ASSIGNMENT 2

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1 Problem

Using Baudhayana's theorem, show that $\begin{pmatrix}-3\\-4\end{pmatrix}$, $\begin{pmatrix}2\\6\end{pmatrix}$ and $\begin{pmatrix}-6\\10\end{pmatrix}$ are the vertices of a right angled triangle. Repeat using orthogonality.

2 Explanation

For points to be vertices of a right angled triangle, square root of the largest distance between two points is equal to the sum of square roots of distances between other two points as per Baudhayana's theorem. So considering largest distance as 'c' and other distances as 'a' and 'b', we have

$$c^2 = a^2 + b^2 \tag{2.0.1}$$

Also, if two vectors, say X and Y are orthogonal, then their dot product ($X^T Y$) equals to 0 indicating that they make a right angle between them. So, proving $X^T Y$=0 shows points that form a right angled triangle. A directional vector connecting two points X, Y(i.e)

$$X - Y$$

is obtained by subtracting initial point(X) from the terminal point(Y).

3 Solution

Say there exists two points $P(x_1, y_1)$ and $Q(x_2, y_2)$. The distance between the points is d.

$$\mathbf{Z} = \mathbf{P} - \mathbf{Q} \tag{3.0.1}$$

Distance between **P** and **Q** is given by

$$d = \|\mathbf{Z}\| \tag{3.0.2}$$

$$d = \|\mathbf{P} - \mathbf{Q}\| \tag{3.0.3}$$

Let $\mathbf{P} = (-3, -4)$, $\mathbf{Q} = (2, 6)$ and $\mathbf{R} = (-6, 10)$. Distance between **P** and **Q** is

$$d = \sqrt{(-3-2)^2 + (-4-6)^2} = \sqrt{125} \tag{3.0.4}$$

Distance between **Q** and **R** is

$$d = \sqrt{(2-(-6))^2 + (6-10)^2} = \sqrt{80} \tag{3.0.5}$$

Distance between P and R is

$$d = \sqrt{(-3-(-6))^2 + (-4-10)^2} = \sqrt{205} \tag{3.0.6}$$

Here, the largest distance is $\sqrt{205}$. To be vertices of a right angled triangle, we should have

$$(\sqrt{205})^2 = (\sqrt{125})^2 + (\sqrt{80})^2 \tag{3.0.7}$$

$$205 = 205 \tag{3.0.8}$$

So, the condition is satisfied. So, using Baudhayana's theorem, it is proved that 3 points given are vertices of a right angled triangle. Now, for orthogonality, let us assume that **X** is the vector connecting P and Q.

$$\mathbf{Q} = \mathbf{P} - \mathbf{Q} \tag{3.0.9}$$

$$\mathbf{X} = \begin{pmatrix}2-(-3),\\6-(-4)\end{pmatrix} \tag{3.0.10}$$

$$\mathbf{X} = \begin{pmatrix}5\\10\end{pmatrix} \tag{3.0.11}$$

Assuming Y is the vector, connecting Q and R.

$$\mathbf{Y} = \mathbf{Q} - \mathbf{R} \tag{3.0.12}$$

$$\mathbf{Y} = \begin{pmatrix}2-(-6)\\6-10\end{pmatrix} \tag{3.0.13}$$

$$\mathbf{Y} = \begin{pmatrix}8\\-4\end{pmatrix} \tag{3.0.14}$$

Assuming Z is the vector connecting P and R.

$$\mathbf{Z} = \mathbf{P} - \mathbf{R} \tag{3.0.15}$$

$$\mathbf{Z} = \begin{pmatrix}-3-(-6)\\-4-10\end{pmatrix} \tag{3.0.16}$$

$$\mathbf{Z} = \begin{pmatrix}3\\-14\end{pmatrix} \tag{3.0.17}$$

For orthogonality, product of transpose of one point and other must be 0. Here, checking for

$$\begin{pmatrix} 8 \\ -4 \end{pmatrix}\begin{pmatrix} 5 \\ 10 \end{pmatrix}^T = \begin{pmatrix} 8 \\ -4 \end{pmatrix}\begin{pmatrix} 5 & 10 \end{pmatrix} = 0 \tag{3.0.18}$$

Hence, using orthogonality, it is proved that the points form a right angled triangle.