

# Assignment 4 Probability

Narsupalli Sai Vamsi

June 2022

## Abstract

This pdf consists the solution to the question 6.46 from in Papoulis pillai

# Outline

- 1 Question 6.46
- 2 Solution
- 3 Solution
- 4 Solution

## Question 6.46

(Q6.46) Let  $x$  and  $y$  be independent Poisson random variables with parameters  $\lambda_1$  and  $\lambda_2$  respectively. Show that the conditional density function of  $x$  given  $x + y$  is binomial

# Solution

## Solution

The moment generating function of X and Y are given by

$$\Gamma_x(z) = e^{\lambda_1(z-1)}$$

$$\Gamma_y(z) = e^{\lambda_1(z-1)}$$

And

$$\Gamma_{x+y}(z) = e^{(\lambda_1 + \lambda_2)(z-1)}$$

$$Z \sim P(\lambda_1 + \lambda_2)$$

# Solution

## Solution

we know that in poissons distribution

$$P(X + Y = k) = e^{-(\lambda_1 + \lambda_2)} \frac{(\lambda_1 + \lambda_2)^k}{k!}$$

and what we need to find is the conditional function so lets take some k

$$\begin{aligned} P(X = k | X + Y = n) &= \frac{P(X = k, X + Y = n)}{P(X + Y = n)} = \frac{P(X = k)P(Y = n - k)}{P(X + Y = n)} \\ &= \frac{e^{-\lambda_1} (\lambda_1^k / k!) e^{-\lambda_2} (\lambda_2^{n-k} / (n - k)!)}{e^{-(\lambda_1 + \lambda_2)} (\lambda_1 + \lambda_2)^n / n!} \end{aligned}$$

# Solution

## Solution

Here we took  $k$  which runs from 0 to  $n$  and we need to prove that the distribution is binomial.

$$= \binom{n}{k} \left( \frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^k \left( \frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{n-k}$$

Binomial  $(n, m)$  where  $m = \frac{\lambda_1}{\lambda_1 + \lambda_2}$