

b) Objective function

MLE

For this, it is simply the -ve log likelihood

$$-\log(L) = \frac{n}{2} \log(\pi) + \frac{1}{2} \sum_{i=1}^n \left[ \log(\sigma_i^2) + \frac{(y_i - \hat{y}_i)^2}{\sigma_i^2} \right]$$

MAP

For this, it is -ve of log posterior

which is 
$$-\log(L) - \log(\text{Prior}(\theta))$$

we know these both