

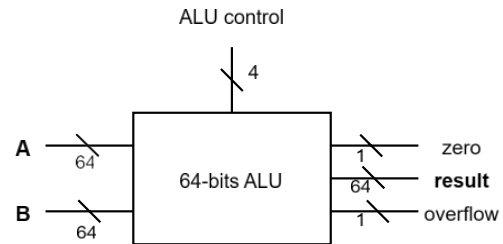
CS4100 Computer Architecture

Spring 2024, Homework 3

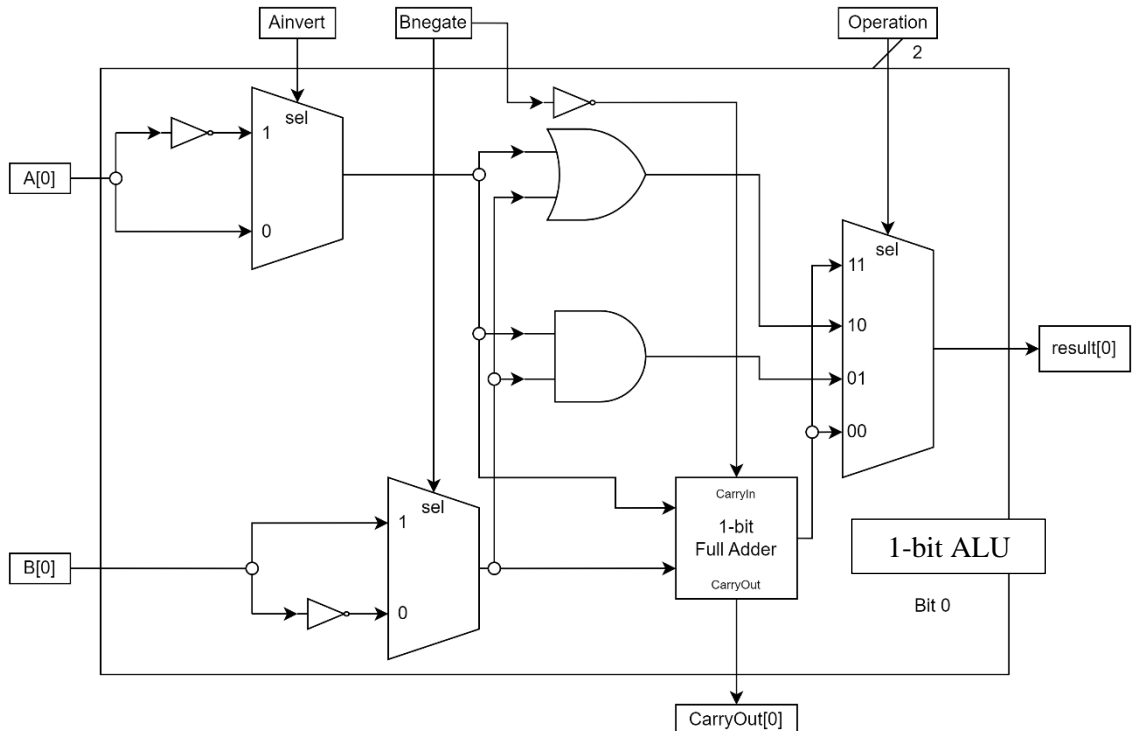
Due: 23:59, 4/21/2024

