FFT

- **The Fourier transform of an analogue signal x(t) is given by:**

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

- **The Discrete Fourier Transform (DFT) of a discrete-time signal x(nT) is given by:**

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

Where:

$$k = 0,1,\ldots N-1$$
$$x(nT) = x[n]$$

$$X(k) = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

x[n] **= input**

X[k] **= frequency bins**

W **= twiddle factors**

$$X(0) = x[0]W_N^{0} + x[1]W_N^{0*1} + \ldots + x[N-1]W_N^{0*(N-1)}$$

$$X(1) = x[0]W_N^{0} + x[1]W_N^{1*1} + \ldots + x[N-1]W_N^{1*(N-1)}$$

$$\vdots$$

$$X(k) = x[0]W_N^{0} + x[1]W_N^{k*1} + \ldots + x[N-1]W_N^{k*(N-1)}$$

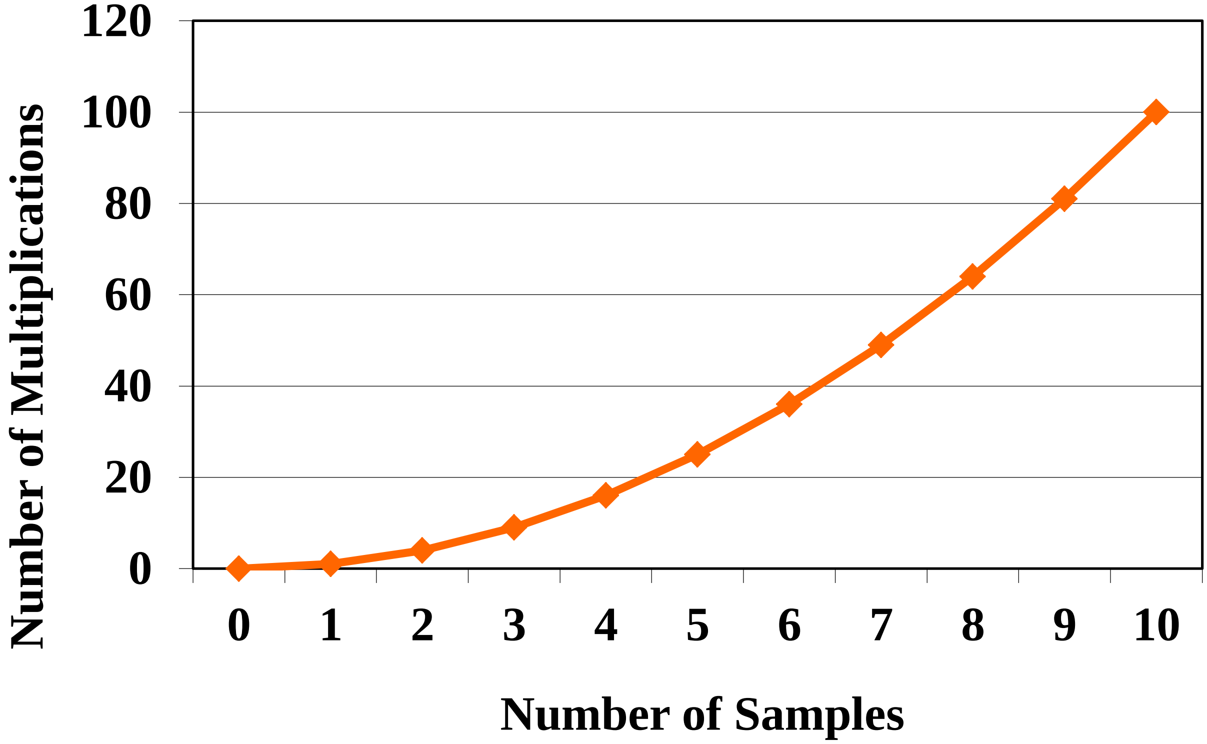
$$\vdots$$

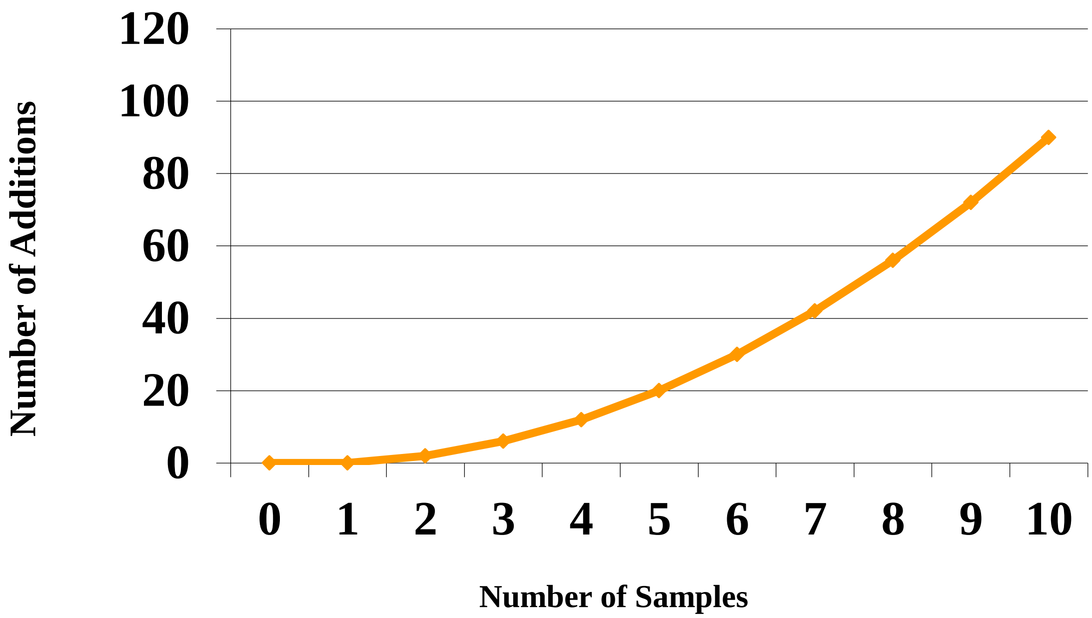
$$X(N-1) = x[0]W_N^{0} + x[1]W_N^{(N-1)*1} + \ldots + x[N-1]W_N^{(N-1)(N-1)}$$

Note: For N samples of x we have N frequencies representing the signal.

Performance of the DFT Algorithm

- **The DFT requires N^2 (NxN) complex multiplications:**
  - **Each X(k) requires N complex multiplications.**
  - **Therefore to evaluate all the values of the DFT (X(0) to X(N-1)) N^2 multiplications are required.**
- **The DFT also requires (N-1)*N complex additions:**
  - **Each X(k) requires N-1 additions.**
  - **Therefore to evaluate all the values of the DFT (N-1)*N additions are required.**





- Can the number of computations required be reduced?

