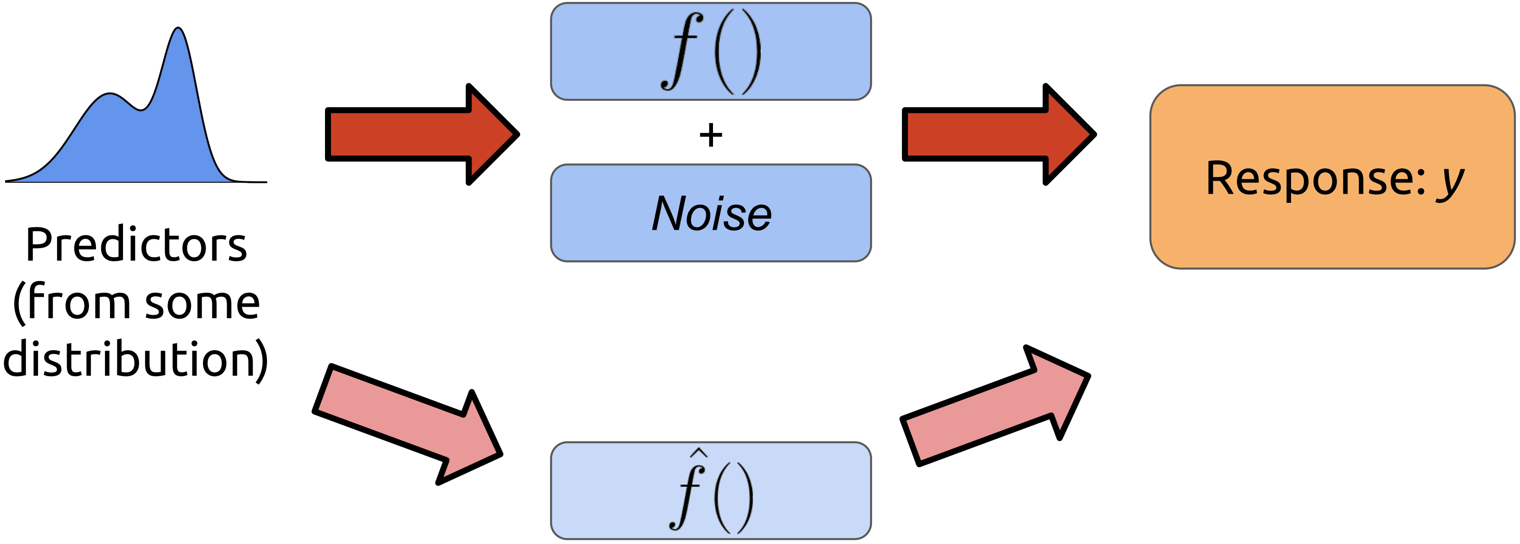
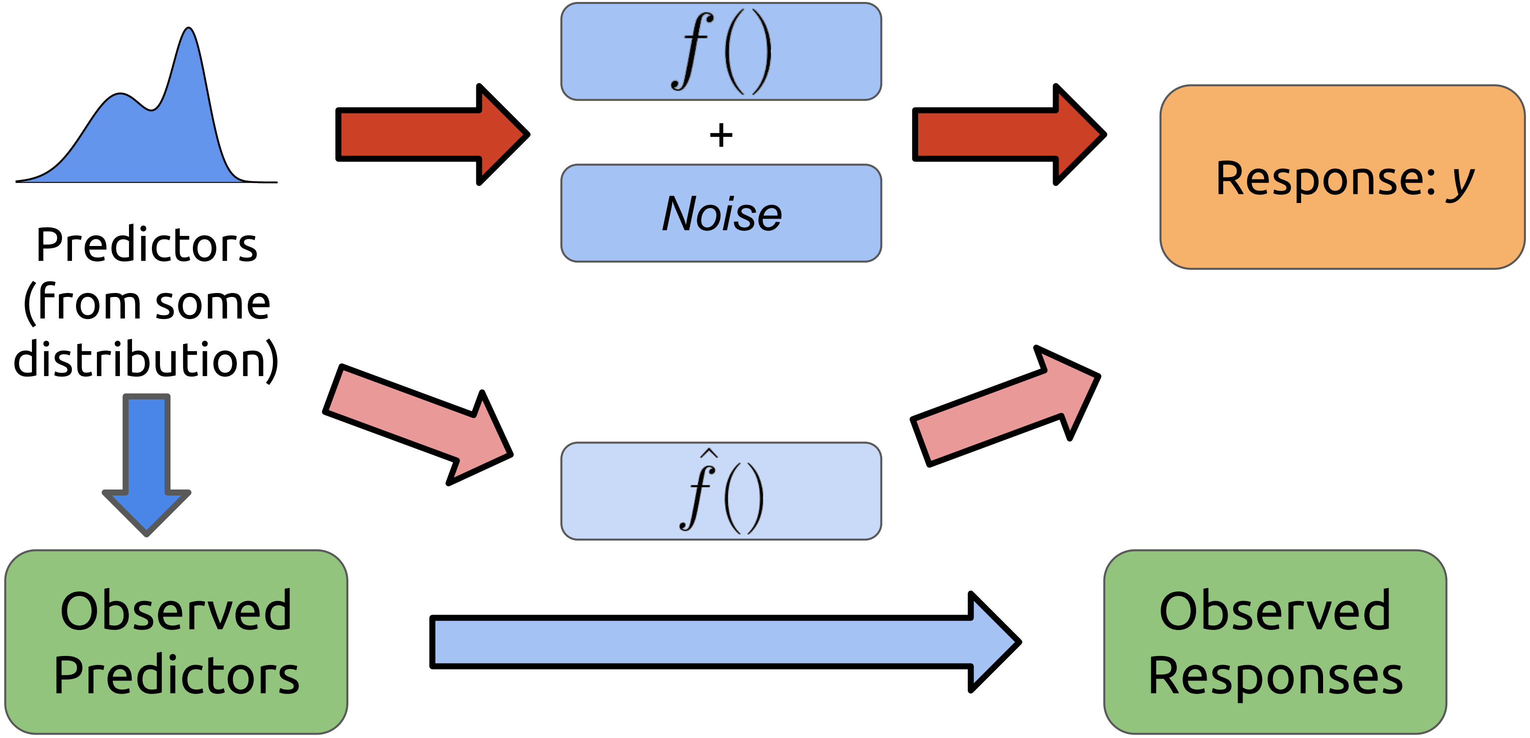
Introduction to Linear Regression

