Hypothesis Testing: Permutation Tests

Recall: Hypothesis Testing Steps

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3. If the probability is low enough, reject the null in favor of the alternative.

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3. If the probability is low enough, reject the null in favor of the alternative.

This is typically done by calculating some statistic (eg. the sample mean) and comparing this value to the corresponding sampling distribution under the assumption of the null hypothesis.

Hypothesis Testing

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Permutation testing allows us to bypass both the assumptions required or the derivation necessary to use the true sampling distribution of our statistic.

Permutation Testing

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Since we are speculating that there are more crashes on Saturdays, we will have a one-tailed alternative hypothesis.

H_1: The distribution of crashes on Saturday has a larger mean than the distribution of crashes on Sunday

Permutation Testing

We also need to set the **significance level**.

That is, how unusual does our observed data need to be under the assumption of the null hypothesis for us to reject that null hypothesis.

For this example, we'll use the 5% significance level.

That is, if what we observe would happen less than 5% of the time when the null hypothesis is true, then we'll reject the null hypothesis in favor of the alternative hypothesis.

Permutation Testing

Original Data

Saturday		
82	61	72
98	69	71
64	81	78
53	83	91

Sunday		
44	75	63
89	68	72
66	73	52
54	76	82

Mean = 75.25

Mean = 67.83

Observed Difference: 75.25 - 67.83 = 7.42

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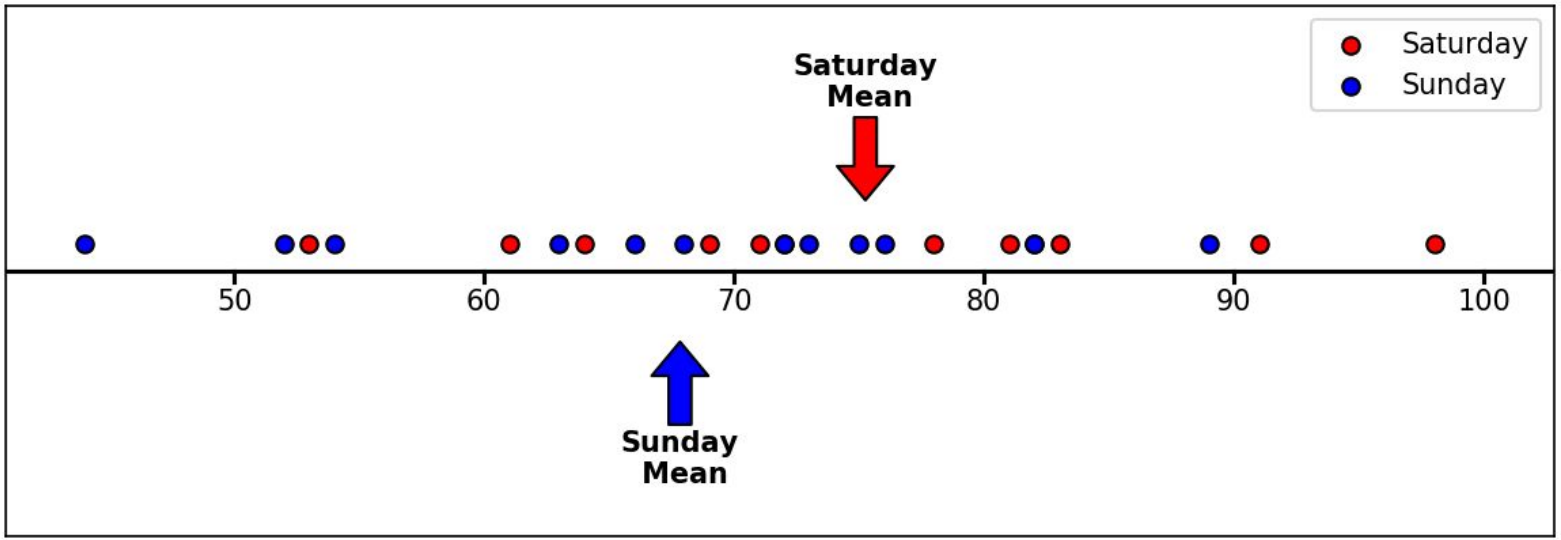
Permutation Testing