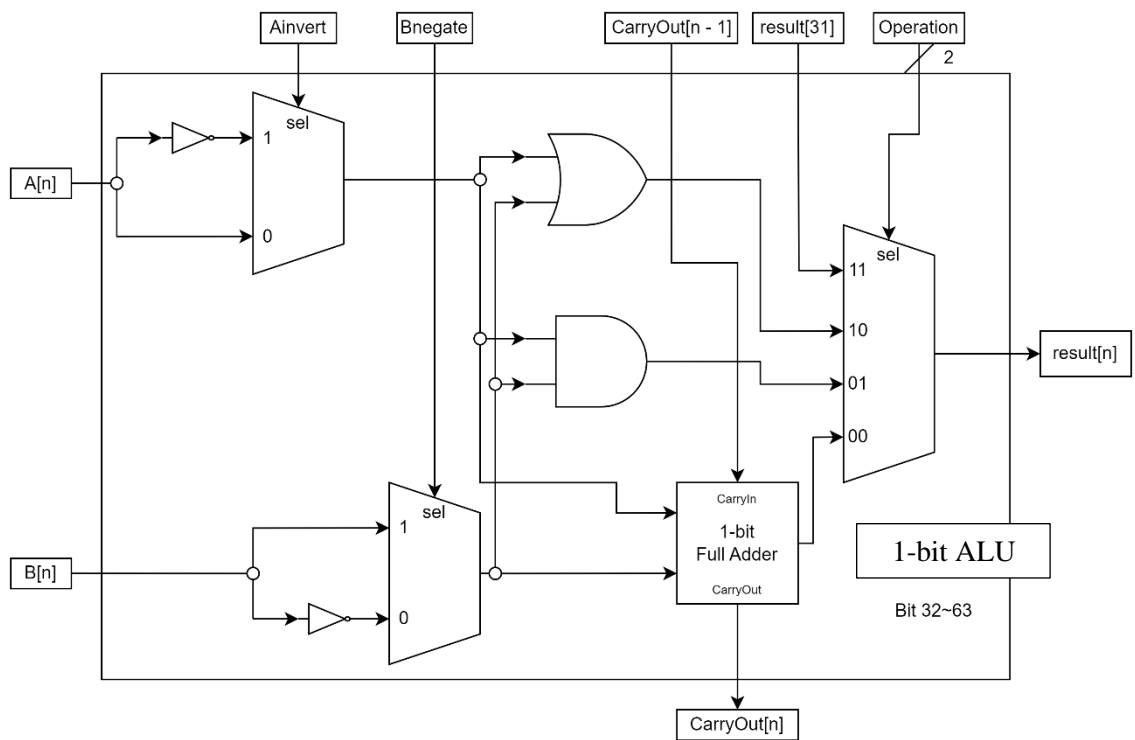
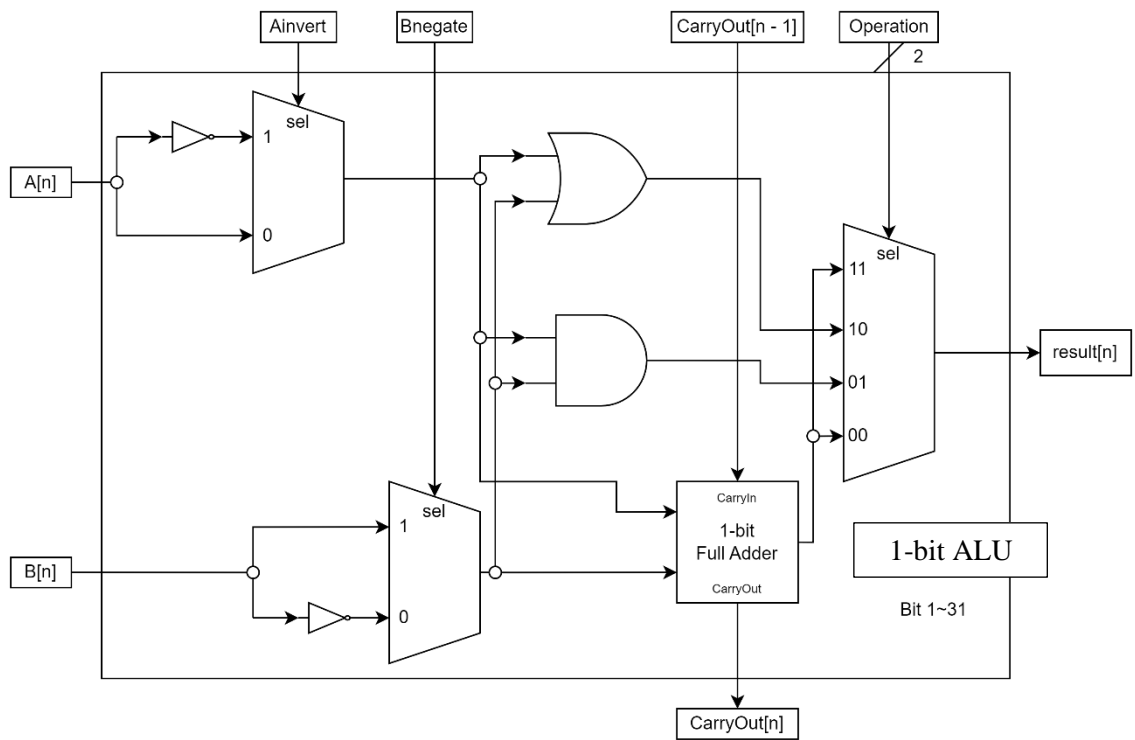
- (18 points) Please modify the ALU design introduced in the class to satisfy the following requirements:

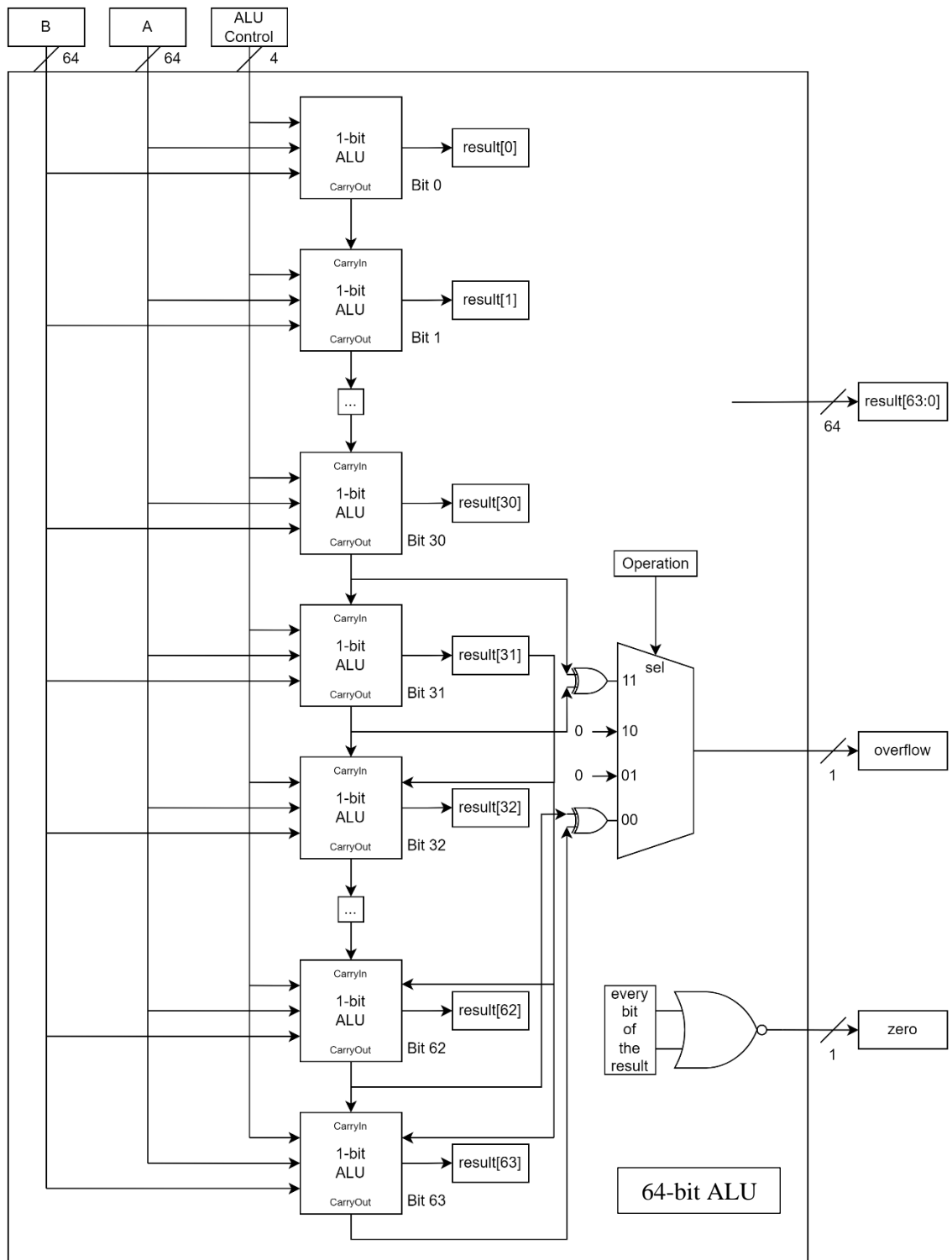
Ainvert	Bnegate	Operation	Function
0	1	01	AND
0	1	10	OR
0	1	00	add
0	0	00	sub
1	0	01	NOR
0	1	11	add-ext
0	0	11	sub-ext



