Chapter 3 Resistive Circuits

Exercises

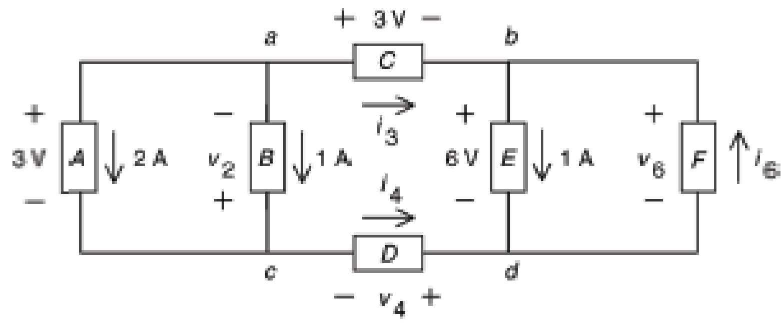


Figure E 3.2-1

Exercise 3.2-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.2-1.

Answer: $i_3 = -3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = -3$ V, $v_4 = -6$ V, $v_6 = 6$ V

Solution:

Apply KCL at node *a* to get $2 + 1 + i_3 = 0 \Rightarrow i_3 = -3$ A

Apply KCL at node *c* to get $2 + 1 = i_4 \Rightarrow i_4 = 3$ A

Apply KCL at node *b* to get $i_3 + i_6 = 1 \Rightarrow -3 + i_6 = 1 \Rightarrow i_6 = 4$ A

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 - 3 = 0 \Rightarrow v_2 = -3 \text{ V}$$

Apply KVL to the loop consisting of elements *C*, *E*, *D*, and *A* to get

$$3 + 6 + v_4 - 3 = 0 \Rightarrow v_4 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements *E* and *F* to get

$$v_6 - 6 = 0 \Rightarrow v_6 = 6 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(3)(2) + (-3)(1) - (3)(-3) + (-6)(3) - (6)(1) + (6)(4) = -6 - 3 + 9 - 18 - 6 + 24 = 0$$

Exercise 3.3-1 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-1*a*.
Hint: Figure E3.3-1*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer : $v_m = 2$ V

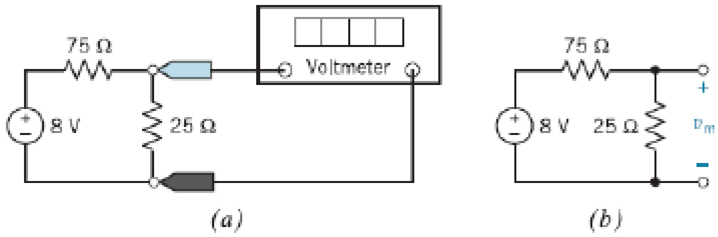


Figure E 3.3-1

Solution: From voltage division $\Rightarrow v_m = \frac{25}{25+75}(8) = 2$ V

Exercise 3.3-2 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.
Hint: Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer: $v_m = -2$ V

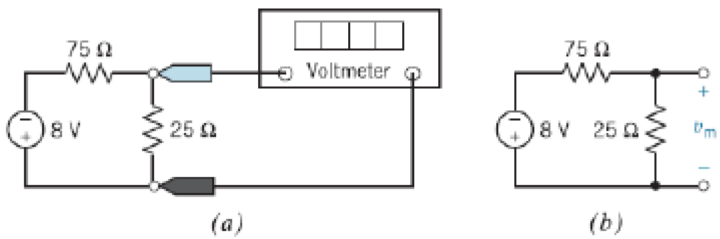
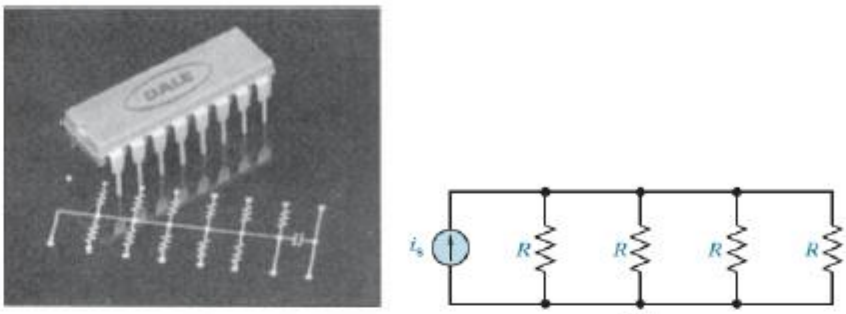


Figure E 3.3-2

Solution: From voltage division $\Rightarrow v_m = \frac{25}{25+75}(-8) = -2$ V

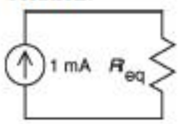
Exercise 3.4-1 A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.4-1*a*. This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.4-1*b*. Find the equivalent circuit for this network. Determine the current in each resistor when i_s = 1 mA.
Answer: $R_p = 250\ \Omega$



(*a*) (*b*)

Figure E 3.4-1

Solution:

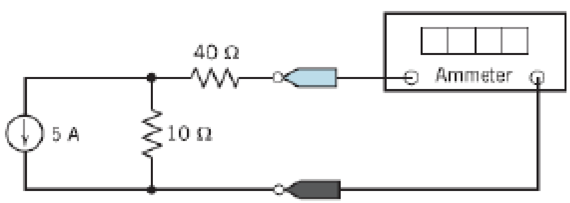


$$\frac{1}{R_{eq}} = \frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3}=\frac{4}{10^3} \Rightarrow R_{eq} = \frac{10^3}{4} = \underline{\frac{1}{4}\ \text{k}\Omega}$$

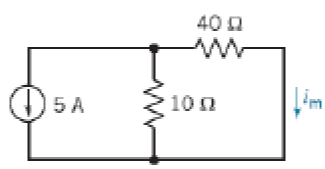
By current division, the current in each resistor $= \frac{1}{4}(10^{-3}) = \underline{\frac{1}{4}\ \text{mA}}$

Exercise 3.4-2 Determine the current measured by the ammeter in the circuit shown in Figure E 3.4-2*a*.

Hint: Figure E 3.4-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.
Answer: $i_m = -1$ A



(*a*)



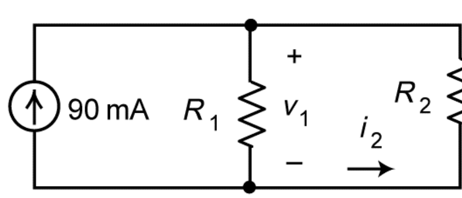
(*b*)

Figure E 3.4-2

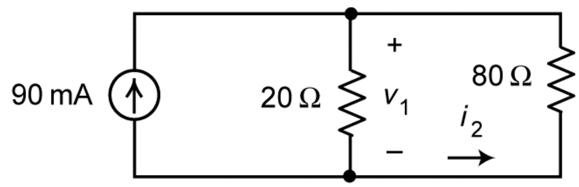
Solution:
From current division

$$i_m = \frac{10}{10+40}(-5) = -1\ \text{A}$$

EXERCISE 3.4-3 Determine the value of the current i_2 and the value of the voltage v_1 in the circuit shown in Figure E 3.4-3, where, R_1 = 20 Ω and R_2 = 80 Ω.



Solution:



The resistors are connected in parallel. Use current division to write

$$i_2 = -\frac{20}{80+20}\left(90\times10^{-3}\right) = -0.018\text{ A} = -18\,\text{mA}$$

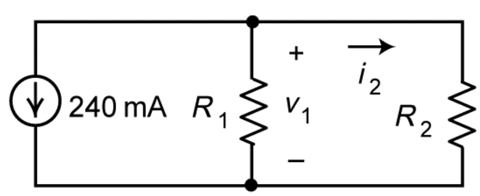
v_1 is the voltage across the parallel resistors. Consequently

$$v_1 = \left(20\,||\,80\right)90\,\text{mA} = \frac{20\left(80\right)}{20+80}\left(90\times10^{-3}\right) = 1.44\text{ V}$$

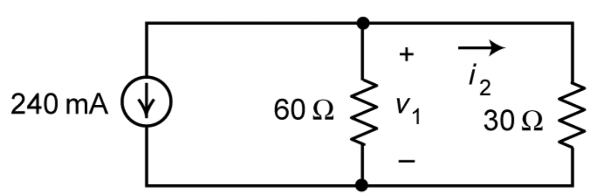
As a check, we can calculate v_1 as

$$v_1 = 20\left(\frac{80}{80+20}90\times10^{-3}\right) = 20\left(72\times10^{-3}\right) = 1.44\,\text{V}$$

EXERCISE 3.4-4 Determine the value of the current i_2 and the value of the voltage v_1 in the circuit shown in Figure E 3.4-4, where, R_1 = 60 Ω and R_2 = 30 Ω.



Solution:



The resistors are connected in parallel. Use current division to write

$$i_2 = -\frac{60}{60+30}\left(240\times10^{-3}\right) = -0.08\text{ A} = -80\,\text{mA}$$

v_1 is the voltage across the parallel resistors. Consequently

$$v_1 = -(60 \| 30)\,240\,\text{mA} = -\frac{60(30)}{60+30}\left(240\times 10^{-3}\right) = -4.8\ \text{V}$$

As a check, we can calculate v_1 as

$$v_1 = -60\left(\frac{30}{60+30}\,240\times 10^{-3}\right) = -60\left(80\times 10^{-3}\right) = -4.8\,\text{V}$$

EXERCISE 3.5-1 Determine the value of the current i_2 and the value of the voltage v_1 in the circuit shown in Figure E 3.5-1, where i_1 = 40 mA and R_2 = 20 Ω.

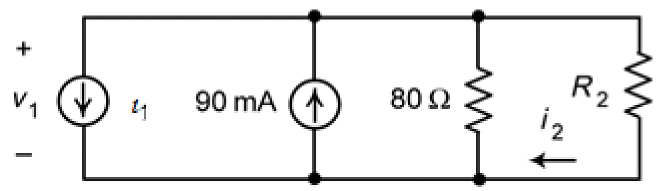
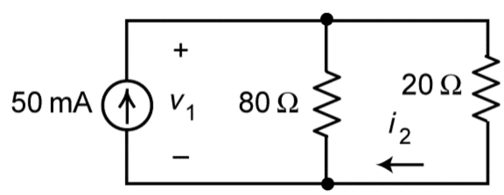


FIGURE E 3.5-1

Answer: i_2 = 40 mA and v_1 = 800 mV

Solution: The current sources are connected in parallel. Replace them by an equivalent current source to obtain



Notice that v_1 is the voltage across the equivalent current source. Use current division to write

$$i_2 = \frac{80}{80+20}50 = 40\,\text{mA}$$

The resistors are connected in parallel and v_1 is the voltage across the parallel resistors. Consequently

$$v_1 = (80 \| 20)\,50\,\text{mA} = \frac{80(20)}{80+20}\,50\times 10^{-3} = 0.8\ \text{V} = 800\ \text{mV}$$

As a check, we can calculate i_2 as

$$i_2 = \frac{v_1}{20} = \frac{0.8}{20} = 0.04 = 40\,\text{mA}$$

EXERCISE 3.5-2 Determine the values of the voltages v_1 and v_2 in the circuit shown in Figure E3.5-2.

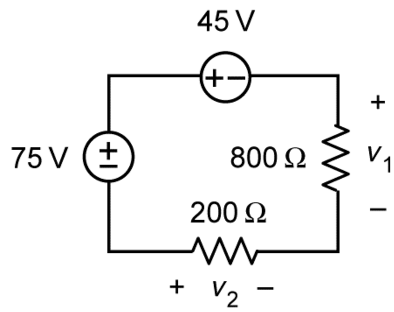
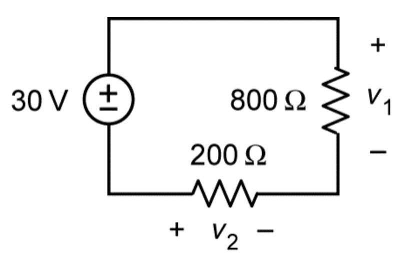


FIGURE E 3.5-2

Answer: v_1 = 24 V and v_2 = –6 V

Solution: The voltage sources are connected in series. Replace them by an equivalent voltage source to obtain



Use voltage division twice to write

$$v_1 = \frac{800}{800+200} 30 = 24\,\mathrm{V} \quad \text{and} \quad v_2 = -\frac{200}{800+200} 30 = -6\,\mathrm{V}$$

EXERCISE 3.5-3 Determine the values of the voltages v_1 and v_2 in the circuit shown in Figure E 3.5-3.

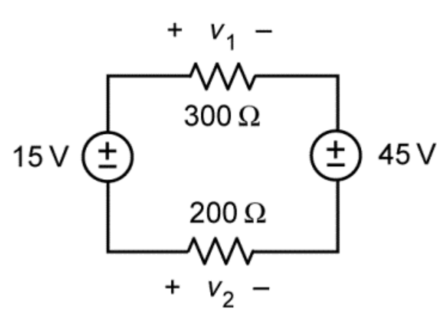


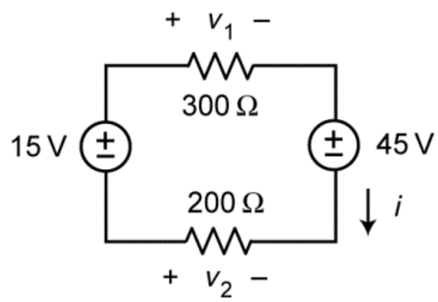
FIGURE E 3.5-3

Answer: v_1 = –18 V and v_2 = 12 V

Solution: The voltage sources and resistors are connected in series. Apply Kirchhoff's Voltage Law to obtain

$$v_1 + 45 - v_2 - 15 = 0 \quad \Rightarrow \quad v_1 + 30 - v_2 = 0$$

Label the current in these series elements to obtain



Apply Ohm's law twice to obtain

$$300i + 30 + 200i = 0 \quad \Rightarrow \quad i = -\frac{30}{500} = -0.06 \text{ A}$$

Apply Ohm's law twice more to obtain

$$v_1 = 300i = 300(-0.6) = -18 \text{ V} \quad \text{and} \quad v_2 = -200i = -200(-0.6) = 12 \text{ V}$$

EXERCISE 3.5-4 Determine the values of the voltages v_1 and v_2 in the circuit shown in Figure E 3.5-4.

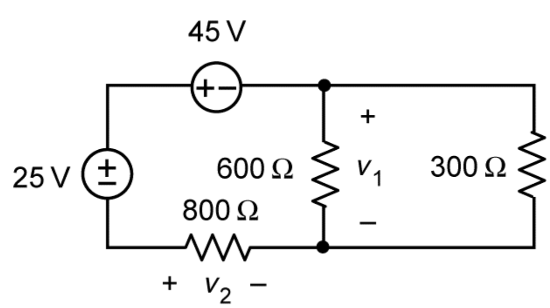
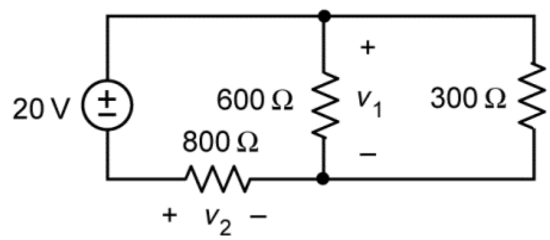


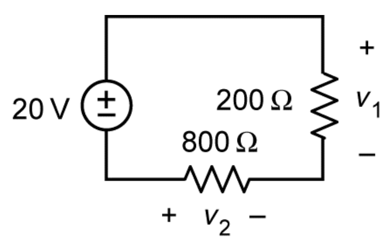
Figure E 3.5-4

Answer: v_1 = 4 V and v_2 = –16 V

Solution: The voltage sources are connected in series. Replace the series voltage sources by an equivalent voltage source to obtain



The 600 Ω and 300 Ω resistors are connected in parallel. Replace the parallel resistors by an equivalent resistor to obtain



Use voltage division twice to write

$$v_1 = \frac{200}{800+200}20 = 4\,\text{V} \quad \text{and} \quad v_2 = -\frac{800}{800+200}20 = -16\,\text{V}$$

EXERCISE 3.6-1 In the circuit shown in Figure E3.6-1 below, R_1 = 500 Ω and R_2 = 120 Ω. Determine the value of resistance measured by the ohmmeter.

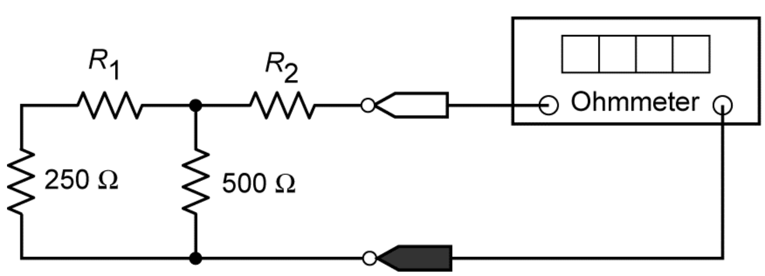
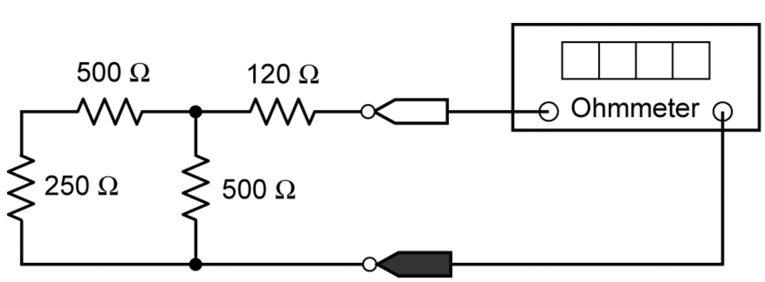
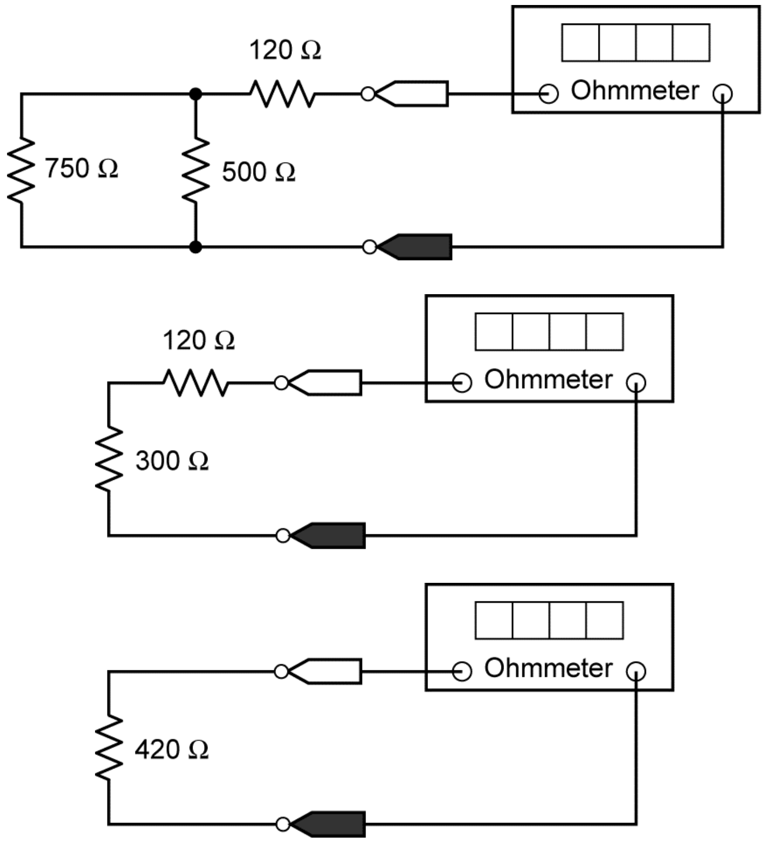


FIGURE E 3.6-1

Answer: 420 Ω

Solution:





EXERCISE 3.6-2 In the circuit shown in Figure E 3.6-2, R_1 = 200 Ω and R_2 = 50 Ω. Determine the value of resistance measured by the Ohmmeter.

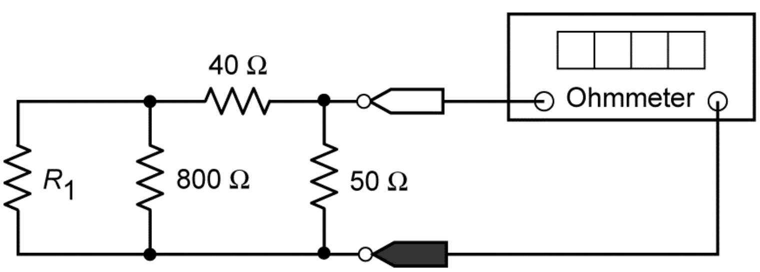
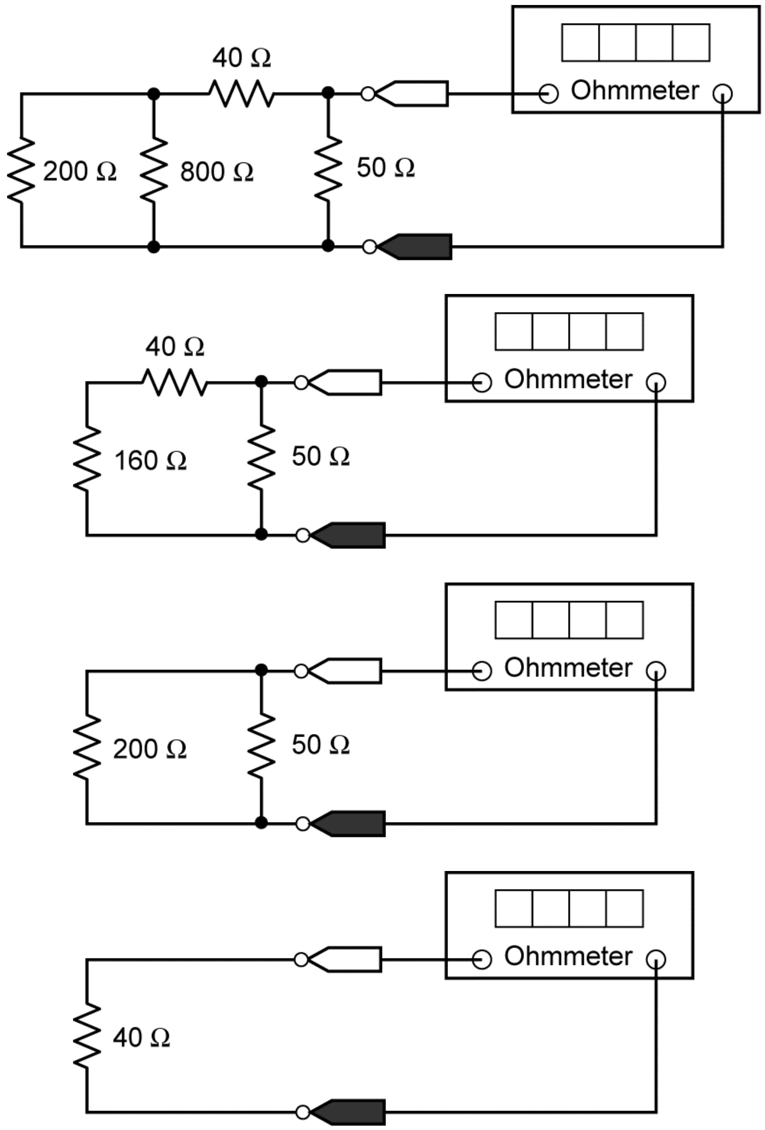


FIGURE E 3.6-2

Answer: 40 Ω

Solution:

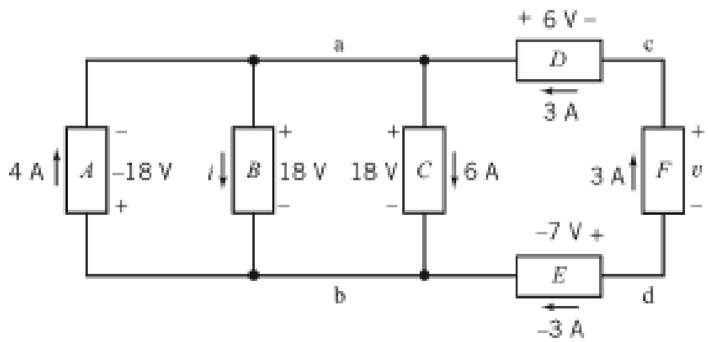


40 Ω
200 Ω
800 Ω
50 Ω
Ohmmeter
40 Ω
160 Ω
50 Ω
Ohmmeter
200 Ω
50 Ω
Ohmmeter
40 Ω
Ohmmeter

Section 3-2 Kirchhoff's Laws

P 3.2-1

Solution:



Apply KCL at node a to get $4+3=i+6 \Rightarrow i=1\text{A}$

The current and voltage of element B adhere to the passive convention so $(18)(1) = 18$ W is power supplied 1 by element B. The power received by element B is $\underline{-18\text{ W}}$.

