# System

A system may also be defined as a physical device that performs an operation on a signal.

For example, a filter used to reduce the noise and interference corrupting a desired information-bearing signal is called a system. In this case the filter performs some operation(s) on the signal, which has the effect of reducing (filtering) the noise and interference from the desired information-bearing signal.

A system is an entity that manipulates one or more input signals to perform a function, which results in a new output signal. A typical prototype of a system is shown in Fig. 2.44.

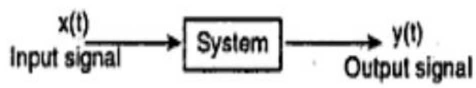


*Fig. 2.44 Basic System*

## Properties of System

Base on the properties, the system can be classified as

1. Continuous-time and Discrete-time system
2. Stable and Unstable system
3. Memory and Memoryless system
4. Invertible and Noninvertible system
5. Time-variant and Time-invariant system
6. Linear and Nonlinear system
7. Causal and Noncausal system

### 2.6.1 Continuous-Time and Discrete-Time System

***Continuous-time system*** If the input and output of the system are continous-time signals, then the system is called 'Continuous-time system'.

Let us consider an input signal $x(t)$ to the system. If the system produces an output signal $y(t)$, then the system is called 'Continuous-time system'.

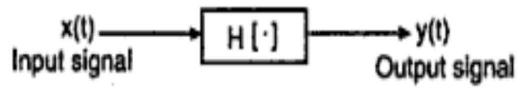


***Fig. 2.47 Continuous-time System***

***Discrete-time system*** If the input and output of the system are discrete-time signals, then the system is called 'Discrete-time system'.

Let us consider an input signal $x(n)$ to the system. If the system produces an output signal $y(n)$, then the system is called 'Discrete-time system'.

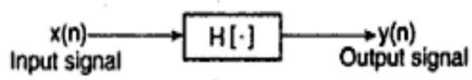


***Fig. 2.48 Discrete-time System***

### 2.2.1 Input–Output Description of Systems

The input–output description of a discrete-time system consists of a mathematical expression or a rule, which explicitly defines the relation between the input and output signals (*input–output relationship*). The exact internal structure of the system is either unknown or ignored. Thus the only way to interact with the system is by using its input and output terminals (i.e., the system is assumed to be a "black box" to the user). To reflect this philosophy, we use the graphical representa-

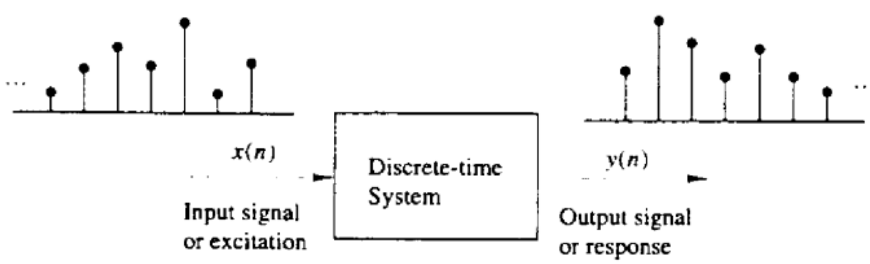


**Figure 2.12** Block diagram representation of a discrete-time system.

### 2.6.2 Stable and Unstable System

***Stable system*** A given system is said to be stable if and only if every bounded input produces a bounded output. The stable system is also known as 'Bounded Input–Bounded Output' (BIBO).

Let us consider a system operated by an operator H[·]. The system is said to be stable if the bounded input $x(t)$ produces bounded output $y(t)$, i.e.

