Assignment 8 Probability

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Abstract

This pdf consists the solution to the question 15.8from in Papoulis pillai

Outline

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Question 15.8

Q 15.8

Time Reversible Markov Chains. Consider a stationary Markov chain $...x_n, x_{n+1}, x_{n+2} \ldots$ with transition probabilities $\{p_{ij}\}$ and steady state probabilities $\{q_{ij}\}$. (a) Show that the reversed sequence $...x_n, x_{n+1}, x_{n+2} \ldots$ is also a stationary Markov process with transition probabilities

$$P\{x_n = j | x_{n+1} = i\} p^*_{ij} = \frac{q_j p_{ij}}{q_i}$$

and steady state probabilities $\{q_i\}$. A Markov chain is said to be time reversible if $p^*_{ij} = p_{ij}$ j for all i. j. (b) Show that a necessary condition for time reversibility is that $p_{ij} p_{jk} p_{ki} = p_{ik} p_{kj} p_{ji}$ for all i,j,k

Question 15.8

Q 15.8

which states that the transition $e_i \to e_j \to e_k \to e_i$ has the same probability as the reversed transition $e_i \to e_k \to e_j \to e_i$. In fact. for a reversible chain starting at any state ell any path back to e; has the same probability as the reversed path.

Solution

Solution

(a) From Bayes theorem

$$P\{x_n = j | x_{n+1} = i\} = \frac{P\{x_{n+1} = i | x_n = j\} P\{x_n = j\}}{P\{x_{n+1} = i\}} = \frac{q_j p_{ij}}{q_i} = p^*_{ij}, \quad (1)$$

Where we assumed the chain to be in steady state. (b)Notice that time$_r$*eversibilityisequivalentto*

$$p^*_{ij} = p_{ij}$$

and using (1) we get

$$p^*_{ij} = \frac{q_j p_{ij}}{q_i} = p_{ij} \quad (2)$$

Solution

Solution

Thus from using (2) we obtain by direct substitution

$$p_{ij}p_{jk}p_{ki} = \left(\frac{q_j}{q_i}p_{ik}\right)\left(\frac{q_k}{q_j}p_{kj}\right)\left(\frac{q_i}{q_k}p_{ji}\right)$$
$$= p_{ik}p_{kj}p_{ji}$$