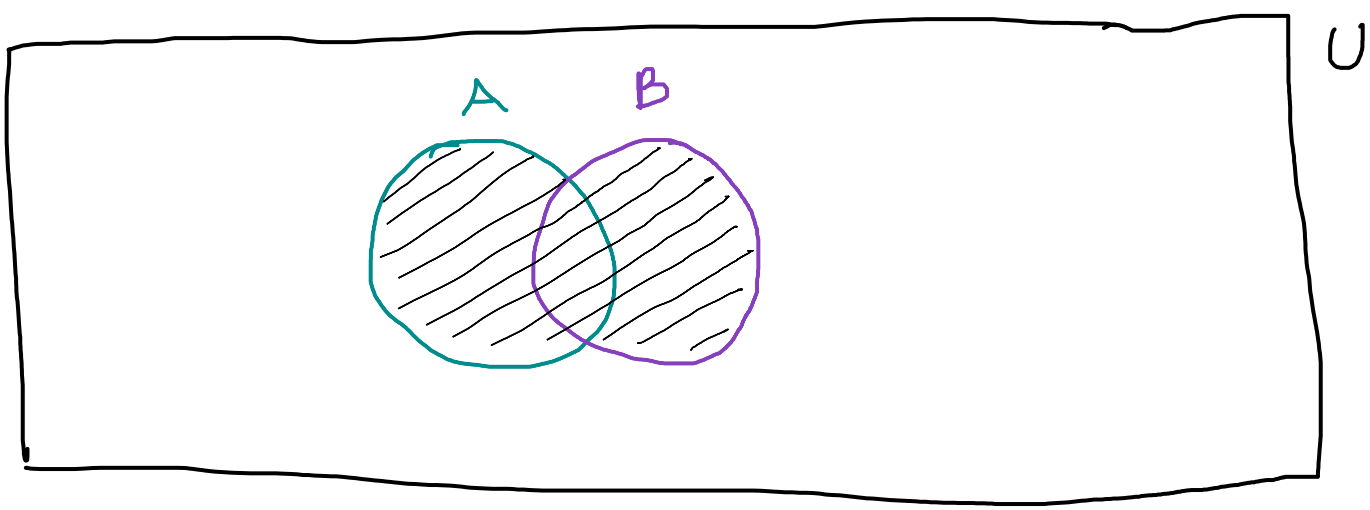
★ **Union illustration** with a Venn diagram:

If A and B are two sets, given in figure 1(a), then $A \cup B$ is the set represented by the shaded region in figure 1(b).

1(a)



1(b)



If A and B are sets, we define their ☆ intersection as the set consisting of all elements that belong to both A and B and denote it by $A \cap B$. Thus,

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

Example 2:

Let $A = \{a, b, c, e, f\}$, $B = \{b, e, f, r, s\}$, and $C = \{a, t, u, v\}$, then

