H.W. 2

109060013 張芯瑜

(1) first (even): $x^2 + x^4 + x^6$

second (odd): $x + x^3 + x^5$

$\Rightarrow GF = (x^2+x^4+x^6)(x+x^3+x^5) = \underline{x^3 + 2x^5 + 3x^7 + 2x^9 + x^{11}}$ #

(2) $\underbrace{\underline{1}\ \underline{1}\ \underline{2}\ \underline{1}\ \underline{3}\ \underline{\cdots\cdots}}_{\substack{n \\ \text{(objects)}}}$ (1、2、3 are colors)

$\Rightarrow EGF_1 = 0 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \cdots = e^x - 1$

$= EGF_2 = EGF_3$

$\Rightarrow$ # 3 colors on not distinct objects $= EGF_1 \times EGF_2 \times EGF_3$

$= (e^x-1)^3 = e^{3x} - 3e^{2x} + 3e^x - 1$

$\Rightarrow$ # 3 colors on n "distinct" objects = coeff of x^n $\times\ n!$

$= \left(\frac{3^n}{n!} - 3\frac{2^n}{n!} + 3\frac{1}{n!}\right)n! = \underline{3^n - 3\cdot 2^n + 3}$ #

(3) $\frac{x+1}{x^2-x-6} = \frac{\frac{4}{5}}{x-3} + \frac{\frac{1}{5}}{x+2}$

$(x-3)^{-1} = \sum_{k=0}^{\infty}\binom{-1}{k}x^k(-3)^{-1-k}$, $(x+2)^{-1} = \sum_{k=0}^{\infty}\binom{-1}{k}x^k(2)^{-1-k}$

$\Rightarrow$ coeff of $x^n = \frac{4}{5}\times\binom{-1}{n}(-3)^{-1-n} + \frac{1}{5}\times\binom{-1}{n}2^{-1-n}$

$= \underline{\frac{4(-3)^{-1-n} + 2^{-1-n}}{5}\binom{-1}{n}}$ #

(4) $\displaystyle\sum_{n=0}^{\infty}\Big(\underbrace{\sum_{m=0}^{n}\binom{n}{m}\omega^m}_{a_n}\Big)\frac{x^n}{n!} = a_0 + \frac{a_1}{1!}x + \frac{a_2}{2!}x^2+\cdots$

$$\Rightarrow \sum_{m=0}^{n}\binom{n}{m}\omega^m = (1+\omega)^n = a_n$$

$$\therefore \text{原式} = (1+\omega)^0 + \frac{(1+\omega)^1}{1!}x + \frac{(1+\omega)^2}{2!}x^2+\cdots\cdots$$

$$= \underline{e^{(1+\omega)x}} \#$$

(5) 1° $(1+x)^{-\frac{5}{4}} = \binom{-\frac{5}{4}}{0} + \binom{-\frac{5}{4}}{1}x + \binom{-\frac{5}{4}}{2}x^2+\cdots$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\frac{1}{0!} \qquad -\frac{\frac{1\times5}{4}}{1!} \qquad \frac{\frac{1\times5\times9}{4^2}}{2!} \qquad \cdots\cdots$$

2° $(1-4x)^b = \binom{b}{0} + \binom{b}{1}(-4x) + \binom{b}{2}(-4x)^2+\cdots$

3° $(1-4x)^{-\frac{5}{4}} = \binom{-\frac{5}{4}}{0} + \binom{-\frac{5}{4}}{1}(-4x) + \binom{-\frac{5}{4}}{2}(-4x)^2+\cdots$

$$= 1 + \frac{\frac{5}{\cancel{4}}}{1!}\cancel{4}x + \frac{\frac{5}{\cancel{4}}\times\frac{9}{\cancel{4}}}{2!}\cancel{4^2}x^2+\cdots$$

$$= 1 + \frac{(1\times5)}{1!}x + \frac{(1\times5\times9)}{2!}x^2+\cdots \frac{(1\times5\times(4r+1))}{r!}x^r+\cdots$$

$$\therefore \underline{(1-4x)^{-\frac{5}{4}}} \#$$