Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University

office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

EE214000 Electromagnetics, Fall, 2020

Homework #6, due in class at 12 pm, noon, Monday, Dec. 28, 2020

Late submission won't be accepted!

80%

Problem 1 A solenoid is a stack of current loops. (1) Given the expression of the magnetic field at the center of a current loop in EXPAMPLE 6-6, would you be able to derive an expression for the solenoid field in EXAMPLE 6-3? (5 points) (2) Use Eq. (6-65) and correct interpretation to derive the solenoid field Eq. (6-14). (5 points)

1.

$$\because B = \int_{-\infty}^{\infty} \frac{\mu_0 I b^2}{2(z^2+b^2)^{\frac{3}{2}}}\, dz \underset{I \to nI}{\approx} \frac{\mu_0 n I b^2}{2} \cdot \left[\frac{z}{b^2 (b^2+z^2)^{\frac{1}{2}}}\right]_{-\infty}^{\infty} = \mu_0 n I$$

∴ We can derive an expression for the Solenoid field in example 6-3 #

2.

$$\vec{B} = \hat{a}_z \lim_{L\to\infty} \int_{-L}^{L} \frac{\mu_0 M_0 b^2}{2[(z-z')^2+b^2]^{\frac{3}{2}}}\, dz' = \hat{a}_z \lim_{L\to\infty} \int_{-L}^{L} \frac{\mu_0 n I b^2}{2[(z-z')^2+b^2]^{\frac{3}{2}}}\, dz'$$

$\vec{J}_{ms} = \hat{a}_\phi M_0 \rightarrow J_{ms} = nI$

$$\underset{z=0}{=} \hat{a}_z \lim_{L\to\infty} \int_{-L}^{L} \frac{\mu_0 n I b^2}{2(z'^2+b^2)^{\frac{3}{2}}}\, dz'$$

$$= \hat{a}_z \left(\frac{\mu_0 n I}{2} + \frac{\mu_0 n I}{2}\right)$$

$$= \hat{a}_z \mu_0 n I \quad \#$$

5%、5%

Problem 2 P. 6-4 in Cheng's textbook (10 points)

P.6–4 A current I flows lengthwise in a very long, thin conducting sheet of width w, as shown in Fig. 6–35.

a) Assuming that the current flows into the paper, determine the magnetic flux density $\mathbf{B}_1$ at point $P_1(0, d)$.

b) Use the result in part (a) to find the magnetic flux density $\mathbf{B}_2$ at point $P_2(2w/3, d)$.

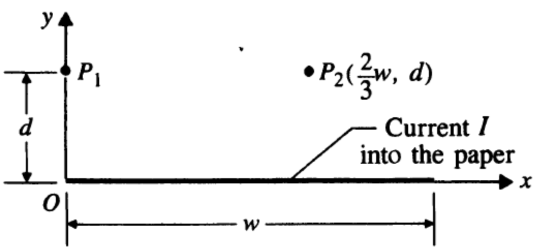
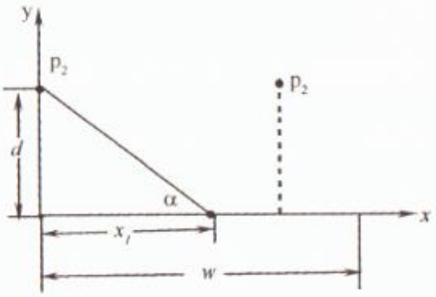


FIGURE 6–35
A thin conducting sheet carrying a current I (Problem P.6–4).

