More Linear (and Logistic) Regression

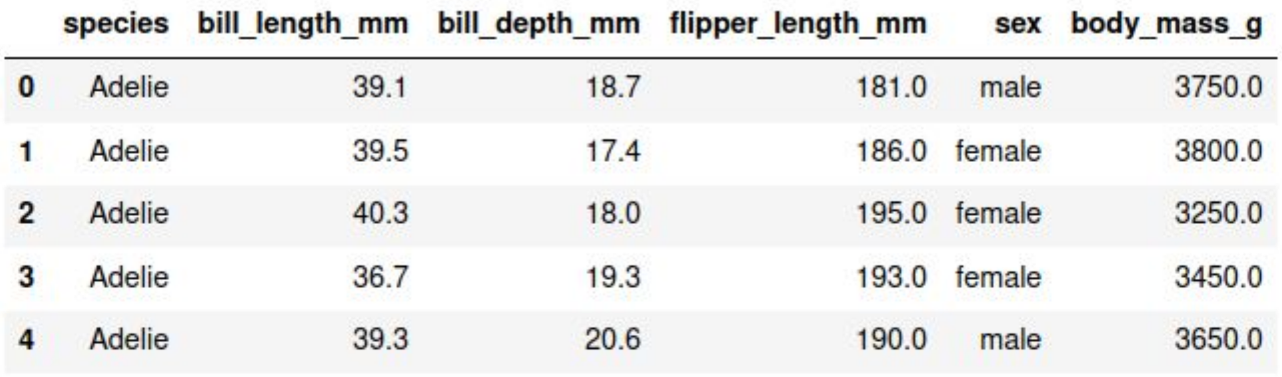
Linear Regression

Given *k* predictors $x^{(1)}$, $x^{(2)}$,...,$x^{(k)}$, linear regression uses the following equation to predict the target variable:

$$\hat{f}(\vec{x}) = \beta_0 + \beta_1 x^{(1)} + \beta_2 x^{(2)} + \cdots + \beta_k x^{(k)}$$

Here, β_0, β_1,...,β_k are constants that are determined by using the available training data.

Linear Regression - Example

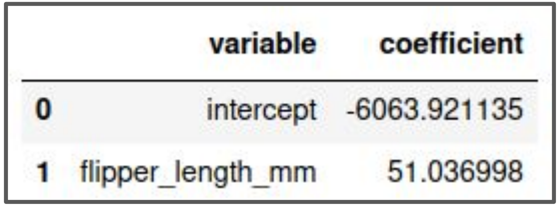


	species	bill_length_mm	bill_depth_mm	flipper_length_mm	sex	body_mass_g
0	Adelie	39.1	18.7	181.0	male	3750.0
1	Adelie	39.5	17.4	186.0	female	3800.0
2	Adelie	40.3	18.0	195.0	female	3250.0
3	Adelie	36.7	19.3	193.0	female	3450.0
4	Adelie	39.3	20.6	190.0	male	3650.0

Consider the penguins dataset.

Say we want to build a linear model to predict body_mass_g.