Supervised Learning - How



Supervised Learning - How



Supervised Learning - How?

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Linear regression use a particularly simple functional form to make predictions - a weighted sum of the predictor variables.

Linear Regression - Example

Let’s say we want to build a model to predict home price.

Starting simple, let’s say we know the square footage of living space and the price of a set of homes.

Linear Regression - Example

	id	sqft_living	price
0	7129300520	1180	221900.0
1	6414100192	2570	538000.0
2	5631500400	770	180000.0
3	2487200875	1960	604000.0
4	1954400510	1680	510000.0
5	7237550310	5420	1225000.0
6	1321400060	1715	257500.0
7	2008000270	1060	291850.0
8	2414600126	1780	229500.0
9	3793500160	1890	323000.0

Here's a sample from our observed data.

Linear Regression - Example

Predictors	**Target**
sqft_living	price

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Approach 1: Multiply sqft_living by a constant to predict price.

$$\text{predicted price} = \boxed{} \cdot \text{sqft\_living}$$

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Determine what goes here
based on the observed data.

Linear Regression - Example

Predictors	**Target**
sqft_living	price

Approach 1.5: Start with a “base price” and then add sqft_living multiplied by a constant to predict price.

$$\text{predicted price} = \boxed{} + \boxed{} \cdot \text{sqft\_living}$$

Determine what goes here
based on the observed data.

Linear Regression

How do we find the values for this coefficient?

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True values

residual for observation *i*: $y_i - \hat{f}(\vec{x}_i)$

Predicted Values

Linear Regression

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True values

squared residual for observation *i*: $$(y_i - \hat{f}(\vec{x}_i))^2$$

Predicted Values

Linear Regression

How do we find the values for this coefficient?

The approach we'll take for this example is to minimize the total squared **residuals** between the predicted and actual values for the data used to fit/train the model.

True values

total squared residuals:

$$\sum_{i=1}^{n}(y_i - \hat{f}(\vec{x}_i))^2$$

Predicted Values

Linear Regression

How do we find the values for this coefficient?

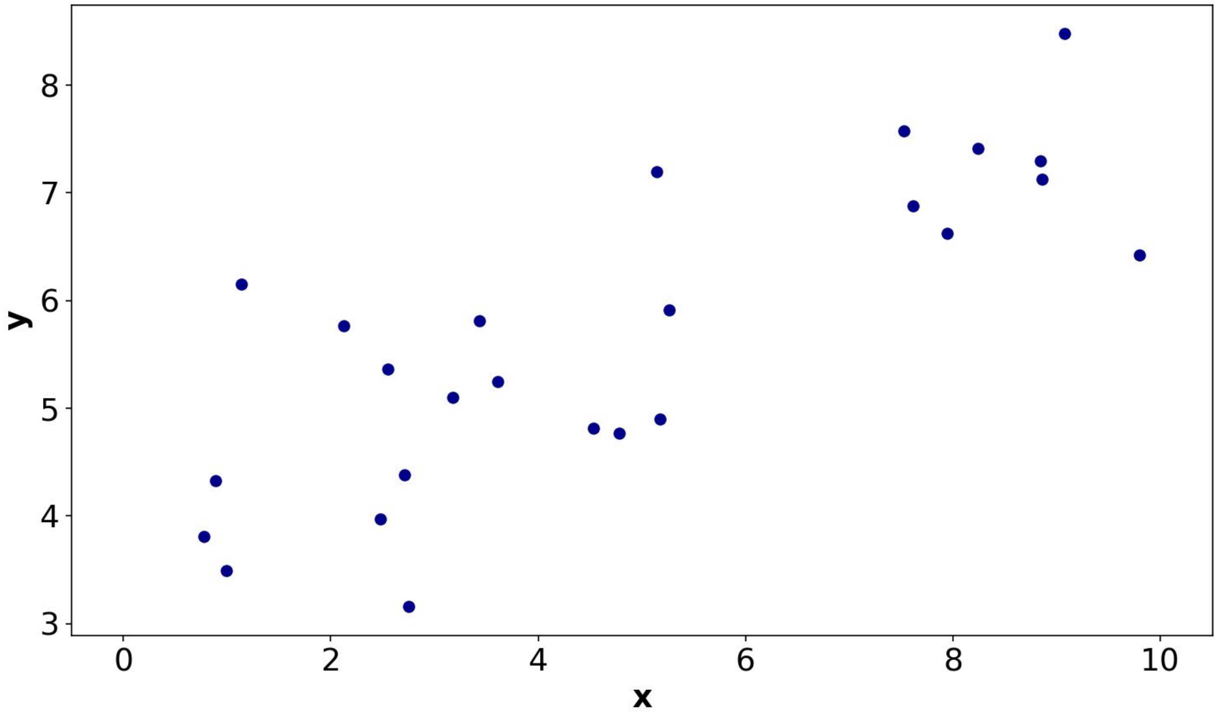
The approach we'll take for this example is to minimize the total squared **residuals** between the predicted and actual values for the data used to fit/train the model.