P.6-4 解：



(a) $d\vec{B}_{p1} = \vec{a}_x dB_x + \vec{a}_y dB_y$

$= \vec{a}_x (dB_{P1})\sin\alpha + \vec{a}_y (db_{p1})\cos\alpha$

$dB_{p1} = \dfrac{\mu_0 (I/W) dx'}{2\pi(x^2+d^2)^{3/2}}\ \sin\alpha = \dfrac{d}{(x^2+d^2)^{1/2}},\ \cos\alpha = \dfrac{x'}{(x^2+d^2)})^{1/2}$

$\therefore \vec{B}_{p_1} = \vec{a}_x B_x + \vec{a}_y By$

其中 $B_x = \dfrac{\mu_0 I d}{2\pi w}\int_0^w \dfrac{dx'}{x^2+d^2} = \dfrac{\mu_0 I}{2\pi w}\tan^{-1}(\dfrac{w}{d})$

$B_y = \dfrac{\mu_0 I}{2\pi w}\int_0^w \dfrac{x' dx'}{x^2+d^2} = \dfrac{\mu_0 I}{\pi w} ln(1+\dfrac{w}{d})$

(b) $\vec{B}_{p_2} = \vec{B}_{2右} + \vec{B}_{2左}$

$\vec{B}_{2右} = \dfrac{\mu_0 I}{2\pi w}[\vec{a}_x \tan^{-1}(\dfrac{w}{3d}) + \vec{a}_y \dfrac{1}{2} ln(1+\dfrac{w^2}{9d^2})]$

$\vec{B}_{2左} = \dfrac{\mu_0 I}{2\pi w}[\vec{a}_x \tan^{-1}(\dfrac{w}{3d}) + \vec{a}_y \dfrac{1}{2} ln(1+\dfrac{w^2}{9d^2})]$

所以 $\vec{B}_{p_2} = \dfrac{\mu_0 I}{2\pi w}[\vec{a}_x(\tan^{-1}\dfrac{w}{3d} + \tan^{-1}\dfrac{2w}{3d}) - \vec{a}_y ln\sqrt{\dfrac{1+(2w/3d)^2}{1+(w/3d)^2}}]$

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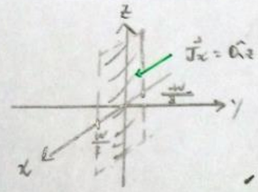
Problem 3 P. 6-10 in Cheng’s textbook. (5 points)

P.6–10 A very long, thin conducting strip of width w lies in the xz-plane between $x = \pm w/2$. A surface current $\mathbf{J}_s = \mathbf{a}_z J_{s0}$ flows in the strip. Find the magnetic flux density at an arbitrary point outside the strip.

5%

10.



let arbitrary point $P(x, y, z)$

$$\rightarrow \quad d\vec{B} = \frac{\mu_0 J_{s0}\, dx'}{2\pi r^2}\left[\hat{a}_x(-y) + \hat{a}_y(x - x')\right]$$

$$r = \sqrt{(x-x')^2 + y^2}$$

then $\vec{B} = \int d\vec{B} = \hat{a}_x B_x + \hat{a}_y B_y$

$$B_x = \frac{-\mu_0 J_{s0}}{2\pi} y \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{dx}{(x-x')^2 + y^2} = \frac{-\mu_0 J_{s0}}{2\pi}\left[\tan^{-1}\left(\frac{x+\frac{w}{2}}{y}\right) + \tan^{-1}\left(\frac{x-\frac{w}{2}}{y}\right)\right]$$

$$B_y = \frac{\mu_0 J_{s0}}{2\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{x - x'}{(x-x')^2 + y^2}\, dx' = \frac{\mu_0 J_{s0}}{4\pi} \ln\left|\frac{(x+\frac{w}{2})^2 + y^2}{(x-\frac{w}{2})^2 + y^2}\right|$$

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Problem 4 P. 6-18 in Cheng's textbook (10 points)

P.6–18 Starting from the expression of **A** in Eq. (6–34) for the vector magnetic potential at a point in the bisecting plane of a straight wire of length $2L$ that carries a current I:

a) Find **A** at point $P(x, y, 0)$ in the bisecting plane of two parallel wires each of length $2L$, located at $y = \pm d/2$ and carrying equal and opposite currents, as shown in Fig. 6–41.

b) Find **A** due to equal and opposite currents in a very long two-wire transmission line.

c) Find **B** from **A** in part (b), and check your answer against the result obtained by applying Ampère's circuital law.

d) Find the equation for the magnetic flux lines in the xy-plane.