Apply KVL to the loop consisting of elements D, F, E, and C to get

$$6+v+(-7)-18=0 \Rightarrow v=19\text{ V}$$

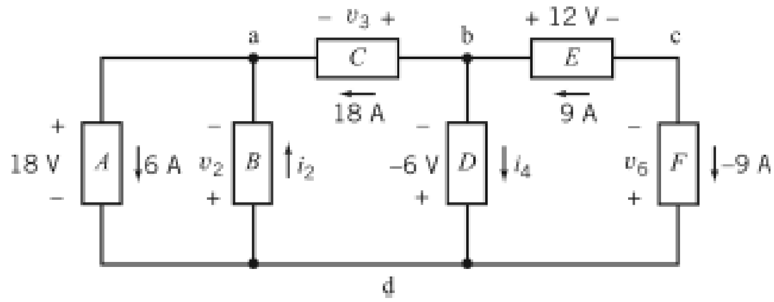
The current and voltage of element F do not adhere to the passive convention so $(19)(3) = \underline{57\text{ W}}$ is the power supplied by element F.

Check: The sum of the power supplied by all branches is

$$-(4)(-18)-\underline{18}-(6)(18)+(3)(6)+\underline{57}-(-3)(-7)=72-18-108+18+57-21=0$$

P 3.2-2

Solution:



Apply KCL at node *a* to get $6 = i_2 + 18 \Rightarrow i_2 = -12\text{A}$

Apply KCL at node *b* to get $9 = i_4 + 18 \Rightarrow i_4 = -9\text{A}$

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 - 18 = 0 \Rightarrow v_2 = -18\text{V}$$

Apply KVL to the loop consisting of elements *C*, *D*, and A to get

$$-v_3 - (-6) - 18 = 0 \Rightarrow v_3 = -12\text{V}$$

Apply KVL to the loop consisting of elements *E*, *F* and *D* to get

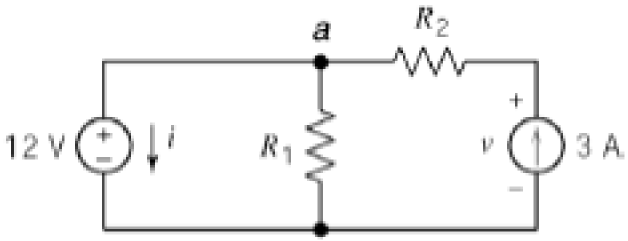
$$12 - v_6 + (-6) \Rightarrow v_6 = 6\text{V}$$

Check: The sum of the power supplied by all branches is

$$-(18)(6) - (-18)(-12) - (-12)(18) + (-6)(-9) + (12)(9) + (6)(-9) = -108 - 216 + 216 + 54 + 108 - 54 = 0$$

P 3.2-3

Solution:



KVL : $-12 - R_2(3) + v = 0$ (outside loop)

$$v = 12 + 3R_2 \quad \text{or} \quad R_2 = \frac{v-12}{3}$$

KCL $\quad i + \frac{12}{R_1} - 3 = 0$ (top node)

$$i = 3 - \frac{12}{R_1} \quad \text{or} \quad R_1 = \frac{12}{3-i}$$

(a) $\quad v = 12 + 3(5) = 27 \text{ V} \quad \text{and} \quad i = 3 - \frac{12}{10} = 1.8 \text{ A}$

(b) $\quad R_2 = \frac{45-12}{3} = \underline{11\,\Omega}\,;\ R_1 = \frac{12}{3-2.75} = \underline{48\,\Omega}$

(c) $\quad 25 = -12\,i$, because 12 and i adhere to the passive convention.

$\therefore \underline{i = -2.08 \text{ A}}$ and $R_1 = \frac{12}{3+2.08} = \underline{2.36\,\Omega}$

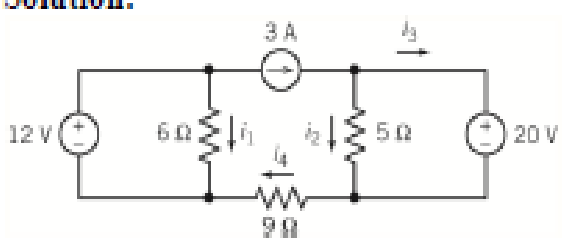
$10 = 3v$, because 3 and v do not adhere to the passive convention

$\therefore \underline{v = 3.3 \text{ V}}$ and $R_2 = \frac{3.3-12}{3} = \underline{-2.9\,\Omega}$

The situations described in (b) and (c) cannot occur if R_1 and R_2 are required to be nonnegative.

P 3.2-4

Solution:



$$i_1 = \frac{12}{6} = 2\text{A}$$

$$i_2 = \frac{20}{5} = 4\text{A}$$

$$i_3 = 3 - i_2 = -1\text{A}$$

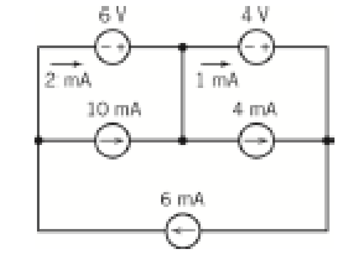
$$i_4 = i_2 + i_3 = 3\text{A}$$

Power absorbed by the 5Ω resistor $= 5 \cdot i_2^2 = \underline{80 \text{ W}}$

Power absorbed by the 6Ω resistor $= 6 i_1^2 = \underline{24 \text{ W}}$

Power absorbed by the 9Ω resistor $= 9 \cdot i_4^2 = \underline{81 \text{ W}}$

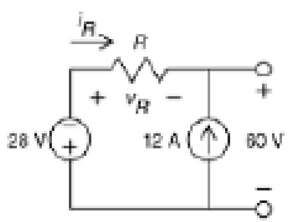
P 3.2-5
Solution:



$$P_{2\,\text{V}} = +\left[4\times\left(1\times10^{-3}\right)\right] = 4\times10^{-3} = 4\ \text{mW}$$
$$P_{3\,\text{V}} = +\left[6\times\left(-2\times10^{-3}\right)\right] = -12\times10^{-3} = -12\ \text{mW}$$

P 3.2-6

Solution:

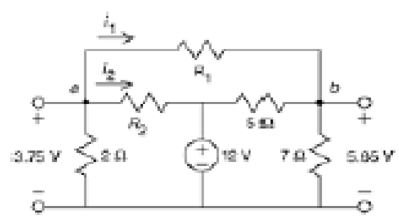


KVL: $v_R + 60 + 28 = 0 \;\Rightarrow\; v_R = -88\ \text{V}$

KCL: $i_R + 12 = 0 \;\Rightarrow\; i_R = -12\ \text{A}$

$$\therefore\quad R = \frac{v_R}{i_R} = \frac{-88}{-12} = 7.3\ \Omega$$

P 3.2-7
Solution:



$$\frac{5.65}{7} = \frac{3.75-5.65}{R_1} + \frac{12-5.65}{5} \;\Rightarrow\; 0.807 = \frac{-1.9}{R_1} + 1.278$$

KCL at node *b*: $\Rightarrow\; R_1 = \dfrac{1.9}{1.27-0.807} = 4.1\ \Omega$

$$\frac{3.75}{2} + \frac{3.75-5.65}{4.1} + \frac{3.75-12}{R_2} = 0 \;\Rightarrow\; 1.875 + (-0.475) + \frac{-8.25}{R_2} = 0$$

KCL at node *a*: $\Rightarrow\; R_2 = \dfrac{8.25}{1.875-0.475} = 5.83$

P 3.2-8

Solution:
The subscripts suggest a numbering of the sources. Apply KVL to get

$$v_1 = v_2 + v_5 + v_9 - v_6$$

i_1 and v_1 do not adhere to the passive convention, so

$$p_1 = i_1 v_1 = i_1 \left(v_2 + v_5 + v_9 - v_6\right)$$

is the power supplied by source 1. Next, apply KCL to get

$$i_2 = -\left(i_1 + i_4\right)$$

i_2 and v_2 do not adhere to the passive convention, so

$$p_2 = i_2 v_2 = -\left(i_1 + i_4\right) v_2$$

is the power supplied by source 2. Next, apply KVL to get

$$v_3 = v_6 - \left(v_5 + v_9\right)$$

i_3 and v_3 adhere to the passive convention, so

$$p_3 = -i_3 v_3 = -i_3 \left(v_6 - \left(v_5 + v_9\right)\right)$$

is the power supplied by source 3. Next, apply KVL to get

$$v_4 = v_2 + v_5 + v_8$$

i_4 and v_4 do not adhere to the passive convention, so

$$p_4 = i_4 v_4 = i_4 \left(v_2 + v_5 + v_8\right)$$

is the power supplied by source 4. Next, apply KCL to get

$$i_5 = i_3 - i_2 = i_3 - \left(-\left(i_1 + i_4\right)\right) = i_1 + i_3 + i_4$$

i_5 and v_5 adhere to the passive convention, so

$$p_5 = -i_5 v_5 = -\left(i_1 + i_3 + i_4\right) v_5$$

is the power supplied by source 5. Next, apply KCL to get

$$i_6 = i_7 - \left(i_1 + i_3\right)$$

i_6 and v_6 adhere to the passive convention, so

$$p_6 = -i_6 v_6 = -\left(i_7 - \left(i_1 + i_3\right)\right) v_6$$

is the power supplied by source 6. Next, apply KVL to get

$$v_7 = -v_6$$

i_7 and v_7 adhere to the passive convention, so

$$p_7 = -i_7 v_7 = -i_7 \left(-v_6\right) = i_7 v_6$$

is the power supplied by source 7. Next, apply KCL to get

$$i_8 = -i_4$$

i_8 and v_8 do not adhere to the passive convention, so

$$p_8 = i_8 v_8 = \left(-i_4\right) v_8 = -i_4 v_8$$

is the power supplied by source 8. Finally, apply KCL to get

$$i_9 = i_1 + i_3$$

i_9 and v_9 adhere to the passive convention, so

$$p_9 = -i_9 v_9 = -\left(i_1 + i_3\right) v_9$$

is the power supplied by source 9.

(Check: $\sum_{n=1}^{9} p_n = 0$.)

P 3.2-9

Solution

The subscripts suggest a numbering of the circuit elements. Apply KCL to get

$$i_2 + 0.4 + 0.6 = 0 \quad \Rightarrow \quad i_2 = -1.0 \text{ A}$$

The power received by the 12 Ω resistor is

$$p_2 = 12 i_2^{\,2} = 12(-1.0)^2 = 12 \text{ W}$$

Next, apply KCL to get

$$i_5 = 0.4 + 0.6 + 1 = 2 \text{ A}$$

The power received by the 16 Ω resistor is

$$p_5 = 16 i_5^{\,2} = 16(2)^2 = 64 \text{ W}$$

Next, apply KVL to get

$$v_7 = 30 \text{ V}$$

The power received by the 40 Ω resistor is

$$p_7 = \frac{v_7^{\,2}}{40} = \frac{30^2}{40} = 22.5 \text{ W}$$

is the power supplied by source 7. Finally, apply KCL to get

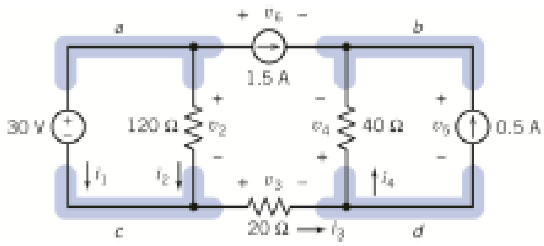
$$i_9 = 0.4 + 1.0 = 1.4 \text{ A}$$

The power received by the 10 Ω resistor is

$$p_9 = 10 i_9^{\,2} = 10(1.4)^2 = 19.6 \text{ W}$$

P 3.2-10

Solution: We can label the circuit as follows:



The subscripts suggest a numbering of the circuit elements. Apply KCL at node *b* to get

$$i_4 + 0.5 + 1.5 = 0 \quad \Rightarrow \quad i_4 = -2.0 \text{ A}$$

Next, apply KCL at node *d* to get

$$i_3 = i_4 + 0.5 = -2.0 + 0.5 = -1.5 \text{ A}$$

Next, apply KVL to the loop consisting of the voltage source and the 120 Ω resistor to get

$$v_2 - 30 = 0 \quad \Rightarrow \quad v_2 = 30 \text{ V}$$

Apply Ohm's law to each of the resistors to get

$$i_2 = \frac{v_2}{120} = \frac{30}{120} = 0.25 \text{ A},$$

$$v_3 = 20\ i_3 = 20(-1.5) = -30 \text{ V}$$

and

$$v_4 = 40 i_4 = 40(-2) = -80 \text{ V}$$

Next, apply KCL at node *c* to get

$$i_1 + i_2 = i_3 \quad \Rightarrow \quad i_1 = i_3 - i_2 = -1.5 - 0.25 = -1.75 \text{ A}$$

Next, apply KVL to the loop consisting of the 1.5 A current source and three resistors to get

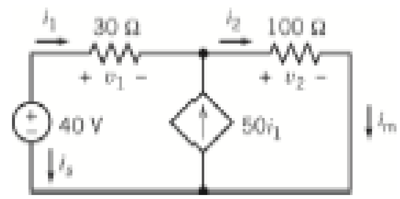
$$v_6 - v_4 - v_3 - v_2 = 0 \quad \Rightarrow \quad v_6 = v_4 + v_3 + v_2 = -80 + (-30) + 30 = -80 \text{ V}$$

Finally, apply KVL to the loop consisting of the 0.5 A current source and the 40 Ω resistor to get

$$v_5 + v_4 = 0 \quad \Rightarrow \quad v_5 = -v_4 = -(-80) = 80 \text{ V}$$

P 3.2-11

Solution:
We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KVL to the left mesh to get

$$30i_1 + 50i_1 - 40 = 0 \quad \Rightarrow \quad i_1 = \frac{40}{80} = 0.5 \text{ A}$$

Apply KVL to the right mesh to get

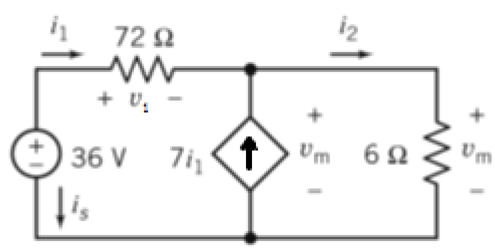
$$v_2 - 50i_1 = 0 \quad \Rightarrow \quad v_2 = 50i_1 = 50(0.5) = 25 \text{ V}$$

Apply KCL to get $i_m = i_2$. Finally, apply Ohm's law to the100Ω resistor to get

$$i_m = i_2 = \frac{v_2}{100} = \frac{25}{100} = 0.25 \text{ A}$$

P 3.2-12

Solution:
We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Ohm's law to the 72 Ω resistor to get

$$v_1 = 72\,i_1$$

Apply KCL at the top node of the CCCS to get

$$i_1 + 7\,i_1 = i_2 \quad \Rightarrow \quad i_2 = 8\,i_1$$

Ohm's law to the 6 Ω resistor to get

$$v_{\text{m}} = 6\,i_2 = 6\left(8\,i_1\right) = 48\,i_1$$

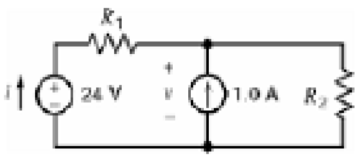
Apply KVL to the outside loop to get

$$v_1 + v_m - 36 = 0 \quad \Rightarrow \quad 72 i_1 + 48 i_1 = 36 \quad \Rightarrow \quad i_1 = \frac{36}{120} = \frac{4}{15}\ \text{A}$$

Finally,
$$v_m = 48 i_1 = 48\left(\frac{4}{15}\right) = \frac{64}{5}\ \text{V}$$

P 3.2-13

Solution:

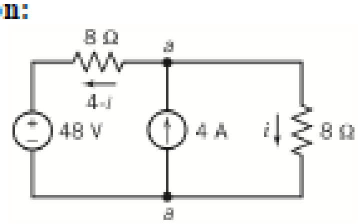


$$i = \frac{3.6}{24} = 0.15\ \text{A} \quad \text{and} \quad v = \frac{5.0}{1.0} = 5\ \text{V}$$

$$R_1 = \frac{24-5}{0.15} = 126.7\ \Omega \quad \text{and} \quad R_2 = \frac{5}{0.15+1.0} = 4.347$$

P 3.2-14

Solution:



Apply KCL at node a to determine the current in the horizontal resistor as shown.

Apply KVL to the loop consisting of the voltage source and the two resistors to get

$-8(4-i) + 8(i) - 48 = 0 \Rightarrow i = 5$ A

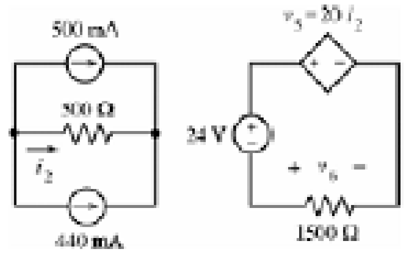
P 3.2-15

Solution:

$$-18 + 0 - 12 - v_a = 0 \quad \Rightarrow \quad v_a = -30\ \text{V} \quad \text{and} \quad i_m = \frac{2}{5} v_a + 3 \quad \Rightarrow \quad i_m = 9\ \text{A}$$

P3.2-16

Solution:
Apply KCL at the left node:



$$0.5 + i_2 + 0.44 = 0 \quad \Rightarrow \quad i_2 = -0.94 \text{ A}$$

Use the element equation of the dependent source:

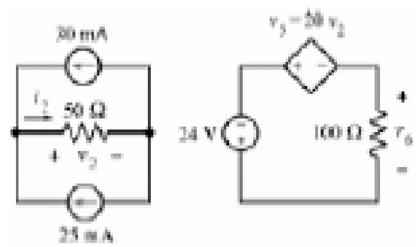
$$v_5 = 20 i_2 = 20(-0.94) = -18.8 \text{ V}$$

Apply KVL to the right mesh

$$v_5 - v_6 - 24 = 0 \quad \Rightarrow \quad v_6 = v_5 - 24 = -18.8 - 24 = -42.8 \text{ V}$$

P3.2-17

Solution:
Apply KCL at the left node:



$$0.03 + 0.025 = i_2 \quad \Rightarrow \quad i_2 = 0.055 \text{ A}$$

From Ohm's law

$$v_2 = 50 i_2 = 50(0.055) = 2.75 \text{ V}$$

Use the element equation of the dependent source:

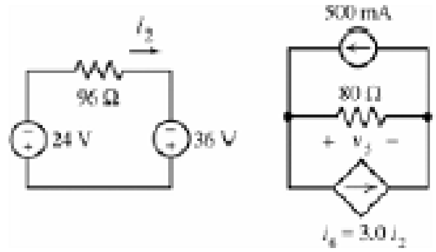
$$v_5 = 20 v_2 = 20(2.75) = 55 \text{ V}$$

Apply KVL to the right mesh

$$v_5 + v_6 + 24 = 0 \quad \Rightarrow \quad v_6 = -v_5 - 24 = -55 - 24 = -79 \text{ V}$$

P3.2-18

Solution:
Apply KVL to the left mesh:



$$96i_2 - 36 + 24 = 0 \quad \Rightarrow \quad i_2 = 0.125 \text{ A}$$

Use the element equation of the dependent source:

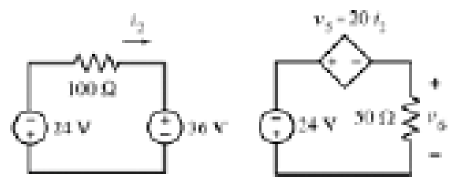
$$i_6 = 3.0i_2 = 3.0(0.125) = 0.375 \text{ A}$$

Apply KCL at the right node

$$\frac{v_5}{80} + i_6 = 0.5 \quad \Rightarrow \quad v_5 = 80(0.5 - i_6) = 80(0.5 - 0.375) = 10 \text{ V}$$

P3.2-19

Solution:
Apply KVL to the left mesh:



$$100i_2 + 36 + 24 = 0 \quad \Rightarrow \quad i_2 = \frac{-60}{100} = -0.6 \text{ A}$$

Use the element equation of the dependent source:

$$v_5 = 20i_2 = 20(-0.6) = -12 \text{ V}$$

Apply KVL to the right mesh

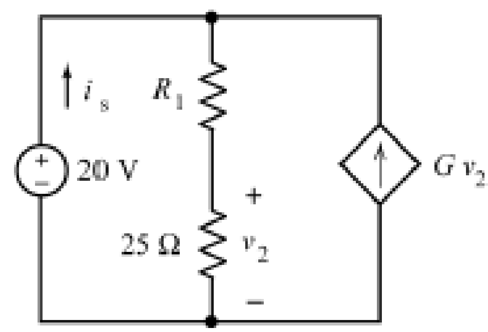
$$v_5 + v_6 + 24 = 0 \quad \Rightarrow \quad v_6 = -v_5 - 24 = -(-12) - 24 = -12 \text{ V}$$

P3.2-20

Solution:

The voltage source current is calculated from the values of the source voltage and power:

$$i_s = \frac{2}{20} = 0.1 \text{ A}$$



Apply KCL at the bottom node to get

$$i_s + Gv_2 = \frac{v_2}{25} \quad \Rightarrow \quad Gv_2 = \frac{4}{25} - 0.1 = 0.06 \text{ A}$$

Then

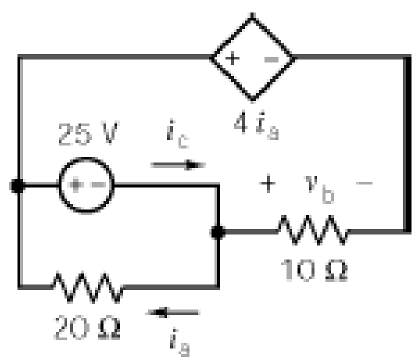
$$G = \frac{Gv_2}{v_2} = \frac{0.06}{4} = 0.015 \text{ A/V} = 15 \text{ mA/V}$$

Next, use Ohm's law to determine the value of the resistance R_1:

$$R_1 = \frac{20 - v_2}{\frac{v_2}{25}} = \frac{20-4}{\frac{4}{25}} = 100\ \Omega$$

P3.2-21

Solution:



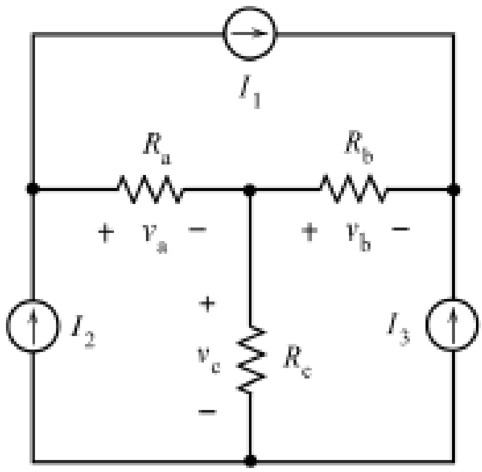
(a) From Ohm's law $i_a = -\frac{25}{20} = -1.25$ A.

