Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

Your name: __________________ ID: ___________________ Sep. 21, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #3-1, Open books, notes (32 points), due 11 pm, Wednesday, Sep. 23, 2020
(email solutions to 劉峰麒 <alex851225@gmail.com>)

Late submission won't be accepted!

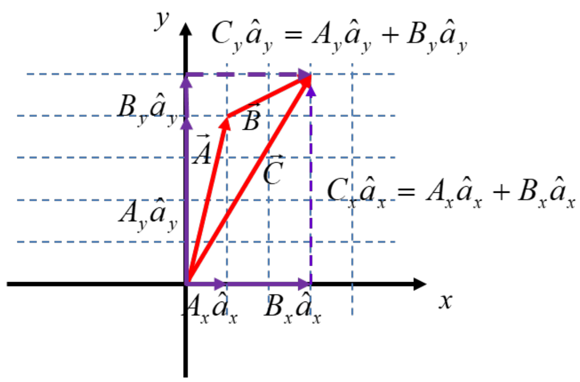
1. Given a vector, $\vec{A}$, how do you calculate its unit vector? (1 point) Suppose $\vec{B} = 2\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z$, what is its unit vector? (1 point)

Ans: The unit vector of $\vec{A}$ is $\hat{a}_u = \vec{A}/|A|$. For $\vec{B} = 2\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z$, $|B| = \sqrt{2^2 + 4^2 + 4^2} = 6$ and its unit vector is

$$\hat{a}_u = \vec{B}/|B| = (2\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z)/6 = \frac{1}{3}\hat{a}_x + \frac{2}{3}\hat{a}_y + \frac{2}{3}\hat{a}_z$$

2. In the *x-y* plane, assume $\vec{A} = \hat{a}_x + 3\hat{a}_y$ and $\vec{B} = 2\hat{a}_x + \hat{a}_y$. Explain that the calculation $\vec{C} = \vec{A} + \vec{B} = (1+2)\hat{a}_x + (3+1)\hat{a}_y = 3\hat{a}_x + 4\hat{a}_y$ is consistent with the head-to-tail construction for $\vec{C}$ in the *x-y* plane. (5 points)

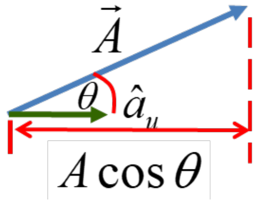
Ans: Suppose, in general, $\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y$, $\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y$, $\vec{C} = C_x\hat{a}_x + C_y\hat{a}_y$. Refer to the following figure. The head-to-tail construction ensures $C_x\hat{a}_x = A_x\hat{a}_x + B_x\hat{a}_x$ and $C_y\hat{a}_y = A_y\hat{a}_y + B_y\hat{a}_y$. Therefore $\vec{C} = (A_x + B_x)\hat{a}_x + (A_y + B_y)\hat{a}_y$ is consistent with the head to tail construction from $\vec{A}, \vec{B}$.



3. What is the physical meaning of the scalar product of a vector $\vec{A}$ and an unit vector $\hat{a}_u$ or $\vec{A}\cdot\hat{a}_u$? Use graph illustration to explain it. (3 points)

Ans: Since $\vec{A}\cdot\hat{a}_u = A\cos\theta$, where θ is the angle between the vector A and the unit vector $\hat{a}_u$, it means the length of $\vec{A}$ projected along the direction of $\hat{a}_u$, as shown below.

Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw



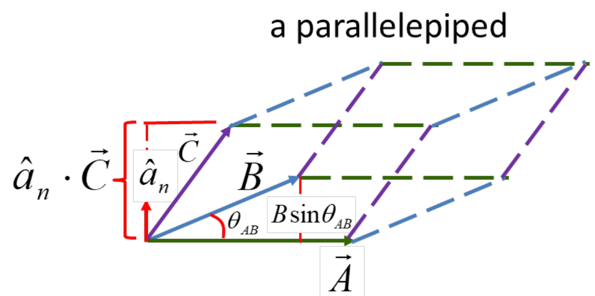
4. What is the area of the parallelogram expanded by the two vectors, $\vec{l}_A = \hat{a}_x + 3\hat{a}_y$ (m) and $\vec{l}_B = 2\hat{a}_x + \hat{a}_y$ (m) ? (3 points)

Ans: The area is equal to $\left|\vec{l}_A \times \vec{l}_B\right| = \left|(\hat{a}_x + 3\hat{a}_y) \times (2\hat{a}_x + \hat{a}_y)\right| = \left|(1-6)\hat{a}_z\right| = 5$ m^2.

5. What is the physical meaning of $(\vec{A}\times\vec{B})\cdot\vec{C}$ in space. Use graphic illustration to explain it. (5 points).

Ans: It is the volume of the parallelepiped expanded by the 3 vectors, $\vec{A},\vec{B},\vec{C}$,

because $\begin{aligned}&(\vec{A}\times\vec{B})\cdot\vec{C} = [AB\sin\theta_{AB}]\times[\hat{a}_n\cdot\vec{C}]\\ &= \text{base area}\times\text{height}\end{aligned}$, as shown below.



6. In an orthogonal coordinate system with a differential length of $d\vec{l} = d\vec{l}_{u_1} + d\vec{l}_{u_2} + d\vec{l}_{u_3}$, what are the expressions for a differential surface (3 points) and a differential volume in this coordinate system? (1 point)

Ans: A differential surface is expressed as

$$d\vec{s} = d\vec{s}_{u_1} + d\vec{s}_{u_2} + d\vec{s}_{u_3} = d\vec{l}_{u_2}\times d\vec{l}_{u_3} + d\vec{l}_{u_3}\times d\vec{l}_{u_1} + d\vec{l}_{u_1}\times d\vec{l}_{u_2}$$

A differential volume is expressed as

$$dv = d\vec{s}_{u_1,u_2,u_3}\cdot d\vec{l}_{u_1,u_2,u_3} = dl_{u_1}dl_{u_2}dl_{u_3}$$

Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

7. What are the 3 differential length vectors in the cylindrical coordinate system? (3 points)

Ans: $d\vec{l}_r = \hat{a}_r dr$, $d\vec{l}_\phi = \hat{a}_\phi r d\phi$, $d\vec{l}_z = \hat{a}_z dz$

8. What are the 3 differential area vectors in the spherical coordinate system? (3 points)

Ans:

$d\vec{s}_R = d\vec{l}_\theta \times d\vec{l}_\phi = R^2 \sin\theta d\theta d\phi \hat{a}_R$, $d\vec{s}_\theta = d\vec{l}_\phi \times d\vec{l}_R = R\sin\theta dR d\phi \hat{a}_\theta$,

$d\vec{s}_\phi = d\vec{l}_R \times d\vec{l}_\theta = RdRd\theta \hat{a}_\phi$

9. Use vector calculus to calculate the surface area of a sphere with radius of a. (2 points)

Ans: The only relevant area is $d\vec{s}_R = d\vec{l}_\theta \times d\vec{l}_\phi = R^2 \sin\theta d\phi d\theta \hat{a}_R$. The total area for sphere with a radius a is therefore

$$a^2 \int_0^{2\pi} \int_0^{\pi} \sin\theta d\theta d\phi = 4\pi a^2$$

10. Use vector calculus to calculate the volume of a sphere with radius a. (2 points)

Ans: The differential volume in the spherical coordinate system is

$dv = R^2 \sin\theta dR d\theta d\phi$. The volume of the hemisphere is the following integration

$$\int_0^a \int_0^{\pi} \int_0^{2\pi} R^2 \sin\theta dR d\theta d\phi = \frac{4}{3}\pi a^3.$$