電磁學（一）
Electromagnetics (I)

1. 電磁學的基本物理量
Basic Electromagnetic Quantities

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This lecture is to introduce basic electromagnetic quantities based on physical observations and intuitions.

- 1.1 Units and Scales 單位與尺度
- 1.2 Electric Force and Electric Field 電力與電場
- 1.3 Electric Current 電流
- 1.4 Magnetic Force and Magnetic Field 磁力與磁場
- 1.5 Connection between Electricity and Magnetism 電與磁的關連
- 1.6 Review 單元回顧

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1.1 單位與尺度
Units and Scales

MKSA Units

In this course, we adopt the International System of Units (SI Units) or the MKSA Unit system.

Length (長度)	Meter (m，公尺，米)
Mass (質量)	Kilogram (kg，公斤)
Time (時間)	Second (s，秒)
Current (電流)	Ampere (A，安培, flow of charges)
Temperature (溫度)	Kelvin (K), water’s freezing temperature is at 273 K

Examples

In the SI Unit System, all the units of physical quantities can be decomposed into M, K, S, A, and Kelvin.

E.g. The SI unit of a force is Newton or ***nt***, which, according to the Newtonian mechanics, can be calculated from $F = ma,$

where the mass $\boldsymbol{m}$ is in ***kg*** and the acceleration $\boldsymbol{a}$ is in $\boldsymbol{m/s^2}$.

As a result, the unit of force, Newton, can be decomposed into $\boldsymbol{kg \cdot m/s^2}$

Scales … sometimes we deal with very large or very small values

E.g. 1,000,000,000 Hz =1 GHz = 10^9 Hz,

1 nm = 10^{-9} m, 1 fs = 10^{-15} s, 1 kA = 1000 A

Multiple prefix			**Sub-multiple prefix**		
Prefix	Symbol	Magnitude	Prefix	Symbol	Magnitude
Exa	***E***	$\mathbf{10^{18}}$	Atto	***a***	$\mathbf{10^{-18}}$
Peta	***P***	$\mathbf{10^{15}}$	Femto	***f***	$\mathbf{10^{-15}}$
Tera	***T***	$\mathbf{10^{12}}$	Pico	***p***	$\mathbf{10^{-12}}$
Giga	***G***	$\mathbf{10^{9}}$	Nano	***n***	$\mathbf{10^{-9}}$
Mega	***M***	$\mathbf{10^{6}}$	Micro	$\boldsymbol{\mu}$	$\mathbf{10^{-6}}$
Kilo	***k***	$\mathbf{10^{3}}$	Milli	***m***	$\mathbf{10^{-3}}$

1.1 單位與尺度

Units and Scales

- In this course, we adopt the SI or MKSA units.
- Sometimes, we use a prefix before a unit to simplify reading and writing.

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1.2 電力與電場
Electric Force and Electric Field

