量子物理一 作業四　　　　　　　　　　111061116 陳亮宇

Problem 8.9

(a) let $|\psi\rangle = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}|m\rangle_1|n\rangle_2$

$[\hat{a}_1, \hat{a}_1^+]$:

$$(\hat{a}_1\hat{a}_1^+ - \hat{a}_1^+\hat{a}_1)|\psi\rangle = \hat{a}_1 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m+1}\,|m+1\rangle_1|n\rangle_2 - \hat{a}_1^+ \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m}\,|m-1\rangle_1|n\rangle_2$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}(m+1)|m\rangle_1|n\rangle_2 - \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}\cdot m|m\rangle_1|n\rangle_2$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}[(\cancel{m}+1)-\cancel{m}]|m\rangle_1|n\rangle_2 + \sum_{n=0}^{\infty} C_{0n}\cdot 0|0\rangle_1|n\rangle_2$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}|m\rangle_1|n\rangle_2$$

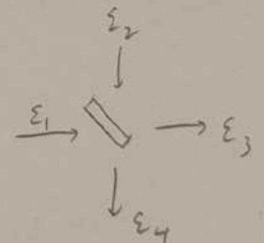
$$= |\psi\rangle \quad \Rightarrow [\hat{a}_1, \hat{a}_1^+] = 1 \ \#$$

$[\hat{a}_1, \hat{a}_2^+]$:

$$(\hat{a}_1\hat{a}_2^+ - \hat{a}_2^+\hat{a}_1)|\psi\rangle = \hat{a}_1 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{n+1}\,|m\rangle_1|n+1\rangle_2 - \hat{a}_2^+ \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m}\,|m-1\rangle_1|n\rangle_2$$

$$= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m}\sqrt{n+1}\,|m-1\rangle_1|n+1\rangle_2 - \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m}\sqrt{n+1}\,|m-1\rangle_1|n+1\rangle_2$$

$$= 0 \quad \Rightarrow [\hat{a}_1, \hat{a}_2^+] = 0 \ \#$$



$[\hat{a}_2, \hat{a}_2^+]$, $[\hat{a}_2, \hat{a}_1^+]$ is on the last page.

(b) When a photon is created at input 1, "t" portion is transmitted to output 3 and "r" portion is reflected to output 4. Similarly, "t" portion of photon is transmitted from input 2 to output 4 and "r" portion is reflected from input 2 to output 4. The same portion relation is true for annihilating photons at the inputs. Hence,

$$\hat{a}_3 = t\hat{a}_1 \quad r\hat{a}_2$$
$$\hat{a}_4 = r\hat{a}_1 \quad t\hat{a}_2$$

← we still don't know the sign.

(the sign can be negative because we square it for probability)

Problem 8.9 (b) 續

Because there is a π phase difference between the two reflected fields, due to energy conservation, one of the r's is negated.

$$\Rightarrow \begin{cases} \hat{a}_3 = t\hat{a}_1 - r\hat{a}_2 \\ \hat{a}_4 = r\hat{a}_1 + t\hat{a}_2 \end{cases} \#$$

(c) Assume: $[\hat{a}_3, \hat{a}_3^\dagger] = [\hat{a}_4, \hat{a}_4^\dagger] = 1$

$[\hat{a}_3, \hat{a}_4^\dagger] = [\hat{a}_4, \hat{a}_3^\dagger] = 0$

$$[\hat{a}_3, \hat{a}_3^\dagger] = (t\hat{a}_1 - r\hat{a}_2)(t^*\hat{a}_1^\dagger - r^*\hat{a}_2^\dagger) - (t^*\hat{a}_1^\dagger - r^*\hat{a}_2^\dagger)(t\hat{a}_1 - r\hat{a}_2)$$

$$= |t|^2 \overbrace{\cancel{(\hat{a}_1\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{a}_1)}}^{1} + |r|^2 \overbrace{\cancel{(\hat{a}_2\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_2)}}^{1}$$

$$- rt^*\hat{a}_2\hat{a}_1^\dagger - tr^*\hat{a}_1\hat{a}_2^\dagger + t^*r\hat{a}_1^\dagger\hat{a}_2 + r^*t\hat{a}_2^\dagger\hat{a}_1$$

