

6/11/24

$$f(x, y) = x + y$$

DNV kunnen we overzetten
in onze karnaugh diagram

x	y	$f(x, y)$	$x + y$
0	0	$f(0, 0)$	0
0	1	$f(0, 1)$	1
1	0	$f(1, 0)$	1
1	1	$f(1, 1)$	1

	\bar{y}	y
\bar{x}	$f(0, 0)$	$f(0, 1)$
x	$f(1, 0)$	$f(1, 1)$

	\bar{y}	y
\bar{x}	0	1
x	1	1

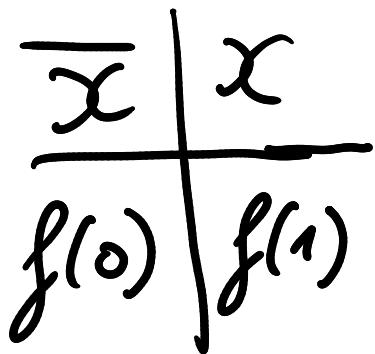
\Downarrow
 xy

$$f(x, y) = f(0, 0)\bar{x}\bar{y} + f(0, 1)\bar{x}y + f(1, 0)x\bar{y} + f(1, 1)xy$$

$$\begin{aligned} \text{DNV} &= 0 \cdot \bar{x}\bar{y} + 1 \cdot \bar{x}y + 1 \cdot x\bar{y} + 1 \cdot xy \\ &= \bar{x}y + x\bar{y} + xy \end{aligned}$$

$f(x)$

x	$f(x)$
0	$f(0)$
1	$f(1)$



$f(x, y, z)$

	\bar{z}	z
$\bar{x} \bar{y}$	1	0
$\bar{x} y$	0	0
$x y$	0	1
$x \bar{y}$	0	1

Rechteck setzen

$$\begin{aligned}f(x, y, z) &= xy\bar{z} + x\bar{y}z + \bar{x}\bar{y}\bar{z} \\&= xz + \bar{x}\bar{y}\bar{z}\end{aligned}$$

DNV

Dijunctive Normal Form

$$f(x, y, z, u) = \overbrace{xyzu} + \overbrace{\bar{x}\bar{y}\bar{z}} + \overbrace{x\bar{y}}$$

	$\bar{z}\bar{u}$	$\bar{z}u$	zu	$z\bar{u}$
$\bar{x}\bar{y}$	1	1	0	0
$\bar{x}y$	0	0	0	0
$x\bar{y}$	0	0	1	0
$x\bar{y}$	1	1	1	1

$$\begin{aligned} \bar{x}\bar{y}\bar{z} &= \bar{x}\bar{y}\bar{z} \cdot 1 \\ &= \bar{x}\bar{y}\bar{z}u + \bar{x}\bar{y}\bar{z}\bar{u} \end{aligned}$$

Wat niet DNV is, is wat je hebt + alle mogelijke combinaties

Oefeningen:

①

	\bar{y}	y
\bar{x}	1	1
x	1	0

$$f(x, y) = x + \bar{x}\bar{y}$$

$$\begin{aligned} DNU &= xy + x\bar{y} + \bar{x}\bar{y} \\ &= x + \bar{y} \end{aligned}$$

②

$$f(x, y, z) = (xyz) + (x\bar{y}z) + (\bar{x}\bar{y}\bar{z}) = DNU$$

Σ	z
$\bar{x}\bar{y}$	1
$\bar{x}y$	0
$x\bar{y}$	0
$x\bar{y}$	1

$$= \bar{x}\bar{y}\bar{z} + xz$$

Den bel 1, geen 0

- ② zo groot als mogelijk
- ③ aantal in rechthoek = macht van 2 of 1
- ④ ver lappen

Regels voor rechthoek

$$\hookrightarrow xz \text{ want } xy + x\bar{y} = x$$

$$③ f(x, y, z, u) = xyzu + \bar{x}\bar{y}\bar{z}u + x\bar{y}u$$

	$\bar{z}\bar{u}$	$\bar{z}u$	zu	$z\bar{u}$
$\bar{x}\bar{y}$	1	1	0	0
$\bar{x}y$	0	0	0	0
$x\bar{y}$	0	0	1	0
$x\bar{y}\bar{u}$	1	1	1	1

$\bar{z}u$ valt weg (complement)

y valt weg
(complement)

$$\text{DNV} = \bar{x}\bar{y}\bar{z}\bar{u} + \bar{x}\bar{y}\bar{z}u + xyzu + x\bar{y}\bar{z}u + x\bar{y}\bar{z}\bar{u} + x\bar{y}zu + x\bar{y}\bar{z}u$$

$$= x\bar{y} + \bar{y}\bar{z} + xzu$$

xu valt weg
(complement)

