Chapter 1 – Electric Circuit Variables

Exercises

Exercise 1.2-1 Find the charge that has entered an element by time t when $i = 8t^2 - 4t$ A, $t \geq 0$. Assume $q(t) = 0$ for $t < 0$.

Answer: $q(t) = \frac{8}{3}t^3 - 2t^2 \quad \text{C}$

Solution:

$$i(t) = 8t^2 - 4t \text{ A}$$

$$q(t) = \int_0^t i\,d\tau + q(0) = \int_0^t (8\tau^2 - 4\tau)\,d\tau + 0 = \frac{8}{3}\tau^3 - 2\tau^2\Big|_0^t = \underline{\frac{8}{3}t^3 - 2t^2 \quad \text{C}}$$

Exercise 1.2-2 The total charge that has entered a circuit element is $q(t) = 4 \sin 3t$ C when $t \geq 0$ and $q(t) = 0$ when $t < 0$. Determine the current in this circuit element for $t > 0$.

Answer: $i(t) = \frac{d}{dt} 4\sin 3t = 12\cos 3t \quad \text{A}$

Solution:

$$i(t) = \frac{dq}{dt} = \frac{d}{dt} 4\sin 3t = 12\cos 3t \quad \text{A}$$

Exercise 1.3-1 Which of the three currents, $i_1 = 45\ \mu$A, $i_2 = 0.03$ mA, and $i_3 = 25 \times 10^{-4}$ A, is largest?

Answer: i_3 is largest.

Solution:

$$i_1 = 45\ \mu\text{A} = 45 \times 10^{-6}\text{ A} < i_2 = 0.03\text{ mA} = .03 \times 10^{-3}\text{ A} = 3 \times 10^{-5}\text{ A} < i_3 = 25 \times 10^{-4}\text{ A}$$

Exercise 1.5-1 Figure E 1.5-1 shows four circuit elements identified by the letters *A*, *B*, *C*, and *D*.

(a) Which of the devices supply 12 W?
(b) Which of the devices absorb 12 W?
(c) What is the value of the power received by device *B*?
(d) What is the value of the power delivered by device *B*?
(e) What is the value of the power delivered by device *D*?

Answers: (a) *B* and *C*, (b) *A* and *D*, (c)–12 W, (c) 12 W, (e)–12 W

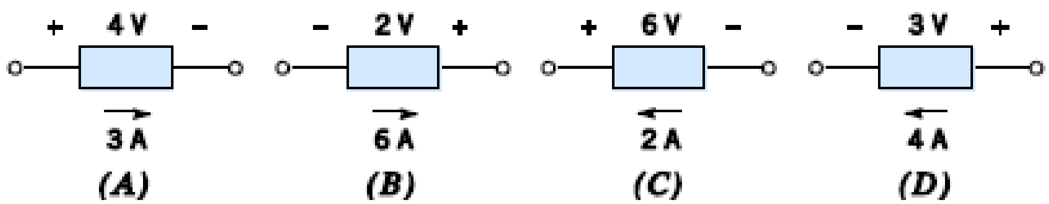


Figure E 1.5-1

Solution:

(a) *B* and *C*. The element voltage and current do not adhere to the passive convention in *B* and *C* so the product of the element voltage and current is the power supplied by these elements.

(b) *A* and *D*. The element voltage and current adhere to the passive convention in *A* and *D* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) −12 W. The element voltage and current do not adhere to the passive convention in *B*, so the product of the element voltage and current is the power received by this element: (2 V)(6 A) = −12 W. The power supplied by the element is the negative of the power delivered to the element, 12 W.

(d) 12 W

(e) −12 W. The element voltage and current adhere to the passive convention in *D*, so the product of the element voltage and current is the power received by this element: (3 V)(4 A) = 12 W. The power supplied by the element is the negative of the power received to the element, −12 W.

