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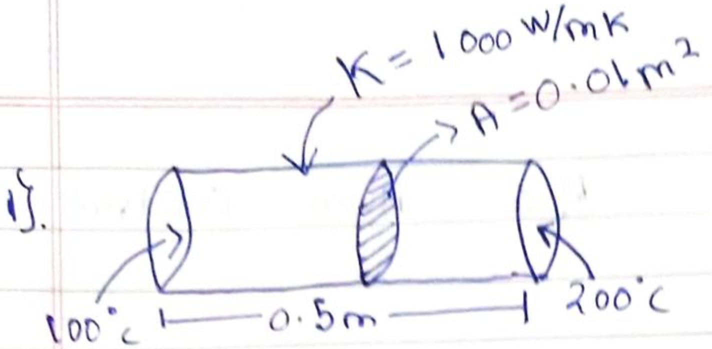
CFD

1)

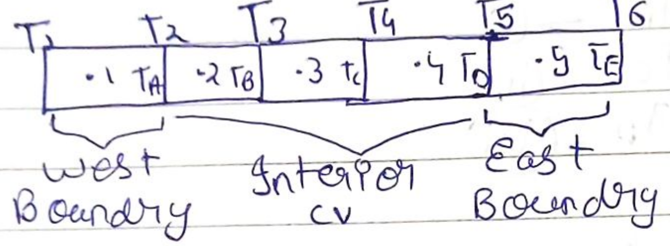


Find steady state temp distribution

Derive discretize equation for :- ① Interior CV ② West Boundry CV ③ East Boundry CV

$\Gamma = K = 1000\ W/mK$

$\Delta y \Delta z = \text{Area}$ $\qquad \Delta x = 0.5/5$



Interior Control Volume

·W	·P	E

$$\int_{cv} \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dv = 0$$

$$\int_{west}^{east} \int_{south}^{north} \int_{bot}^{top} \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dx\, dy\, dz = 0$$

$$\Delta y \Delta z \int_{w}^{e} \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dx = 0$$

$$\Gamma \frac{d\phi}{dx}\Big|_e \Delta y \Delta z - \Gamma \frac{d\phi}{dx}\Big|_w \Delta y \Delta z = 0$$

$$\Gamma_e \left(\frac{\phi_E - \phi_P}{\Delta x_{PE}}\right) \Delta y \Delta z - \Gamma_w \left(\frac{\phi_P - \phi_W}{\Delta x_{PW}}\right) \Delta y \Delta z = 0$$

$$\underbrace{\Gamma\left(\frac{\Delta y \Delta z}{\Delta x}\right)}_{A_E} \phi_E + \underbrace{\Gamma\left(\frac{\Delta y \Delta z}{\Delta x}\right)}_{A_W} \phi_W = \underbrace{\left[\Gamma\left(\frac{\Delta y \Delta z}{\Delta x}\right) + \Gamma\left(\frac{\Delta y \Delta z}{\Delta x}\right)\right]}_{A_P} \phi_P$$

$$A_E \phi_E + A_W \phi_W = A_P \phi_P$$

$; A_P = A_E + A_W$

$\Gamma = k = 1000\ W/mK \qquad \Delta y \Delta z = \text{Area} = 0.01 m^2$

$\Delta x = \frac{0.5}{5} = 0.1 m$

$A_E = \frac{\Gamma \Delta y \Delta z}{\Delta x} = 100 \qquad A_W = 100 \qquad A_P = 200$

$\therefore \boxed{100\phi_E + 100\phi_W = 200\phi_P}$

$\phi_E + \phi_W = 2\phi_P$

$\therefore T_C + T_A = 2T_B$ — (i)

$T_D + T_B = 2T_C$ — (ii)

$T_C + T_E = 2T_D$ — (iii)

