Exercise 4

$$z_{1,i} = w_i \cdot x + b_{1,i} \qquad w_i \cdot x + b_{1,i} = 0$$

$\downarrow$ boundary line

$$a_1 = x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad W_1 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \\ -1 & -1 \end{pmatrix}, \quad b_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

For each hidden unit $i = 1, 2, 3$

$$z_{1,i} = W_{1,i} \cdot x + b_{i,x}$$

$$z_{11} = 0 \cdot x_1 + 1 \cdot x_2 + 0 = x_2$$

$$z_{12} = 1 \cdot x_1 - 1 \cdot x_2 + 1 = x_1 - x_2 + 1$$

$$z_{13} = -1 \cdot x_1 - 1 \cdot x_2 + 1 = -x_1 - x_2 + 0$$

Each boundary line is $z_{1,i} = 0$

$L_1 : x_2 = 0$

$L_2 : x_2 = x_1 + 1$

$L_3 : x_2 = -x_1 + 1$

Then $a_{2,i} = H(z_{1,i})$ which means $a_{2,i} = 1$ if the point (x_1, x_2) is on or above the corresponding line (depending on sign of coefficients)

Output layer:

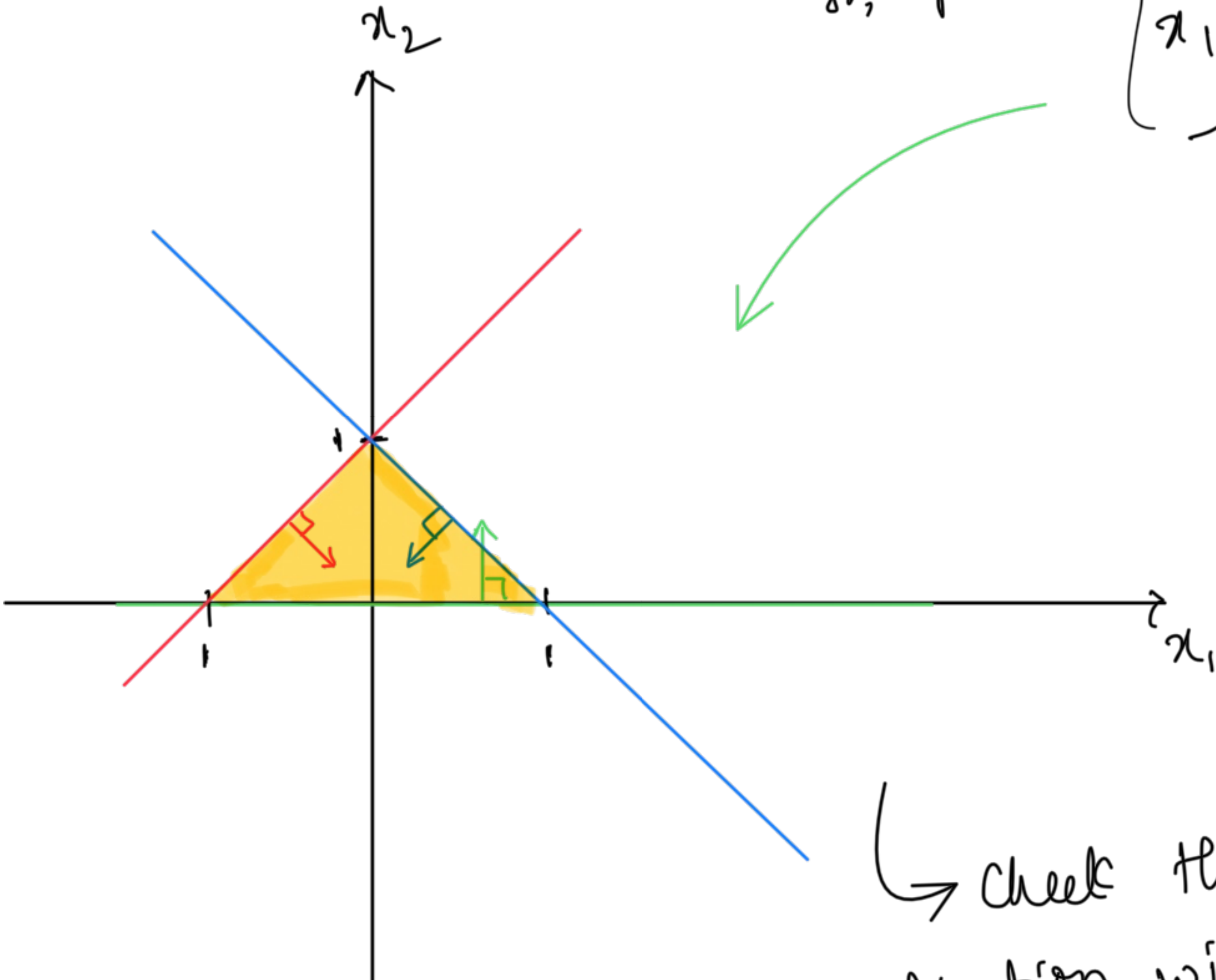
$$z_2 = [1, 1, 1]\, a_2 + (-3) = a_{2,1} + a_{2,2} + a_{2,3} - 3$$

Hence

$f(x) = H(z_2) = 1$ if and only if

$a_{2,1} = a_{2,2} = a_{2,3} = 1$

So, $f(x) = 1 \iff \begin{cases} x_2 \geq 0 \\ x_1 - x_2 + 1 \geq 0 \\ -x_1 - x_2 + 1 \geq 0 \end{cases}$



Check the activation with all the lines using heaviside function and use it same for next layer too.

No need of math directly proceed by plotting boundary lines