Introduction to Supervised Learning

Question - Is it going to rain today?

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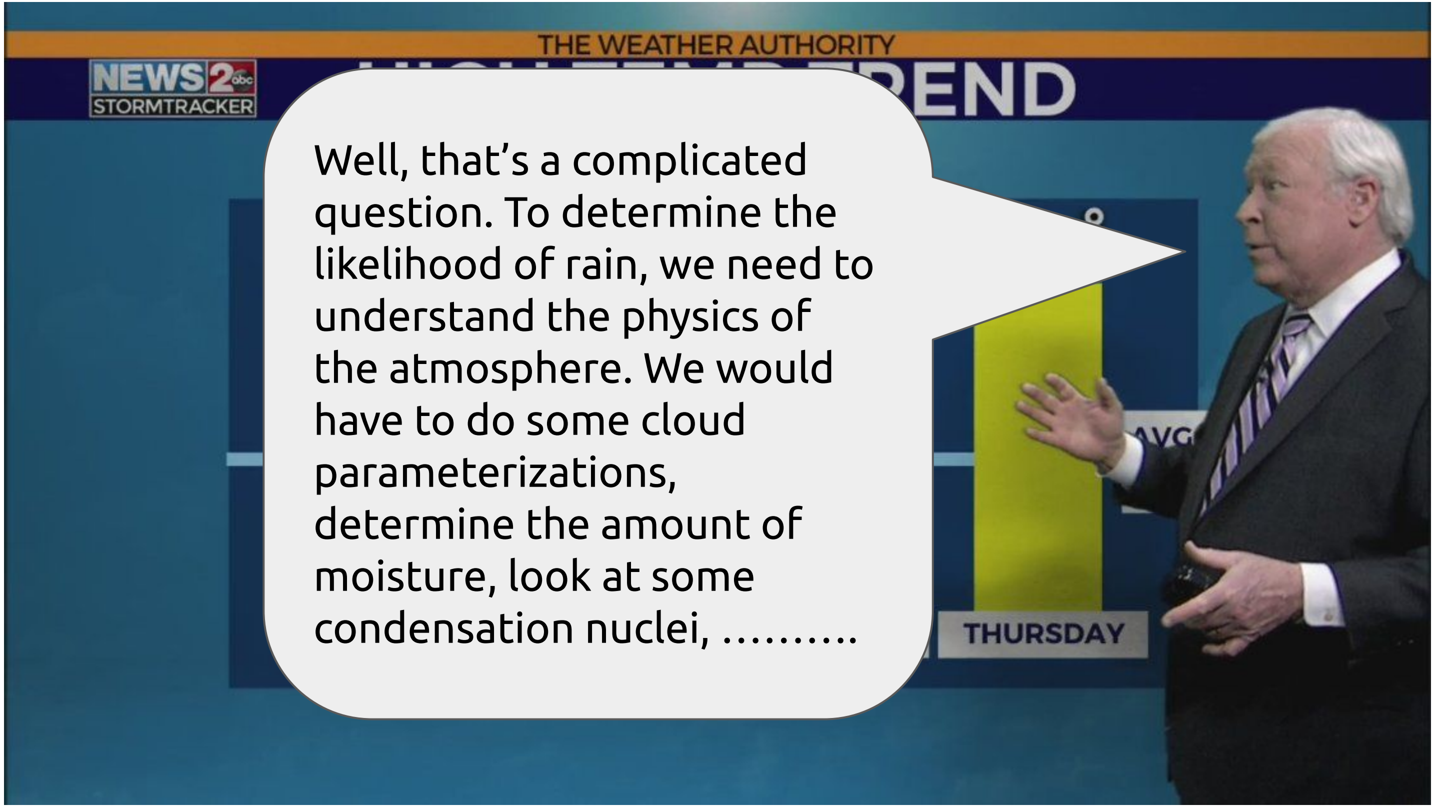

Question - Is it going to rain today?

You look outside and it looks like this:

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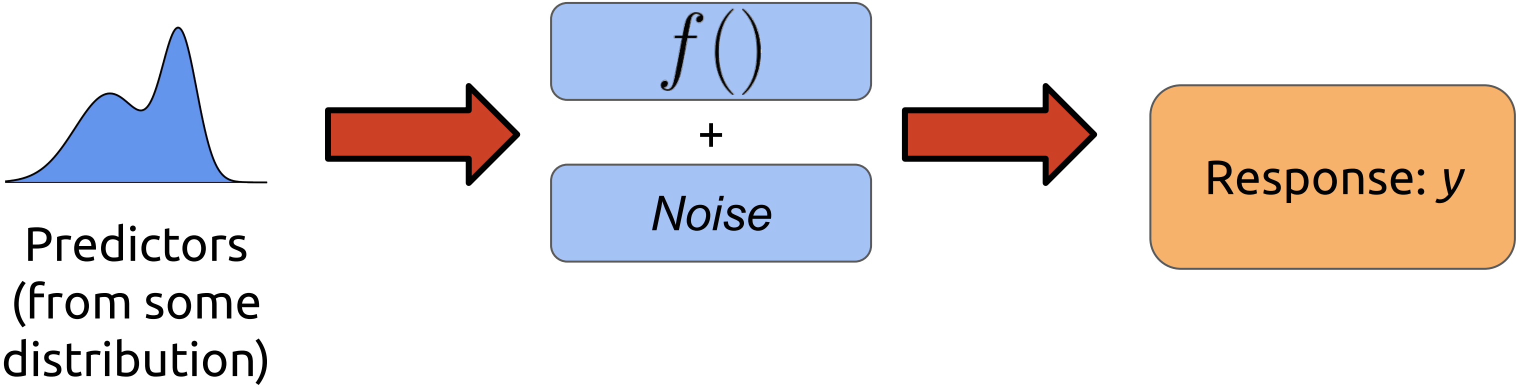
Supervised Learning - Setup



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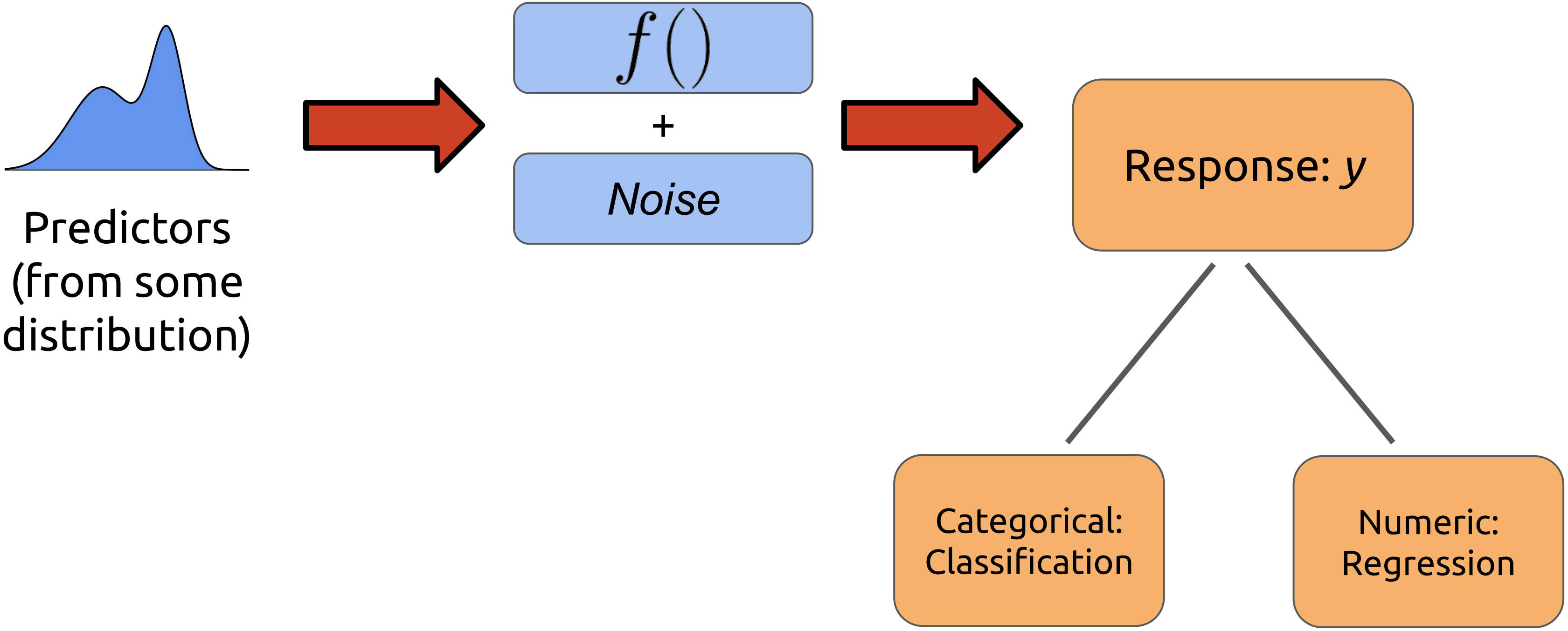