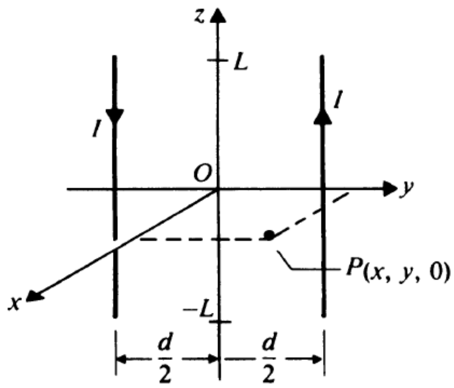

FIGURE 6–41
Parallel wires carrying equal and opposite currents (Problem P.6–18).

P.6-18 解：由(6-34) $\vec{A}=\vec{a}_z\frac{\mu_0 I}{4\pi}ln\frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}$

對兩個帶有大小相同，方向相反電流的導綫：

(a) $$\vec{A}=\vec{a}_z\frac{\mu_0 I}{4\pi}ln[\frac{\sqrt{L^2+r_2^2}+L}{\sqrt{L^2+r_2^2}-L}\cdot\frac{\sqrt{L^2+r_1^2}-L}{\sqrt{L^2+r_1^2}+L}]$$

$$=\vec{a}_z\frac{\mu_0 I}{2\pi}ln[\frac{r_1}{r_2}\cdot\frac{\sqrt{L^2+r_2^2}-L}{\sqrt{L^2+r_1^2}+L}]$$

(b)對於很長的雙綫傳輸綫，$L\rightarrow W$：

$$\vec{A}=\vec{a}_z\frac{\mu_0 I}{2\pi}ln(\frac{r_1}{r_2})=\vec{a}_z\frac{\mu_0 I}{4\pi}ln\frac{(\frac{d}{2}+y)^2+x^2}{(\frac{d}{2}-y)^2+x^2}$$

(c) $$\vec{B}=\nabla\times\vec{A}=\vec{a}_x\frac{\partial A_z}{\partial y}-\vec{a}_y\frac{\partial A_z}{\partial x}$$

$$=\vec{a}_x\frac{\mu_0 I}{2\pi}[\frac{\frac{d}{2}+y}{(\frac{d}{2}+y)^2+x^2}-\frac{\frac{d}{2}-y}{(\frac{d}{2}-y)^2+x^2}]-\vec{a}_y$$

$$\frac{\mu_0 I}{2\pi}[\frac{x}{(\frac{d}{2}+y)^2+x^2}-\frac{x}{(\frac{d}{2}-y)^2+x^2}]$$

$$=\frac{\mu_0 I}{2\pi}[\vec{a}_{\phi 1}\frac{1}{r_1}-\vec{a}_{\phi 2}\frac{1}{r_2}]$$

(d) $$\frac{dx}{B_x}=\frac{dy}{B_y}\Rightarrow\frac{\partial A}{\partial x}dx+\frac{\partial A}{\partial y}dy=0$$

$\Rightarrow dA=0\Rightarrow A$ 爲常量.

所以，$$\frac{r_1^2}{r_2^2}=\frac{(\frac{d}{2}+y)^2+x^2}{(\frac{d}{2}-y)^2+x^2}=K$$

2%、2%、2%、4%

Problem 5 P. 6-22 in Cheng's textbook (10 points)

P.6–22 A circular rod of magnetic material with permeability μ is inserted coaxially in the long solenoid of Fig. 6–4. The radius of the rod, a, is less than the inner radius, b, of the solenoid. The solenoid's winding has n turns per unit length and carries a current I.

a) Find the values of **B**, **H**, and **M** inside the solenoid for $r < a$ and for $a < r < b$.

b) What are the equivalent magnetization current densities $\mathbf{J}_m$ and $\mathbf{J}_{ms}$ for the magnetized rod?

