Exam 1

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(1a) 1° For each pair, we can choose 0, 1 or 2, so we can use ternary string.

$\because 3^5 > 100$

$\therefore$ 100 can be written by 5 digits in ternary num.

$\Rightarrow$ at least 5 pairs of measuring weights are needed.

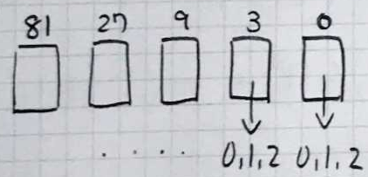
(1b) 1° According to (1a), we know that we should choose power of 3 for the pairs.

$\therefore (1,1)\ (3,3)\ (9,9)\ (27,27)\ (81,81)$

2° Justify:

Seeing the 5 pairs above as the ternary string below.

5-digits



We can calculate $1 \sim 100$ by using this 5-digits ternary string uniquely, so it serves our purpose.

(3) $$\frac{x+1}{(x-2)(1+5x)} = \frac{\frac{3}{11}}{x-2} + \frac{-\frac{4}{11}}{1+5x} = \frac{\frac{3}{-22}}{(1-\frac{1}{2}x)} + \frac{-\frac{4}{11}}{(1+5x)}$$

$$= -\frac{3}{22}\sum_{i=0}^{\infty}\binom{-1}{i}\left(-\frac{1}{2}x\right)^i + \left(-\frac{4}{11}\right)\sum_{j=0}^{\infty}\binom{-1}{j}(5x)^j$$

$$\therefore \text{coeff of } x^n = \left(-\frac{3}{22}\right)\times\frac{(-1)(-2)\cdots(-\cancel{1}-n+1)}{\cancel{n!}}\times\left(\cancel{-}\frac{1}{2}\right)^n$$

$$+\left(-\frac{4}{11}\right)\times\underbrace{\frac{(-1)(-2)\cdots(-\cancel{1}-n+1)}{\cancel{n!}}}_{=(-1)^n}\times(5)^n$$

$$= \left(-\frac{3}{11}\right)\times\left(\frac{1}{2}\right)^{n+1} + \left(-\frac{4}{11}\right)\times(-5)^n$$

$$= \frac{(-3)\times 2^{-n-1} + (-4)\times(-5)^n}{11}$$ #

(4a) $\frac{1}{1}$ $\frac{2^{(1)}}{}$ $\frac{3^{(1)}}{}$ $\frac{1}{r}$ .... $\frac{3^{(?)}}{)}$

$\therefore \ EGF_1 = 0 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots = e^x - 1$

$EGF_2 = 0 + \frac{1!}{1!}x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 + \cdots = \frac{1}{1-x} - 1$

$EGF_3 = 1 + \frac{1!}{1!}x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 + \cdots = \frac{1}{1-x}$

$\Rightarrow \underline{EGF = (e^x - 1)\left(\frac{x}{1-x}\right)\left(\frac{1}{1-x}\right)}$ #

(4b) $a_6 = [\text{coeff of } x^6] \times 6!$

$(e^x-1)\frac{x}{(1-x)^2} = (e^x-1)\,x\,(1+x+x^2+\cdots)^2 \rightarrow$ 找 x^6

$\Rightarrow (e^x-1)(1-x)^{-2} \rightarrow$ 找 x^5

$0 + \frac{1}{1!}x + \frac{1}{2!}x^2 \cdots$ $\qquad \sum_{i=0}^{\infty}\binom{-2}{i}(-x)^i$

$\Rightarrow \text{coeff} = \overset{0}{\cancel{0\binom{-2}{5}(-1)^5}} + \frac{1}{1!}\times\binom{-2}{4}(-1)^4 + \frac{1}{2!}\times\binom{-2}{3}(-1)^3$

$+ \frac{1}{3!}\binom{-2}{2}(-1)^2 + \frac{1}{4!}\binom{-2}{1}(-1)^1 + \frac{1}{5!}\binom{-2}{0}(-1)^0$

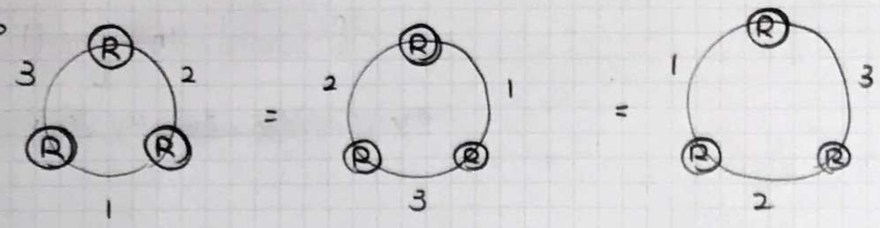
$= 0 + \frac{\cancel{2\cdot3\cdot4\cdot5}}{\cancel{4!}} + \frac{\cancel{2\cdot3\cdot4}}{\cancel{2!\,3!}}\,2$

$+ \frac{\cancel{2\times3}}{\cancel{3!}\,2!} + \frac{2}{4!\,1!} + \frac{1}{5!} = 5 + 2 + \frac{1}{2!} + \frac{1}{12} + \frac{1}{5!})$

$\therefore a_6 = 6! \times \text{coeff}$

$= 6 + 60 + 6\cdot5\cdot4\cdot3 + 7! = \underline{5466}$ #

(2a) 1°



∴ count 3 times

2° $\dfrac{C^{10+2}_{10}}{3} = \dfrac{C^{12}_{2}}{3} = \underline{22}$ #

3° check :

‿	R	‿	R	‿
10		0		0
9		1		0
9		0		1
8		2		0
8		0		2
8		1		1
7		3		0
7		0		3
7		2		1
7		1		2
6		0		4
6		4		0
6		3		1
6		1		3
6		2		2
5		5		0
5		4		1
5		1		4
5		2		3
5		3		2
4		4		2
4		3		3

⇒ $\underline{22}$ ※

(2b) case 1 : ‿ R ‿ R ‿ ⇒ 1

✓ × ×

case 2 : ‿ R ‿ R ‿ ⇒ 99

✓ ✓ ×

case 3 : ‿ R ‿ R ‿

✓ ✓ ✓

(6) $e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$e^x - 1 = \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$\Rightarrow \frac{e^x-1}{x} = 1 + \frac{1}{2!}x + \frac{1}{3!}x^2 + \cdots$

$\Rightarrow \left(\frac{e^x-1}{x}\right)' = \frac{1}{2!} + \frac{2}{3!}x + \frac{3}{4!}x^2 + \frac{4}{5!}x^3 + \cdots = F(x)$

$\therefore F(x) = \left((e^x-1)x^{-1}\right)'$

$= e^x \cdot x^{-1} + (e^x-1)(-x^{-2}) = \frac{e^x}{x} - \frac{e^x-1}{x^2}$

$\Rightarrow F(1) = \frac{e^1}{1} - \frac{e^1-1}{1} = e - (e-1) = \underline{1}$ #

(5) a_n