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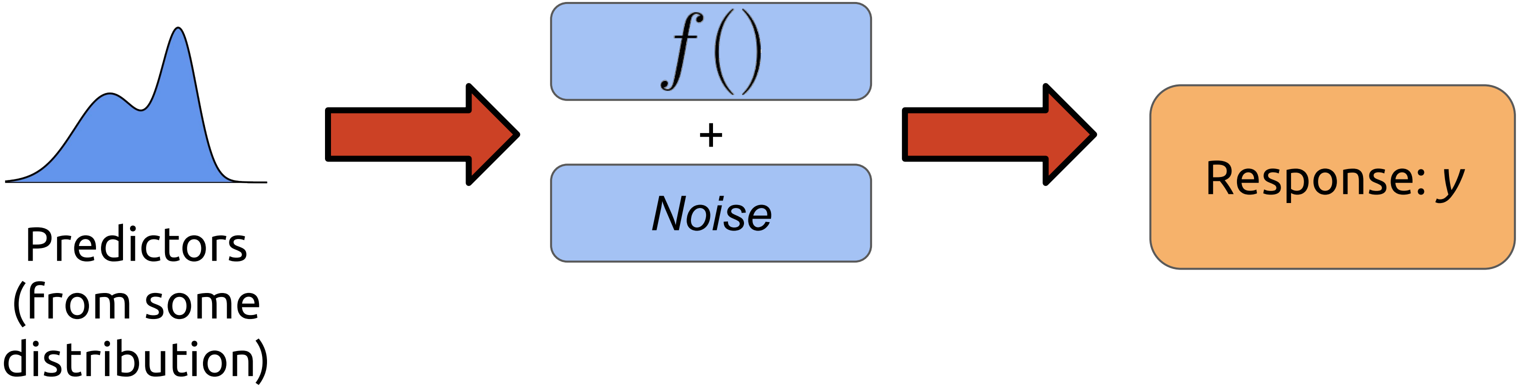
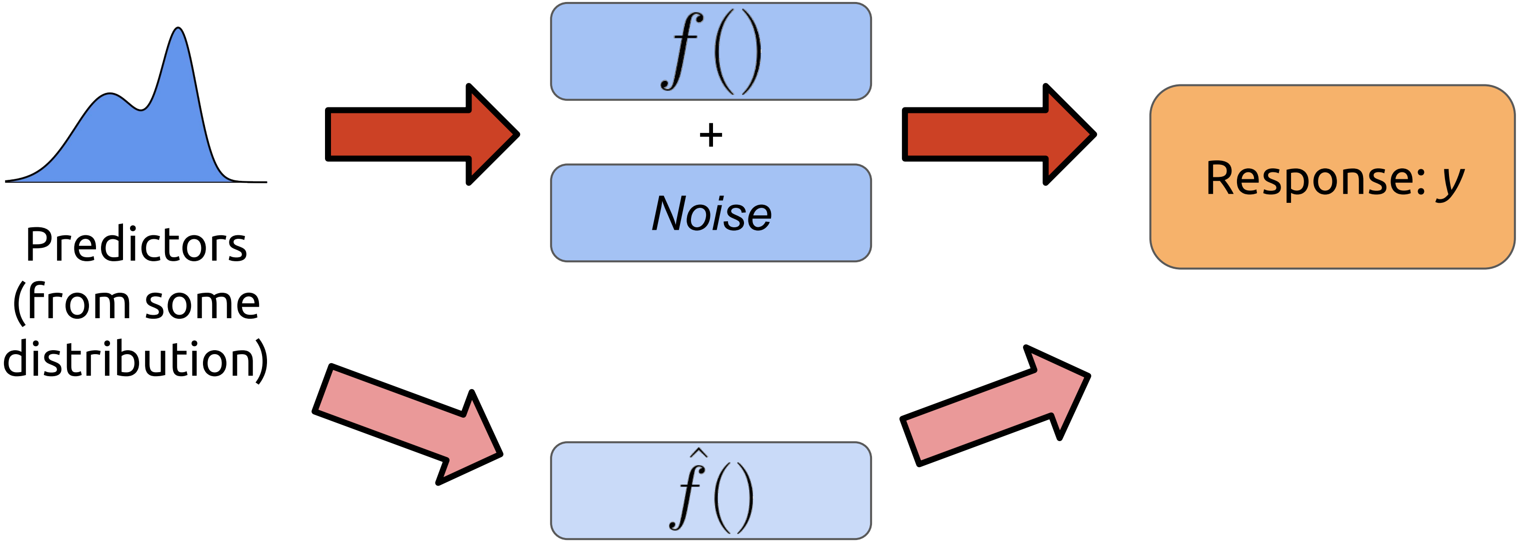