(b) From KVL $4i_a - v_b + 20i_a = 0 \quad \Rightarrow \quad v_b = 24i_a = 24(-1.25) = -30$ V

(c) From KCL $i_c = i_a + \frac{v_b}{10} = -1.25 + \frac{-30}{10} = -4.25$ A

P3.2-22

Solution:



Using KCL and Ohm's law:

$$v_a = R_a(I_2 - I_1),\ \ v_b = -R_b(I_1 + I_3)$$

and

$$v_c = R_c(I_2 + I_3)$$

Using KVL, the power supplied the current sources are:

$I_2(v_a + v_c)$, $-I_1(v_a + v_b)$ and $I_3(-v_b + v_c)$

The power received the resistors are:

$$v_a(I_2 - I_1),\ \ -v_b(I_1 + I_3)\ \text{ and }\ v_c(I_2 + I_3)$$

a.) $R_a = \dfrac{4}{3-2} = 4\ \Omega$, $R_b = -\dfrac{-3}{2+(-1.5)} = 6\ \Omega$

and $R_c = \dfrac{12}{3+(-1.5)} = 8\ \Omega$

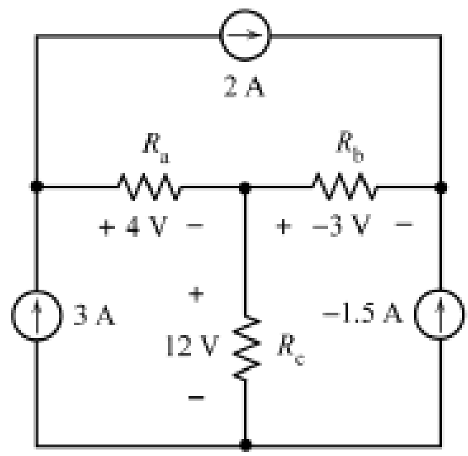
b.) $3(4+12) = 48$ W, $-2(4+(-3)) = -2$ W

and

$-1.5(-(-3)+12) = -22.5$ W

c.) $4(3-2) = 4$ W, $-(-3)(2+(-1.5)) = 1.5$ W

and $12(3+(-1.5)) = 18$ W



LNAPDC (8/30/10):

P3.2-23

Solution: Notice that *i(t)* is the current in the 20 Ω resistor. Apply KCL at the top node of the 80 Ω resistor to get

$$\frac{v(t)}{80} = 10 - \left(8 - 6e^{-25t}\right) \Rightarrow v(t) = 80\left[10 - \left(8 - 6e^{-25t}\right)\right] = 160 + 480e^{-25t} \text{ for } t \ge 0$$

P3.2-24

Solution:

The power supplied by the 12-V voltage source is given by

$$P_{vs} = -12\, i_4$$

Apply KCL at node *e* to obtain

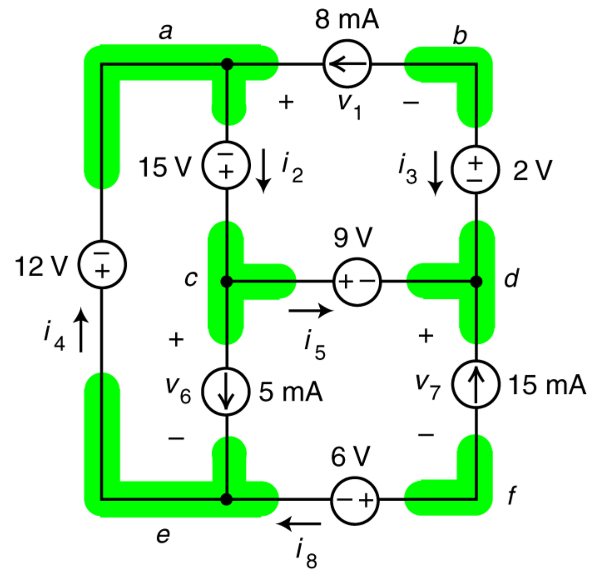
$$i_8 + 0.005 = i_4$$

Apply KCL at node *f* to obtain

$$0 = i_8 + 0.015$$

Combining these equations yields

$$P_{vs} = -12(-0.010) = 120 \text{ mW}$$



The power supplied by the 8-mA current source is given by

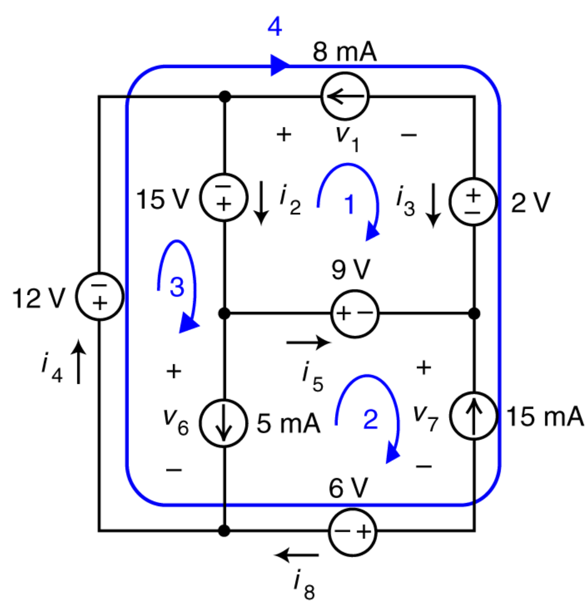
$$P_{cs} = 0.008\, v_1$$

Apply KVL to mesh 1 to obtain

$$v_1 + 2 - 9 + 15 = 0$$

Combining these equations yields

$$P_{cs} = 0.008(-8) = -64 \text{ mW}$$



P3.2-25

Solution:

The power supplied by the 9-V voltage source is given by

$$P_{vs} = -9\,i_6$$

Apply KVL to loop 3 to obtain

$$-v_2 + 9 + 12 = 0 \quad \Rightarrow \quad v_2 = 21\text{ V}$$

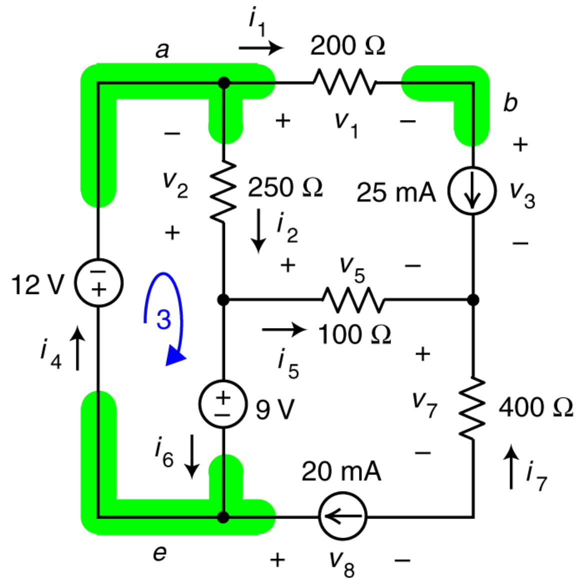
Apply Ohm's law to the 250 Ω resistor to obtain

$$i_2 = -\frac{v_2}{250} = -\frac{21}{250} = -84\text{ mA}$$

Apply KCL at nodes *a* and *b* to obtain

$$i_4 = i_1 + i_2 = 0.025 + (-0.084) = -59\text{ mA}$$

Apply KCL at node *e* to obtain



$$P_{vs} = -9\,i_6 = -9(i_4 - 0.020) = -9(-0.059 - 0.020) = 0.711 = 711\text{ mW}$$

The power supplied by the 20-mA current source is given by

$$P_{cs} = 0.020\,v_8$$

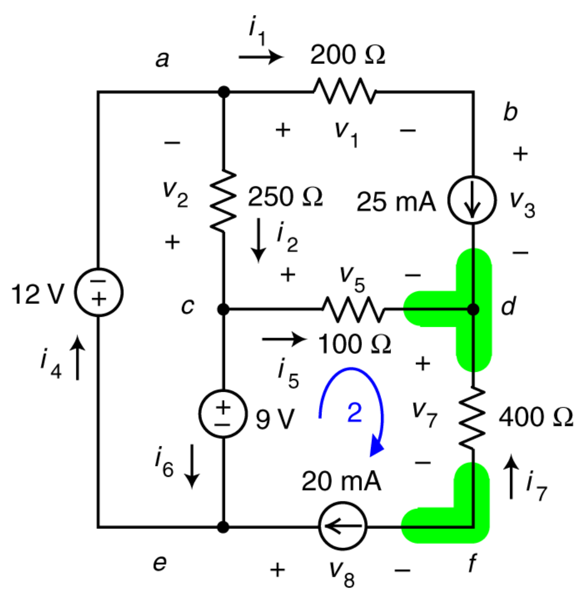
Apply KCL at node *f* and then at node *d* to obtain

$$i_7 = -20\text{ mA}$$

and

$$i_5 = -(i_7 + 25\text{ mA}) = -5\text{ mA}$$

Apply Ohm's law to the 100 Ω and 400 Ω resistors to obtain



$$v_5 = 100(i_5) = -0.5\text{ V}$$

and

$$v_7 = -400(i_7) = 8\text{ V}$$

Apply KVL to loop 2 to obtain

$$v_5 + v_7 - v_8 - 9 = 0 \quad \Rightarrow \quad v_8 = -0.5 + 8 - 9 = -1.5\text{ V}$$

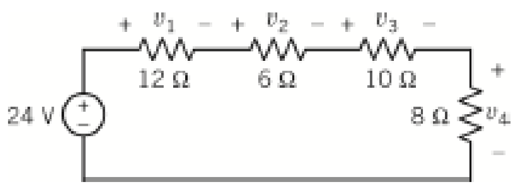
Finally

$$P_{cs} = 0.020(-1.5) = -30 \text{ mW}$$

Section 3-3 Series Resistors and Voltage Division

P 3.3-1

Solution:



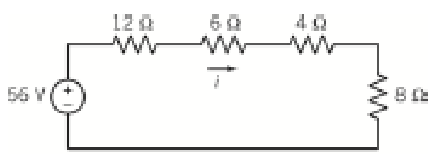
$$v_1 = \frac{12}{12+6+10+8} 24 = \frac{12}{36} 24 = \underline{8 \text{ V}}$$

$$v_2 = \frac{6}{36} 24 = \underline{4 \text{ V}} \; ; \; v_3 = \frac{10}{36} 24 = \underline{\frac{20}{3} \text{ V}}$$

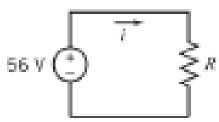
$$v_4 = \frac{8}{36} 24 = \underline{\frac{16}{3} \text{ V}}$$

P 3.3-2

Solution:



(a)



(b)

(a) $R = 12+6+4+8 = \underline{30\ \Omega}$

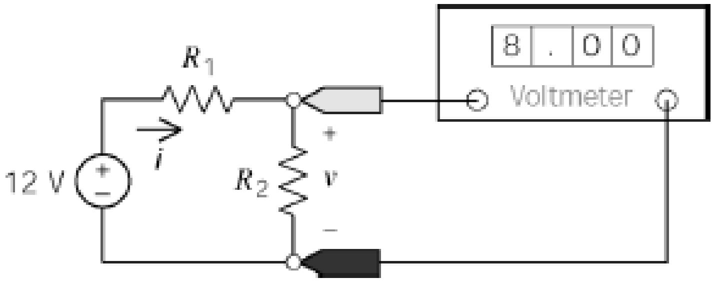
(b) $i = \dfrac{56}{R} = \dfrac{56}{30} = \underline{1.867 \text{ A}}$

(c) $p = 56 \cdot i = 56(1.867) = \underline{104.55 \text{ W}}$

(56 V and i do not adhere to the passive convention.)

P 3.3-3

Solution:



$i\,R_2 = v = 8\text{ V}$

$12 = i\,R_1 + v = i\,R_1 + 8$

$\Rightarrow 4 = i\,R_1$

$(a)\ i = \dfrac{8}{R_2} = \dfrac{8}{50}\ ;\ R_1 = \dfrac{4}{i} = \dfrac{4 \cdot 50}{8} = \underline{25\,\Omega}$

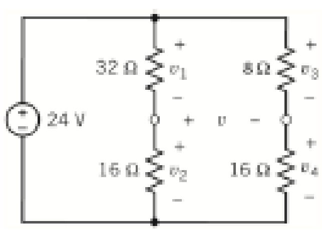
$(b)\ i = \dfrac{4}{R_1} = \dfrac{4}{50}\ ;\ R_2 = \dfrac{8}{i} = \dfrac{8 \cdot 50}{4} = \underline{100\,\Omega}$

$(c)\ 1.2 = 12\,i \Rightarrow i = 0.1\text{ A}\ ;\ R_1 = \dfrac{4}{i} = \underline{40\,\Omega};\ R_2 = \dfrac{8}{i} = \underline{80\,\Omega}$

(Checked using LNAP 8/16/02)

P 3.3-4

Solution:



Voltage division

$v_1 = \dfrac{32}{32+16}24 = 16\text{ V}$

$v_3 = \dfrac{8}{8+16}24 = 8\text{ V}$

KVL: $v_3 - v - v_1 = 0$

$\underline{v = -8\text{ V}}$

(checked using LNAP 8/16/02)

P 3.3-5

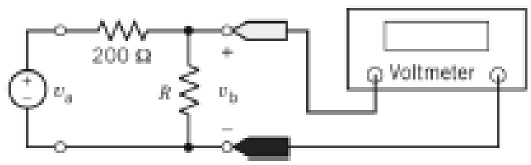
Solution:

$$\text{using voltage divider: } v_0 = \left(\frac{100}{100+2R}\right) v_s \Rightarrow R = 50\left(\frac{v_s}{v_0} - 1\right)$$

$$\left.\begin{array}{l} \text{with } v_s = 19 \text{ V and } v_0 = 8 \text{ V}, \quad R = 68.7\Omega \\ \text{with } v_s = 29 \text{ V and } v_0 = 14 \text{ V}, R = 53.5\Omega \end{array}\right\} \quad \underline{R = 61\Omega}$$

P 3.3-6

Solution:



a.) $\left(\frac{240}{200+240}\right) 24 = 13.09 \text{ V}$

b.) $24\left(\frac{24}{200+240}\right) = 1.31 \text{ W}$

c.) $\left(\frac{R}{R+200}\right) 24 = 3 \quad \Rightarrow \quad 24\,R = 3R + 3\,(200) \quad \Rightarrow \quad R = 28.57 \;\Omega$

d.) $0.2 = \frac{R}{R+200} \quad \Rightarrow \quad (0.2)(200) = 0.8\,R \quad \Rightarrow \quad R = 50 \;\Omega$

(Checked using LNAP 8/16/02)

P 3.3-7

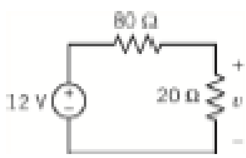
Solution:
All of the elements are connected in series.

Replace the series voltage sources with a single equivalent voltage having voltage

$$24 + 24 - 36 = 12 \text{ V}.$$

Replace the series 15 Ω, 5 Ω and 20 Ω resistors by a single equivalent resistance of

$$30 + 10 + 40 = 80 \ \Omega.$$

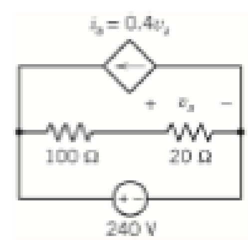


By voltage division

$$v = \left(\frac{20}{20+80}\right)12 = \frac{12}{5} = 2.4 \text{ V}$$

P 3.3-8

Solution:
Use voltage division to get



$$v_a = \left(\frac{20}{20+100}\right)(240) = 40 \text{ V}$$

Then

$$i_a = 0.4(40) = 16 \text{ A}$$

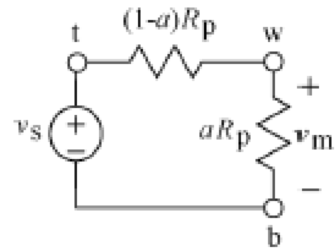
The power supplied by the dependent source is given by

$$p = (240)i_a = 3840 \text{ W}$$

P 3.3-9

Solution:

(a) Use voltage division to get



$$v_m = \frac{aR_p}{(1-a)R_p + R_p} v_s = a v_s$$

Therefore $$v_m = \left(\frac{v_s}{360}\right)\theta$$

So the input is proportional to the input.

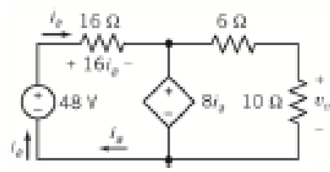
(b) When $v_s = 24$ V then $v_m = \left(\frac{1}{15}\right)\theta$. When $\theta = 45^\circ$ then $v_m = 3$ V. When $v_m = 10$ V then $\theta = 150^\circ$.

(Checked: LNAP 6/12/04)

P 3.3-10

Solution:

Replace the (ideal) voltmeter with the equivalent open circuit. Label the voltage measured by the meter. Label some other element voltages and currents.



Apply KVL the left mesh to get

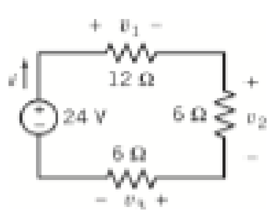
$$16 i_a + 8 i_a - 48 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

Use voltage division to get

$$v_m = \frac{10}{10+6} 8 i_a = \frac{10}{10+6} 8(2) = 10 \text{ V}$$

P 3.3-11

Solution:



From voltage division $v_3 = 24\left(\frac{6}{6+18}\right) = \underline{6\text{ V}}$

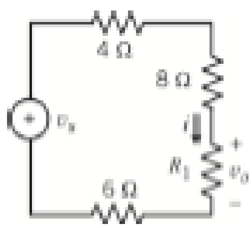
then $i = \frac{v_3}{6} = \underline{1\text{ A}}$

The power absorbed by the resistors is: $(1^2)(12)+(1^2)(6)+(1^2)(6) = 24$ W

The power supplied by the source is (24)(1) = 24 W.

P 3.3-12

Solution:



$P = 9$ W and $R_1 = 16\ \Omega$

$$i^2 = \frac{P}{R_1} = \frac{9}{16} = 0.5625 \text{ or } i = 0.75 \text{ A}$$

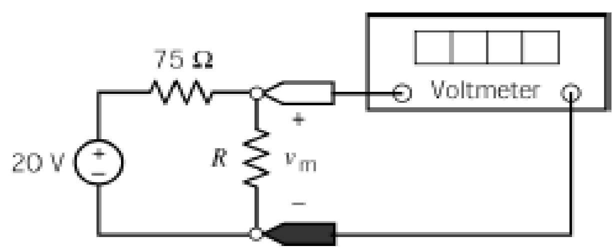
$v_0 = i\,R_1 = (0.75)\,(16) = \underline{12 \text{ V}}$

from KVL: $-v_s + i\,(4+8+16+4) = 0$

$\Rightarrow\ v_s = 32\,i = 32(0.75) = 24$ V

P3.3-13

Solution



Using voltage division $\qquad v_m = \left(\frac{R}{75+R}\right)20$

Solving for R yields $\qquad R = \frac{75\,v_m}{20 - v_m}$

The temperature can be calculated from the resistance using

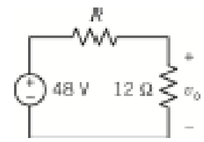
$$T = 2(R-50) = 2\left(\frac{75\,v_m}{20-v_m} - 50\right) = \frac{150\,v_m}{20-v_m} - 100$$

a) At 25 °C the resistance is $R = 62.5\ \Omega$ so $v_m = \left(\frac{62.5}{75+62.5}\right)20 = 9.1$ V. At 100 °C the resistance is $R = 100\ \Omega$ so $v_m = \left(\frac{100}{75+100}\right)20 = 11.4$ V. At 125 °C the resistance is $R = 112.5\ \Omega$ so $v_m = \left(\frac{112.5}{75+112.5}\right)20 = 12$ V.

b) When $v_m = 10$ V, the temperature is $T = \dfrac{150(10)}{20-10} - 100 = 50$ °C. When $v_m = 12$ V, the temperature is $T = \dfrac{150(12)}{20-12} - 100 = 125$ °C. When $v_m = 14$ V, the temperature is $T = \dfrac{150(14)}{20-14} - 100 = 350$ °C.

P3.3-14

Solution:



Voltage division indicates that $v_p = \left(\dfrac{12}{12+R}\right)48$.

(a) When $v_o = 17.07$ V, then $17.07 = \left(\dfrac{12}{12+R}\right)48 \;\Rightarrow\; R = \dfrac{(12)48}{17.07} - 12 = 21.7\ \Omega$.

(b) When $R = 21\ \Omega$ then $v_o = \left(\dfrac{12}{12+21}\right)48 = 17.5$ V.

(c) The power supplied by the voltage source is given by $48\dfrac{v_o}{12} = 4v_o$ since $\dfrac{v_o}{12}$ is the current in each of the series elements. When $v_o = 14.22$ V the voltage source supplies 56.88 W.

P3.3-15.

