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Classical boundary-value problems

A lot of physics/engineering systems involve the following three types of processes, which yield three classical boundary-value problems:

Type I:

ex:

Type II:

ex:

Type III:

☆ In these cases, there are more than

where DEs are presented as

How to solve

Solve PDEs by sov + Fourier series

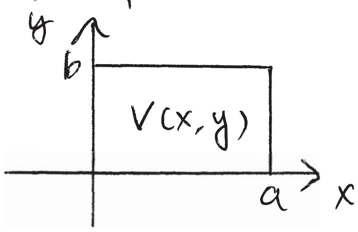
Idea: By sov, we can

Then,

In the following, we will show how to use this method to solve PDEs.

Example I: Solve Laplace's equation

ex: electric potential $V(x,y)$ in a rectangular region.



Laplace's eq: $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

$0 < x < a,\ 0 < y < b$

with B.C.

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✳ Trick of SOV : Assume the solution $V(x,y)$ can be

Step 1 : Assume $V(x,y) =$

Step 2 : Separate the variables of PDE and

Take (✳) into PDE :

$\frac{\partial^2}{\partial x^2} \qquad \rightarrow$

Step 3 : List possible solutions of ODEs

Step4: Apply conditions to delete unsatisfying terms

From ①, ②

From ③

$\Rightarrow V(x, y) =$

Step5: Use superposition and proper conditions to get the solution as

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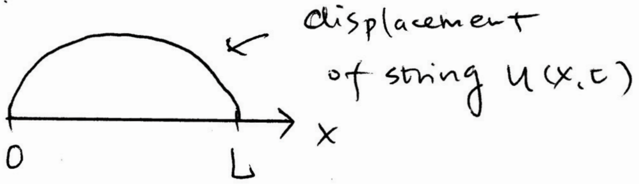
Example II: Solve wave equation

ex:



how the displacement $u(x,t)$ varies with x and t is described by "wave equation"

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

a: positive constant

with conditions:

Step 1: Assume $u(x,t) =$

Step 2: Separate the variables and get a set of ODEs

Take (*) into PDE $\Rightarrow$

$\Rightarrow$

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \xrightarrow{\text{by SOV}} \left\{ \vphantom{\frac{a}{b}} \right.$$

Step 3: List possible solutions of ODEs