Supervised Learning - Goals



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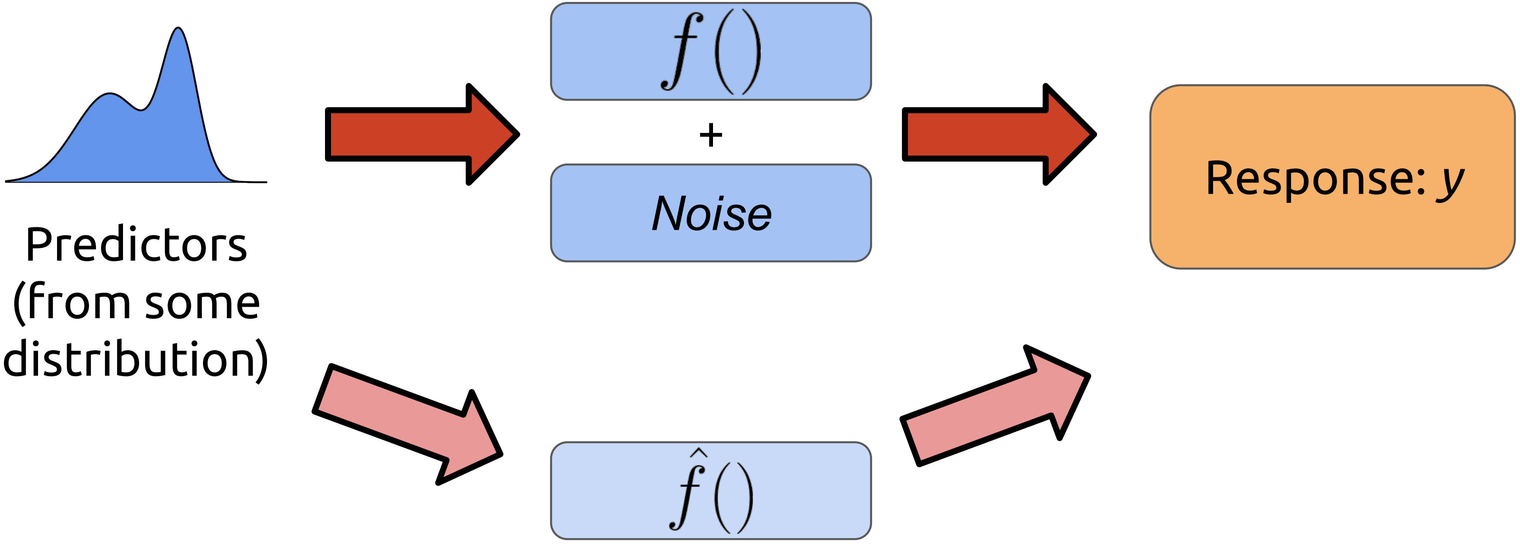
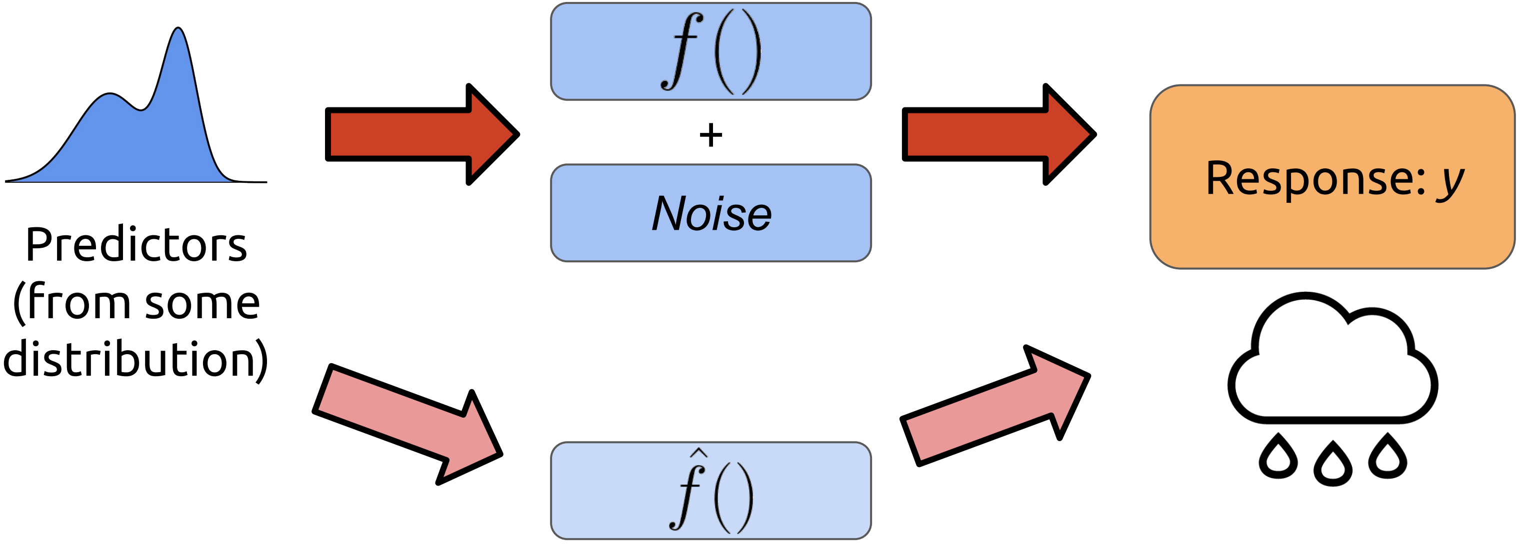
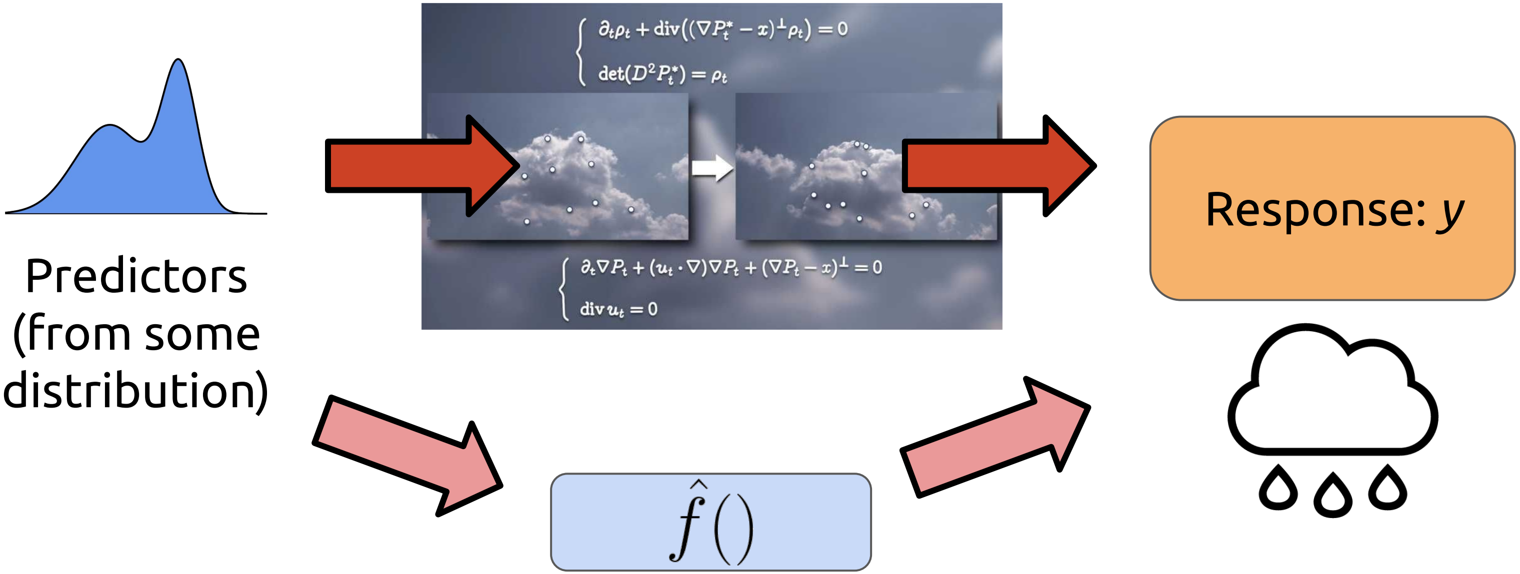


Goal: Choose a function so that the our predictions are close (on average) to the true values.

Supervised Learning - Goals



Supervised Learning - Grossly Simplified



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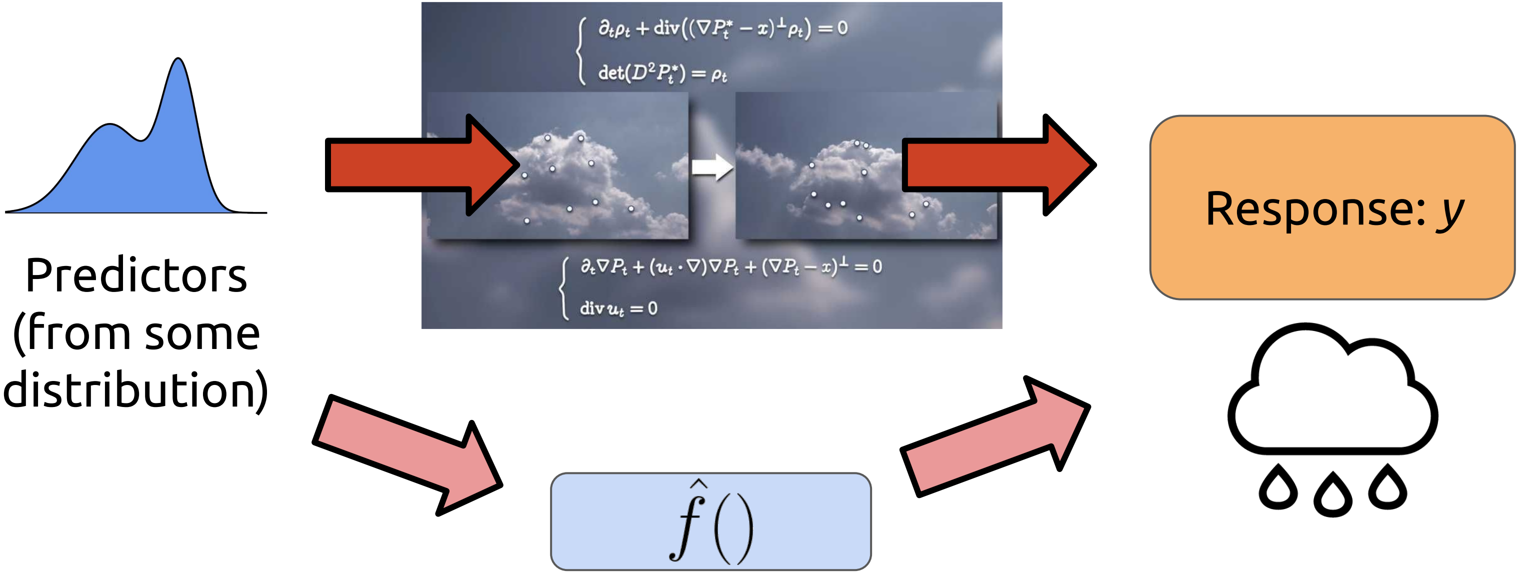