解：(a) $r<a$: $\vec{H}=\vec{a}_z nI$

$$\vec{B}=\vec{a}_z \mu nI$$

$$\vec{M}=\frac{\vec{B}}{\mu_0}-\vec{H}=\vec{a}_z\left(\frac{\mu}{\mu_0}-1\right)nI$$

$a<r<b$: $\vec{H}=\vec{a}_z nI$

$$\vec{B}=\vec{a}_z \mu_0 nI$$

$$\vec{M}=0$$

(b) $\vec{J}_m=\vec{\nabla}\times\vec{M}=0$;

$$\vec{J}_{ms}=\vec{M}\times\vec{a}_n=(\vec{a}_z\times\vec{a}_r)\left(\frac{\mu}{\mu_0}-1\right)nI=\vec{a}_\phi\left(\frac{\mu}{\mu_0}-1\right)nI$$

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Problem 6 P. 6-27 in Cheng's textbook. (15 points)

P.6–27 A toroidal iron core of relative permeability 3000 has a mean radius $R = 80$ (mm) and a circular cross section with radius $b = 25$ (mm). An air gap $\ell_g = 3$ (mm) exists, and a current I flows in a 500-turn winding to produce a magnetic flux of 10^{-5} (Wb). (See Fig. 6–44.) Neglecting flux leakage and using mean path length, find

a) the reluctances of the air gap and of the iron core,
b) $\mathbf{B}_g$ and $\mathbf{H}_g$ in the air gap, and $\mathbf{B}_c$ and $\mathbf{H}_c$ in the iron core,
c) the required current I.

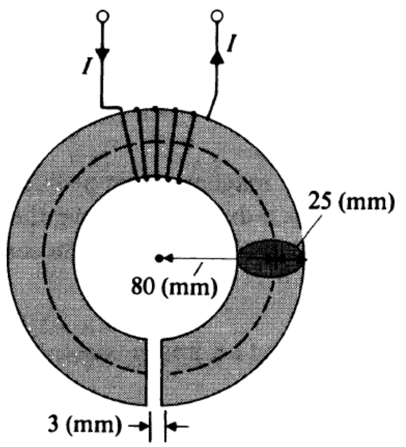

FIGURE 6–44
A toroidal iron core with air gap (Problem P.6–27).

P. 6—27 解：(a)$R_g=\frac{\ell_g}{\mu_0 s}=\frac{3\times10^{-3}}{4\pi\times10^{-7}(\pi\times0.025)^2}=1.21\times10^6(H^{-1})$

$$R_c=\frac{2\pi\times0.08-0.003}{3000\times(4\pi\times10^{-7})\times(\pi\times0.025)^2}=6.75\times10^4(H^{-1})$$

$$\text{(b)}\vec{B}_g=\vec{B}_C=\vec{a}_\phi\frac{10^{-5}}{\pi\times0.025^2}=\vec{a}_\phi5.09\times10^{-3}(T)$$

$$\vec{H}_g=\frac{1}{\mu_0}\vec{B}_g=\vec{a}_\phi\frac{5.09\times10^{-3}}{4\pi\times10^{-7}}=\vec{a}_\phi4.05\times10^3(A/m)$$

$$\vec{H}_c=\frac{1}{\mu_0\mu_r}\vec{B}_c=\vec{a}_\phi\frac{4.05\times10^3}{3000}=\vec{a}_\phi1.35(A/m)$$

$$\text{(c)}NI=\phi(R_c+R_g),$$

$$I=\frac{1}{n}\phi(R_c+R_g)=0.025(A)=25.6(mA)$$

5%、5%、5%

Problem 7 (20 points) Visit a few electronics stores to find out at least 5 kinds of magnetic circuit devices. Use your cell phone camera to take photographs for those components and show them in your homework report. Describe the specifications, purposes, materials, and functioning principles of those components that you find in the stores. DO NOT COPY ANYTHING FORM THE INTERNET. Direct copying is an academic crime!