True values

$$RSS = \sum_{i=1}^{n} (y_i - \hat{f}(\vec{x}_i))^2$$

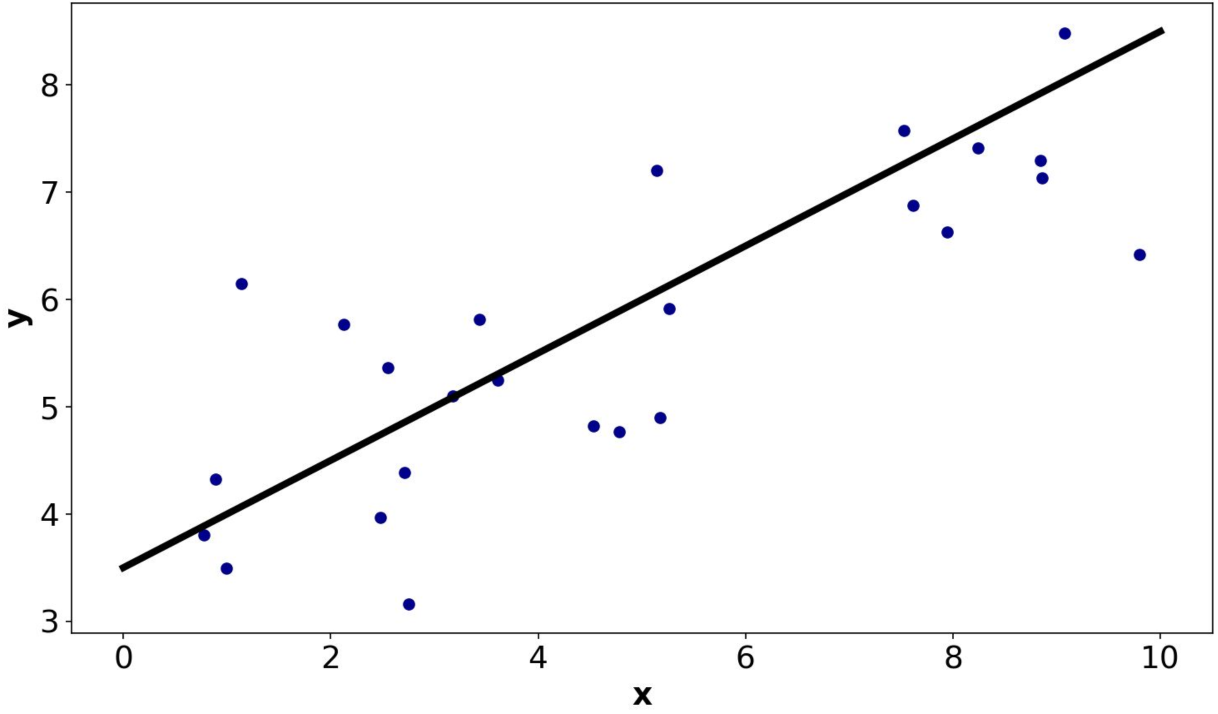
Predicted Values

Linear Regression



Example: Let's say we have this data available. We want to predict *y* based on our one predictor, *x*.

