New York City Taxi Fare Prediction

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Abstract

This project aims to predict taxi fares in New York City using machine learning models, leveraging a comprehensive dataset from Kaggle, which contains records of historical taxi trips. By addressing challenges like data preprocessing, outlier management, and feature engineering, the project focuses on deriving meaningful insights and enhancing predictive accuracy. A variety of models were explored, including Linear Regression, Ridge Regression, Decision Trees, Random Forests, and Gradient Boosting Machines (GBM). Among these, GBM emerged as the best-performing model, achieving an RMSE of 2.72 and an R² score of 81.7%, indicating a strong predictive capability. The project emphasizes real-world applications such as improving fare transparency for passengers, optimizing routes for drivers, and integrating dynamic pricing systems.

1 Introduction

Urban transportation systems, particularly taxi services, play a crucial role in the daily lives of residents and visitors in bustling cities like New York [1, 2]. With over 485,000 taxi trips occurring daily, fare transparency and predictability are pivotal for enhancing user experience and operational efficiency [3]. However, existing systems often lack the ability to provide accurate fare estimations, leading to confusion for passengers and inefficiencies for drivers [4, 5]. This project addresses these challenges by leveraging machine learning to predict taxi fares in New York City.

The prediction of taxi fares is a regression problem that involves understanding the

relationships between various features such as trip distance, time of day, passenger count, and geographic locations [6, 7]. This project utilizes a comprehensive dataset from Kaggle, which contains over 55 million records of historical taxi rides [1]. The dataset includes critical attributes like pickup and drop-off coordinates, timestamps, passenger counts, and the corresponding fare amounts. By analyzing and modeling this data, we aim to predict fares with high accuracy and uncover patterns that influence pricing [8].

The significance of this work extends beyond fare prediction. Accurate models can help passengers anticipate costs, enabling informed travel decisions and fostering trust in taxi services [9, 2]. For drivers, precise fare predictions aid in planning routes and estimating earnings, improving their efficiency [3]. Furthermore, the insights derived can be leveraged for dynamic pricing strategies and integration into ride-sharing platforms, contributing to smart transportation systems [10, 5].

This project applies advanced machine learning techniques, including Gradient Boosting Machines, Random Forests, and Ridge Regression, to analyze the dataset [10, 11, 12]. By implementing robust preprocessing steps such as feature engineering, outlier handling, and normalization, the models capture complex relationships in the data [13, 6]. Evaluation metrics like Root Mean Square Error (RMSE) and R² Score are used to assess the performance of the models, ensuring their reliability for real-world applications [14, 15].

In addition to its technical merits, this project aligns with the broader goals of data science in urban development. By enhancing transparency and optimizing resources, it demonstrates how machine learning can be harnessed to address real-world problems and improve quality of life in dense metropolitan areas [16, 17].

2 MODELS

• Linear Regression: Linear regression is a type of supervised machine learning algorithm that maps data points to optimized linear functions for prediction on new datasets [6, 7]. It is simple and interpretable, making it easy to understand feature importance while effectively identifying linear patterns in the data [18]. However, it fails to capture non-linear relationships or complex feature interactions [12].

This model assumes a linear relationship between the independent variables (e.g., distance, pickup time) and the target variable (fare). It serves as a simple baseline model to understand how features like trip distance influence fare, making it a starting point for comparison against more complex models [4].

• Ridge Regression: Ridge Regression extends Linear Regression by adding L2 regularization, which penalizes large coefficients to prevent overfitting [12]. This regularization improves generalization by controlling model complexity, particularly when multicollinearity is present [19]. However, it remains unsuitable for handling non-linear patterns in the data [6].

Ridge Regression is relevant for this dataset if features such as trip distance and time are correlated. By improving model stability and interpretability, Ridge Regression can help predict fares with reduced variance compared to ordinary least squares methods [7, 5].

• Decision Trees: Decision Trees are non-linear models that recursively split data into subsets based on feature values, creating a tree-like structure for decision-making [20]. This approach is suitable for capturing complex interactions between features like distance, time, and location, making it effective for identifying patterns that influence fare prices in diverse scenarios [5].

However, Decision Trees are prone to overfitting, especially with deep trees, and are highly sensitive to small changes in the dataset [21, 22]. Techniques such as pruning and ensemble methods can help mitigate these issues.

