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<u>Case II</u> : $\frac{dy}{dt} = f(y) \overset{ex:}{=\!=} 4y(1-y)$

(* Such DE is called "   " DE, where the RHS does not depend on the independent variable t.

** Many of the DEs that arise in application are "   ")

— for   , slope is the same.

— geometrically, all the slopes on each   .   line are parallel.

Feature:

① We can get infinite solutions from one solution curve by translating the curve

(ex:

② Depending on the values of the slopes, we can divide the slope field into the following regions:

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Remarks:

① We can use the phase line to roughly and quickly predict the system

Ex: If a system can be modeled by an ODE as

$$\frac{dy}{dt} = \left(1-\frac{y}{20}\right)^3\left(\frac{y}{5}-1\right)y^7 \quad , \quad y(0)=8$$

② Classification of critical points:

③ For a system with "growing and bounded" feature, the response normally exhibits an , and yields an ODE with a math form of

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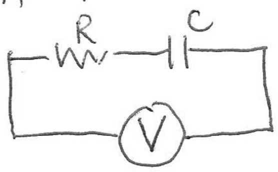
Example: Use qualitative approach to analyze a circuit problem.

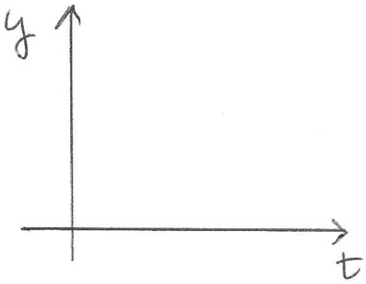
An RC circuit



II. Numerical approach (ch 2-6)

The geometric concept of the is closely related to the ideas used in numerical methods for approximating solutions to a DE. In this class, we will discuss a numerical method called

Example : Solve $\frac{dy}{dt} = 0.2ty$, $y(t_0) = y_0$ by Euler's method.



procedures :

① Start at (t_0, y_0) in the slope field

② Move a tiny step

③ Use the slope at to get the

④ At (t_1, y_1), the procedures are repeated. Take a step whose size is Δt and whose direction is determined by

General expression of Euler's method for $\frac{dy}{dt} = f(t, y)$

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Remarks:

① In Euler's method, we always make an error in each step. (called "   ").

② More numerical methods, such as

are used with improved accuracy.

③ Numerical methods are commonly used to solve Complicated DEs and are widely used in many fields