(3)

a) Expressions

Likelihood :

$$PDF = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i^2}} \times \exp\left(-\frac{\varepsilon^2}{2\sigma_i^2}\right)$$

$$\Rightarrow \quad \mathcal{L} = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i^2}} \times \exp\left(-\frac{(y_i - \hat{y}_i)^2}{2\sigma_i^2}\right)$$

For a Single point $(x_n, t_n)$

$$\mathcal{L} = \frac{1}{\sqrt{2\pi\sigma_i^2}} \times \exp\left(-\frac{(t_n - t(x_n))^2}{2\sigma_i^2}\right)$$

Prior :

$$Prior(\theta_i) = N(\mu_{\theta_i}, \sigma_{\theta_i}^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-1}{2}\left(\frac{(\omega-\mu)}{\sigma_i}\right)^2\right)$$

and it varies for each $\theta_i$

$$p(\omega) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \times \exp\left(-\frac{1}{2}\left(\frac{\omega-\mu}{\sigma_i}\right)^2\right)$$