Homework 5

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p.5-8

P.5–8 A d-c voltage of 6 (V) applied to the ends of 1 (km) of a conducting wire of 0.5 (mm) radius results in a current of 1/6 (A). Find

a) the conductivity of the wire,
b) the electric field intensity in the wire,
c) the power dissipated in the wire,
d) the electron drift velocity, assuming electron mobility in the wire to be 1.4×10^{-3} ($\text{m}^2/\text{V}\cdot\text{s}$).

P.5—8 解：(a)$R=\dfrac{l}{\sigma s}=\dfrac{V}{I}$ 則 $\sigma=\dfrac{lI}{VS}=3.54\times10^{7}(\text{s/m})$

(b)$E=\dfrac{V}{l}=6\times10^{-3}(\text{V/m})$

(c)$p=VI=1(\text{W})$

(d)$\rho_l=-\dfrac{\sigma}{\mu_l}$

$$\mu=\left|\frac{J}{\rho_l}\right|=\left|\frac{\mu_l\cdot J}{\sigma}\right|=|\mu_l E|=1.4\times10^{-3}\times6\times10^{-3}$$

$$=8.4\times10^{-6(\text{m/s})}$$

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p.5-13

P.5–13 A d-c voltage V_0 is applied across a cylindrical capacitor of length L. The radii of the inner and outer conductors are a and b, respectively. The space between the conductors is filled with two different lossy dielectrics having, respectively, permittivity ϵ_1 and conductivity σ_1 in the region $a < r < c$, and permittivity ϵ_2 and conductivity σ_2 in the region $c < r < b$. Determine

a) the current density in each region,
b) the surface charge densities on the inner and outer conductors and at the interface between the two dielectrics.

P. 5—13 解：(a) $G_1=\frac{2\pi\sigma_1 L}{ln(c/a)}, G_2=\frac{2\pi\sigma_2 L}{ln(b/c)}$

$$I=V_0G=V_0\frac{G_1G_2}{G_1+G_2}=\frac{2\pi\sigma_1\sigma_2LV_0}{\sigma_1 ln(b/c)+\sigma_2 ln(c/a)}$$

$$J_1=J_2=\frac{I}{2\pi rl}=\frac{\sigma_1\sigma_2V_0}{r[\sigma_1 ln(b/c)+\sigma_2 ln(c/a)]}$$

$$(b)\ \rho_{sa}=\varepsilon_1E_1|_{r=a}=\frac{\varepsilon_1\sigma_2V_0}{a[\sigma_1 ln(b/c)+\sigma_2(c/a)]}$$

$$\rho_{sb}=-\varepsilon_2E_2|_{r=d}=-\frac{d_2\sigma_1V_0}{Bb[\sigma_1 ln(b/c)+\sigma_2 ln(c/a)]}$$

$$\rho_{sc}=-(\varepsilon_1E_1-\varepsilon_2E_2)|_{r=c}=\frac{(\varepsilon_2\sigma_1-\varepsilon_1\sigma_2)V_0}{c[\sigma_1 n(b/c)+\sigma_2 ln(c/a)]}$$

另解

in circuit: $C_1=\frac{\varepsilon_1 2\pi L}{ln\frac{c}{a}} \rightarrow R_1=\frac{1}{C_1}\frac{\varepsilon_1}{\sigma_1}=\frac{ln\frac{c}{a}}{\sigma_1 2\pi L}$

then $R_2=\frac{ln\frac{b}{c}}{\sigma_2 2\pi L}$

$\hookrightarrow I=\frac{V_0}{R_1+R_2}=\frac{V_0}{\frac{ln\frac{c}{a}}{\sigma_1 2\pi L}+\frac{ln\frac{b}{c}}{\sigma_2 2\pi L}}$

B.C. $J_1=J_2=\frac{I}{2\pi rL}=\frac{V_0}{r\left(\frac{ln\frac{c}{a}}{\sigma_1}+\frac{ln\frac{b}{c}}{\sigma_2}\right)}$

$\rho_{sa}=\varepsilon_1E_1=\varepsilon_1\frac{J_1}{\sigma_1}(r=a)$

$\rho_{sb}=-\varepsilon_2E_2=-\varepsilon_2\frac{J_2}{\sigma_2}(r=b)$

$\rho_{sc}=\varepsilon_2E_2-\varepsilon_1E_1=\varepsilon_2\frac{J_2}{\sigma_2}(r=c)-\varepsilon_1\frac{J_1}{\sigma_1}(r=c)$

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p.5-14

P.5–14 Refer to the flat conducting quarter-circular washer in Example 5–6 and Fig. 5–8. Find the resistance between the curved sides.