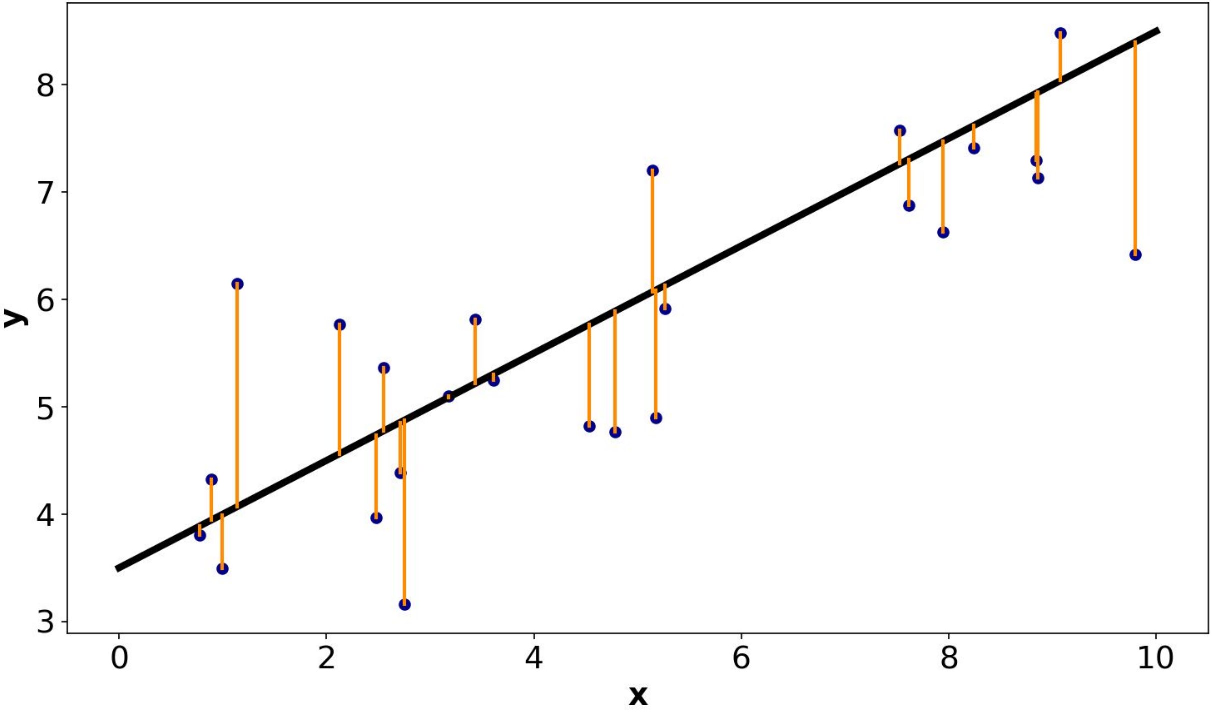
Linear Regression



One possible line:

y = 3.5 + 0.5x

Linear Regression



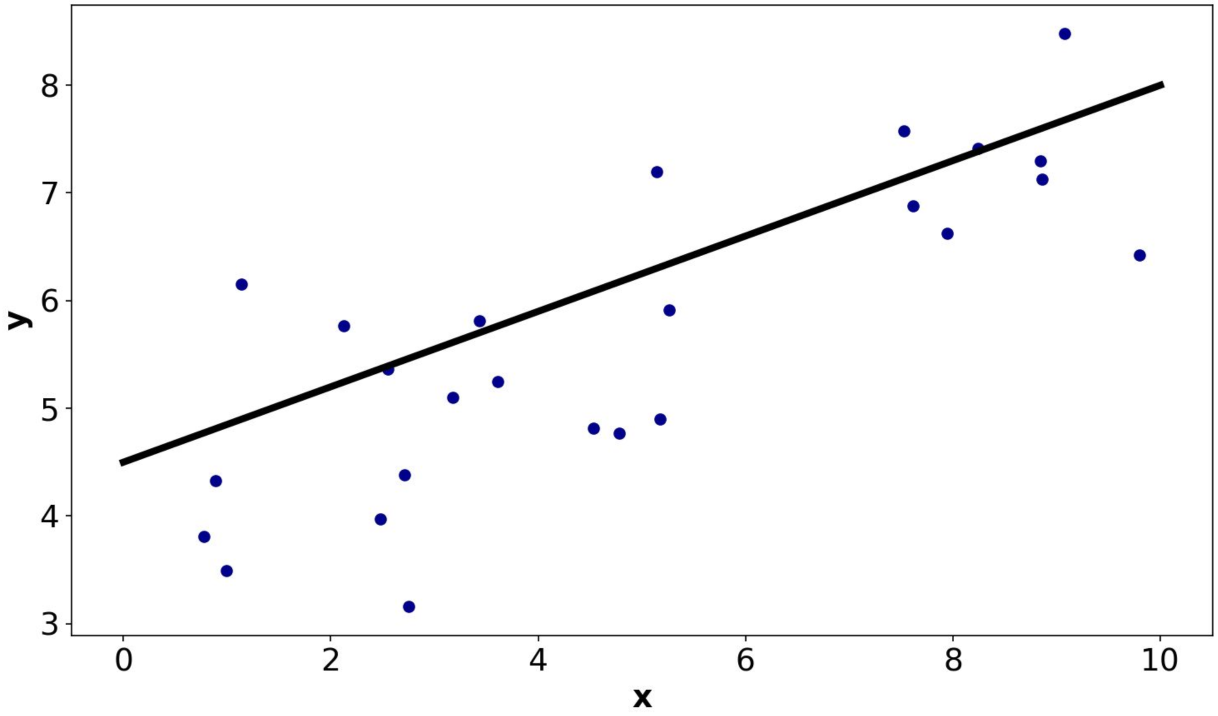
One possible line:

y = 3.5 + 0.5x

For this line,

RSS = 20.36

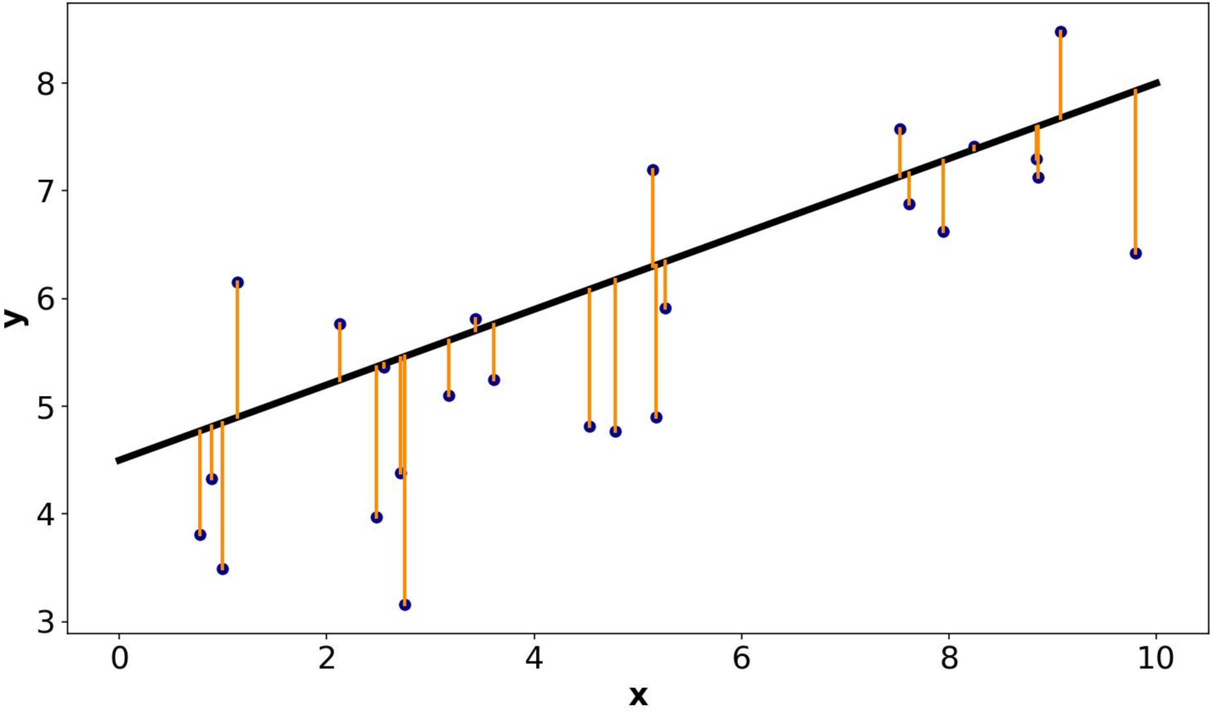
Linear Regression



Another possibility:

$y = 4.5 + 0.35x$

Linear Regression



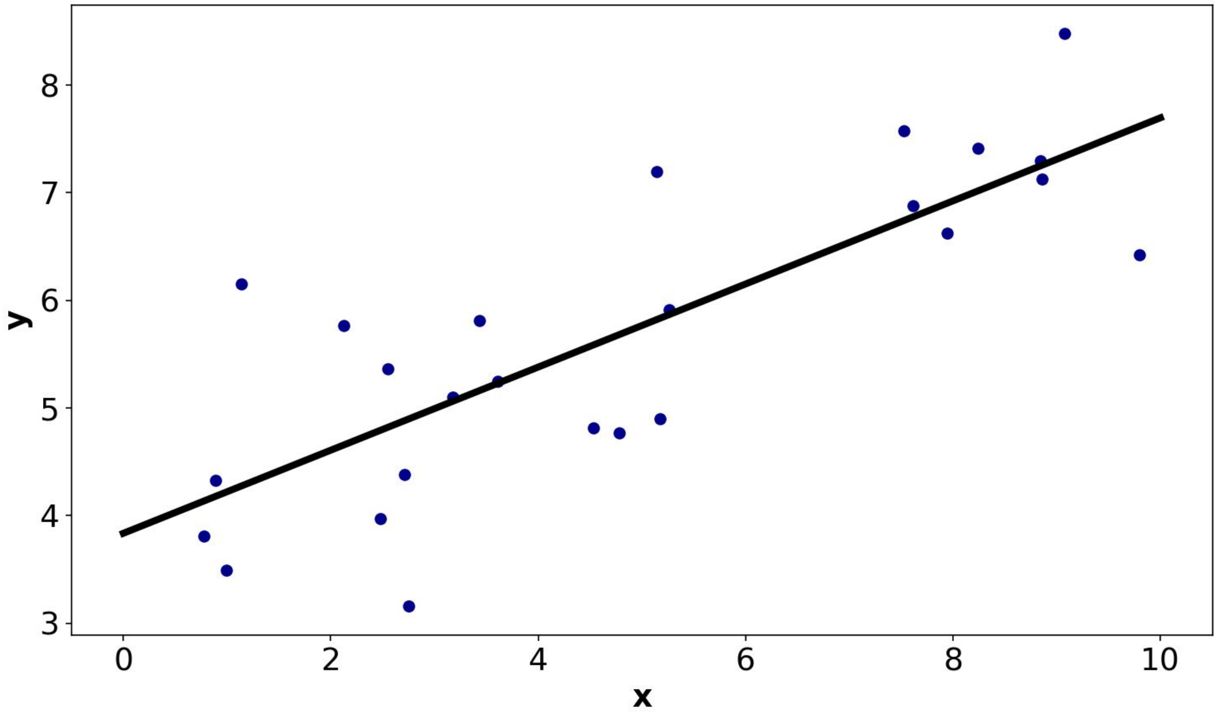
Another possibility:

y = 4.5 + 0.35x

Here,

RSS = 24.28

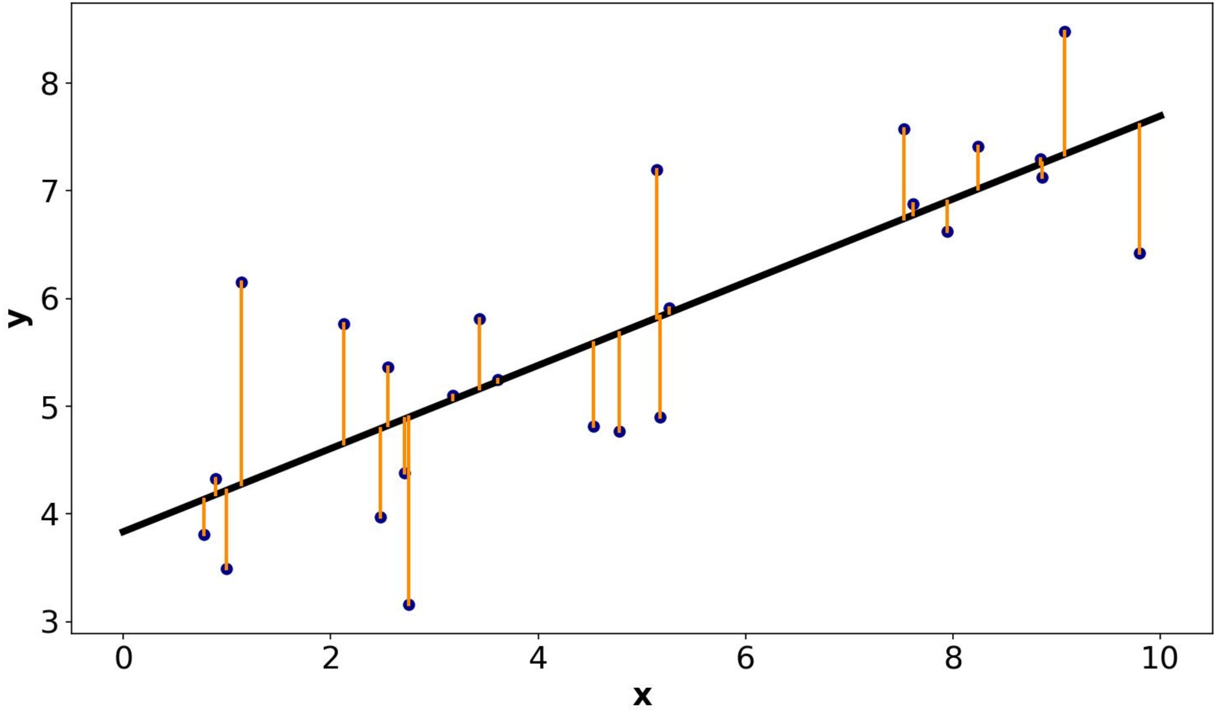
Linear Regression



The best possible:

$y = 3.84 + 0.386x$

Linear Regression



The best possible:

$y = 3.84 + 0.386x$

Here,

RSS = 17.97

Linear Regression

How *exactly* do we minimize RSS?

Linear Regression

How *exactly* do we minimize RSS?

Through some kind of optimization algorithm:

- Analytical solution could be used in this case (using matrix algebra tricks)
- Limited-memory BFGS (https://en.wikipedia.org/wiki/Limited-memory_BFGS)
- Gradient descent (https://en.wikipedia.org/wiki/Gradient_descent)

Linear Regression

$$RSS = \sum_{i=1}^{n} (y_i - \hat{f}(\vec{x}_i))^2$$

If we have a lot of observed points, we might instead use **mean squared error (MSE)**.

$$\mathrm{MSE} = \frac{\sum_{i=1}^{n} (y_i - \hat{f}(x_i))^2}{n}$$

Minimizing RSS is equivalent to minimizing MSE (Why?)

Supervised Learning - Goals

Very Important Note: For linear regression, we find the coefficients by minimizing RSS/MSE on the training data, but this does not guarantee that the model will generalize well.

It is important to do a train/test split and estimate the *generalization error* - the only thing that we care about in evaluating a machine learning model.

Linear Regression - Example

Predictors	**Target**
sqft_living	price

Approach 1.5: Start with a “base price” and then add sqft_living multiplied by a constant to predict price.

$$\text{predicted price} = \boxed{} + \boxed{} \cdot \text{sqft\_living}$$

Determine what goes here based on the observed data.