DFT $\rightarrow$ FFT

- A large amount of work has been devoted to reducing the computation time of a DFT.
- This has led to efficient algorithms which are known as the Fast Fourier Transform (FFT) algorithms.

$$X(k) = \sum_{n=0}^{N-1} x[n] W_N^{nk}; \quad 0 \le k \le N-1 \tag{1}$$

$$x[n] = x[0], x[1], \ldots, x[N-1]$$

- Lets divide the sequence x[n] into even and odd sequences:
  - x[2n] = x[0], x[2], …, x[N-2]
  - x[2n+1] = x[1], x[3], …, x[N-1]
- Equation 1 can be rewritten as:

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x[2n] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] W_N^{(2n+1)k} \qquad (2)$$

- **Since:**

$$W_N^{2nk} = e^{-j\frac{2\pi}{N}2nk} = e^{-j\frac{2\pi}{N/2}nk}$$
$$= W_{\frac{N}{2}}^{nk}$$

$$W_N^{(2n+1)k} = W_N^k \cdot W_{\frac{N}{2}}^{nk}$$

- **Then:**

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x[2n] W_{\frac{N}{2}}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] W_{\frac{N}{2}}^{nk}$$
$$= Y(k) + W_N^k Z(k)$$

- **The result is that an N-point DFT can be divided into two N/2 point DFT's:**

$$X(k) = \sum_{n=0}^{N-1} x[n] W_N^{nk}; \quad 0 \le k \le N-1$$

N-point DFT

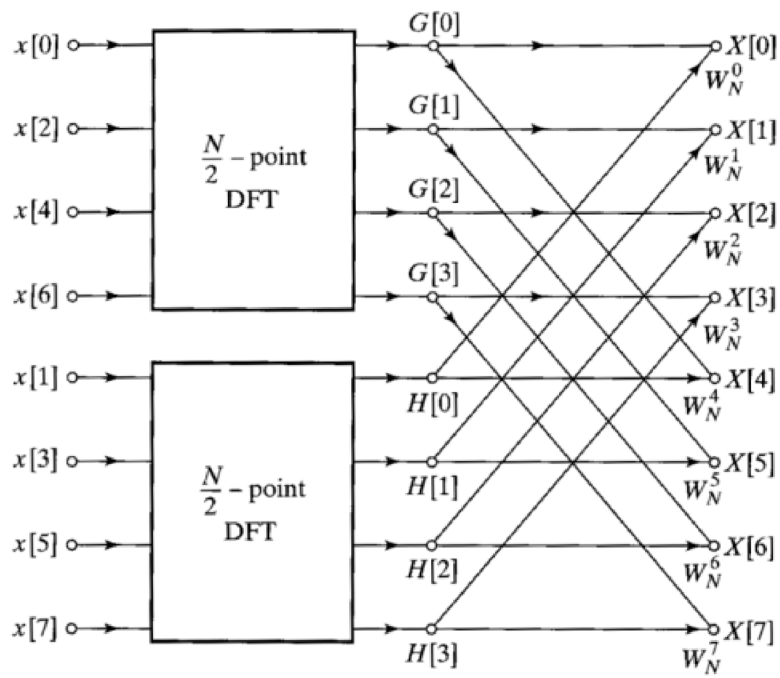
- **Where Y(k) and Z(k) are the two N/2 point DFTs operating on even and odd samples respectively:**

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x_1[n] W_{\frac{N}{2}}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_2[n] W_{\frac{N}{2}}^{nk}$$
$$= Y(k) + W_N^k Z(k)$$

Two N/2-point DFTs

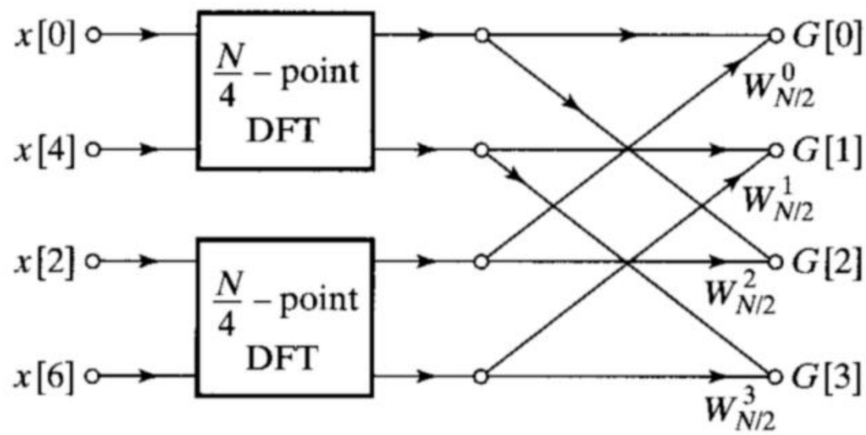
Signal flowgraph representation of 8-point DFT

- Recall that the DFT is now of the form $X[k] = G[k] + W_N^k H[k]$
- The DFT in (partial) flowgraph notation:

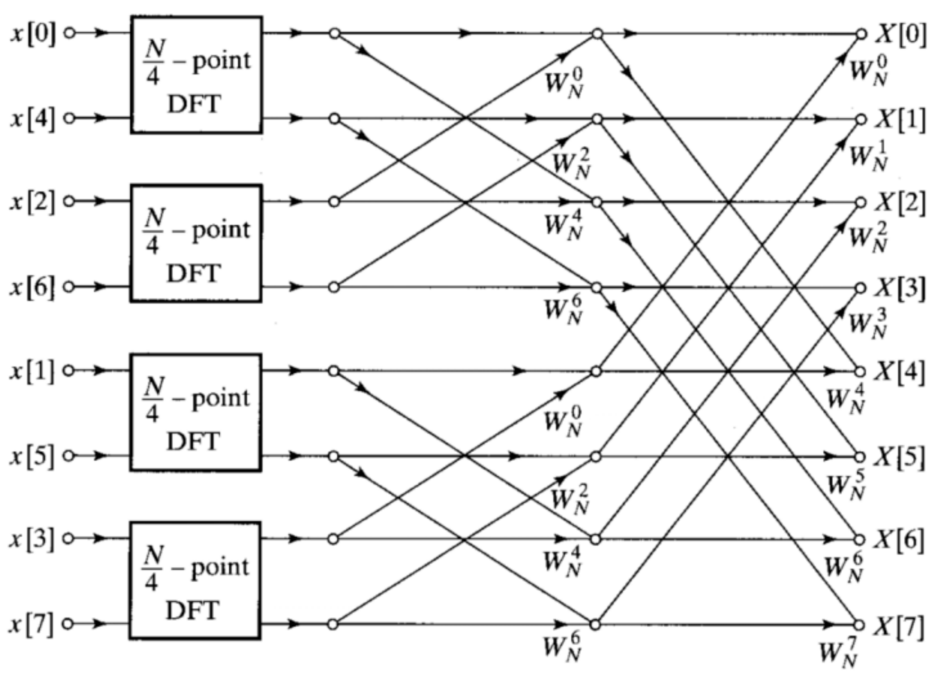


Continuing with the decomposition …

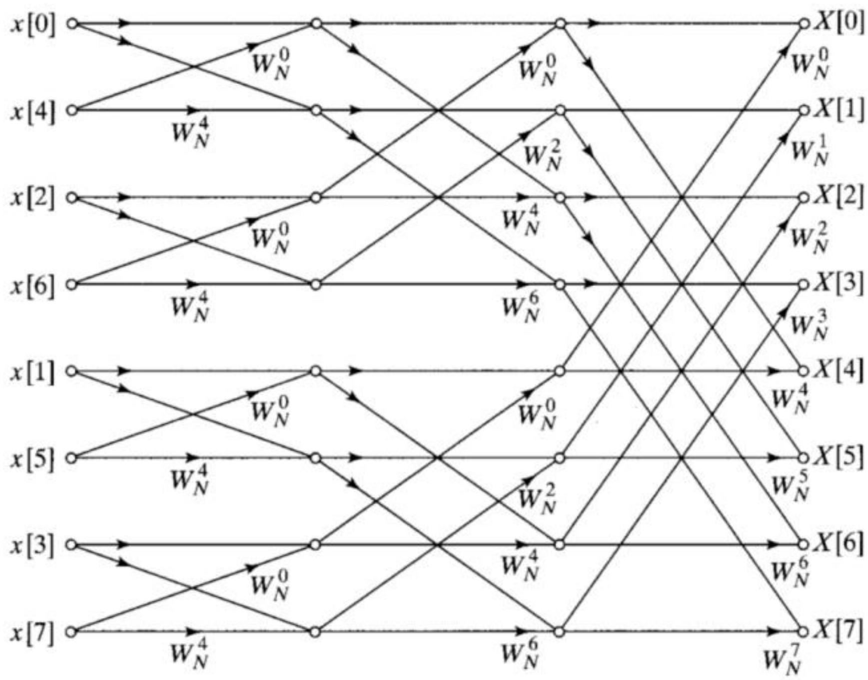
- **So why not break up into additional DFTs? Let's take the upper 4-point DFT and break it up into two 2-point DFTs:**



The complete decomposition into 2-point DFTs



The complete 8-point decimation-in-time FFT



X(n) length N

Radix-2 DIT-FFT Algorithm:

DIT → Decimation in Time
FFT → Fast Fourier Transform.

$x(n) \rightarrow$ lenth N

$x(n) = \{x(0), x(1), x(2), x(3) \ldots . x(N-2), x(N-1)\}$

even indexed seq. : $\{x(0), x(2), x(4) \ldots . \; x(N-2)\}$

odd indexed seq : $\{x(1), x(3), x(5) \ldots . \; x(N-1)\}$

W.K.T. N-Point DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{Kn} \; ; \quad 0 \le K \le N-1 \longrightarrow ①$$

Decimation → even, odd seq.

eqn ① → Decimation $2r = N-2 \Rightarrow r = \frac{N}{2}-1$

$$X(K) = \sum_{n=0}^{N-2} x(n)\, W_N^{Kn} + \sum_{n=1}^{N-1} x(n)\, W_N^{Kn} \longrightarrow ②$$

$n \rightarrow$ even. $\qquad n \rightarrow$ odd. $\quad 2r+1=1,\; 2r=0,\; r=0 \qquad 2r+1=N-1,\; 2r=N-2,\; r=\frac{N}{2}-1$

$2r=0,\; r=0$

Put $n = 2r$ in 1^{st} term, $n = 2r+1$ in 2^{nd} term.

$$X(K) = \sum_{r=0}^{N/2-1} \underbrace{x(2r)}_{g(r)}\, W_N^{2Kr} + \sum_{r=0}^{N/2-1} \underbrace{x(2r+1)}_{h(r)}\, W_N^{K(2r+1)} \longrightarrow ③$$

$$X(K) = \sum_{r=0}^{N/2-1} g(r)\, W_N^{2Kr} + \sum_{r=0}^{N/2-1} h(r)\, \boxed{W_N^{K(2r+1)}}$$

$W_N^{2Kr} \cdot W_N^{K}$

$$\because W_N = e^{\frac{-j2\pi}{N}} \Rightarrow W_N^2 = e^{\frac{-j2\pi \cdot 2}{N}} = e^{-j\frac{2\pi}{N/2}} = W_{N/2}$$

Rearrange,

$$X(K) = \underbrace{\sum_{r=0}^{N/2-1} g(r)\, W_{N/2}^{Kr}}_{N/2 \text{ DFT eqn seq}} + W_N^K \underbrace{\sum_{r=0}^{N/2-1} h(r)\, W_{N/2}^{Kr}}_{N/2 \text{ DFT odd}} \rightarrow (4) \quad (1)$$

$$X(K) = G(K) + W_N^K H(K) \; ; \; 0 \le K \le \frac{N}{2} - 1 \rightarrow (5)$$

$G(K)$ & $H(K) \rightarrow$ periodic with period $\frac{N}{2}$

$$X(K) = G\left(K - \frac{N}{2}\right) + W_N^K H\left(K - \frac{N}{2}\right) \; ; \; \frac{N}{2} \le K \le N-1 \rightarrow (6)$$

$G(K-4)$ $\quad$ $H(K-4)$

Ex:- $N = 8$ $\quad \therefore K \rightarrow 0$ to 7

$K = 0$ to $3 \rightarrow (5)$ $\quad K = 4$ to 7 (6)

(5) ⇒

$K=0;\; X(0) = G(0) + W_8^0 H(0)$

$K=1;\; X(1) = G(1) + W_8^1 H(1)$

$K=2;\; X(2) = G(2) + W_8^2 H(2)$

$K=3;\; X(3) = G(3) + W_8^3 H(3)$

(6) ⇒

$K=4;\; X(4) = G(0) + W_8^4 H(0)$

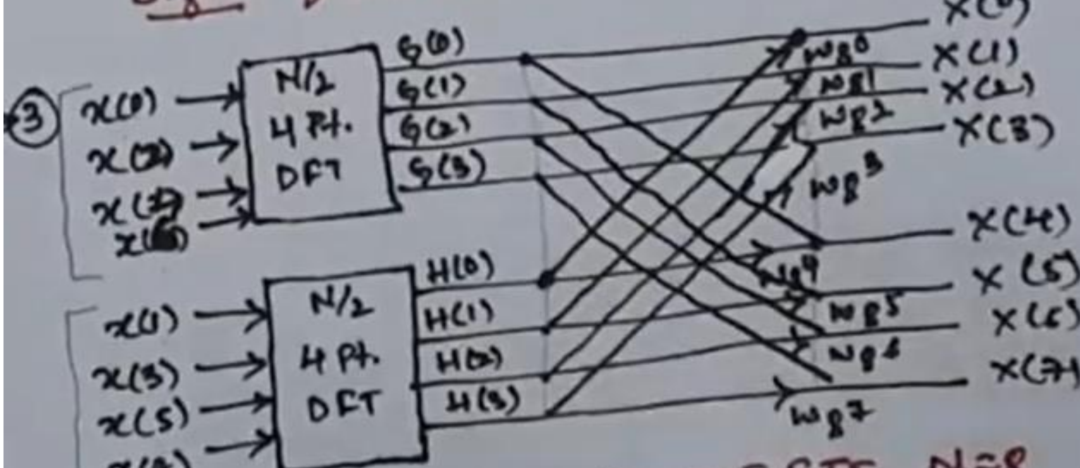
$K=5;\; X(5) = G(1) + W_8^5 H(1)$

$K=6;\; X(6) = G(2) + W_8^6 H(2)$

$K=7;\; X(7) = G(3) + W_8^7 H(3)$

$K=3;\; X(3) =$

Signal flow graph



First stage in DIT FFT $N=8$

$G(K)$ & $H(K) \rightarrow \frac{N}{2}$ point

combination of two $\frac{N}{4}$ points. (8)

$$G(K) = \sum_{r=0}^{N/2-1} g(r)\, W_{N/2}^{Kr} \rightarrow (7)$$

$2l = \frac{N}{2}-2 \Rightarrow l = \frac{N}{4}-1$ $\quad 2l+1 = \frac{N}{2}-1$ $\quad 2l = \frac{N}{2}-2$ $\quad l = \frac{N}{4}-1$

$$G(K) = \sum_{r=0}^{N/2-2} g(r)\, W_{N/2}^{Kr} + \sum_{r=1}^{N/2-1} g(r)\, W_{\frac{N}{2}}^{Kr}$$ (9) ⇒