$$= |t|^2 + |r|^2 + t^*r\underbrace{\cancel{[\hat{a}_1^\dagger, \hat{a}_2]}}_{0} + tr^*\underbrace{\cancel{[\hat{a}_2^\dagger, \hat{a}_1]}}_{0}$$

$$= 1 \qquad \Rightarrow |t|^2 + |r|^2 = 1 \ \#$$

$$[\hat{a}_3, \hat{a}_4^\dagger] = (t\hat{a}_1 - r\hat{a}_2)(r^*\hat{a}_1^\dagger + t^*\hat{a}_2^\dagger) - (r^*\hat{a}_1^\dagger + t^*\hat{a}_2^\dagger)(t\hat{a}_1 - r\hat{a}_2)$$

$$= |t|^2 \underbrace{\cancel{(\hat{a}_1\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_1)}}_{0} - |r|^2 \underbrace{\cancel{(\hat{a}_2\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{a}_2)}}_{0}$$

$$+ tr^* \underbrace{\cancel{(\hat{a}_1\hat{a}_1^\dagger - \hat{a}_1^\dagger\hat{a}_1)}}_{1} - t^*r \underbrace{\cancel{(\hat{a}_2\hat{a}_2^\dagger - \hat{a}_2^\dagger\hat{a}_2)}}_{1}$$

$$= tr^* - t^*r$$

$$= 0 \qquad \Rightarrow r^*t - rt^* = 0 \ \#$$

Problem 8.10

Use :

$$\begin{cases} \hat{a}_3 = \frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2) \\ \hat{a}_4 = \frac{1}{\sqrt{2}}(-\hat{a}_1+\hat{a}_2) \end{cases}$$

$$g^{(2)}(0) = \frac{\langle \hat{n}_3 \hat{n}_4 \rangle}{\langle \hat{n}_3\rangle\langle \hat{n}_4\rangle} = \frac{\langle \hat{a}_3^{+} \hat{a}_4^{+} \hat{a}_4 \hat{a}_3 \rangle}{\langle \hat{a}_3^{+}\hat{a}_3\rangle\langle \hat{a}_4^{+}\hat{a}_4\rangle}$$

$$= \frac{\cancel{\frac{1}{4}}\langle (\hat{a}_1^{+}+\hat{a}_2^{+})(-\hat{a}_1^{+}+\hat{a}_2^{+})(-\hat{a}_1+\hat{a}_2)(\hat{a}_1+\hat{a}_2)\rangle}{\cancel{\frac{1}{2}}\langle (\hat{a}_1^{+}+\hat{a}_2^{+})(\hat{a}_1+\hat{a}_2)\rangle \cdot \cancel{\frac{1}{2}}\langle (-\hat{a}_1^{+}+\hat{a}_2^{+})(-\hat{a}_1+\hat{a}_2)\rangle}$$

$$= \frac{\langle (-\hat{a}_1^{+}\hat{a}_1^{+} + \cancel{\hat{a}_1^{+}\hat{a}_2^{+}}^{\,0} \cancel{- \hat{a}_2^{+}\hat{a}_1^{+}} + \hat{a}_2^{+}\hat{a}_2^{+})(-\hat{a}_1\hat{a}_1 - \cancel{\hat{a}_1\hat{a}_2}^{\,0} \cancel{+ \hat{a}_2\hat{a}_1} + \hat{a}_2\hat{a}_2)\rangle}{\langle \hat{a}_1^{+}\hat{a}_1 + \hat{a}_1^{+}\hat{a}_2 + \hat{a}_2^{+}\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2\rangle\langle \hat{a}_1^{+}\hat{a}_1 - \hat{a}_1^{+}\hat{a}_2 - \hat{a}_2^{+}\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2\rangle}$$

$$= \frac{\langle \psi_1, 0_2 | \hat{a}_1^{+}\hat{a}_1^{+}\hat{a}_1\hat{a}_1 - \hat{a}_1^{+}\hat{a}_1^{+}\hat{a}_2\hat{a}_2 - \hat{a}_2^{+}\hat{a}_2^{+}\hat{a}_1\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2^{+}\hat{a}_2\hat{a}_2 | \psi_1, 0_2\rangle}{\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1 + \hat{a}_1^{+}\hat{a}_2 + \hat{a}_2^{+}\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2 |\psi_1,0_2\rangle\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1 - \hat{a}_1^{+}\hat{a}_2 - \hat{a}_2^{+}\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2 |\psi_1,0_2\rangle}$$