Electric Phenomena

Glow discharge powering a fluorescence light bulb

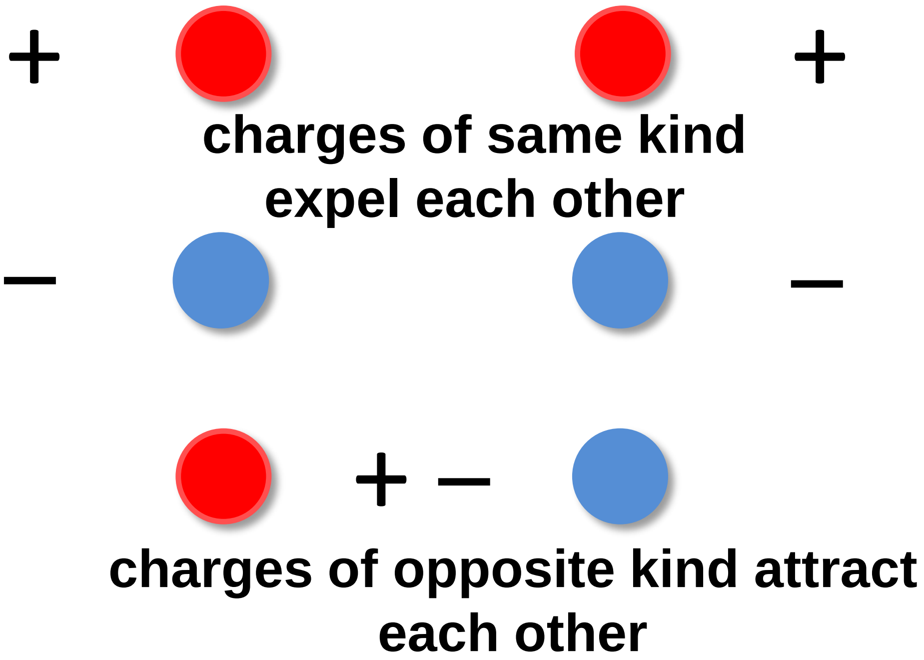
Glow discharge

lightning

Electric Force and Electric Field
電力與電場

Observations on *Charges*

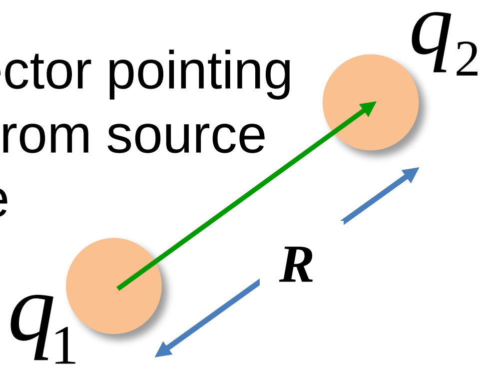
(SI unit of electric charge is **Coulomb or C**)



Coulomb Force (庫倫力)
– electric force between charges

Test charge

unit vector pointing away from source charge



Reference/source charge

q : amount of charge

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon R^2}\hat{a}_R$$

R : distance between charges

ε : permittivity, modified by materials

*Note the inverse square dependence of the force

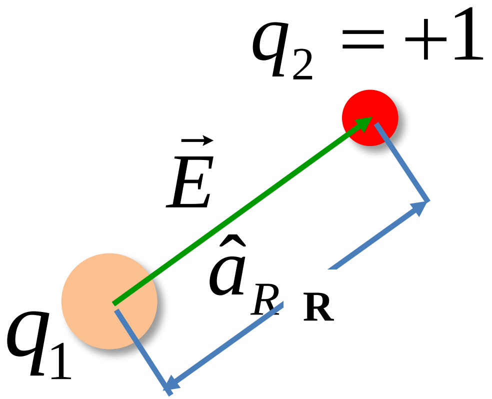
$$F \propto \frac{1}{R^2}$$

– same as the gravitational force

Electric Field Intensity

E – **electric force experienced by a unit positive charge**

電場強度 *E*：單位正電荷所受的力



Source charge

$$\vec{E} = \vec{F}\Big|_{q_2=+1} = \frac{q_1 \times (+1)}{4\pi\varepsilon R^2}\hat{a}_R$$

or

$$\vec{E} = \frac{\vec{F}}{q_2} = \frac{q_1}{4\pi\varepsilon R^2}\hat{a}_R$$

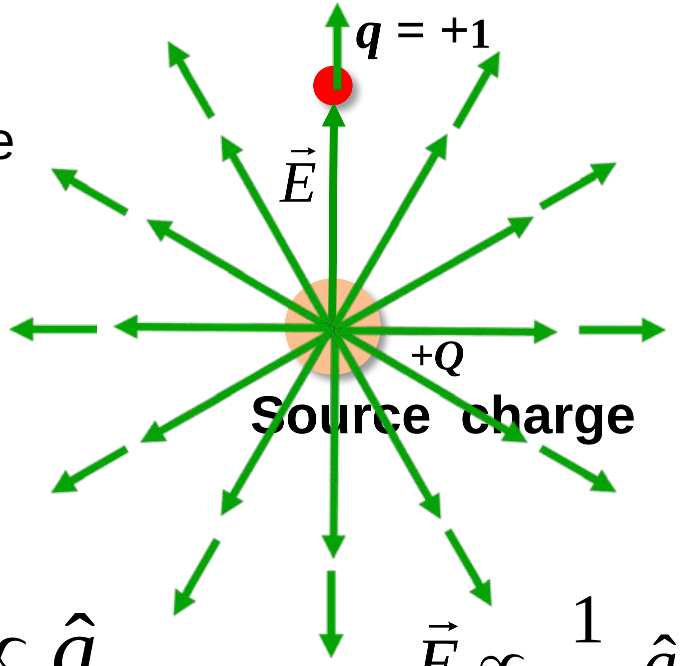
(SI unit of electric field is **Volt/m**)

Electric Field Lines

Electric field lines: lines of force experienced by a positive point charge

- Electric field lines help to visualize the direction and magnitude of an electric force in space.
- Arrow: direction of force
  Length: strength of force
- A point charge has a radially symmetric electric field lines

$$\vec{E} \propto \hat{a}_R$$



$$\vec{E} \propto \frac{1}{R^2}\hat{a}_R$$

Flux (通量)

Flux: a physical quantity penetrating through or across an area *S* (think about water flux, air flux etc.)

