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EE214000 Electromagnetics, Fall, 2020
Quiz #1-1, Open books, notes (21 points), due 11 pm, Wednesday, Oct. 21st, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. When you design a capacitor, what are the key parameters to increase its capacitance? (6 points)

From the formula of a parallel-plate capacitor: $C = \frac{\varepsilon S}{d}$
one can in general increase the capacitance of a capacitor by increasing the electrode areas and the permittivity of the dielectric between the electrodes, and decreasing the seperation of the electrodes.

2. In Sec. 7.3, for a cylindrical capacitor of length L, we derive a capacitance given by

$$C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln(b/a)}.$$

Show that, when the radii a and b are big, the expression converges to $C = \frac{\varepsilon S}{d}$ with $d = b - a$. (5 points)

We know $\ln(\frac{b}{a})$ can be expanded by Taylor series.

$\Rightarrow \ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

$\Rightarrow \ln(\frac{b}{a}) = \ln(1+\frac{b-a}{a}) = (\frac{b-a}{a}) - \frac{1}{2}(\frac{b-a}{a})^2 + \frac{1}{3}(\frac{b-a}{a})^3 - \cdots$

By ignoring high order terms in Taylor series, we can get

$\ln(\frac{b}{a}) = \ln(1+\frac{b-a}{a}) \doteqdot \frac{b-a}{a}$, since a and b are extremely big.

$\Rightarrow C = \frac{2\pi\varepsilon L}{\ln(\frac{b}{a})} = \frac{\varepsilon \cdot 2\pi L}{(\frac{b-a}{a})} = \frac{\varepsilon \cdot 2\pi(a)L}{b-a}$, for $2\pi aL$ is the surface area of the side of the cylinder. Since both a and b are big, the cylinder can be seen as a parallel-plate capacitor with $a \doteqdot b$, which means $2\pi aL \doteqdot 2\pi bL \doteqdot S$ is the area of the side surface of the parallel-plate capacitor. In addition, we know $d = b-a$.

$\Rightarrow C = \frac{\varepsilon \cdot 2\pi aL}{b-a} = \frac{\varepsilon S}{d}$.

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3. In Sec. 7.4, we derive the expression $C \equiv \frac{Q}{V} = \frac{4\pi\varepsilon}{(1/a - 1/b)}$ for the capacitance of a spherical capacitor. Argue that when radii *a* and *b* are big, the expression also converge to $C = \frac{\varepsilon S}{d}$ (4 points)

$C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}} = \frac{\varepsilon \cdot 4\pi}{\frac{b-a}{ab}} = \frac{\varepsilon \cdot 4\pi(ab)}{b-a}$, for $4\pi ab$ is the surface area of the sphere. Since both a and b are extremely big, the sphere can be seen as parallel-plate capacitor with $a \doteq b$, which means $4\pi ab \doteq 4\pi a^2 \doteq 4\pi b^2 \approx S$ is the area of the surface area of the parellel-plate capacitor. Besides, we know $d = b - a$.

$\Rightarrow C = \frac{\varepsilon \cdot 4\pi ab}{b-a} = \frac{\varepsilon S}{d}$

4. If you have a few capacitors in your hands and you want to connect them together to have a high capacitance for your circuit, would you choose serial or parallel connections for your capacitors? (3 points)

To solve this problem, one could prove from the following two formulas for serial and parallel capacitors:

$$\left.\begin{aligned} \frac{1}{C_{sr}} &= \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \\ C_p &= C_1 + C_2 + C_3 + \cdots + C_n \end{aligned}\right\} \quad C_p > C_{sr}$$

We can see that the parallel capacitors store more charges from a increased area. Therefore, to increase the capacitance, parallel connection is the choice.

5. For a high-speed circuit containing *R* and *C*, if you would like to have a signal bandwidth > 1 GHz, what is the requirement on the RC time constant of the circuit? (3 points)

Consider a sinusoidal signal of 1 GHz in the circuit. Since the charging and discharging time in the circuit has to be less than $\frac{1}{1\,\text{GHz}} = 1\,\text{ns}$ to support the 1 GHz signal, the RC constant of the circuit has to be less than 1 ns.