Original Data



Observed Difference: 75.25 - 67.83 = 7.42

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Permutation Testing

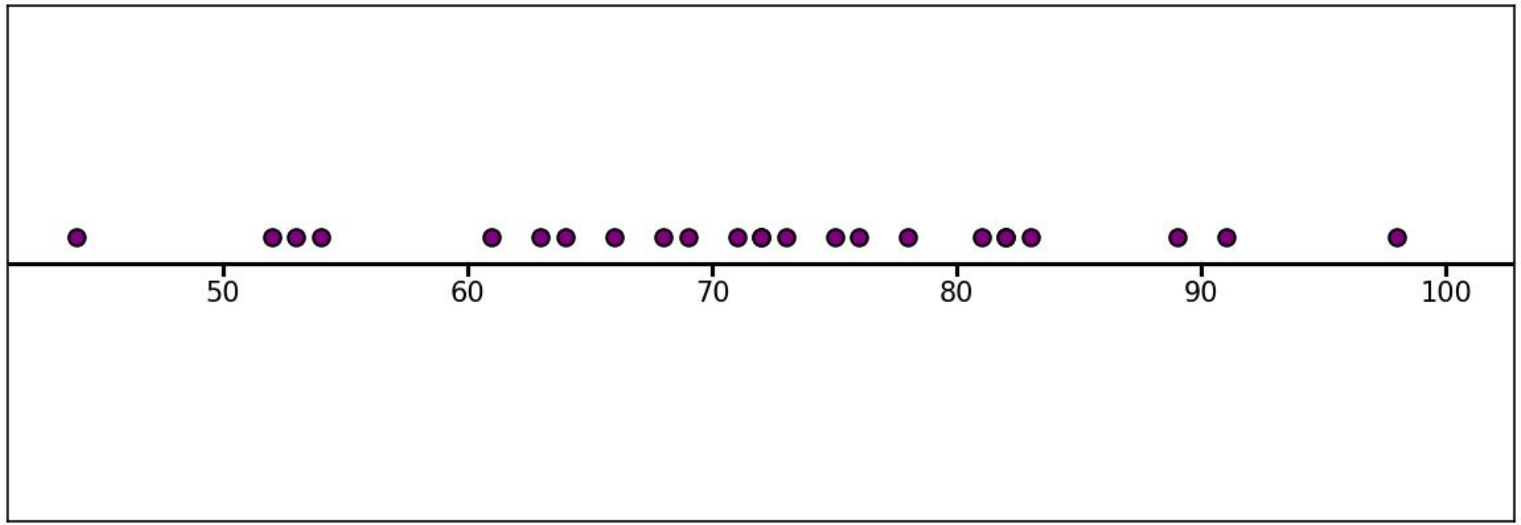
Saturday/Sunday					
82	61	72	44	75	63
98	69	71	89	68	72
64	81	78	66	73	52
53	83	91	54	76	82

Under the assumptions of the null hypothesis we are not looking at two samples but instead a single sample from the Saturday/Sunday distribution.

That is, the label does not impart any additional information.

Permutation Testing

Original Data Under Null Hypothesis



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Permutation Testing

Another way to interpret the null hypothesis is to say that the labels are irrelevant - even if we randomly assigned labels, we would still have a valid sample.

