

# ASSIGNMENT 2

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## 1 PROBLEM

Using Baudhayana's theorem, show that  $\begin{pmatrix} -3 \\ -4 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix}$  and  $\begin{pmatrix} -6 \\ 10 \end{pmatrix}$  are the vertices of a right angled triangle. Repeat using orthogonality.

## 2 SOLUTION

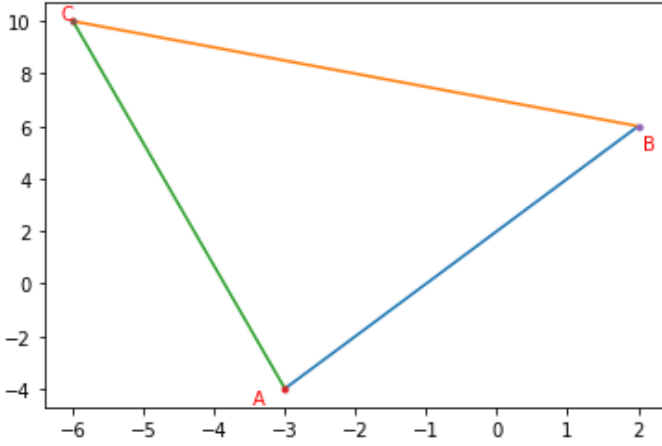


Fig. 0: Right Angled Triangle

Say there exists two points  $\mathbf{P}(x_1, y_1)$  and  $\mathbf{Q}(x_2, y_2)$ . The distance between the points is:

$$\mathbf{Z} = \mathbf{P} - \mathbf{Q} \quad (2.0.1)$$

Distance between  $\mathbf{P}$  and  $\mathbf{Q}$  is given by

$$\|\mathbf{Z}\| = \|\mathbf{P} - \mathbf{Q}\| \quad (2.0.2)$$

Let  $\mathbf{P} = (-3, -4)$ ,  $\mathbf{Q} = (2, 6)$  and  $\mathbf{R} = (-6, 10)$ .

1) Distance between  $\mathbf{P}$  and  $\mathbf{Q}$  is

$$\|\mathbf{P} - \mathbf{Q}\| = \sqrt{(-3 - 2)^2 + (-4 - 6)^2} = \sqrt{125} \quad (2.0.3)$$

2) Distance between  $\mathbf{Q}$  and  $\mathbf{R}$  is

$$\|\mathbf{Q} - \mathbf{R}\| = \sqrt{(2 - (-6))^2 + (6 - 10)^2} = \sqrt{80} \quad (2.0.4)$$

3) Distance between  $\mathbf{P}$  and  $\mathbf{R}$  is

$$\|\mathbf{P} - \mathbf{R}\| = \sqrt{(-3 - (-6))^2 + (-4 - 10)^2} = \sqrt{205} \quad (2.0.5)$$

Here, the largest distance is  $\sqrt{205}$ . To be vertices of a right angled triangle, we should have

$$\|\mathbf{P} - \mathbf{Q}\|^2 + \|\mathbf{Q} - \mathbf{R}\|^2 = \|\mathbf{R} - \mathbf{P}\|^2 \quad (2.0.6)$$

$$(\sqrt{205})^2 = (\sqrt{125})^2 + (\sqrt{80})^2 \quad (2.0.7)$$

$$205 = 205 \quad (2.0.8)$$

So, the condition is satisfied. So, using Baudhayana's theorem, it is proved that 3 points given are vertices of a right angled triangle. Now, for orthogonality,

$$(\mathbf{P} - \mathbf{Q})^T (\mathbf{Q} - \mathbf{R}) = 0 \quad (2.0.9)$$

We have

1)

$$\mathbf{P} - \mathbf{Q} = (2 - (-3), 6 - (-4)) \quad (2.0.10)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} \quad (2.0.11)$$

2)

$$\mathbf{Q} - \mathbf{R} = (2 - (-6), 6 - 10) \quad (2.0.12)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 8 \\ -4 \end{pmatrix} \quad (2.0.13)$$

3)

$$\mathbf{P} - \mathbf{R} = (-3 - (-6), -4 - 10) \quad (2.0.14)$$

$$\mathbf{P} - \mathbf{R} = \begin{pmatrix} 3 \\ -14 \end{pmatrix} \quad (2.0.15)$$

For orthogonality, product of transpose of one point and other must be 0. Here, checking for

$$\begin{pmatrix} 5 \\ 10 \end{pmatrix}^T \begin{pmatrix} 8 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}^T \begin{pmatrix} 8 \\ -4 \end{pmatrix} = 0 \quad (2.0.16)$$

Hence, using orthogonality, it is proved that the

points form a right angled triangle.

### 3 FIGURE

Figure 0 Right angled triangle where  $A=P$  and  $B=Q$  and  $C=R$