Here, "add-ext" and "sub-ext" refer to 32-bit addition and subtraction with sign-extension to 64 bits. You are required to draw the circuit diagrams for each 1-bit ALU and the 64-bit ALU. For each 1-bit ALU, use only one full adder to perform an addition or subtraction operation, similar to the method demonstrated in class. Additionally, show the ALU control signals for "add-ext" and "sub-ext" in your design.







2. (14 points) Consider two unsigned binary numbers: $M = 1110$ and $N = 1001$.
- (a) (7 points) Write down each step of $M \times N$ according to version 1 of the multiply algorithm.

Iteration	Step	Multiplier	Multiplicand	Product
0	Initial values	1001	0000 1110	0000 0000
1	Prod = Prod + Mcand	1001	0000 1110	0000 1110
	Shift left multiplicand	1001	0001 1100	0000 1110
	Shift right multiplier	0100	0001 1100	0000 1110
2	Prod = Prod + 0	0100	0001 1100	0000 1110
	Shift left multiplicand	0100	0011 1000	0000 1110
	Shift right multiplier	0010	0011 1000	0000 1110
3	Prod = Prod + 0	0010	0011 1000	0000 1110
	Shift left multiplicand	0010	0111 0000	0000 1110
	Shift right multiplier	0001	0111 0000	0000 1110
4	Prod = Prod + Mcand	0001	0111 0000	0111 1110
	Shift left multiplicand	0001	1110 0000	0111 1110
	Shift right multiplier	0000	1110 0000	0111 1110

$M * N = 0111\ 1110$

- (b) (7 points) Write down each step of $M \times N$ according to version 2 of the multiply algorithm.

Iteration	Step	Multiplicand	Product
0	Initial values	1110	0000 1001
1	LeftProd += Mcand	1110	1110 1001
	Shift right multiplier	1110	0111 0100
2	LeftProd += 0	1110	0111 0100
	Shift right multiplier	1110	0011 1010
3	LeftProd += 0	1110	0011 1010
	Shift right multiplier	1110	0001 1101
4	LeftProd += Mcand	1110	1111 1101
	Shift right multiplier	1110	0111 1110

$M * N = 0111\ 1110$

3. (14 points) Consider two unsigned binary numbers: $M = 0111$ and $N = 0101$.
- (a) (7 points) Write down each step of $M \div N$ according to version 1 of the division algorithm.

Iteration	Step	Quotient	Divisor	Remainder
0	Initial values	0000	0101 0000	0000 0111
1	Rem = Rem - Div	0000	0101 0000	1011 0111
	Rem < 0			
	⇒ Rem += Div	0000	0101 0000	0000 0111
	⇒ Shift left quotient			
	Shift right divisor	0000	0010 1000	0000 0111
2	Rem = Rem - Div	0000	0010 1000	1101 1111
	Rem < 0			
	⇒ Rem += Div	0000	0010 1000	0000 0111
	⇒ Shift left quotient			
	Shift right divisor	0000	0001 0100	0000 0111
3	Rem = Rem - Div	0000	0001 0100	1111 0011
	Rem < 0			
	⇒ Rem += Div	0000	0001 0100	0000 0111
	⇒ Shift left quotient			
	Shift right divisor	0000	0000 1010	0000 0111
4	Rem = Rem - Div	0000	0000 1010	1111 1101
	Rem < 0			
	⇒ Rem += Div	0000	0000 1010	0000 0111
	⇒ Shift left quotient			
	Shift right divisor	0000	0000 0101	0000 0111
5	Rem = Rem - Div	0000	0000 0101	0000 0010
	Rem >= 0			
	⇒ Shift left quotient	0001	0000 0101	0000 0010
	⇒ quotient[0] = 1			
	Shift right divisor	0001	0000 0010	0000 0111

$M / N = 0001...0010$

- (b) (7 points) Write down each step of $M \div N$ according to version 2 of the division algorithm.