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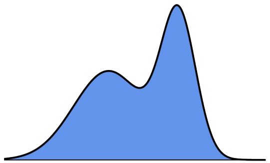
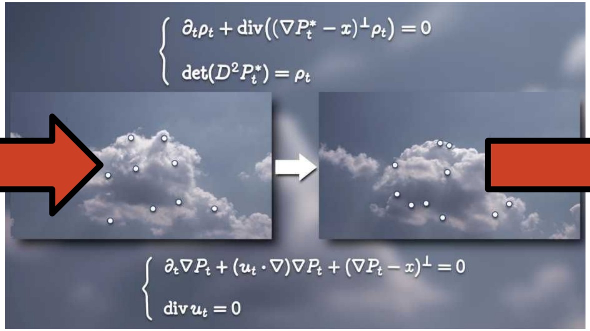


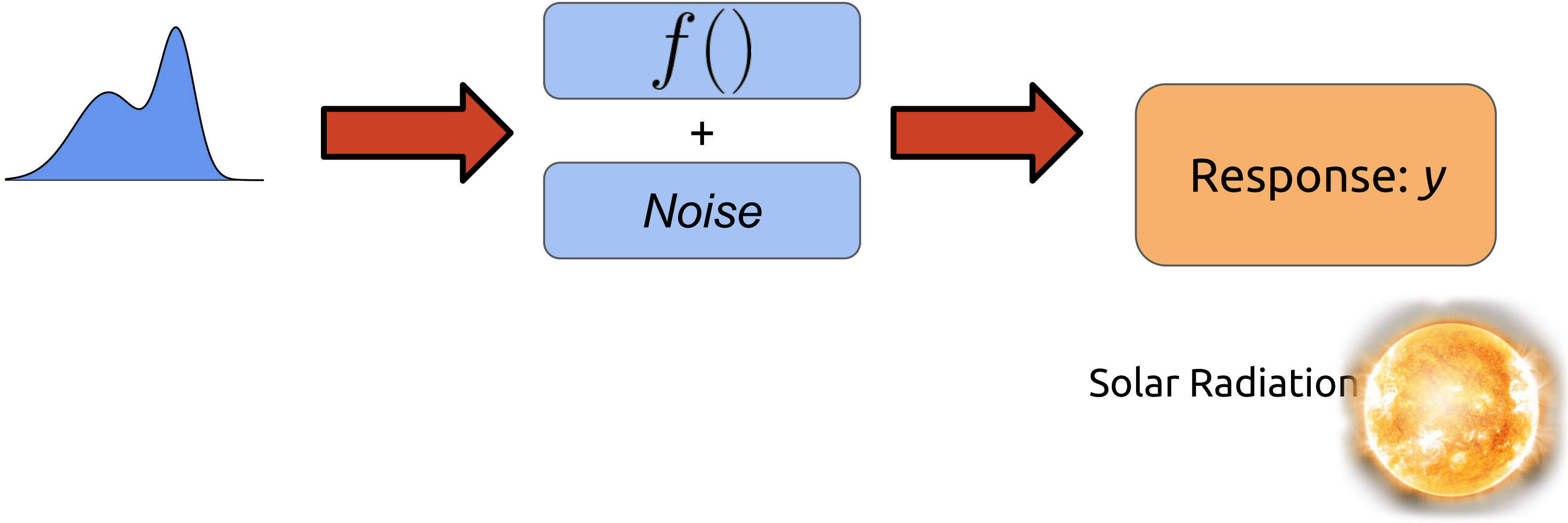
Predictors (from some distribution)



My knee is acting up. Must be rain coming.

Response: *y*

Example - Weather Prediction



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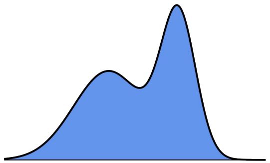

air temperature
precipitation rate
visibility
wind speed
wind direction
dew point
temperature
air pressure
relative humidity

$f()$

+

Noise

Response: *y*

Solar Radiation

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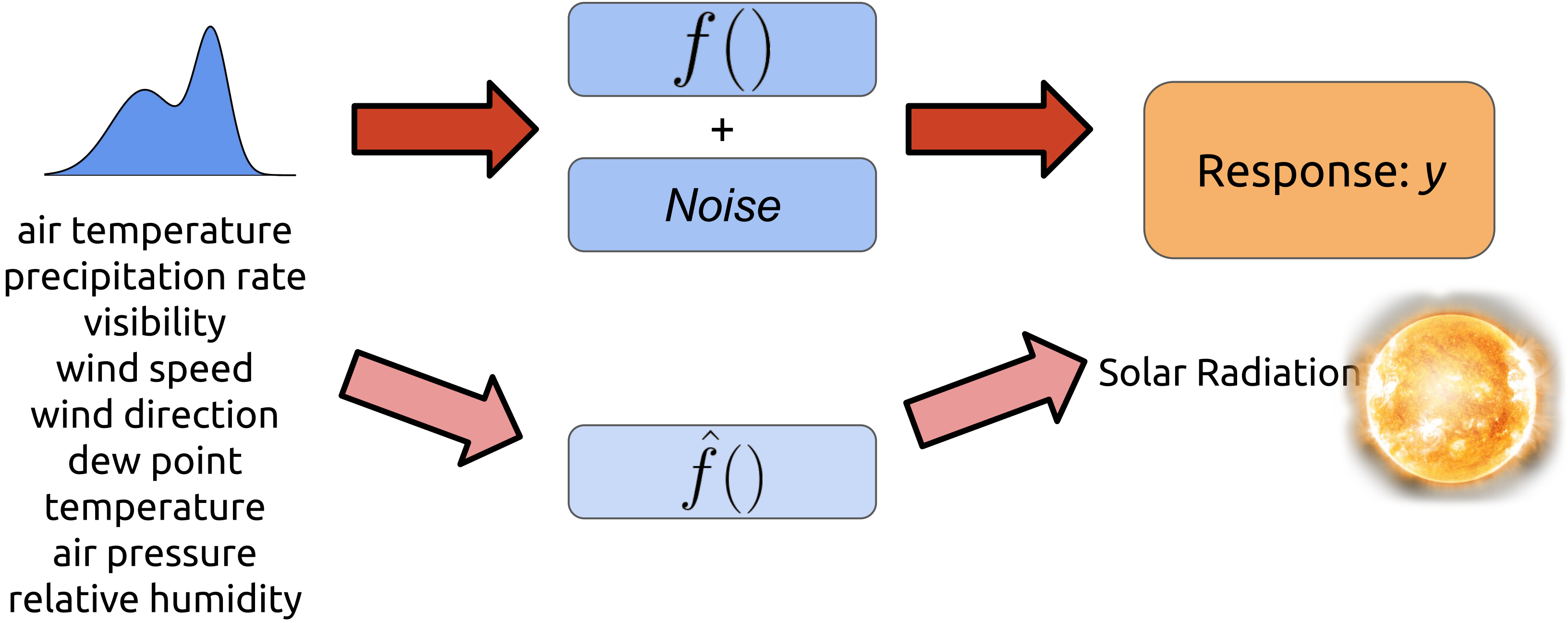

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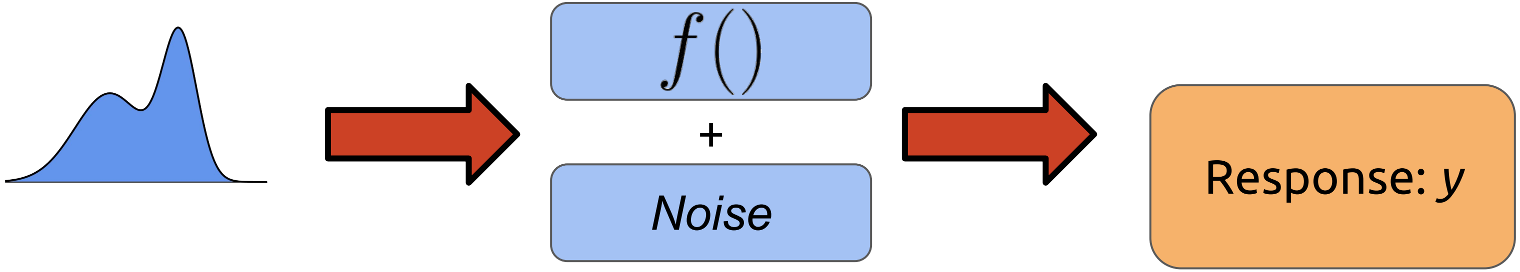
Example - Readmission or Death of Stroke Patients