$2l = 0 \Rightarrow l = 0$ $\quad 2l+1 = 1 \Rightarrow l = 0$

$$g(r) = \{g(0), g(1), g(2) \cdots g(\tfrac{N}{2}-2), g(\tfrac{N}{2}-1)\}$$

Put $r = 2l$ in 1st term, $r = 2l+1$ in 2nd (7)

$$G(K) = \sum_{l=0}^{N/4-1} \underbrace{g(l)}_{a(l)}\, W_{N/2}^{2Kl} + \sum_{l=0}^{N/4-1} \underbrace{g(2l+1)}_{b(l)} \boxed{W_{N/2}^{K(2l+1)}}$$

$W_{N/2}^{2Kl} \cdot W_{N/2}^{K}$

$$G(K) = \underbrace{\sum_{l=0}^{N/4-1} a(l)\, W_{N/4}^{2Kl}}_{A(K)} + W_{N/2}^{K} \underbrace{\sum_{l=0}^{N/4-1} b(l)\, W_{N/4}^{2Kl}}_{B(K)}$$ eq (10)

$W_{N/2}^{2} = W_{N/4}$

$$G(K) = \underbrace{A(K)}_{N/4-} + W_{N/2}^{K} \underbrace{B(K)}_{N/4} \; ; \; 0 \le K \le \frac{N}{4}-1 \rightarrow (8)$$

lly

$$H(K) = C(K) + W_{N/2}^{K} D(K) \; ; \; 0 \le K \le \frac{N}{4}-1 \rightarrow (9)$$

$A(K), B(K), C(K)$ & $D(K) \rightarrow$ periodic $\frac{N}{4}$.

(2)

(8) => $G(K) = A(K-\frac{N}{4}) + W^{K}_{N/2}\, B(K-\frac{N}{4})$

$A(K-2)$ $\quad B(K-2)$

; $\frac{N}{4} \le K \le \frac{N}{2}-1 \rightarrow$ (10)

(9) => $H(K) = C(K-\frac{N}{4}) + W^{K}_{N/2}\, D(K-\frac{N}{4})$;

$C(K-2)$ $\quad D(K-2)$

$\frac{N}{4} \le K \le \frac{N}{2}-1 \rightarrow$ (11)

1)}

(7)

2l+1)

Kl . $W^{K}_{N/2}$

/2

2Kl

N/4

K => 0 & 1 → eqn (8) & (9)

2 & 3 → eqn (10) & (11)

eqn (8) =>

$K=0;\ G(0) = A(0) + W_4^0 B(0)$

$K=1;\ G(1) = A(1) + W_4^1 B(1)$

eqn (10) $K=2;\ G(2) = A(0) + W_4^2 B(0)$

$K=3;\ G(3) = A(1) + W_4^3 B(1)$

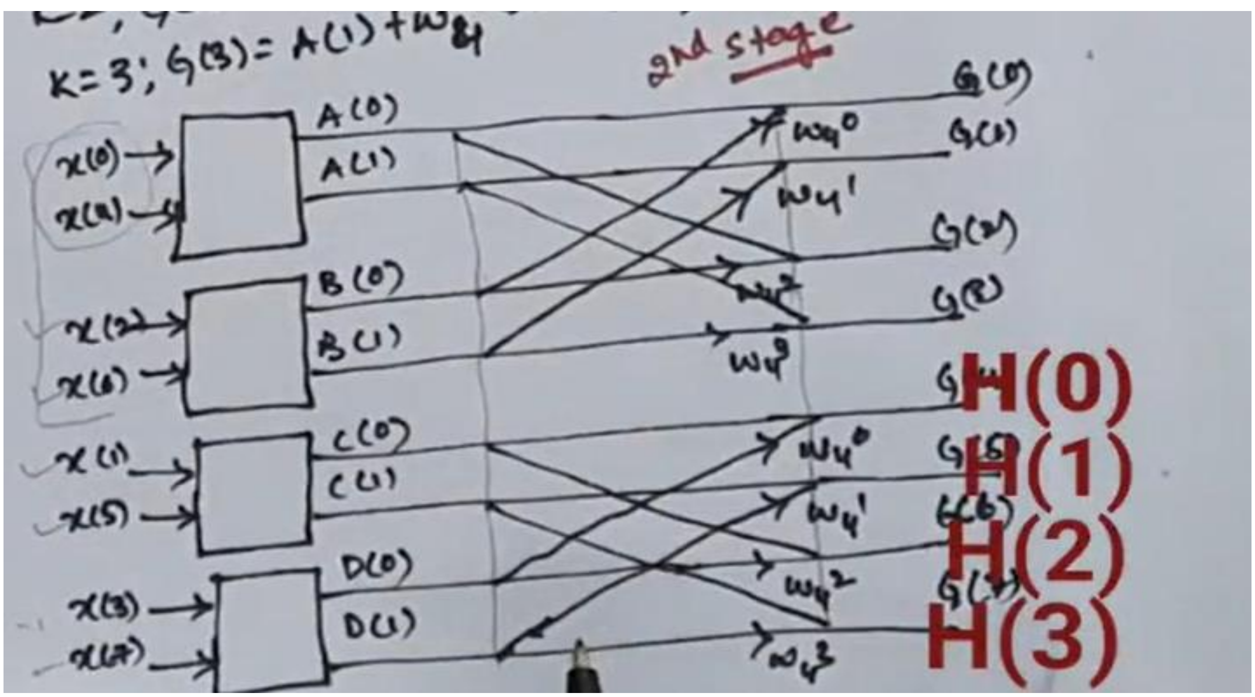
eqn (9)

$K=0;\ H(0) = C(0) + W_4^0 D(0)$

$K=1;\ H(1) = C(1) + W_4^1 D(1)$

eqn (11) $K=2;\ H(2) = C(0) + W_4^2 D(0)$

$K=3;\ H(3) = C(1) + W_4^3 D(1)$



Radix-2 DIT-FFT Algorithm:

each $\frac{N}{4}$ DFT as two $\frac{N}{8}$ point DFTs.