Problems

Section 1-2 Electric Circuits and Current Flow

P1.2.1

Solution: $$i(t) = \frac{d}{dt}0.30\left(1-e^{-5t}\right) = 1.5e^{-5t} \text{ A}$$

P 1.2-2

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 6\left(1-e^{-5\tau}\right)d\tau + 0 = \int_0^t 6\,d\tau - \int_0^t 6e^{-5\tau}\,d\tau = 6t + 1.2e^{-5t} - 1.2 \text{ C}$$

P 1.2-3

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau + q(0) = \int_0^t 5\sin 6\tau\,d\tau + 0 = -\frac{5}{6}\cos 3\tau\Big|_0^t = -\frac{5}{6}\cos 6t + \frac{5}{6} \text{ C}$$

P 1.2-4

Solution:

$q(t) = \int_{-\infty}^t i(\tau)\,d\tau = \int_{-\infty}^t 0\,d\tau = \underline{0 \text{ C for } t \le 2}$ so $q(2) = 0$.

$q(t) = \int_2^t i(\tau)\,d\tau + q(2) = \int_2^t 2\,d\tau = 2\tau\Big|_2^t = \underline{2t-4 \text{ C for } 2 \le t \le 4}$. In particular, $q(4) = 4$ C.

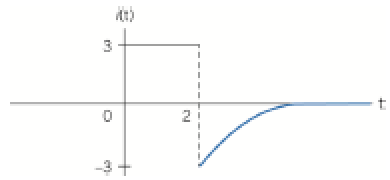
$q(t) = \int_4^t i(\tau)\,d\tau + q(4) = \int_4^t -1\,d\tau + 4 = -\tau\Big|_4^t + 4 = \underline{8-t \text{ C for } 4 \le t \le 8}$. In particular, $q(8) = 0$ C.

$q(t) = \int_8^t i(\tau)\,d\tau + q(8) = \int_8^t 0\,d\tau + 0 = \underline{0 \text{ C for } 8 \le t}$.

P 1.2-5

Solution:

$$i(t) = \frac{dq(t)}{dt} \qquad i(t) = \begin{cases} 0 & t<0 \\ 3 & 0<t<2 \\ \underline{-2e^{-2(t-2)}} & t>2 \end{cases}$$



P 1.2-6

Solution:

$$i = 460A = 460\ \frac{C}{s}$$

$$\text{Silver deposited} = 460\frac{C}{s}\times 30\,\text{min}\times 60\,\frac{s}{\text{min}}\times 1.120\,\frac{\text{mg}}{C} = 9.2736\times 10^5\ \text{mg} = \underline{927.36\ g}$$

P1.2-7

Solution:

$$i(t) = \begin{cases} 1 \quad \text{when} \quad 0 < t \le 4 \\ \dfrac{t}{2} - 1 \quad \text{when} \quad 4 \le t \end{cases}$$

and $$q(t) = \int_0^t i(t)dt + q(0) = \int_0^t i(t)dt$$

since $q(0) = 0$.

When $0 < t \le 4$, we have

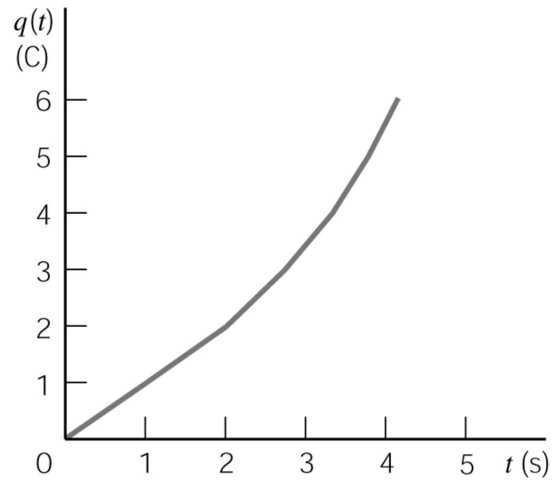
$$q = \int_0^t 1\,dt = t\,\text{C}$$

When $t \ge 2$, we have

$$q = \int_0^t i\,(t)dt = \int_0^4 1\,dt + \int_4^t (\frac{t}{2} - 1)\,dt$$

$$= t\Big|_0^4 + \tfrac{t^2}{4}\Big|_4^t - t\Big|_4^t = \frac{t^2}{4} - t + 4\,\text{C}$$

The sketch of $q(t)$ is shown to the right:.



