Probability Part 3:
The Poisson Distribution

The Poisson Distribution

The Poisson distribution expresses the probability of a given number of events occurring in a fixed interval of time or space.

There are certain assumptions to be met for a variable to follow a Poisson distribution:

- The events must occur at a known, constant mean rate.
- The events must occur independently of the time since the last event.

The Poisson Distribution

Examples:

- Customer arrivals at a restaurant
- Phone calls at a call center
- Number of flaws on a length of cable

The Poisson Distribution

The PMF for the Poisson distribution is:

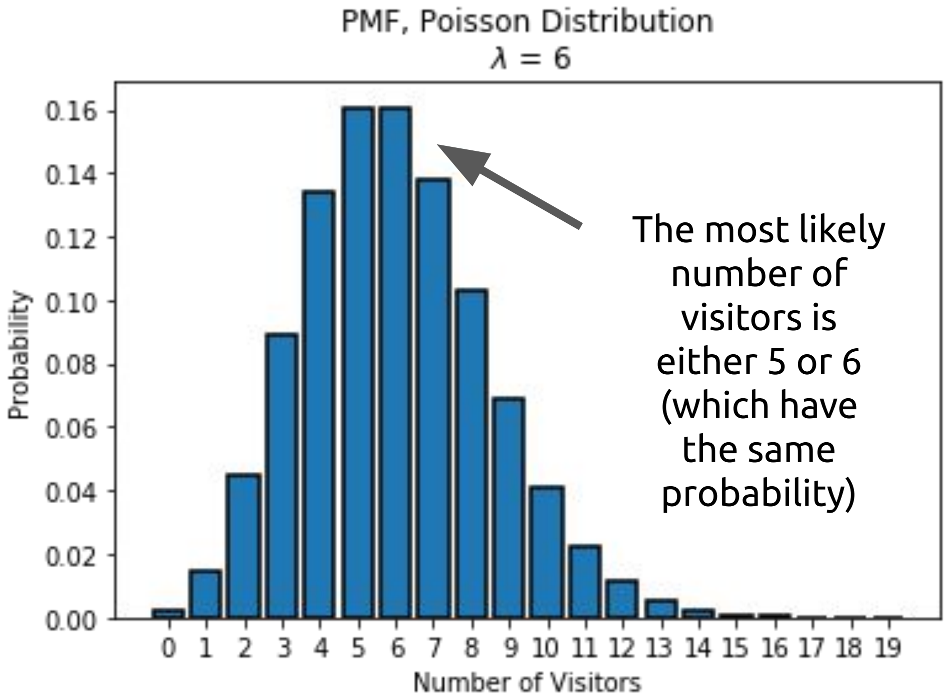
$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where λ is the average number of occurrences in a given interval and *e* is Euler's number (approximately 2.71828), and k = 0, 1, 2, 3, ...

The Poisson Distribution

Example: The number of visitors to a website per minute follows a Poisson distribution, with the average number of visitors per minute being 6.