the 2-point DFT of $x(0)$ & $x(4)$

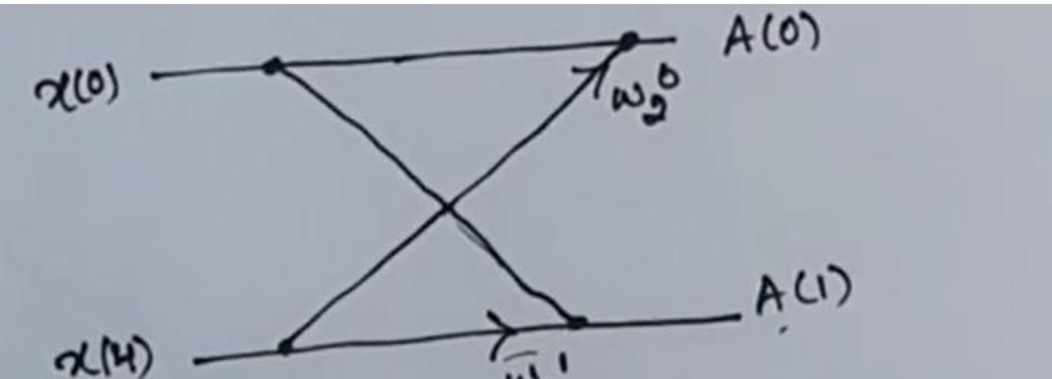
$$A(K) = \sum_{n=0}^{N/4-1} x(n)\, W_{N/4}^{Kn} \quad ; \ 0 \le K \le \frac{N}{4} - 1$$

$\boxed{N=8}$

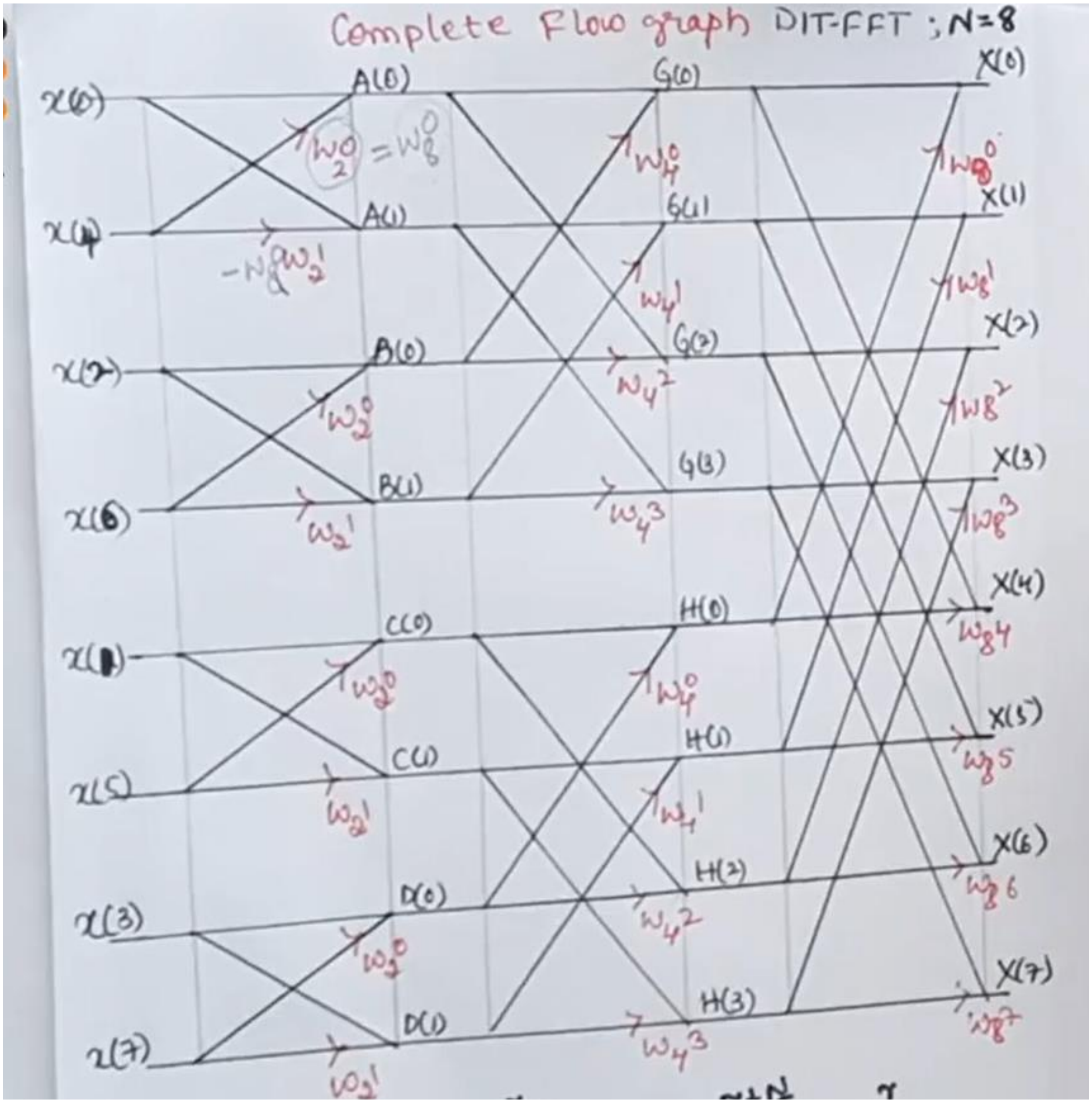
$$A(K) = \sum_{n=0}^{1} x(n)\, W_{2}^{Kn} \quad ; \ 0 \le K \le 1$$

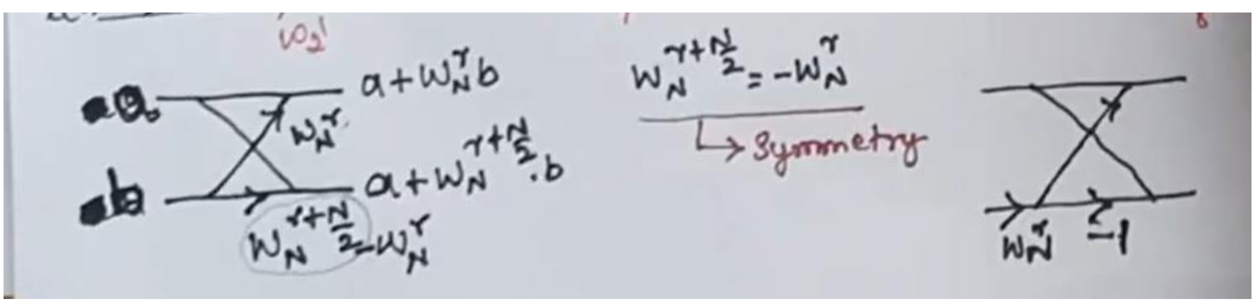
For ; $K=0 \Rightarrow A(0) = x(0) + W_2^0\, x(4)$

