The Second Exam on Linear Algebra Dec 6, 2021

(1) (20%) Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

(a) Determine a basis for the subspace of $R(A^T)$
(b) Determine a basis for the subspace of $N(A)$
(c) Determine a basis for the subspace of $N(A^T)$
(d) Determine a basis for the subspace of $R(A)$.

(2) (20%) (a) Find the best least squares fit to the following data by a quadratic polynomial

x	-1	0	1	2
y	0	1	3	9

(3) (20%) Let $u_1=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $u_2=\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ and $v_1=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $v_2=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

and let L be a linear operator from R^2 whose matrix representation with respect to the ordered basis $[\ u_1,\ \ u_2]$ is $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(a) Determine the transition matrix from the basis $[\ v_1,\ \ v_2]$ to the basis $[\ u_1,\ u_2]$.
(b) Find the matrix representation L with respect to $[\ v_1,\ $ v2].

4. (10%) Let L be the linear operator on R^3 defined by

L: (**x**)=$\begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \end{bmatrix}$ and let S=Span ($[1 \quad 0 \quad 1]^T$)

(a) (5%)Determine the kernel of L
(b) (5%) Determine L(S)

5.(20%) The linear transformation L on P_3 defined by $L(p(x)) = xp'(x) + p''(x)$

(a) Find the matrix A representation L with respect to the ordered basis $[1, x, x^2]$
(b) Find the matrix B representation L with respect to the ordered basis $[1, x, 1+x^2]$
(c) Find the matrix S such that $B = S^{-1}AS$
(d) If p(x) $=a_0 + a_1x + a_2(1+ x^2)$ calculate L^n(p(x))

6. (10%) Given $\mathbf{x} = [3 \quad 4]^T$ $and\ \mathbf{y} = [1 \quad 0]^T$, find the vector projection **p** of **x** onto **y** and verify that **p** and **x-p** are orthogonal.