90 day
readmission or
mortality

https://www.mdpi.com/2076-3417/10/18/6337/pdf

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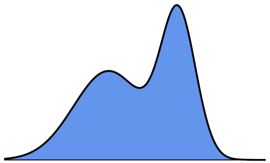

demographic data
initial vital signs
laboratory results
past medical history
comorbidities
treatment-seeking behavior
pre-stroke functional status

$f()$
+
Noise

Response: *y*

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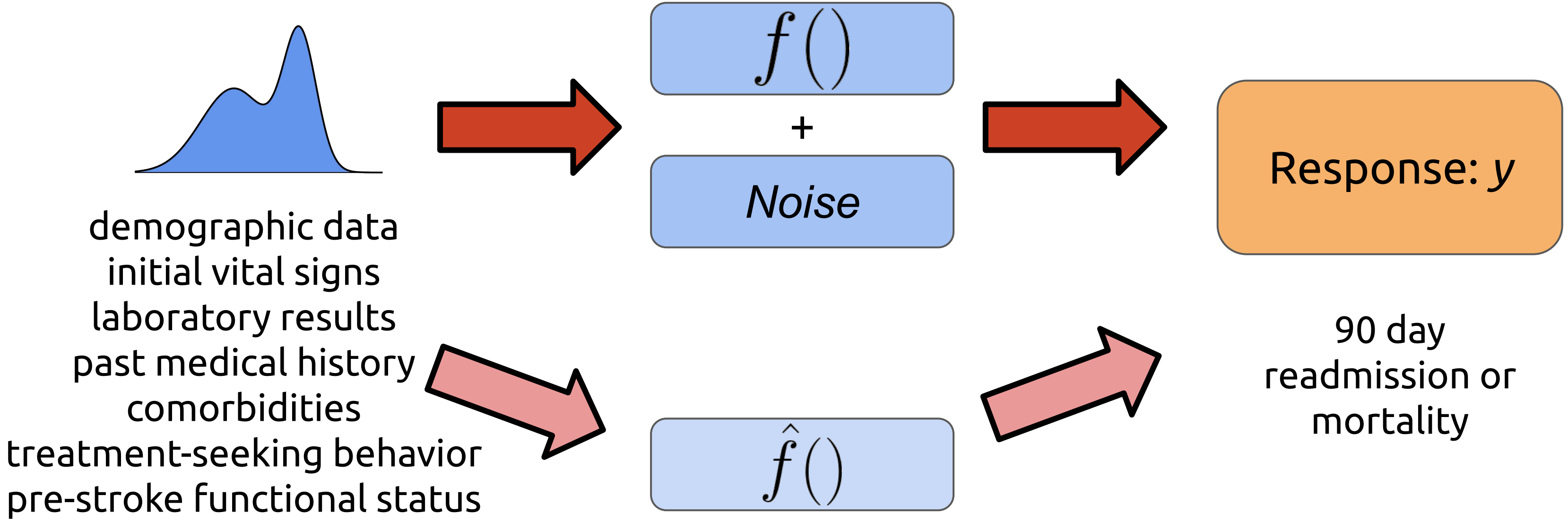

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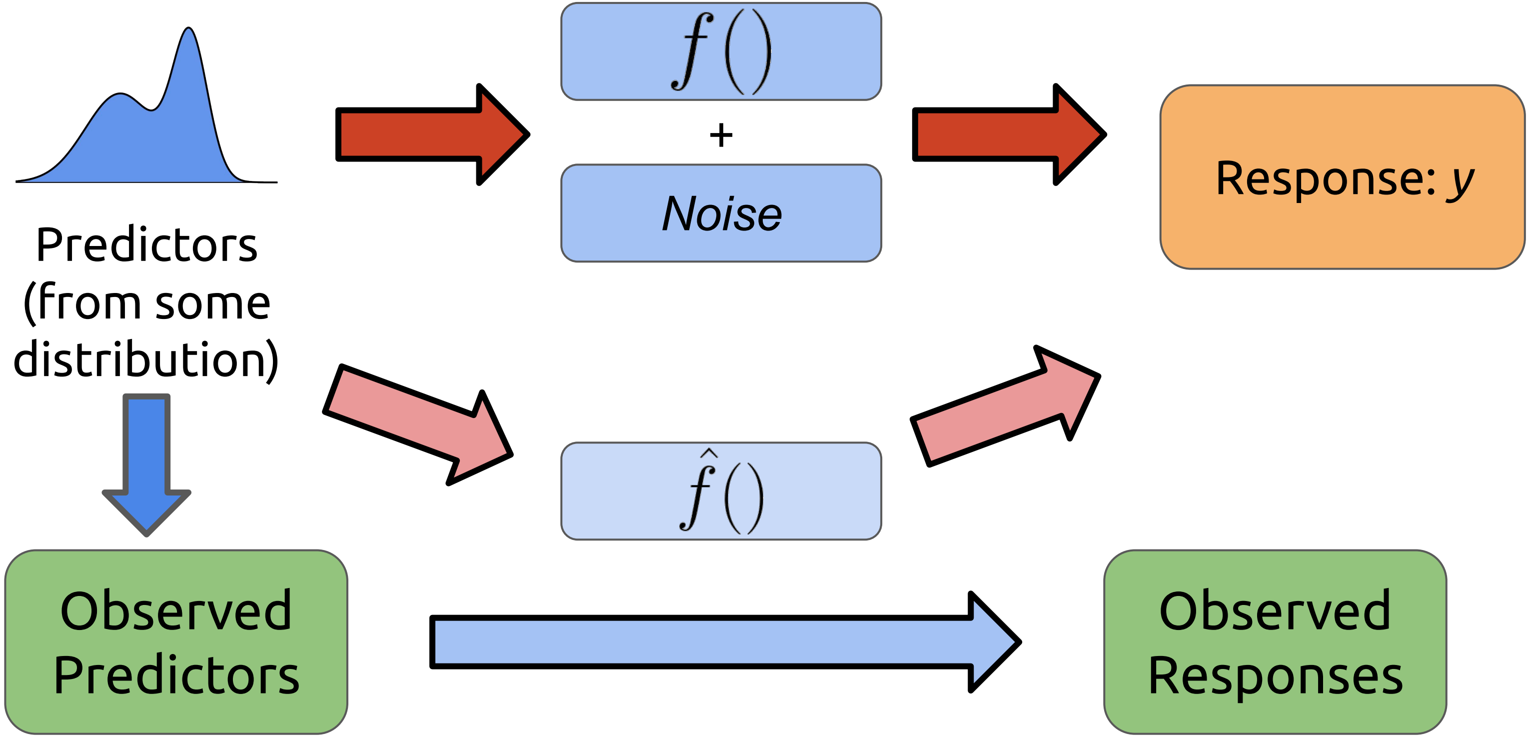
Supervised Learning - How



Supervised Learning - Goals

To measure how “good” our model is, we need some way to measure “error” (eg. mean squared error).

Our goal is to minimize the expected loss over *new* data.

Important: We are not trying to minimize loss over the observed data (which is often very easy to do), but to minimize the *generalization error* - the performance on unseen data.

Supervised Learning - How?

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For example, we can pick a functional form for $\hat{f}()$

Linear regression is a way to make predictions where we pick a particular functional form for our predictor function.

Linear Regression

Given *k* predictors $x^{(1)}$, $x^{(2)}$,...,$x^{(k)}$, linear regression uses the following equation to predict the target variable:

$$\hat{f}(\vec{x}) = \beta_0 + \beta_1 x^{(1)} + \beta_2 x^{(2)} + \cdots + \beta_k x^{(k)}$$

Here, β_0, β_1,...,β_k are constants that are determined by using the available training data.