for ; $K=1 \Rightarrow A(1) = x(1) + W_2^1\, x(4)$

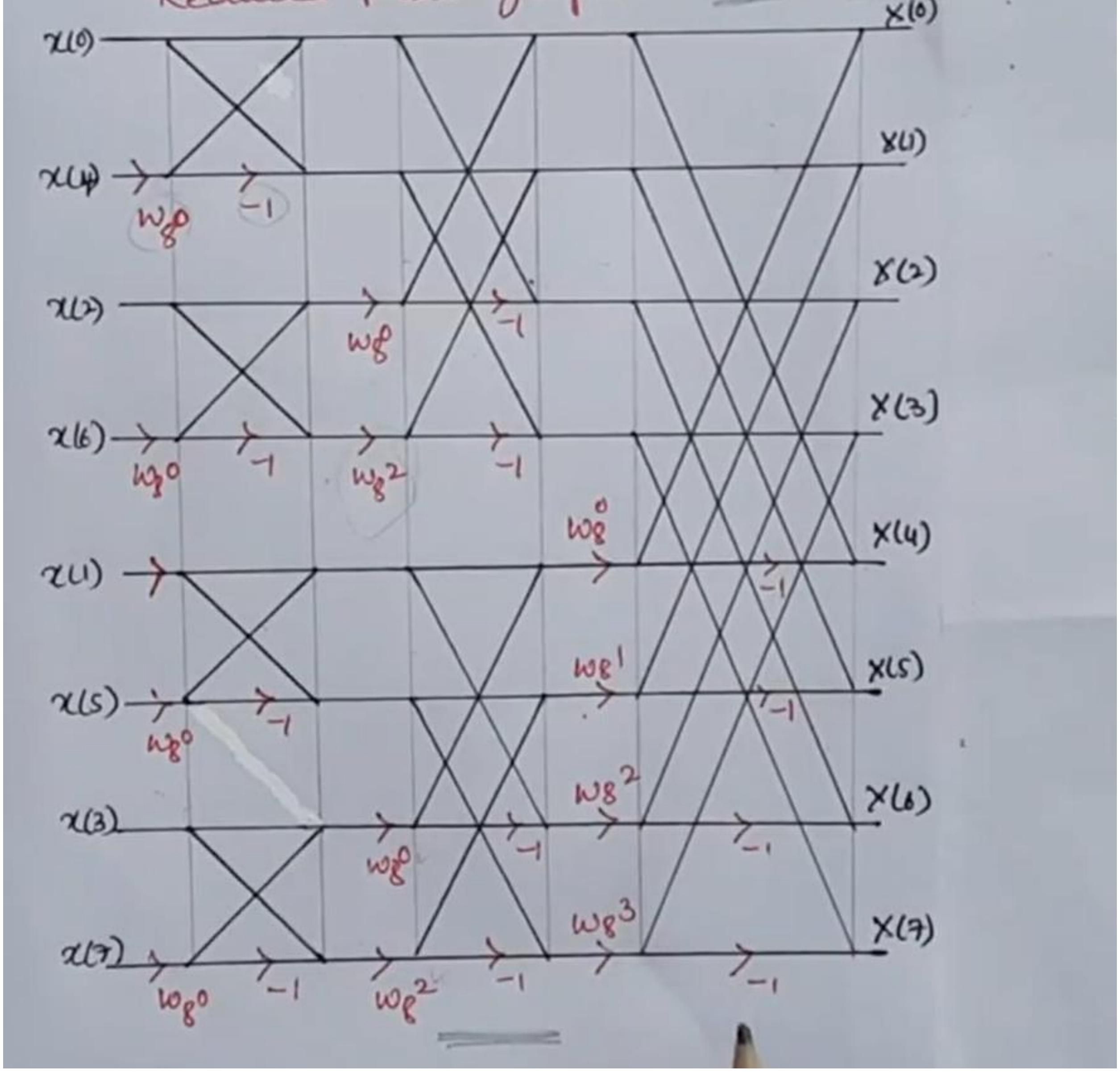


3rd stage in DIT FFT $N=8$


Complete Flow graph DIT-FFT ; N=8
x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
A(0)
A(1)
B(0)
B(1)
C(0)
C(1)
D(0)
D(1)
G(0)
G(1)
G(2)
G(3)
H(0)
H(1)
H(2)
H(3)
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
W20 = W80
-W80 W21
W20
W21
W40
W41
W42
W43
W80
W81
W82
W83
W84
W85
W86
W87


a
b
a + WNr b
a + WN r+N/2 . b
WN r+N/2 = -WNr
↳ Symmetry
WNr
-1

Reduced Flow graph: DIT-FFT N=8



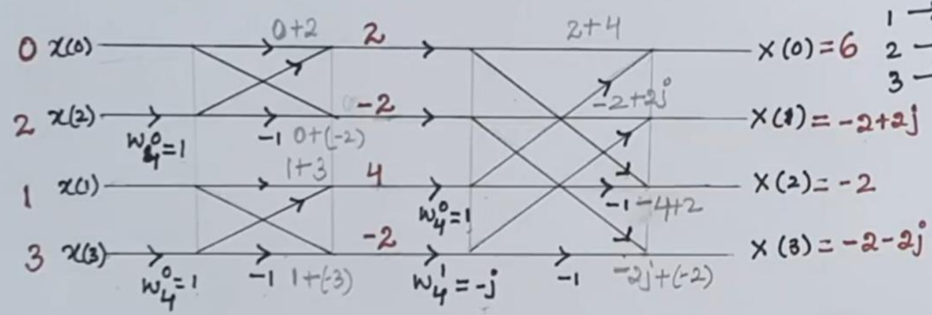
Given $x(n) = \{0, 1, 2, 3\}$, find $X(K)$ using DIT-FFT Algorithm.

$\therefore N = 4$ $\qquad W_4^0 = 1 \qquad W_4^1 = -j$

bit reversal

$4 = 2^{2}$

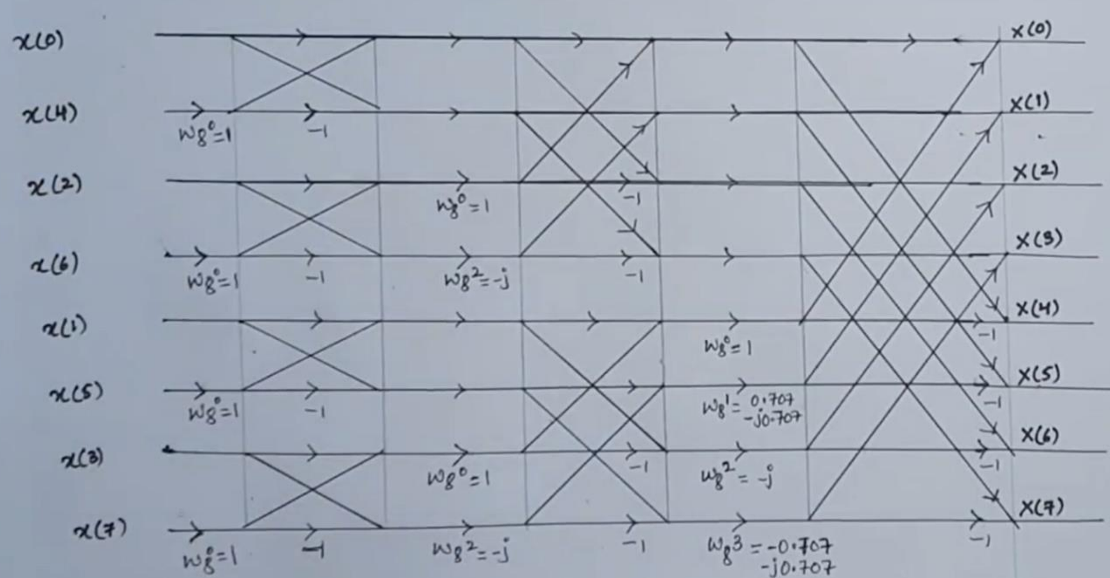
	BR
$0 \to 00$	$00 \to 0$
$1 \to 01$	$10 \to 2$
$2 \to 10$	$01 \to 1$
$3 \to 11$	$11 \to 3$