• Random Forest Regressor: Random Forest is an ensemble model that combines multiple decision trees to capture non-linear relationships and feature interactions [11]. By aggregating predictions from diverse trees, Random Forest reduces overfitting while improving predictive robustness [23]. For this dataset, it models complex relationships between pickup/drop-off coordinates, time, and fare amounts, resulting in more reliable predictions [4, 3].

Random Forest is particularly well-suited for high-dimensional data, as it can analyze intricate patterns in diverse features like date, time, and geographic coordinates, making it highly effective for fare prediction tasks [21].

• Gradient Boosting Machines (GBM): GBM iteratively builds decision trees, each one correcting errors from the previous model, making it a powerful tool for regression tasks [10, 24]. GBM excels in capturing subtle patterns and interactions within features, providing more accurate predictions when fine-tuned with hyperparameters like learning rate and tree depth [14, 25].

In this dataset, GBM demonstrates its strength by modeling complex interactions between trip distance, time, and other variables, outperforming simpler linear models [8, 13]. Additionally, GBM's flexibility with various loss functions makes it suitable for optimization in real-world applications [9].

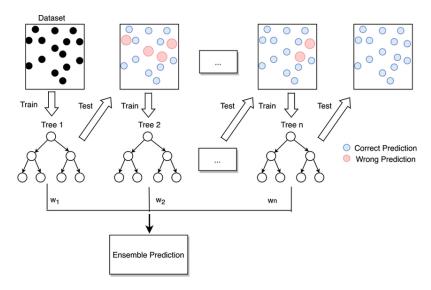


Figure 1: Flow diagram of GBM

3 EXPERIMENT

3.1 Data

The dataset to be utilized for this project is the New York City Taxi Fare Prediction dataset. The data utilized in this research concern historical data from 2009 to 2016 of New York Taxi fares in USD, which is publicly available on Kaggle at Kaggle URL. This dataset comprises over 55 million records of taxi rides collected from New York City. Each record includes essential attributes such as the pickup and drop-off locations (in latitude and longitude coordinates), the timestamp of the trip, the number of passengers, and the corresponding fare amount.

The left plot below is a histogram of taxi fares, showing how often different fare amounts occur. The x-axis represents the fare amounts (in dollars), and the y-axis represents the frequency of fares in the dataset.

- The distribution is *right-skewed*, meaning most fares are low (clustered around \$5-\$15), with fewer high fares.
- The highest frequency of fares is concentrated around \$7-\$10, indicating this is the typical fare range.
- There are some outliers with fares above \$30, but these are rare.

This scatter plot below on the right shows the relationship between the distance traveled (in miles) on the x-axis and the corresponding fare amount (in dollars).

- There is a positive correlation between distance and fare; longer distances generally result in higher fares.
- For short distances (around 0–5 miles), fares vary widely, indicating possible additional charges (e.g., base fare or surcharges).
- Outliers are visible, such as rides with unusually high fares for relatively short distances
 or low fares for long distances. These might result from data errors, discounts, or fixed
 pricing scenarios.

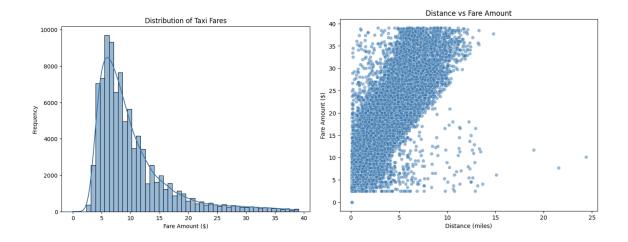


Figure 2: (Left) Histogram of Distribution of Taxi Fares (Right) Distance vs Fare amount

The graph on the left below shows the distribution of taxi rides across different hours of the day. It can be observed that the number of rides is lowest between 2 AM and 5 AM (likely due to reduced demand during late-night hours), and there is a steady increase in rides during the morning hours, peaking around 6 PM to 8 PM, indicating rush hour demand.

The graph on the right below illustrates the average fare amounts based on the day of the week (y-axis) and the hour of the day (x-axis). Higher fares (indicated in red) are observed early in the morning (around 4–5 AM) and during specific rush hour periods. The heatmap highlights fare variability influenced by factors like demand, time of day, and weekday versus weekend travel patterns.

