Basic Relationships between Pixels-

Dr. N.Subhash Chandra

Outline:

- **Neighbourhood**
- **Adjacency**
- **Connectivity**
- **Paths**
- **Regions and boundaries**
- **Distance Measures**

Neighbors of a Pixel

1. **N_4 (p) : 4-neighbors of p.**
   - Any pixel p(x, y) has two vertical and two horizontal neighbors, given by **(x+1,y), (x-1, y), (x, y+1), (x, y-1)**
   - This set of pixels are called the **<u>4-neighbors</u>** of P**, and is denoted by** N_4(P)
   - Each of them is at a **<u>unit distance</u>** from P.

2. **N_D(p)**
   - This set of pixels, called 4-neighbors and denoted by N_D (p).
   - N_D(p): four diagonal neighbors of p have coordinates: **(x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1,y-1)**
   - Each of them are at **<u>Euclidean distance</u>** of **1.414** from P.

3. **N_8 (p):** 8-neighbors of p.

- $N_4(P)$ and $N_D(p)$ together are called 8-neighbors of p, denoted by $N_8(p)$.
- $N_8 = N_4 \cup N_D$
- Some of the points in the N_4 , N_D and N_8 may fall **outside** image when P lies on the **border** of image.

F(x-1, y-1)	F(x-1, y)	F(x-1, y+1)
F(x, y-1)	F(x,y)	F(x, y+1)
F(x+1, y-1)	F(x+1, y)	F(x+1, y+1)

N_8 (p)

Adjacency

- Two pixels are **connected** if they are neighbors and their gray levels satisfy some specified criterion of similarity.
- For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1)
- **Let v:** a set of intensity values used to ***define adjacency*** and ***connectivity***.
- In a **binary Image** **v={1},** if we are referring to adjacency of pixels with value 1.
- In a **Gray scale image**, the idea is the same, but **v** typically contains more elements, for example **v= {180, 181, 182, ,200}.**
- If the possible intensity values 0 to 255, **v** set could be any subset of these 256 values.

Types of adjacency

1. **4-adjacency:** Two pixels p and q with values from v are **4-adjacent** if q is in the set N_4 (p).

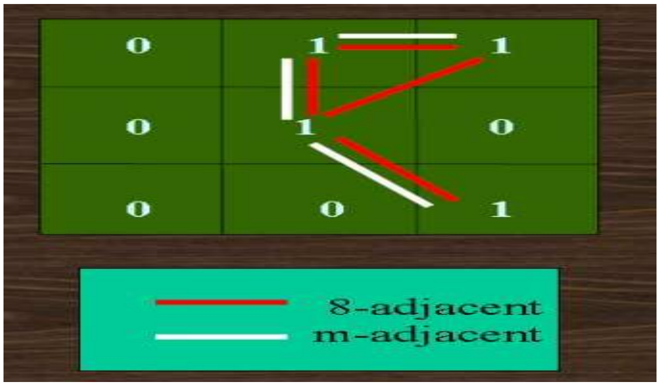
2. **8-adjacency:** Two pixels p and q with values from v are **8-adjacent** if q is in the set N_8 (p).

3. **m-adjacency** (mixed): two pixels p and q with values from v are **m-adjacent** if:

- q **is in** N_4 (p) or
- q **is in** N_D (P) and
- **The set** $N_4(p) \cap N_4(q)$ has no pixel whose values are from v (**No intersection**).

- **Mixed adjacency** is a modification of 8-adjacency "introduced to eliminate the ambiguities that often arise when 8- adjacency is used. (eliminate multiple path connection)
- Pixel arrangement as shown in figure for **v= {1}**

Example:



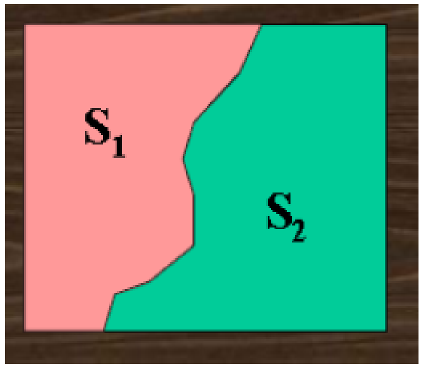
Path

- A ***digital path*** (or curve) from pixel p with coordinate (x,y) to pixel q with coordinate (s,t) is a sequence of ***distinct*** pixels with coordinates $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$, where $(x_0, y_0) = (x,y)$, $(x_n, y_n) = (s,t)$
- (x_i, y_i) is adjacent pixel (x_{i-1}, y_{i-1}) for $1 \le j \le n$,
- n- The ***length*** of the path.
- If $(x_0, y_0) = (x_n, y_n)$:the path is ***closed path***.

- We can define ***4- ,8- , or m-paths*** depending on the type of adjacency specified.

Connectivity

- Let S represent a subset of pixels in an image, Two pixels p and q are said to be connected in S if there exists a path between them.
- Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2



There are three types of connectivity on the basis of adjacency. They are:

a) 4-connectivity: Two or more pixels are said to be 4-connected if they are 4-adjacent with each others.

b) 8-connectivity: Two or more pixels are said to be 8-connected if they are 8-adjacent with each others.

c) m-connectivity: Two or more pixels are said to be m-connected if they are m-adjacent with each others.

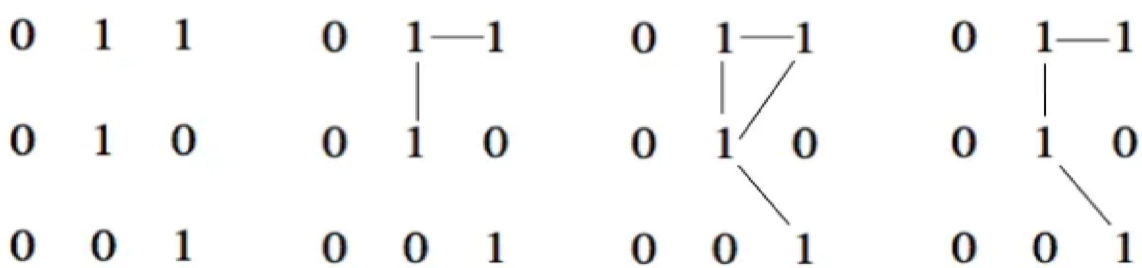


Fig: An arrangement of pixels **Fig: 4-connectivity of pixels** **Fig: 8-connectivity of pixels** **Fig: m-connectivity of pixels**

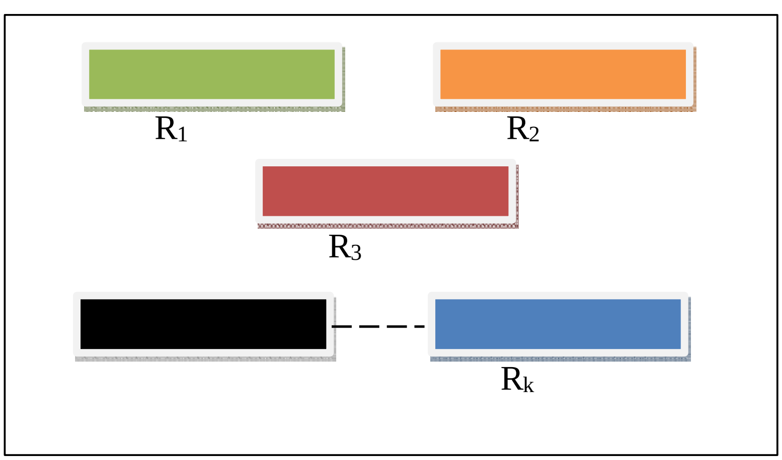
Region

- Let R to be a subset of pixels in an image, we call a R a region of the image. If R is a ***connected*** set.
- Region that are not adjacent are said to be **disjoint**.
- ***Example***: the two regions (of Is) in figure, are adjacent only if 8-adjacany is used.

1	1	1	
1	0	1	Ri
0	1	0	
0	0	1	
1	1	1	Rj
1	1	1	

- ***4-path*** between the two regions does not exist, (so their union in not a connected set).