Iteration	Step	Divisor	Remainder / Quotient
0	Initial values	0101	0000 0111
1	Shift left remainder	0101	0000 1110
	LeftRem -= Dsivisor	0101	1011 1110
	Rem < 0		
	⇒ LeftRem += Divisor ⇒ Shift left Rem	0101	0001 1100
2	LeftRem -= Dsivisor	0101	1100 1100
	Rem < 0		
	⇒ LeftRem += Divisor	0101	0011 1000
	⇒ Shift left Rem		
3	LeftRem -= Dsivisor	0101	1110 1000
	Rem < 0		
	⇒ LeftRem += Divisor	0101	0111 0000
	⇒ Shift left Rem		
4	LeftRem -= Dsivisor	0101	0010 0000
	Rem >= 0		
	⇒ Shift left Rem	0101	0100 0001
	⇒ Rem[0] = 1		
5	Shift right leftRem	0101	0010 0001

$M / N = 0001...0010$

4. (12 points) Answer the following questions in detail. You will receive 0 point if you only write down the answers.

- (a) (4 points) What decimal number does the bit pattern $05948DEC_{16}$ represent if it's a two's complement integer? If it's an unsigned number, is the result the same as the two's complement? If they are different, why?

$$\begin{aligned} 05948DEC_{16} &= 12 + 14 \cdot 16 + 13 \cdot 16^2 + 8 \cdot 16^3 + 4 \cdot 16^4 + 9 \cdot 16^5 + 5 \cdot 16^6 \\ &= 93621740_{10} \end{aligned}$$

因為在 signed 的情況下 sign bit 是 0，因此 signed 與 unsigned 的結果會是一樣的。

- (b) (4 points) Answer problem (a) with a different bit pattern $FA6B7214_{16}$.
signed number:

$$\begin{aligned} FA6B7214_{16} &= 1111_1010_0110_1011_0111_0010_0001_0100_2 \\ &= - (0000_0101_1001_0100_1000_1101_1110_1011_2 + 1) \\ &= - (0000_0101_1001_0100_1000_1101_1110_1100_2) \\ &= - 93621740_{10} \end{aligned}$$

unsigned number:

$$\begin{aligned} FA6B7214_{16} &= 1111_1010_0110_1011_0111_0010_0001_0100_2 \\ &= 4201345556_{10} \end{aligned}$$

以 signed 的方式來看，sign bit 為 1 代表是負數，因此結果會與 unsigned 的結果不一樣。

- (c) (4 points) What decimal numbers do $05948DEC_{16}$ and $FA6B7214_{16}$ represent if they are IEEE 754 floating point numbers.

$$05948DEC_{16} = 0000_0101_1001_0100_1000_1101_1110_1100_2$$

$$\Rightarrow 0_00001011_00101001000110111101100$$

$$\Rightarrow \text{sign bit: } 0, \text{ exponent: } 11_{10} - 127_{10} (\text{bias}) = -116_{10},$$

$$\text{fraction: } 00101001000110111101100_2$$

$$\begin{aligned} \Rightarrow 1 + 2^{-3} + 2^{-5} + 2^{-8} + 2^{-12} + 2^{-13} + 2^{-15} + 2^{-16} + 2^{-17} + 2^{-18} + 2^{-20} + 2^{-21} \\ = 1 + 0.160581 = 1.160581 \end{aligned}$$