Linear Regression

Example: We might want to try and predict home price (our target) based on square footage (sqft), number of bedrooms (br), and number of floors (floors).

The model we will use to make predictions will look like:

$$\hat{f}(\vec{x}) = \beta_0 + \beta_1 \cdot (\mathrm{sqft}) + \beta_2 \cdot (\mathrm{br}) + \beta_3 \cdot (\mathrm{floors})$$

Linear Regression

Example: We might want to try and predict home price (our target) based on square footage (sqft), number of bedrooms (br), and number of floors (floors).

The model we will use to make predictions will look like:

$$\hat{f}(x) = 40000 + 180 \cdot (\text{sqft}) + 15000 \cdot (\text{br}) + 30000 \cdot (\text{floors})$$

Linear Regression

How do we find the values for the coefficients?

Linear Regression

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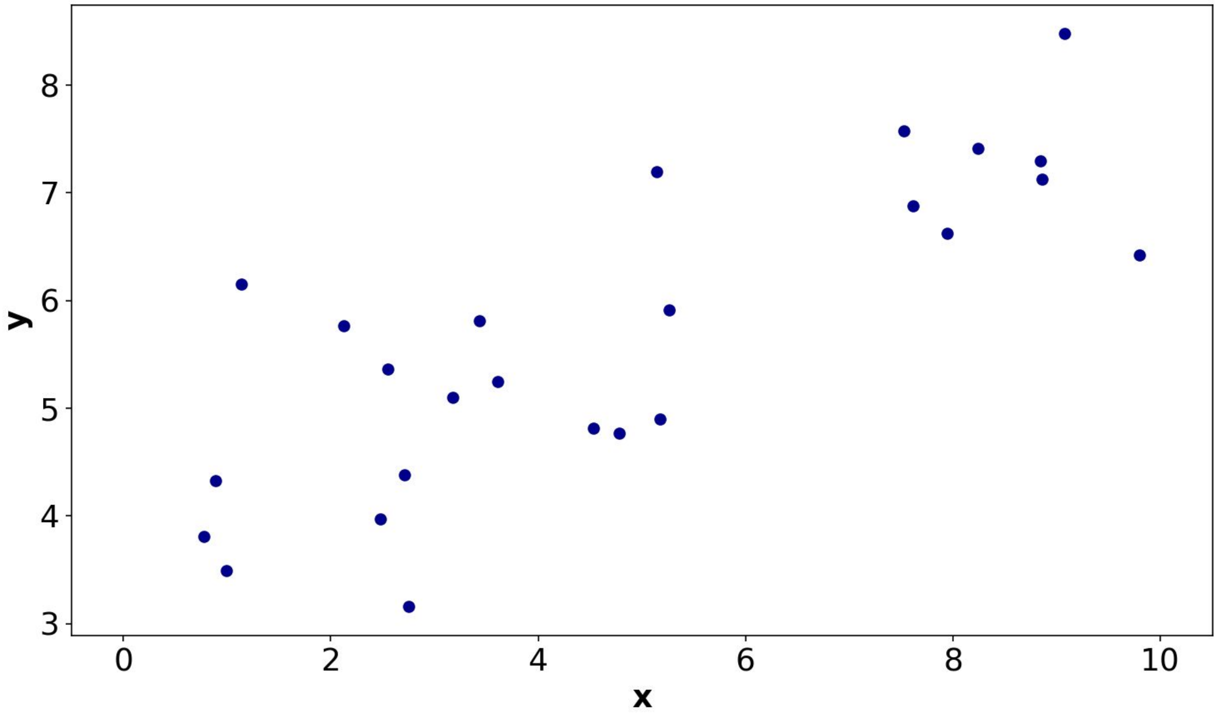
The usual way to do it is to minimize the total squared residuals between the predicted and actual values for the data used to fit/train the model.

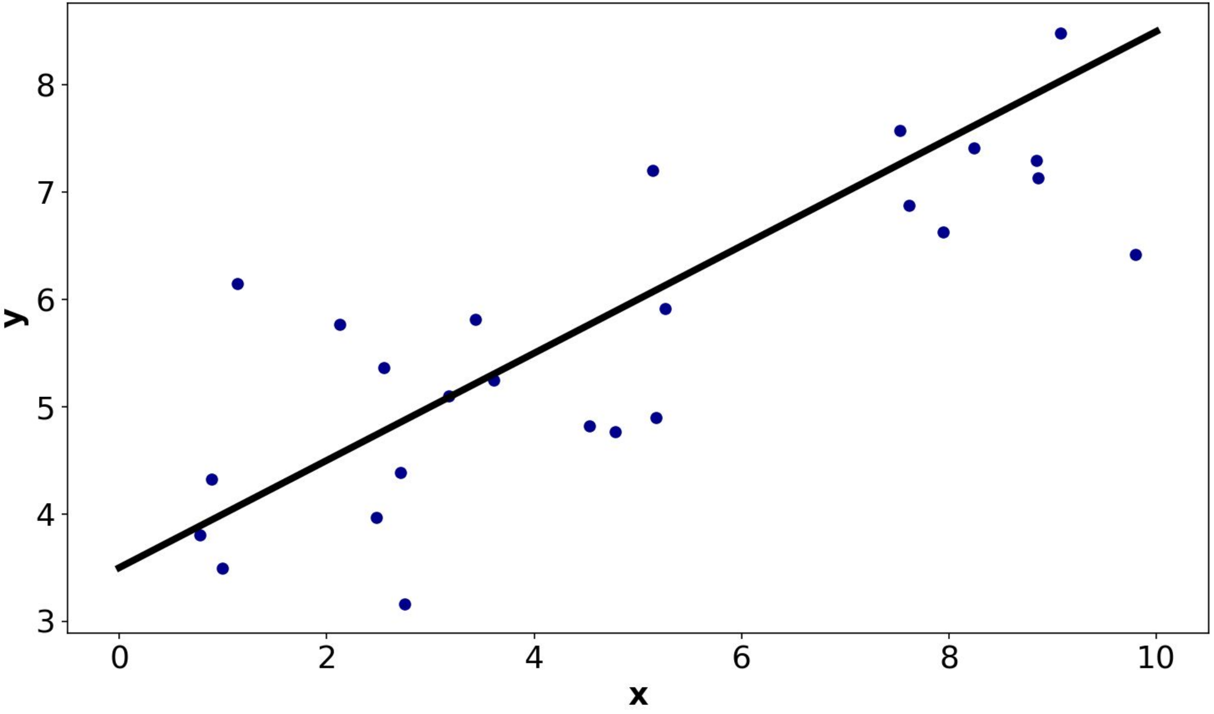
$$RSS = \sum_{i=1}^{n} (y_i - \hat{f}(\vec{x}_i))^2$$

Linear Regression



Example: Let's say we have this data available. We want to predict *y* based on our one predictor, *x*.