Linear Regression - Example

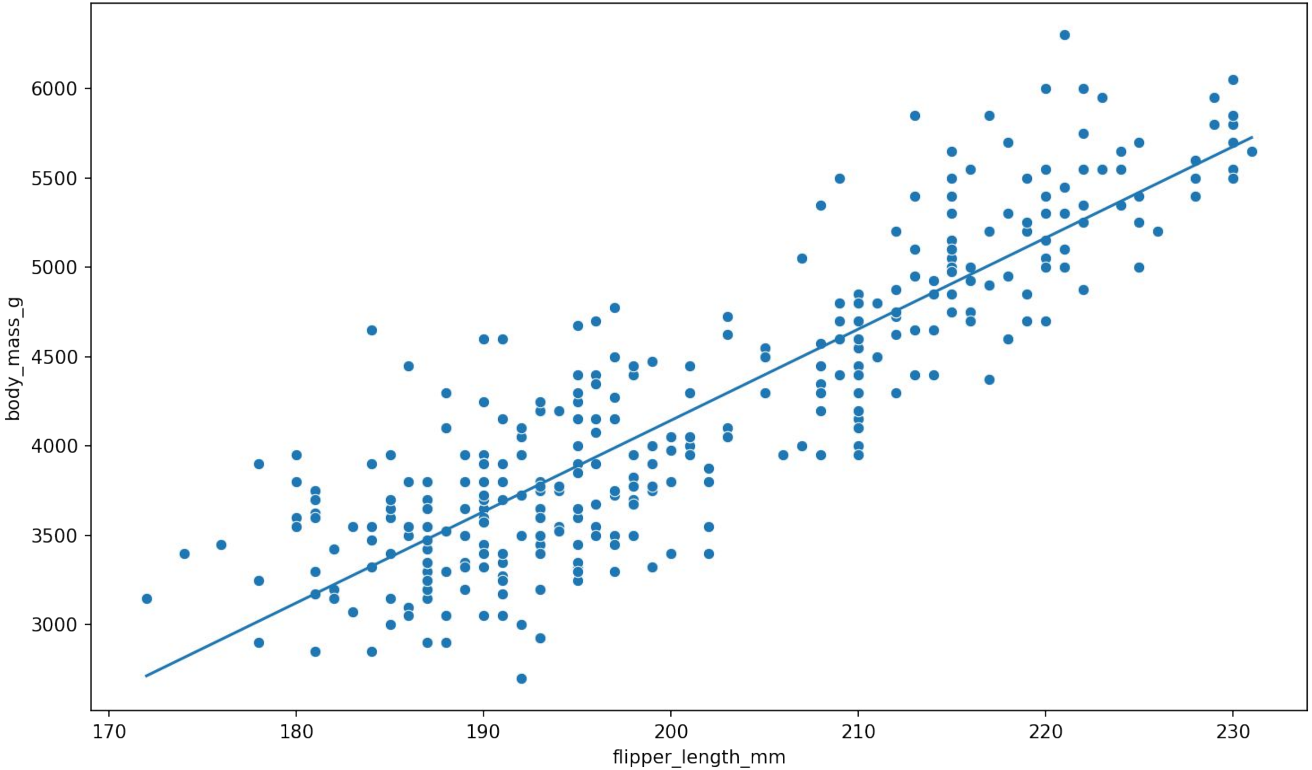


	variable	coefficient
0	intercept	-6063.921135
1	flipper_length_mm	51.036998

Using just flipper length gives these coefficients.

$$\text{predicted body\_mass} = -6064 + 51 \cdot \text{flipper\_length}$$

Linear Regression - Example



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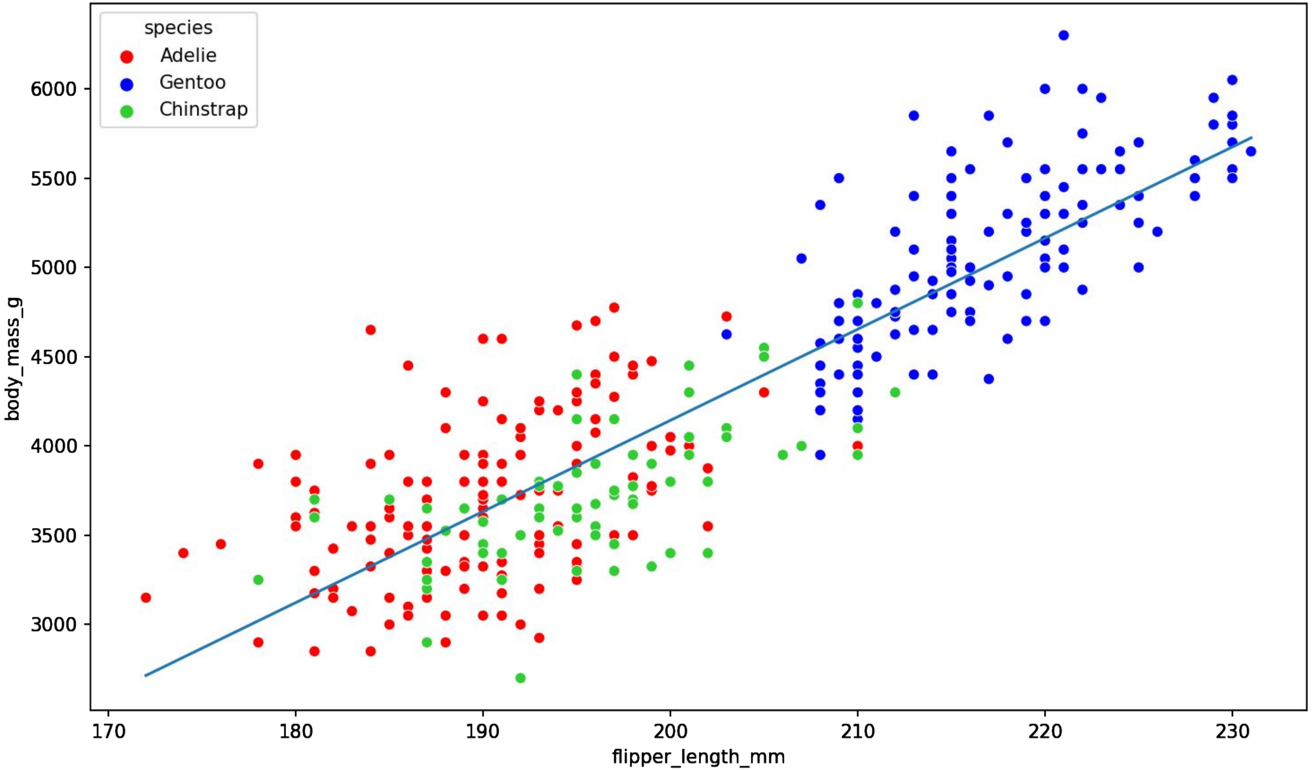