$A \cap B = \{ b, e, f \}$

$A \cap C = \{ a \}$

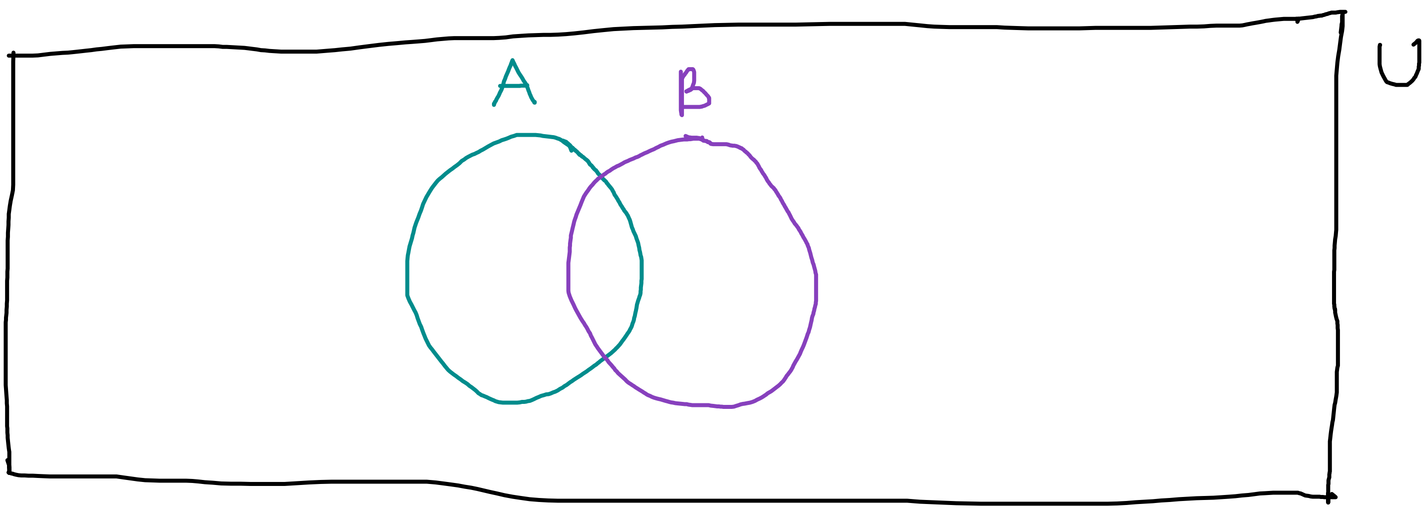
$B \cap C = \varnothing$

Two sets that have no common elements, such as B and C in example 2, are called ☆ disjoint sets.

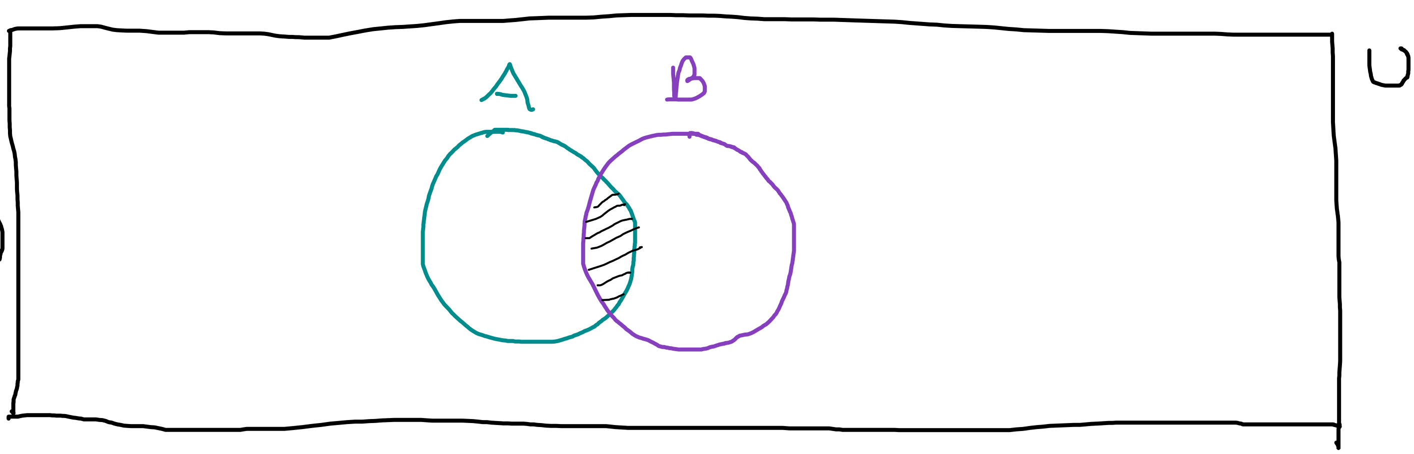
✯ Intersection illustration with a Venn diagram:

If A and B are two sets, given in figure 2(a), then $A \cap B$ is the set represented by the shaded region in figure 2(b).

2(a)



2(b)



Union and intersection on three or more sets:

The operation of union and intersection can be defined for three or more sets in an obvious maner:

$$A \cup B \cup C = \{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\}$$

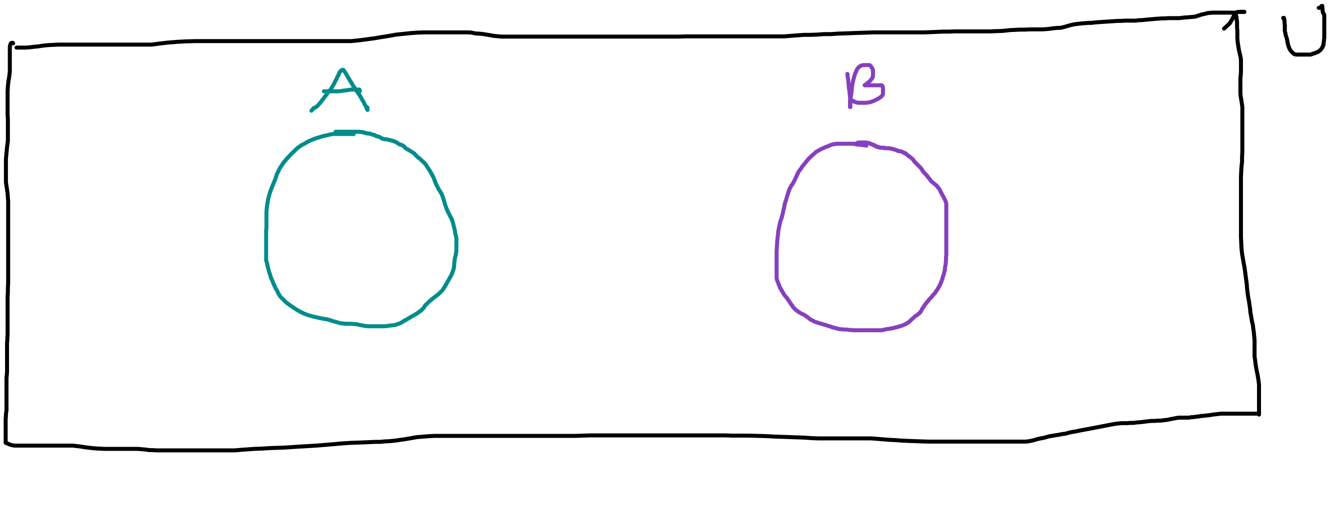
and

$$A \cap B \cap C = \{x \mid x \in A \text{ and } x \in B \text{ and } x \in C\}.$$

★ **Disjoint sets** illustration with a Venn diagram:

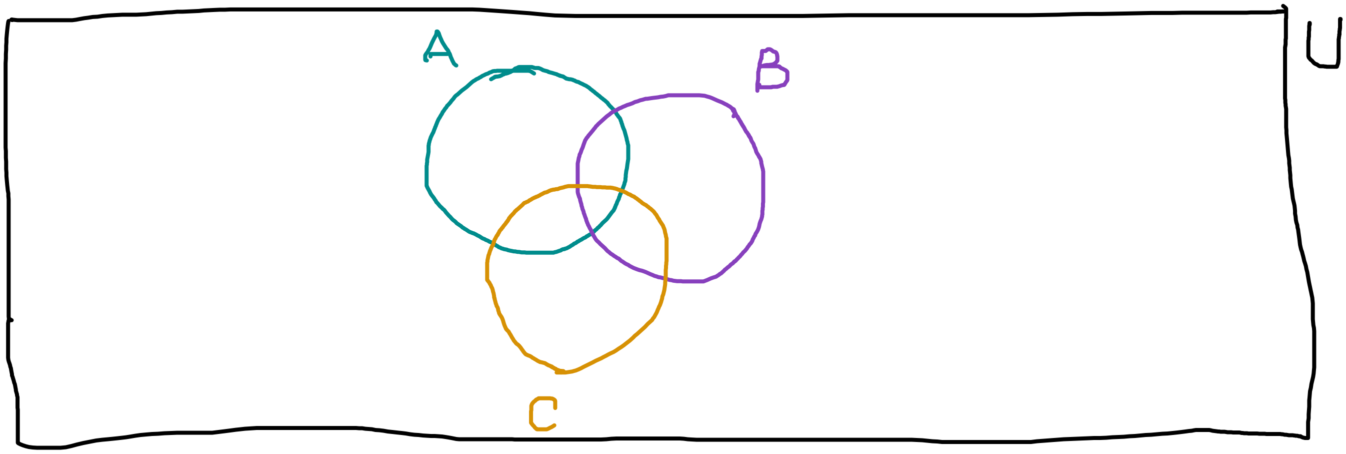
Figure 3 illustrates a Venn diagram for two disjoint sets:

3

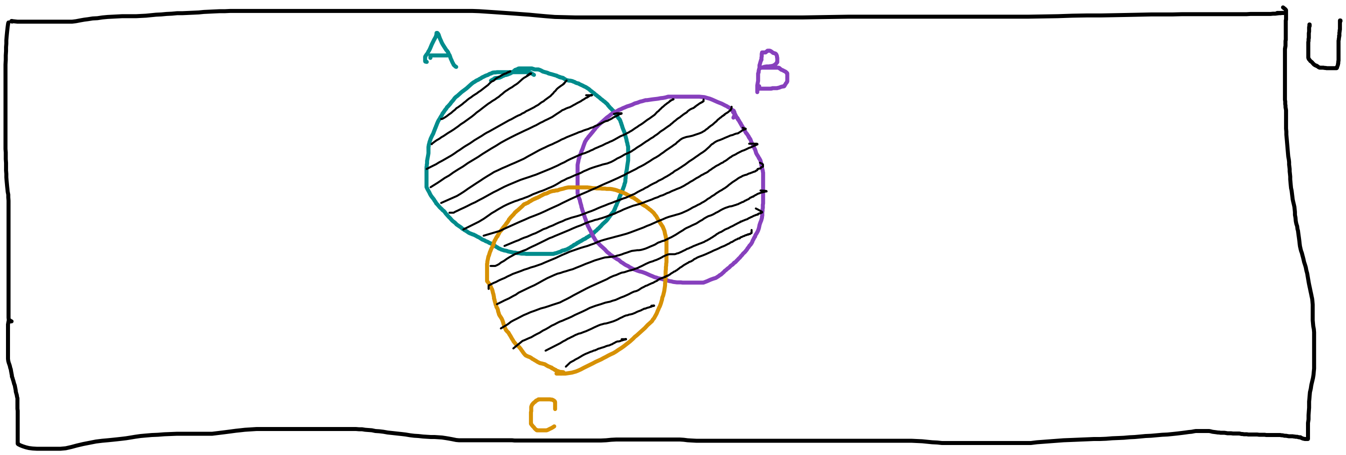


The Venn diagram in figure 4(a) represents three sets A, B and C, the union and intersection of them are illustrated in figure 4(b) and respectively 4(c)

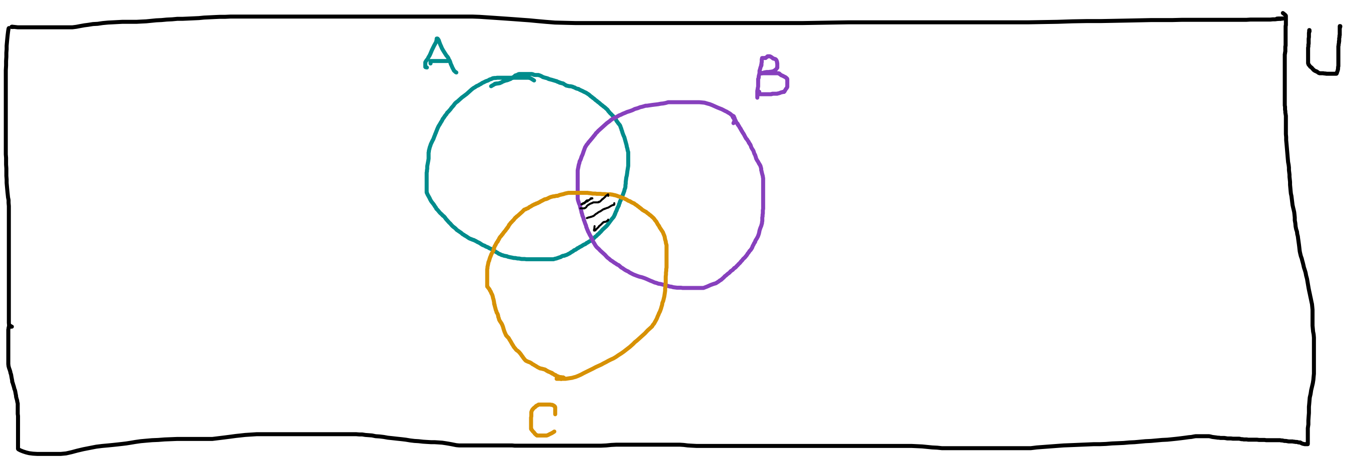
4(a)



4(b)



4(c)



If $A_1, A_2, A_3, \dots, A_n$ are subsets of U, then $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$ will be denoted by $\bigcup_{k=1}^{n} A_k$ and $A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$ will be denoted by $\bigcap_{k=1}^{n} A_k$

Example 3:

Let $A = \{1,2,3,4,5,7\}$, $B = \{1,3,8,9\}$, and $C = \{1,3,6,8\}$, then

$A \cap B \cap C = \{1,3\}$

If A and B are two sets, we define the *complement of B with respect to A* as the set of all elements that belong to A but not to B, and we denote it by $A - B$. Thus

