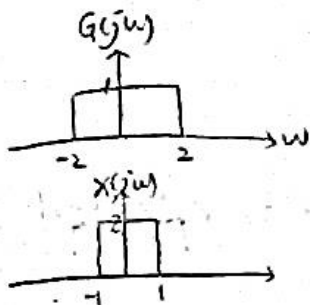


EE 361002 Signal and System HW8 Answer

4.30

$$(a) \quad g(t) = x(t) \cos t \xleftrightarrow{\mathcal{F}} G(j\omega) = \frac{1}{2\pi} [X(j\omega) * \pi[\delta(\omega+1) + \delta(\omega-1)]]$$

$$= \frac{1}{2} [X(j(\omega+1)) + X(j(\omega-1))]$$

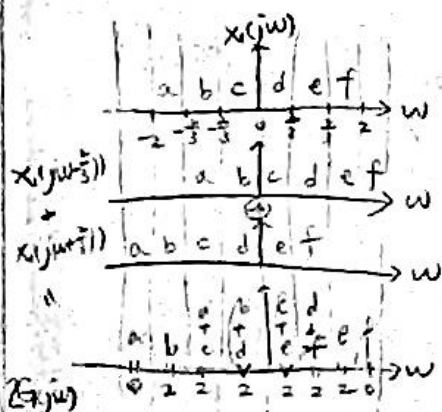


$$= \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow X(j\omega) = \begin{cases} 2, & |\omega| \leq 1 \\ 0, & \text{otherwise} \end{cases} \xleftrightarrow{\mathcal{F}} x(t) = \frac{2 \sin t}{\pi t}$$

$$(b) \quad g(t) = x_1(t) \cos\left(\frac{\pi}{3}t\right) \xleftrightarrow{\mathcal{F}} G(j\omega) = \frac{1}{2\pi} [X_1(j\omega) * \pi[\delta(\omega+\frac{\pi}{3}) + \delta(\omega-\frac{\pi}{3})]]$$

$$= \frac{1}{2} [X_1(j(\omega+\frac{\pi}{3})) + X_1(j(\omega-\frac{\pi}{3}))]$$



因為 $X_1(j\omega)$ 求不出來，(b) 小題送分

$\therefore X_1(j\omega)$ 畫不出來

4.32

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)} \Rightarrow H(j\omega) = \begin{cases} e^{-j\omega} & |\omega| < 4 \\ 0 & \text{o.w.} \end{cases}$$

$$(a) x_1(t) = \cos\left(6t + \frac{\pi}{2}\right) = \cos\left(6\left(t + \frac{\pi}{12}\right)\right)$$

$$\Rightarrow X_1(j\omega) = \pi e^{j\frac{\pi}{12}\omega} [\delta(\omega-6) + \delta(\omega+6)]$$

$$\Rightarrow Y_1(j\omega) = H(j\omega) \cdot X_1(j\omega) = 0$$

$$\Rightarrow y_1(t) = 0$$

$$(b) x_2(t) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \sin(3kt)$$

$$\Rightarrow X_2(j\omega) = \frac{\pi}{j} \left\{ \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k [\delta(\omega-3k) - \delta(\omega+3k)] \right\}$$

$$= \frac{\pi}{j} \left\{ [\delta(\omega) - \delta(\omega)] + \left(\frac{1}{2}\right) [\delta(\omega-3) - \delta(\omega+3)] + \left(\frac{1}{2}\right)^2 [\delta(\omega-6) - \delta(\omega+6)] + \dots \right\}$$

$$\Rightarrow Y_2(j\omega) = H(j\omega) \cdot X_2(j\omega) = \frac{\pi}{j} e^{-j\omega} \frac{1}{2} [\delta(\omega-3) - \delta(\omega+3)]$$

$$\Rightarrow y_2(t) = \frac{1}{2} \sin(3(t-1))$$

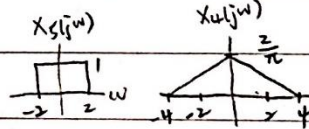
$$(c) x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)} \Rightarrow X_3(j\omega) = \begin{cases} e^{j\omega} & |\omega| < 4 \\ 0 & \text{o.w.} \end{cases}$$

$$\Rightarrow Y_3(j\omega) = H(j\omega) \cdot X_3(j\omega) = \begin{cases} 1 & |\omega| < 4 \\ 0 & \text{o.w.} \end{cases}$$