Linear Regression - Example

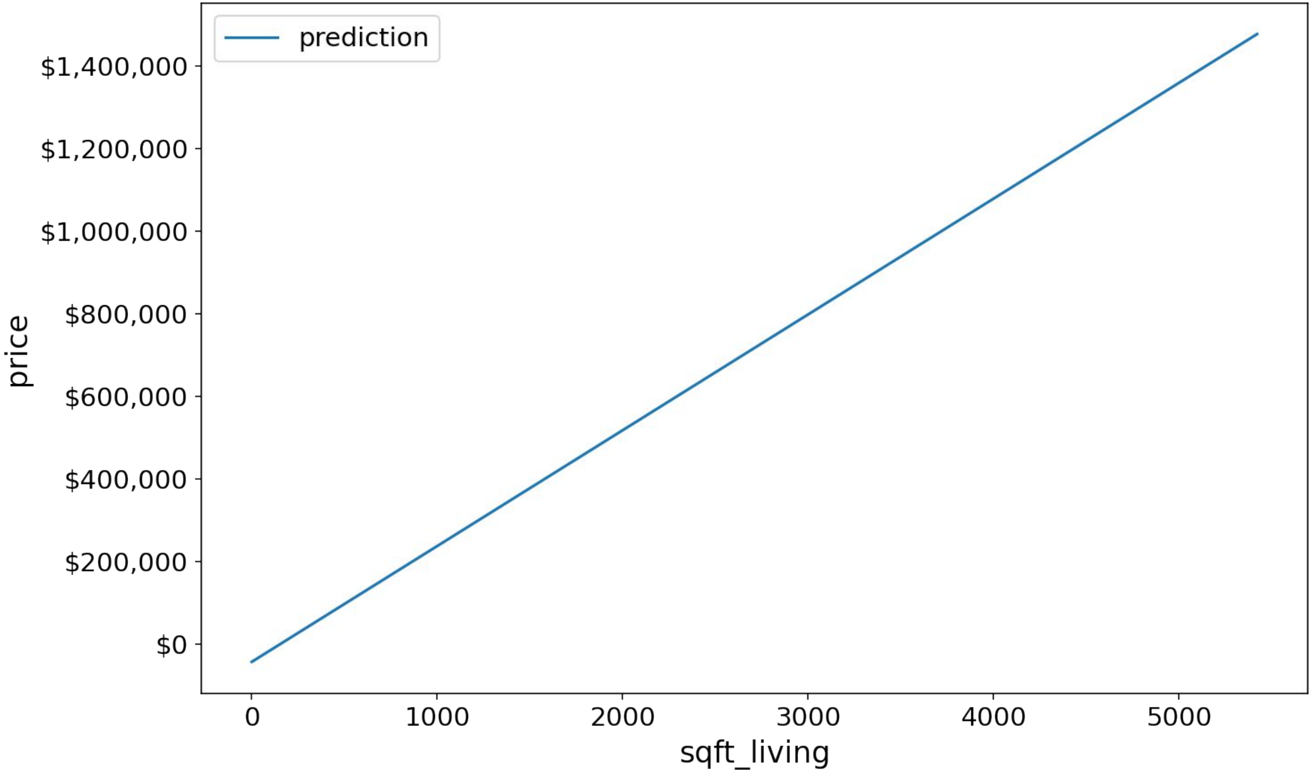
Predictors	**Target**
sqft_living	price

Approach 1.5: Start with a “base price” and then add sqft_living multiplied by a constant to predict price.

$$\text{predicted price} = \boxed{-42123} + \boxed{281} \cdot \text{sqft\_living}$$

Determine what goes here based on the observed data.

Linear Regression - Example



Linear Regression - Example

	id	sqft_living	price	predicted_price
0	7129300520	1180	221900.0	289118.0
1	6414100192	2570	538000.0	679309.0
2	5631500400	770	180000.0	174026.0
3	2487200875	1960	604000.0	508074.0
4	1954400510	1680	510000.0	429474.0
5	7237550310	5420	1225000.0	1479340.0
6	1321400060	1715	257500.0	439299.0
7	2008000270	1060	291850.0	255433.0
8	2414600126	1780	229500.0	457546.0
9	3793500160	1890	323000.0	488424.0

Applying this to our data gives these results.

Linear Regression - Example

	id	sqft_living	price	predicted_price
0	7129300520	1180	221900.0	289118.0

Let’s see how we arrived at this predicted price.

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	id	sqft_living	price	predicted_price
0	7129300520	1180	221900.0	289118.0

Let’s see how we arrived at this predicted price.

$$\text{predicted price} = -42123 + 281 \cdot \text{sqft\_living}$$

Linear Regression - Example

	id	sqft_living	price	predicted_price
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$$\text{predicted price} = -42123 + 281 \cdot (1180)$$

$$\text{predicted price} = -42123 + 331580$$

Linear Regression - Example

	id	sqft_living	price	predicted_price
0	7129300520	1180	221900.0	289118.0

Let's see how we arrived at this predicted price.

