P.2–2 Given

$$\mathbf{A} = \mathbf{a}_x - \mathbf{a}_y 2 + \mathbf{a}_z 3,$$
$$\mathbf{B} = \mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z 2,$$

find the expression for a unit vector **C** that is perpendicular to both **A** and **B**.

評分標準

1. 向量符號誤用(Ex. $\hat{a}_x \neq a_x$) → 扣 2 分(在原文書裏頭會將向量符號以粗體字表示，可能是因為印刷或打字問題，但是手寫並未能被分辨，還須將向量符號表示出來)
2. 答案錯 → 全扣
3. 最後答案∓均給分

4%

[參考解答]

P. 2-2 解：令 $\vec{C}=\vec{a}_x C_x+\vec{a}_y C_y+\vec{a}_z C_z$，

其中 $C_x^2+C_y^2+C_z^2=1$ ①

由 $\vec{C}\perp\vec{A}$ 即 $\vec{C}\cdot\vec{A}=0$ 則 $C_x-2C_y+3C_z=0$ ②

又由 $\vec{C}\perp\vec{B}$ 即 $\vec{C}\cdot\vec{B}=0$ 則 $C_x-C_y+2C_z=0$ ③

聯解①②③可得：$C_x=\dfrac{1}{\sqrt{35}}, C_y=\dfrac{5}{\sqrt{35}}, C_z=\dfrac{3}{\sqrt{35}}$

即 $\vec{C}=\dfrac{1}{\sqrt{35}}(\vec{a}_x+\vec{a}_y 5+\vec{a}_z 3)$

P.2–14

a) Prove that the equation of any plane in space can be written in the form $b_1x + b_2y + b_3z = c$. (*Hint:* Prove that the dot product of the position vector to any point in the plane and a normal vector is a constant.)

b) Find the expression for the unit normal passing through the origin.

c) For the plane $3x - 2y + 6z = 5$, find the perpendicular distance from the origin to the plane.

a. 答案錯 → 全扣

沒在解答加上適量文字敘述→ 扣 1 分

b. 答案錯 → 全扣

c. 答案錯 → 全扣

未化簡 → 扣 1 分

2%、2%、2%

[參考解答]

P. 2-14 解：(a)令平面上任一點的位置向量爲：$\vec{R}=\vec{a}_x x+\vec{a}_y y+\vec{a}_z z$

并引入 $\vec{N}=\vec{a}_x b_1+\vec{a}_y b_2+\vec{a}_z b_3$

給定的方程可表示爲：$\vec{R}\cdot\vec{N}=C$（C 爲常數）

即位置向量到平面上任一點的投影爲常量，則 $\vec{N}$ 爲法向量

(b) $\vec{a}_N=\dfrac{\vec{N}}{|\vec{N}|}=\dfrac{\vec{a}_x b_1+\vec{a}_y b_2+\vec{a}_z b_3}{\sqrt{b_1^2+b_2^2+b_3^2}}$

(c)原點到平面的距離爲：

$$\vec{a}_N\cdot\vec{R}=\frac{C}{|\vec{N}|}$$

而 $C=5$，$|\vec{N}|=\sqrt{3^2+(-2)^2+6^2}=7$

$$\therefore \vec{a}_N\cdot\vec{R}=\frac{5}{7}$$

P.2–21 Given a vector function $\mathbf{E} = \mathbf{a}_x y + \mathbf{a}_y x$, evaluate the scalar line integral $\int \mathbf{E} \cdot d\boldsymbol{\ell}$ from $P_1(2, 1, -1)$ to $P_2(8, 2, -1)$

a) along the parabola $x = 2y^2$,

b) along the straight line joining the two points.

Is this **E** a conservative field?

P.2–22 For the **E** of Problem P.2–21, evaluate $\int \mathbf{E} \cdot d\boldsymbol{\ell}$ from $P_3(3, 4, -1)$ to $P_4(4, -3, -1)$ by converting both **E** and the positions of P_3 and P_4 into cylindrical coordinates.

