Assignment 9 Probability

Narsupalli Sai Vamsi

June 2022

Abstract

This pdf consists the solution to the question 15.15 from in Papoulis pillai

Outline

Question 15.15

Q 15.15

Determine the mean time to absorption for the random walk model in Example 15-25. In the context of the gambler's ruin problem discussed there, show that the mean time to absorption for player A (starting with \$a) reduces to Eq. (3-53).

Solution

Solution

The mean time to absorption satisfies (15-240). From there

$$m_i = 1 + \sum_{k \in T} p_{ik} m_k = 1 + p_{i,i+1} m_{i+1} + p_{i,i-1} m_{i-1}$$

$$m_i = 1 + pm_{i+1} + qm_{i-1}$$

replacing i with k we get

$$m_k = 1 + pm_{k+1} + qm_{k-1}$$

as p+q =1, we can write

$$p(m_{k+1} - m_k) = q(m_k - m_{k-1}) - 1$$

Solution

Solution

Let

$$M_{k+1} = m_{k+1} - m_k$$

so now we get

$$M_{k+1} = \frac{q}{p} M_k - \frac{1}{p}$$

$$\implies M_{k+1} = \left(\frac{q}{p}\right)^k M_1 - \frac{1}{p}\left[1 + \left(\frac{q}{p}\right)^k + \left(\frac{q}{p}\right)^k + \cdots + \left(\frac{q}{p}^{k-1}\right)\right]$$

Solution

Solution

From this we get

$$M_{k+1} = \begin{cases} \left(\frac{q}{p}\right)^k M_1 - \frac{1}{p-q}\left\{1 - \left(\frac{q}{p}\right)^k\right\}, & p \neq q \\ M_1 - \frac{k}{p}, & p = q \end{cases} \tag{1}$$

From this we get

$$m_k = \sum_{0}^{i-1} M_{k+1}$$

$$m_k = \begin{cases} \left(M_1 + \frac{1}{p-q}\right) \sum_{k=0}^{i-1} \left(\frac{q}{p}\right)^k - \frac{i}{p-q}, & p \neq q \\ iM_i - \frac{i(i-1)}{2p}, & p = q \end{cases} \tag{2}$$

Solution

Solution

$$m_k = \begin{cases} \left(M_1 + \frac{1}{p-q}\right)\left(\frac{1-(\frac{q}{p})^i}{1-(\frac{q}{p})}\right) - \frac{i}{p-q}, & p \neq q \\ iM_i - \frac{i(i-1)}{2p}, & p = q \end{cases} \tag{3}$$

where we have used $m_o = 0$.Similarly $m_{a+b} = 0$ gives

$$M_1 + \frac{1}{p-q} = \frac{a+b}{p-q} \cdot \frac{1-(\frac{q}{p})^i}{1-(\frac{q}{p})^{a+b}} \tag{4}$$

$$m_i = \begin{cases} \frac{a+b}{p-q} \cdot \frac{1-(\frac{q}{p})^i}{1-(\frac{q}{p})^{a+b}} - \frac{i}{p-q}, & p \neq q \\ i(a+b-i), & \text{p = q} \end{cases} \tag{5}$$

Solution

Solution

Which gives for i = a

$$m_a = \begin{cases} \frac{a+b}{p-q} \cdot \frac{1-(\frac{q}{p})^a}{1-(\frac{q}{p})^{a+b}} - \frac{a}{p-q}, & p \neq q \\ ab, & \text{p = q} \end{cases}$$

$$= \begin{cases} \frac{a}{p-q} - \frac{a+b}{2p-1} \cdot \frac{1-(\frac{p}{q})^a}{1-(\frac{q}{p})^{a+b}}, & p \neq q \\ ab, & \text{p = q} \end{cases} \quad (6)$$

by writing

$$\frac{1-(\frac{q}{p})^a}{1-(\frac{q}{p})^{a+b}} = 1 - \frac{(\frac{q}{p})^a - (\frac{q}{p})^{a+b}}{1-(\frac{q}{p})^{a+b}} = 1 - \frac{1-(\frac{p}{q})^k}{1-(\frac{p}{q})^{a+b}}$$