CS5319 Advanced Discrete Structure

Exam 2 – December 07, 2021 (13:20–15:10)

Answer all questions. Total marks = 100. Maximum score = 100 (out of 100).

1. Solve the following recurrence relations:

   (a) (15%) $a_n = 2a_{n-1} + 3a_{n-2} + 25 \times 4^{n-2}$ for $n \geq 2$, and $a_0 = 1$, $a_1 = 32$.

   (b) (15%) $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$, and $a_0 = 1$, $a_1 = 15$.

2. (20%) Solve the following recurrence relation for $n \geq 0$:

$$a_n a_{n-2} = (a_{n-1})^2 + 2a_{n-1}a_{n-2} \qquad \text{for } n \geq 2$$

   with initial conditions
$$a_0 = 2, \quad a_1 = 4.$$

3. (20%) Show that for any positive integer k, $2020^{2k} - 1$ is a multiple of 2021.

4. (20%) Show that when 16 distinct numbers are selected from 1 to 100, we must be able to find four distinct numbers w, x, y, z such that they can form two pairs, say (w, x) and (y, z), such that for each pair, either the sum or the difference, is a multiple of 25.

5. (10%) Find four different ways to select five distinct integers from 1 to 9, such that the sum of their squares is a square number.

   Hint: You may write a program if you have time.