if $$|x(t)| \le M_x < \infty \text{ for all } t \tag{2.74a}$$

then $$|y(t)| \le M_y < \infty \text{ for all } t \tag{2.74b}$$

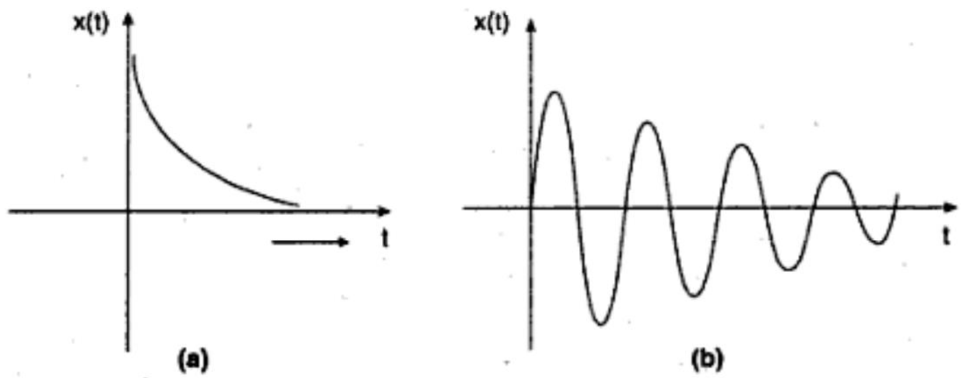


***Fig. 2.49 Bounded Signals***
***(a) Decay Exponential (b) Sinusoidal Signal***

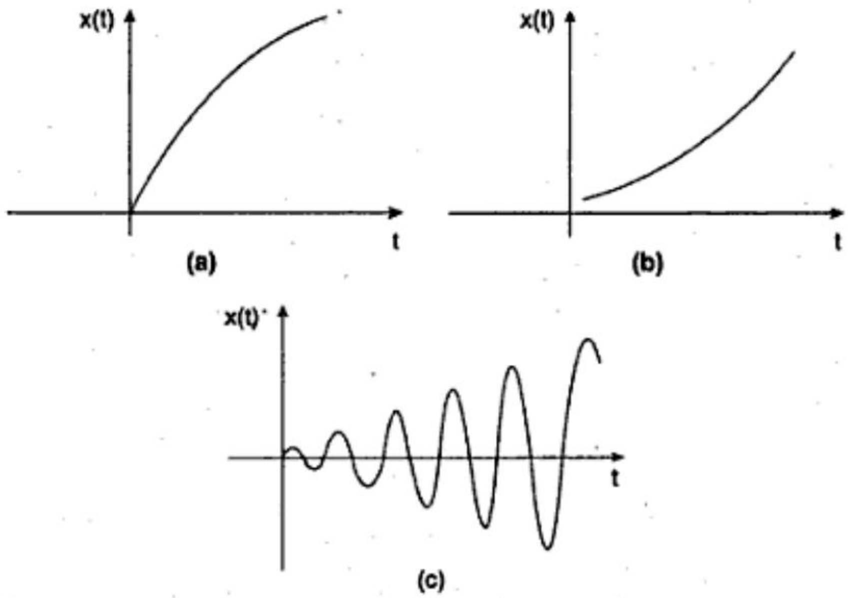


***Fig. 2.50 Unbounded Signals***

***(a), (b) Rising Exponential***
***(c) Sinusoidal Signal with Exponential Rising Magnitude***

**Static versus dynamic systems:** A discrete-time system is called static or memoryless if its output at any instant *n* depends at most on the input sample at the same time, but not on past or future samples of the input. In any other case, the system is said to be dynamic or to have memory.

### 2.6.3 Memory and Memoryless System

***Memory system*** The given system is said to possess memory if the output of the system depends on past and future values.

***Examples***

$$y(t) = x(t) + x(t-1) + x(t+1)$$

***Memoryless system*** The given system is said to be memoryless if the output of the system depends solely on the present value.

***Examples***

$$y(t) = x(t)$$
$$y(t) = x^2(t)$$
$$y(n) = x(n)$$
$$y(n) = nx(n)$$

### 2.6.4 Invertible and Noninvertible System

***Invertible system*** A system is said to be an invertible system if the input signal given to the system can be recovered.

The concept of invertibility is illustrated in Fig. 2.51.

