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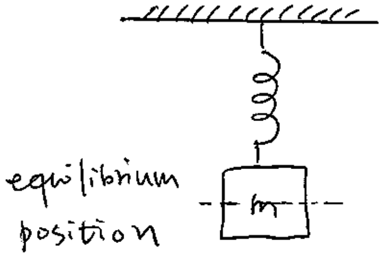
Introduction

Q1:

Example: A spring-mass system (Ch 5.1.1)



A mass m is attached to a spring with a spring constant K.

The motion of the mass follows Newton's 2nd law:

$$\sum F = ma = m\frac{d^2y}{dt^2}$$

What are the forces that the mass experiences?

— "external force": F_{ext}

— "restoring force" of the spring (Hooke's law)

— "damping force" due to the resistance from air

$$\sum F = F_{ext} + F_{restoring} + F_{damping} = m\frac{d^2y}{dt^2}$$

$$\Rightarrow F_{ext} - Ky - b\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

$\Rightarrow$

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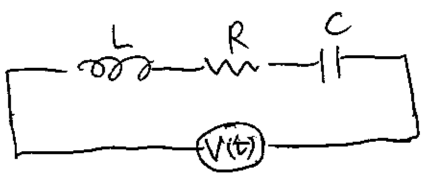
(Note: In this system, a driving force F_{ext} is applied, causing the changes of the displacement y. So we call

F_{ext} is the of the system.

and y is the of the system.

Now, consider a totally different system. In a circuit class, you will be dealing with circuit problems with R, L, C components.

For a LRC-series circuit driven by a voltage source $V(t)$.



By Kirchhoff's 2nd law, the charges of the circuit can be related to the driving voltage by

$\Rightarrow$

This is the circuit analogue of the spring-mass system. In the two completely different systems, not only the

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resulting DEs have the same form, but each term has similar physical interpretation.

ex: mass-spring $\longleftrightarrow$ LRC ckt

From the above discussion, we then know

$\Rightarrow$ Many different engineering/physics systems involve changes of quantities under some operation conditions, that can be described by DEs.

$\Rightarrow$ Many systems yield the same form of DE with similar physical interpretations.

In this class, we will learn how to solve some of the most commonly used DEs in engineering/physics.

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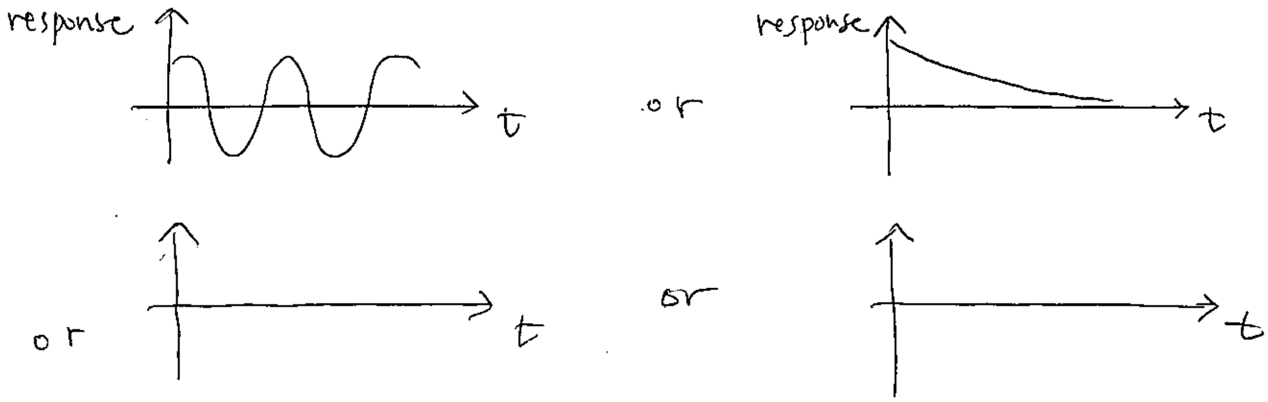
Q2: What are we interested in these systems?

→ How do the responses of the systems change (with time).

ex:



(Note: Because y or q is a function of time (depend on time), we call "t" is the independent variable.
"y" "q" is the dependent variable.)

→ Will this system be stable?
(What is the "long-term behavior" of the system?)

→ How do the responses change under
- different driving forces? (inputs)
- different values of parameters?

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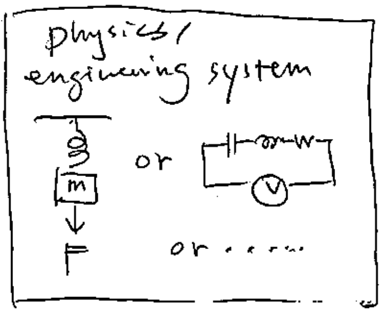
Q3 : What do we need to learn in this class ?

→ learn how to "model" a system



1) Identity the quantities

- indep variables
- dep variables
- input (driving force)
- output (response)
- parameters

→

DE relating the quantities

$$a\frac{d^2y}{dt^2}+b\frac{dy}{dt}+cy=V$$

2) Based on the system descriptions, create the DE.

→ learn how to solve DEs

Three types of approaches to solve DEs