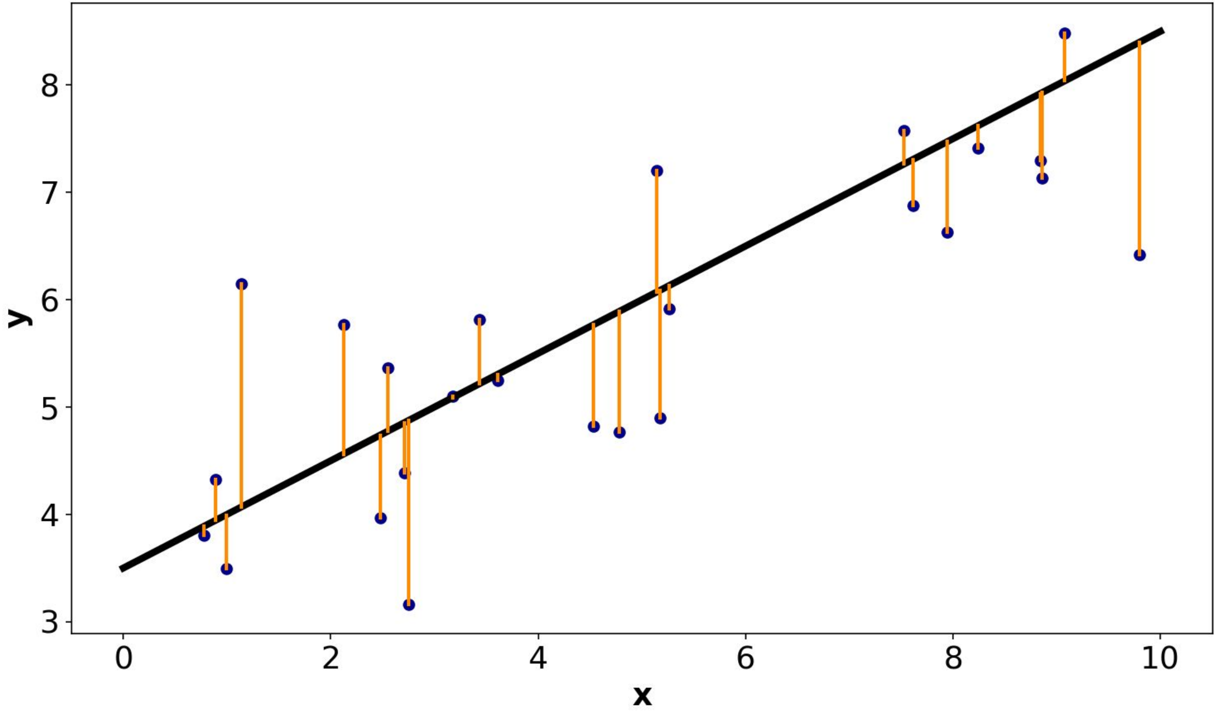
Linear Regression



One possible line:

$y = 3.5 + 0.5x$

Linear Regression



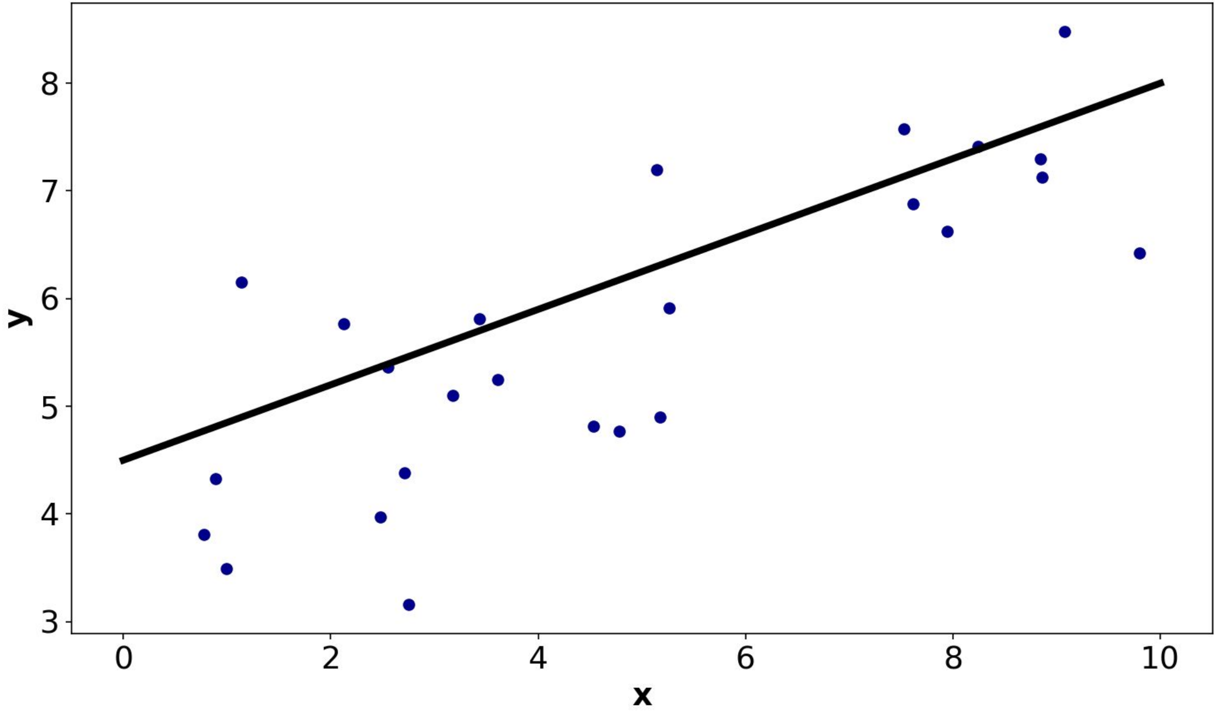
One possible line:

$y = 3.5 + 0.5x$

For this line,

RSS = 20.36

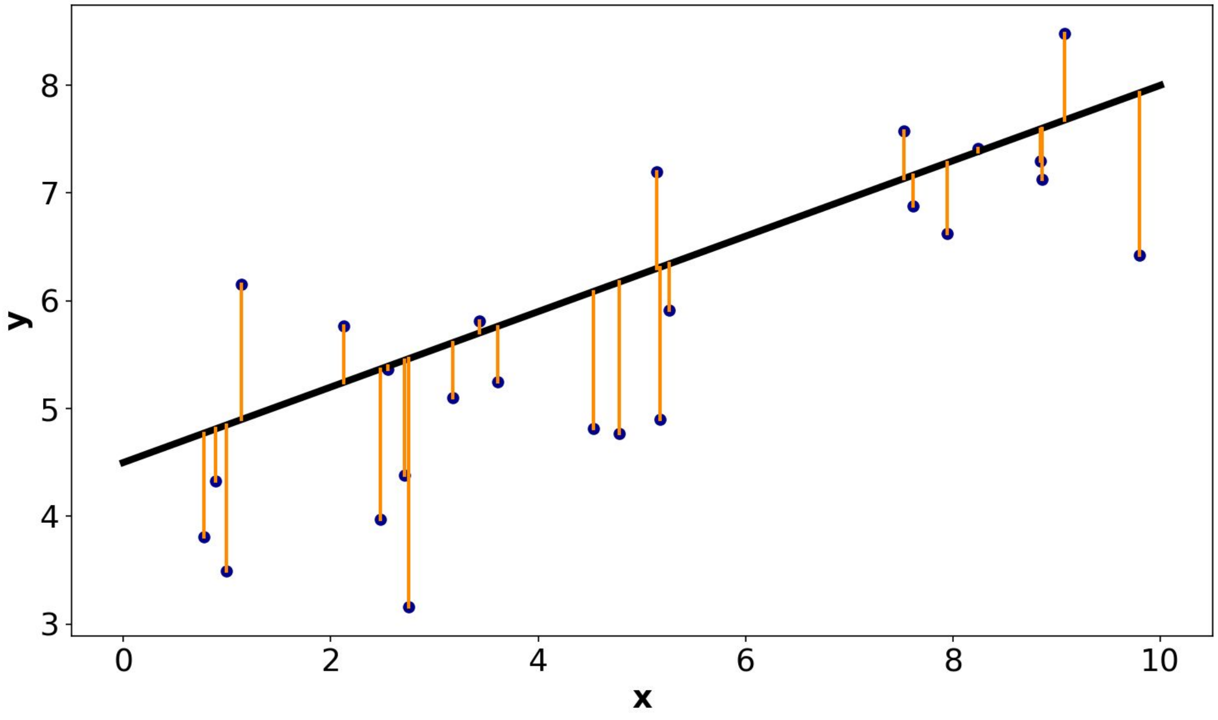
Linear Regression



Another possibility:

$y = 4.5 + 0.35x$

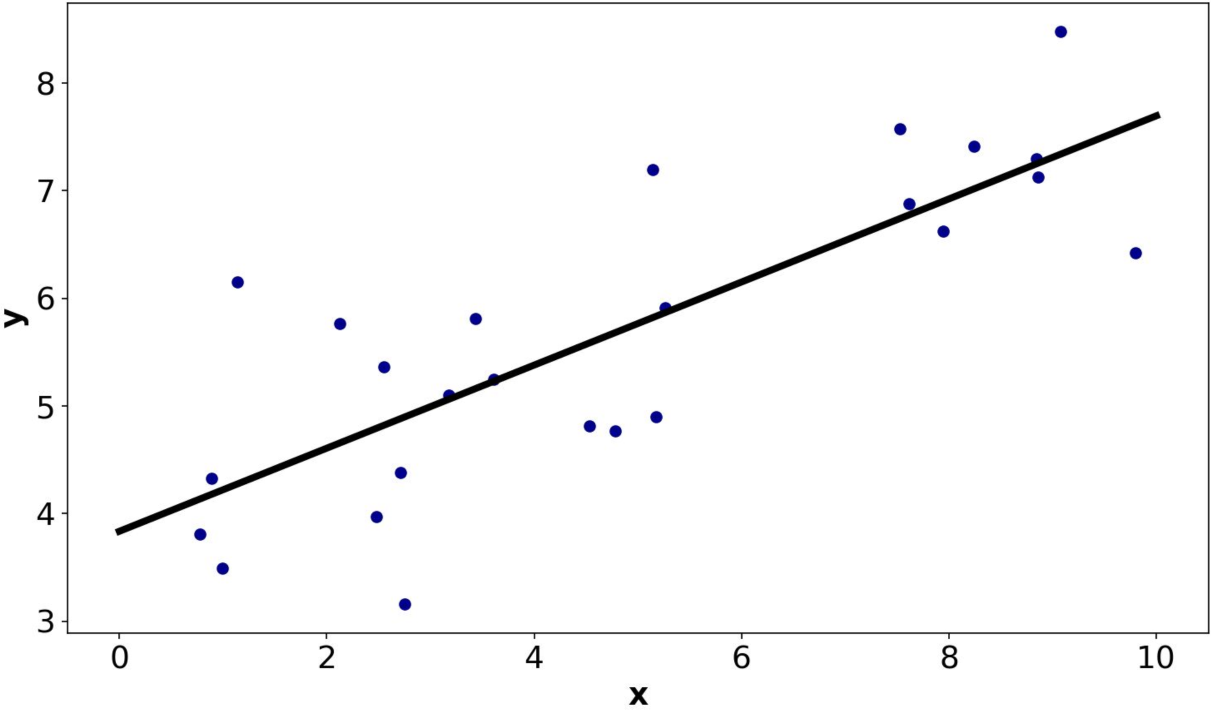
Linear Regression



Another possibility:
y = 4.5 + 0.35x

Here,
RSS = 24.28

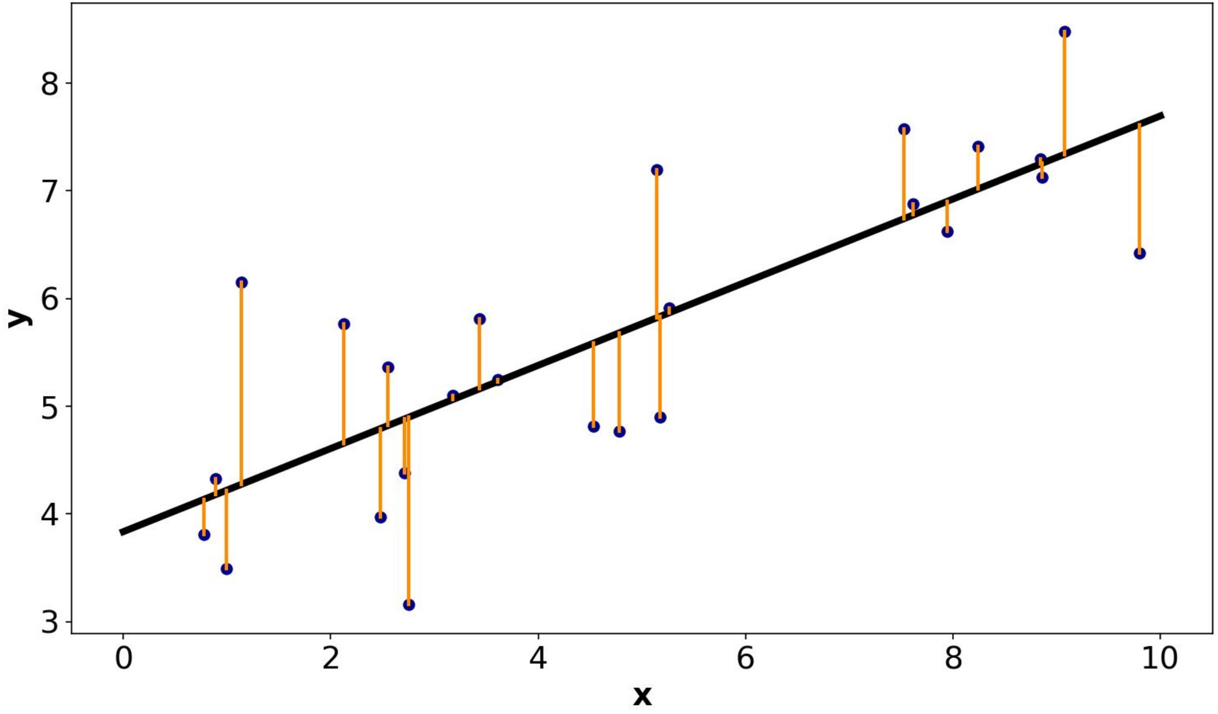
Linear Regression



The best possible:

$y = 3.84 + 0.386x$

Linear Regression



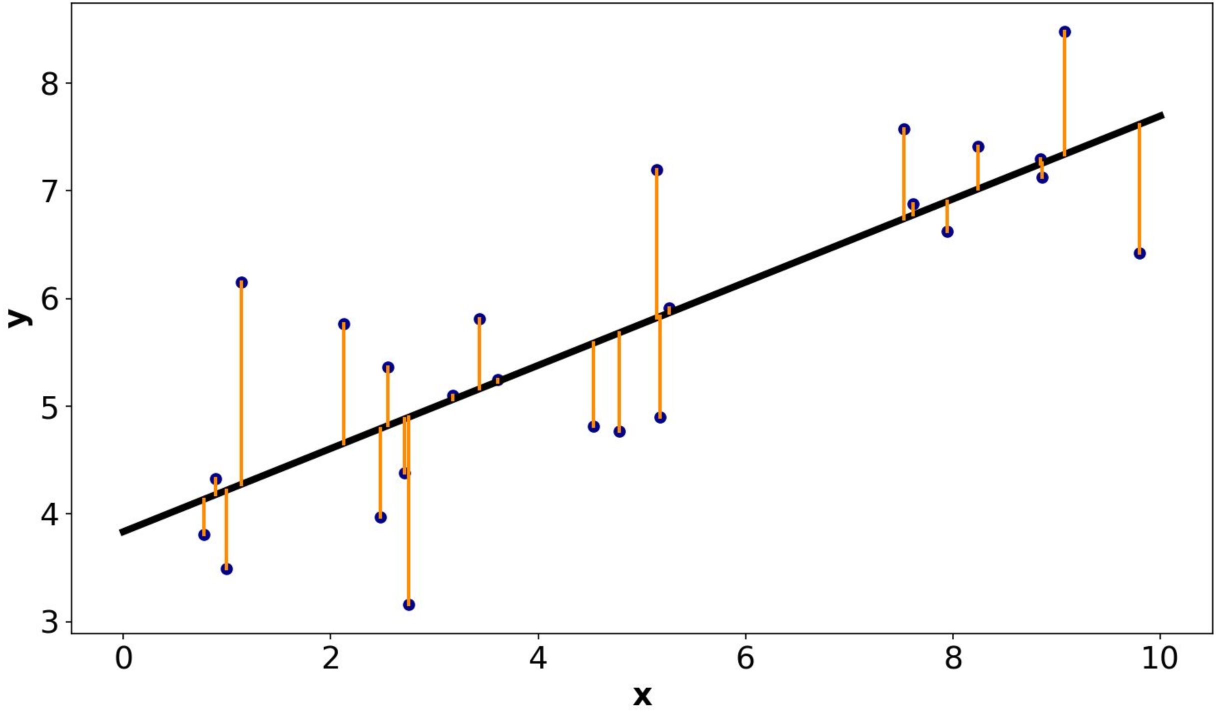
The best possible:

$y = 3.84 + 0.386x$

Here,

RSS = 17.97

Linear Regression



For the best-fitting line, the average (absolute) residual is equal to 0.67.

Can we expect that on new data generated by the same process, we will be off on average by 0.67 still?