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Species is a categorical variable, but we can include it if we make dummy columns. This is also known as one-hot encoding.

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Species is a categorical variable, but we can include it if we make dummy columns. This is also known as one-hot encoding.

We have 3 species (Adelie, Chinstrap, and Gentoo), so we’ll create two new 0/1 columns.

Linear Regression - Example

species
Adelie
Chinstrap
Gentoo

species_Chinstrap	**species_Gentoo**
0	0
1	0
0	1

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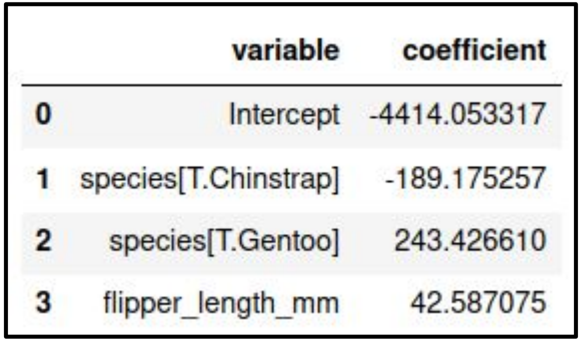

	variable	coefficient
0	Intercept	-4414.053317
1	species[T.Chinstrap]	-189.175257
2	species[T.Gentoo]	243.426610
3	flipper_length_mm	42.587075

$$y = -4414 - 189(\text{Chinstrap}) + 243 \cdot (\text{Gentoo}) + 43 \cdot \text{flipper\_length}$$

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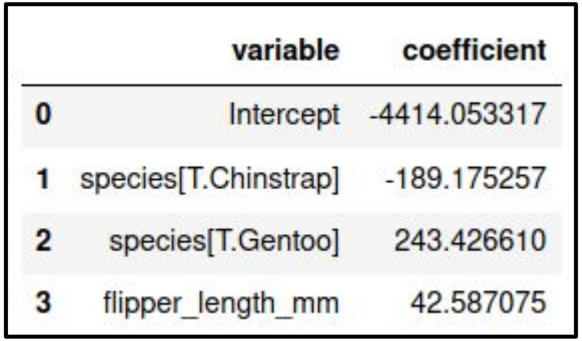

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We have three different parallel lines, one per species.