Flow-graph for DIT-FFT : N=4

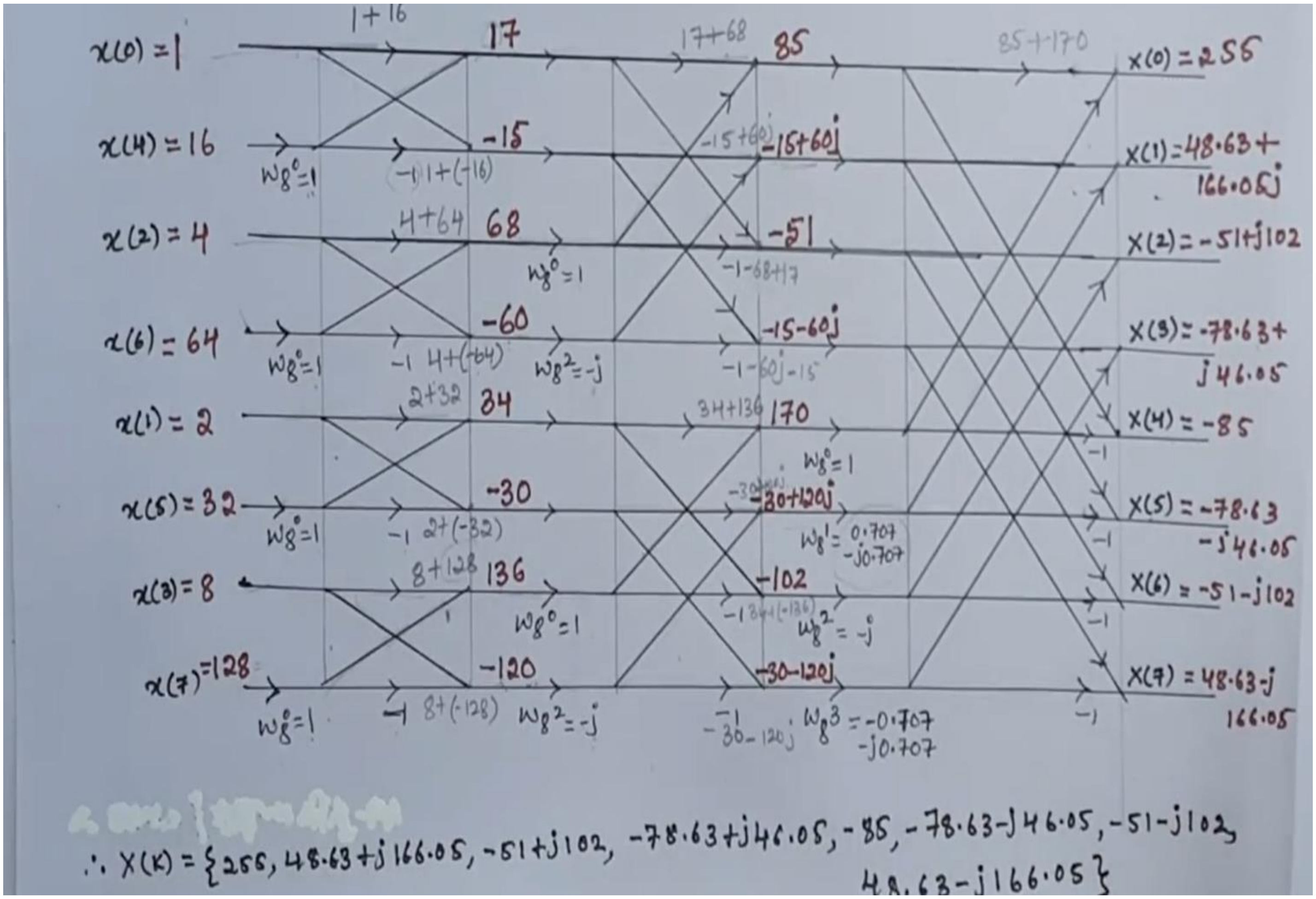
$\therefore X(K) = \{6, -2+2j, -2, -2-2j\}$

Given $x(n) = \{1, 2, 4, 8, 16, 32, 64, 128\}$
Find $X(K)$ using DIT-FFT.



BIT REVERSAL

000 --> 000 --> 0
001 --> 100 --> 4
010 --> 010 --> 2
011 --> 110 --> 6
100 --> 001 --> 1
101 --> 101 --> 5
110 --> 011 --> 3
111 --> 111 --> 7



Example 1: Consider a sequence $x[n] = \{1, 1, -1, -1, -1, 1, 1, -1\}$

Determine DFT X[k] of x[n] using the decimation-in-time FFT algorithm.

$$f[n] = x[2n] = \{x[0], x[2], x[4], x[6]\} \quad = \{1, -1, -1, 1\}$$

$$g[n] = x[2n+] = \{x[1], x[3], x[5], x[7]\} = \{1, -1, 1, -1\}$$

X= W_x

$$\begin{bmatrix} F(0) \\ F(1) \\ F(2) \\ F(3) \end{bmatrix} = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 2+j2 \\ 0 \\ 2-j2 \end{bmatrix}$$

$$\begin{bmatrix} G(0) \\ G(1) \\ G(2) \\ G(3) \end{bmatrix} = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \begin{bmatrix} g(0) \\ g(1) \\ g(2) \\ g(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$X[0] = F[0] + W_8^0 G[0] = 0$

$X[1] = F[1] + W_8^1 G[1] = 2 + j2$

$X[2] = F[2] + W_8^2 G[2] = -j4$

$X[3] = F[3] + W_8^3 G[3] = 2 - j2$

$X[4] = F[0] - W_8^0 G[0] = 0$

$X[5] = F[1] - W_8^1 G[1] = 2 + j2$

$X[6] = F[2] - W_8^2 G[2] = j4$

$X[7] = F[3] - W_8^3 G[3] = 2 - j2$

6.57. Consider a sequence

$$x[n] = \{1, 1, -1, -1, -1, 1, 1, -1\}$$

Determine the DFT $X[k]$ of $x[n]$ using the decimation-in-time FFT algorithm.