Solution: Using voltage division:

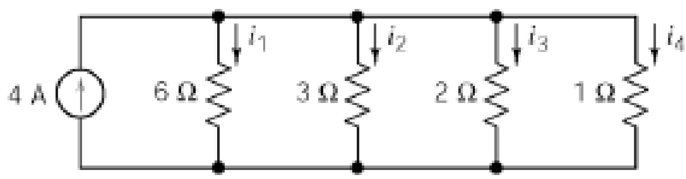
$$v_1 = \left(\frac{60}{60+70}\right)26 = 12\text{ V},\quad v_2 = -\left(\frac{130}{70+130}\right)26 = -16.9\text{ V},$$

$$v_3 = -\left(\frac{30}{70+30}\right)26 = -7.8\text{ V} \quad \text{and} \quad v_4 = -\left(\frac{70}{30+70}\right)26 = 18.2\text{ V}$$

Section 3-4 Parallel Resistors and Current Division

P 3.4-1

Solution:



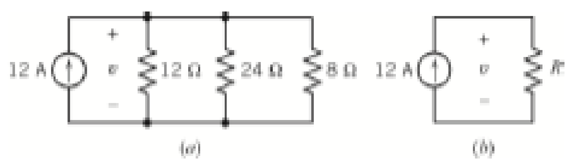
$$i_1 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} 4 = \frac{1}{1+2+3+6} \; 4 = \underline{\frac{1}{3}\text{ A}}$$

$$i_2 = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} \; 4 = \underline{\frac{2}{3}\text{ A}};$$

$$i_3 = \frac{\frac{1}{2}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}} \; 4 = \underline{1\text{ A}}$$

$$i_4 = \frac{1}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+1} \; 4 = \underline{2\text{ A}}$$

P 3.4-2



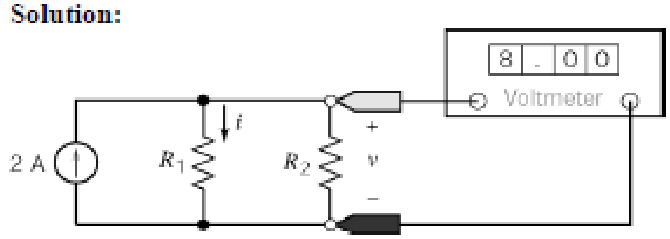
$(a)\;\; \frac{1}{R} = \frac{1}{12}+\frac{1}{24}+\frac{1}{8} = \frac{1}{4} \Rightarrow \underline{R = 4\,\Omega}$

$(b)\;\; v = 12 \times 4 = \underline{48\text{ V}}$

$(c)\;\; p = 12 \cdot 48 = \underline{576\text{ W}}$

P 3.4-3

Solution:



$$i = \frac{8}{R_1} \text{ or } R_1 = \frac{8}{i}$$

$$8 = R_2(2-i) \Rightarrow i = 2 - \frac{8}{R_2} \text{ or } R_2 = \frac{8}{2-i}$$

$(a)\;\; i = 2 - \frac{8}{6} = \underline{\frac{2}{3}\text{ A}} \;;\; R_1 = \frac{8}{\frac{2}{3}} = \underline{12\,\Omega}$

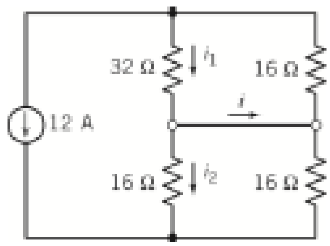
$(b)\;\; i = \frac{8}{6} = \underline{\frac{4}{3}\text{ A}} \;;\; R_2 = \frac{8}{2-\frac{4}{3}} = \underline{12\,\Omega}$

$(c)\;\; R_1 = R_2$ will cause $i = \frac{1}{2} 2 = 1$ A. The current in both R_1 and R_2 will be 1 A.

$$2 \cdot \frac{R_1 R_2}{R_1 + R_2} = 8 ; \quad R_1 = R_2 \quad \Rightarrow \quad 2 \cdot \frac{1}{2} R_1 = 8 \Rightarrow R_1 = 8 \;\therefore \underline{R_1 = R_2 = 8\,\Omega}$$

P 3.4-4

Solution:



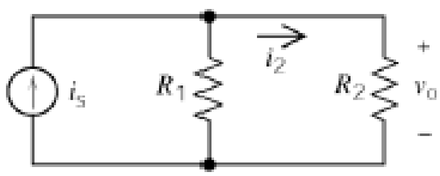
Current division:

$$i_1 = \frac{16}{32+16}(-12) = -4\text{ A}$$

$$i_2 = \frac{16}{16+16}(-12) = -6\text{ A}$$

$$i = i_1 - i_2 \quad = \underline{+2\text{ A}}$$

P3.4-5



current division: $i_2 = \left(\dfrac{R_1}{R_1 + R_2}\right) i_s$ and

Ohm's Law: $v_o = i_2 R_2$ yields

$$i_s = \left(\frac{v_o}{R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right)$$

plugging in $R_1 = 4\Omega$, $v_o > 8$ V gives $i_s > 2.67$ A

and $R_1 = 6\Omega$, $v_o < 14$ V gives $i_s < 3.5$ A

So any $\underline{2.67\text{ A} < i_s < 3.5\text{ A}}$ keeps $8\text{ V} < v_o < 14\text{ V}$

P 3.4-6

Solution:

(a) To insure that i_b is negligible we require

$$i_1 = \frac{15}{R_1 + R_2} \geq 10\left(10\times10^{-6}\right) = 10^{-3}$$

so

$$R_1 + R_2 \leq 150 \text{ k}\Omega$$

To insure that the total power absorbed by R_1 and R_2 is no more than 5 mW we require

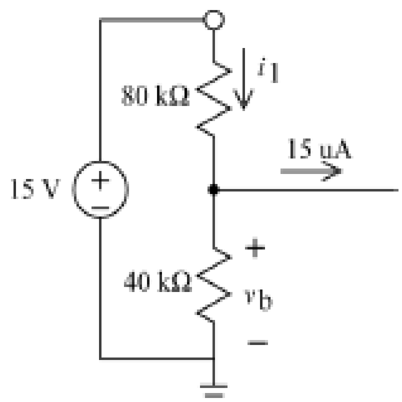
$$\frac{15^2}{R_1 + R_2} \leq 5\times10^{-3} \quad \Rightarrow \quad R_1 + R_2 \geq 45 \text{ k}\Omega$$

Next to cause v_b = 5 V we require

$$5 = v_b = \frac{R_2}{R_1 + R_2} 15 \quad \Rightarrow \quad R_1 = 2R_2$$

For example, $R_1 = 40 \text{ k}\Omega,\ R_2 = 80 \text{ k}\Omega,$ satisfy all three requirements.

(b)



KVL gives $\left(80\times10^3\right)i_1 + v_b - 15 = 0$

KCL gives $i_1 = \dfrac{v_b}{40\times10^3} + 15\times10^{-6}$

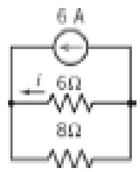
Therefore $\left(80\times10^3\right)\left(\dfrac{v_b}{40\times10^3} + 15\times10^{-6}\right) + v_b = 15$

Finally

$$3v_b + 1.2 = 15 \quad \Rightarrow \quad v_b = \frac{13.8}{3} = 4.6 \text{ V}$$

P 3.4-7

Solution:
All of the elements of this circuit are connected in parallel. Replace the parallel current sources by a single equivalent 4 – 1 + 3 = 6 A current source. Replace the parallel 24 Ω and 12 Ω resistors by a single $\frac{24 \times 12}{24+12} = 8\ \Omega$ resistor.

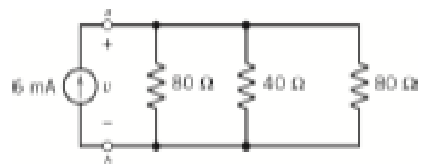


By current division

$$i = -\left(\frac{8}{6+8}\right)6 = -\frac{24}{7} = -3.429\text{ A}$$

P3.4-8

Solution:
Each of the resistors is connected between nodes *a* and *b*. The resistors are connected in parallel and the circuit can be redrawn like this:



Then

$$80 \,||\, 40 \,||\, 80 = 20\ \Omega$$

So

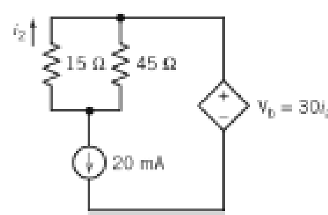
$$v = 20(0.006) = 0.12 = 120\text{ mV}$$

P 3.4-9

Solution:
Use current division to get

$$i_a = -\frac{45}{15+45}\left(20 \times 10^{-3}\right) = -15\text{ mA}$$

so $\quad v_b = 30\left(-15 \times 10^{-3}\right) = -0.45\text{ V}$



The power supplied by the dependent source is given by

$$p = -\left(20 \times 10^{-3}\right)(-0.45) = 9\text{ mW}$$

(checked: LNAP 6/12/04)

P 3.4-10

Solution:

Using Voltage Division

$$8 = \frac{R_1}{R_1 + \dfrac{80R_2}{R_2 + 80}} \times 18$$

$$= \frac{R_1(R_2 + 80)}{R_1R_2 + 80(R_1 + R_2)}$$

$$5R_1R_2 + 400R_2 = 320R_2$$

$$R1(5R_2 + 400) = 320R_2$$

$$R_1 = \frac{320R_2}{5R_2 + 400}$$

Using KVL

$$18 = 8 + 3.2R2$$

$$R2 = \frac{10}{3.2} = 3.125\Omega$$

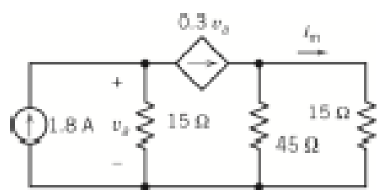
$$R1 = \frac{320 \times 3.125}{(5 \times 3.125) + 400}$$

$$R1 = 12.41\Omega$$

P 3.4-11

Solution:

Replace the (ideal) ammeter with the equivalent short circuit. Label the current measured by the meter.



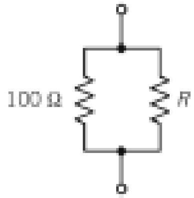
Apply KCL at the left node of the VCCS to get

$$1.8 = \frac{v_a}{15} + 0.3v_a = 0.37v_a \quad \Rightarrow \quad v_a = \frac{1.8}{0.37} = 4.9 \text{ V}$$

Use current division to get

$$i_m = \frac{45}{45+15}0.3v_a = \frac{45}{45+15}0.3(4.9) = 1.1 \text{ A}$$

P3.4-12



Solution:

(a) First, when $R = 25\ \Omega$ then $R_p = 100 \,||\, 25 = \dfrac{100(25)}{100+25} = 20\ \Omega$. Next, when $R = 400\ \Omega$ then

$R_p = 100 \,||\, 400 = \dfrac{100(400)}{100+400} = 80\ \Omega$. Consequently

$$20\ \Omega \le R_p \le 80\ \Omega.$$

(b) When $R = 0$ then $R_p = 100 \,||\, 0 = \dfrac{100(0)}{100+0} = 0\ \Omega$.

(c) When $R = \infty$ then $R_p = \dfrac{100(\infty)}{100+\infty} = \dfrac{100}{\dfrac{100}{\infty}+1} = 100\ \Omega$.

(d) When $R_p = 50\ \Omega$ then $50 = 100 \,||\, R = \dfrac{100R}{100+R} \Rightarrow 100+R = 2R \Rightarrow R = 100\ \Omega$

P3.4-13

Solution:

(a) First, when $R = 40\ \Omega$ then $R_p = 160 \,||\, (40+40) = 160 \,||\, 80 = \dfrac{160(80)}{160+80} = \dfrac{160}{3} = 53.33\ \Omega$. Next, when $R = 400\ \Omega$ then $R_p = 160 \,||\, (40+400) = 160 \,||\, 440 = \dfrac{160(440)}{160+440} = \dfrac{16(44)}{6} = 117.33\ \Omega$.

Consequently $$53.33\ \Omega \le R_p \le 117.33\ \Omega.$$

(b) When $R = 0$ then $R_p = 160 \,||\, (0+40) = 160 \,||\, 40 = \dfrac{160(40)}{160+40} = \dfrac{16(4)}{2} = 32\ \Omega$.

(c) When $R = \infty$ then $R_p = 160 \,||\, (\infty+40) = 160 \,||\, \infty = \dfrac{1}{\dfrac{1}{160}+\dfrac{1}{\infty}} = 160\ \Omega$.

(d) When $R_p = 80\ \Omega$ then

$$80 = 160 \,||\, (R+40) = \frac{160(R+40)}{160+R+40} \Rightarrow R+200 = \frac{160}{80}(R+40) = 2R+80 \Rightarrow R = 120\ \Omega.$$

P3.4-13

Solution:

(a) First, when $R = 40\ \Omega$ then $R_p = 160 \,||\, (40+40) = 160 \,||\, 80 = \dfrac{160(80)}{160+80} = \dfrac{160}{3} = 53.33\ \Omega$. Next, when $R = 400\ \Omega$ then $R_p = 160 \,||\, (40+400) = 160 \,||\, 440 = \dfrac{160(440)}{160+440} = \dfrac{16(44)}{6} = 117.33\ \Omega$.

Consequently $\quad 53.33\ \Omega \le R_p \le 117.33\ \Omega$.

(b) When $R = 0$ then $R_p = 160 \,||\, (0+40) = 160 \,||\, 40 = \dfrac{160(40)}{160+40} = \dfrac{16(4)}{2} = 32\ \Omega$.

(c) When $R = \infty$ then $R_p = 160 \,||\, (\infty+40) = 160 \,||\, \infty = \dfrac{1}{\dfrac{1}{160}+\dfrac{1}{\infty}} = 160\ \Omega$.

(d) When $R_p = 80\ \Omega$ then

$$80 = 160 \,||\, (R+40) = \frac{160(R+40)}{160+R+40} \quad \Rightarrow \quad R+200 = \frac{160}{80}(R+40) = 2R+80 \quad \Rightarrow \quad R = 120\ \Omega.$$

P3.4-14

Solution:

From current division, $i_o = \left(\dfrac{R_1}{R_1+R_2}\right) i_s$. When $i_s = 5$ A and $i_o = 2$ A then $\dfrac{2}{5} = \dfrac{R_1}{R_1+R_2}$ so $2(R_1+R_2) = 5R_1$ or $2R_2 = 3R_1$.

The power supplied by the source is given by $i_s\left[\left(\dfrac{R_1 R_2}{R_1+R_2}\right) i_s\right]$. When $i_s = 2$ A the source supplies 48 W, so $48 = 2\left[\left(\dfrac{R_1 R_2}{R_1+R_2}\right) 2\right] \quad \Rightarrow \quad 12 = \dfrac{R_1 R_2}{R_1+R_2}$.

Combining these results gives $12 = \dfrac{R_1\left(\frac{3}{2}R_1\right)}{R_1+\left(\frac{3}{2}R_1\right)} = \dfrac{\frac{3}{2}R_1}{\frac{5}{2}} = \dfrac{3}{5}R_1 \quad \Rightarrow \quad R_1 = \dfrac{5}{3}(12) = 20\ \Omega$ and $\dfrac{3R_1}{2} = 30\ \Omega$.

P3.4-15

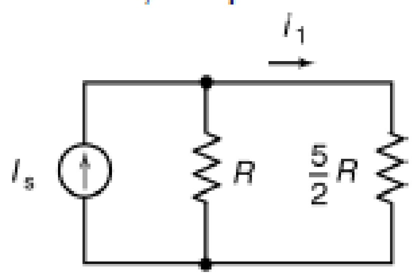
Solution:
Using current division:

$$i_1 = \left(\frac{30}{30+60}\right)240 = 80 \text{ mA}, \quad i_2 = -\left(\frac{60}{60+40}\right)240 = -144 \text{ mA},$$

$$i_3 = \left(\frac{20}{60+20}\right)240 = 60 \text{ mA} \quad \text{and} \quad i_4 = -\left(\frac{60}{60+15}\right)240 = -192 \text{ mA}$$

P3.4-16

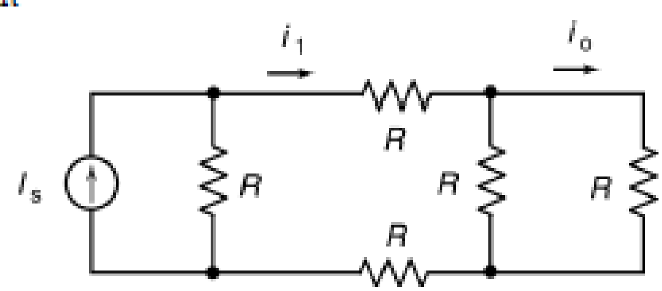
Solution:
Replace six resistors at the right of the circuit by an equivalent resistance to get



Using current division:

$$i_1 = \left(\frac{\frac{5}{2}R}{R+\frac{5}{2}R}\right)I_s = \frac{2}{7}I_s$$

Return to the original circuit



Using current division:

$$i_o = \frac{R}{R+R}i_1 = \frac{1}{2}i_1 = \frac{1}{2}\left(\frac{2}{7}I_s\right) = \frac{1}{7}I_s$$

Therefore $\quad k = \frac{1}{7}$

P3.4-17

Solution:

Current Division: $$4 = \frac{20}{20+20}\left(a\,v_2\right) \quad \Rightarrow \quad a\,v_2 = 8 \text{ A}$$

Voltage Division: $$v_2 = \frac{56}{56+56}20 = 10 \text{ V}$$

$$a = \frac{a\,v_2}{v_2} = \frac{8}{10} = 0.8 \ \frac{\text{A}}{\text{V}} = 800 \ \frac{\text{mA}}{\text{V}}$$

P3.4-18

Solution:

$$20 = \left(5000 \,\|\, 5000\right)\left(a\,v_2\right) \quad \Rightarrow \quad a\,v_2 = \frac{20}{2500} = 0.008 \text{ A} = 8 \text{ mA}$$

$$v_2 = \left(600 \,\|\, 1200\right)\left(0.005\right) = 2 \text{ V}$$

$$a = \frac{a\,v_2}{v_2} = \frac{0.008}{2} = 0.004 \ \frac{\text{A}}{\text{V}} = 4 \ \frac{\text{mA}}{\text{V}}$$

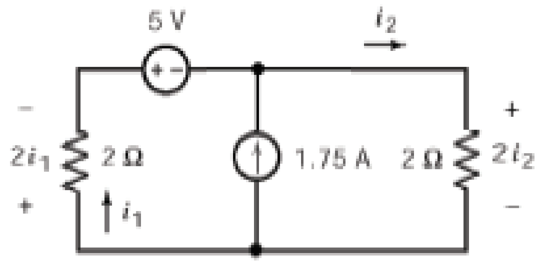
Section 3-5 Series Voltage Sources and Parallel Current Sources

P 3.5-1

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + 2i_1 = 0$$

so

$$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and

$$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

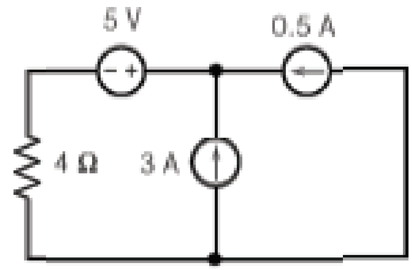
P 3.5-2

Solution:

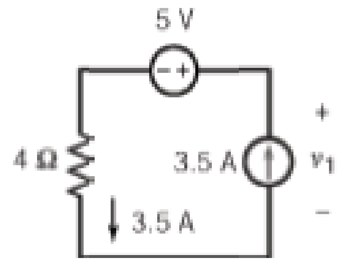
The 20-Ω and 5-Ω resistors are connected in parallel. The equivalent resistance is $\frac{20\times 5}{20+5}=4\ \Omega$. The 7-Ω resistor is connected in parallel with a short circuit, a 0-Ω resistor. The equivalent resistance is $\frac{0\times 7}{0+7}=0\ \Omega$, a short circuit.

The voltage sources are connected in series and can be replaced by a single equivalent voltage source.

After doing so, and labeling the resistor currents, we have the circuit shown.



The parallel current sources can be replaced by an equivalent current source.



Apply KVL to get

$$-5+v_1-4(3.5)=0 \quad \Rightarrow \quad v_1=19\ \text{V}$$

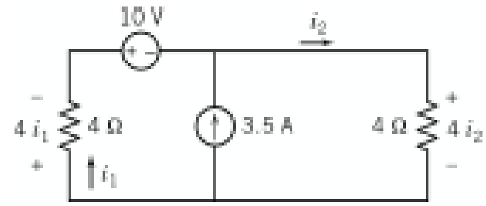
The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-2(3.5)=-7$ W
3-V voltage source	$-3(3.5)=-10.5$ W
3-A current source	$3\times 19=57$ W
0.5-A current source	$0.5\times 19=9.5$ W

P 3.5-3

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.



After doing so, and labeling the resistor currents, we have the circuit shown.