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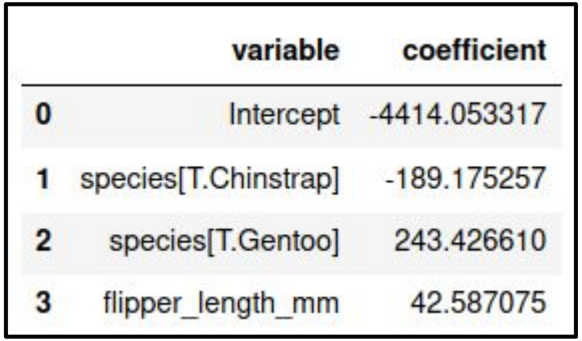

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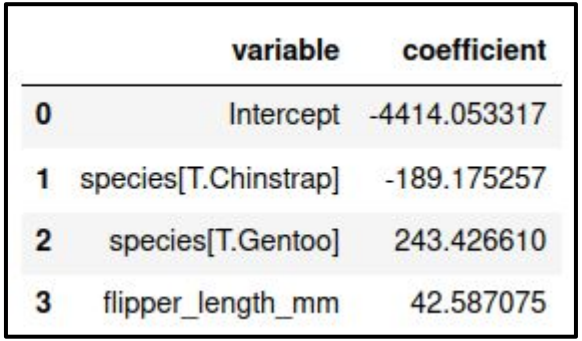

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Adelie: $y = -4414 + 43 \cdot \text{flipper\_length}$

Chinstrap: $y = -4603 + 43 \cdot \text{flipper\_length}$

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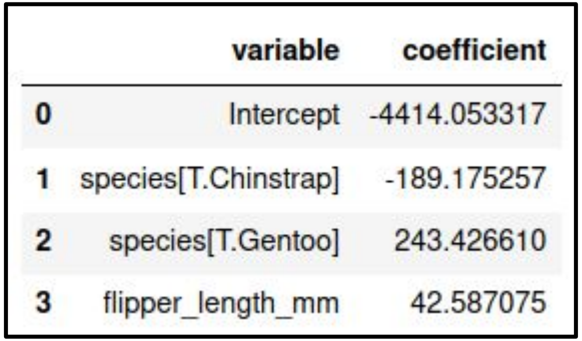

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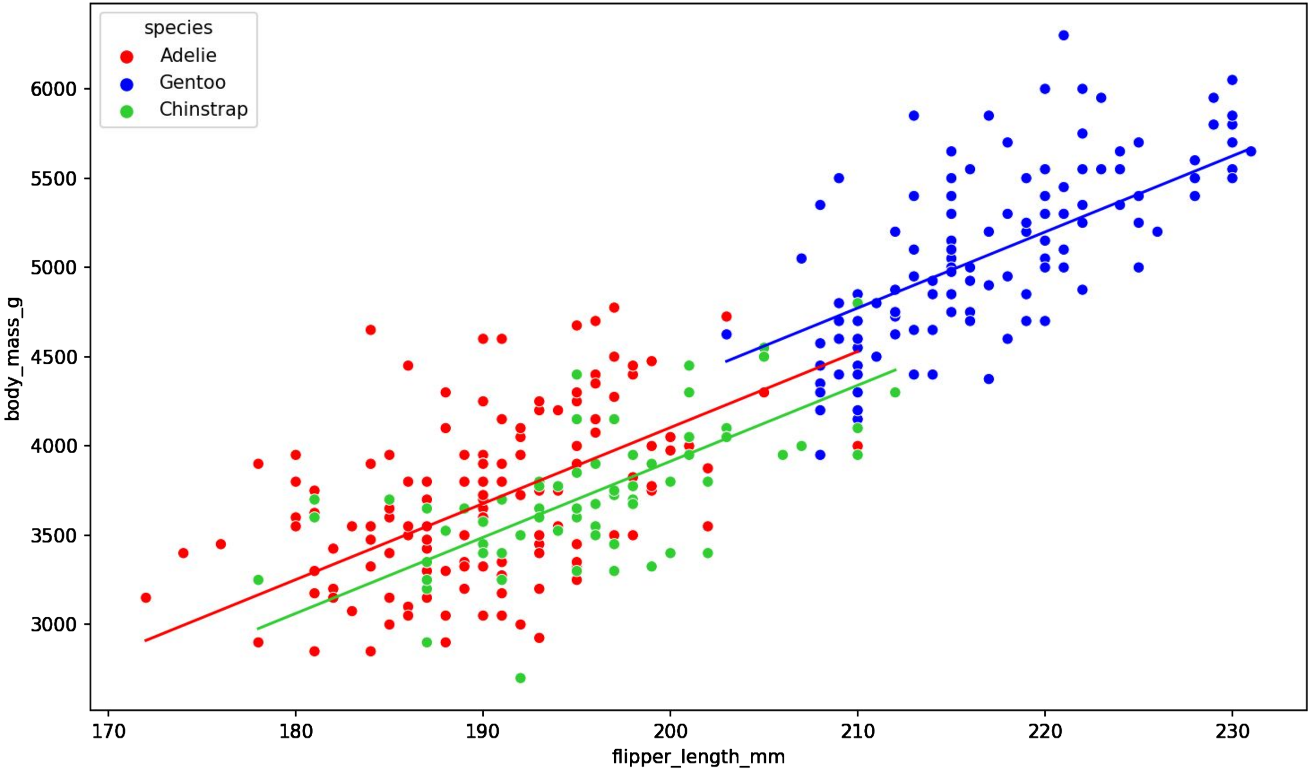
Gentoo: $y = -4171 + 43 \cdot \text{flipper\_length}$