- **Boundary (border)** image contains K disjoint regions, R_k, k=1, 2, ..., k, none of which touches the image border.

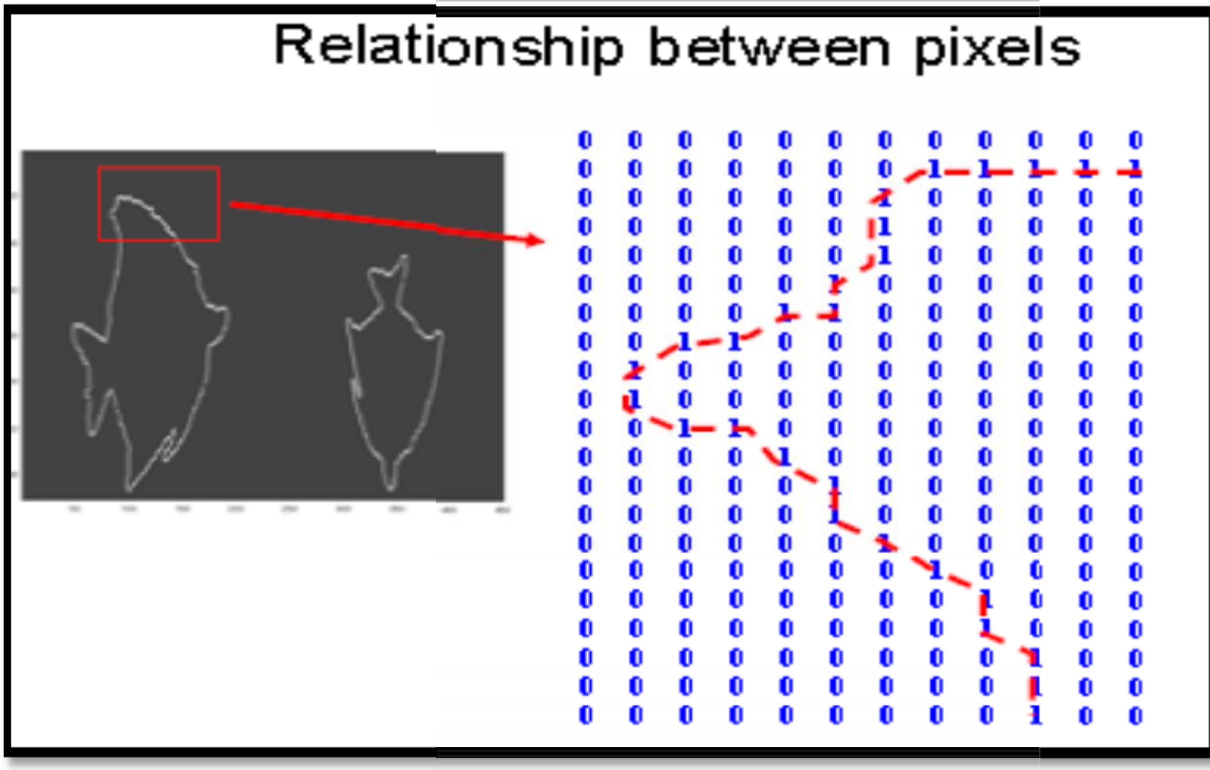


- **Let:** R_u - denote the **union** of all the K regions, $(R_u)^c$- denote its **complement**. (**Complement** of a set S is the set of points that are not in s).

R_u - called **foreground**; $(R_u)^c$ - called **background** of the image.

- **Boundary (border or contour)** of a region R is the set of points that are adjacent to points in the **complement** of R (another way: the border of a region is the set of pixels in the region that have at least are background neighbor).