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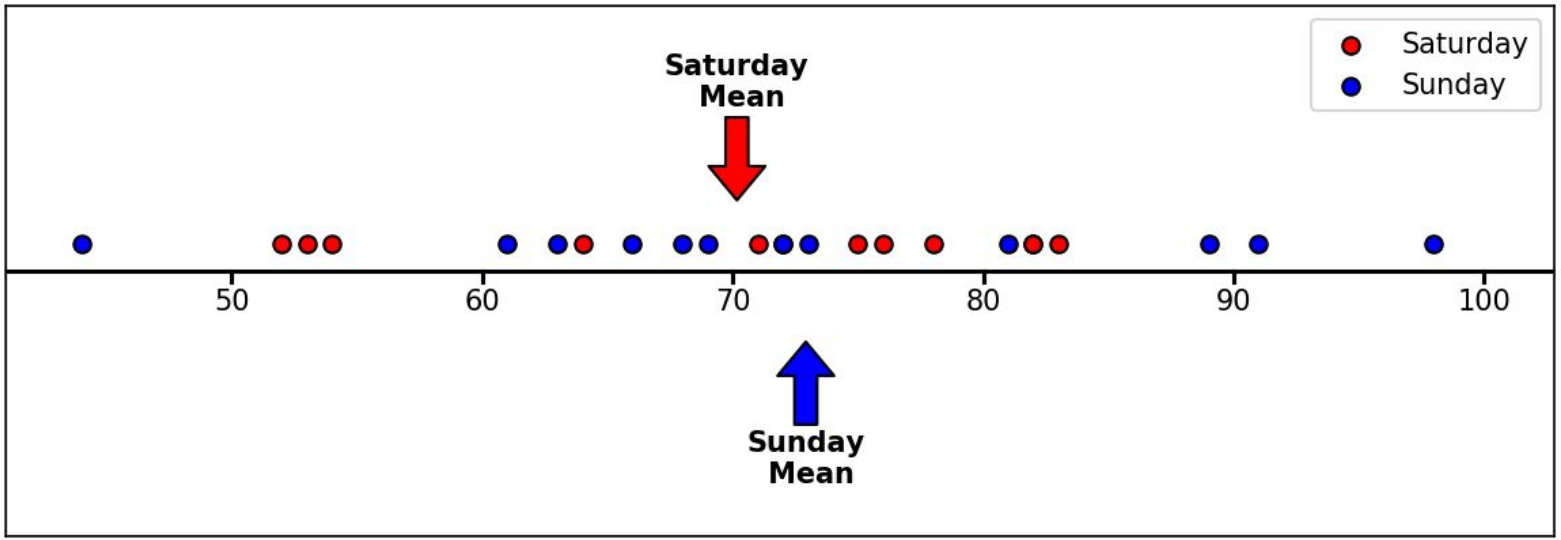
Let’s see what it would look like in a couple of cases if we did just randomly assign labels.

This technique is called **permutation testing** because we are permuting the observed values.

Permutation Testing

Permuted Data:



Difference: 70.17 - 72.92 = -2.75

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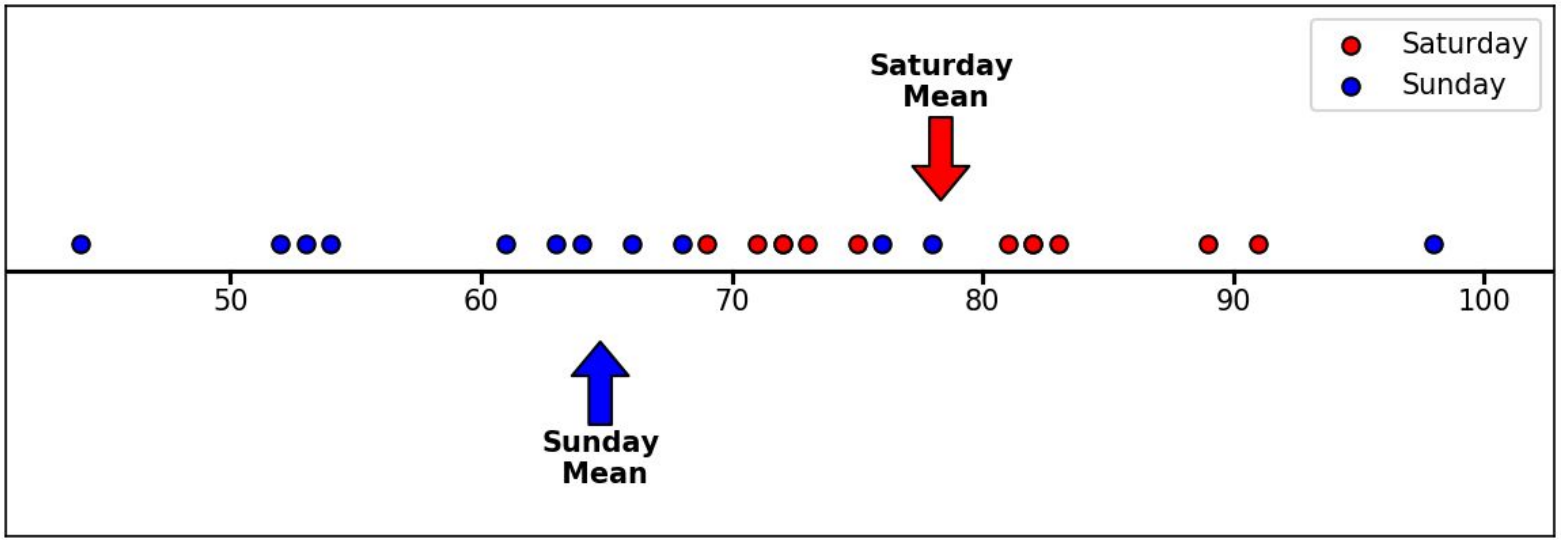
Permutation Testing