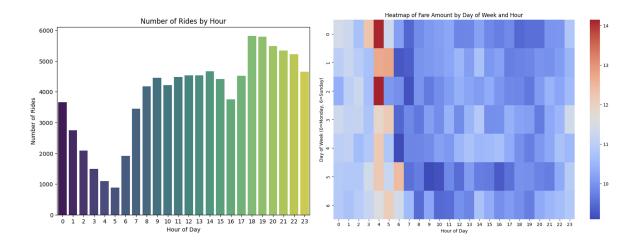


Figure 3: (Left) Number of rides by Hour (Right) Heatmap of fare amount by day of week and hour

These graphs below in Figure analyze average taxi fare trends:

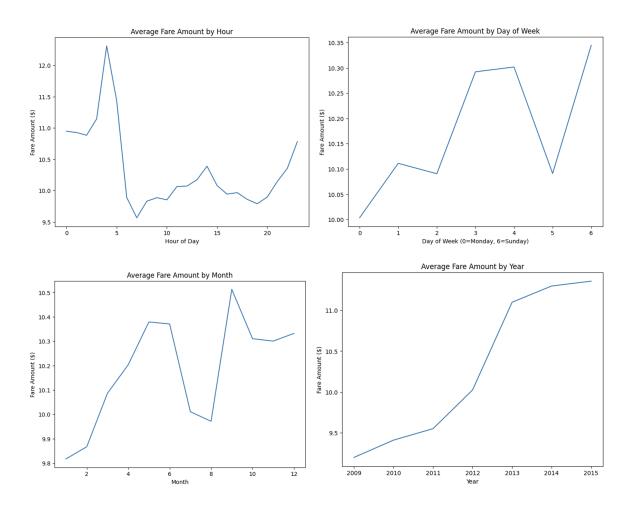


Figure 4: Average Taxi fares

- By Hour (Top Left): Fares are highest around early morning (2–3 AM), likely due to limited demand/supply, and increase slightly late at night.
- By Day of Week (Top Right): Fares peak on weekends (Saturdays and Sundays), reflecting higher demand.
- By Month (Bottom Left): Fares rise in the spring and summer months (April–July), possibly due to tourism and better weather.
- By Year (Bottom Right): A steady increase in average fares is observed over time, possibly due to inflation or fare adjustments.

3.1.1 Data Preprocessing

- Data Cleaning: The dataset may contain missing or incomplete records, which need to be handled appropriately. We will address missing values by either imputing data or removing incomplete rows that cannot be corrected.
- Feature Engineering: One of the most important steps in this project will involve engineering new features that could improve model performance. For example, trip distance will be calculated using the haversine formula to derive the distance between the pickup and drop-off points. Additional features, such as the time of day, day of the week, and proximity to notable landmarks (e.g., airports), will also be extracted from the data.
- Handling Outliers: Taxi fare data can often include outliers—such as abnormally high or low fares—that could skew predictions. These outliers will need to be detected and managed carefully, either by removing them or transforming the data so that the model is not disproportionately affected.
- Normalization and Scaling: In order to ensure that models like linear regression and neural networks perform optimally, normalization or scaling may be required. For instance, the latitude and longitude values, which represent geographic coordinates, will be scaled appropriately so that all input features are on a similar scale.

3.2 Parameters

The table below is a structured table summarizing the hyperparameters for each model:

Model	Hyperparameters		
Linear Regression	Default parameters (no specific hyperparameters)		
Ridge Regression	Default parameters (no specific hyperparameters)		
Decision Tree	Default parameters (no specific hyperparameters)		
Random Forest	<pre>n_estimators=100 , random_state=42</pre>		
Gradient Boosting	<pre>n_estimators=100 , random_state=42</pre>		

Figure 5: Hyperparameters for each model

3.3 Method

Root Mean Square Error (RMSE)

Definition and Significance

RMSE is defined as the square root of the average squared differences between the predicted and actual values:

RMSE =
$$\sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2}$$
 (1)

where N is the number of predictions, \hat{y}_i is the predicted fare, and y_i is the actual fare.

RMSE is the primary metric for this project because it penalizes larger errors more than smaller ones, making it sensitive to outliers. In the context of taxi fare prediction, significant prediction errors (i.e., predicting a fare far from the actual value) can substantially degrade the user experience for passengers.