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Section 1-3 Systems of Units

P 1.3-1

Solution:

$$\Delta q = i\,\Delta t = (3.5\times10^{-6}\ \text{A})(1\times10^{-3}\ s) = 3.5\times10^{-9}\,\text{C} = \underline{3.5\,\text{nC}}$$

P 1.3-2

Solution:

$$i = \frac{\Delta q}{\Delta t} = \frac{50 \times 10^{-9}}{8 \times 10^{-3}} = 6.25 \times 10^{-6} = 6.25\ \mu\text{A}$$

P 1.3-3

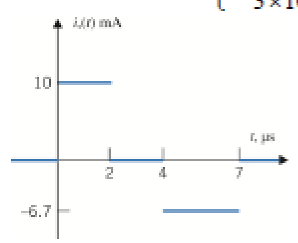
Solution

$$\begin{aligned} i &= \left[20\,\text{billion}\ \frac{\text{electron}}{s}\right]\left[1.602\times10^{-19}\ \frac{C}{\text{electron}}\right] = \left[20\times10^{9}\,\frac{\text{electron}}{\text{s}}\right]\left[1.602\times10^{-19}\,\frac{\text{C}}{\text{electron}}\right] \\ &= 2\times10^{10}\times1.602\times10^{-19}\,\frac{\text{electron}}{\text{s}}\,\frac{\text{C}}{\text{electron}} \\ &= 3.204\times10^{-9}\,\frac{\text{C}}{\text{s}} = \underline{3.204\,\text{nA}} \end{aligned}$$

P1.3-4

Solution

$$i(t) = \frac{d}{dt}q(t) = \text{the slope of the } q \text{ versus } t \text{ plot} = \begin{cases} \dfrac{20\times10^{-9}}{2\times10^{-6}} = 10\times10^{-3} = 10\,\text{mA} \\ -\dfrac{20\times10^{-9}}{3\times10^{-6}} = -6.7\times10^{-3} = -6.7\ \text{mA} \end{cases}$$



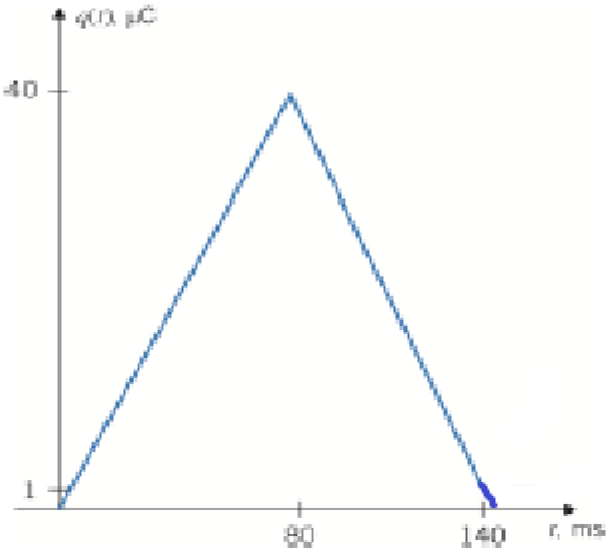
P1.3-5

Solution:

$$q(t) = \int_0^t i(\tau)\,d\tau = \begin{cases} \int_0^t 500\,\mu\text{A}\,d\tau \quad \text{when } 0 < t < 80\ \text{ms} \\ \left(500\times10^{-6}\right)\left(80\times10^{-3}\right) + \int_{80\ ms}^t (-650\,\mu\text{A})\,d\tau \quad \text{when } 80\ \text{ms} < t < 140\ \text{ms} \\ \left(500\times10^{-6}\right)\left(80\times10^{-3}\right) + \left(-650\times10^{-6}\right)\left(60\times10^{-3}\right) + \int_{140\ \text{ms}}^t 0\,d\tau \end{cases}$$

$$= \begin{cases} \left(500\times10^{-6}\right)t \quad \text{when } 0 < t < 80\ \text{ms} \\ \left(40\times10^{-6}\right) + \left(-650\times10^{-6}\right)t \quad \text{when } 80\ \text{ms} < t < 140\ \text{ms} \\ 1\ C \quad \text{when } 140\ \text{ms} < t \end{cases}$$

