

Problem 1 (10%)

$$f(t) = a\delta(t - b)$$

Find the Fourier Transform of $f(t)$

Problem 2 (10%)

$$F^{-1} \left\{ \frac{e^{-j4\pi f}}{5 + j2\pi f} \right\}$$

Problem 3 (15%)

$$f(t) = \begin{cases} e^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Find the Fourier Transform of $f(t)$

Problem 4 (15%)

$$f(t) = e^{-a|t|}, a > 0$$

Find the Fourier Transform of $f(t)$

Problem 5 (20%)

$$f(x) = \begin{cases} x^2 e^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

Find the Fourier transform of the function.

Problem 6 (10%)

We shall find the Fourier transform of $f(x) = e^{-ax^2}, a > 0$

Problem 7 (20%)

$$F(xe^{-x^2})$$

Find the Fourier transform of the function.

Problem 1 Solution

$$F(f) = ae^{-j2\pi fb}$$

Problem 2 Solution

$$f(t) = e^{-5(t-2)}u(t-2)$$

Problem 3 Solution

$$F(f) = \frac{1}{a + j2\pi f}$$

Problem 4 Solution

$$F(f) = \frac{2}{a^2 + (2\pi f)^2}$$

Problem 5 Solution

$$F(f) = \frac{2}{(1 + i2\pi f)^3}$$

Problem 6 Solution

$$F(f) = \sqrt{\frac{\pi}{a}} e^{-\frac{(\pi f)^2}{a}}$$

Problem 7 Solution

$$F\left(-\frac{1}{2}(e^{-x^2})'\right) = -if(\pi)^{\frac{3}{2}}e^{-(\pi f)^2}$$

1. $\mathcal{L}\{u(t) - u(t - \pi) + \sin(t) u(t - 2\pi)\} = ?$
2. $\mathcal{L}\{\delta'(t - a)\} = ?$
3. $\mathcal{L}\{(t^3 - 3t^2 - 3t + 1)u(t)\} = ?$
4. $\mathcal{L}\{t^2 \sin(t) u(t)\} = ?$
5. $\mathcal{L}\{te^{at} \cos(bt)\} = ?$
6. $\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+6s+25}\right\} = ?$
7. $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+2^2)^2}\right\} = ?$
8. $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s^2}e^{-2s} - \frac{1}{s}e^{-2s} + \frac{2s}{s^2+1}e^{-\pi s}\right\} = ?$
9. $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = ?$
10. $\mathcal{L}^{-1}\left\{\tan^{-1}\frac{1}{s}\right\} = ?$
11. $y'' + 3y' + 2y = u(t) - u(t - 1), y(0) = 0, y'(0) = 0$
12. $y'' + 2y' + 2y = \cos t \delta(t - 3\pi), y(0) = 0, y'(0) = 0$
13. $y'' + 4y = \cos 2t u(t), y(0) = 1, y'(0) = 0$
14. $y'' + y = \delta'(t - 2\pi), y(0) = 0, y'(0) = 0$
15. $y'' + y = \sin 2t u(t) + \cos 2t u(t), y(0) = 0, y'(0) = 0$
16. $ty'' + y' = -u(t), y(0) = 0, y'(0) = 1$
17. $y'' + 2ty' - 4y = u(t), y(0) = 0, y'(0) = 0$
18. $\begin{cases} y_1' = -y_1 + y_2 \\ y_2' = -y_1 - y_2 \end{cases}, y_1(0) = 1, y_2(0) = 0$
19. $\begin{cases} y_1' + 2y_2' - y_2 = u(t) \\ y_1' + y_2 = 0 \end{cases}, y_1(0) = 0, y_2(0) = 0$
20. $\begin{cases} y_1'' = y_1 + 3y_2 \\ y_2'' = 4y_1 - 4e^t \end{cases}, y_1(0) = 2, y_1'(0) = 3, y_2(0) = 1, y_2'(0) = 2$