-> west Boundry

week Boundry

$$\int \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dV = 0$$

$$\int_{west}^{east\ CV} \int_{sout}^{north} \int_{b}^{t} \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dx\, dy\, dz = 0$$

$$\Delta y \Delta z \int_w^e \frac{\partial}{\partial x}\left(\Gamma \frac{d\phi}{dx}\right) dx = 0$$

$$\left(\Gamma \frac{d\phi}{dx}\right)_e \Delta y \Delta z - \left(\Gamma \frac{d\phi}{dx}\right)_w \Delta y \Delta z = 0$$

$$\therefore \frac{\Gamma \Delta y \Delta z}{\Delta x}\phi_e - \frac{\Gamma \Delta y \Delta z}{\Delta x}\phi_P - \frac{2\Gamma \Delta y \Delta z}{\Delta x}\phi_P + \frac{2\Gamma \Delta y \Delta z}{\Delta x}\phi_w = 0$$

$$\therefore \underbrace{\frac{\Gamma \Delta y \Delta z}{\Delta x}}_{A_E}\phi_e + \underbrace{\frac{2\Gamma \Delta y \Delta z}{\Delta x}}_{A_W}\phi_w = \left[\underbrace{\frac{2\Gamma \Delta y \Delta z}{\Delta x}}_{A_W} + \underbrace{\frac{\Gamma \Delta y \Delta z}{\Delta x}}_{A_E}\right]\phi_P$$

$$A_E \phi_E + A_W \phi_W = A_P \phi_P$$

$; A_P = A_E + A_W \qquad \Gamma = 1000\ W/mK \qquad \Delta y \Delta z = 0.01 m^2 \quad \Delta x = 0.1 m$

$A_E = \frac{1000 \times 0.01}{0.1} = 100 \qquad A_E = \frac{2 \times 1000 \times 0.01}{0.1} = 200$

$A_P = 300 \qquad 100\phi_E + 200\phi_W = 300\phi_P$

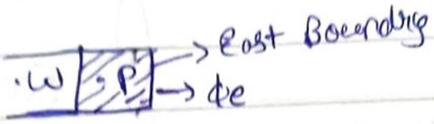
$$\boxed{\phi_E + 2\phi_W = 3\phi_P}$$

Given

$\phi_w = 100^\circ C$

$T_B + 2\times 100 = 3T_A \Rightarrow T_B + 200 = 3T_A$

→ East Boundary



$$\int_{cv} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dV = 0$$

$$\int_{west}^{east}\int_{south}^{north}\int_{b}^{t} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dx\, dy\, dz = 0$$

$$\Delta y \Delta z \int_{w}^{e} \frac{d}{dx}\left(\Gamma \frac{d\phi}{dx}\right) dx = 0$$

$$\frac{\Gamma \Delta y \Delta z\,(\phi_e - \phi_P)}{\Delta x/2} - \cancel{2}\frac{\Gamma \Delta y \Delta z}{\Delta x}(\phi_P - \phi_w)$$

$$\underbrace{\frac{2\Gamma \Delta y \Delta z}{\Delta x}}_{A_E}\phi_e + \underbrace{\frac{\Gamma \Delta y \Delta z}{\Delta x}}_{A_W}\phi_w = \left[\underbrace{\frac{2\Gamma \Delta y \Delta z}{\Delta x}}_{A_E} + \underbrace{\frac{\Gamma \Delta y \Delta z}{\Delta x}}_{A_W}\right] A_P$$

$$A_E = \frac{2\Gamma \Delta y \Delta z}{\Delta x} \qquad A_W = \frac{\Gamma \Delta y \Delta z}{\Delta x} \qquad A_P = A_E + A_W$$

$$A_E \phi_e + A_W \phi_w = A_P \phi_P$$

$$A_E = \frac{2\times 1000 \times 0.1}{0.1} = 200 \qquad A_W = 100 \qquad A_P = A_E + A_W = 300$$