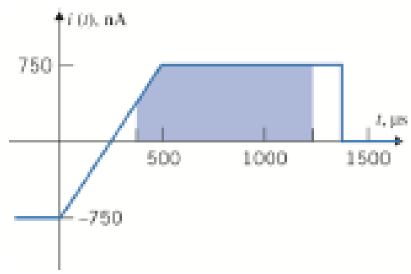
While $0 < t < 80$ ms $q(t)$ increases linearly from 0 to 40 μC and while $80 < t < 140$ ms $q(t)$ decreases linearly from 40 to 0 μC. Here's the sketch:



P1.3-6

Solution:

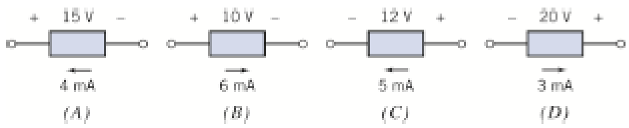
$$q(t)=\int_{300\ \mu s}^{1250\ \mu s} i(\tau)\,d\tau = \text{"area under the curve between 300 } \mu\text{s and 1250 } \mu\text{s"}$$



$$q(t)=\left(\frac{375+750}{2}\times10^{-9}\right)\left(100\times10^{-6}\right)+\left(750\times10^{-9}\right)\left(750\times10^{-6}\right)=(112.5+562.5)\times10^{-12}=675\ \text{pC}$$

Section 1-5 Power and Energy

P1.5-1



Solution:

(a) *A* and *D*. The element voltage and current do not adhere to the passive convention in Figures P1.5- *A* and *D* so the product of the element voltage and current is the power supplied by these elements.

(b) *B* and *C*. The element voltage and current adhere to the passive convention in Figures P1.5-1 *B* and *C* so the product of the element voltage and current is the power delivered to, or absorbed by these elements.

(c) 60 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*B*, so the product of the element voltage and current is the power received by this element: (10**V**)(6 mA) = 60 mW. The power supplied by the element is the negative of the power received to the element, −60 mW.

(d) −60 mW

(e) −60 mW. The element voltage and current adhere to the passive convention in Figure P1.5-1*C*, so the product of the element voltage and current is the power received by this element: (12 V)(5 mA) = 60 mW. The power supplied by the element is the negative of the power received to the element, −60 mW.

P 1.5-2

Solution:

a.) $q = \int i\,dt = i\Delta t = (10\,\text{A})(2\,\text{hrs})(3600\,\text{s/hr}) = \underline{7.2\times 10^4\,\text{C}}$

b.) $P = v\,i = (110\,\text{V})(10\,\text{A}) = \underline{1100\ \text{W}}$

c.) $\text{Cost} = \dfrac{0.12\,\$}{\text{kWh}}\times 1.1\,\text{kW}\times 2\,\text{hr} = \underline{0.264\,\$}$

P 1.5-3

Solution:

$$P = (8\text{V})(10\text{ mA}) = 0.08\ \text{W}$$

$$\Delta t = \frac{\Delta w}{P} = \frac{220\ \text{W}\cdot\text{s}}{0.08\ \text{W}} = \underline{2.75\times10^3\ \text{s}}$$

P 1.5-4

Solution:

for $0 \le t \le 10$ s: $\quad v = 30\text{ V}$ and $i = \frac{30}{15}t = 2t\text{ A}$ $\quad\therefore\ \underline{P = 30(2t) = 60t\text{ W}}$

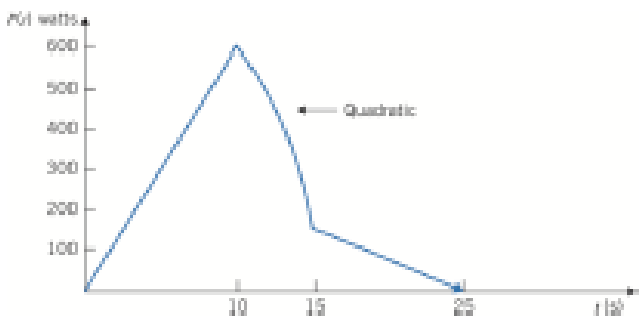
for $10 \le t \le 15$ s: $v(t) = -\frac{25}{5}t + b \Rightarrow v(10) = 30\text{ V} \Rightarrow b = 80\text{ V}$