E.g. Water flux $\propto$ strength of water source

Total flux Φ = surface integration of density of flux D

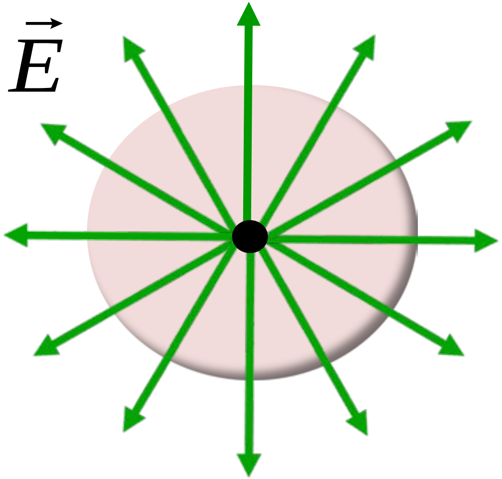
$$\Phi = \int_S \vec{D} \cdot ds$$

S: surface

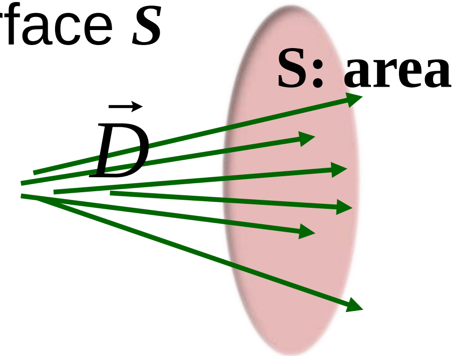
Electric Flux (電通量)

Define **electric flux density** $\vec{D} \equiv \varepsilon \vec{E}$ **(電通密度)** (SI unit: C/m^2)

E.g. For a point charge $\boldsymbol{q}$, the electric flux density is $\vec{D} = \varepsilon \vec{E} = \dfrac{q}{4\pi R^2} \hat{a}_R$



Electrical Flux Φ_e: vector dot product of $\boldsymbol{D}$ over a surface $\boldsymbol{S}$



Φ_e over a sphere of radius $\boldsymbol{R}$ is therefore the charge inside

$$\Phi_e = 4\pi R^2 \times D = q$$

1.2 電力與電場

Electric Force and Electric Field

- A charge generates an electric force on another charge.
- An electric field intensity is the force experienced by a unit positive charge.
- An electric flux is proportional to the amount of charge that generates it.

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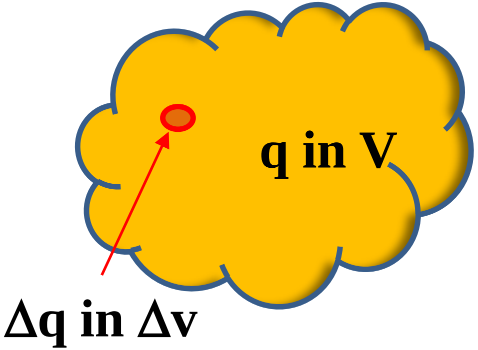
1.3 電流
Electric Current

Description of Charges
– volume charge

Volume Charge: charges distributed in a volume V



Volume charge density $\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v}$

(SI unit: C/m^3)

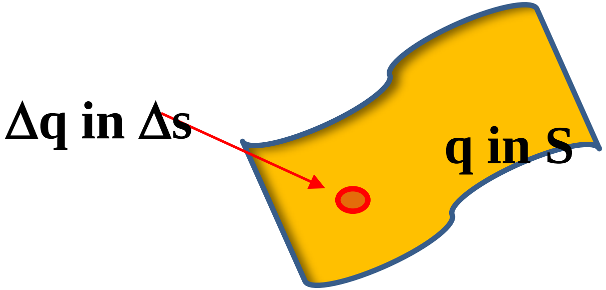
Total charge $q = \int_V \rho_v dv$

Description of Charges

– surface charge

Surface Charge: charges distributed on a surface S (occurs in a perfect conductor – infinite volume charge density $\rho_v \to \infty$ multiplying a zero thickness $dw \to \infty$ results in a finite surface charge density ρ_s)

