

# EE 361002 Signal and System HW7 Answer

4.21

(a) The given signal is

$$e^{-\alpha t} \cos(\omega_0 t) u(t) = \frac{1}{2} e^{-\alpha t} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-\alpha t} e^{-j\omega_0 t} u(t).$$

Therefore,

$$X(j\omega) = \frac{1}{2(\alpha - j\omega_0 + j\omega)} + \frac{1}{2(\alpha + j\omega_0 + j\omega)}.$$

(c) Using the Fourier transform analysis equation (4.9) we have

$$X(j\omega) = \frac{2 \sin \omega}{\omega} + \frac{\sin \omega}{\pi - \omega} - \frac{\sin \omega}{\pi + \omega}.$$

(e) We have

$$x(t) = (1/2j)te^{-2t}e^{j4t}u(t) - (1/2j)te^{-2t}e^{-j4t}u(t).$$

Therefore,

$$X(j\omega) = \frac{1/2j}{(2 - j4 + j\omega)^2} - \frac{1/2j}{(2 + j4 - j\omega)^2}.$$

(g) Using the Fourier transform analysis eq. (4.9) we obtain

$$X(j\omega) = \frac{2j}{\omega} \left[ \cos 2\omega - \frac{\sin \omega}{\omega} \right].$$

4.22

(a)  $x(t) = \begin{cases} e^{j2\pi t}, & |t| < 3 \\ 0, & \text{otherwise} \end{cases}$

(b)  $x(t) = \frac{1}{2}e^{-j\pi/3}\delta(t-4) + \frac{1}{2}e^{j\pi/3}\delta(t+4).$

(c) The Fourier transform synthesis eq. (4.8) may be written as

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)| e^{j\omega t} d\omega.$$

From the given figure we have

$$x(t) = \frac{1}{\pi} \left[ \frac{\sin(t-3)}{t-3} + \frac{\cos(t-3)-1}{(t-3)^2} \right].$$

(d)  $x(t) = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos(2\pi t)$

For the given signal  $x_0(t)$ , we use the Fourier transform analysis eq. (4.8) to evaluate the corresponding Fourier transform

$$X_0(j\omega) = \frac{1 - e^{-(1+j\omega)}}{1 + j\omega}.$$

(i) We know that

$$x_1(t) = x_0(t) + x_0(-t).$$

Using the linearity and time reversal properties of the Fourier transform we have

$$X_1(j\omega) = X_0(j\omega) + X_0(-j\omega) = \frac{2 - 2e^{-1} \cos \omega - 2\omega e^{-1} \sin \omega}{1 + \omega^2}.$$

(ii) We know that

$$x_2(t) = x_0(t) - x_0(-t).$$

Using the linearity and time reversal properties of the Fourier transform we have

$$X_2(j\omega) = X_0(j\omega) - X_0(-j\omega) = j \left[ \frac{-2\omega + 2e^{-1} \sin \omega + 2\omega e^{-1} \cos \omega}{1 + \omega^2} \right].$$

(iii) We know that

$$x_3(t) = x_0(t) + x_0(t+1).$$

Using the linearity and time shifting properties of the Fourier transform we have

$$X_3(j\omega) = X_0(j\omega) + e^{j\omega} X_0(-j\omega) = \frac{1 + e^{j\omega} - e^{-1}(1 + e^{-j\omega})}{1 + j\omega}.$$

(iv) We know that

$$x_4(t) = tx_0(t).$$

Using the differentiation in frequency property

$$X_4(j\omega) = j \frac{d}{d\omega} X_0(j\omega).$$

Therefore,

$$X_4(j\omega) = \frac{1 - 2e^{-1}e^{-j\omega} - j\omega e^{-1}e^{-j\omega}}{(1 + j\omega)^2}.$$

(a) From Table 4.2 we know that

$$p(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t} \xleftrightarrow{FT} P(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0).$$

From this,

$$Y(j\omega) = \frac{1}{2\pi} \{X(j\omega) * H(j\omega)\} = \sum_{k=-\infty}^{\infty} a_k X(j(\omega - k\omega_0)).$$

(b) The spectra are sketched in Figure S4.28.