Apply KCL at the top node of the current source to get

$$i_1 + 3.5 = i_2$$

Apply KVL to the outside loop to get

$$10 + 4\,i_2 + 4i_1 = 0$$

so

$$10 + 4\left(i_1 + 3.5\right) + 4\,i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{24}{8} = -3 \text{ A}$$

and

$$i_2 = -3 + 3.5 = 0.5 \text{ A}$$

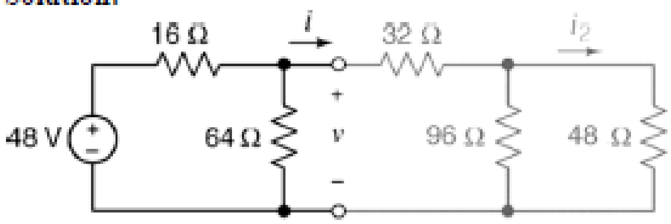
The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 24$ W
3-V voltage source	$3i_1 = -9$ W
3-A current source	$3 \times 2i_2 = 3$ W
1.25-A current source	$-1.25 \times 2i_2 = -1.25$ W

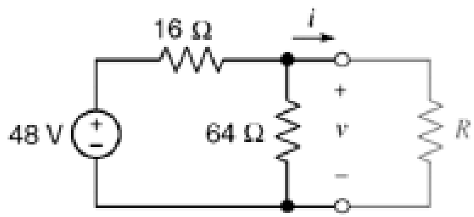
Section 3-6 Circuit Analysis

P 3.6-1

Solution:



(a)



(b)

$$(a)\quad R = 32+\frac{96\cdot 48}{96+48} = \underline{64\ \Omega}$$

$$(b)\quad v = \frac{\frac{64\cdot 64}{64+64}}{16+\frac{64\cdot 64}{64+64}} = \underline{32\ \text{V}}\ ;$$

$$i=\frac{32}{64}=\underline{\frac{1}{2}\ \text{A}}$$

$$(c)\quad i_2 = \frac{96}{96+48}\cdot\frac{1}{2} = \underline{\frac{1}{3}\ \text{A}}$$

P 3.6-2

Solution:

$$(a)\quad R_1 = 4+\frac{3\cdot 6}{3+6} = \underline{6\ \Omega}$$

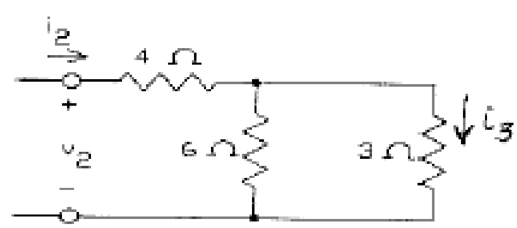
$$(b)\quad \frac{1}{R_p} = \frac{1}{12}+\frac{1}{6}+\frac{1}{6} \Rightarrow R_p = 2.4\ \Omega \quad \text{then} \quad R_2 = 8+R_p = \underline{10.4\ \Omega}$$

(c) KC[illegible] i + [illegible] = i [illegible] $-$[illegible]$+$[illegible]i $+R$ i = [illegible]

$\Rightarrow$ $-$ [illegible] $+$ [illegible] $i-$ [illegible] $+$ [illegible] $i =$ [illegible]

$\Rightarrow$ i_1 [illegible] 16.4 [illegible] $\Rightarrow$ v_1 [illegible] $i_1 R_2$ [illegible]

$$(d)\quad i_2 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{12}}(2.195) = \underline{0.878\ \text{A}},$$

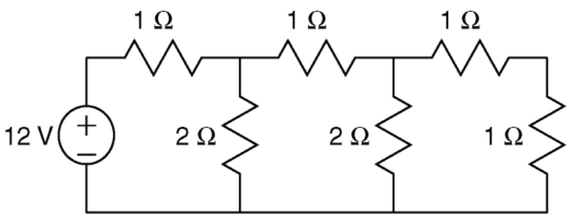


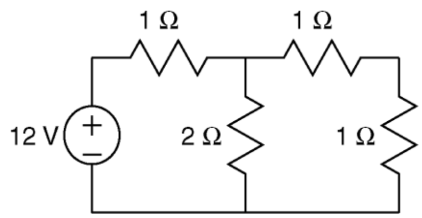
$$v_2 = (0.878)(6) = \underline{5.3\ \text{V}}$$

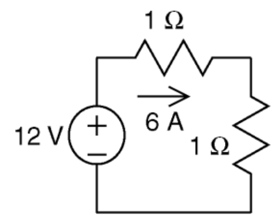
$$(e)\quad i_3 = \frac{6}{3+6}i_2 = 0.585\ \text{A} \quad \Rightarrow \quad P = 3\,i_3^{\,2} = \underline{1.03\ \text{W}}$$

P 3.6-3
Solution:

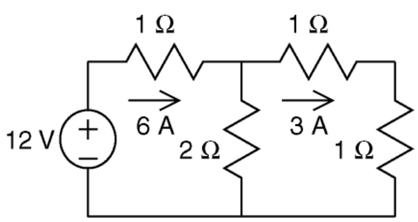
Reduce the circuit from the right side by repeatedly replacing series 1 Ω resistors in parallel with a 2 Ω resistor by the equivalent 1 Ω resistor

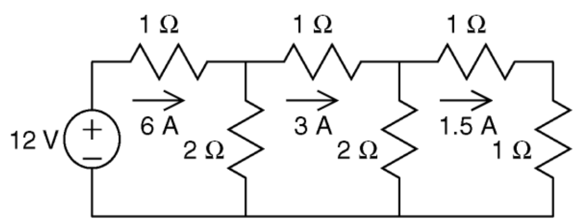


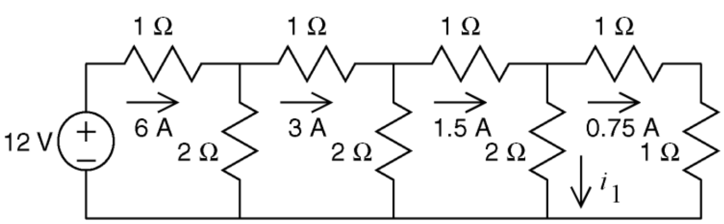




This circuit has become small enough to be easily analyzed. The vertical 1 Ω resistor is equivalent to a 2 Ω resistor connected in parallel with series 1 Ω resistors:







$$i_1 = \frac{1+1}{2+(1+1)}(1.5) = 0.75 \text{ A}$$

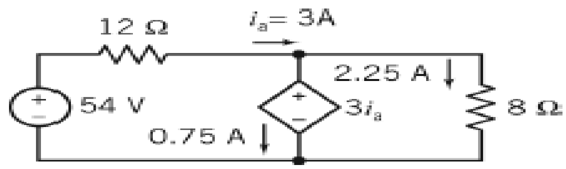
P 3.6-4

Solution:

(a)
$$\frac{1}{R_2}=\frac{1}{48}+\frac{1}{24}+\frac{1}{16} \quad \Rightarrow \quad R_2=8\,\Omega \qquad \text{and}$$

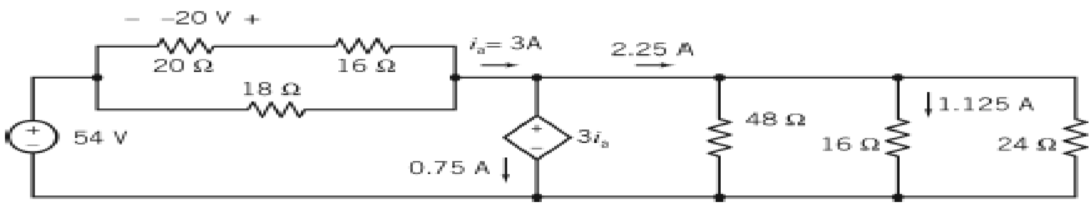
$$R_1=\frac{(20+16)\cdot 18}{(20+16)+18}=12\,\Omega$$

(b)



First, apply KVL to the left mesh to get $-54+12i_a+6i_a=0 \quad \Rightarrow \quad i_a=3\text{ A}$.
Next, apply KVL to the left mesh to get $8i_b-6i_a=0 \quad \Rightarrow \quad i_b=2.25\text{ A}$.

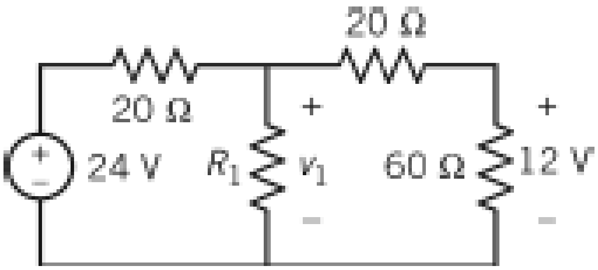
(c)



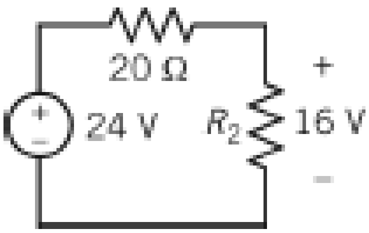
$$i_2=\frac{\frac{1}{16}}{\frac{1}{48}+\frac{1}{16}+\frac{1}{24}}2.25=1.125\text{ A} \qquad \text{and} \qquad v_1=-(20)\left[\frac{18}{(20+16)+18}3\right]=-20\text{ V}$$

P 3.6-5

Solution:



$$\frac{60}{20+60} v_1 = 12 \quad \Rightarrow \quad v_1 = 16\text{ V}$$

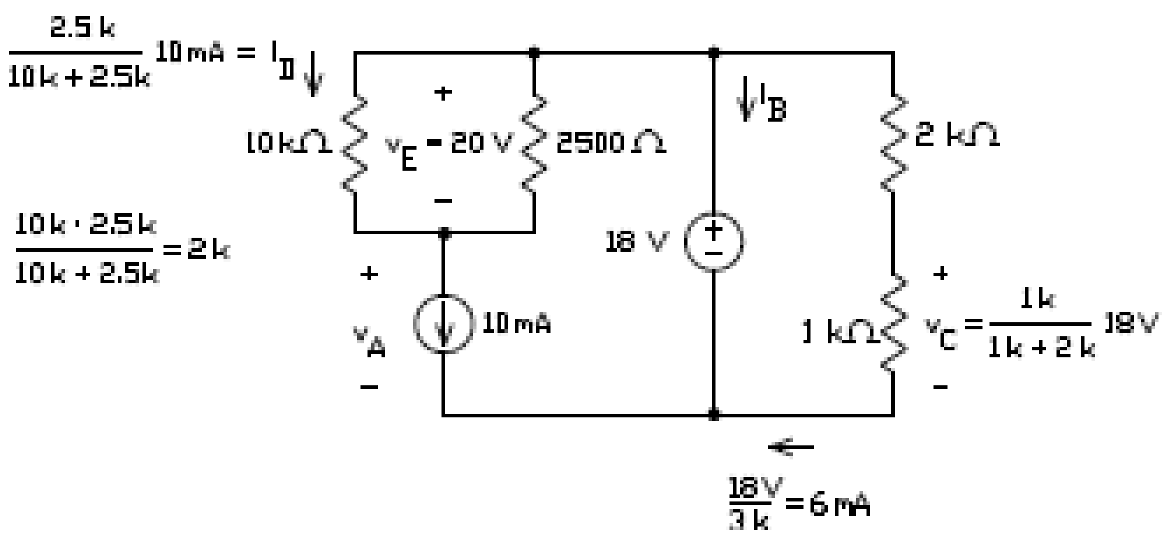


$$\frac{R_2}{R_2+20} 24 = 16 \quad \Rightarrow \quad R_2 = 40\,\Omega$$

$$40 = \frac{R_1(20+60)}{R_1+(20+60)} \quad \Rightarrow \quad R_1 = 80\,\Omega$$

P 3.6-6

Solution:



P 3.6-7

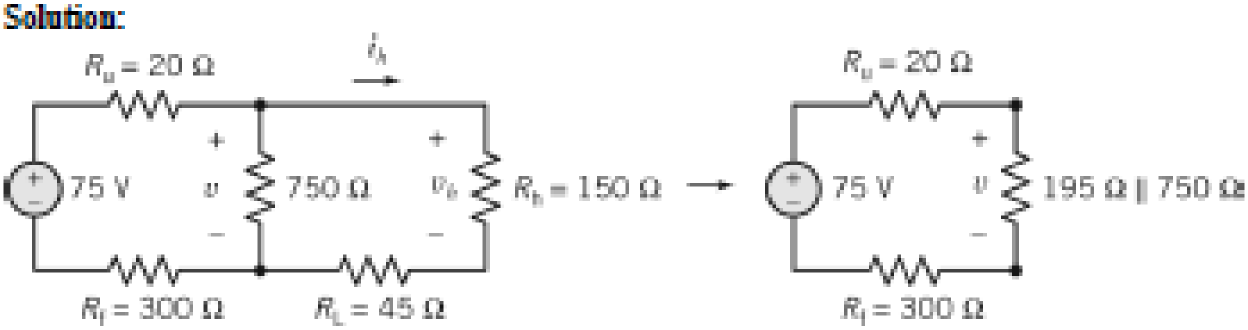
Solution:

$$1\times10^{-3} = \frac{48}{24\times10^{3}+R_p} \Rightarrow R_p = 24\times10^{3} = 24\ \text{k}\Omega$$

$$24\times10^{3} = R_p = \frac{\left(42\times10^{3}\right)R}{\left(42\times10^{3}\right)+R} \Rightarrow R = 56\ \text{k}\Omega$$

P 3.6-8

Solution:



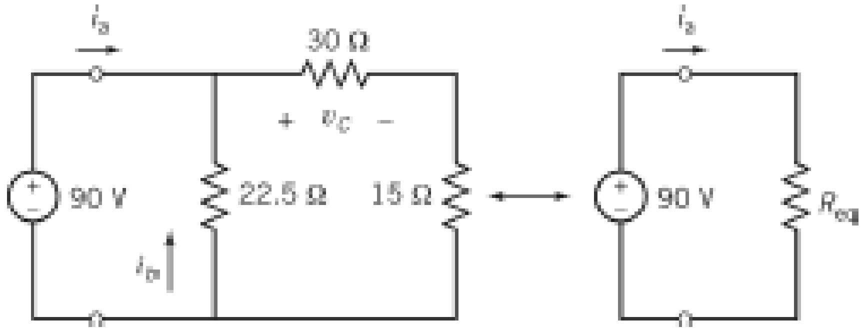
Voltage divison $\Rightarrow v = 75\left(\frac{195 \| 750}{195 \| 750+300+20}\right) = 24.5\ \text{V}$

$$\therefore v_h = v\left(\frac{150}{150+45}\right) = 24.5\left(\frac{10}{13}\right) = \underline{18.8\ \text{V}}$$

$$\therefore\ i_h = \frac{v_h}{150} = \underline{0.12533\ A}$$

P 3.6-9

Solution:



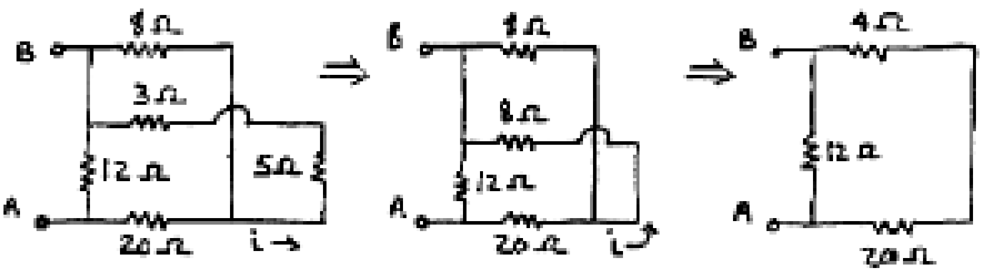
$$R_{eq} = \frac{22.5\,(30+15)}{22.5+(30+15)} = 15\ \Omega$$

$$i_a = -\frac{90}{R_{eq}} = -6\text{ A},\quad i_b = \left(\frac{45}{45+22.5}\right)\left(\frac{90}{R_{eq}}\right) = 4\text{ A},\quad v_c = \left(\frac{30}{30+15}\right)(-90) = -60\text{ V}$$

P 3.6-10

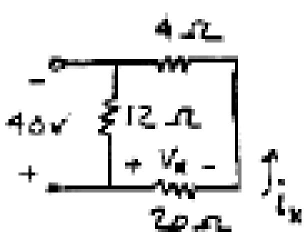
Solution:

a)



$$R_{eq} = 24\|12 = \frac{(24)(12)}{24+12} = \underline{8\,\Omega}$$

b)



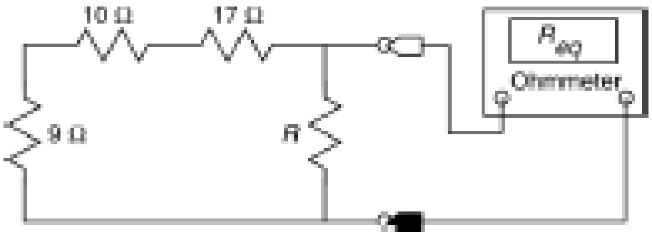
from voltage division:

$$v_x = 40\left(\frac{20}{20+4}\right) = \frac{100}{3}\text{V} \therefore i_x = \frac{\frac{100}{3}}{20} = \frac{5}{3}\text{A}$$

from current division: $i = i_x\left(\frac{8}{8+8}\right) = \underline{\frac{5}{6}\text{ A}}$

P 3.6-11

Solutipn:



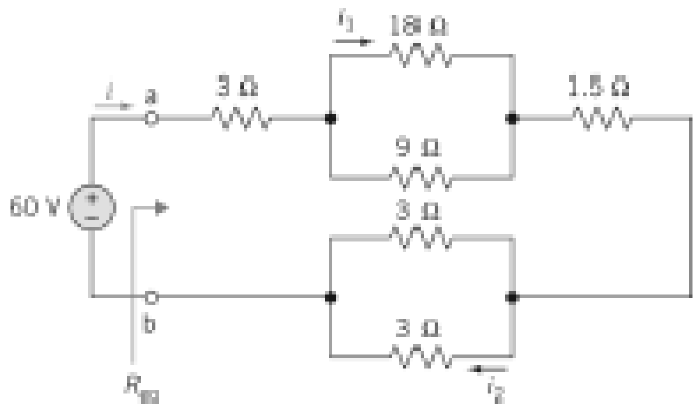
$9+10+17=36\ \Omega$

a.) $\dfrac{36(9)}{36+9}=7.2\ \Omega$

b.) $\dfrac{36\,\text{R}}{36+\text{R}}=12 \;\Rightarrow\; 24\,R=(12)(36) \;\Rightarrow\; R=18\ \Omega$

P 3.6-12

Solution:



$\text{R}_{\text{eq}}=3+1.5+\left(9\|18\right)+\left(3\|3\right)=4.5+6+1.5=\underline{12\ \Omega}$ so $i=\dfrac{60}{R_{eq}}=\dfrac{60}{12}=\underline{5\text{ A}}$

Using current division

$\text{i}_1=\text{i}\left(\dfrac{9}{9+18}\right)=(5)\left(\tfrac{1}{3}\right)=\underline{\tfrac{5}{3}\text{ A}}$ and $i_2=i\left(\dfrac{3}{3+3}\right)=(5)\left(\tfrac{1}{2}\right)=\underline{\tfrac{5}{2}\text{ A}}$

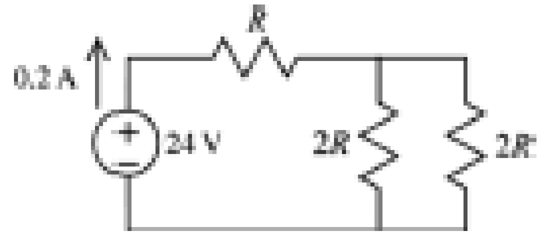
P 3.6-13

Solution:

$$(R \| 4R) + (2R \| 3R) = \frac{4}{5}R + \frac{6}{5}R = 2R$$

$$R + \left(2R \| \left(R + (2R \| 2R)\right)\right) = R + (2R \| 2R) = 2R$$

So the circuit is equivalent to

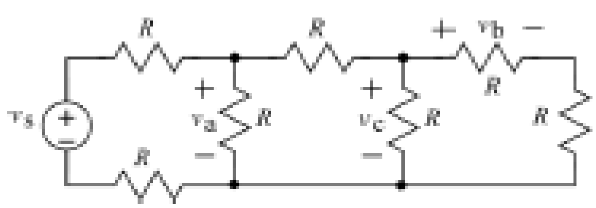


Then

$$24 = 0.2\left(R + (2R \| 2R)\right) = 0.2(2R) \quad \Rightarrow \quad R = 60\ \Omega$$

P 3.6-14

Solution:
The circuit can be redrawn as



$$v_a = \frac{R \| \left(R + (R \| 2R)\right)}{2R + R \| \left(R + (R \| 2R)\right)} v_s = \frac{5}{21} v_s$$

$$v_c = \frac{R \| 2R}{R + (R \| 2R)} v_a = \frac{2}{5} v_a = \frac{2}{21} v_s$$

$$v_b = \frac{R}{R + R} v_c = \frac{1}{2} v_c = \frac{1}{21} v_s$$

(Checked using LNAP 5/23/04)

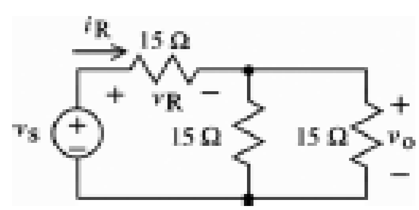
P 3.6-15

Solution:

$$v_o = \frac{(15 \| 15)}{15 + (15 \| 15)} v_s = \frac{7.5}{22.5} v_s = \frac{v_s}{3}$$

$$v_R + v_o - v_s = 0 \quad \Rightarrow \quad v_R = \frac{2}{3} v_s$$

$$i_R = \frac{v_R}{15} = \frac{2}{45} v_s$$



$$P = \left(\frac{2}{45} v_s\right)^2 (15) = \frac{4}{135} v_s^2 \le \frac{1}{4} \quad \Rightarrow \quad |v_s| \le \sqrt{\frac{135}{16}} = \frac{3\sqrt{15}}{4} = 2.9\ \text{V}$$

P 3.6-16

Solution:

The voltage across each strain gauge is $\frac{v_s}{2}$ so the current in each strain gauge is $\frac{v_s}{400}$. The power dissipated by each resistor is given by $\frac{v_s}{2}\left(\frac{v_s}{400}\right)=\frac{v_s^{\,2}}{800}$ so we require $0.5\times10^{-3}\leq\frac{v_s^{\,2}}{800}$ or $\left|v_s\right|\leq\sqrt{0.4}=0.6325\text{ V}$.