$(\infty \times 0 \sim \text{a finite value})$



Surface charge density

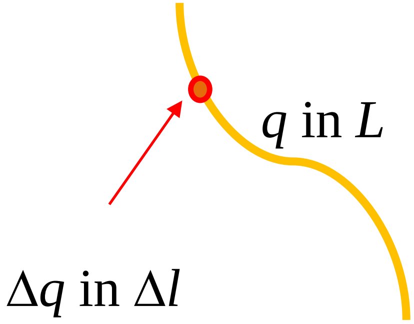
$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s}$$

(SI unit: C/m^2)

Total charge $q = \int_S \rho_s ds$

Description of Charges
– line charge

Line Charge: charges distributed along a line L



Line charge density

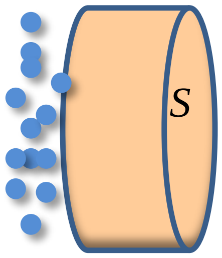
$$\rho_l = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l}$$

(SI unit: C/m)

Total charge $q = \int_L \rho_l \, dl$

Current – flow of charges

Current = amount charges crossing an area per unit time



$\vec{u}$: velocity of charges

$$I = \frac{dq}{dt} = \int_S \vec{J} \cdot d\vec{s}$$

(SI unit: Ampere ≡ Coulomb/sec)

Define **Volume Current Density**,

$$\vec{J} = \rho_v \vec{u} \quad \text{(SI unit: A/m}^2\text{)}\ ,$$

where ρ_v is the volume charge density.

$$dI = \vec{J} \cdot ds \Rightarrow I = \int_S \vec{J} \cdot d\vec{s}$$

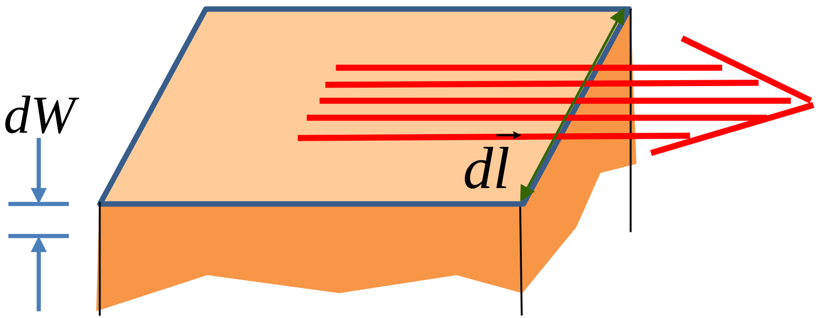
* Dot product: Only the charges flowing along the normal direction of the cross sectional area will move down a wire effectively.

Surface Current Density –
only exists on a conducting surface

In a perfect conductor, the volume charge density $\rho_v \rightarrow \infty$.
Thus, the volume current density $\mathbf{J} = \rho_v \mathbf{u} \rightarrow \infty$

There exists a finite **surface current density** in the limit

$$\vec{J}_s = \lim_{dW \rightarrow 0} \vec{J} \times dW \qquad \text{(SI unit: A/m)}$$



The total current on the surface is the line integration

$$I = \int_S J_s dl$$

1.3 電流
Electric Current

- Charges can be distributed in a volume, on a surface, and a line
- Flow of charges generates a current.
- Surface charge and surface current exist on a perfect conductor.

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1.4 磁力與磁場
Magnetic Force and Magnetic Field