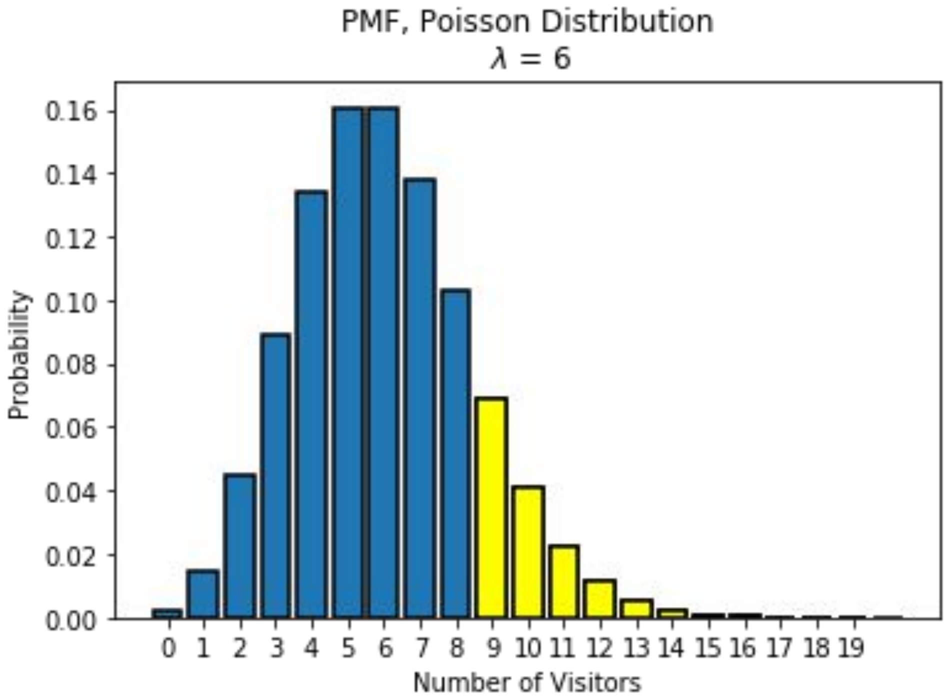
Let's look at the PMF for this distribution.



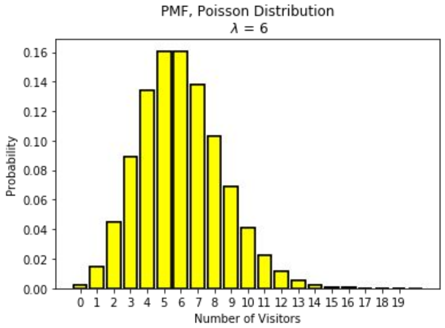
The Poisson Distribution

For this website, how likely is it to receive more than 8 visitors in a minute?

To answer this, we can use the CDF.



Poisson CDF



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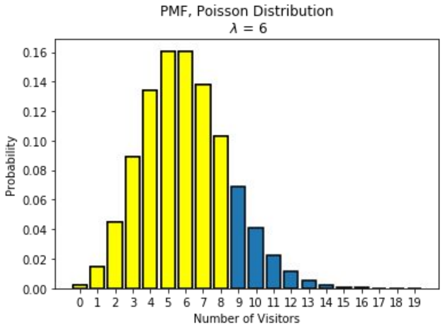

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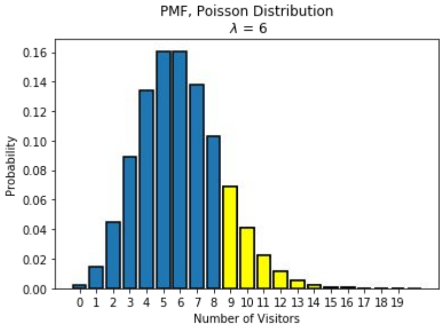


