ASSIGNMENT 2

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1 Problem

Using Baudhayana's theorem, show that $\begin{pmatrix}-3\\-4\end{pmatrix}, \begin{pmatrix}2\\6\end{pmatrix}$ and $\begin{pmatrix}-6\\10\end{pmatrix}$ are the vertices of a right angled triangle. Repeat using orthogonality.

2 Solution

Say there exists two points $\mathbf{P}(x_1, y_1)$ and $\mathbf{Q}(x_2, y_2)$. The distance between the points is:

$$\mathbf{Z = P - Q} \tag{2.0.1}$$

Distance between $\mathbf{P}$ and $\mathbf{Q}$ is given by

$$\|\mathbf{Z}\| = \|\mathbf{P\text{-}Q}\| \tag{2.0.2}$$

Let $\mathbf{P} = (-3, -4)$, $\mathbf{Q} = (2, 6)$ and $\mathbf{R} = (-6, 10)$.

1) Distance between $\mathbf{P}$ and $\mathbf{Q}$ is
$$\|\mathbf{P\text{-}Q}\| = \sqrt{(-3-2)^2 + (-4-6)^2} = \sqrt{125} \tag{2.0.3}$$
2) Distance between $\mathbf{Q}$ and $\mathbf{R}$ is
$$\|\mathbf{Q\text{-}R}\| = \sqrt{(2-(-6))^2 + (6-10)^2} = \sqrt{80} \tag{2.0.4}$$
3) Distance between $\mathbf{P}$ and $\mathbf{R}$ is
$$\|\mathbf{P\text{-}R}\| = \sqrt{(-3-(-6))^2 + (-4-10)^2} = \sqrt{205} \tag{2.0.5}$$

Here, the largest distance is $\sqrt{205}$. To be vertices of a right angled triangle, we should have

$$\|\mathbf{P\text{-}Q}\|^2 + \|\mathbf{Q\text{-}R}\|^2 = \|\mathbf{R\text{-}P}\|^2 \tag{2.0.6}$$

$$(\sqrt{205})^2 = (\sqrt{125})^2 + (\sqrt{80})^2 \tag{2.0.7}$$

$$205 = 205 \tag{2.0.8}$$

So, the condition is satisfied. So, using Baudhayana's theorem, it is proved that 3 points given are vertices of a right angled triangle. Now, for orthogonality,

$$(\mathbf{P} - \mathbf{Q})^T(\mathbf{Q} - \mathbf{R}) = 0 \tag{2.0.9}$$

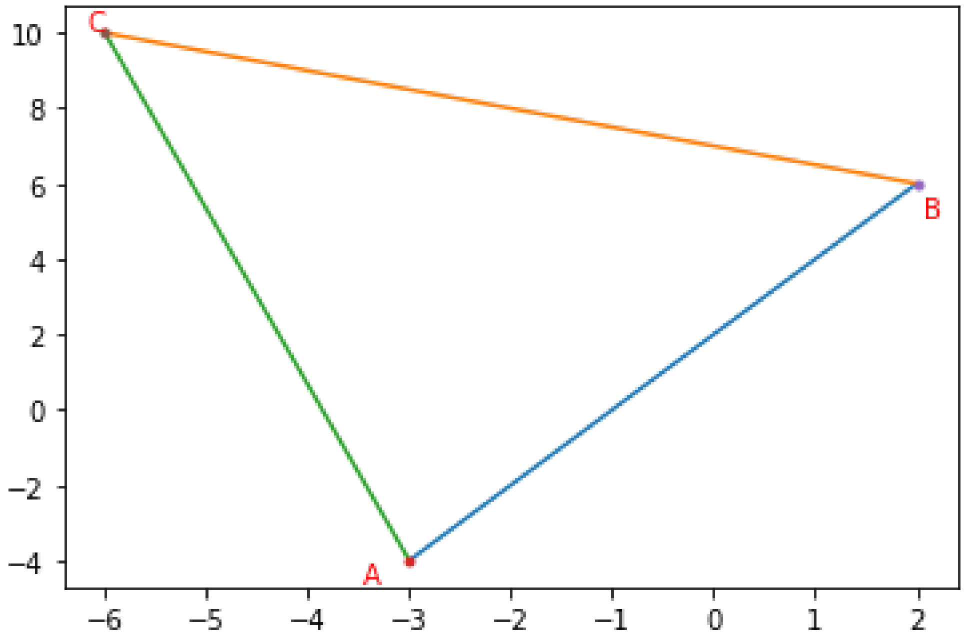
We have

1)
$$\mathbf{P} - \mathbf{Q} = (2-(-3), 6-(-4)) \tag{2.0.10}$$
$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix}5\\10\end{pmatrix} \tag{2.0.11}$$
2)
$$\mathbf{Q} - \mathbf{R} = (2-(-6), 6-10) \tag{2.0.12}$$
$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix}8\\-4\end{pmatrix} \tag{2.0.13}$$
3)
$$\mathbf{P} - \mathbf{R} = (-3-(-6), -4-10) \tag{2.0.14}$$
$$\mathbf{P} - \mathbf{R} = \begin{pmatrix}3\\-14\end{pmatrix} \tag{2.0.15}$$

For orthogonality, product of transpose of one point and other must be 0. Here, checking for

$$\begin{pmatrix}5\\10\end{pmatrix}^T \begin{pmatrix}8\\-4\end{pmatrix} = \begin{pmatrix}5\\10\end{pmatrix}^T \begin{pmatrix}8\\-4\end{pmatrix} = 0 \tag{2.0.16}$$

Hence, using orthogonality, it is proved that the points form a right angled triangle.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