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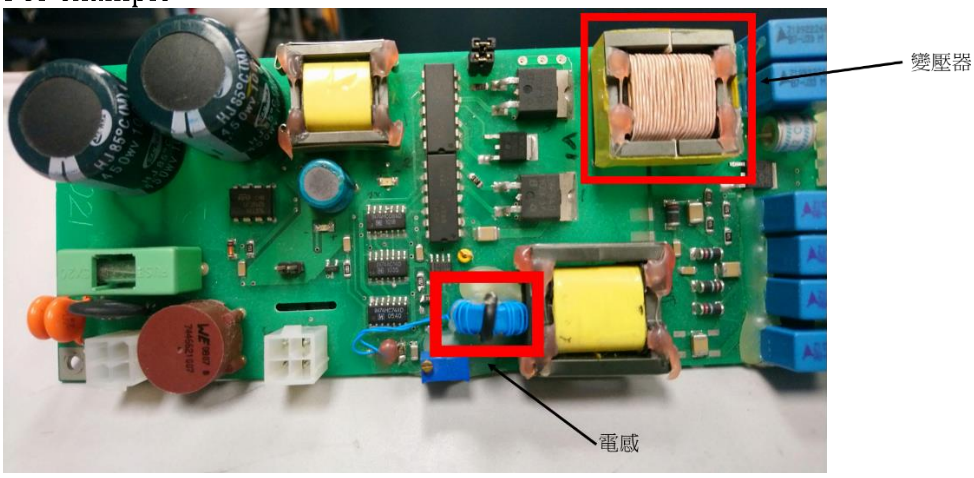
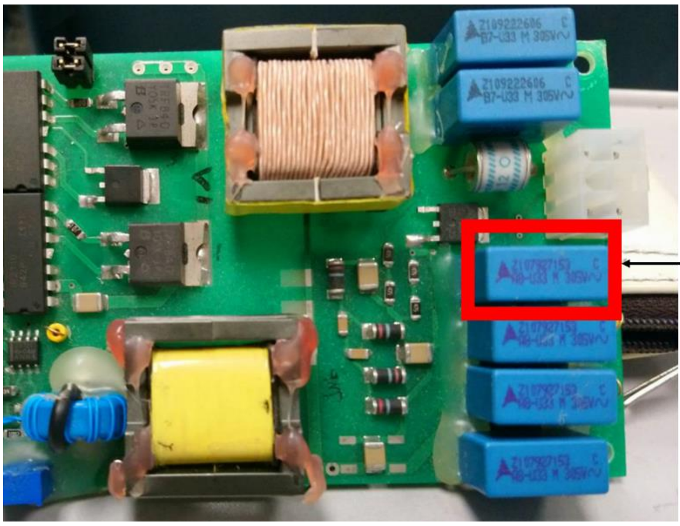
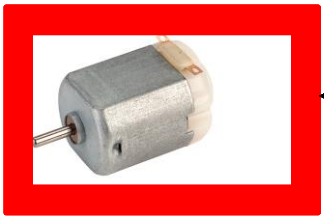
For example





繼電器



馬達

Functioning principle

1. 變壓器(voltage transformer)

藉由加上交流電壓於第一級線圈中並使其中通有電流(AC)時，會讓鐵(磁)芯中產生磁通(AC)，便讓第二級線圈中感應出相對應的電壓（或電流）(AC)。

2. 電感(inductance)

電感為應用自”電磁感應”之電子材料，會因為通過的電流的改變而產生相對應的電動勢(emf)，從而抵抗電流的改變，其中又可因為作用的對象差異分為自感(self inductance)以及互感(mutual inductance)。

3. 繼電器(relay)

繼電器為一種電子控制元件,常用於自動控制的電路中，實際上是利用較小的電流去控制較大的電流，達到開關的效果，繼電器中有個磁簧開關，即，利用電的訊號控制磁場，進而控制磁簧開關的開或關。

4. 馬達(motor)

在磁場內放置一導線(具有旋轉軸)並通以電流，會使得導線產生磁場並與原來之磁場相互作用下，使此導線產生旋轉的作用。

5. 電磁式蜂鳴器(Buzzer)

將繞有線圈的鐵心放置於磁環內，而後再放置導磁鐵片及振動膜片。當通電後其電路會向線圈發出訊號，線圈會產生出隨時間替換的磁場，使得受磁場影響的鐵(膜)片會吸引向鐵心,使得電能轉換為聲能。