$\langle 0_2| \hat{a}_2^{+} \to 0$

$\hat{a}_2 |0_2\rangle \to 0$

$$= \frac{\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1^{+}\hat{a}_1\hat{a}_1 |\psi_1,0_2\rangle}{\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1 |\psi_1,0_2\rangle\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1 |\psi_1,0_2\rangle}$$

$$= \frac{\langle \psi_1,0_2| \hat{a}_1^{+}(\hat{a}_1\hat{a}_1^{+}-1)\hat{a}_1 |\psi_1,0_2\rangle}{\langle \hat{n}_1\rangle^2}$$

$$= \frac{\langle \psi_1,0_2| \hat{a}_1^{+}\hat{a}_1\hat{a}_1^{+}\hat{a}_1 - \hat{a}_1^{+}\hat{a}_1 |\psi_1,0_2\rangle}{\langle \hat{n}_1\rangle^2}$$

$$= \frac{\langle \hat{n}_1^2 - \hat{n}_1\rangle}{\langle \hat{n}_1\rangle^2} = \frac{\langle \hat{n}(\hat{n}-1)\rangle}{\langle \hat{n}\rangle^2} \quad \#$$

Problem 8.11

$$\begin{cases} \hat{a}_3 = \frac{1}{\sqrt{2}}(\hat{a}_1-\hat{a}_2) \\ \hat{a}_4 = \frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2) \end{cases} \Rightarrow \begin{cases} \hat{a}_1 = \frac{1}{\sqrt{2}}(\hat{a}_3+\hat{a}_4) \\ \hat{a}_2 = \frac{1}{\sqrt{2}}(-\hat{a}_3+\hat{a}_4) \end{cases}$$

(a) input $|0\rangle_1 |0_2\rangle \Rightarrow$ Output $|0\rangle_3 |0\rangle_4$

No input photon $\Rightarrow$ no output photon #

(b) input $|1\rangle_1 |0\rangle_2 = \hat{a}_1^{+}|0\rangle_1|0\rangle_2$

$\Rightarrow$ Output : $\frac{1}{\sqrt{2}}(\hat{a}_3^{+}+\hat{a}_4^{+})|0\rangle_3|0\rangle_4 = \frac{1}{\sqrt{2}}|1\rangle_3|0\rangle_4 + \frac{1}{\sqrt{2}}|0\rangle_3|1\rangle_4$ #

input $|0\rangle_1|1\rangle_2 = \hat{a}_2^{+}|0\rangle_1|0\rangle_2$

$\Rightarrow$ Output : $\frac{1}{\sqrt{2}}(-\hat{a}_3^{+}+\hat{a}_4^{+})|0\rangle_3|0\rangle_4 = \frac{-1}{\sqrt{2}}|1\rangle_3|0\rangle_4 + \frac{1}{\sqrt{2}}|0\rangle_3|1\rangle_4$ #

Problem 8.11 (c)

input $|1\rangle_1 |1\rangle_2 = \hat{a}_1^{+} \hat{a}_2^{+} |0\rangle_1 |0\rangle_2$

Output: $\frac{1}{\sqrt{2}}(\hat{a}_3^{+} + \hat{a}_4^{+}) \frac{1}{\sqrt{2}}(-\hat{a}_3^{+} + \hat{a}_4^{+}) |0\rangle_3 |0\rangle_4$

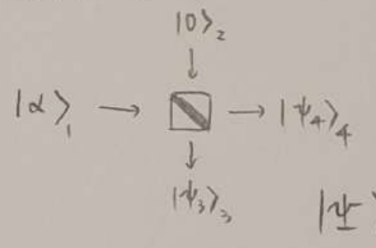
$$= \frac{1}{2}\left(-\hat{a}_3^{+}\hat{a}_3^{+} + \overbrace{\cancel{\hat{a}_3^{+}\hat{a}_4^{+} - \hat{a}_4^{+}\hat{a}_3^{+}}}^{0} + \hat{a}_4^{+}\hat{a}_4^{+}\right)|0\rangle_3|0\rangle_4$$

$$= \frac{1}{2}\left(-\sqrt{2}\cdot\sqrt{1}\,|2\rangle_3|0\rangle_4 + \sqrt{2}\cdot\sqrt{1}\,|0\rangle_3|2\rangle_4\right)$$

$$= \frac{-1}{\sqrt{2}}|2\rangle_3|0\rangle_4 + \frac{1}{\sqrt{2}}|0\rangle_3|2\rangle_4 \quad \#$$