Why It is Suitable for This Problem

Since fare prediction is a continuous regression problem, RMSE serves as an appropriate metric for evaluating how closely the model's predictions match the actual fare amounts.

Interpreting RMSE in This Context

A lower RMSE indicates that the model's predictions are closer to the actual fare amounts on average, reflecting higher accuracy.

Mean Absolute Error (MAE)

Definition and Significance

MAE is defined as the average absolute difference between the predicted and actual values:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |\hat{y}_i - y_i|$$
 (2)

Unlike RMSE, MAE treats all errors equally, without penalizing larger errors more heavily.

Why It is Suitable for This Problem

MAE provides a more interpretable metric for average prediction errors, expressed in the same units as the predicted fare (i.e., dollars).

Interpreting MAE in This Context

A lower MAE indicates better model performance, with the average prediction being closer to the actual fare.

R-squared (R^2)

Definition and Significance

 R^2 , or the coefficient of determination, measures the proportion of variance in the dependent variable that is predictable from the independent variables:

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} (\hat{y}_{i} - y_{i})^{2}}{\sum_{i=1}^{N} (y_{i} - \bar{y})^{2}}$$
(3)

where \bar{y} is the mean of the actual values.

Why It is Suitable for This Problem

 \mathbb{R}^2 helps to assess how well the features used in the model explain the variability in taxi fares.

Interpreting R^2 in This Context

An \mathbb{R}^2 value closer to 1 indicates a better model fit, implying that the independent variables adequately capture the variation in taxi fares.

Mean Squared Error (MSE)

Definition and Significance

MSE is the average of the squared differences between the predicted and actual values:

$$MSE = \frac{1}{N} \sum_{i=1}^{N} (\hat{y}_i - y_i)^2$$
 (4)

Why It is Suitable for This Problem

MSE is sensitive to outliers and is useful during the training phase to evaluate model convergence and guide hyperparameter tuning.

Interpreting MSE in This Context

A lower MSE indicates that the model is generally making predictions that are close to the actual values, with less variance in the errors.

3.4 Results

The models were evaluated on the New York City taxi fare dataset using three key metrics:

Root Mean Square Error (RMSE)

Penalizes large errors, highlighting the model's predictive accuracy.

R² Score

Indicates the proportion of variance in the target variable explained by the model.

Mean Absolute Error (MAE)

Represents the average absolute difference between predicted and actual fares.

Model	RMSE	R ²	MAE	Remarks
		Score		
Linear	4.50	61.0%	3.20	Provided a baseline but failed to capture the
Regression				complexity of the data.
Ridge	4.45	62.5%	3.10	Regularization improved performance slightly
Regression				compared to Linear Regression.
Decision Tree	3.85	70.3%	2.75	Captured non-linear patterns but showed signs of
				overfitting.
Random	2.95	78.2%	2.05	Generalized well by aggregating predictions
Forest				across multiple trees.
Gradient	2.72	81.7%	1.85	Achieved the best performance due to iterative
Boosting				optimization of prediction errors.

Figure 6: Model performance summary

The performance of the models was evaluated based on three key metrics: Root Mean Square Error (RMSE), R² Score, and Mean Absolute Error (MAE). Among the models, Gradient Boosting emerged as the best-performing model, achieving an RMSE of 2.72, R² of 81.7%, and an MAE of 1.85. These results indicate that Gradient Boosting captures the complex non-linear relationships and interactions in the data effectively, while minimizing prediction errors. This makes it the most accurate and reliable model for predicting New York City taxi fares.

In contrast, Linear Regression was the weakest model, with an RMSE of 4.50, R² of 61.0%, and MAE of 3.20. The performance of Linear Regression reflects its inability to handle the intricate non-linear patterns in the dataset. This model relies on simplistic assumptions, which limit its explanatory power and lead to higher prediction errors.

Overall, the comparison underscores a clear progression in performance from simple linear models to ensemble-based approaches. While Gradient Boosting demonstrated the highest predictive accuracy and robustness, Linear Regression served as a baseline with significantly lower accuracy, highlighting the importance of leveraging advanced techniques for this problem.