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

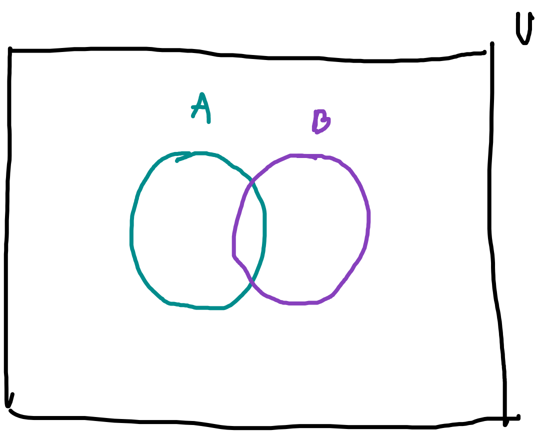
Example 4:

Let $A = \{a, b, c\}$, and $B = \{b, c, d, e\}$, then $A - B = \{a\}$ and $B - A = \{d, e\}$

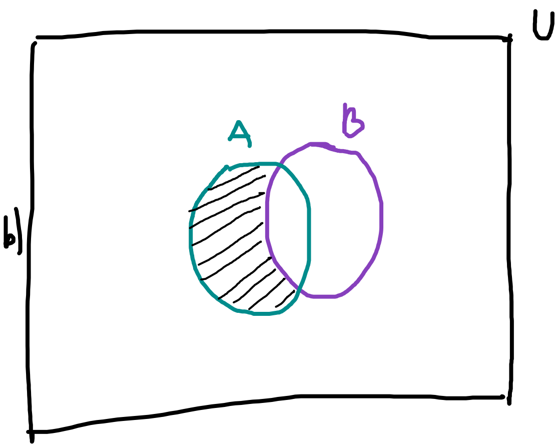
☆ Complement illustration with a Venn diagram:

If A and B are the sets in figure 5(a), then $A - B$ and $B - A$ are represented by the shaded regions in figures 5(b) and 5(c), respectively.

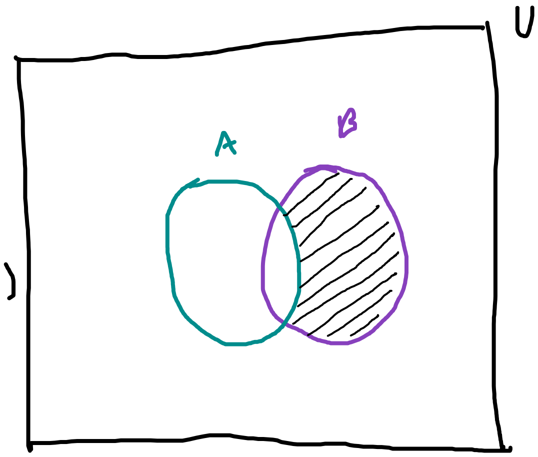
5(a)



5(b)



5(c)



If U is a universal set containing A, then $U - A$ is called the ☆ complement of A and is denoted by $\bar{A}$. Thus,