When we shoot 1 photon into each input at the same time, both photons will appear at the same output. #

Problem 8.12



$$|\alpha\rangle_1 = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{(n!)^{1/2}} |n\rangle_1 = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} (\hat{a}_1^{+})^n |0\rangle_1$$

$\hat{a}^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$\sqrt{n}\,|n\rangle = \hat{a}^{+}|n-1\rangle$

$|n\rangle = \frac{1}{\sqrt{n}}\hat{a}^{+}|n-1\rangle$

$|\Psi\rangle \equiv |\psi_3\rangle_3 |\psi_4\rangle_4$

$\hat{a}_1^{+} = \frac{1}{\sqrt{2}}\hat{a}_3^{+} + \frac{1}{\sqrt{2}}\hat{a}_4^{+}$

$\Rightarrow$ Output: $e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} \left(\frac{1}{\sqrt{2}}\hat{a}_3^{+} + \frac{1}{\sqrt{2}}\hat{a}_4^{+}\right)^n |0\rangle_3 |0\rangle_4 \equiv |\Psi\rangle$

$g^{(2)}(0) = \frac{\langle \hat{a}_3^{+} \hat{a}_4^{+} \hat{a}_4 \hat{a}_3 \rangle}{\langle \hat{a}_3^{+} \hat{a}_3 \rangle \langle \hat{a}_4^{+} \hat{a}_4 \rangle}$

$$g^{(2)}(0) = \frac{\langle \alpha_1, 0_2 | \hat{a}_3^{+} \hat{a}_4^{+} \hat{a}_4 \hat{a}_3 | \alpha_1, 0_2 \rangle}{\langle \alpha_1, 0_2 | \hat{a}_3^{+} \hat{a}_3 | \alpha_1, 0_2 \rangle \langle \alpha_1, 0_2 | \hat{a}_4^{+} \hat{a}_4 | \alpha_1, 0_2 \rangle}$$

$$= \frac{\langle \alpha_1, 0_2 | \cancel{\tfrac{1}{4}} (\hat{a}_1^{+} - \cancel{\hat{a}_2^{+}})(\hat{a}_1^{+} + \hat{a}_2^{+})(\hat{a}_1 + \hat{a}_2)(\hat{a}_1 - \cancel{\hat{a}_2}) | \alpha_1, 0_2 \rangle}{\langle \alpha_1, 0_2 | \cancel{\tfrac{1}{2}} (\hat{a}_1^{+} - \cancel{\hat{a}_2^{+}})(\hat{a}_1 - \cancel{\hat{a}_2}) | \alpha_1, 0_2 \rangle \langle \alpha_1, 0_2 | \cancel{\tfrac{1}{2}} (\hat{a}_1^{+} + \cancel{\hat{a}_2^{+}})(\hat{a}_1 + \cancel{\hat{a}_2}) | \alpha_1, 0_2 \rangle}$$

$\langle 0_2 | \hat{a}_2^{+} \to 0$

$\hat{a}_2 |0_2\rangle \to 0$

$$= \frac{\langle \alpha_1 0_2 | \hat{a}_1^{+} (\hat{a}_1^{+}\hat{a}_1 + \hat{a}_1^{+}\hat{a}_2 + \hat{a}_2^{+}\hat{a}_1 + \hat{a}_2^{+}\hat{a}_2) \hat{a}_1 | \alpha_1, 0_2 \rangle}{\langle \alpha, 0_2 | \hat{a}_1^{+}\hat{a}_1 | \alpha, 0_2 \rangle \langle \alpha, 0_2 | \hat{a}_1^{+}\hat{a}_1 | \alpha, 0_2 \rangle}$$