$$2\phi_e + \phi_w = 3\phi_P$$

we Know that. $\phi_e = 200^\circ C$

$\therefore \quad 2\times 200 + T_D = 3T_E$

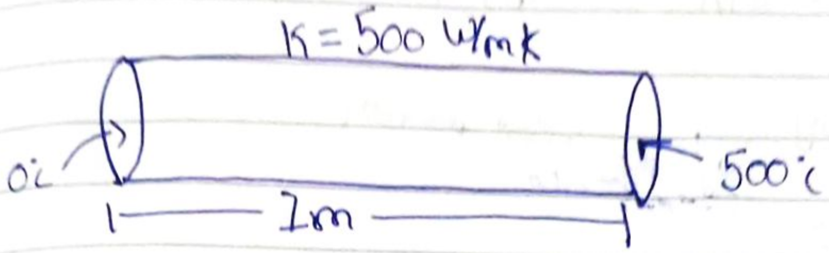
$\therefore \quad 400 + T_D = 3T_E$

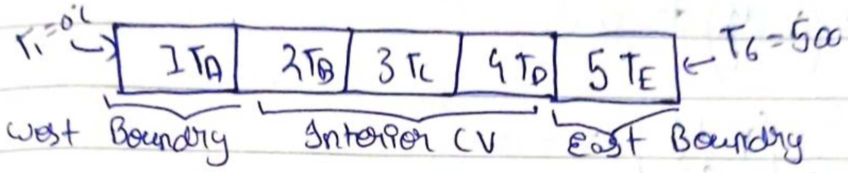
Now we have,

$$\left.\begin{aligned} T_B + 200 &= 3T_A \\ T_C + T_A &= 2T_B \\ T_D + T_B &= 2T_C \\ T_C + T_E &= 2T_D \\ 400 + T_D &= 3T_E \end{aligned}\right\} \begin{array}{l}\text{5 equations} \\ \text{5 variables}\end{array}$$

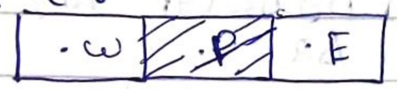
$\therefore T_A = 110°C \quad T_B = 130°C \quad T_C = 150°C \quad T_D = 170°C \quad T_E = 190°C$

2).





For interior CV



$A_E\phi_E + A_W\phi_W = A_P\phi_P$ {Derived Before} ; $A_E = \frac{\Gamma \Delta y \Delta z}{\Delta x}$ $\quad A_W = \frac{\Gamma \Delta y \Delta z}{\Delta x}$ $\quad A_P = A_E + A_W$

$\Gamma = K = 500$ W/mK $\qquad \Delta y \Delta z$ = Area = $0.01 m^2$

$\Delta x = 1/5 = 0.2m$

$A_E = \frac{500 \times 0.01}{0.2} = 25$. $\quad A_W = 25 \quad A_P = A_E + A_W = 50$

$25\phi_E + 25\phi_W = 50\phi_P$

$\phi_E + \phi_W = 2\phi_P$

$\therefore T_C + T_A = 2T_B$ —(1)

$T_D + T_B = 2T_C$ —(2)

$T_E + T_C = 2T_D$ —(3)

For West Boundry.

$A_E\phi_E + A_W\phi_W = A_P\phi_P$ {Derived before}

$A_E = \frac{\Gamma \Delta y \Delta z}{\Delta x}$ $\quad A_W = \frac{2\Gamma \Delta y \Delta z}{\Delta x}$, $A_P = A_E + A_W$

$A_E = \frac{500 \times 0.01}{0.1} = 25$ $\quad A_W = \frac{2 \times 500 \times 0.01}{0.2} = 50$

$A_E + A_W = A_P = 75$

$25\phi_E + 50\phi_W = 75\phi_P$

$\therefore \phi_E + 2\phi_W = 3\phi_P$

$\phi_W = 0^\circ C$

$\therefore \quad T_B + 2\times 0^\circ C = 3T_A$

$T_B = 3T_A \quad$ — (iv)

For East boundry

$A_E\phi_E + A_W\phi_W = A_P\phi_P$

$A_E = \dfrac{2\gamma\Delta y\Delta z}{\Delta x} \qquad A_W = \dfrac{\gamma\Delta z\Delta y}{\Delta x} \qquad A_P = A_W + A_E$

$A_E = \dfrac{2\times 500\times 0.01}{0.2} = 50 \qquad A_W = 25 \qquad A_P = 75$

$50\phi_E + 25\phi_W = 75\phi_P$

$2\phi_E + \phi_W = 3\phi_P$

we know that,

$\phi_E = 500^\circ C$

$2\times 500 + T_D = 3T_E \quad$ — (v)

Now we have 5 ~~equilibrium~~ equations & variables

$T_B = 3T_A \qquad T_C + T_A = 2T_B \qquad T_D + T_B = 2T_C \qquad T_E + T_C = 2T_D$

$1000 + T_D = 3T_E$

$\therefore T_A = 50^\circ C \quad T_B = 150^\circ C \quad T_C = 250^\circ C \quad T_D = 350^\circ C \quad T_E = 450^\circ C$