P 3.6-17

Solution:

(a)

$$R_1=15\,||\,(45+15)=12\,\Omega,\quad R_2=6+(27\,||\,13.5)=15\,\Omega$$

$$\text{and }\ R_3=9\,||\,(9+9)=6\,\Omega$$

(b)

$$i=1\text{ A},\quad v_1=12\text{ V}\ \text{ and }\ v_2=6\text{ V}$$

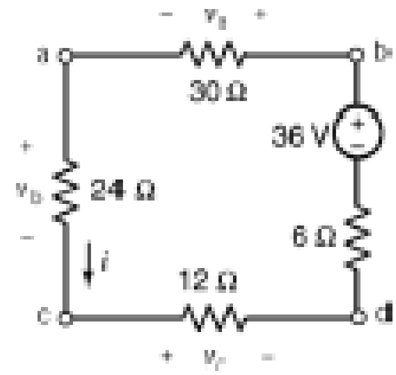
(c)

$$v_4=-\frac{15}{15+45}12=-3\text{ V},\quad i_5=-\frac{13.5}{13.5+27}1=-\frac{1}{3}\text{ A},$$

$$v_5=-27\left(-\frac{1}{3}\right)=+9\text{ V}\ \text{ and }\ i_6=\frac{6}{18}=\frac{1}{3}\text{ A}$$

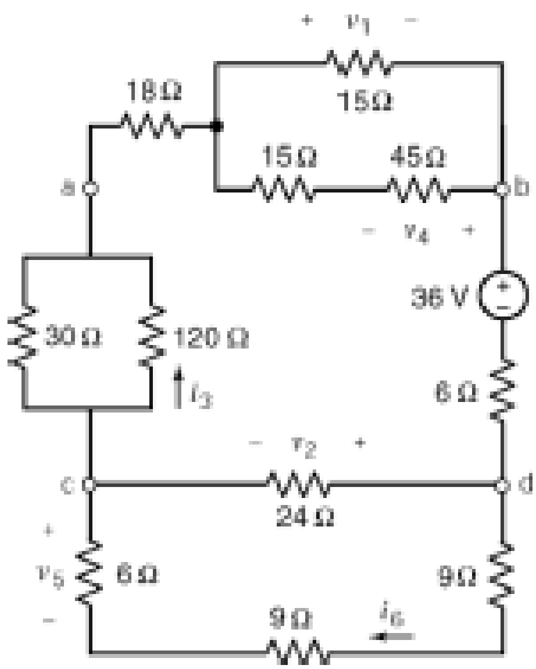
P3.6-18

Solution:
Replace series and parallel combinations of resistances by equivalent resistances. Then KVL gives



$$(30+6+12+24)i = 36 \quad \Rightarrow \quad i = 0.5 \text{ A}$$

$$v_a = 30i = 15 \text{ V}, \; v_b = 24i = 12 \text{ V and } v_c = 12i = 6 \text{ V}$$



Compare the original circuit to the equivalent circuit to get

$$v_1 = -\left(\frac{15 \| (15+45)}{18+15 \| (15+45)}\right) v_a = -\left(\frac{12}{18+12}\right) 15 = -6 \text{ V}$$

$$v_2 = -v_c = -6 \text{ V}$$

$$i_3 = -\left(\frac{30}{30+120}\right) i = -\left(\frac{1}{5}\right)(0.5) = -0.1 \text{ A}$$

$$v_4 = -\left(\frac{45}{15+45}\right) v_1 = -\left(\frac{3}{4}\right)(-4) = 3 \text{ V}$$

$$v_5 = \left(\frac{6}{6+9+9}\right) v_c = \left(\frac{1}{4}\right)(6) = 1.5 \text{ V}$$

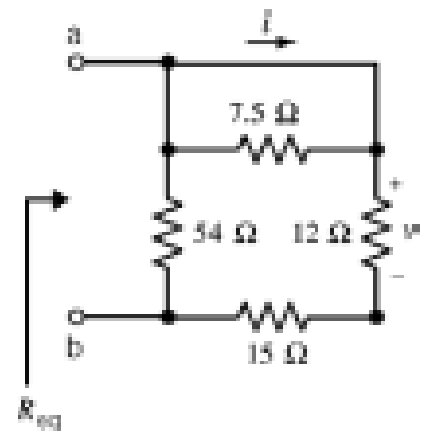
$$i_6 = -\left(\frac{24}{24+(6+9+9)}\right) i = -\left(\frac{1}{2}\right)(0.5) = 0.25 \text{ A}$$

P3.6-19

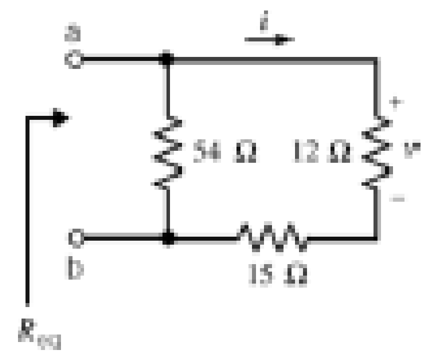
Solution:

Replace parallel resistors by equivalent resistors:

$$9 \parallel 45 = 7.5\ \Omega \text{ and } 108 \parallel 13.5 = 12\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.



$$R_{eq} = 54 \parallel (12+15) = 18\ \Omega$$

$$v = \frac{12}{12+15} v_{ab} = \frac{12}{27}(27) = 12\text{ V}$$

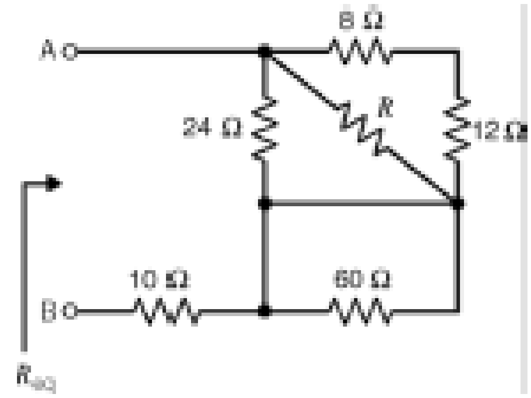
$$i = \frac{v}{12} = 1\text{ A}$$

P 3.6-20

Solution:

Replace parallel resistors by an equivalent resistor:

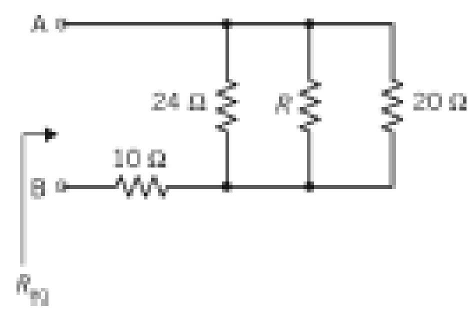
$$16 \,||\, 48 = 12\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.

Replace series resistors by an equivalent resistor:

$$8+12 = 20\ \Omega$$



Now

$$18 = R_{eq} = 10 + (24 \,||\, R \,||\, 20)$$

so

$$8 = \frac{R \times \frac{120}{11}}{R + \frac{120}{11}} \quad \Rightarrow \quad R = 30\ \Omega$$

P 3.6-21

Solution:

$$R_{eq} = (R \,||\, (R+R) \,||\, R) \,||\, (R \,||\, (R+R) \,||\, R)$$

$$R \,||\, (R+R) \,||\, R = 2R \,||\, \frac{R}{2} = \frac{2}{5} R$$

$$R_{eq} = \frac{2}{5} R \,||\, \frac{2}{5} R = \frac{R}{5} \quad \Rightarrow \quad R = 5\, R_{eq} = 200\ \Omega$$

(checked: LNAP 6/21/04)

P 3.6-22

Solution:

$$i_a = \frac{9.74}{8} = 1.2175 \text{ A}$$

$$9.74 - 6.09 = r\,i_a = r\left(\frac{9.74}{8}\right) \quad \Rightarrow \quad r = \left(\frac{9.74\text{-}6.09}{9.74}\right)8 = 3 \ \frac{\text{V}}{\text{A}}$$

$$v_b = 12 - 9.74 = 2.26 \text{ V}$$

$$g v_b + \frac{6.09}{8} + \frac{9.74}{8} - \frac{2.26}{8} = 0 \quad \Rightarrow \quad g v_b = -1.696 \text{ A}$$

$$g = \frac{g v_b}{v_b} = \frac{-6.696}{2.26} = -0.75$$

(checked: LNAP 6/21/04)

P 3.6-23

Solution:

$$v_a = \frac{30 \| 30}{30 + (30 \| 30)} v_s = \frac{1}{3} v_s$$

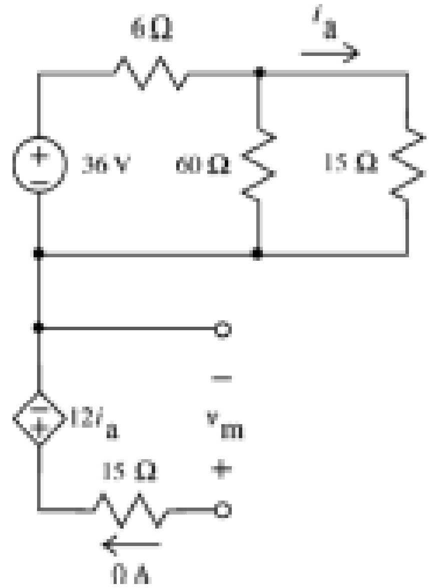
$$v_o = \left(\frac{18}{18+12}\right)(15 v_a) = \frac{3}{5} \times 15 \times \frac{1}{3} v_s = 3 v_s$$

So v_o is proportional to v_s and the constant of proportionality is $3 \ \frac{\text{V}}{\text{V}}$.

P 3.6-24

Solution:

Replace the voltmeter by the equivalent open circuit and label the voltage measured by the meter as v_m.

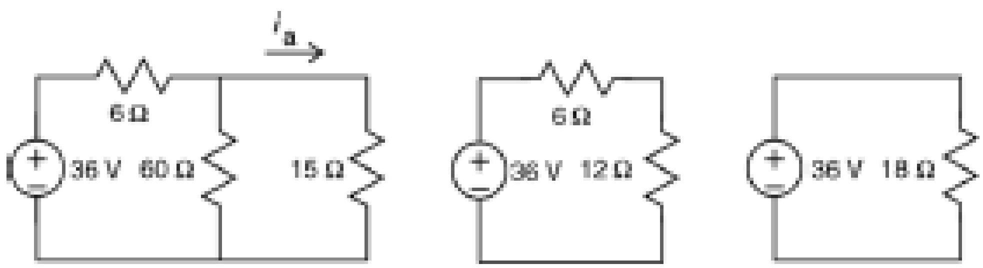


The 15-Ω resistor at the right of the circuit is in series with the open circuit that replaced the voltmeter so its current is zero as shown. Ohm's law indicates that the voltage across that 15-Ω resistor is also zero. Applying KVL to the mesh consisting of the dependent voltage source, 15-Ω resistor and open circuit shows that

$$v_m = 12\, i_a$$

The 15-Ω resistor and 60-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

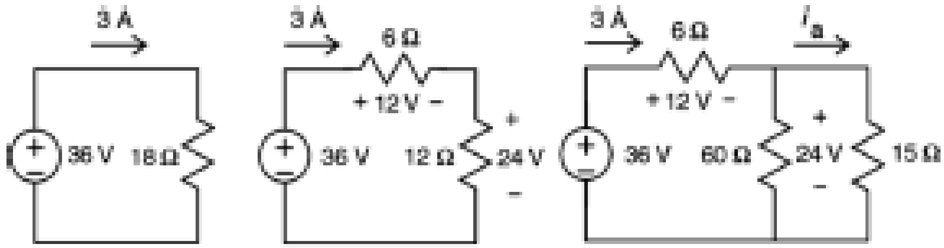
$$\frac{60 \times 15}{60+15} = 12\ \Omega$$



(a) (b) (c)

Figure a shows part of the circuit. In Figure b, an equivalent resistor has replaced the parallel resistors. Now the 6-Ω resistor and 12-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single resistor with a resistance equal to $6+12=18\ \Omega$. In Figure c, an equivalent resistor has replaced the series resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



(a) (b) (c)

Consider Figure a. Using Ohm's law, we see that the current in the 18-Ω resistor is 3 A. The current in the voltage source is also 3 A. Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the voltage source must also be 3 A in Figure b. The currents in resistors in Figure b are equal to the current in the voltage source. Next, Ohm's law is used to calculate the resistor voltages as shown in Figure b.

Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the 6-Ω resistor in Figure c must be equal to the current in the 6-Ω resistor in Figure b. Using current division in Figure c are yields

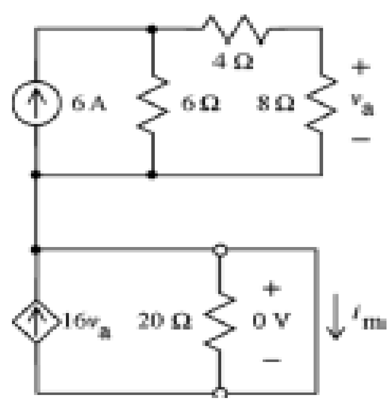
$$i_a = \left(\frac{60}{60+15}\right)3 = 2.4 \text{ A}$$

Finally,

$$v_m = 12\, i_a = 12 \times 2.4 = 28.8 \text{ V}$$

P 3.6-25

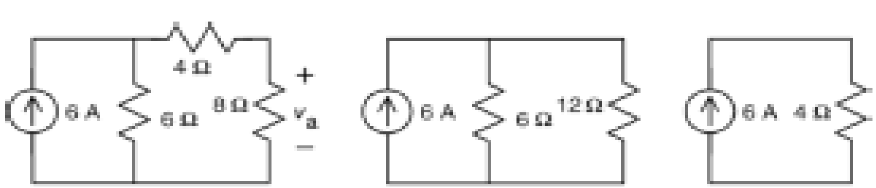
Solution:
Replace the ammeter by the equivalent short circuit and label the current measured by the meter as i_m.



The 20-Ω resistor at the right of the circuit is in parallel with the short circuit that replaced the ammeter so its voltage is zero as shown. Ohm's law indicates that the current in that 20-Ω resistor is also zero. Applying KCL at the top node of that 20-Ω resistor shows that

$$i_m = 16\, v_a$$

Figure a shows part of the circuit. The 4-Ω resistor and 8-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single 12-Ω resistor.



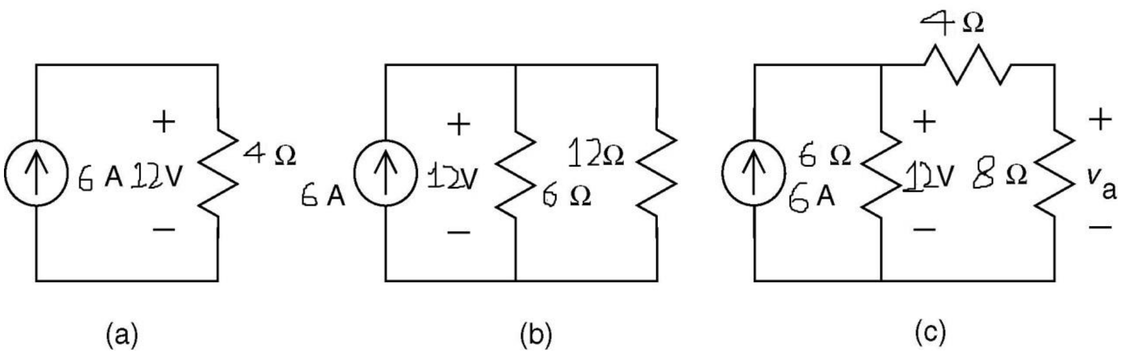
(a) (b) (c)

In Figure b, an equivalent resistor has replaced the series resistors. Now the 3-Ω resistor and 6-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

$$\frac{6\times 12}{6+12}=4\ \Omega$$

In Figure c, an equivalent resistor has replaced the parallel resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



Consider Figure a. Using Ohm's law, we see that the voltage across the 2-Ω resistor is 6 V. The voltage across the current source is also 6 V. Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the current source must also be 6 V in Figure b. The voltage across each resistor in Figure b is equal to the voltage across the current source.

Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the 3-Ω resistor in Figure c must be equal to the voltage across the 3-Ω resistor in Figure b. Using voltage division in Figure c yields

$$v_a=\left(\frac{8}{4+8}\right)12=8\ \mathrm{V}$$

Finally,

$$i_m=16\,v_a=16\times 8=128\ \mathrm{V}$$

P 3.6-26

Solution:
Use current division in the top part of the circuit to get

$$i_a = \left(\frac{80}{80+20}\right)(-6) = -4.8 \text{ A}$$

Next, denote the voltage measured by the voltmeter as v_m and use voltage division in the bottom part of the circuit to get

$$v_m = \left(\frac{R}{36+R}\right)(-10\, i_a) = \left(\frac{-10\, R}{36+R}\right) i_a$$

Combining these equations gives:

$$v_m = \left(\frac{-10\, R}{36+R}\right)(-4.8) = \frac{48\, R}{36+R}$$

When $v_m = 6$ V,

$$6 = \frac{48\, R}{36+R} \quad \Rightarrow \quad R = \frac{6\times 36}{48-6} = 5.1\ \Omega$$

P 3.6-27

Soluton:
Use voltage division in the top part of the circuit to get

$$v_a = \left(\frac{16}{16+22}\right)(-v_s) = -\frac{8}{19} v_s$$

Next, use current division in the bottom part of the circuit to get

$$i_m = -\left(\frac{20}{20+R}\right)(5\, v_a) = \left(-\frac{100}{20+R}\right) v_a$$

Combining these equations gives:

$$i_m = \left(-\frac{100}{20+R}\right)\left(-\frac{8}{19} v_s\right) = \left(\frac{800}{380+19R}\right) v_s$$

a. When $v_s = 20$ V and $i_m = 17$ A

$$17 = \left(\frac{800}{380+19R}\right) 20 \quad \Rightarrow \quad 6460 + 323R = 16000 \quad \Rightarrow \quad R = 29.54\,\Omega$$

b. When $v_s = 20$ V and $R = 85\ \Omega$

$$i_m = \left(\frac{800}{380+19(85)}\right)(20) = 8.02 \text{ A}$$

c. When $i_m = 4$ A and $R = 32\ \Omega$

$$4 = \left(\frac{800}{380+19(32)}\right) v_s = 0.8\, v_s \quad \Rightarrow \quad v_s = \frac{4}{0.8} = 5 \text{ V}$$

P 3.6-28

Solution:

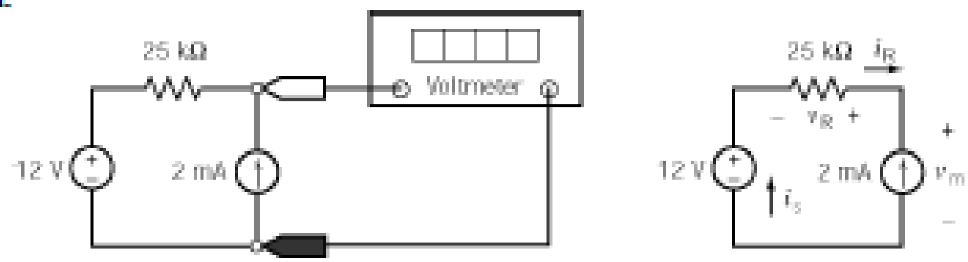
$$R_{eq} = \left((R+8)\,\|\,40\right)+4 = \frac{(R+8)\times 40}{(R+8)+40}+4 = \frac{40\,R+320}{R+48}+4$$

a. $$24 = \frac{40\,R+160}{R+48}+4 \;\Rightarrow\; 20 = \frac{40\,R+160}{R+48} \;\Rightarrow\; R+48 = 2\,R+8 \;\Rightarrow\; R = 40\ \Omega$$

b. $$R_{eq} = \frac{40(14)+320}{14+48}+4 = 11.7\ \Omega$$

P 3.6-29

Solution:



(a) (b)

Replace the ideal voltmeter with the equivalent open circuit and label the voltage measured by the meter. Label the element voltages and currents as shown in (*b*).

Using units of V, A, Ω and W:	**Using units of V, mA, kΩ and mW:**
a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R = v_m$ and $-i_R = -i_s = 2\times10^{-3}$ A Ohm's law gives $v_R = -\left(25\times10^{3}\right) i_R$ Then $v_R = -\left(25\times10^{3}\right) i_R = -\left(25\times10^{3}\right)\left(-2\times10^{-3}\right) = 50$ V $v_m = 12+v_R = 12+50 = 62$ V	a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R = v_m$ and $-i_R = -i_s = 2$ mA Ohm's law gives $v_R = -25\,i_R$ Then $v_R = -25\,i_R = -25(-2) = 50$ V $v_m = 12+v_R = 12+50 = 62$ V

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2\times10^{-3})$ $= -24\times10^{-3}$ W
current source	$62(2\times10^{-3}) = 124\times10^{-3}$ W
resistor	$v_R\, i_R = 50(-2\times10^{-3})$ $= -100\times10^{-3}$ W
total	0

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2)$ $= -24$ mW
current source	$62(2) = 124$ mW
resistor	$v_R\, i_R = 50(-2)$ $= -100$ mW
total	0

P3.6-30

Solution:

$$12 + \frac{40\times10}{40+10} + 4 = 12\Omega$$

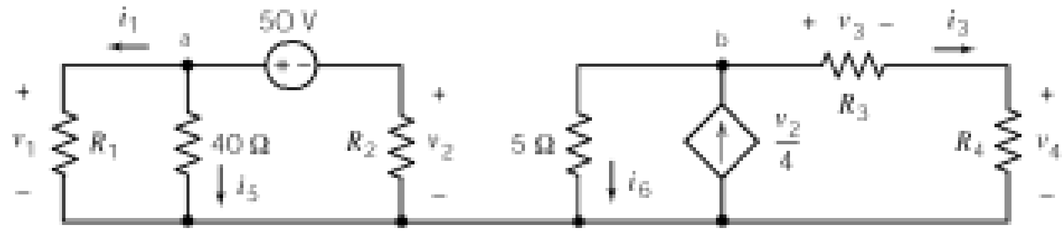
P3.6-31

Solution:

$$\frac{(60+60+60)\times60}{(60+60+60)+60} = 45\Omega$$

P3.6-32

Solution:



From KVL $\quad 50+v_2-v_1=0 \;\Rightarrow\; v_1=50+(-25)=25\text{ V}$

From Ohm's law $\quad R_1=\dfrac{v_1}{i_1}=\dfrac{25}{0.625}=40\;\Omega$

From KCL

$$i_1+i_5+i_2=0 \;\Rightarrow\; i_2=-(i_1+i_5)=-\left(0.625+\frac{v_1}{40}\right)=-\left(0.625+\frac{25}{40}\right)=-1.25\text{ A}$$

From Ohm's law $\quad R_2=\dfrac{v_2}{i_2}=\dfrac{-25}{-1.25}=20\;\Omega$

From KCL $\quad \dfrac{v_2}{4}=i_6+i_3 \;\Rightarrow\; i_6=-i_3+\dfrac{v_2}{4}=-(-1.25)+\dfrac{-25}{4}=-5\text{ A}$

From KVL $\quad v_3+v_4-5i_6=0 \;\Rightarrow\; v_3=-v_4+5i_6=-(-18.75)+5(-5)=-6.25\text{ V}$

From Ohm's law $\quad R_3=\dfrac{v_3}{i_3}=\dfrac{-6.25}{-1.25}=5\;\Omega$ and $R_4=\dfrac{v_4}{i_3}=\dfrac{-18.75}{-1.25}=15\;\Omega$

P3.6-33

Solution:

$$R_1 \,\|\, 18=6 \;\Rightarrow\; \frac{18R_1}{18+R_1}=6 \;\Rightarrow\; 3R_1=18+R_1 \;\Rightarrow\; R_1=9\;\Omega$$

$$R_2+10=28 \;\Rightarrow\; R_2=18\;\Omega$$

$$40=(6+28+R_3)i_a \;\Rightarrow\; \frac{40}{0.8}=34+R_3 \;\Rightarrow\; R_3=50-34=16\;\Omega$$

$$v_2=-10i_a=-10(0.8)=-8\text{ V and } i_3=-\frac{32}{32+32}i_a=-\frac{0.8}{2}=-0.4\text{ A}$$

P3.6-34

Solution:

From voltage division $v_4 = \frac{R_4}{50+R_4} v_2$

so $$\frac{R_4}{50+R_4} = \frac{3}{8} \quad \Rightarrow \quad 8R_4 = 3\left(50+R_4\right) \quad \Rightarrow \quad R_4 = \frac{150}{8-3} = 30\ \Omega\,.$$

From current division $i_3 = \frac{R_2}{R_2+\left(50+R_4\right)} i_1 = \frac{R_2}{R_2+80} i_1$

so $$\frac{R_2}{R_2+80} = \frac{1}{5} \quad \Rightarrow \quad 5R_2 = R_2+80 \quad \Rightarrow \quad R_2 = 20\ \Omega\,.$$

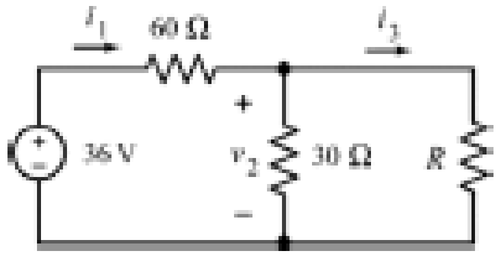
Notice that $R_2 \,||\left(50+R_4\right) = 20\,||\left(50+30\right) = 20\,||\,80 = 16\ \Omega$. From voltage division

$$v_1 = \frac{R_2 \,||\left(50+R_4\right)}{R_1+\left(R_2 \,||\left(50+R_4\right)\right)} v_\text{s} = \frac{16}{R_1+16} v_\text{s}$$

so $$\frac{16}{R_1+16} = \frac{2}{3} \quad \Rightarrow \quad 48 = 2\left(R_1+16\right) \quad \Rightarrow \quad R_1 = \frac{48-32}{2} = 8\ \Omega\,.$$

P3.6-35

Solution:



(a) From current division $i_3 = \frac{30}{30+R} i_1$ so $\frac{30}{30+R} = \frac{1}{3} \Rightarrow 90 = 30 + R \Rightarrow R = 60\,\Omega$.