$v(t) = -5t + 80$ and $i(t) = 2t$ A $\Rightarrow$ $P = (2t)(-5t+80) = \underline{-10t^2 + 160t\text{ W}}$

for $15 \le t \le 25$ s: $v = 5$ V and $i(t) = -\frac{30}{10}t + b$ A

$i(25) = 0 \Rightarrow b = 75 \Rightarrow i(t) = -3t + 75$ A

$\therefore\ \underline{P = (5)(-3t+75) = -15t + 375\text{ W}}$



$$\text{Energy} = \int P\,dt = \int_0^{10} 60t\,dt + \int_{10}^{15}\left(160t - 10t^2\right)dt + \int_{15}^{25}(375 - 15t)\,dt$$

$$= 30t^2\Big|_0^{10} + 80t^2 - \frac{10}{3}t^3\Big|_{10}^{15} + 375t - \frac{15}{2}t^2\Big|_{15}^{25} = \underline{5833.\bar{3}\ \text{J}}$$

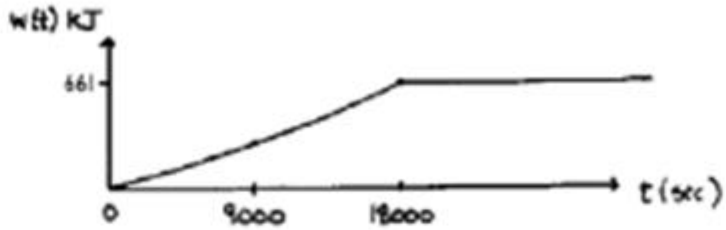
P 1.5-5

Solution:

a.) Assuming no more energy is delivered to the battery after 5 hours (battery is fully charged).

$$w = \int P\,dt = \int_0^t vi\,d\tau = \int_0^{5(3600)} 2\left(11+\frac{0.5\,\tau}{3600}\right)d\tau = 22t + \frac{0.5}{3600}t^2\Big|_0^{5(3600)}$$

$$= 661\times10^3 J = \underline{661\text{ kJ}}$$



b.) $\text{Cost} = 661\text{kJ}\times\frac{1\text{ hr}}{3600\text{s}}\times\frac{15¢}{\text{kWhr}} = \underline{2.76¢}$

P 1.5-6

Solution:

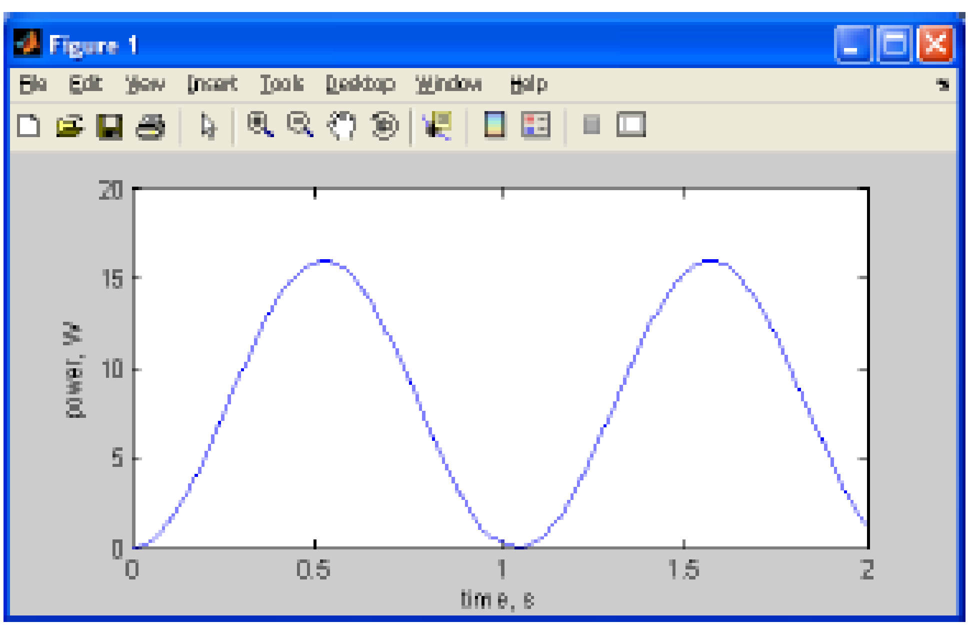
$$p(t)=v(t)i(t)=(4\cos 3t)\left(\frac{1}{12}\sin 3t\right)=\frac{1}{6}(\sin 0+\sin 6t)=\underline{\frac{1}{6}\sin 6t \quad \text{W}}$$