答案錯 → 全扣

向量符號誤用 → 扣 1 分

5%

[參考解答]

P. 2-22 解：$\begin{bmatrix} E_\gamma \\ E_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \gamma\sin\phi \\ \gamma\cos\phi \end{bmatrix}$

$$\vec{E} = \vec{a}_\gamma \gamma \sin 2\phi + \vec{a}_\Phi \gamma \cos 2\phi$$

$$\vec{E}\vec{d}l = \gamma \sin 2\phi d\gamma + \gamma^2 \cos 2\phi d\phi$$

$P_3(3,4,-1) = P_3(5, 53.1^\circ, -1)$;

$P_4(4,-3,-1) = P_4(5, -36.9^\circ, -1)$

P_3 到 P_4 的距離 $\gamma = 5$

$\therefore \int_{P_3}^{P_4} \vec{E} \cdot \vec{d}l = 5^2 \int_{53.1^\circ}^{-36.9^\circ} \cos 2\phi d\phi = -24.$

P.2–23 Given a scalar function

$$V = \left(\sin \frac{\pi}{2} x\right)\left(\sin \frac{\pi}{3} y\right) e^{-z},$$

determine

a) the magnitude and the direction of the maximum rate of increase of V at the point $P(1, 2, 3)$,

b) the rate of increase of V at P in the direction of the origin.

P.2–23 **a)** $(\nabla V)_P = -(\mathbf{a}_y 0.026 + \mathbf{a}_z 0.043)$. **b)** 0.0485.

a. 答案錯 → 全扣

向量符號誤用 → 扣 1 分

b. 答案錯 → 全扣

向量符號誤用 → 扣 1 分

2%、2%

[參考解答]

P. 2-23 解：(a) $\vec{\nabla} V=[\vec{a}_x(\frac{\pi}{2}\cos\frac{\pi}{2}x)(\sin\frac{\pi}{3}y)+\vec{a}_y(\sin\frac{\pi}{2}x)(\frac{\pi}{3}\cos\frac{\pi}{3}y)$

$$-\vec{a}_z(\sin\frac{\pi}{2}x)(\sin\frac{\pi}{3}y)]e^{-z}$$

$$(\nabla V)_P=-(\vec{a}_y\frac{\pi}{6}+\vec{a}_z\frac{\sqrt{3}}{2})e^{-3}=-(\vec{a}_y 0.026+\vec{a}_z 0.043)$$

(b) $\vec{PO}=-\vec{a}_x-2\vec{a}_y-\vec{a}_z 3;\ \vec{a}_{po}=\frac{1}{\sqrt{14}}(\vec{a}_x+\vec{a}_y 2+\vec{a}_z 3)$

$$\therefore(\vec{\nabla} V)_p\cdot\vec{a}_{po}=\frac{1}{\sqrt{14}}(\frac{\pi}{3}+\frac{3\sqrt{3}}{2})e^{-3}=0.0485.$$

P.2–29 For vector function $\mathbf{A}=\mathbf{a}_r r^2+\mathbf{a}_z 2z$, verify the divergence theorem for the circular cylindrical region enclosed by $r=5$, $z=0$, and $z=4$.

P.2–29 $\oint_S \mathbf{A}\cdot d\mathbf{s}=\int_V \nabla\cdot\mathbf{A}\,dv=1{,}200\pi$.

答案錯 → 全扣

未寫出 divergence theorem → 扣 3 分

divergence theorem 寫錯(包含公式的向量符號) → 扣 3 分

其他向量符號誤用 → 扣 2 分

5%

[參考解答]

P. 2-29 解：$\oint \vec{A}\cdot\vec{ds}=(\int_{上表面}+\int_{下表面}+\int_{壁})\vec{A}\cdot\vec{ds}.$

上表面$(z=4)$：$\vec{A}=\vec{a}_{\gamma}\gamma^2+\vec{a}_z 8$；$\vec{ds}=\vec{a}_z ds.$

$\int_{上表面}\vec{A}\cdot\vec{ds}=\int_{上表面}8ds=8(\pi 5^2)=200\pi$

下表面$(z=0)$：$\vec{A}=\vec{a}_{\gamma}\gamma^2, \vec{ds}=-\vec{a}_z ds$

$$\int_{下表面}\vec{A}\cdot d\vec{s}=0$$

壁$(\gamma=5)$：$\vec{A}=\vec{a}_{\gamma}25+\vec{a}_z 2z, d\vec{s}=\vec{a}_{\gamma} ds$