$$\Rightarrow \text{Decimal number: } 1.160581 * 2^{-116}$$

$$FA6B7214_{16} = 1111_1010_0110_1011_0111_0010_0001_0100_2$$

$$\Rightarrow 1_11110100_11010110111001000010100_2$$

$$\Rightarrow \text{sign bit: } 1, \text{ exponent: } 244_{10} - 127_{10} = 117_{10}$$

$$\text{fraction: } 11010110111001000010100_2$$

$$\begin{aligned} \Rightarrow 1 + 2^{-1} + 2^{-2} + 2^{-4} + 2^{-6} + 2^{-7} + 2^{-9} + 2^{-10} + 2^{-11} + 2^{-14} + 2^{-19} + 2^{-21} \\ = 1 + 0.839419 = 1.839419 \end{aligned}$$

$$\Rightarrow \text{Decimal number: } -1.839419 * 2^{117}$$

5. (10 points) Consider two decimal numbers: $X = 88.4375$ and $Y = -7.3125$.

- (a) (6 points) Write down X and Y in the IEEE 754 single precision format. You must detail how you get your answer, or you will receive 0 point.

$$88.4375_{10} = 88_{10} + 0.25_{10} + 0.125_{10} + 0.0625_{10}$$

$$= 01011000.0111_2 = 1.0110000111_2 * 2_{10}^6$$

X is positive \Rightarrow sign bit = 0

$$\text{exponent} = 6_{10} + 127_{10} = 133_{10} = 10000101_2$$

$$\text{fraction} = 01100001110000000000000_2$$

$$X = 01000010101100001110000000000000$$

$$7.3125_{10} = 7_{10} + 0.25_{10} + 0.0625_{10}$$

$$= 00000111_2 + 0.0101_2 = 00000111.0101_2$$

$$= 1.110101_2 * 2_{10}^2$$

Y is negative \Rightarrow sign bit = 1

$$\text{exponent} = 2_{10} + 127_{10} = 10000001_2$$

$$\text{fraction} = 11010100000000000000000_2$$

$$Y = 11000000111010100000000000000000$$

- (b) (4 points) Assuming X and Y are given in the IEEE 754 single precision format. Show all the steps to perform $X \times Y$ and write the solution in the IEEE 754 single precision format.

Add exponent: $2 + 6 = 8$, 加上 bias 為 135

$$1.01100001110000000000000_2 * 1.11010100000000000000000_2$$

$$= 10.1000011010110011000000000000000000000000000_2$$

$$\Rightarrow 10.10000110101100110000000_2 * 2^8 = 1.01000011010110011000000_2 * 2^9$$

$$\text{sign bit: } 1, \text{ exponent: } 9 + 127 = 136_{10} = 10001000_2,$$

$$\text{fraction: } 01000011010110011000000$$

$$X * Y = 11000100001000011010110011000000$$

6. (20 points) Consider a new floating-point number representation that is only 16 bits wide. The leftmost bit is still the sign bit, the exponent is 9 bits wide and has a bias of 255, and the fraction is 6 bits long. A hidden 1 to the left of the binary point is assumed. In this representation, any 16-bit binary pattern having 000000000 in the exponent field and a non-zero fraction indicates a denormalized number: $(-1)^s \times (0 + \text{Fraction}) \times 2^{-254}$. Write the answers of (a), (b) and (c) in scientific notation, e.g., 1.0101×2^2 .

- (a) (3 points) What is the smallest positive “normalized” number, denoted as a0?

sign bit: 0, exponent: 000000001, fraction: 000000
 $\text{exponent} - \text{bias} = 1 - 255 = -254 \Rightarrow a0 = 1.000000_2 \times 2^{-254}$.