Linear Regression - Example



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We get these by multiplying the value across two variables.

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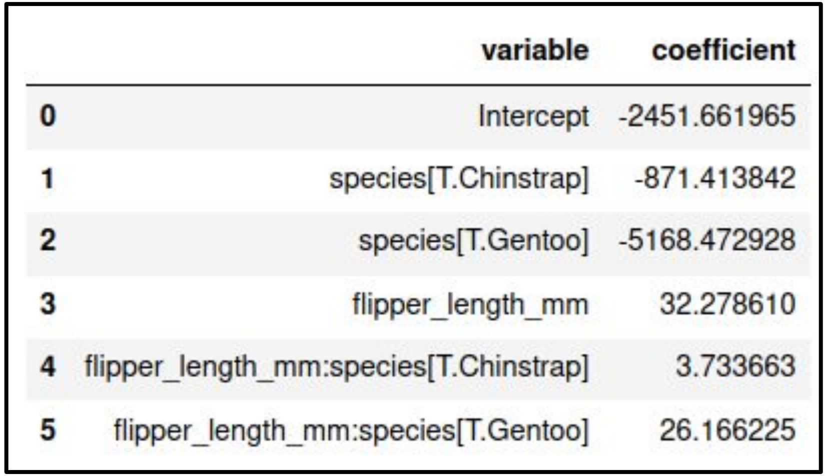

	variable	coefficient
0	Intercept	-2451.661965
1	species[T.Chinstrap]	-871.413842
2	species[T.Gentoo]	-5168.472928
3	flipper_length_mm	32.278610
4	flipper_length_mm:species[T.Chinstrap]	3.733663
5	flipper_length_mm:species[T.Gentoo]	26.166225

Now, we have 3 different lines, one per species:

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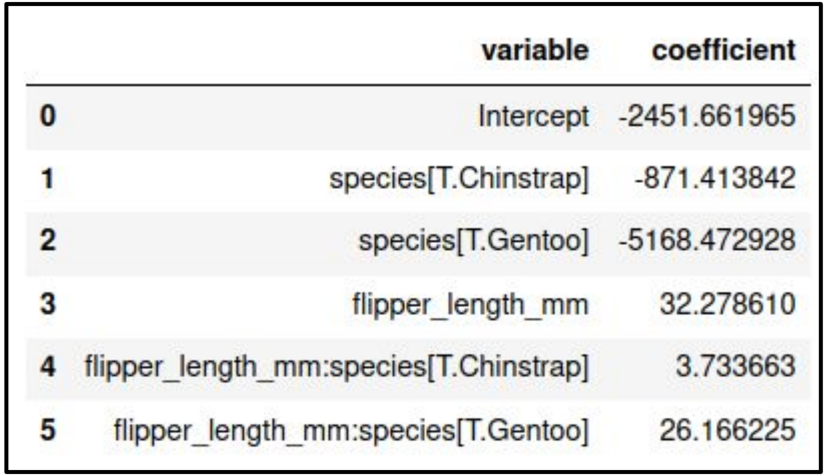

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Now, we have 3 different lines, one per species:

Adelie: $y = -2452 + 32 \cdot \text{flipper\_length}$

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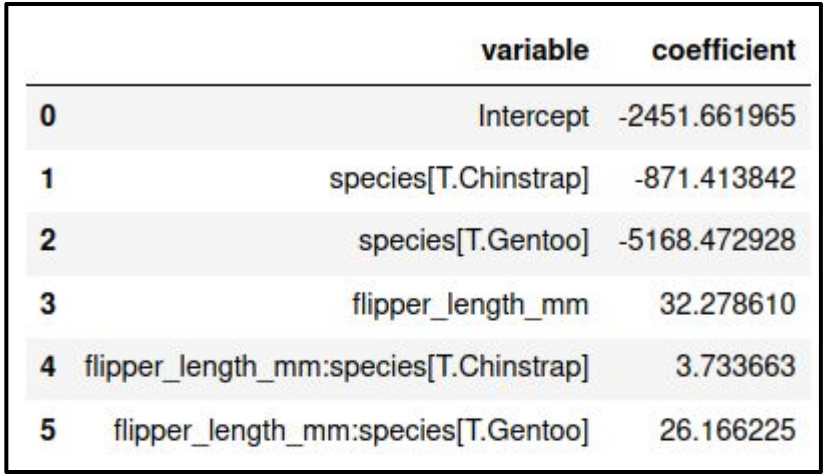

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Now, we have 3 different lines, one per species:

Adelie: $y = -2452 + 32 \cdot \text{flipper\_length}$

Chinstrap: $y = -3323 + 36 \cdot \text{flipper\_length}$

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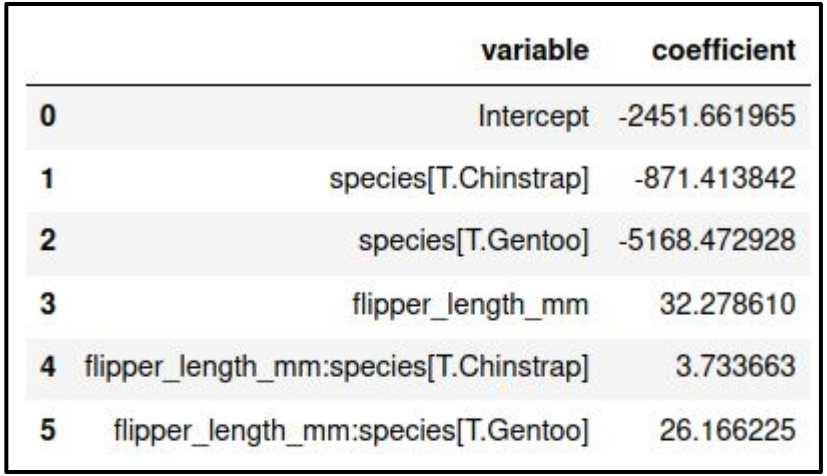