Another possible permutation:



Difference: 78.33 - 64.75 = 13.58

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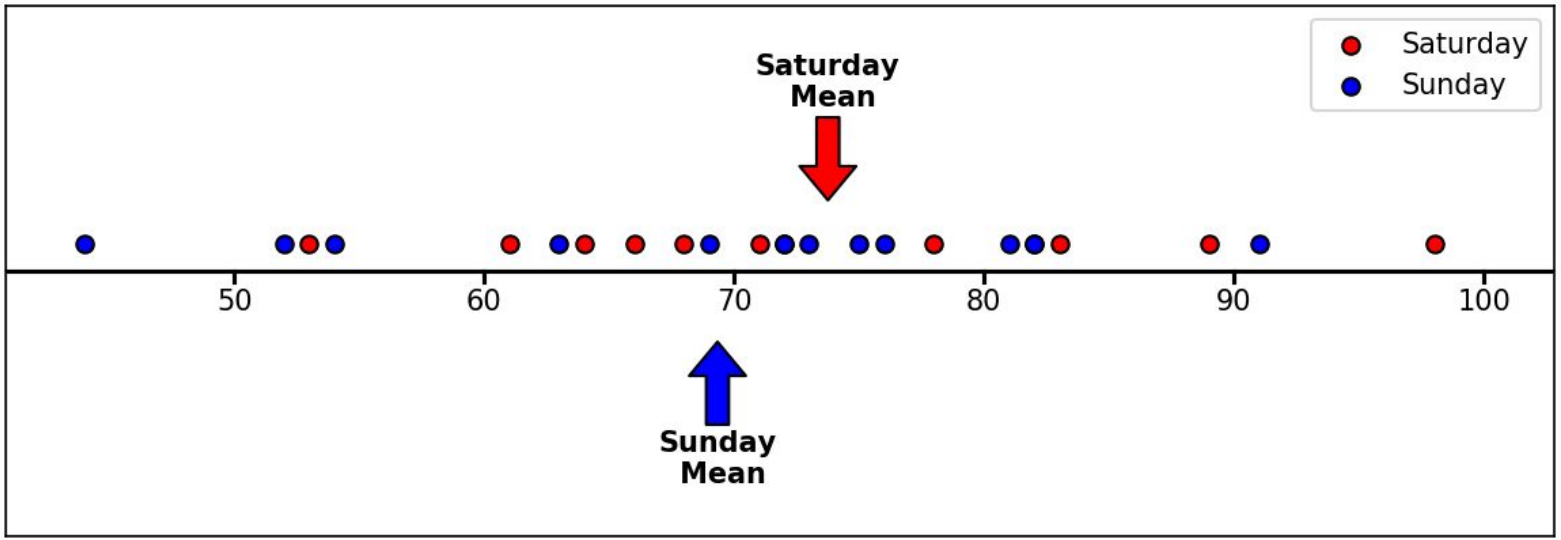
Permutation Testing

Another possible permutation:



Difference: 73.75 - 69.33 = 4.42

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Permutation Testing

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In permutation testing, we do this by looking at all of the possible assignments of labels (here, Saturday/Sunday) and see for what percentage of the time we get a more extreme difference in means between groups than what we observed.