$\hat{a}_1 |\alpha_1\rangle = \alpha_1 |\alpha_1\rangle$

$\langle \alpha_1 | \hat{a}_1^{+} = \langle \alpha_1 | \alpha_1^{*}$

$$= \frac{\cancel{\alpha^{*}\alpha} \langle \alpha, 0_2 | \hat{a}_1^{+}\hat{a}_1 + \cancel{\hat{a}_1^{+}\hat{a}_2} + \cancel{\hat{a}_2^{+}\hat{a}_1} + \cancel{\hat{a}_2^{+}\hat{a}_2} | \alpha, 0_2 \rangle}{\cancel{\alpha^{*}\alpha} \quad \alpha^{*}\alpha}$$

$$= \frac{\alpha^{*}\alpha}{\alpha^{*}\alpha}$$

$$= 1 \quad \#$$

Problem 12.1

Message: 11111 00001 10111 00110 11000 10000 10011 01011

Key: 11010 01000 11001 10101 01100 10101 11010 00101

XOR: 00101 01001 01110 10011 10100 00101 01001 01110

Letter: E I N S T E I N #

Problem 12.2

A's basis: $\oplus$ $\oplus$ $\otimes$ $\oplus$ $\otimes$ $\otimes$ $\otimes$ $\oplus$ $\oplus$ $\otimes$ $\oplus$ $\otimes$

A's message: 0 0 1 0 1 1 0 0 1 0 1 1

A's photons: $|\leftrightarrow\rangle\, |\leftrightarrow\rangle\, |\nearrow\rangle\, |\leftrightarrow\rangle\, |\nearrow\rangle\, |\nearrow\rangle\, |\nwarrow\rangle\, |\leftrightarrow\rangle\, |\updownarrow\rangle\, |\nwarrow\rangle\, |\updownarrow\rangle\, |\nearrow\rangle$

B's basis: $\otimes$ $\oplus$ $\oplus$ $\otimes$ $\otimes$ $\oplus$ $\oplus$ $\oplus$ $\otimes$ $\oplus$ $\otimes$ $\otimes$

same basis: ✓ ✓ ✓ ✓

sifted data set: 0 1 0 1 #

Problem 12.8

$\bar{n} = x$, $\eta = P(n>1)/P(1)$

$P(1) = e^{-\bar{n}} \cdot \frac{\bar{n}^1}{1!} = x e^{-x}$, $P(0) = e^{-\bar{n}} \cdot \frac{\bar{n}^0}{0!} = e^{-x}$

$\Rightarrow P(n>1) = 1 - P(1) - P(0) = 1 - xe^{-x} - e^{-x}$

$\lim_{x\to 0} \frac{1-xe^{-x}-e^{-x}}{xe^{-x}} = \lim_{x\to 0} \frac{\cancel{-e^{-x}}+xe^{-x}\cancel{+e^{-x}}}{e^{-x}-xe^{-x}} = \frac{0}{1-0} = 0$

$\eta = \frac{P(n>1)}{P(1)} = \frac{1-xe^{-x}-e^{-x}}{xe^{-x}} = \frac{e^x-1}{x} - 1 \approx \frac{1}{2}x$ when $x \ll 1$

$\frac{d\eta}{dx} = \frac{e^x \cdot x - (e^x-1)\cdot 1}{x^2} \Rightarrow \left.\frac{d\eta}{dx}\right|_{x=0} = \frac{1}{2}$

$\lim_{x\to 0} \frac{xe^x - e^x + 1}{x^2} = \lim_{x\to 0} \frac{\cancel{e^x}+xe^x-\cancel{e^x}}{2x} = \frac{1}{2}$

$\eta < 1\% \Rightarrow \bar{n} = x < 0.02$ #

$xe^{-x} \to e^{-x} - xe^{-x} \to -e^{-x} - e^{-x} + xe^{-x}$

$= -2e^{-x} + xe^{-x}$

0 1 -2

$0 + \frac{1}{1!}x + \frac{-2}{2!}x^2 = x - x^2$

$e^{-x} \to -e^{-x} \to e^{-x}$

1 -1 1

$1 + \frac{-1}{1!}x + \frac{1}{2!}x = 1 - x + \frac{1}{2}x^2$

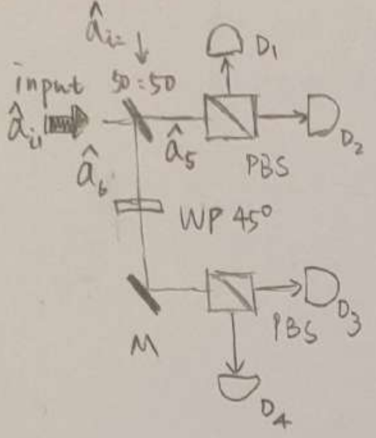
$P(n>1) = \cancel{1} - (\cancel{x} - x^2) - (\cancel{1} - \cancel{x} + \frac{1}{2}x^2)$