(b) From voltage division $v_2 = \frac{R_p}{60+R_p} 36$

so $$4.8 = \frac{R_p}{60+R_p} 36 \Rightarrow \frac{4.8}{36}\left[60 + R_p\right] = R_p \Rightarrow R_p = .13 \times 60 + .13 R_p = 9\,\Omega.$$

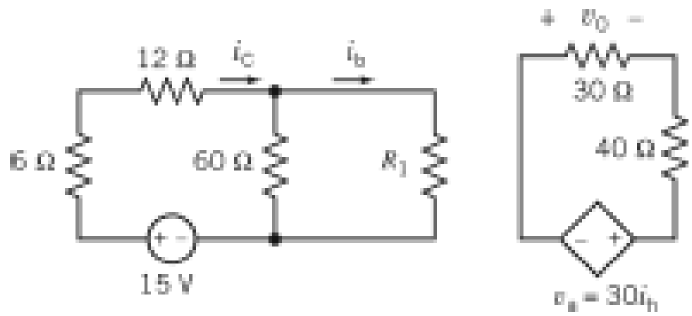
(c) $$i_1 = \frac{36}{60 + (30 \| 30)} = \frac{36}{60+15} = \frac{36}{75} = 0.48\text{ A}$$

P3.6-36

Solution:

First,
$$v_o = -\frac{30}{30+40}(12) = -5.14 \text{ V}$$

Next,



$$\frac{12}{30} = i_b = \frac{60}{60+R_1} i_c = \frac{60}{60+R_1}\left(\frac{15}{18+60 \parallel R_1}\right)$$

$$= \frac{60}{60+R_1}\left(\frac{15}{18+\dfrac{60 R_1}{60+R_1}}\right)$$

$$= \frac{900}{18(60+R_1)+60R_1}$$

$$= \frac{900}{1080+78R_1}$$

then

$$\frac{12}{30} = \frac{900}{1080+78R_1} \Rightarrow \frac{900(30)}{12} = 1080+78R_1$$

$$\Rightarrow 2250 = 1080+78R_1$$

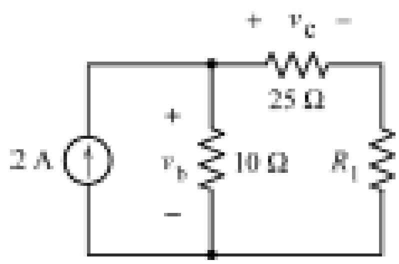
$$\Rightarrow R_1 = 15\ \Omega$$

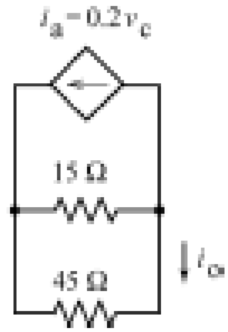
P3.6-37

Solution:
First,

$$i_o = -\frac{15}{15+45}2 = -0.5 \text{ A}$$

Next,





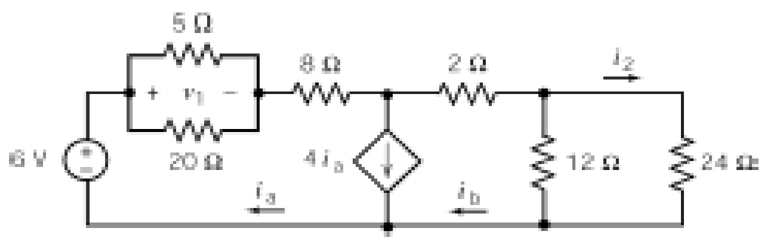
$$\frac{2}{0.2} = v_c = \frac{25}{25+R_1}v_b = \frac{25}{25+R_1}\left(2\left(10 \| 25+R_1\right)\right) = \frac{50}{25+R_1}\left(\frac{10\left(25+R_1\right)}{10+\left(25+R_1\right)}\right) = \frac{500}{35+R_1}$$

then

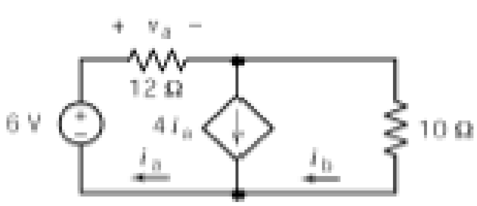
$$\frac{2}{0.2} = \frac{500}{35+R_1} \quad \Rightarrow \quad 35+R_1 = 50 \quad \Rightarrow \quad R_1 = 15\ \Omega$$

P3.6-38

Solution:



Use equivalent resistances to reduce the circuit to



From KCL $i_b = 4i_a + i_a \quad \Rightarrow \quad i_b = -3i_a$.

From KVL
$12i_a + 10i_b - 6 = 0 \quad \Rightarrow \quad 12i_a + 10\left(-3i_a\right) = 6$

So $i_a = -\frac{1}{3}$ A, $v_a = -4$ A and $i_b = 1$ A .

Returning our attention to the original circuit, notice that i_a and i_b were not changed when the circuit was reduced. Now $v_1 = (5 \| 20)i_a = (4)(-0.333) = -1.333$ V and $i_2 = \frac{12}{12+24}i_b = 0.333$ A .

P3.6-39

Solution:
Using voltage division

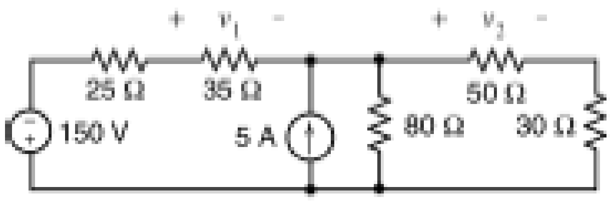
$$10.38 = v_a = \frac{12}{R_1+12}(32) \Rightarrow R_1+12 = \frac{12(32)}{10.38} = 36.9942 \approx 37\ \Omega \Rightarrow R_1 = 25\ \Omega$$

Using current division

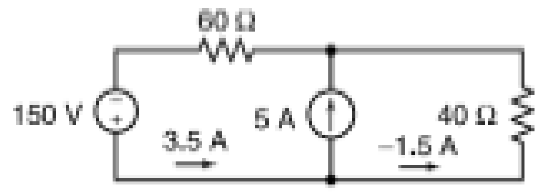
$$0.4151 = i_o = \frac{40}{40+10}G v_a = (0.8)G(10.38) \Rightarrow G = \frac{0.4151}{(0.8)10.38} = 0.05\ \frac{\text{A}}{\text{V}}$$

P3.6-40

Solution:

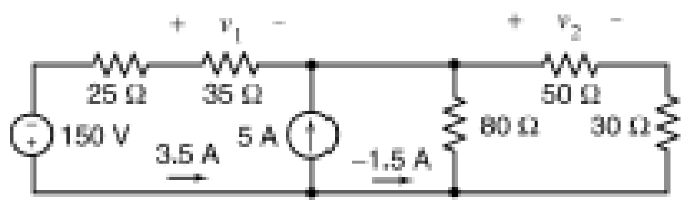


original circuit



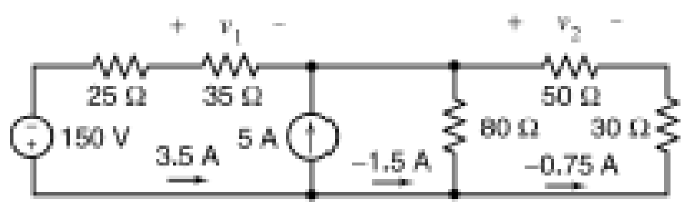
equivalent circuit

Label the currents in the equivalent circuit that correspond to the give currents in the equivalent circuit:



original circuit

Use current division:



original circuit

Using Ohm's law:

$$v_1 = -35 i_a = -35(3.5) = -122.5\ \text{V} \quad \text{and} \quad v_2 = -50(-0.75) = 37.5\ \text{V}$$

P3.6-41 Determine the values of the currents i_1 and i_2 and the voltages v_3 and v_4 in the circuit shown in Figure P3.6-41.

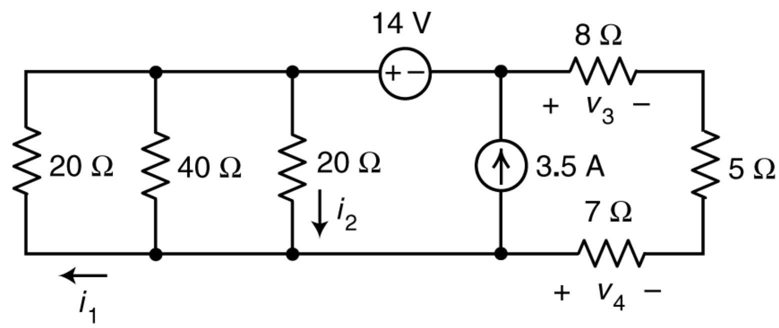
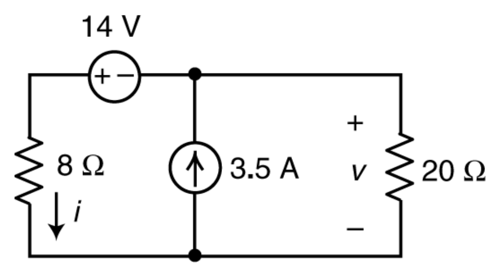


Figure P3.6-41

Solution: Replace series and parallel combinations of resistors with equivalent resistors to obtain

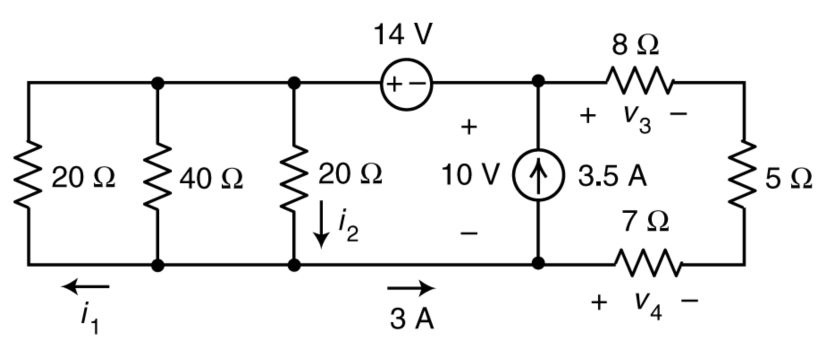


KVL: $$14+v-8i=0 \quad \Rightarrow \quad v=8i-14$$

KCL: $$i+\frac{v}{20}=3.5$$

Solving $$i=3\text{ A} \quad \text{and} \quad v=10\text{ V}$$

Labeling the current *i* and voltage *v* on the original circuit, we have



Using current division and voltage division

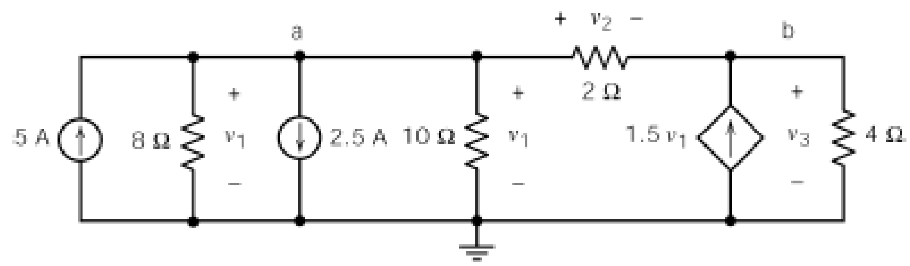
$$i_1 = -\frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{40}+\frac{1}{20}}(3) = -\frac{2}{5}(3) = -1.2 \text{ A}\text{ , } \quad i_2 = \frac{\frac{1}{20}}{\frac{1}{20}+\frac{1}{40}+\frac{1}{20}}(3) = \frac{2}{5}(3) = 1.2 \text{ A}\text{ ,}$$

and $\quad v_3 = \frac{8}{8+5+7}(10) = \frac{8}{20}(10) = 4 \text{ V}$, and $v_4 = -\frac{7}{8+5+7}(10) = -\frac{7}{20}(10) = -3.5 \text{ V}$

Section 3-7 Analyzing Resistive Circuits using MATLAB

P3.7-1

Solution:
We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element voltages:



Notice that the 8 Ω resistor, the 10 Ω resistor and the two independent current sources are all connected in parallel. Consequently, the element voltages of theses elements can be labeled so that they are equal. Similarly, the 4 Ω resistor and the dependent current source are connected in parallel so their voltages can be labeled so as to be equal.

Using Ohm's Law we see that the current directed downward in the 8 Ω resistor is $\frac{v_1}{8}$, current directed downward in the 10 Ω resistor is $\frac{v_1}{10}$, and the current directed from left to right in the 2 Ω resistor is $\frac{v_2}{2}$. Applying Kirchhoff's Current Law (KCL) at node a gives

$$5 = \frac{v_1}{8} + 2.5 + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225 v_1 + 0.5 v_2 = 2.5 \tag{1}$$

Using

Kirchhoff's Current Law (KCL) at node a gives

$$\frac{v_2}{2} + 1.5 v_1 = \frac{v_3}{4} \;\Rightarrow\; 1.5 v_1 + 0.5 v_2 - 0.25 v_3 = 0 \qquad (2)$$

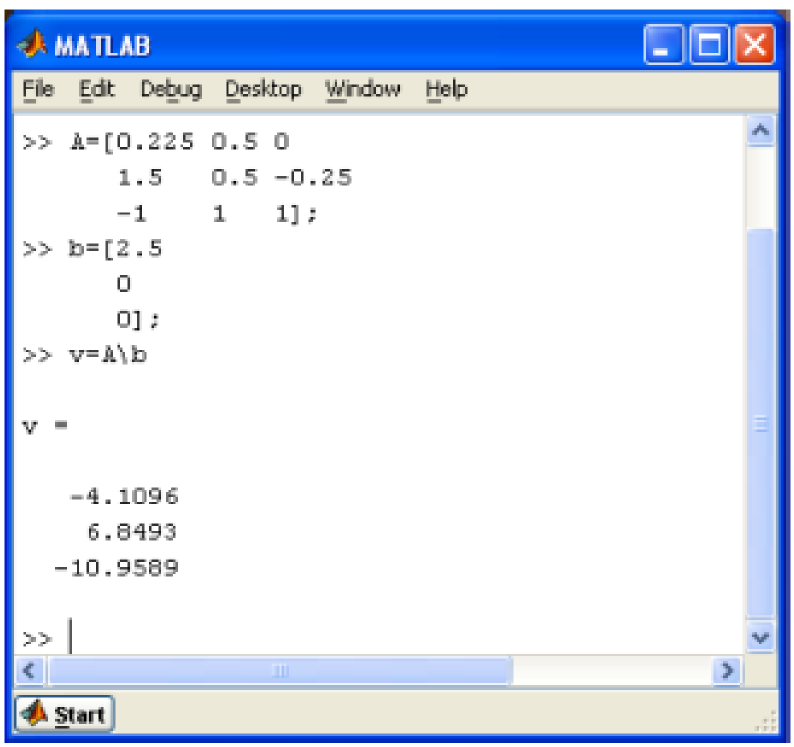
Applying Kirchhoff's Voltage Law (KVL) to the mesh consisting of the 10 Ω resistor, the 2 Ω resistor and the dependent source to get

$$v_2 + v_3 - v_1 = 0 \qquad (3)$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 0.225 & 0.5 & 0 \\ 1.5 & 0.5 & -0.25 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:



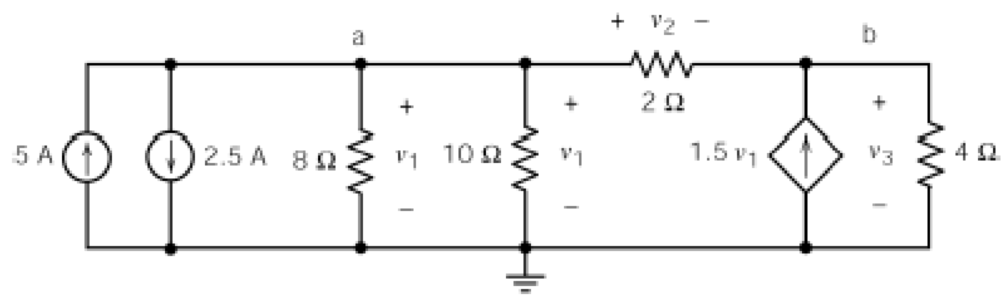
Hence

$$v_1 = -4.1096 \text{ V}, \quad v_2 = 6.8493 \text{ V} \quad \text{and} \quad v_3 = -10.9589 \text{ V}$$

The power supplied by the 5 A current source is $5v_1 = 5(-4.1096) = -20.548 \text{ W}$. The power supplied by the 2.5 A current source is $-2.5v_1 = -2.5(-4.1096) = 10.247 \text{ W}$. The power supplied by the dependent current source is $(1.5v_1)v_3 = 1.5(-4.1096)(-10.9589) = 67.555 \text{ W}$.

Observation: Changing the order of the 8 Ω resistor, the 10 Ω resistor and the two independent current sources only changes the order of the terms in the KCL equation at node a. We know that addition is commutative, so change the order of the terms will not affect the values of the voltages v_1, v_2 and v_3.

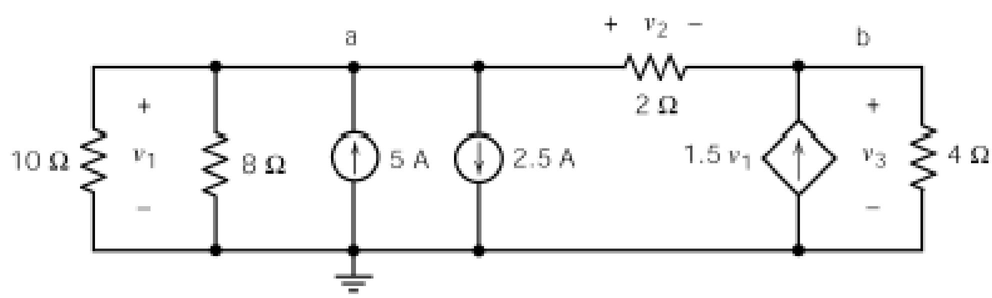
For example, if the positions of the 2.5 A current source and 8 Ω resistor are switched:



The KCL equation at node a is

$$5 = 2.5 + \frac{v_1}{8} + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

Similarly, when the circuit is drawn as



The KCL equation at node a is

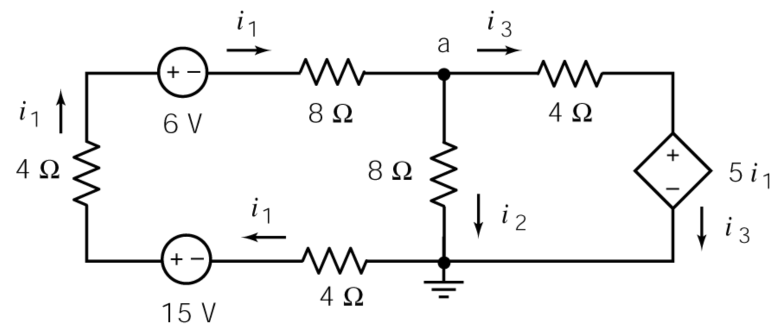
$$5 = \frac{v_1}{10} + \frac{v_1}{8} + 2.5 + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

The changes do not affect the values of the voltages v_1, v_2 and v_3.

P3.7-2

Solution:

We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element currents:



Notice that two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources are all connected in series. Consequently, the element currents of theses elements can be labeled so that they are equal. Similarly, a 4 Ω resistor and the dependent voltage source are connected in series so their currents can be labeled so as to be equal.

The current in each resistor has been labeled so we can use Ohm's Law to calculate resistor voltages from the resistor currents and the resistances. Apply Kirchhoff's Voltage Law (KVL) to the left mesh to get

$$6+8i_1+8i_2+4i_1-15+4i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9 \tag{1}$$

Apply Kirchhoff's Voltage Law (KVL) to the right mesh to get

$$4i_3+5i_1-8i_2=0 \tag{2}$$

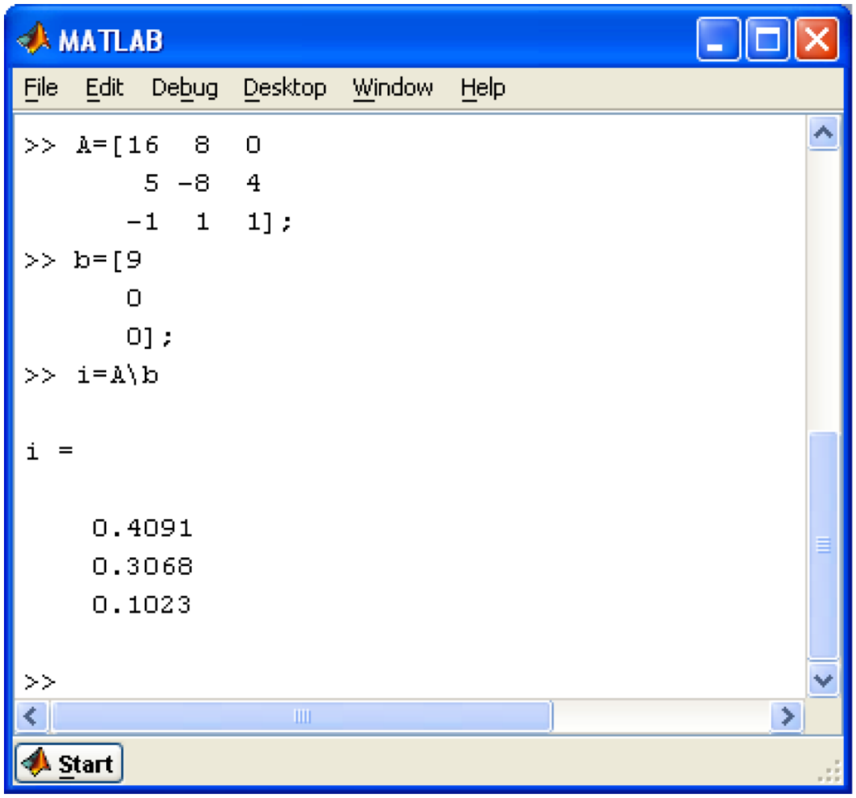
Applying Kirchhoff's Current Law (KCL) at node a to get

$$i_1=i_2+i_3 \quad \Rightarrow \quad -i_1+i_2+i_3=0 \tag{3}$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 16 & 8 & 0 \\ 5 & -8 & 4 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:

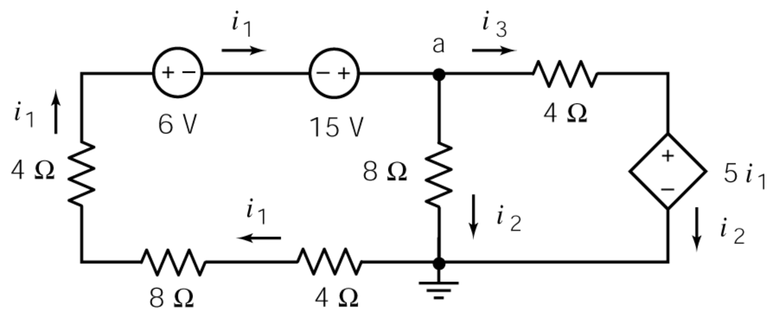


Hence

$$i_1 = 0.4091 \text{ A}, \quad i_2 = 0.3068 \text{ A} \quad \text{and} \quad i_3 = 0.1023 \text{ A}$$

The power supplied by the 15 V voltage source is $15 i_1 = 15(0.4091) = 6.1365 \text{ W}$. The power supplied by the 6 V voltage source is $-6 i_1 = -6(0.4091) = -2.4546 \text{ W}$. The power supplied by the dependent voltage source is $-(5 i_2) i_3 = -5(0.3068)(0.1023) = 0.1569 \text{ W}$.