From Figs. 6-38(a) and (b), the phase factors W_4^k and W_8^k are easily found as follows:

$$W_4^0 = 1 \qquad W_4^1 = -j \qquad W_4^2 = -1 \qquad W_4^3 = j$$

and

$$W_8^0 = 1 \qquad W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \qquad W_8^2 = -j \qquad W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^4 = -1 \qquad W_8^5 = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} \qquad W_8^6 = j \qquad W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

Next, from Eqs. (*6.215a*) and (*6.215b*)

$$f[n] = x[2n] = \{x[0], x[2], x[4], x[6]\} = \{1, -1, -1, 1\}$$
$$g[n] = x[2n+1] = \{x[1], x[3], x[5], x[7]\} = \{1, -1, 1, -1\}$$

Then, using Eqs. (*6.206*) and (*6.212*), we have

$$\begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2+j2 \\ 0 \\ 2-j2 \end{bmatrix}$$

$$\begin{bmatrix} G[0] \\ G[1] \\ G[2] \\ G[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

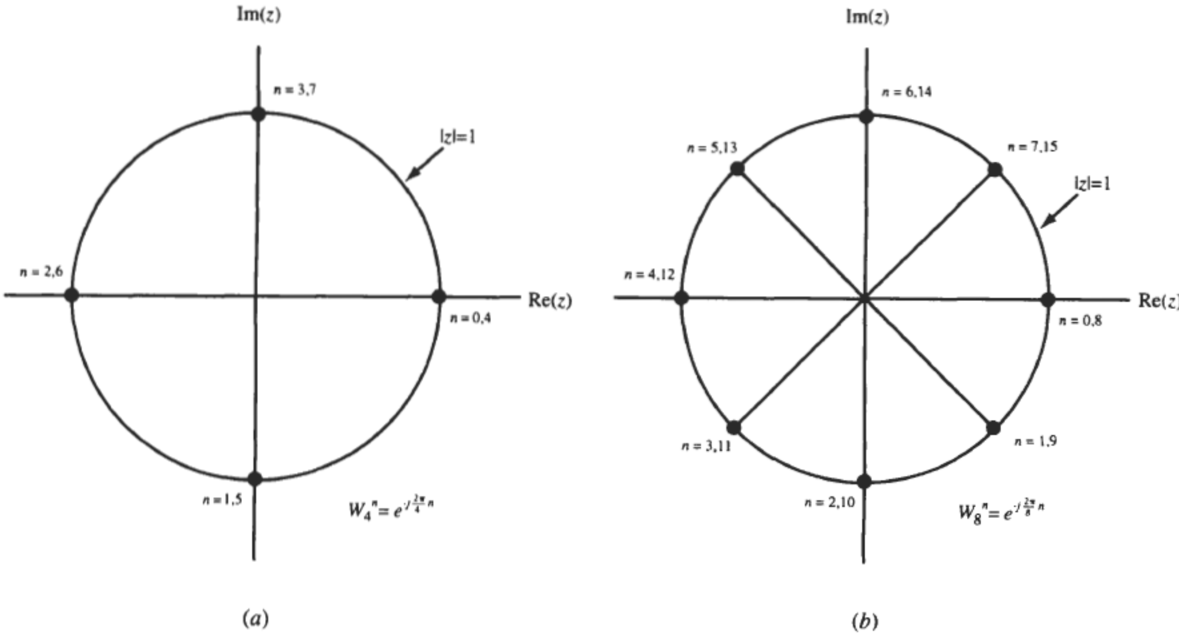


Fig. 6-38 Phase factors W_4^n and W_8^n.

and by Eqs. (*6.217a*) and (*6.217b*) we obtain

$$X[0] = F[0] + W_8^0 G[0] = 0 \qquad X[4] = F[0] - W_8^0 G[0] = 0$$
$$X[1] = F[1] + W_8^1 G[1] = 2 + j2 \qquad X[5] = F[1] - W_8^1 G[1] = 2 + j2$$
$$X[2] = F[2] + W_8^2 G[2] = -j4 \qquad X[6] = F[2] - W_8^2 G[2] = j4$$
$$X[3] = F[3] + W_8^3 G[3] = 2 - j2 \qquad X[7] = F[3] - W_8^3 G[3] = 2 - j2$$

Noting that since $x[n]$ is real and using Eq. (*6.204*), $X[7]$, $X[6]$, and $X[5]$ can be easily obtained by taking the conjugates of $X[1]$, $X[2]$, and $X[3]$, respectively.

Example 1: Consider a sequence $x[n] = \{1, 1, -1, -1, -1, 1, 1, -1\}$

Determine DFT X[k] of x[n] using the decimation-in-frequency FFT algorithm.

$$p[n] = x[n] + x\left[n + \frac{N}{2}\right]$$

$$= \{(1-1), (1+1), (-1+1), (-1-1)\} \quad = \{0, 2, 0, 2\}$$

$$q[n] = \left(x[n] - x\left[n + \frac{N}{2}\right]\right) W_8^n$$

$$= \{(1+1)W_8^0, (1-1)W_8^1, (-1-1)W_8^2, (-1+1)W_8^3\}$$

$$= \{2, 0, j2, 0\}$$

$X = Wx$

$$\begin{bmatrix} P(0) \\ P(1) \\ P(2) \\ P(3) \end{bmatrix} = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \begin{bmatrix} p(0) \\ p(1) \\ p(2) \\ p(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -j4 \\ 0 \\ j4 \end{bmatrix}$$

$$\begin{bmatrix} Q(0) \\ Q(1) \\ Q(2) \\ Q(3) \end{bmatrix} = \begin{bmatrix} w_4^0 & w_4^0 & w_4^0 & w_4^0 \\ w_4^0 & w_4^1 & w_4^2 & w_4^3 \\ w_4^0 & w_4^2 & w_4^4 & w_4^6 \\ w_4^0 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \begin{bmatrix} q(0) \\ q(1) \\ q(2) \\ q(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ j2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+j2 \\ 2-j2 \\ 2+j2 \\ 2-j2 \end{bmatrix}$$

$X[0] = P[0] = 0$

$X[1] = Q[0] = 2 + j2$

$X[2] = P[1] + W_8^2 G[2] = -j4$

$X[3] = Q[1] + W_8^3 G[3] = 2 - j2$

$X[4] = P[2] = 0$

$X[5] = Q[2] = 2 + j2$

$X[6] = P[3] = j4$

$X[7] = Q[3] = 2 - j2$

6.59. Using the decimation-in-frequency FFT technique, redo Prob. 6.57.

From Prob. 6.57

$$x[n] = \{1, 1, -1, -1, -1, 1, 1, -1\}$$

By Eqs. (*6.225a*) and (*6.225b*) and using the values of W_8^n obtained in Prob. 6.57, we have

$$p[n] = x[n] + x\left[n + \frac{N}{2}\right]$$

$$= \{(1-1), (1+1), (-1+1), (-1-1)\} = \{0, 2, 0, 2\}$$

$$q[n] = \left(x[n] - x\left[n + \frac{N}{2}\right]\right) W_8^n$$

$$= \{(1+1)W_8^0, (1-1)W_8^1, (-1-1)w_8^2, (-1+1)W_8^3\}$$

$$= \{2, 0, j2, 0\}$$

Then using Eqs. (*6.206*) and (*6.212*), we have

$$\begin{bmatrix} P[0] \\ P[1] \\ P[2] \\ P[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ -j4 \\ 0 \\ j4 \end{bmatrix}$$

$$\begin{bmatrix} Q[0] \\ Q[1] \\ Q[2] \\ Q[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ j2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+j2 \\ 2-j2 \\ 2+j2 \\ 2-j2 \end{bmatrix}$$

and by Eqs. (*6.226a*) and (*6.226b*) we get

$$X[0] = P[0] = 0 \qquad X[4] = P[2] = 0$$

$$X[1] = Q[0] = 2 + j2 \qquad X[5] = Q[2] = 2 + j2$$

$$X[2] = P[1] = -j4 \qquad X[6] = P[3] = j4$$

$$X[3] = Q[1] = 2 - j2 \qquad X[7] = Q[3] = 2 - j2$$

which are the same results obtained in Prob. 6.57.