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EE214000 Electromagnetics, Fall 2020

Your name: ________________ ID: __________________ Sep. 29, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #4-1, Open books, notes (26 points), due 11 pm, Wednesday, Sep. 30th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. What is the physical meaning of the gradient of a scalar? (3 point)

Ans: The gradient of a scaler is a vector, whose magnitude is the maximum rate of change of a scalar in space and its direction is along the maximum change.

2. What is the physical meaning of the divergence of a vector? (3 points)

Ans: The divergence of a vector is a scalar, which equals the net outward flux of the vector per unit volume at an infinitesimal point volume in space.

3. What is the physical meaning of the curl of a vector? (3 points)

Ans: The curl of a vector is the maximum circulation of the vector per unit area along an infinitesimal point area in space. The direction of it is chosen to be along the surface normal direction of the infinitesimal area with which the net circulation is a maximum.

4. Verbally describe the meaning of the Stokes theorem? (3 points)

Ans: The Stokes theorem states that the surface integration of the curl of a vector is equal to the circulation of the vector along the path enclosing the surface.

5. Verbally describe the meaning of the divergence theorem? (3 points)

Ans: The divergence theorem states that the volume integration of the divergence of a vector is equal to the total outward flux of the vector over the surface enclosing the volume.

6. In the Cartesian coordinate system, what are the mathematic expressions of

$\nabla V, \nabla \times \vec{A}, \nabla \cdot \vec{B}, \nabla^2 V, \nabla^2 \vec{A}$? (5 points) *Please get familiar with the expressions, because you will be using them quite often in this class.

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Ans:

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla \times \vec{A} = (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{a}_x + (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{a}_y + (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{a}_z$$

$$\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 \vec{A} = \hat{a}_x \nabla^2 A_x + \hat{a}_y \nabla^2 A_y + \hat{a}_z \nabla^2 A_z$$

7. Explain intuitively why the two null identities:

$\nabla \times (\nabla V) = 0$ and $\nabla \cdot (\nabla \times \vec{A}) = 0$. (3+3 points)

- Note that proof of the two null identities needs some mathematically skills. As an engineer, you would at least remember the two expression from some intuitive arguments.

Ans: ∇V is the maximum change of V in space and is perpendicular to the equal potential surface in space. One would expect no component of ∇V on any infinitesimal surface on it when you calculate the circulation of ∇V or calculating $\nabla \times (\nabla V) = 0$.

$\nabla \times A$ is the maximum circulation of $\vec{A}$ over an infinitesimal area. One would expect an infinitesimal point volume (think about a disk volume with $\nabla \times A$ along the positive and next surface directions) enclosing $\nabla \times \vec{A}$ ends up cancelling the positive and negative fluxes when you calculate $\nabla \cdot (\nabla \times \vec{A}) = 0$.