$$\text{predicted price} = -42123 + 281 \cdot \text{sqft\_living}$$

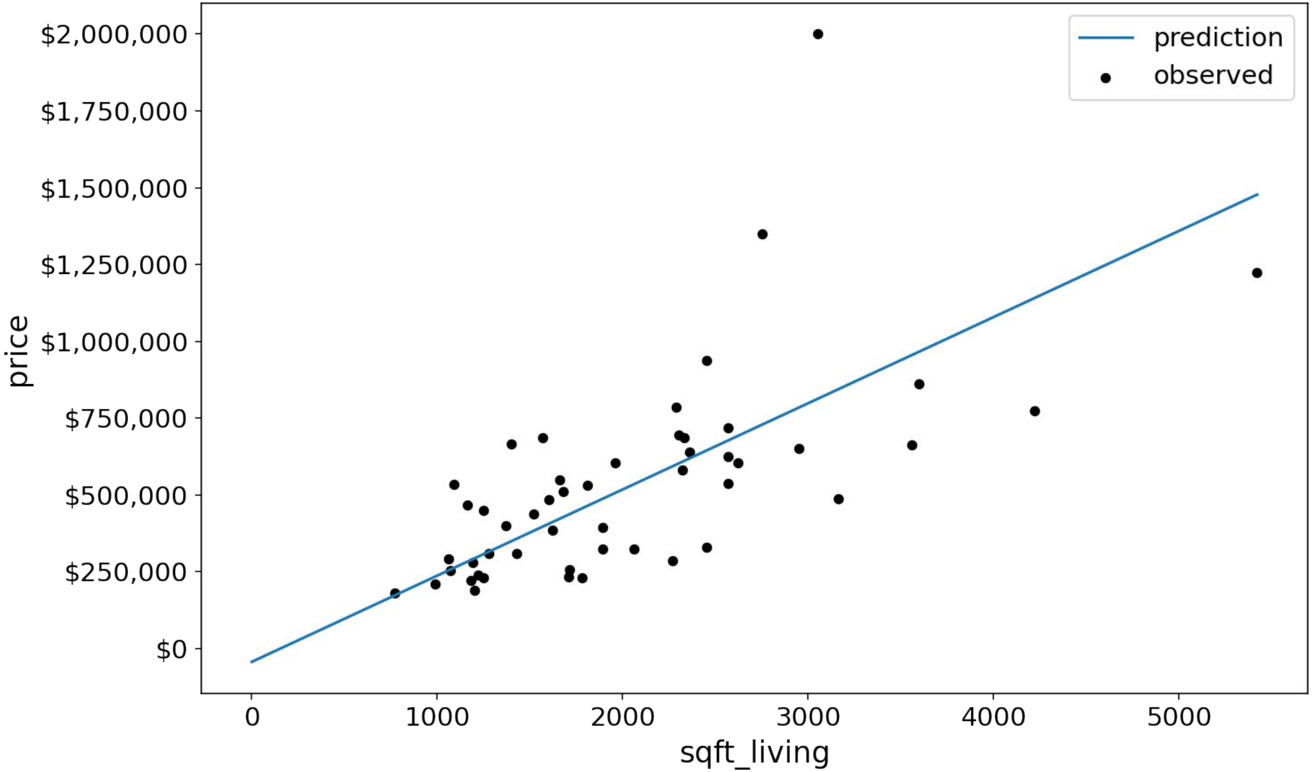
$$\text{predicted price} = -42123 + 281 \cdot (1180)$$

$$\text{predicted price} = -42123 + 331580$$

$$\text{predicted price} = 289457^{*}$$

* This number is slightly different due to rounding the coefficients

Linear Regression - Example



Linear Regression - Example

This looks okay, but could potentially be improved. What if we add in more information about our observations?

Linear Regression - Example

	id	sqft_living	condition	price
0	7129300520	1180	3	221900.0
1	6414100192	2570	3	538000.0
2	5631500400	770	3	180000.0
3	2487200875	1960	5	604000.0
4	1954400510	1680	3	510000.0
5	7237550310	5420	3	1225000.0
6	1321400060	1715	3	257500.0
7	2008000270	1060	3	291850.0
8	2414600126	1780	3	229500.0
9	3793500160	1890	3	323000.0

This looks okay, but could potentially be improved. What if we add in more information about our observations?

Linear Regression - Example

<u>Predictors</u>

sqft_living
condition

<u>Target</u>

price

$$\text{predicted price} = \boxed{} + \boxed{} \cdot \text{sqft\_living} + \boxed{} \cdot \text{condition}$$

Determine what goes here
based on the observed data.

Linear Regression - Example

Predictors

sqft_living
condition

Target

price

$$\text{predicted price} = \boxed{-192628} + \boxed{282} \cdot \text{sqft\_living} + \boxed{43067} \cdot \text{condition}$$

Determine what goes here
based on the observed data.

Linear Regression - Example

$$\text{predicted price} = -192628 + 282 \cdot \text{sqft\_living} + \boxed{43067} \cdot \text{condition}$$

How do we interpret
this value?

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$$\text{predicted price} = -192628 + 282 \cdot \text{sqft\_living} + \boxed{43067} \cdot \text{condition}$$

How do we interpret
this value?

If two houses have the same sqft_living, for every one unit difference in condition, their predicted prices will be differ by $43,067.

Linear Regression - Example

$$\text{predicted price} = -192628 + \boxed{282} \cdot \text{sqft\_living} + 43067 \cdot \text{condition}$$

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$$\text{predicted price} = -192628 + \boxed{282} \cdot \text{sqft\_living} + 43067 \cdot \text{condition}$$

How do we interpret
this value?

If two houses have the same condition for every unit of difference in sqft_living, their predicted prices will differ by $282.

Linear Regression - Example

What if we have even more predictors?

Linear Regression

Given k predictors $x^{(1)}$, $x^{(2)}$,...,$x^{(k)}$, linear regression uses the following equation to predict the target variable:

$$\hat{f}(\vec{x}) = \beta_0 + \beta_1 x^{(1)} + \beta_2 x^{(2)} + \cdots + \beta_k x^{(k)}$$

Here, β_0, β_1,...,β_k are constants that are determined by using the available training data.