\Rightarrow Som van de
Recht hoeken

5.3 voorbeeld 3 : $f(x, y, z, u) = (y \cdot u) + (x \bar{y} z) + (x \bar{y}) + (\bar{x} \bar{y})$

$\bar{z} \bar{u}$	$\bar{z} u$	$z u$	$z \bar{u}$
$\bar{x} \bar{y}$	1	1	1
$\bar{x} y$	0	1	1
$x y$	0	1	1
$x \bar{y}$	1	1	1

$$DNF = (\bar{x} \bar{y} \bar{z} \bar{u}) + (\bar{x} \bar{y} \bar{z} u) + (\bar{x} \bar{y} z u) + (\bar{x} \bar{y} z \bar{u}) + (\bar{x} y \bar{z} u) + (\bar{x} y z \bar{u}) + (x y \bar{z} u) + (x \bar{y} z \bar{u}) + (x \bar{y} \bar{z} u)$$

$$+ (x \bar{y} \bar{z} \bar{u}) + (x \bar{y} z u) + (x y z \bar{u})$$

$$= u + \bar{y}$$

$$CNF = (x + \bar{y} + z + u) \cdot (x + \bar{y} + \bar{z} + u) \cdot (\bar{x} + \bar{y} + z + u) \cdot (\bar{x} + \bar{y} + \bar{z} + u)$$

$$= \bar{u} + \bar{y}$$

Voorbeeld 4

$$f(x, y, z, u) = \bar{y} \bar{z} \bar{u} + \bar{y} z \bar{u} + y \bar{z} \bar{u} + \bar{x} \bar{y} z u$$

	$\bar{z}\bar{u}$	$\bar{z}u$	zu	$z\bar{u}$
$\bar{x} \bar{y}$	1	0	1	1
$\bar{x} y$	—	0	0	0
$x y$	1	0	0	0
$x \bar{y}$	1	0	0	1

16 poorten

- 4 niet poorten
(elke niet letter man)
1 heen
- 9 en poorten
- 3 of poorten

DNU =

$$= (\bar{z}\bar{u}) + (\bar{y}\bar{u}) + (\bar{x}\bar{y}z) \rightarrow 10 \text{ poorten}$$

CNU =

$$= (z+u) \cdot (\bar{y}+\bar{z}) \cdot (\bar{x}+\bar{u}) \rightarrow 9 \text{ poorten}$$

↳ efficiëntste
↳ 4 niet + 3 of + 2 en
↳ 4 niet + 3 of + 1 en

$f(x, y, z, u, v)$

kan want met 5 of meer variabelen

	$\bar{u}\bar{v}$	$\bar{u}v$	uv	$u\bar{v}$
$\bar{x}\bar{y}\bar{z}$				
$\bar{x}\bar{y}z$				
$\bar{x}yz$				
$\bar{x}y\bar{z}$				
$x\bar{y}\bar{z}$				
$xy\bar{z}$				
$x\bar{y}z$				
$x\bar{\bar{y}}z$				
$x\bar{y}\bar{z}$				

vanaf \bar{x} is tijd om x te veranderen

5.3 oef 1

lossen we
na op

$$f(x,y,z) = xyz + \bar{x}\bar{y}z + x\bar{y}z + \bar{x}yz$$

$$\bar{f}(x,y,z) = xy\bar{z} + \bar{x}\bar{y}z + x\bar{y}z + \bar{x}yz$$

$$\begin{array}{c|cc} f(x,y,z) & \bar{z} & z \end{array}$$

$\bar{x}\bar{y}$	1	0
$\bar{x}y$	1	0
$x\bar{y}$	1	0
xy	1	0

$\bar{1}$ wordt 0
 $\bar{0}$ wordt 1

is
gelijk

$\bar{f}(x,y,z)$	\bar{z}	z
$\bar{x}\bar{y}$	0	1
$\bar{x}y$	0	1
$x\bar{y}$	0	1
xy	0	1

Om van \bar{f} naar f te gaan

oplossing DNV = \bar{z}

$$③ f(x,y,z,u,v) = x\bar{z}\bar{v} + z\bar{u}\bar{v} + z\bar{u}v + xy\bar{z}u$$

	$\bar{u}\bar{v}$	$\bar{u}v$	uv	$u\bar{v}$
$\bar{x}\bar{y}\bar{z}$	0	0	0	0
$\bar{x}\bar{y}z$	1	0	0	1
$\bar{x}yz$	1	0	0	1
$\bar{x}y\bar{z}$	0	0	0	0
$xy\bar{z}$	1	0	0	1
xyz	1	1	1	1
$x\bar{y}z$	1	0	0	1
$x\bar{y}\bar{z}$	1	0	0	1

$$\text{DNF} = (z\bar{v}) + (x\bar{v}) + (xyz)$$

$$⑥ f(x, y, z) = \underbrace{(x + \bar{y}) \cdot (x + z) \cdot (\bar{x} + \bar{z}) \cdot (\bar{y} + \bar{z})}_{CNU}$$

