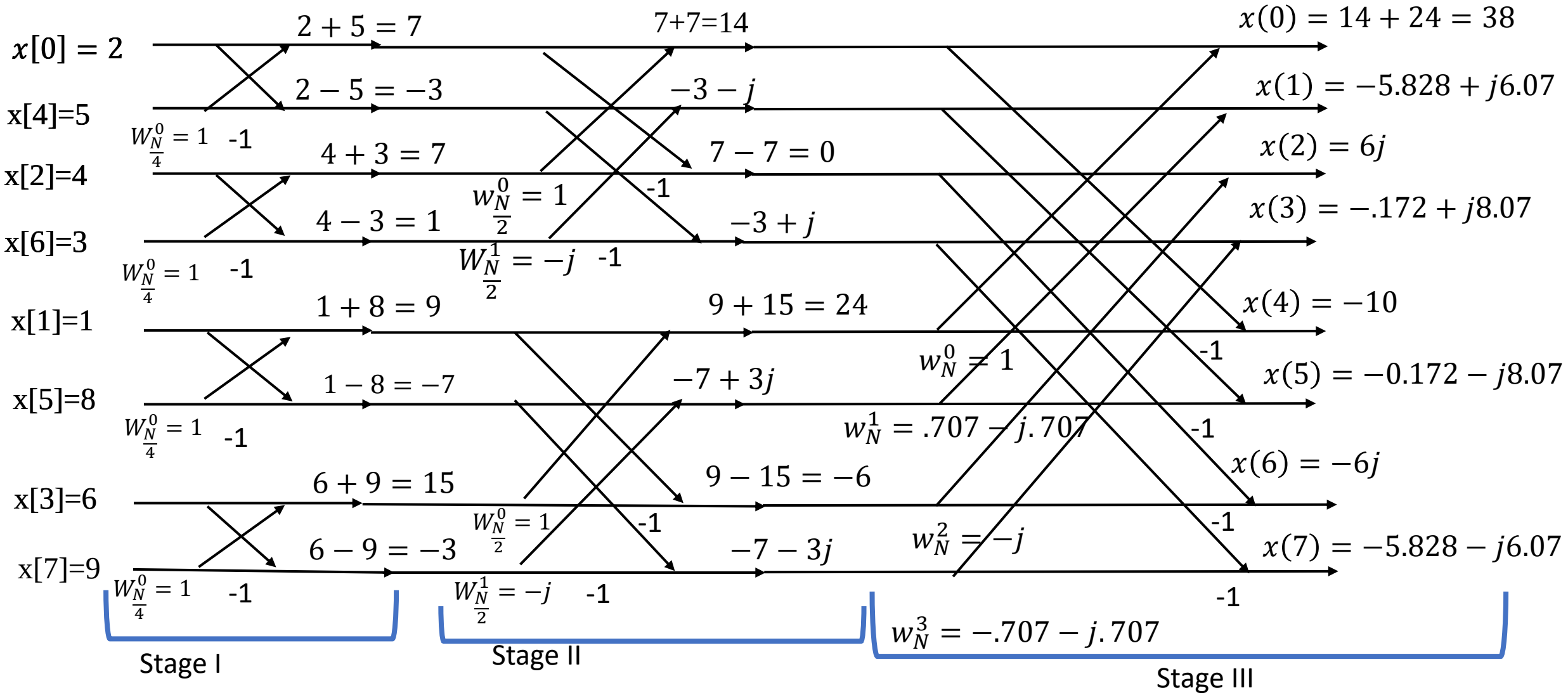
44.FFT DIT Algorithm

Determine DFT of the sequence of the given data sequence x(n)={2,1,4,6,5,8,3,9}

by decimation in frequency FFT.

Solution.



- Stage I Weight Analysis

$$W_{\frac{N}{4}}^{0} = e^{-j\left(\frac{2\pi}{\frac{N}{4}}\right)0} = 1$$

- Stage II Weight Analysis

$$W_{\frac{N}{4}}^{0} = e^{-j\left(\frac{2\pi}{\frac{N}{4}}\right)0} = 1$$

$$W_{\frac{N}{2}}^{1} = e^{-j\left(\frac{2\pi}{\frac{N}{2}}\right)1} = e^{-j\left(\frac{2\pi}{\frac{8}{2}}\right)1} = e^{-j\left(\frac{\pi}{2}\right)} = \cos\left(\frac{\pi}{2}\right) - \mathrm{j}\sin\left(\frac{\pi}{2}\right) = -j$$