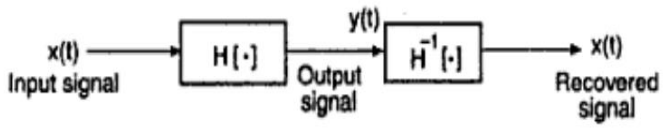


*Fig. 2.51 Invertible System*

***Noninvertible system*** A system is said to be noninvertible if the input signal given to the system cannot be recovered from the output signal of the system.

### 2.6.5 Time-invariant and Time-variant System

***Time-invariant system*** A system is said to be time-invariant if the input signal is delayed or advanced by any factor that leads to some delay or advancement in the time scale by the same factor, i.e. the system responds to an input which is given at any instant of time and results in an output.

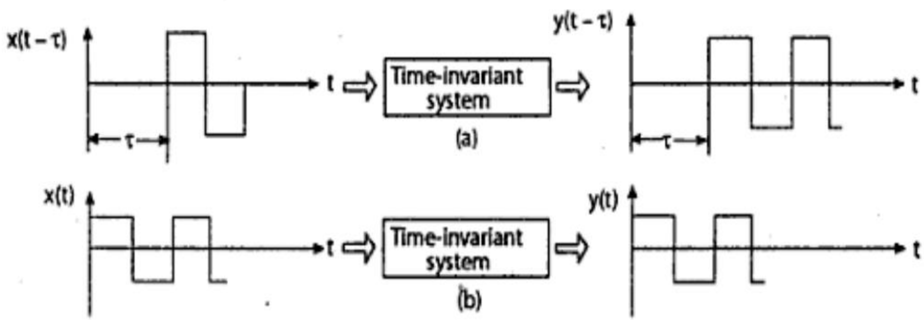


***Fig. 2.53 Basic Time-invariant Systems***
***(a) Delayed Time-invariant System (b) Time-invariant System***

**Problem 2.41** The input-output relation is given by $y(t)=\sin[x(t)]$. Determine whether the system is time-invariant or not.

**Solution**

$$y(t)=\sin[x(t)]$$

Let us assume the signal of the form

$$y_1(t)=\sin[x_1(t)] \tag{1}$$

Let us introduce time delay $t_0$ in the input signal in equation (1), then

$$x_2(t)=x_1(t-t_0)$$

The delay input therefore results in the output

$$y_2(t)=\sin[x_2(t)]=\sin[x_1(t-t_0)] \tag{2}$$

Let us introduce the same time delay $t_0$ in the output of the equation, i.e.

$$y_1(t-t_0)=\sin[x_1(t-t_0)] \tag{3}$$

On comparing equations (2) and (3),

$$y_2(t)=y_1(t-t_0)$$

Hence, system is time-invariant.

***Time-variant system*** A system is said to be time-variant if the output signal is delayed or advanced with respect to input signal as shown in Fig. 2.54.

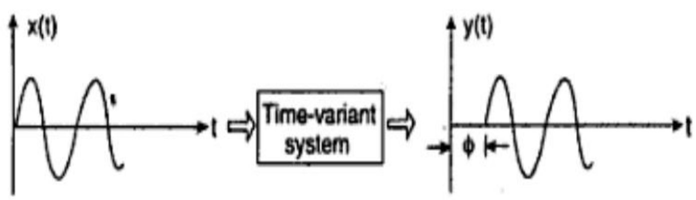


*Fig. 2.54 Time-variant System*

**Problem 2.42** The input-output relation is given by $y(t) = t\,x(t)$. Determine whether the system is time-variant or not.

**Solution**

$$y(t) = t\,x(t)$$

Let us assume the signal of the form

$$y_1(t) = t\,x_1(t) \tag{1}$$

Let us introduce time delay $t_0$ in the input signal in equation (1)

$$x_2(t) = x_1(t - t_0)$$

The delay in input results in an output

$$y_2(t) = t\,x_2(t) = t\,x_1(t - t_0) \tag{2}$$

Let us introduce the same delay $t_0$ in the output of the system

$$y(t-t_0) = (t-t_0)x_1(t-t_0) \tag{3}$$

On comparing equation (2) and (3),

$$y_2(t) \neq y_1(t - t_0)$$

Hence, the system is time-variant.