$$\bar{A} = \{x \mid x \notin A\}$$

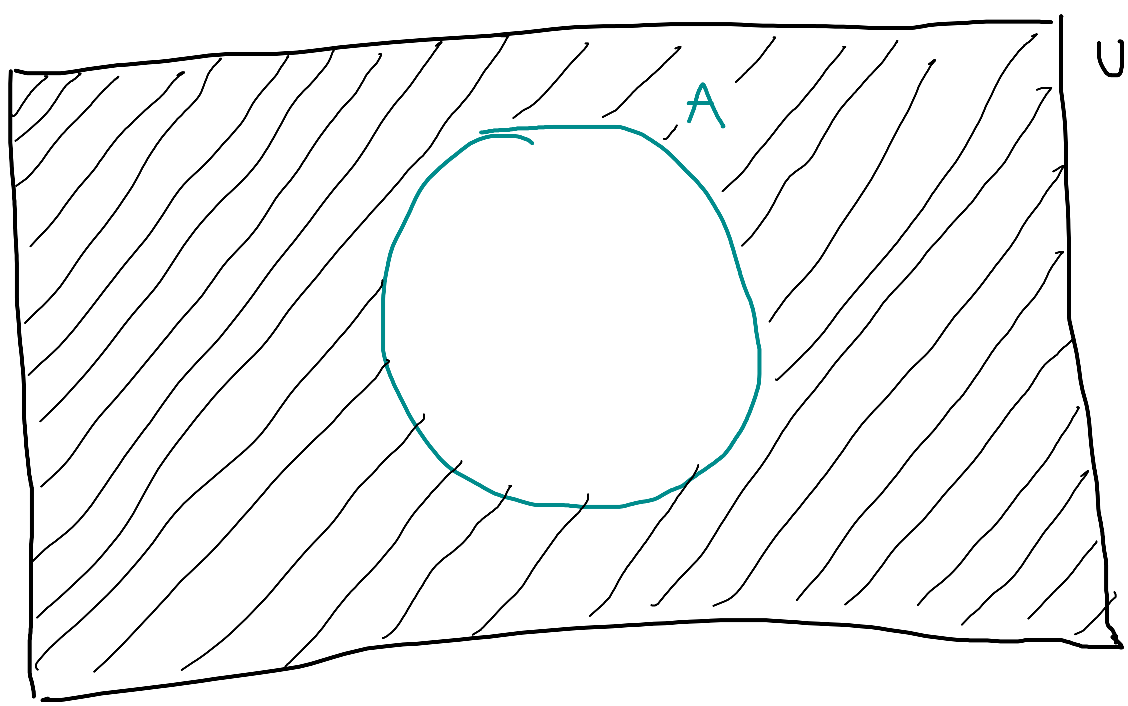
Example 5:

$A = \{x \mid x \text{ is an integer and } x \leq 4\}$,

$U = \mathbb{Z}$;

$\bar{A} = \{x \mid x \text{ is an integer and } x > 4\}$;

If A is the set in figure 6, its complement is the shaded region in that figure



6

If A and B are two sets, we define ☆ their symmetric difference as the set of all elements that belong to A or to B, but to both A and B, and we denote it by $A \oplus B$. Thus,

$$A \oplus B = \{ x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A) \}$$

Example 6:

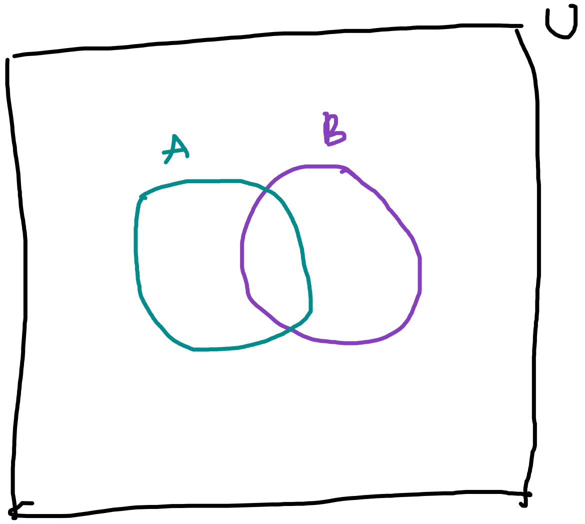
$A = \{ a, b, c, d \}$

$B = \{ a, c, e, f, g \}$

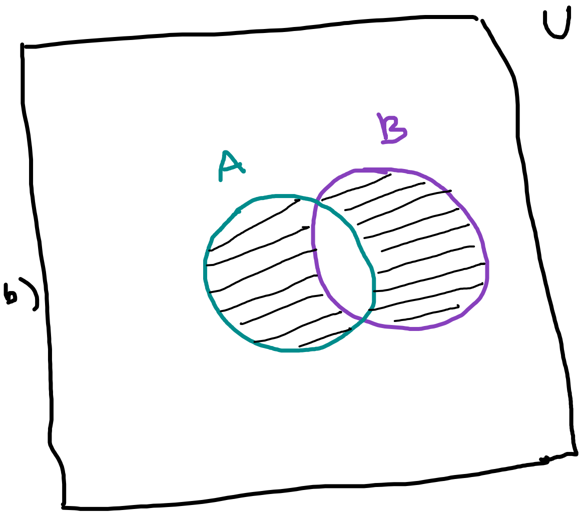
$A \oplus B = \{ b, d, e, f, g \}$

Figure 7(b) represents the symmetric difference of the sets in figure 7(a). ☆ Symmetric difference illustration:

7(a)



7(b)



It is easy to see that.

$$A \oplus B = (A - B) \cup (B - A)$$