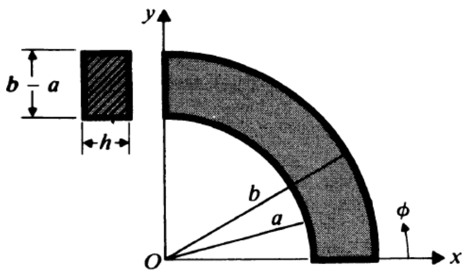

FIGURE 5–8
A quarter of a flat conducting circular washer (Example 5–6).

P. 5—14 解：$\vec{\nabla}^2 V=0$ 则 $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right)=0$

解得：$V(r)=C_1 lnr+C_2$.

由邊界條件：$V(a)=V_0$；$V(b)=0$

则 $V(r)=V_0\,\frac{ln(b/r)}{ln(b/m)}$

$$\vec{E}(r)=-\vec{a}_r\frac{\partial v}{\partial r}=\vec{a}_r\frac{V_0}{rln(b/a)}$$

$$\vec{J}(r)=\sigma\vec{E}(r)$$

$$I=\int_s\vec{J}\cdot\vec{ds}=\int_0^{\pi/2}\vec{J}\cdot(\vec{a}_r hrd\phi)$$

$$=\frac{\pi\sigma hV_0}{2ln(b/a)}$$

$$R=\frac{V_0}{I}=\frac{2ln(b/a)}{\pi\sigma h}$$

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p.5-22

P.5–22 Assume a rectangular conducting sheet of conductivity σ, width a, and height b. A potential difference V_0 is applied to the side edges, as shown in Fig. 5–14. Find

a) the potential distribution,

b) the current density everywhere within the sheet. (*Hint:* Solve Laplace's equation in Cartesian coordinates subject to appropriate boundary conditions.)

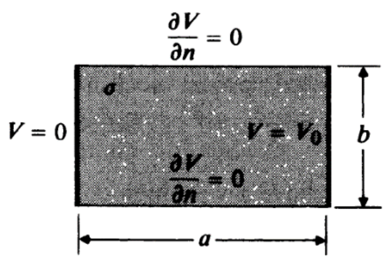
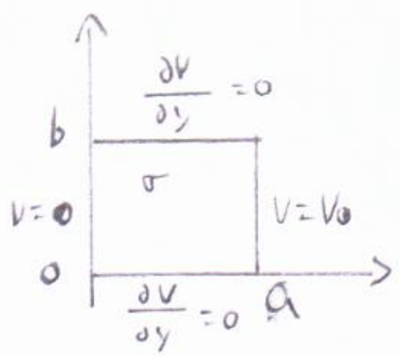


FIGURE 5–14
A conducting sheet (Problem P.5–22).