1 - 0.8472 = 0.1528

Exponential Distribution

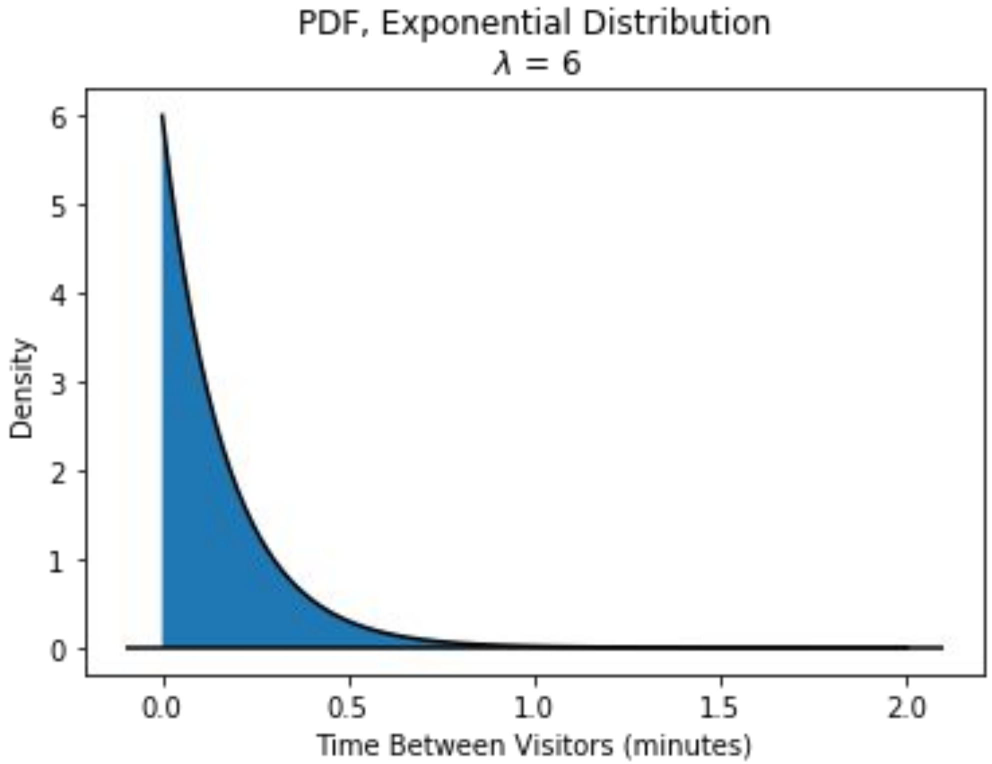
If we are interested not in the number of occurrences, but instead the time until the next occurrence, we can use an **exponential** distribution.

This is a *continuous* probability distribution and has pdf

$$f(x) = \lambda \cdot e^{-\lambda \cdot x}, \quad x \geq 0$$

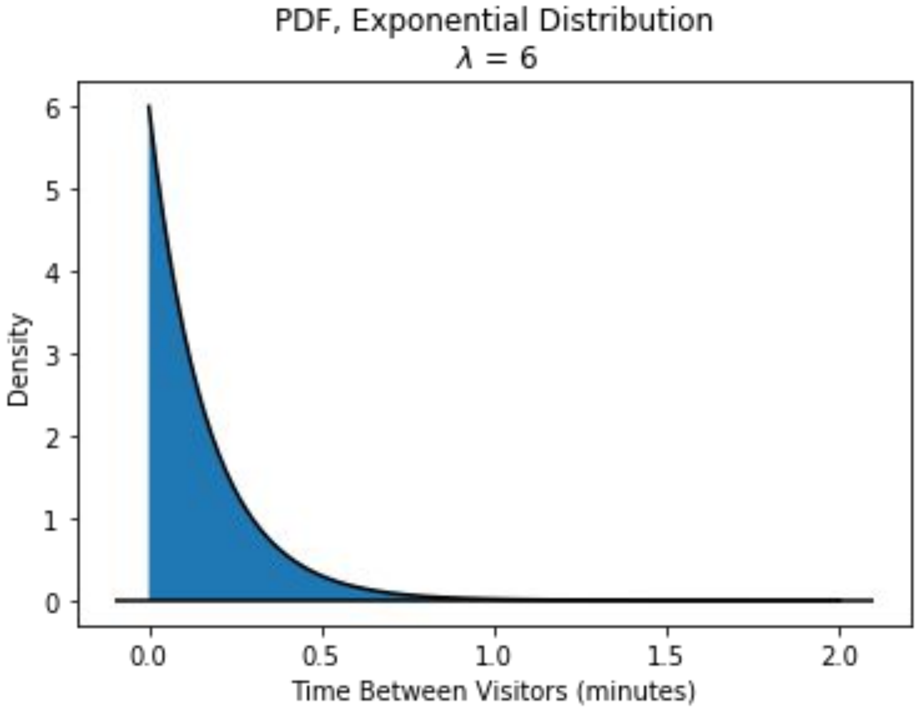
λ is the average number of occurrences in a one-unit time interval (the same as for the related Poisson distribution)

Exponential Distribution



Exponential Distribution

What's the probability that the next visitor will arrive in the next 30 seconds?



Exponential Distribution

What's the probability that the next visitor will arrive in the next 30 seconds?

Ans: 0.9502