ASSIGNMENT 3

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1 Problem

Find the equation of the circle that passes through the points $\begin{pmatrix}2\\3\end{pmatrix}, \begin{pmatrix}3\\2\end{pmatrix}, \begin{pmatrix}5\\1\end{pmatrix}$.

2 Solution

The general equation of circle is represented as

$$\boldsymbol{x}^T\boldsymbol{x} - 2\boldsymbol{c}^T\boldsymbol{x} + f = 0 \tag{2.0.1}$$

where $\mathbf{c}$ is the center of the circle. Substituting the given points in the equation (2.0.1), we obtain

$$2\begin{pmatrix}2 & 3\end{pmatrix}\mathbf{c} - f = 13 \tag{2.0.2}$$

$$2\begin{pmatrix}3 & 2\end{pmatrix}\mathbf{c} - f = 13 \tag{2.0.3}$$

$$2\begin{pmatrix}5 & 1\end{pmatrix}\mathbf{c} - f = 13 \tag{2.0.4}$$

can be expressed in matrix form as

$$\begin{pmatrix}4 & 6 & -1\\6 & 4 & -1\\10 & 2 & -1\end{pmatrix}\begin{pmatrix}\mathbf{c}\\f\end{pmatrix} = \begin{pmatrix}13\\13\\26\end{pmatrix} \tag{2.0.5}$$

The augmented matrix for (2.0.5) can be row reduced as follows

$$\begin{pmatrix}4 & 6 & -1 & 13\\6 & 4 & -1 & 13\\10 & 2 & -1 & 26\end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_2\leftarrow 4R_3-10R_1]{R_3\leftarrow 4R_3-10R_1}\begin{pmatrix}4 & 6 & -1 & 13\\0 & -20 & 2 & -26\\0 & -52 & 6 & -26\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_3\leftarrow 5R_3-13R_2}\begin{pmatrix}4 & 6 & -1 & 13\\0 & -20 & 2 & -26\\0 & 0 & 4 & 208\end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_1\leftarrow 4R_1+R_3]{R_2\leftarrow 2R_2-R_3}\begin{pmatrix}16 & 24 & 0 & 260\\0 & -40 & 0 & -260\\0 & 0 & 4 & 208\end{pmatrix} \tag{2.0.9}$$

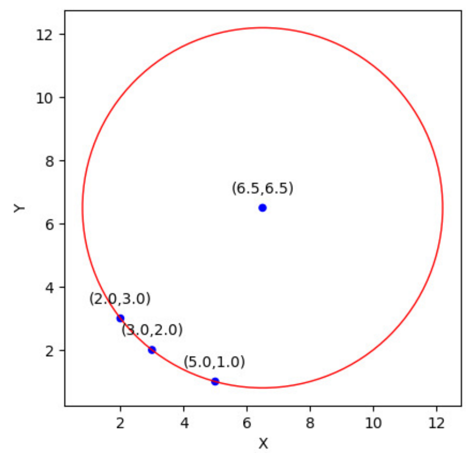

Fig. 0: Circle passing through the points A, B, C with center O

$$\xleftrightarrow{R_1\leftarrow 5R_1+3R_2}\begin{pmatrix}80 & 0 & 0 & 52\\0 & -40 & 0 & -260\\0 & 0 & 4 & 208\end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_1\leftarrow \frac{R_1}{40}]{R_2\leftarrow \frac{R_2}{-20}, R_3\leftarrow \frac{R_1}{4}}\begin{pmatrix}2 & 0 & 0 & 13\\0 & 2 & 0 & 13\\0 & 0 & 1 & 52\end{pmatrix} \tag{2.0.11}$$

From the matrix (2.0.11),

$$\mathbf{c} = \begin{pmatrix}\frac{13}{2}\\\frac{13}{2}\end{pmatrix} \tag{2.0.12}$$

$$k = 52 \tag{2.0.13}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = 11 \tag{2.0.14}$$

Hence the circle equation can be written as,

$$\boldsymbol{x}^T\boldsymbol{x} - 2\begin{pmatrix}\frac{13}{2} & \frac{13}{2}\end{pmatrix}^T\boldsymbol{x} + 52 = 0 \tag{2.0.15}$$