- Stage III Weight Analysis

$$W_N^0 = e^{-j\left(\frac{2\pi}{N}\right)0} = 1$$

$$W_N^1 = e^{-j\left(\frac{2\pi}{N}\right)1} = e^{-j\left(\frac{2\pi}{8}\right)1} = e^{-j\left(\frac{\pi}{4}\right)} = \cos\left(\frac{\pi}{4}\right) - \mathrm{j}\sin\left(\frac{\pi}{4}\right) = 0.707 - j0.707$$

$$W_N^2 = e^{-j\left(\frac{2\pi}{N}\right)2} = e^{-j\left(\frac{2\pi}{8}\right)2} = e^{-j\left(\frac{\pi}{2}\right)} = \cos\left(\frac{\pi}{2}\right) - \mathrm{j}\sin\left(\frac{\pi}{2}\right) = -j$$

$$W_N^3 = e^{-j\left(\frac{2\pi}{N}\right)3} = e^{-j\left(\frac{2\pi}{8}\right)3} = e^{-j\left(\frac{3\pi}{4}\right)} = \cos\left(\frac{3\pi}{4}\right) - \mathrm{j}\sin\left(\frac{3\pi}{4}\right) = -0.707 - j0.707$$

Input x(n)	$Stage\ I$ $Computation$	$Stage\ II$ $Computation$	$Stage\ III$ $Computation\ , X(k)$
2	$2+5=7$	$7+7=14$	$14+24=38$
5	$2-5=-3$	$-3+[(-j)1]=-3-j$	$[(-3-j)+\{(0.707-j.707)(-7+j3)]$ $=-5.828+j6.07$
4	$4+3=7$	$7-7=0$	$0+(-j)(-6)=j6$
3	$4-3=1$	$-3-[(-j)1]=-3-j$	$[(-3-j)+\{(0.707-j.707)(-7-j3)]$ $=-0.172+j8.07$
1	$1+8=9$	$9+15=24$	$14-24=-10$
8	$1-8=-7$	$-7+[(-j)(-3)]=-7+j3$	$[(-3-j)-\{(0.707-j.707)(-7+j3)]$ $=-0.172-j8.07$
6	$6+9=15$	$9-15=-6$	$0-(-j)(-6)=-j6$
9	$6-9=-3$	$-7-[(-j)(-3)]=-7-j3$	$[(-3-j)-\{(0.707-j.707)(-7-j3)]$ $=-5.828-j6.07$

Input x(n)	$Stage\ I$ $Computation$	$Stage\ II$ $Computation$	$Stage\ III$ $Computation\ , X(k)$
2			
5			
4			
3			
1			
8			
6			
9			

Input x(n)	$Stage\ I$ $Computation$	$Stage\ II$ $Computation$	$Stage\ III$ $Computation\ , X(k)$
2	$2 + 5 = 7$		
5	$2 - 5 = -3$		
4	$4 + 3 = 7$		
3	$4 - 3 = 1$		
1	$1 + 8 = 9$		
8	$1 - 8 = -7$		
6	$6 + 9 = 15$		
9	$6 - 9 = -3$		

Input x(n)	$Stage\ I$ $Computation$	$Stage\ II$ $Computation$	$Stage\ III$ $Computation\ , X(k)$
2	$2 + 5 = 7$	$7 + 7 = 14$	
5	$2 - 5 = -3$	$-3 + [(-j)1] = -3 - j$	
4	$4 + 3 = 7$	$7 - 7 = 0$	
3	$4 - 3 = 1$	$-3 - [(-j)1] = -3 - j$	
1	$1 + 8 = 9$	$9 + 15 = 24$	
8	$1 - 8 = -7$	$-7 + [(-j)(-3)] = -7 + j3$	
6	$6 + 9 = 15$	$9 - 15 = -6$	
9	$6 - 9 = -3$	$-7 - [(-j)(-3)] = -7 - j3$	

Input x(n)	$Stage\ I$ $Computation$	$Stage\ II$ $Computation$	$Stage\ III$ $Computation\ , X(k)$
2	$2+5=7$	$7+7=14$	$14+24=38$
5	$2-5=-3$	$-3+[(-j)1]=-3-j$	$[(-3-j)+\{(0.707-j.707)(-7+j3)]$ $=-5.828+j6.07$
4	$4+3=7$	$7-7=0$	$0+(-j)(-6)=j6$
3	$4-3=1$	$-3-[(-j)1]=-3-j$	$[(-3-j)+\{(0.707-j.707)(-7-j3)]$ $=-0.172+j8.07$
1	$1+8=9$	$9+15=24$	$14-24=-10$
8	$1-8=-7$	$-7+[(-j)(-3)]=-7+j3$	$[(-3-j)-\{(0.707-j.707)(-7+j3)]$ $=-0.172-j8.07$
6	$6+9=15$	$9-15=-6$	$0-(-j)(-6)=-j6$
9	$6-9=-3$	$-7-[(-j)(-3)]=-7-j3$	$[(-3-j)-\{(0.707-j.707)(-7-j3)]$ $=-5.828-j6.07$