$$V = c_1 x + c_2$$

1t'λ B.C. $\begin{cases} V = V_0 = c_1 a + c_2 \\ V = 0 = c_2 \end{cases}$

$$\therefore V = \frac{V_0}{a} x$$

$$\rightarrow \quad E = -\nabla V = -\frac{V_0}{a}$$

then $J = \sigma E \rightarrow \vec{J} = -\hat{a}_x \frac{\sigma V_0}{a}$

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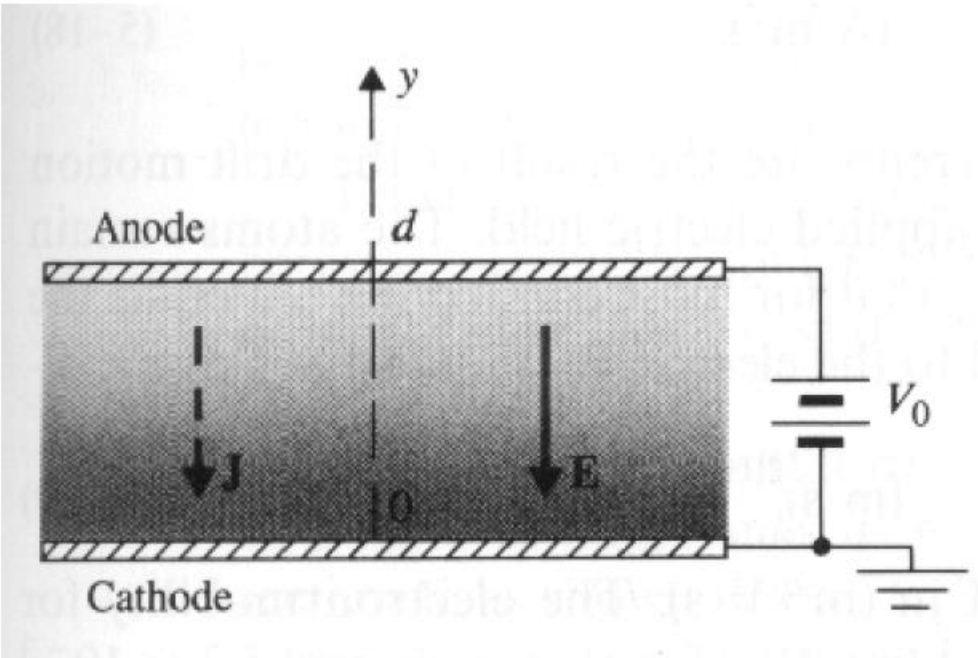
5. (20 points) Consider the following device having a top flat electrode at $y = d$ biased at V_0 and a bottom flat electrode at $y = 0$ grounded to a zero potential. Ignore the fringe fields near the edge of the electrodes. The electrons boil off the bottom electrode and propagate upward, while colliding with some neutral molecules (for example, air molecules) and moving at a constant speed $\vec{v} = -\mu_e \vec{E}$, where μ_e is the mobility of the electrons, and $\vec{E} = E_y \hat{a}_y$. In the space-charge limited regime, the electric field at $y = 0$ is zero. At a steady state, the current density between the two electrodes is a constant $\vec{J} = -J_0 \hat{a}_y$. (1) Express the volume charge density ρ as a function of J_0 and E_y. (2) Insert ρ into the Poisson's equation to write a differential equation for E_y. (3) Find E_y as a function of y. (4) Find V_0 as a function of J_0.



Ans: (1) Use $v\hat{a}_y = -\mu_e E_y \hat{a}_y$ in $\vec{J} = \rho \vec{v}$ or $-J_0 = \rho v = -\rho \mu_e E_y$ to obtain $\rho = \dfrac{J_0}{\mu_e E_y}$. (2) Insert ρ into the Poisson's equation $\nabla^2 V = -\dfrac{\rho}{\varepsilon}$ to obtain $\dfrac{d^2 V}{dy^2} = \dfrac{-J_0}{\varepsilon \mu_e E_y}$. With $\dfrac{dV}{dy} = -E_y$, one has $\dfrac{dE_y}{dy} = \dfrac{J_0}{\varepsilon \mu_e E_y}$. (3) Apply the boundary condition of E = 0 at y = 0 to solve E_y from $\dfrac{dE_y}{dy} = \dfrac{J_0}{\varepsilon \mu_e E_y}$. The result is

$$E_y = -\sqrt{\frac{2J_0}{\varepsilon\mu_e}y}$$ (note the choice of the negative sign to denote a downward field).

(4) Use $\frac{dV}{dy} = -E_y$ and the boundary conditions for *V* at *y* = 0 and *y* = *d* to obtain

$$V_0 = \frac{2}{3}\sqrt{\frac{2J_0}{\varepsilon\mu_e}}d^{3/2}$$. (note the departure of this expression from the Ohm's law)

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