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Sep. 21, 2020

EE214000 Electromagnetics, Fall, 2020

Quiz #2-2, Open books, notes (39 points), due in class, Monday, Sep. 21st, 2020

1. What is the phase angle of the imaginary unit $-\sqrt{-1}$. (2 points)

$-\sqrt{-1} = -j$  Ans: 270° or 90°

2. A and B are real numbers. What is the complex conjugate of $z = \frac{1 - Ae^{j\phi}}{A + jB}$. (2 points)

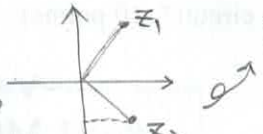
"complex conjugate: 共轭复数即可"

Ans: $z^* = \frac{1 - Ae^{-j\phi}}{A - jB}$

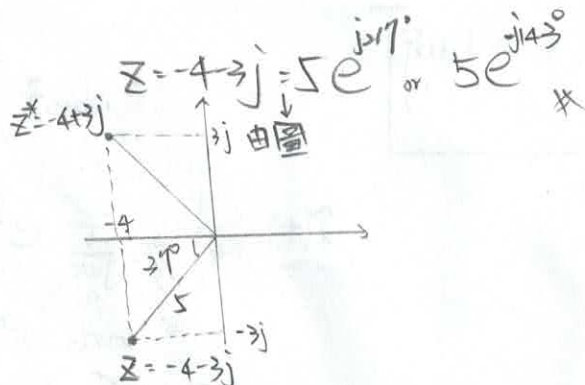
3. If you rotate the complex number $z_1 = x_1 + jy_1$ on the polar-coordinate plane by -90° , what is the resulting complex number z_2 ? (5 points)

旋轉 $-90^\circ \Rightarrow z_1$ 與 z_2 正交

幼稚園時學過 = 垂直 = 係數對調, 且其中一項加負號即可 (斜率相乘 = -1)

$\Rightarrow z_2 = y_1 - jx_1$  前係數為負

4. Express $z = -4 - 3j$ in the polar form (3 points) and mark it (2 points) and its complex conjugate (2 points) on the polar coordinate system.



5. Calculate the division $z_3 = z_1 / z_2$ and express the result in polar form, $z_2 = 1 + j$ and $z_1 = 4 + 3j$. (5 points)

$$z_3 = \frac{z_1}{z_2} = \frac{4+3j}{1+j} = \frac{5e^{j37^\circ}}{\sqrt{2}e^{j45^\circ}} = \frac{5}{\sqrt{2}}e^{-j8^\circ}$$

6. For a harmonic wave expressed as $A(z, t) = A_0 \sin(\omega t - kz + \phi)$, what is the phasor of this wave? (3 points)

$$A(z, t) = A_0 \cos\left(\frac{\pi}{2} - \omega t + kz - \phi\right) = \text{Re}\left[A_0 e^{j(\frac{\pi}{2} + kz - \phi)} e^{-j\omega t}\right]$$

ps: $\sin\theta = \cos(90^\circ - \theta)$ 很多人錯在這步!! $\therefore A_0 e^{j(\frac{\pi}{2} + kz - \phi)}$

7. For a time-harmonic wave function expressed by A , what is the phasor expression of the wave equation $\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = 0$, where c is a constant? (5 points)

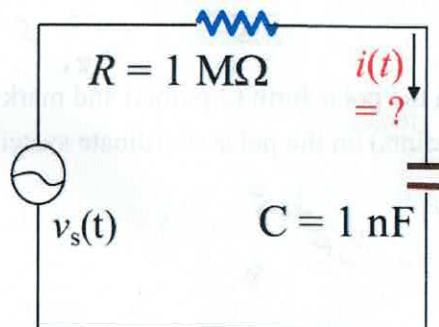
$$\nabla^2 \hat{A} - \frac{\omega^2}{c^2} \hat{A} = 0$$

$$\Rightarrow \nabla^2 \hat{A} + \left(\frac{\omega}{c}\right)^2 \hat{A} = 0$$

8. For the RC circuit shown below, if the driving voltage is a sinusoidal input with a frequency of 60 Hz, given by

$$\tilde{v}_s(t) = 100 \cos(2\pi \times 60t + \pi/6) \text{ volts}$$

what is the current in the circuit? (10 points)



$$\tilde{v}_s(t) = \text{Re}\left[100 e^{j\frac{\pi}{6}} e^{j2\pi \times 60t}\right]$$

\hat{v}_s

by KVL

$$\Rightarrow R\hat{i}(t) + \frac{\hat{q}(t)}{C} = \hat{v}_s(t)$$

$$\Rightarrow R\hat{i}(t) + \frac{1}{C} \int \hat{i}(t) dt = \hat{v}_s(t) \quad \text{僅以 phasor 作計算}$$

$$\Rightarrow \hat{I}\left(R + \frac{1}{j\omega C}\right) = \hat{v}_s$$

$$\Rightarrow \hat{I} = \frac{\hat{v}_s}{R + \frac{1}{j\omega C}}$$

$$\therefore \hat{i}(t) = \text{Re}\left[\frac{\hat{v}_s}{R + \frac{1}{j\omega C}} e^{j\omega t}\right]$$

\hat{v}_s

$10^6 \quad 2\pi \times 60 \quad 10^{-9}$

$$= 3.5 \times 10^{-5} \angle (2\pi \times 60t + 99.3^\circ)$$