**Problem 2.43** Determine whether the following systems are time-variant or not.

(i) $y(t) = x(t)\sin \omega t$ (ii) $y(t) = x(4t)$

(iii) $y(t) = e^{x(t)}$ (iv) $y(t) = t^2 x(t)$

**Solution**

(i) $y(t) = x(t)\sin \omega t$ (1)

$$y(t) = T[x(t)] = \sin \omega t\; x(t)$$

Introduce time delay $t_0$ in the input, i.e.

$$x_1(t) = x(t - t_0)$$

$$y_1(t) = \sin \omega t\; x_1(t) = \sin \omega t\; x(t - t_0) \tag{2}$$

Introduce time delay $t_0$ in the output of the equation, i.e.

$$y(t - t_0) = \sin \omega t\,(t - t_0)\,x(t - t_0) \tag{3}$$

On comparing equations (2) and (3),

$$y_1(t) \neq y(t - t_0)$$

The system is time-variant.

### 2.6.6 Linear and Nonlinear System

Let us consider an input $x_1(t)$ given to a continuous-time system which responds with $y_1(t)$. Similarly, let us consider another signal $x_2(t)$ given to the same continuous-time system which results in a response $y_2(t)$. Then the system is said to be linear if

1. The response to $x_1(t)+x_2(t)$ is $y_1(t)+y_2(t)$ (additive property)
2. The response to $ax_1(t)+bx_2(t)$ is $ay_1(t)+by_2(t)$, where $a$ and $b$ are complex constants (scaling property)

Mathematically,

$$T[ax_1(t)+bx_2(t)]=aT[x_1(t)]+bT[x_2(t)]$$
$$T[ax_1(t)+bx_2(t)]=ay_1(t)+by_2(t) \tag{2.76}$$

***Problem 2.47*** Determine whether the given continuous-time system is linear or not.

$$y(t)=t\,x(t)$$

**Solution** Let us define the input signal $x_1(t)$ whose response $y_1(t)$ is given by

$$y_1(t)=t_1x(t)$$

Similarly, let us define another signal $x_2(t)$ whose response $y_2(t)$ is

$$y_2(t)=t\,x_2(t)$$

The two defined signals are related by

$$x_3(t)=ax_1(t)+bx_2(t)$$

Then the output $y_3(t)$ is defined as

$$y_3(t)=t\,x_3(t)$$
$$y_3(t)=t[ax_1(t)+bx_2(t)]$$
$$y_3(t)=at\,x_1(t)+bt\,x_2(t)$$
$$y_3(t)=ay_1(t)+by_2(t)$$

Therefore, the system is linear.

***Problem 2.50*** Determine whether the given system is linear or not.

$$y(n) = x^2(n)$$

**Solution** Let us define the signal $x_1(n)$ whose response is given by

$$y_1(n) = x_1^2(n)$$

Similarly, let us define another signal $x_2(n)$ whose response is given by

$$y_2(n) = x_2^2(n)$$

These two signals are related as

$$x_3(n) = ax_1(n) + bx_2(n)$$

where $a$, $b$ = constants

Then the output $y_3(n)$ becomes

$$y_3(n) = x_3^2(n)$$
$$y_3(n) = [ax_1(n) + bx_2(n)]^2$$
$$y_3(n) = a^2x_1^2(n) + b^2x_2^2(n) + 2x_1(n)x_2(n)$$
$$y_3(n) \neq ay_1(n) + by_2(n)$$

Therefore, the system is nonlinear.

### 2.6.7 Causal and Noncausal System

***Causal system*** In a causal system, the output response of the system at any time depends only on the present input and/or on the past input, but not on the future inputs.

In a causal system, the next input cannot be predicted. Hence, this may not be an essential condition for all systems.

Examples for causal system are

$$y(n) = x(n) - x(n-1)$$
$$y(t) = tx(t)$$

In the noncausal system, the output response of the system depends on the future input values also.

Examples for noncausal system are

$$y(n) = n^2x(n)$$
$$y(t) = x(n+1) - x(n)$$