- (b) (6 points) What is the largest positive “denormalized” number, denoted as a1? What is the second largest positive “denormalized” number, denoted as a2?

sign bit: 0, exponent: 000000000, fraction: 111111

$$0.111111_2 \times 2^6 = 111111_2 = 63_{10}$$

$$a1 = 63 \times 2^{-260} = 111111_2 \times 2^{-260} = 1.111110_2 \times 2^{-255}$$

sign bit: 0, exponent: 000000000, fraction: 111110

$$0.111110_2 \times 2^6 = 111110_2 = 62_{10}$$

$$a2 = 62 \times 2^{-260} = 111110_2 \times 2^{-260} = 1.111100_2 \times 2^{-255}$$

- (c) (4 points) Find the differences between a0 and a1, and between a1 and a2.

$$a0 - a1 = 1.000000_2 \times 2^{-254} - 1.111110_2 \times 2^{-255} = 0.000001_2 \times 2^{-254} = 1.000000_2 \times 2^{-260}$$

$$a1 - a2 = 1.111110_2 \times 2^{-255} - 1.111100_2 \times 2^{-255} = 0.000010_2 \times 2^{-255} = 1.000000_2 \times 2^{-260}$$

- (d) (3 points) What binary number does the binary pattern 1011110110100111 represent?

sign bit: 1, exponent: 011110110, fraction: 100111

$$\text{exponent} - \text{bias} = 011110110_2 - 255_{10} = 246_{10} - 255_{10} = -9_{10}$$

$$1.100111_2 = 1100111_2 \times 2_{10}^{-6} = 103_{10} \times 2_{10}^{-6}$$

This binary number is $-1100111_2 \times 2_{10}^{-15}$.

- (e) (4 points) Let U be the nearest representation of the decimal number 1.31; that is, U has the smallest approximation error. What is U? What is the actual decimal number represented by U?

$$0.31 \times 2 \Rightarrow (0).62 \times 2 \Rightarrow (1).24 \times 2 \Rightarrow (0).48 \times 2 \Rightarrow (0).96 \times 2 \Rightarrow (1).92 \times 2 \Rightarrow (1).84 \times 2 \Rightarrow (1).68 \text{ (進位)} \circ \Rightarrow 1.31_{10} = 1.010100_2 \circ 255 + 0 = 255_{10}$$

\Rightarrow sign bit: 0, exponent: 011111111 (255_{10}), fraction: 010100

U is 0011111111010100. Its actual decimal number is 1.3125

7. (12 points) X is a 32-bit signed integer variable, $\&$ is the bitwise-AND operator, and \gg is the sra (shift right arithmetic) operator. For the following options, determine whether they provide the correct result for $(X / 4)$ and explain the reasons.

- (a) $(X + 3) \gg 2$
- (b) $((X \geq 0) ? X \gg 2 : (X + 3) \gg 2)$
- (c) $X \gg 2$
- (d) $(X + ((X \gg 31) \& 3)) \gg 2$

Let A be a nonnegative 32-bit signed integer. Obviously, $A \gg 2 = A / 4$.

$$\begin{aligned} (-A) / 4 &= -(A / 4) = -(A \gg 2) = \sim(A \gg 2) + 1 = ((\sim A) \gg 2) + 1 = (\sim A + 4) \gg 2 \\ &= (\sim A + 1 + 3) \gg 2 = (-A + 3) \gg 2 \end{aligned}$$

Therefore, $X / 4 = X \gg 2$ if $X \geq 0$. Otherwise, $X / 4 = (X + 3) \gg 2$.

Counter case:

- \Rightarrow Use $X / 4 = (X + 3) \gg 2$: For $X = 1 \geq 0$, $(X + 3) \gg 2 = 1 \neq X / 4 = 0$
- \Rightarrow Use $X / 4 = X \gg 2$: For $X = -1 < 0$, $X \gg 2 = -1 \neq X / 4 = 0$

(a) 由上述可知，若 $X \geq 0$ 則 $(X + 3) \gg 2$ 不一定等於 $X / 4$

(b) 由上述可知， $((X \geq 0) ? X \gg 2 : (X + 3) \gg 2) = X / 4$

(c) 由上述可知，若 $X < 0$ 則 $X \gg 2$ 不一定等於 $X / 4$

(d) 若 $X < 0$ 則 $(X + ((X \gg 31) \& 3)) \gg 2 = (X + 3) \gg 2 = X / 4$

若 $X \geq 0$ 則 $(X + ((X \gg 31) \& 3)) \gg 2 = (X + 0) \gg 2 = X / 4$

$(X + ((X \gg 31) \& 3)) \gg 2 = X / 4$