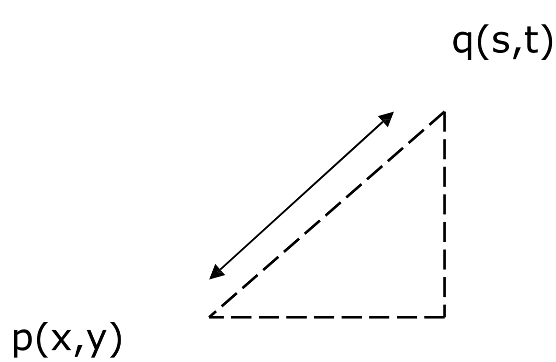
We must specify the connectivity being used to define adjacency



Distance Measures

For pixels p, q and z, with coordinates (x,y), (s,t) and (u,v), respenctively, D is a **distance function** or metric if:

D(p,q)≥0, D(p,q) = 0 if p=q
D(p,q) = D(q,p), and
D(p,z) ≤D(p,q) + D(q,z)



The following are the different *Distance measures*:

1. **Euclidean Distance (D_e)**

$$D_e(p,q) = \sqrt{[(x-s)^2 + (y-t)^2]}$$

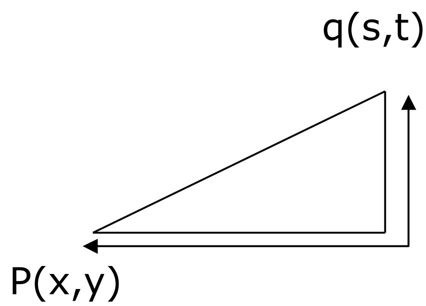
- The points contained in a **disk** of radius r centred at (x,y).

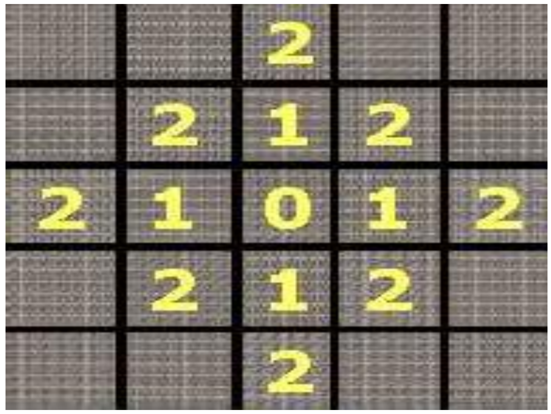
2. **D_4 distance (city-block distance)**

$$D_4(p,q) = |x-s| + |y-t|$$

- Pixels having a D_4 distance from (x,y) less than or equal to some value r form a

  Diamond centred (x,y),.





		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

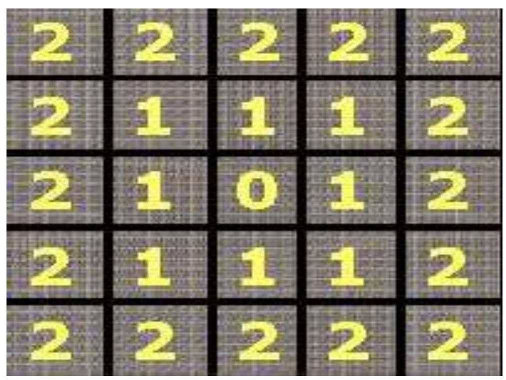
Example 1: the pixels with $D_4=1$ are the **4-nighbors** of (x, y).

3. D_8 distance (chess board distance)

$$D_8(p, q) = \max(|x - s|, |y - t|)$$

- **square** – centred at (x, y)
- $D_8 = 1$ are 8-neighbors of (x,y)

Example: D_8 **distance** $\leq$**2**



2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

4. D_m distance:

- Is defined as the **shortest m-path** between the points.
- The distance between pixels depends only on the values of pixels.

Example: consider the following arrangement of pixels

$$\begin{array}{ccc} & P_3 & P_4 \\ P_1 & P_2 & \\ P & & \end{array}$$

and assume that P, P_2 **have value 1 and that** P_1 **and** P_3 **can have a value of 0 or 1**

Suppose, that we consider adjacency of pixels value 1 (**v={1})**

a) ***if P_1 and P_3 are 0:***

Then D_m distance = 2

b) ***if $P_1 = 1$ and $P_3 = 0$***

m-distance = 3;

c) ***if $P_1=0$; and $P_3 = 1$***

d) if P1=P3 =1 ;

m-distance=4 path = p p_1 p_2 p_3 p_4