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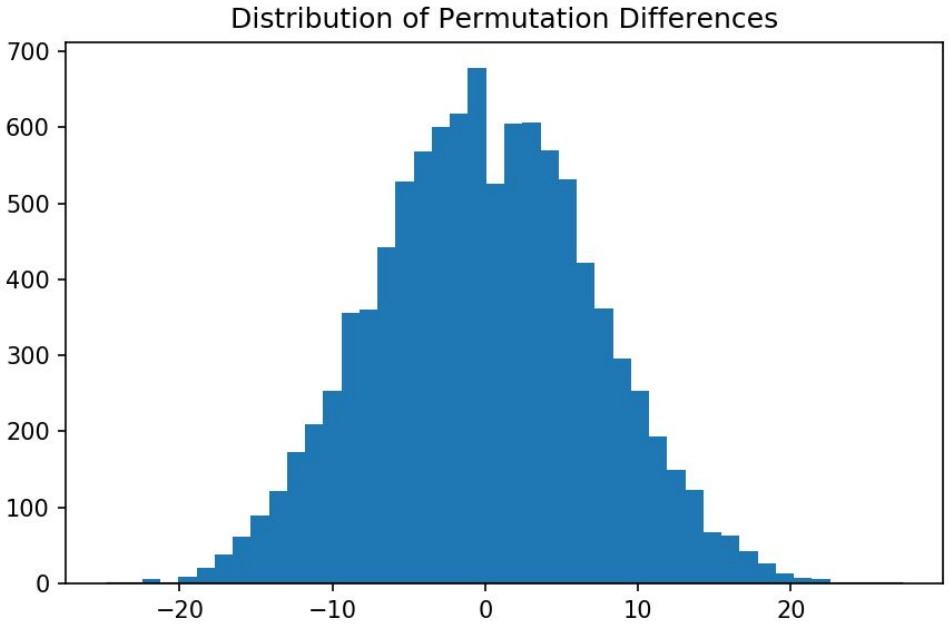
In permutation testing, we do this by looking at all of the possible assignments of labels (here, Saturday/Sunday) and see for what percentage of the time we get a more extreme difference in means between groups than what we observed.

Note that there are 2,704,156 possible ways to shuffle the Saturday/Sunday labels, so we usually just take 10,000 or so random shufflings and use this to approximate the distribution of differences.

Permutation Testing

If we shuffle labels 10,000 times, we end up with a distribution of differences that looks like this:



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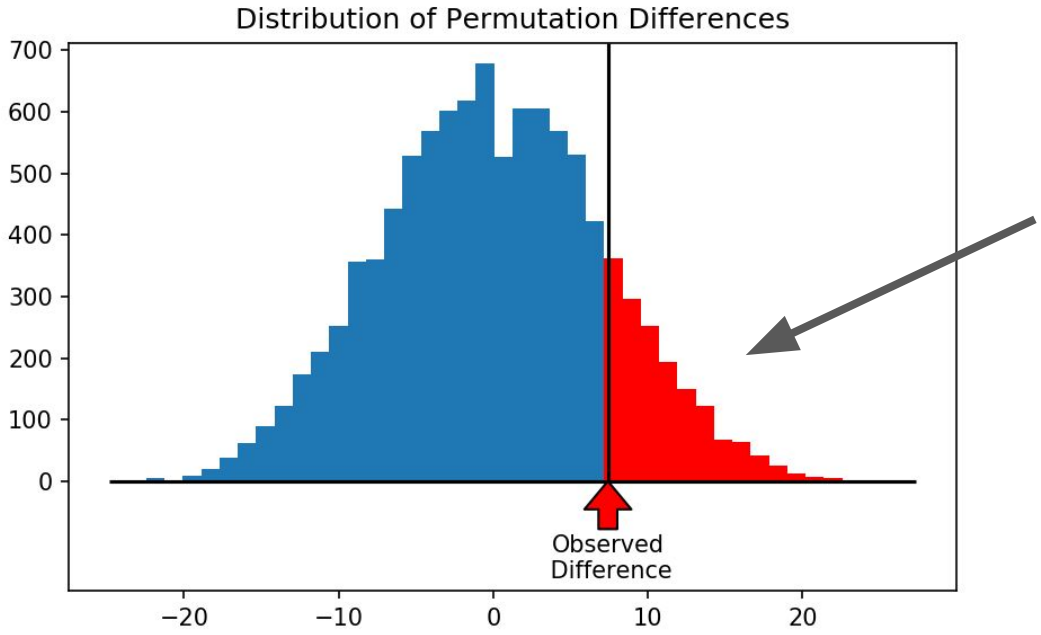
Permutation Testing

We need to ask how unusual what we observed was. That is, how often did we end up with a difference at least as large as what we observed.



Proportion More Extreme = 0.1509

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Permutation Testing

What we see is that if the distribution on Saturday and Sunday was identical, we would obtain a difference at least as large more that 15% of the time.

This is not *that* unusual and does not meet the 5% threshold to reject the null hypothesis.

We must conclude that our data does not provide enough evidence to conclude that there is a difference in the distribution of crashes on Saturdays vs. Sundays.