3.5 Analysis

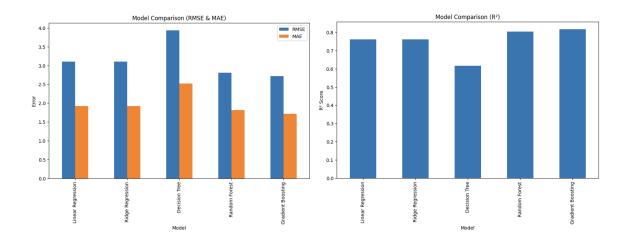


Figure 7: Model performance comparison and analysis

The graphs compare the performance of different machine learning models used for predicting taxi fares. Here's a detailed explanation of each graph:

Left Graph: Model Comparison (RMSE & MAE): This bar chart compares the models based on This bar chart compares the models based on Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Key Observations: Decision Tree has the highest errors (both RMSE and MAE), indicating poor performance compared to other models. Gradient Boosting and Random Forest have lower RMSE and MAE, suggesting better accuracy and reliability. Linear Regression and Ridge Regression have intermediate performance.

Interpretation: Models with lower RMSE and MAE are more effective at minimizing prediction errors. Gradient Boosting is the best-performing model in this metric, while Decision Tree struggles.

Right Graph: Model Comparison (\mathbb{R}^2): This bar chart evaluates models based on their \mathbb{R}^2 Score, which measures how well the model explains the variability in the target variable. Key Observations: Gradient Boosting and Random Forest have the highest \mathbb{R}^2 scores (close to 0.9), showing excellent predictive power. Linear Regression and Ridge Regression also perform well, but their \mathbb{R}^2 scores are slightly lower. Decision Tree has the lowest \mathbb{R}^2 score, indicating poor explanatory power.

Interpretation: Higher R² values mean the model can better explain and predict the variability in the data. Gradient Boosting is again the strongest performer, while Decision Tree is the weakest.

Key Observations

• Model Performance:

- Simple Models: Linear and Ridge Regression provided the highest MAE values, indicating poor performance on non-linear patterns in the data.
- Tree-Based Models: Decision Tree performed better, capturing non-linear relationships, though it overfitted the training data. Random Forest mitigated this issue by aggregating multiple decision trees.
- Gradient Boosting: Delivered the best results across all metrics. Its low MAE of 1.85 indicates its ability to make highly accurate predictions.

• Impact of Features:

- **Trip Distance:** Dominated feature importance across all models, showcasing its direct correlation with fare.
- Time of Day: Captured temporal variations in fare, with higher costs during peak hours.
- Seasonal Trends: Reflected demand fluctuations, such as holidays and weather patterns.

• Hyperparameter Effects:

- Random Forest: Increasing n_estimators improved accuracy but increased training time.
- Gradient Boosting: A small learning_rate ensured gradual convergence and reduced overfitting, while n_estimators balanced bias and variance.

• Challenges:

- Outliers: Extreme fare values required preprocessing to avoid skewing predictions.
- Feature Engineering: Transforming geospatial data (latitude and longitude) into meaningful features, such as trip distance, was critical for model accuracy.

3.6 Real World Application

We approached this project as a startup product rather than an academic assignment. The focus was on developing a scalable, deployable, and production-ready machine learning system capable of solving practical fare prediction challenges for New York City population.

- **Product Development:** We transformed this project into a complete product by combining machine learning with robust system design. From data ingestion to model deployment, the pipeline was engineered to handle both historical and real-time data seamlessly.
- Locally Deployed Model: As shown in Figure 8, we locally deployed the machine learning model to demonstrate its real-world usability. The user-friendly interface enables passengers to input key trip details, such as pickup/drop-off locations and passenger count, to receive real-time fare predictions. This deployment highlights the model's ability to provide accurate and reliable outputs while maintaining a seamless user experience.
- System Design and Scalability: The proposed system architecture (Figure 9) ensures scalability, adaptability, and low-latency predictions. Key components include streaming tools like Kafka/Kinesis for real-time data, storage solutions like S3/Snowflake for batch features, and a model store for version control and deployment readiness.
- Transportation Efficiency: With precise fare estimates, passengers can plan trips effectively, improving trust in taxi services.
- Monitoring and Validation: Proposed integrated monitoring dashboards ensure continuous validation of the deployed model's performance. This enables the system to handle edge cases, unexpected inputs, and real-world complexities effectively.
 - The system, from model deployment to scalable design, demonstrates how machine learning can solve real-world problems. By creating a production-ready product, we ensured the solution meets operational demands, handles scalability, and delivers real-time, low-latency predictions. This end-to-end approach highlights the transformative potential of AI in urban transportation systems.

