Assignment 5 Probability

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Abstract

This pdf consists the solution to the question 6.47 from in Papoulis pillai

Outline

Question 6.47

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The random variables X_l and X_2 are jointly normal with zero mean. Show that their density can be written in the form.

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\Delta}} exp\left\{-\frac{1}{2}XCX^{-1}\right\}$$

where C is given by

$$C = \begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}$$

where $X : [x_1, x_2]$,$\mu_{ij} = E(x_i x_j)$ and $\Delta = \mu_{11}\mu_{22} - \mu_{12}^2$

Solution

Solution

$$C = \begin{bmatrix} {\sigma_1}^2 & r\sigma_1\sigma_2 \\ r\sigma_1\sigma_2 & {\sigma_2}^2 \end{bmatrix} \qquad \Delta = {\sigma_1}^2{\sigma_2}^2(1 - r^2)$$

$$C^{-1} = \begin{bmatrix} \frac{1}{(1-r^2){\sigma_1}^2} & \frac{r}{(1-r^2)\sigma_1\sigma_2} \\ \frac{r}{(1-r^2)\sigma_1\sigma_2} & \frac{1}{(1-r^2){\sigma_2}^2} \end{bmatrix}$$

We know that $X : [x_1, x_2]$

$$XC^{-1}X^t = \frac{1}{(1 - r^2)}\left(\frac{{x_1}^2}{{\sigma_1}^2} - 2r\frac{x_1x_2}{\sigma_1\sigma_2} + \frac{{x_2}^2}{{\sigma_2}^2}\right)$$