$$p(0.5)=\frac{1}{6}\sin 3=\underline{0.0235 \quad \text{W}}$$

$$p(1)=\frac{1}{6}\sin 6=\underline{-0.0466 \quad \text{W}}$$

Here is a MATLAB program to plot *p*(*t*):

```
clear
t0=0;                    % initial time
tf=2;                    % final time
dt=0.02;                 % time increment
t=t0:dt:tf;              % time

v=4*cos(3*t);            % device voltage
i=(1/12)*sin(3*t);       % device current

for k=1:length(t)
   p(k)=v(k)*i(k);       % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```



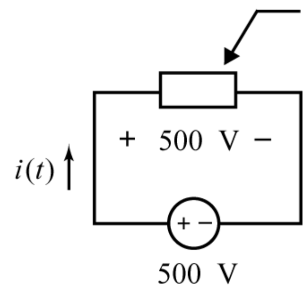
P 1.5-7

Solution:
The power is $P = V\,I = 5 \times 0.021 = 0.105$ W Next, the energy is $w = P\Delta t = 0.105 \times 5 \times 60 = 31.5$J.

P1.5-8

Solution:

$$
\begin{aligned}
energy = w(t) = \int_0^T p(t)\,dt &= \int_0^T v(t)\,i(t)\,dt \\
&= \frac{500}{1000}\int_0^3 \left(2 + 30\,e^{-0.85t}\right) dt \\
&= \int_0^3 dt + 15\int_0^3 \left(e^{-0.85t}\right) dt \\
&= (3-0) + \frac{15}{-0.85}\left(e^{-2.55} - 1\right) \\
&= 3 + 16.3 = 19.3 \text{ J}
\end{aligned}
$$



Section 1.7 How Can We Check…?

P 1.7-1

Solution:
Notice that the element voltage and current of each branch adhere to the p
sum of the powers absorbed by each branch is:

$(-2V)(5A)+(6V)(2A)+(3V)(4A)+(4V)(-5A)+(1V)(6A) = -10W + 12W + 1$
$= 0 W$

The element voltages and currents satisfy conservation of energy and may

P 1.7-2

Solution:
Notice that the element voltage and current of some branches do not adhe
convention. The sum of the powers absorbed by each branch is:

$-(3V)(6A)+(3V)(2A)+ (5V)(2A)+(4V)(6A)+(-3V)(-3A)+(4V)(-3A)$
$= -18W + 6W + 10W$
$\neq 0$

The element voltages and currents do not satisfy conservation of energy a

Design Problems

DP 1-1

Solution:
The voltage may be as large as 20(1.25) = 25 V and the current may be as large as (0.008)(1.25
= 0.01 A. The element needs to be able to absorb (25 V)(0.01 A) = 0.25 W continuously. A Grade B element is adequate, but without margin for error. Specify a Grade B device if you trus
the estimates of the maximum voltage and current and a Grade A device otherwise.

DP 1-2

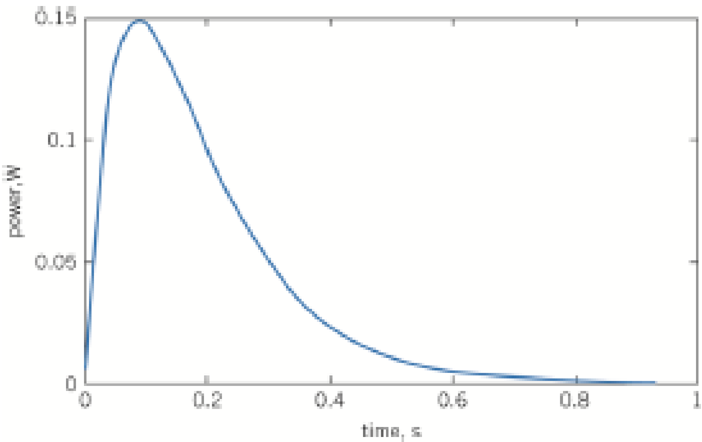
Solution:

$$p(t)=20\left(1-e^{-8t}\right)\times 0.03\,e^{-8t}=\underline{0.6\left(1-e^{-8t}\right)e^{-8t}\quad\text{W}}$$

Here is a MATLAB program to plot $p(t)$:

```
clear

t0=0;                  % initial time
tf=1;                  % final time
dt=0.02;               % time increment
t=t0:dt:tf;            % time

v=20*(1-exp(-8*t));    % device voltage
i=.030*exp(-8*t);      % device current

for k=1:length(t)
   p(k)=v(k)*i(k);     % power
end

plot(t,p)
xlabel('time, s');
ylabel('power, W')
```

Here is the plot:



The circuit element must be able to absorb 0.15 W.

DP 1-3

Solution:

(a) $v_{\mathrm{m}}=\dfrac{\theta}{60} \;\Rightarrow\; \theta=60\,v_{\mathrm{m}}$ then $v_{\mathrm{a}}=\dfrac{\theta+90°}{18}=\dfrac{60\,v_{\mathrm{m}}+90°}{18}=\dfrac{10}{3}v_{\mathrm{m}}+5$ where the units of where the units of both v_a and v_m are Volts. and the units of θ are degrees.

(b) $\theta=-8° \;\Rightarrow\; v_{\mathrm{a}}=\dfrac{\theta+90°}{18}=\dfrac{-8°+90°}{18}=4.556\text{ V}$ and

$\theta=8° \;\Rightarrow\; v_{\mathrm{a}}=\dfrac{\theta+90°}{18}=\dfrac{8°+90°}{18}=5.444\text{ V}$

DP1-4

Solution:

(a) v_m =   50(5) =   250 mV when x = 5 cm and v_m =   50(18) =   900 mV when x = 18 cm.
v_a = (5   5)/1.3 = 0 V when x = 5 cm and v_m = (18   5)/1.3 = 10 V when x = 18 cm.

(b) $v_a = m v_m + b$ where the slope is $m = \dfrac{10-0}{-0.9-(-0.25)} = -15.385$. When x = 5 cm, v_a= 0 and v_m = -0.25 V. Consequently $0 = -15.385(-0.25) + b \quad \Rightarrow \quad b = -3.846 \text{ V}$. We have $v_a = -15.385 v_m - 3.846$ where the units of both v_a and v_m are Volts.

DP1-5

Solution: $v_m = 8\varepsilon \quad \Rightarrow \quad \varepsilon = \dfrac{v_m}{8}$ so $v_a = 1000\,\varepsilon = 1000\dfrac{v_m}{8} = 125 v_m$ where the units of both v_a and v_m are Volts.

DP1-6.

Solution:

Step 1: v_m = 9(25) = 225 mV when T = 25 °C and v_m = 9(200) = 1800 mV when T = 200 °C .

v_a = (25   5)/17.5 = 0 V when T = 25 °C cm and v_a = (200   5)/17.5 = 10 V when T = 200 °C.

Step 2: $v_a = mv_m + b$ where the slope is $m = \frac{10-0}{1.8-0.225} = 6.35$. When T = 25 °C v_a= 0 and v_m = 1.8 V. Consequently, $0 = 6.35(0.225) + b \quad \Rightarrow \quad b = -1.429$ V. We have $v_a = 6.35 v_m - 1.429$ where the units of both v_a and v_m are Volts.