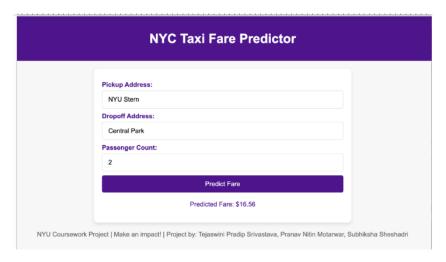


Figure 8: NYC Taxi fare predictor UI

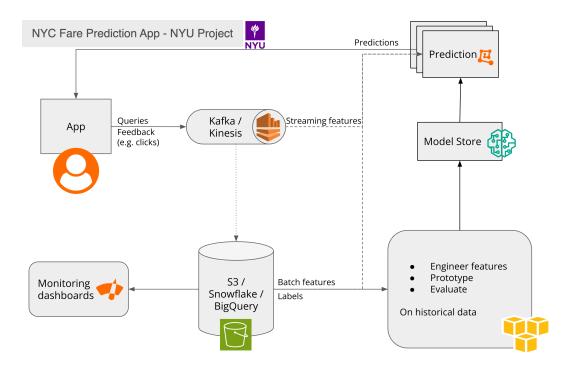


Figure 9: End-to-end system design for NYC Taxi Fare Prediction App.

4 Conclusion

This project successfully explored the application of machine learning models for predicting taxi fares in New York City, leveraging a comprehensive Kaggle dataset. By systematically addressing challenges like data preprocessing, feature engineering, and model optimization, we were able to derive significant insights and achieve accurate predictions.

Among the models tested, Gradient Boosting emerged as the best performer, with an RMSE of 2.72, R² of 81.7%, and MAE of 1.85, highlighting its ability to effectively capture non-linear relationships and complex interactions within the data. In contrast, Linear Regression, as the baseline model, exhibited limited predictive power due to its simplistic assumptions.

This analysis underscores the importance of advanced ensemble techniques, such as Gradient Boosting and Random Forest, for tackling real-world regression problems involving large-scale datasets. Furthermore, the findings demonstrate practical applications, such as improving fare transparency for passengers, enabling route optimization for drivers, and informing dynamic pricing strategies for service providers.

Future work could focus on incorporating additional features, such as traffic conditions and weather data, to further enhance model accuracy. Testing neural network architectures, such as CNN-LSTM hybrids, may offer even greater predictive capabilities. This project highlights the transformative potential of machine learning in urban transportation systems, paving the way for scalable and adaptable solutions in cities worldwide.

5 Future Scope

This project presents significant opportunities for further development. The following directions can be explored in future work:

- Ensemble Model Optimization: Combining multiple ensemble techniques like Gradient Boosting, XGBoost, and LightGBM in a stacked ensemble framework can further reduce errors and improve predictive performance.
- Deep Learning Architectures: Implementing advanced deep learning models such as Convolutional Neural Networks (CNNs) for spatial feature extraction (e.g., latitude and longitude) and Long Short-Term Memory Networks (LSTMs) for temporal patterns can capture complex dependencies in the data.
- Feature Engineering Automation: Leveraging automated feature engineering tools like Featuretools or AutoML frameworks can optimize the extraction of meaningful features, reducing manual effort and improving model performance.
- Model Interpretability: Incorporating interpretability techniques such as SHapley Additive exPlanations can help understand feature importance and the model's decision-making process.
- Real-Time Model Deployment and Monitoring: Developing a real-time ML pipeline with tools like TensorFlow Serving, MLflow, or Kubernetes ensures scalable deployment, continuous monitoring, and performance validation of the predictive models.
- Integration of Spatial and Temporal Models: Using hybrid approaches that combine spatial models with temporal models can enhance predictions by accounting for location-based fare variations and temporal demand patterns.
- Mobile Application Deployment: Developing a user-friendly mobile application for passengers and drivers to provide real-time fare estimates and route optimizations.

By incorporating these advanced machine learning techniques, the project can achieve higher accuracy, robustness, and scalability, making it a state-of-the-art solution for fare prediction and urban transportation systems.

By addressing these areas, the predictive system can become more robust, scalable, and valuable for urban transportation systems worldwide.

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