$= \frac{1}{2}x^2$

$\frac{\frac{1}{2}x^2}{x - x^2} = \frac{\frac{1}{2}x}{1-x} \approx \frac{1}{2}x(1 + x + x^2 + \cdots)$

$\approx \frac{1}{2}x$

Problem 12.10



(a) 3 different detector fires

$\Rightarrow$ 2 photons go to the wrong polarisation and each goes to a different detector

let $\begin{cases} \hat{a}_5 = \frac{1}{\sqrt{2}}(\hat{a}_{i1} + \hat{a}_{i2}) \\ \hat{a}_6 = \frac{1}{\sqrt{2}}(-\hat{a}_{i1} + \hat{a}_{i2}) \end{cases} \Rightarrow \hat{a}_{i1} = \frac{1}{\sqrt{2}}(\hat{a}_5 + \hat{a}_6)$

Assume the 3 input photons are in $\oplus$ basis.

Then exactly 2 of them need to go to WP 45°.

3 input photons $\Rightarrow \frac{1}{\sqrt{3!}}(\hat{a}_{i1}^{+})^3 |0_{i1}, 0_{i2}\rangle \Rightarrow \frac{1}{\sqrt{3!}}\left(\frac{1}{\sqrt{2}}(\hat{a}_5^{+} + \hat{a}_6^{+})\right)^3 |0_5, 0_6\rangle$

$= \frac{1}{\sqrt{6}}\frac{1}{\sqrt{8}}(\hat{a}_5^{+}\hat{a}_5^{+} + \hat{a}_5^{+}\hat{a}_6^{+} + \hat{a}_6^{+}\hat{a}_5^{+} + \hat{a}_6^{+}\hat{a}_6^{+})(\hat{a}_5^{+} + \hat{a}_6^{+})|0_5, 0_6\rangle$

$= \frac{1}{\sqrt{6}}\frac{1}{\sqrt{8}}(\hat{a}_5^{+}\hat{a}_5^{+}\hat{a}_5^{+} + \hat{a}_5^{+}\hat{a}_6^{+}\hat{a}_5^{+} + \hat{a}_6^{+}\hat{a}_5^{+}\hat{a}_5^{+} + \hat{a}_6^{+}\hat{a}_6^{+}\hat{a}_5^{+}$
$+ \hat{a}_5^{+}\hat{a}_5^{+}\hat{a}_6^{+} + \hat{a}_5^{+}\hat{a}_6^{+}\hat{a}_6^{+} + \hat{a}_6^{+}\hat{a}_5^{+}\hat{a}_5^{+} + \hat{a}_6^{+}\hat{a}_6^{+}\hat{a}_6^{+})|0_5, 0_6\rangle$

$= \frac{1}{\sqrt{6}}\cdot\frac{1}{\sqrt{8}}(\sqrt{6}|3_5 0_6\rangle + \sqrt{2}|2_5 1_6\rangle + \sqrt{2}|2_5 1_6\rangle + \sqrt{2}|1_5 2_6\rangle$
$+ \sqrt{2}|2_5 1_6\rangle + \sqrt{2}|1_5 2_6\rangle + \sqrt{2}|1_5 2_6\rangle + \sqrt{6}|0_5 3_6\rangle)$

$= \frac{1}{\sqrt{8}}|3_5 0_6\rangle + \frac{1}{\sqrt{24}}|2_5 1_6\rangle \times 3 + \frac{1}{\sqrt{24}}|1_5 2_6\rangle \times 3 + \frac{1}{\sqrt{8}}|0_5 3_6\rangle$

$\Rightarrow$ probability $= \left(\frac{3}{\sqrt{24}}\right)^2 = \frac{9}{24} = \frac{3}{8}$

Then, one photon needs to go to D_3 and one needs to go to D_4.