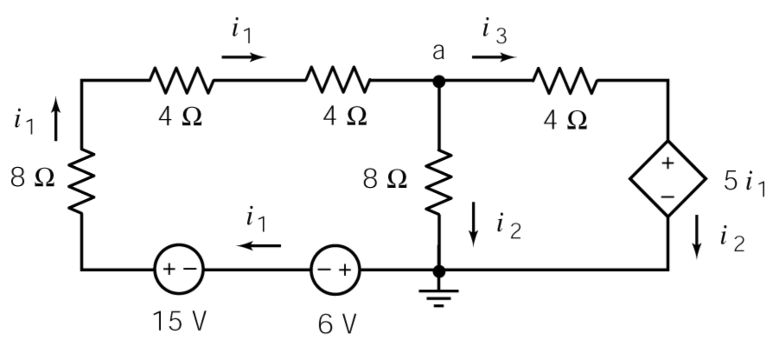
Observation: Changing the order of the two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources in the left mesh changes the order of the terms in the KVL equation for that mesh. We know that addition is commutative, so change the order of the terms will not affect the values of the currents i_1, i_2 and i_3. For example, when the circuit is drawn as



The KVL equation for the left mesh is

$$6-15+8i_2+4i_1+8i_1+4i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9$$

When the circuit is drawn as



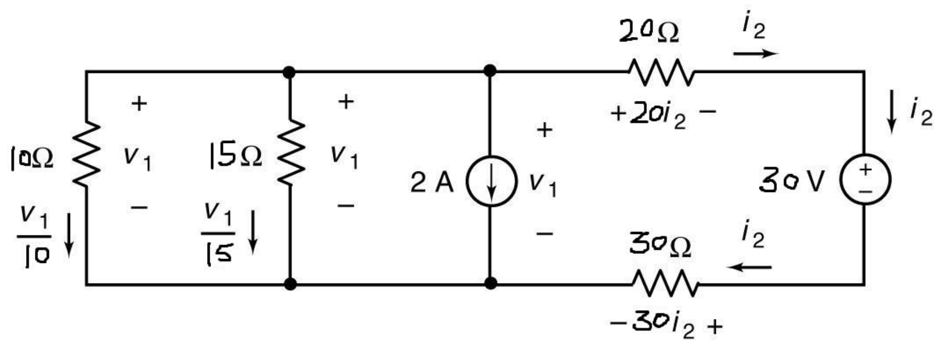
The KVL equation for the left mesh is

$$4i_1+4i_1+8i_2+6-15+8i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9$$

These changes do not affect the values of the currents i_1, i_2 and i_3.

P3.7-3

Label the element currents and voltages as suggested in Table 3.7-1 Guidelines for Labeling Circuit Variables:



Apply KCL at the top left node: $\frac{v_1}{10}+\frac{v_1}{15}+2+i_2=0$

Apply KVL to the left mesh: $20i_2-30+30i_2-v_1=0$

In matrix form:

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{15} & 1 \\ -1 & 20+30 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -2 \\ 30 \end{bmatrix}$$

Solving using MATLAB:

$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -7.5 \\ 0.75 \end{bmatrix}$$

The current source supplies $-2v_1=-2(-7.5)=11.1428\text{ W}$

The voltage source supplies $30i_2=30(0.75)=22.5$ W

Section 3-8 How Can We Check …

P 3.8-1

Solution:

KCL at node a: $i_3 = i_1 + i_2$

$$-1.167 = -0.833 + (-0.333)$$

$$-1.167 = -1.166 \text{ OK}$$

KVL loop consisting of the vertical 6 Ω resistor, the 3 Ω and4Ω resistors, and the voltage source:

$$6i_3 + 3i_2 + v + 12 = 0$$

yields $\underline{v = -4.0 \text{ V}}$ $\underline{\text{not}}$ $v = -2.0$ V

The answers are not correct.

P 3.8-2

Solution:

Apply current division to get: $i = \left(\dfrac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{5+5}} \right) 5 = \left(\frac{1}{4} \right) 5 = 1.25 \text{ A}$

The answer is correct.

P 3.8-3

Solution:

KVL bottom loop: $-14 + 0.1 i_A + 1.2 i_H = 0$

KVL right loop: $-12 + 0.05 i_B + 1.2 i_H = 0$

KCL at left node: $i_A + i_B = i_H$

Solving the three above equations yields:

$\underline{i_A = 16.8 \text{ A}}$ $\underline{i_H = 10.3 \text{ A}}$ and $\underline{i_B = -6.49 \text{ A}}$

These are not the given values. Consequently, ***the report is incorrect.***

P 3.8-4

Solution:

Top mesh: $0 = 4\,i_a + 4\,i_a + 2\left(i_a + \frac{1}{2} - i_b\right) = 10\,(-0.5) + 1 - 2\,(-2)$

Lower left mesh: $v_s = 10 + 2\left(i_a + 0.5 - i_b\right) = 10 + 2\left(2\right) = 14\text{ V}$

Lower right mesh: $v_s + 4\,i_a = 12 \quad \Rightarrow \quad v_s = 12 - 4\,(-0.5) = 14\text{ V}$

The KVL equations are satisfied so ***the analysis is correct***.

P 3.8-5

Solution:

(a)

$$\begin{aligned} 7 + \left(-3\right) &= 4 \qquad \text{(node } a\text{)} \\ 4 + \left(-2\right) &= 2 \qquad \text{(node } b\text{)} \\ -5 &= -2 + \left(-3\right) \quad \text{(node } c\text{)} \end{aligned}$$

(b)

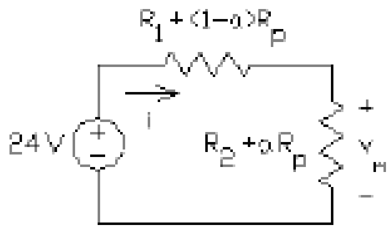
$$\begin{aligned} -1 - \left(-6\right) + \left(-8\right) + 3 = 0 \quad &\text{(loop } a\text{ - }b\text{ - }d\text{ - }c\text{ - }a\text{)} \\ -1 - 2 - \left(-8\right) - 5 = 0 \quad &\text{(loop } a\text{ - }b\text{ - }c\text{ - }d\text{ - }a\text{)} \end{aligned}$$

The given currents and voltages satisfy these five Kirchhoff's laws equations

Design Problems

DP 3-1

Solution:



Using voltage division:

$$v_m = \frac{R_2 + aR_p}{R_1 + (1-a)R_p + R_2 + aR_p} 24 = \frac{R_2 + aR_p}{R_1 + R_2 + R_p} 24$$

$$v_m = 8 \text{ V when } a = 0 \Rightarrow \frac{R_2}{R_1 + R_2 + R_p} = \frac{1}{3}$$

$$v_m = 12 \text{ V when } a = 1 \Rightarrow \frac{R_2 + R_p}{R_1 + R_2 + R_p} = \frac{1}{2}$$

The specification on the power of the voltage source indicates

$$\frac{24^2}{R_1 + R_2 + R_p} \le \frac{1}{2} \quad \Rightarrow \quad R_1 + R_2 + R_p \ge 1152\ \Omega$$

Try $R_p = 2000\ \Omega$. Substituting into the equations obtained above using voltage division gives $3R_2 = R_1 + R_2 + 2000$ and $2\left(R_2 + 2000\right) = R_1 + R_2 + 2000$. Solving these equations gives $R_1 = 6000\ \Omega$ and $R_2 = 4000\ \Omega$.

With these resistance values, the voltage source supplies 48 mW while R_1, R_2 and R_p dissipate 24 mW, 16 mW and 8 mW respectively. Therefore the design is complete.

DP 3-2

Solution:

Try $R_1 = \infty$. That is, R_1 is an open circuit. From KVL, 8 V will appear across R_2. Using voltage division, $\frac{200}{R_2 + 200} 12 = 4 \quad \Rightarrow \quad R_2 = 400\ \Omega$. The power required to be dissipated by R_2 is $\frac{8^2}{400} = 0.16 \text{ W} < \frac{1}{8} \text{ W}$.

To reduce the voltage across any one resistor, let's implement R_2 as the series combination of two 200 Ω resistors. The power required to be dissipated by each of these resistors is $\frac{4^2}{200} = 0.08 \text{ W} < \frac{1}{8} \text{ W}$.

Now let's check the voltage:

$$11.88 \frac{190}{190 + 420} < v_0 < 12.12 \frac{210}{210 + 380}$$

$$3.700 < v_0 < 4.314$$

$$4 - 7.5\% < v_0 < 4 + 7.85\%$$

Hence, $v_o = 4 \text{ V} \pm 8\%$ and the design is complete.

DP 3-3

Solution:

$$V_{ab} \cong 200 \text{ mV}$$

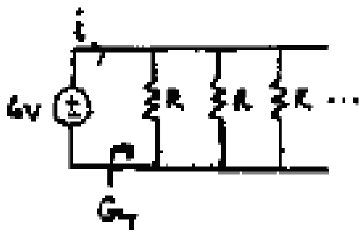
$$v = \frac{10}{10+R}\ 120\, V_{ab} = \frac{10}{10+R}\ (120)\ (0.2)$$

$$\text{let } v = 16 = \frac{240}{10+R} \Rightarrow \underline{R = 5\ \Omega}$$

$$\therefore\ \text{P} = \frac{16^2}{10} = \underline{25.6\text{W}}$$

DP 3-4

Solution:



$$i = G_T v = \frac{N}{R} v \quad \text{where } G_T = \sum_{n=1}^{N} \frac{1}{R_n} = N\left(\frac{1}{R}\right)$$

$$\therefore\ N = \frac{iR}{v} = \frac{(9)(12)}{6} = \underline{18 \text{ bulbs}}$$

DP 3-5

Solution:

Notice that $$g = \frac{R_2}{R_1 + R_2} \Rightarrow g\, R_1 = (1-g) R_2$$

Thus either resistance can be determined from the other resistance and the gain of the voltage divider. Also

$$g = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_{in}} \Rightarrow R_2 = g\, R_{in}$$

Consequently $$g\, R_1 = (1-g) R_2 = (1-g)\, g\, R_{in} \Rightarrow R_1 = (1-g) R_{in}$$

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then $$g\, R_1 = (1-g) R_2 \Rightarrow R_2 = \frac{g\, R_1}{1-g} = \frac{0.65 \times 100}{1-0.65} = 186\ \Omega$$

(b) $$R_2 = g\, R_{in} = 0.65 \times 2500 = 1625\ \Omega$$

and $$R_1 = (1-g) R_{in} = (1-0.65) 2500 = 875\ \Omega$$

DP 3-6

Solution:

Notice that $$g = \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad g\,R_2 = (1-g)\,R_1$$

Thus either resistance can be determined from the other resistance and the gain of the current divider. Also

$$g = \frac{R_1}{R_1 + R_2} = \frac{R_{in}}{R_2} \quad \Rightarrow \quad R_2 = \frac{R_{in}}{g}$$

Consequently $$(1-g)\,R_1 = g\,R_2 = R_{in} \quad \Rightarrow \quad R_1 = \frac{R_{in}}{(1-g)}$$

Thus specified values of g and R_{in} uniquely determine the required values of R_1 and R_2.

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then $$g\,R_2 = (1-g)\,R_1 \quad \Rightarrow \quad R_2 = \frac{(1-g)\,R_1}{g} = \frac{(1-0.65)\times 100}{0.65} = 54\ \Omega$$

(b) $$R_2 = \frac{R_{in}}{g} = \frac{10000}{0.65} = 15385\ \Omega$$

and $$R_1 = \frac{R_{in}}{(1-g)} = \frac{10000}{(1-0.65)} = 28571\ \Omega$$

DP 3-7

Solution:

The required gain is $$g = \frac{v_o}{v_s} = \frac{8.5}{12} = 0.7083$$

Consequently, $$0.7083\,R_1 = (1-0.7083)\,R_2 \quad \Rightarrow \quad R_2 = 2.428\,R_1$$

It is also required that

$$0.001 \ge \frac{12^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \ge 144000 \quad \Rightarrow \quad 3.428\,R_1 \ge 144000 \quad \Rightarrow \quad R_1 \ge 42007\ \Omega$$

For example, $$R_1 = 45\ \text{k}\Omega \quad \text{and} \quad R_2 = 109.26\ \text{k}\Omega$$

DP 3-8

Solution:

The required gain is $$g = \frac{i_o}{i_s} = \frac{1.8}{5} = 0.36$$

Consequently, $$0.36\,R_2 = (1-0.36)\,R_1 \quad \Rightarrow \quad R_2 = 1.778\,R_1$$

It is also required that

$$0.001 \ge 0.005^2 \frac{R_1 R_2}{R_1+R_2} \quad \Rightarrow \quad \frac{R_1+R_2}{R_1 R_2} = \frac{2.778\,R_1}{1.778\,R_1^{\,2}} \ge \frac{0.005^2}{0.001} = 0.025$$

$$\Rightarrow \quad R_1 \le \frac{2.778}{1.778(0.025)} = 62.5\ \Omega$$

For example, $$R_1 = 60\ \Omega \quad \text{and} \quad R_2 = 106.7\ \Omega$$

DP 3-9

Solution:

Specification A requires that v_4 = 2 V when v_2 = 5 V. Using voltage division

$$2 = \frac{R_4}{R_3+R_4}5 \quad \Rightarrow \quad 5\,R_4 = 2\left(R_3+R_4\right) \quad \Rightarrow \quad R_3 = \frac{3}{2}R_4$$

Following the advice in **Hint 1** we have,

$$R_2 = R_3 + R_4 = \frac{5}{2}R_4$$

Specification A requires that v_2 = 5 V when the battery voltage is 9 V. Using equivalent resistance and voltage division

$$5 = \frac{R_2 \,\|\, \left(R_3+R_4\right)}{R_1 + R_2 \,\|\, \left(R_3+R_4\right)}9 \quad \Rightarrow \quad 5 = \frac{\frac{5}{2}R_4 \,\|\, \frac{5}{2}R_4}{R_1 + \frac{5}{2}R_4 \,\|\, \frac{5}{2}R_4}9 = \frac{\frac{5}{4}R_4}{R_1 + \frac{5}{4}R_4}9$$

$$5\left(R_1 + \frac{5}{4}R_4\right) = \left(\frac{5}{4}R_4\right)9 \quad \Rightarrow \quad 5\left(4\,R_1 + 5R_4\right) = \left(5R_4\right)9 \quad \Rightarrow \quad 20\,R_1 = 45R_4 - 25R_4$$

In summary $$R_1 = R_4,\ \ R_2 = R_3 + R_4 = \frac{5}{2}R_4 \ \text{ and } \ R_3 = \frac{3}{2}R_4$$

Notice that this result is consistent with Hint 2.

The power supplied by the 9-V battery is given by

$$9\,i = 9\left(\frac{9-v_2}{R_1}\right) = 9\left(\frac{9-5}{R_1}\right) = \frac{36}{R_1}$$

Specification B requires that this power is no more than 0.25 W. Consequently

$$0.25 \ge \frac{36}{R_1} \quad \Rightarrow \quad R_1 \ge \frac{36}{0.25} = 144\ \Omega$$

Choose R_1 = 200 Ω. Specification B is satisfied. Also $R_4 = R_1$ = 200 Ω, $R_3 = \frac{3}{2}R_4 = 300\ \Omega$ and $R_2 = \frac{5}{2}R_4 = 500\ \Omega$. The power received by each of the 4 resistors is by

$$p_1 = \frac{(9-5)^2}{200} = 0.08\text{ W}, \quad p_2 = \frac{5^2}{500} = 0.05\text{ W}, \quad p_3 = \frac{(5-2)^2}{300} = 0.03\text{ W} \text{ and } \quad p_4 = \frac{2^2}{200} = 0.02\text{ W}$$

The power received by each resistor is less than ¼ watt so specification C is satisfied.

All of the specifications are satisfied so the design is complete.

DP 3-10

Solution:

Specification A requires that v_4 = 5 V when v_2 = 2 V. Using voltage division

$$5 = \frac{R_4}{R_3+R_4}2 \quad \Rightarrow \quad 2\,R_4 = 5\left(R_3+R_4\right) \quad \Rightarrow \quad R_3 = -\frac{3}{2}R_4$$

Since it’s expected that resistors all have positive values of resistance **Specification A cannot be satisfied.** Following the advice in **Hint 3** we report this result as soon as possible and check with your supervisor to see if the specifications need to be corrected.

DP 3-11

Solution:

Specification A requires that v_4 = 12 V when v_2 = 4 V. Using voltage division

$$12 = \frac{R_4}{-R_3 + R_4} 4 \quad \Rightarrow \quad 4 R_4 = 12\left(-R_3 + R_4\right) \quad \Rightarrow \quad -R_3 = -\frac{2}{3} R_4$$

Following the advice the Hint, we have,

$$R_2 = -R_3 + R_4 = \frac{R_4}{3}$$

Specification A requires that v_2 = 4 V when the battery voltage is 9 V. Using equivalent resistance and voltage division

$$4 = \frac{R_2 \,\|\, \left(-R_3 + R_4\right)}{R_1 + R_2 \,\|\, \left(-R_3 + R_4\right)} 9 \quad \Rightarrow \quad 5 = \frac{\dfrac{R_4}{3} \,\|\, \dfrac{R_4}{3}}{R_1 + \left(\dfrac{R_4}{3} \,\|\, \dfrac{R_4}{3}\right)} 9 = \frac{\dfrac{R_4}{6}}{R_1 + \dfrac{R_4}{6}} 9$$

$$4\left(R_1 + \frac{R_4}{6}\right) = \left(\frac{R_4}{6}\right) 9 \quad \Rightarrow \quad 4\left(6 R_1 + R_4\right) = \left(R_4\right) 9 \quad \Rightarrow \quad 24 R_1 = 5 R_4 \quad \Rightarrow \quad R_1 = \frac{5}{24} R_4$$

In summary $R_1 = \frac{5}{24} R_4,\ R_2 = \frac{R_4}{3}$ and $-R_3 = -\frac{2}{3} R_4$

The power supplied by the 9-V battery is given by

$$9 i = 9\left(\frac{9 - v_2}{R_1}\right) = 9\left(\frac{9 - 4}{R_1}\right) = \frac{45}{R_1}$$

Specification B requires that this power is no more than 0.5 W. Consequently

$$0.5 \geq \frac{45}{R_1} \quad \Rightarrow \quad R_1 \geq \frac{45}{0.5} = 90\ \Omega$$

Choose R_1 = 125 Ω. Specification B is satisfied. Also $R_4 = \frac{24}{5} R_1 = 600\ \Omega$, $-R_3 = -\frac{2}{3} R_4 = -400\ \Omega$ and $R_2 = \frac{R_4}{3} = 200\ \Omega$. The power **supplied** by the dependent source is

$$v_3\left(\frac{v_3}{R_3}\right) = (4 - 12)\left(\frac{4 - 12}{400}\right) = 0.16\ \text{W}$$

so the power supplied by the dependent source satisfies specification C.

The power received by each of the 3 resistors is by

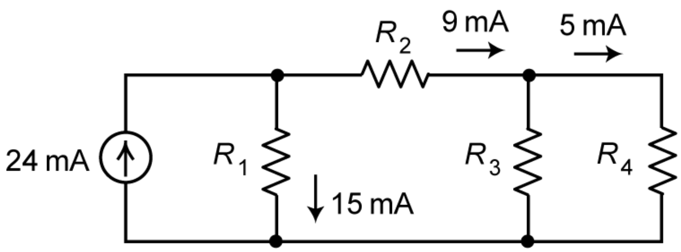
$$p_1 = \frac{(9-4)^2}{125} = 0.2 \text{ W}, \quad p_2 = \frac{4^2}{200} = 0.08 \text{ W}, \quad \text{and} \quad p_4 = \frac{12^2}{600} = 0.24 \text{ W}$$

The power received by each resistor is less than ¼ watt so specification D is satisfied.

All of the specifications are satisfied so the design is complete.

DP 3-12

Solution:



Specification A requires that i_4 = 5 mA when i_1 = 15 mA. Using current division

$$5 = \frac{R_3}{R_3 + R_4} 9 \quad \Rightarrow \quad 9R_3 = 5\left(R_3 + R_4\right) \quad \Rightarrow \quad R_3 = \frac{5}{4} R_4$$

Following the advice in **Hint 1** we have,

$$R_2 = R_3 \,||\, R_4 = \frac{\left(\frac{5}{4} R_4\right) R_4}{\frac{5}{4} R_4 + R_4} = \frac{5}{9} R_4$$

Specification A requires that v_2 = 5 V when the battery voltage is 9 V. Using equivalent resistance and voltage division

$$9 = \frac{R_1}{R_1 \,||\, \left(R_2 + \left(R_3 \,||\, R_4\right)\right)} 24 \quad \Rightarrow \quad 9 = \frac{R_1}{R_1 \,||\, \left(\frac{5}{9} R_4 + \frac{5}{9} R_4\right)} 24 = \frac{R_1}{R_1 \,||\, \frac{10}{9} R_4} 24 \quad \Rightarrow \quad R_1 = \frac{2}{3} R_4$$

In summary $$R_1 = \frac{2}{3} R_4, \;\; R_2 = R_3 \,||\, R_4 = \frac{5}{9} R_4 \;\; \text{and} \;\; R_3 = \frac{5}{4} R_4$$

Notice that this result is consistent with Hint 2.

The power supplied by the current source is given by

$$v\,i = \left(0.015\,R_1\right)\left(0.024\right) = \left(0.00036\right)R_1$$

Specification B requires that this power is no more than 0.25 W. Consequently

$$0.25 \geq 0.0036\,R_1 \quad \Rightarrow \quad R_1 \leq \frac{0.25}{0.00036} = 694.4\ \Omega$$

Choose R_1 = 480 Ω. **Specification B is satisfied**.

Specification A is satisfied by choosing

$$R_4 = \frac{3}{2}R_1 = 720\ \Omega\,,\ R_2 = \frac{5}{9}R_4 = 400\ \Omega \text{ and } R_3 = \frac{5}{4}R_4 = 900\ \Omega\,.$$

The power received by each of the 4 resistors is by

$$p_1 = \left(0.015\right)^2 480 = 0.108\ \text{W}, \quad p_2 = \left(0.009\right)^2 400 = 0.0324\ \text{W}, \quad p_3 = \left(0.004\right)^2 900 = 0.0144\ \text{W}$$

and

$$p_4 = \left(0.005\right)^2 720 = 0.018\ \text{W}$$

The power received by each resistor is less than ¼ watt so **Specification C is satisfied**.

All of the specifications are satisfied so the design is complete.