$\int_{壁}\vec{A}\cdot d\vec{s}=25\int_{壁}ds=25(2\pi 5\times 4)=1000\pi$

$\therefore \oint\vec{A}\cdot d\vec{s}=200\pi+1000\pi+0=1200\pi$

$\nabla\cdot\vec{A}=3\gamma+2, \int\nabla\cdot\vec{A}d\gamma=\int_0^4\int_0^{2\pi}\int_0^5\nabla\cdot\vec{A}\gamma d\gamma d\phi dz=1200\pi$

散度定理得證。

P.2–36 Given the vector function $\mathbf{A}=\mathbf{a}_{\phi}\sin(\phi/2)$, verify Stokes's theorem over the hemispherical surface and its circular contour that are shown in Fig. 2–37.

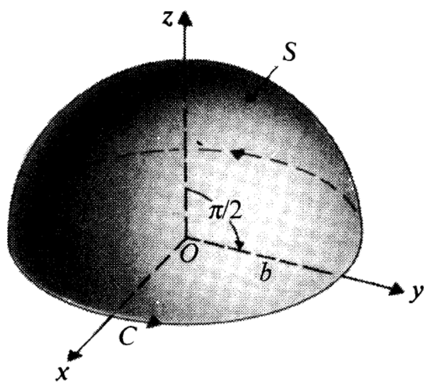

FIGURE 2–37
Graph for Problem P.2–36.

答案錯(計算錯誤、未寫出 Stokes's theorem) → 全扣
未寫出 Stokes's theorem → 扣 3 分
Stokes's theorem 寫錯(包含公式的向量符號) → 扣 3 分
其他向量符號誤用 → 扣 2 分

5%
[參考解答]

P. 2-36 解：$\vec{\nabla}\times\vec{A}=\frac{1}{R\sin\theta}(\vec{a}_R\cos\theta\sin\frac{\phi}{2}-A_{\vec{\theta}}\sin\theta\sin\frac{\phi}{2})$

$$\int_s(\vec{\Delta}\times\vec{A})\ ds=\int_0^{2\pi}\int_0^{\frac{\pi}{2}}(\vec{\Delta}\times\vec{A})_{R=b}(\vec{a}_R b^2\sin\theta d\theta d\phi)=4b$$

$$\oint_c\vec{A}\ d\vec{l}=\int_0^{2\pi}(A)R-b\theta=\pi/2\cdot(\vec{a}_\phi bd\phi)=\int_0^{2\pi}b\sin\frac{\phi}{2}d\phi=4b.$$

P.3–5 Two point charges, Q_1 and Q_2, are located at (1, 2, 0) and (2, 0, 0), respectively. Find the relation between Q_1 and Q_2 such that the total force on a test charge at the point $P(-1, 1, 0)$ will have

a) no x-component, **b)** no y-component.

P.3–5 a) $Q_1/Q_2=-3/4\sqrt{2}$. **b)** $Q_1/Q_2=1/2\sqrt{2}$.

答案錯 → 全扣

3%、3%

P. 3-5 解：$\vec{Q_1p}=-\vec{a}_x2-\vec{a}_y$；$\vec{Q_2p}=-\vec{a}_x3+\vec{a}_y$

$$\vec{E}_{P1}=\frac{Q_1}{4\pi t_0(\sqrt{5})^3}(-\vec{a}_x2-\vec{a}_y);\ \vec{E}_{P_2}=\frac{Q_2}{4\pi E_0(\sqrt{10})}(-\vec{a}_x3+\vec{a}_y)$$

(a)没有 x 分量：$-\frac{2Q_1}{(\sqrt{5})^3}-\frac{3Q_2}{(\sqrt{10})^3}=0$，或$\frac{Q_1}{Q_2}=-\frac{3}{4\sqrt{2}}$

(b)没有 y 分量：$-\frac{Q_1}{(\sqrt{5})^3}+\frac{Q_2}{(\sqrt{10})^3}=0$，或$\frac{Q_1}{Q_2}=\frac{1}{2\sqrt{2}}$.

總分:35