

HW4-2 solution

1. Design a combinational circuit that compares two 4-bit unsigned numbers A and B to see whether A is greater than B. The circuit has one output X, so that $X = 0$ if $A \leq B$ and $X = 1$ if $A > B$.

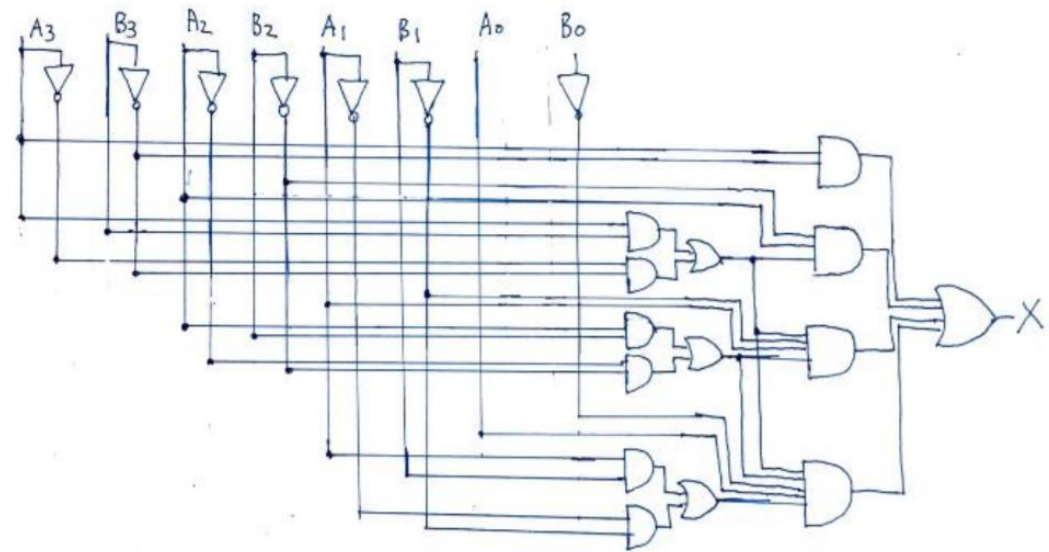
$A > B$: ① $A_3=1$ and $B_3=0$

② $A_3=B_3$ and $A_2=1, B_2=0$

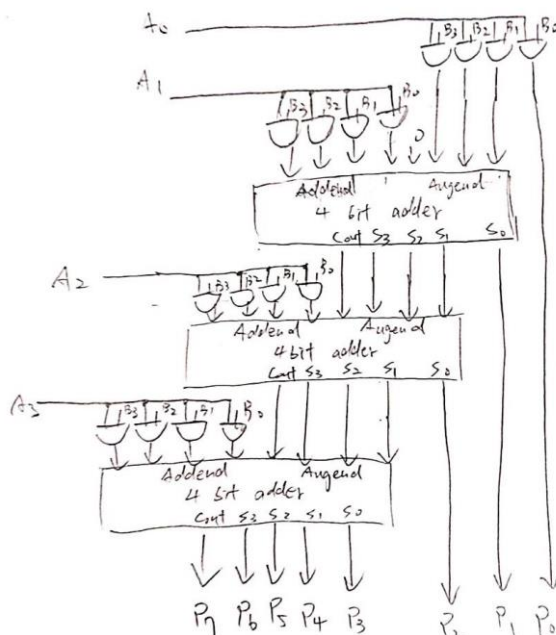
③ $A_3=B_3, A_2=B_2, A_1=1, B_1=0$

④ $A_3=B_3, A_2=B_2, A_1=B_1, A_0=1, B_0=0$

$$X = A_3\bar{B}_3 + (A_3B_3 + \bar{A}_3\bar{B}_3)A_2\bar{B}_2 + (A_3B_3 + \bar{A}_3\bar{B}_3)(A_2B_2 + \bar{A}_2\bar{B}_2)A_1\bar{B}_1 + (A_3B_3 + \bar{A}_3\bar{B}_3)(A_2B_2 + \bar{A}_2\bar{B}_2)(A_1B_1 + \bar{A}_1\bar{B}_1)A_0\bar{B}_0$$

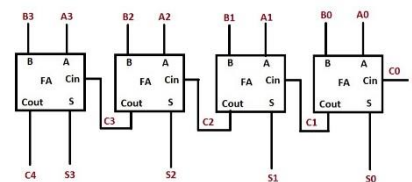


2. Design a 4x4 multiplier using four-bit adders (Ripple-Carry adders) and other logic gates.



$$A_3A_2A_1A_0 \times B_3B_2B_1B_0 = P_7P_6P_5P_4P_3P_2P_1P_0$$

4 bits adder block



3. Design a three-way magnitude comparator that outputs true if its three inputs are in strict order: $a < b < c$. a , b , and c are all three-bit unsigned numbers.

$$a < b < c \Rightarrow (a < b) \text{ AND } (b < c)$$

$$\begin{aligned} a < b &: a_2' b_2 + (a_2 b_2 + a_2' b_2') a_1' b_1 + (a_2 b_2 + a_2' b_2') (a_1 b_1 + a_1' b_1') a_0' b_0 \\ &= a_2' b_2 + \overline{a_2 \oplus b_2} \cdot a_1' b_1 + \overline{a_2 \oplus b_2} \cdot \overline{a_1 \oplus b_1} \cdot a_0' b_0 \quad (\text{參考講義 P.58}) \end{aligned}$$

$$\begin{aligned} b < c &: b_2' c_2 + (b_2 c_2 + b_2' c_2') b_1' c_1 + (b_2 c_2 + b_2' c_2') (b_1 c_1 + b_1' c_1') b_0' c_0 \\ &= b_2' c_2 + \overline{b_2 \oplus c_2} \cdot b_1' c_1 + \overline{b_2 \oplus c_2} \cdot \overline{b_1 \oplus c_1} \cdot b_0' c_0 \end{aligned}$$

$$\therefore \text{Output } F = (a_2' b_2 + \overline{a_2 \oplus b_2} \cdot a_1' b_1 + \overline{a_2 \oplus b_2} \cdot \overline{a_1 \oplus b_1} \cdot a_0' b_0) \cdot (b_2' c_2 + \overline{b_2 \oplus c_2} \cdot b_1' c_1 + \overline{b_2 \oplus c_2} \cdot \overline{b_1 \oplus c_1} \cdot b_0' c_0)$$

4. Design a 4→2 priority encoder with input $D[3:0]$ and output $A[1:0]$ where D_0 has the highest priority and D_3 has the lowest priority.