	\bar{z}	z	CNU
$\bar{x} \bar{y}$	0	1	
$\bar{x} y$	0	0	
$x y$	1	0	
$x \bar{y}$	1	0	

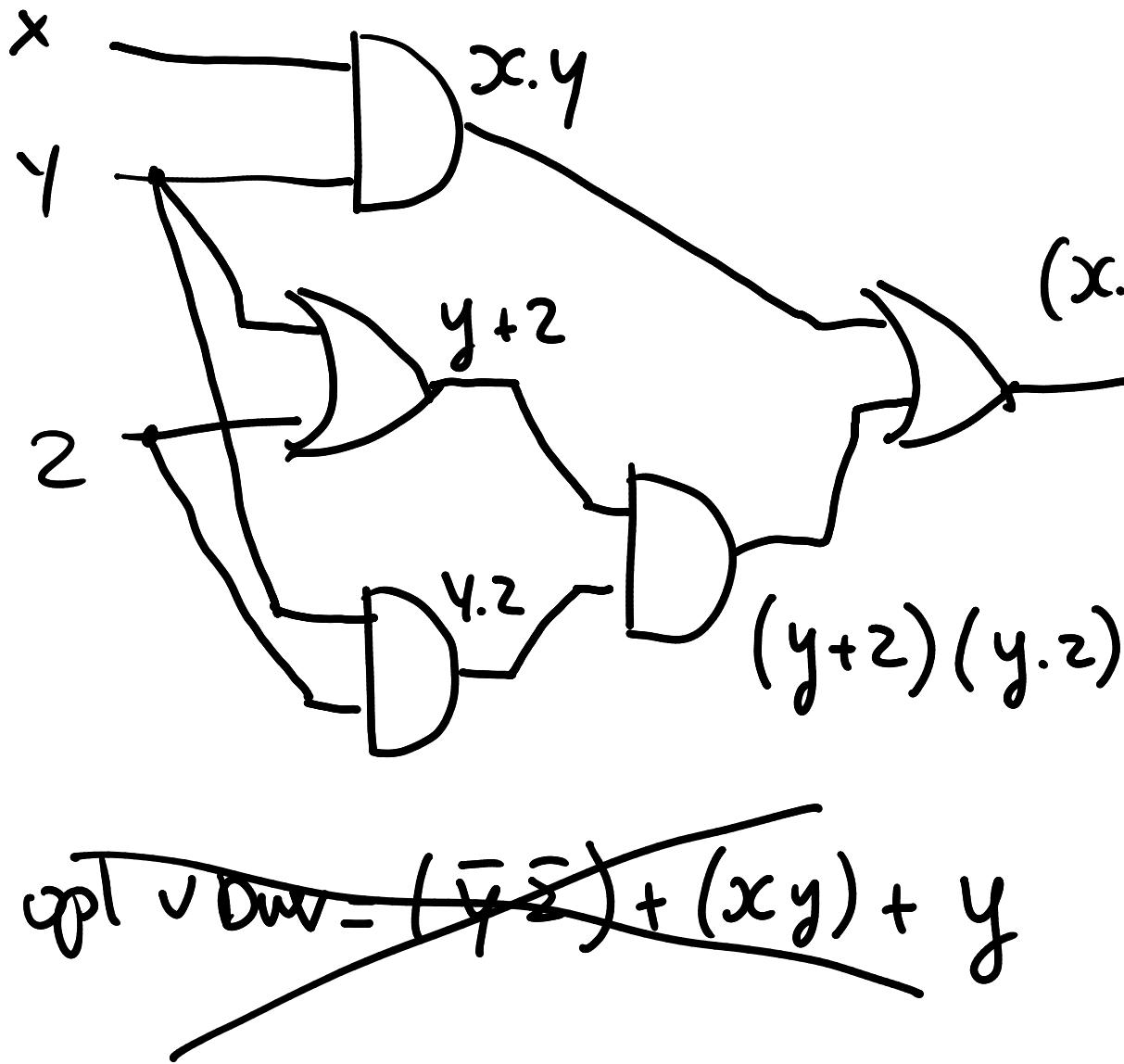
we zetten een ϕ waar het voor komt bij de complementen

$$\text{opl. DNV} = (x \bar{z}) + (\bar{x} \bar{y} z)$$

$$\text{opl. CNU} = (x + z) \cdot (x + \bar{y}) \cdot (\bar{x} + \bar{z})$$

$$5.4 \quad f(x, y, z) = (x \cdot y) + (y + z) + (y \cdot z)$$

DNF =



$$\text{opt DNF} = (\bar{x} \bar{z}) + (xy) + y$$

