Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University

office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

EE214000 Electromagnetics, Fall 2020

Your name: ________________ ID: ________________ Nov. 2nd, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #8-2, Open books, notes (10 points), due in class, Monday, Nov. 2nd, 2020

solution 2. in PPT

1. Complete the calculation $W_e = \frac{1}{2}\int_{V'} \rho V dv = \frac{1}{2}\rho\int_0^b V(R')4\pi R'^2 dR'$ below and verify that the answer is the same as that derived from assembling the charges layer by layer. (5 points)

soluton 1.

Ans 在第2頁

2. Given a voltage V and a charge Q on a capacitor, what is the electrostatic energy stored in this capacitor? (2 points)

$W = \frac{1}{2}QV$ #

3. Use the constant-charge techniques to calculate the force between two parallel plates of a capacitor holding a charge of $\pm Q$. Assume the capacitor has an area of S, electrode separation of d, and permittivity of ε. (3 points)

$C = \frac{\epsilon S}{d}$

$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2 d}{\epsilon S}$

$\vec{F} = -\nabla W = -\frac{1}{2}\frac{Q^2}{\epsilon S}\hat{a}_d$ #

sol 2:

$$W_e = \frac{1}{2}\int_{v'} \rho V dv = \frac{\rho}{2}\int_0^b V(R')\, 4\pi (R')^2 dR'$$

$$V(R') = \frac{\frac{4}{3}\pi b^3 \rho}{4\pi\epsilon_0 b} - \int_b^{R'} \frac{\frac{4}{3}\pi R^3 \rho}{4\pi \epsilon_0 R^2} dR$$

$$= \frac{\rho b^2}{3\epsilon_0} - \int_b^{R'} \frac{\rho}{3\epsilon_0} R\, dR$$

$$= \frac{\rho b^2}{3\epsilon_0} - \frac{\rho}{6\epsilon_0}\left[(R')^2 - b^2\right]$$

$$= \frac{\rho b^2}{3\epsilon_0} - \frac{\rho}{6\epsilon_0}(R')^2 + \frac{\rho b^2}{6\epsilon_0}$$

$$= \frac{\rho b^2}{2\epsilon_0} - \frac{\rho}{6\epsilon_0}(R')^2$$

$$\therefore W_e = \frac{\rho}{2}\int_0^b \left[\frac{\rho b^2}{2\epsilon_0} - \frac{\rho}{6\epsilon_0}(R')^2\right] 4\pi (R')^2 dR'$$

$$= \frac{\rho}{2}\int_0^b \left[\frac{\rho b^2}{\cancel{2}\epsilon_0} \overset{2\pi}{\cancel{4\pi}}(R')^2 - \frac{\rho}{\underset{3}{\cancel{6}}\epsilon_0} \overset{2\pi}{\cancel{4\pi}}(R')^4\right] dR'$$

$$= \frac{\rho}{2}\left(\frac{2\pi\rho b^2}{3\epsilon_0}(R')^3 - \frac{2\pi\rho}{15\epsilon_0}(R')^5\right)\Bigg|_0^b$$

$$= \frac{\rho}{2}\left(\frac{2\pi\rho b^5}{3\epsilon_0} - \frac{2\pi\rho b^5}{15\epsilon_0}\right)$$

$$= \frac{4\pi\rho^2 b^5}{15\epsilon_0} \quad \# \quad \text{—①}$$

sol 1:

$$dW_e = V dq$$

$$dq = \rho dv = \rho\, 4\pi (R')^2 dR'$$

$$V(R') = \frac{\frac{4}{3}\pi (R')^3 \rho}{4\pi\epsilon_0 R'}$$

$$\therefore W_e = \int_0^b V dq = \int_0^b \frac{\frac{4}{3}\pi (R')^3 \rho}{\cancel{4\pi}\epsilon_0 R'} \times \rho\, \cancel{4\pi}(R')^2 dR'$$

$$= \int_0^b \frac{4\pi\rho^2 (R')^4}{3\epsilon_0} dR'$$

$$= \frac{4\pi\rho^2 b^5}{15\epsilon_0} \quad \text{—②}$$

由①②

⇒解相同 #