3c

For MLE, we got

$$-\log(L) = \frac{n}{2}\log(\pi) + \frac{1}{2}\left[\sum_{i=1}^{N}\left(\log(\sigma_i^2) + \frac{(y_i-\hat{y}_i)^2}{\sigma_i^2}\right)\right]$$

as our objective function.

we can drop $\left(\frac{1}{2}\log(\sigma_i^2)\right)$ as it is a constant

$$\Rightarrow \quad -\log(L) = \sum_{i=1}^{N} \frac{(y_i-\hat{y}_i)^2}{2\sigma_i^2}$$

$$= \frac{1}{2}\sum_{i=1}^{N} r_i\,(t_n - f(x_i))^2 \quad \left(\text{Let } r_i = \frac{1}{\sigma_i^2}\right)$$

$f(x_i)$ can be written as $w^T \times \phi(x_n)$

$$\Rightarrow \boxed{-\log(L) = E_D(w) = \frac{1}{2}\sum_{i=1}^{N} r_i\,(t_n - w^T \times \phi(x_n))^2}$$

where $\boxed{r_i = \frac{1}{\sigma_i^2}}$

→ Find the expression for the solution of ω that minimizes the Error func

$$E_D(\omega) = \frac{1}{2}\sum_{i=1}^{N} r_i \{t_i - \omega^T \phi(x_i)\}^2$$

on applying derivate w.r.to ω

$$\frac{\partial E_D(\omega)}{\partial \omega} = \frac{1}{2}\sum_{i=1}^{N} r_i (2)(t_i - \omega^T \phi(x_i)) \times (-1)\, \phi(x_i) \times \frac{\partial \omega^T}{\partial \omega}$$

$$= 0$$

we know $\phi(x_i) \neq 0$ & $r_i \neq 0$

we know $\frac{\partial w^T}{\partial w} = 1$ for a row matrix w

$$\Rightarrow \sum_{i=1}^{N} r_i \left(t_i - w^T \phi(x_i)\right) \times \left(-\phi(x_i)\right) = 0$$

$$\Rightarrow \sum_{i=1}^{N} r_i \left(w^T \phi(x_i)\right) \cdot \phi(x_i) = \sum_{i=1}^{N} r_i t_i \phi(x_i)$$

Comparing constants

$\Rightarrow \quad w^T \phi(x_i) = t_i$

$\Rightarrow \quad \boxed{w\, \phi^T(x_i) = t_i}$ for all i in 1 to n

$\Rightarrow \quad \boxed{w A = T}$

where $A = [\phi^T(x_1) \;\; \phi^T(x_2) \; \cdots \; \phi^T(x_n)]$

$T = [t_1 \;\; t_2 \; \cdots \; t_n]$

$$\phi^T(x_i)\, w = t_i$$

$\Rightarrow \quad \boxed{A w = T}$ where $A = \begin{bmatrix} \phi^T(x_1) \\ \phi^T(x_2) \\ \vdots \\ \phi^T(x_n) \end{bmatrix}$

$T = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$