Assignment 4 Probability

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Abstract

This pdf consists the solution to the question 6.46 from in Papoulis pillai

Outline

Question 6.46

(Q6.46) Let” and y be independent Poisson random variables with parameters λ_1 and λ_2 respectively. Show that the conditional density function of x given x + y is binomial

Solution

Solution

The moment generating function of X and Y are given by

$$\Gamma_x(z) = e^{\lambda_1(z-1)}$$
$$\Gamma_y(z) = e^{\lambda_1(z-1)}$$

And

$$\Gamma_{x+y}(z) = e^{(\lambda_1+\lambda_2)(z-1)}$$
$$Z \sim P(\lambda_1 + \lambda_2)$$

Solution

Solution

we know that in poissons distribution

$$P(X+Y=k) = e^{-(\lambda_1+\lambda_2)}\frac{(\lambda_1+\lambda_2)^k}{k!}$$

and what we need to find is the conditional function so lets take some k

$$P(X=k|X+Y=n) = \frac{P(X=k, X+Y=n)}{P(X+Y=n)} = \frac{P(X=k)P(Y=n-k)}{P(X+Y=n)}$$

$$= \frac{e^{-\lambda_1}(\lambda_1^k/k!)e^{-\lambda_2}(\lambda_2^{n-k}/(n-k)!)}{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n/n!}$$

Solution

Solution

Here we took k which runs from 0 to n and we need to prove that the distribution is binomial.

$$= \binom{n}{k}\left(\frac{\lambda_1}{\lambda_1 + \lambda_2}\right)^k \left(\frac{\lambda_2}{\lambda_1 + \lambda_2}\right)^{n-k}$$

$$= \binom{n}{k} m^k (1 - m)^{n-k}$$

Binomial (n,m) where m $= \frac{\lambda_1}{\lambda_1+\lambda_2}$