Since $\hat{a}_6^{+} = \frac{1}{\sqrt{2}}(\hat{a}_3^{+} + \hat{a}_4^{+})$

$\Rightarrow \frac{1}{\sqrt{2!}}\hat{a}_6^{+}\hat{a}_6^{+}|0_6\rangle \Rightarrow \frac{1}{\sqrt{2!}}\left(\frac{1}{\sqrt{2}}(\hat{a}_3^{+} + \hat{a}_4^{+})\right)^2|0_3 0_4\rangle$

$= \frac{1}{\sqrt{8}}(\hat{a}_3^{+}\hat{a}_3^{+} + \hat{a}_3^{+}\hat{a}_4^{+} + \hat{a}_4^{+}\hat{a}_3^{+} + \hat{a}_4^{+}\hat{a}_4^{+})|0_3 0_4\rangle$

$= \frac{1}{\sqrt{8}}(\sqrt{2}|2_3 0_4\rangle + |1_3 1_4\rangle + |1_3 1_4\rangle + \sqrt{2}|0_3 2_4\rangle)$

$= \frac{1}{\sqrt{4}}|2_3 0_4\rangle + \frac{1}{\sqrt{2}}|1_3 1_4\rangle + \frac{1}{\sqrt{4}}|0_3 2_4\rangle \quad \Rightarrow$ probability $= \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

Problem 12.10 (a) 續

The probability of 2 photons going to the wrong polarisation and went to different detectors is $\frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$.

The result will be the same if the incoming photon is $\otimes$ basis

Hence, the probability of 3 detectors firing is $3/16$ #

(b) If 3 detector fired, we can first check which polarisation fired twice and know that it's the wrong basis.

Then, we can look at the remaining detector, since it must be the correct basis, it will reveal the value of the bit (i.e. the polarisation angle)

Say $D_1 \leftrightarrow 0°$, $D_2 \leftrightarrow 90°$, $D_3 \leftrightarrow 45°$, $D_4 \leftrightarrow 135°$, then we can find the result:

Detectors fired	D_1, D_2, D_3	D_1, D_2, D_4	D_1, D_3, D_4	D_2, D_3, D_4
Basis w/ 2 photons	$\oplus$	$\oplus$	$\otimes$	$\otimes$
Correct basis	$\otimes$	$\otimes$	$\oplus$	$\oplus$
Polarisation	45°	135°	0°	90°
Bit value	1	0	1	0

#

Problem 8.9 (a) 續

$$[\hat{a}_2, \hat{a}_2^\dagger]|\psi\rangle = \hat{a}_2 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{n+1}\,|m\rangle_1|n+1\rangle_2 - \hat{a}_2^\dagger \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sqrt{n}\,|m\rangle_1|n-1\rangle_2$$

$$= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{n+1}\sqrt{n+1}\,|m\rangle_1|n\rangle_2 - \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sqrt{n}\sqrt{n}\,|m\rangle_1|n\rangle_2$$

$$= \sum_{m=0}^{\infty}\left(C_{m0}\sqrt{1}\sqrt{1}\,|m\rangle_1|0\rangle_2 + \sum_{n=1}^{\infty} C_{mn}\overset{1}{\left(\cancel{(n+1)}-n\right)}\,|m\rangle_1|n\rangle_2\right)$$

$$= \sum_{m=0}^{\infty} C_{mn}|m\rangle_1|n\rangle_2$$

$$= |\psi\rangle \Rightarrow [\hat{a}_2, \hat{a}_2^\dagger] = 1 \quad \#$$

$$[\hat{a}_2, \hat{a}_1^\dagger]|\psi\rangle = \hat{a}_2 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_{mn}\sqrt{m+1}\,|m+1\rangle_1|n\rangle_2 - \hat{a}_1^\dagger \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sqrt{n}\,|m\rangle_1|n-1\rangle_2$$

$$= \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sqrt{m+1}\sqrt{n}\,|m+1\rangle_1|n-1\rangle_2 - \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sqrt{n}\sqrt{m+1}\,|m+1\rangle_1|n-1\rangle_2$$

$$= 0 \Rightarrow [\hat{a}_2, \hat{a}_1^\dagger] = 0 \quad \#$$