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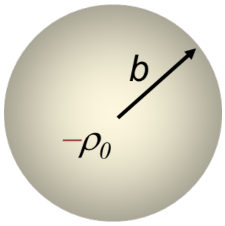
EE214000 Electromagnetics, Fall 2020

Your name: ________________ ID: __________________ Nov. 2nd, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #9-1, Open books, notes (27 + 3 free points), due 11 pm, Wednesday, Nov. 4th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. Continue the charge ball calculation in the lecture to determine the electric potential and electric field outsider the ball (R > b). Verify the results by using the Gauss-law calculations. (9 points)



Ans: Outside the ball, the charge is zero and $\frac{dV}{dR} = \frac{C_2}{R^2} = -E$. At R = b, the electric field (normal component) is continuous. Therefore $-\frac{C_2}{b^2} = -\frac{\rho_0}{3\varepsilon_0} b \Rightarrow C_2 = \frac{\rho_0}{3\varepsilon_0} b^3$

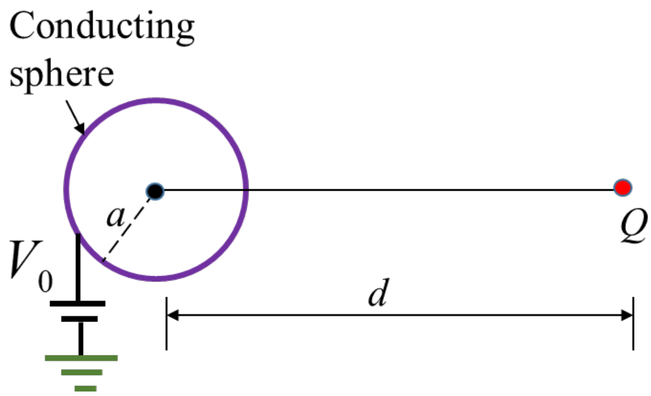
Substitute C_2 into $\frac{C_2}{R^2} = -E$ to obtain $\vec{E} = \frac{-\rho_0 b^3}{3\varepsilon_0 R^2} \hat{a}_R = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R$ with $Q = \frac{-4\pi b^3}{3} \rho_0$, as expected from the calculation of Gauss law. (5 points)

Continue the calculation for $V \to \frac{dV}{dR} = \frac{C_2}{R^2} \Rightarrow V = -\frac{C_2}{R} + C_3$. Use the boundary condition $V = 0$ when $R = \infty$ to obtain $C_3 = 0$. Therefore,

$V = -\frac{C_2}{R} = -\rho_0 \frac{b^3}{3\varepsilon_0 R} = \frac{Q}{4\pi\varepsilon_0 R}$, as expected from the calculation of Gauss law. (4 points)

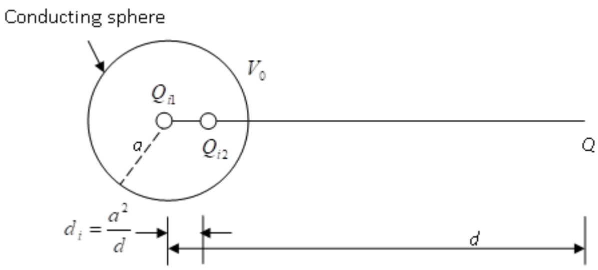
2. A conducting sphere of radius a is maintained at a constant voltage of V_0. A point charge of Q is placed at a distance d from the center of the conducting sphere, where d > a. Find out the locations and values of the image charges that can replace the spherical boundaries. (8 points) What is the total charge induced on the surface of the

sphere? (4 points)



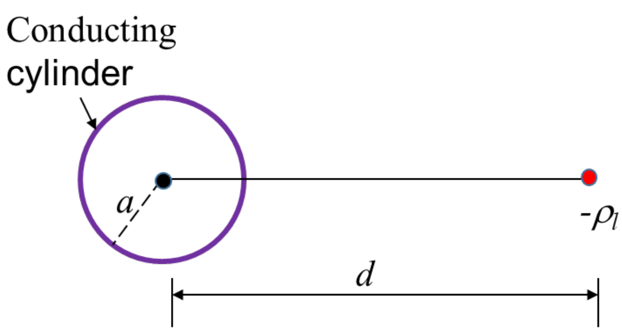
Ans: (a) One of the two image charges is located at the **center** of the conducting sphere (2 points) with an amount of charge equals to $Q_{i1} = 4\pi\varepsilon_0 a V_0$ (2 points) for maintaining a constant voltage of V_0 on the conducting surface.

(b) The other image charge is located at a distance $d_i = a^2/d$ (2 points) with an amount of charge equal to $Q_{i2} = -\frac{a}{d}Q$ (2 points)



According to the Gauss law, the total charge induced on the surface of the sphere must be the sum of the two image charges. (4 points)

3. An infinitely long conducting cylinder of radius of a is installed in parallel with an infinitely long wire with a line charge density of $-\rho_l$, as shown below. The separation of the two objects is d.



(a) What is the electric potential on the conduction cylinder? (3 points)
(b) What is the capacitance per unit length of this signal line? (free credit: 3 points)
(c) What is the total surface charge per unit length along the longitudinal direction induced on the conducting cylinder? (3 points)

Ans: The purpose of this problem is just to ask you to go through the content of the lecture. (a) Follow the same derivation in the lecture, the electric potential on the conducting cylinder is $V_M = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{a}{d}$

(b) $V_M = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{a}{d}$

(b) To know the voltage between the wire carrying $-\rho_l$ and the cylinder, you need to calculate the voltage on the wire. Intuitively, similar to a parallel plate capacitor, the wire is connected to the ground of a battery at a voltage of zero. Alternatively, recall the voltage at r with reference to r_0, given by $V(r) = -\int_{r_0}^{r} E_r dr = \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$. Let both $r \to r_0 \to 0$ for the wire carrying $-\rho_l$, the voltage on the wire is therefore $V \to 0$. Therefore, the wire is considered grounded on one end of a battery.

The voltage difference between the two electrodes is then

$$V_M = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{a}{d}$$

The capacitance per unit length is then $C_l = \frac{2\pi\varepsilon_0}{|\ln(d/a)|}$

(c) Again, from Gauss law, the surface charge per unit length is just the same as the image charge, which is $+\rho_l$.

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