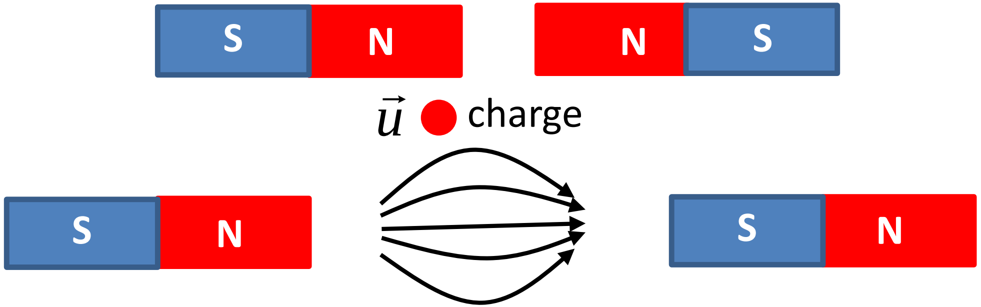
Magnetic Phenomena

Magnets attract screws and nails

A compass is aligned along the earth magnetic field

Magnetic Force and Magnetic Field
磁力與磁場

Observations on magnetic force



Magnetic field lines to help visualize the effect between N & S

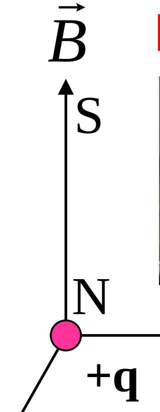
- Arrow: direction from N to S
- density of lines: strength of the magnetic effect

B: magnetic flux density (磁通密度) (SI unit: Tesla = Weber/m^2)

Magnetic Force

Magnetic force

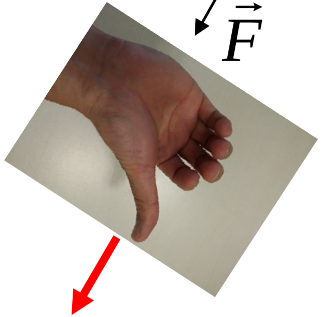
$$\vec{F} = q\vec{u} \times \vec{B}$$



Right-hand rule

Point your figures toward $\vec{u}$

$\vec{u}$: velocity



Curl up your figures toward $\vec{B}$

Your thumb is then along the direction of $\vec{F}$ for a positive charge.

1.4 磁力與磁場

Magnetic Force and Magnetic Field

- A magnet has two poles, N and S, with magnetic flux lines connected from N to S.
- A magnetic force on a moving charge is perpendicular to the motion of the charge and the direction of the magnetic field line.

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1.5 電與磁的關連

Connection between Electricity and Magnetism

Connection between Electricity and Magnetism – Electromagnetic Force

Both electric force and magnetic force act on a charge through the so-called **Lorentz force equation**

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$$

- The electric force is collinear to the electric field direction
- The magnetic force is perpendicular to the magnetic field direction
- The magnetic force only acts on a moving charge, perpendicular to the motion of the charge

Connection between Electricity and Magnetism – current as a source of magnetic field

Analogy

$\vec{u}$ ● charge

A magnet generates a magnetic force

A moving charge generates a magnetic force

$\Rightarrow$ Moving charges or current $\equiv$ a magnet $\Rightarrow$ magnetic field $\propto qu$

Recall the current density $\vec{J} = \rho_v \vec{u}$

Recall the electric current $I = \int_S \vec{J} \cdot d\vec{s}$

Define **magnetic field intensity** $H \propto I$, with

$\vec{H} \equiv \vec{B} / \mu,$ (SI unit: A/m)

where μ is called permeability.

1.5 電與磁的關連

Connection between Electricity and Magnetism

- A current, having moving charges in it, experiences a magnetic force perpendicular to it.
- A current generates a magnetic field intensity.

電磁學的基本物理量
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1.6 單元回顧 Review

單元回顧

1. SI units = MKSA units: meter, kilogram, sec, Ampere are used as units for length, mass, time, and current.

2. A charge exerts a force on a charge. Charges of different signs attract and charges of same signs expel each other.

3. $\vec{E}$, electric field intensity, generated by a source charge, is the electric force experienced by a unit positive charge.

4. $\vec{D}$, electric flux density, is an invented symbol to calculate the amount of charge enclosed in a surface.

單元回顧

5. An electric current I is the amount of charges flowing across an area per unit time.

6. Surface charges and surface current only exist on the surface of an ideal conductor.

7. A magnet has two poles, north (N) pole and south (S) pole, generating a magnetic flux density and pointing from N to S .

8. The magnetic flux density, B, exerts a force on a moving charge. The force is transverse to the motion of the charge.

單元回顧

9. Moving charges or a current generates a magnetic field intensity, $\vec{H}$, which is proportional to the magnetic flux density.

10. The Lorentz force equation, $\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$, governs the total electromagnetic force acting on a charge *q*.

11. An electric force is collinear to an electric field; whereas a magnetic force, following the right-hand rule, is perpendicular to a magnetic field and to the moving direction of a charge.

THANK YOU FOR YOUR ATTENTION