

Quantum Physics I Midterm Exam

2023/11/10 8:00~9:50

1. The wave function of a particle is given by

$$\psi(x) = \begin{cases} Ae^{\alpha x}, & x < 0 \\ Be^{-\alpha x}, & x > 0 \end{cases}$$

where A , B , and α are positive real constants.

- Calculate A and B in terms of α so that $\psi(x)$ is normalized. (5 pts.)
- Where is the particle most likely to be found? (5 pts.)
- What is the probability of finding the particle between $x = 0$ and $1/\alpha$? (5 pts.)
- What is the expectation value of the particle's position x ? (5 pts.)
- What is the expectation value of the particle's momentum p ? (5 pts.)
- Calculate the particle's wave function in the momentum space $\phi(p)$. (5 pts.)

2. A particle in the harmonic potential $\frac{1}{2}m\omega^2 x^2$ starts out in the state

$$\Psi(x, 0) = A[\psi_1(x) + 2\psi_2(x) + 2\psi_3(x)],$$

where $\psi_n(x)$ is the n th excited state of the quantum oscillator.

- Calculate A so that $\Psi(x, 0)$ is normalized. (5 pts.)
- If you measure the particle's energy, what values might you get and with what probabilities? (5 pts.)
- Suppose the energy is measured (at time t_0) to be the highest value in (b). What is the expectation value of x after the energy measurement (for time $t > t_0$)? (5 pts.)
- What is the probability density of finding the particle at position x and time $t > t_0$ after the energy measurement in (c)? (5 pts.)

$$\hat{H} = \left(\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 \hat{x}^2 \right)$$

3. Consider the potential

$$V(x) = \begin{cases} 0, & x \leq 0 \\ V_0, & x > 0 \end{cases}$$

$$ik \left(\frac{ik-l}{-ik+l} \right) - l \left(\frac{2ik}{ik+l} \right)$$

$$= \frac{-k^2 - ikl - 2ik}{ik+l} \stackrel{?}{=} ik$$

where V_0 and a are positive real numbers.

- Calculate the transmission coefficient of a particle with energy $E < V_0$. (10 pts.)
- Repeat (a) for $E > V_0$. (10 pts.)

$$ik(ik+l) = -k^2 + ikl$$

25 4. The Hamiltonian for a certain three-level system is represented by the matrix

$$\hat{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix},$$

where a , b , and c are real numbers.

(a) If you measure the energy, what are the possible values? (10 pts.)

(b) If the system starts out in the state

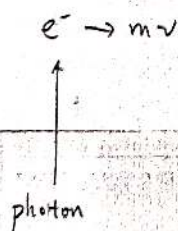
$$|S(0)\rangle = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = f_1 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} + f_2 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} + f_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(f_1 + f_2) = d_1 \quad \sqrt{2}f_1 = d_1 + d_2$$

$$\frac{1}{\sqrt{2}}(f_1 - f_2) = d_2$$

what is $|S(t)\rangle$? What are the probabilities of measuring the energy at time t to be the values in (a)? (15 pts.)

10 5. Consider an experiment in which the position of an electron is measured by viewing the electron with a microscope while illuminating it. Use such a thought experiment to estimate the ultimate limit (minimal value) of $\Delta p_x \Delta x$ set by the uncertainty principle. (10 pts.)



$$\Delta p_x = |\vec{p}'| \sin \theta \sim |\vec{p}'| \theta \quad \theta \sim \frac{h}{\lambda}$$

$$\Delta x \sim |\vec{p}'| \cos \theta \sim |\vec{p}'|$$

$$\Delta x \sim \frac{p_e}{m_e} \cdot \frac{d}{p_h \cos \theta}$$