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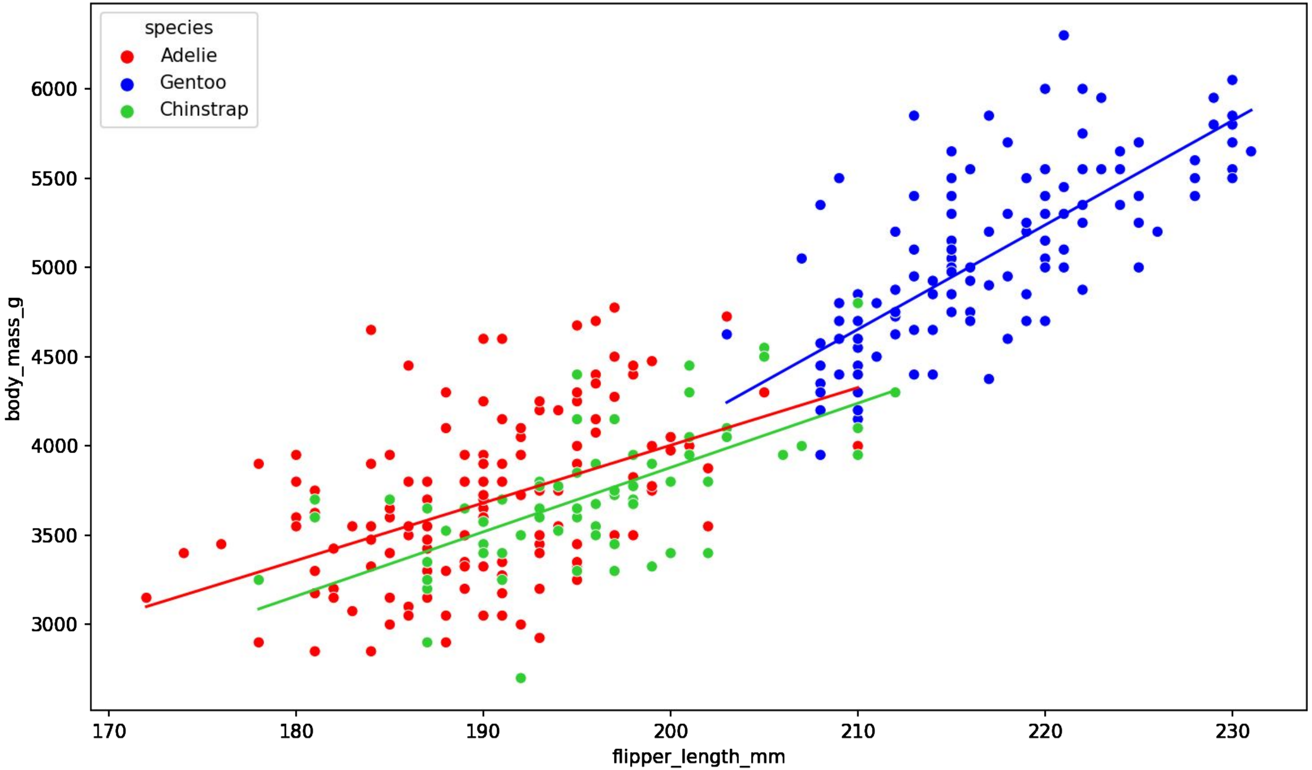
Now, we have 3 different lines, one per species:

Adelie: $y = -2452 + 32 \cdot \text{flipper\_length}$

Chinstrap: $y = -3323 + 36 \cdot \text{flipper\_length}$

Gentoo: $y = -7620 + 58 \cdot \text{flipper\_length}$

Linear Regression - Example



Linear Regression - Example

What if we also include the sex variable?

We’ll add it and the interactions with the flipper length.

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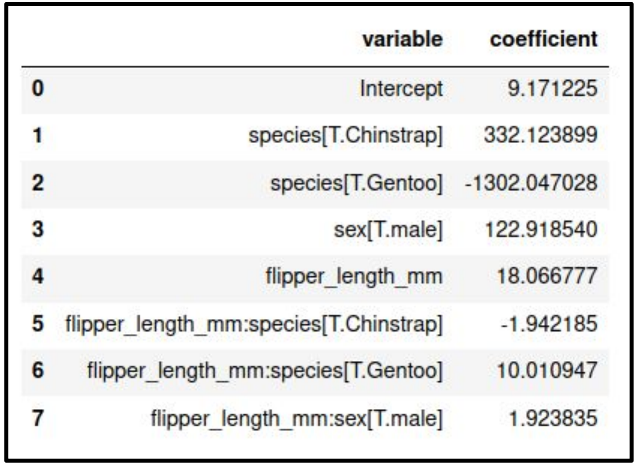

	variable	coefficient
0	Intercept	9.171225
1	species[T.Chinstrap]	332.123899
2	species[T.Gentoo]	-1302.047028
3	sex[T.male]	122.918540
4	flipper_length_mm	18.066777
5	flipper_length_mm:species[T.Chinstrap]	-1.942185
6	flipper_length_mm:species[T.Gentoo]	10.010947
7	flipper_length_mm:sex[T.male]	1.923835

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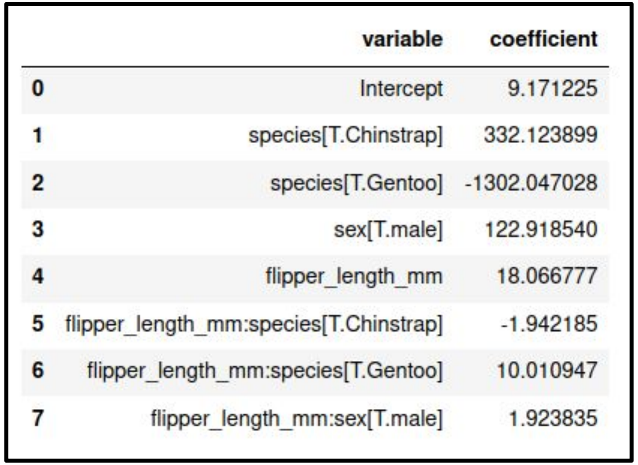