$$\Rightarrow y_3(t) = \frac{\sin(4t)}{\pi t}$$

$$(d) x_4(t) = \left(\frac{\sin 2t}{\pi t} \right)^2$$

$$\text{let } x_5(t) = \frac{\sin 2t}{\pi t} \Rightarrow X_5(j\omega) = \begin{cases} 1 & |\omega| < 2 \\ 0 & \text{o.w.} \end{cases}$$



$$X_4(j\omega) = \frac{1}{\pi} X_5(j\omega) * X_5(j\omega) = \frac{1}{\pi} |\omega| + \frac{2}{\pi} \Rightarrow \text{within } |\omega| < 4, H(j\omega) \text{ causes time-shift}$$

$$\Rightarrow y_4(t) = \left(\frac{\sin 2(t-1)}{\pi(t-1)} \right)^2$$

in time domain

(a)

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{(j\omega)^2 + 6j\omega + 8} = \frac{2}{(j\omega + 2)(j\omega + 4)} = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}$$

$$\rightarrow h(t) = e^{-2t} u(t) - e^{-4t} u(t) \quad \times$$

$$b) \quad x(t) = t e^{-2t} u(t) \Rightarrow X(j\omega) = \frac{1}{(j\omega + 2)^2}$$

$$\begin{aligned} Y(j\omega) &= X(j\omega) H(j\omega) = \frac{1}{(2 + j\omega)^2} \left(\frac{1}{(2 + j\omega)} - \frac{1}{4 + j\omega} \right) \\ &= \frac{1}{(2 + j\omega)^3} - \frac{1}{(2 + j\omega)^2 (4 + j\omega)} \\ &= \frac{1}{(2 + j\omega)^3} + \frac{\frac{1}{4}}{j\omega + 2} - \frac{\frac{1}{2}}{(j\omega + 2)^2} - \frac{\frac{1}{4}}{j\omega + 4} \end{aligned}$$

$$y(t) = \frac{t^2}{2} e^{-2t} u(t) + \frac{1}{4} e^{-2t} u(t) - \frac{1}{2} t e^{-2t} u(t) - \frac{1}{4} e^{-4t} u(t) \quad \times$$

$$\begin{aligned} (c) \quad H(j\omega) &= \frac{-2\omega^2 - 2}{-\omega^2 + \sqrt{2}j\omega + 1} = 2 + \frac{-2\sqrt{2}j\omega - 4}{-\omega^2 + \sqrt{2}j\omega + 1} \\ &= 2 + \frac{-\sqrt{2} + \sqrt{2}j}{(j\omega - \frac{-\sqrt{2} + \sqrt{2}j}{2})} + \frac{-\sqrt{2} - \sqrt{2}j}{(j\omega - \frac{-\sqrt{2} - \sqrt{2}j}{2})} \end{aligned}$$

$$\Rightarrow h(t) = 2\delta(t) + (-\sqrt{2} + \sqrt{2}j) e^{\frac{(-\sqrt{2} + \sqrt{2}j)t}{2}} u(t) + (-\sqrt{2} - \sqrt{2}j) e^{\frac{(-\sqrt{2} - \sqrt{2}j)t}{2}} u(t) \quad \times$$

$g(t)$ denotes the inverse Fourier Transform of $\frac{1}{2\pi} \{X(j\omega) * Y(j\omega)\}$

(a) Using $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

$$\begin{aligned} g(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{2\pi} \{X(j\omega) * Y(j\omega)\} e^{j\omega t} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{2\pi} \left[\int_{-\infty}^{\infty} X(j\theta) \cdot Y(j(\omega-\theta)) d\theta \right] e^{j\omega t} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta) \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} Y(j(\omega-\theta)) e^{j\omega t} d\omega \right] d\theta \quad \text{--- (1)} \end{aligned}$$

(b)

Freq - shift $e^{j\omega_0 t} x(t) \xrightarrow{F} X(j(\omega-\omega_0))$

$$\Rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(j(\omega-\theta)) e^{j\omega t} d\omega = e^{j\theta t} y(t) \quad \text{--- (2)}$$

(c)

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta) e^{j\theta t} y(t) d\theta \quad (\text{2 if } \lambda \text{ 1})$$

$$\begin{aligned} F^{-1} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta) e^{j\theta t} d\theta \cdot y(t) \\ &= \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta) e^{j\theta t} d\theta}_{x(t)} \cdot y(t) \quad \# \end{aligned}$$