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5	flipper_length_mm:species[T.Chinstrap]	-1.942185
6	flipper_length_mm:species[T.Gentoo]	10.010947
7	flipper_length_mm:sex[T.male]	1.923835

Now, we have 6 different lines, one per species/sex combination:

	female	male
Adelie	y = 9 + 18 · (flipper length)	y = 132 + 20 · (flipper length)
Chinstrap	y = 341 + 16 · (flipper length)	y = 464 + 18 · (flipper length)
Gentoo	y = -1293 + 28 · (flipper length)	y = -1170 + 30 · (flipper length)

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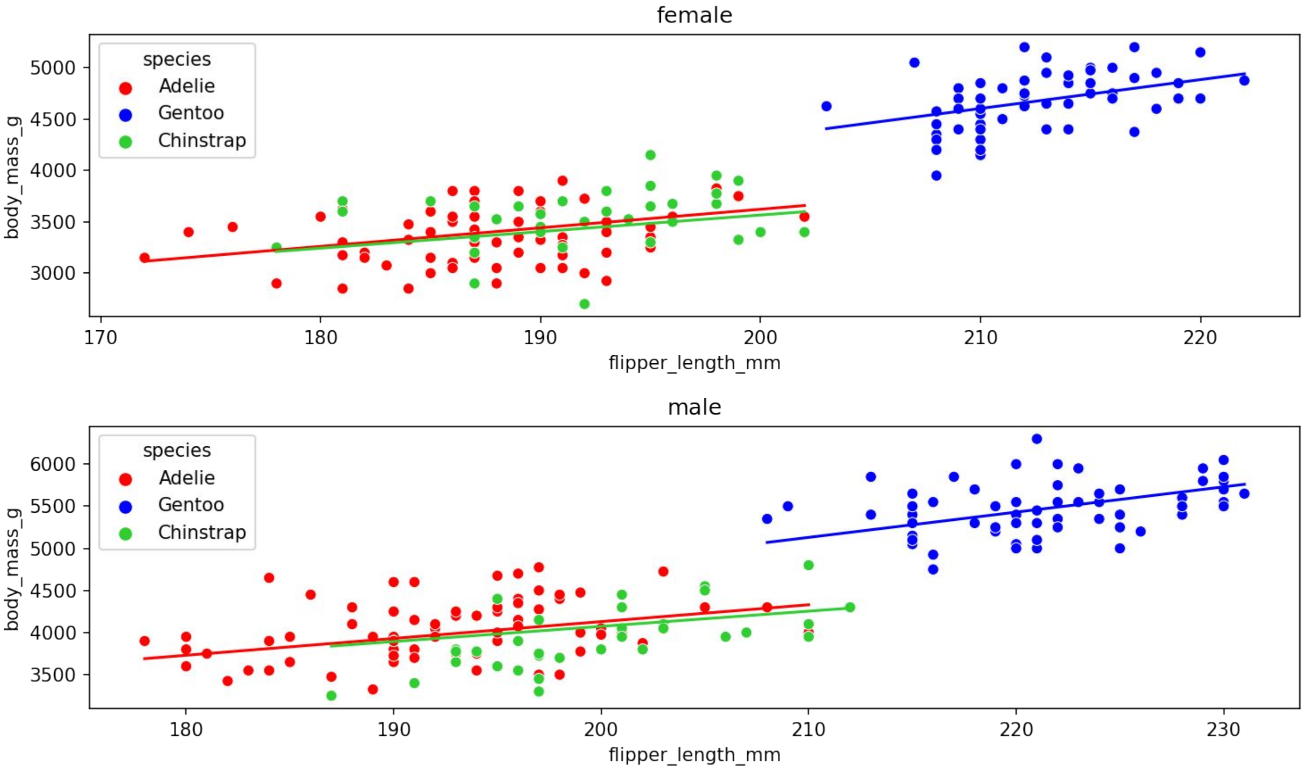

Linear Regression - Example

Question: Would we ever not want to do this? What are the potential downsides?