Chapter 8 Exercises

Exercise 8.3-1 The circuit shown in Figure E 8.3-1 is at steady state before the switch closes at time t = 0. Determine the capacitor voltage, $v(t)$, for $t \geq 0$.

Answer: $v(t) = 2 + e^{-2.5t}$ V for $t > 0$

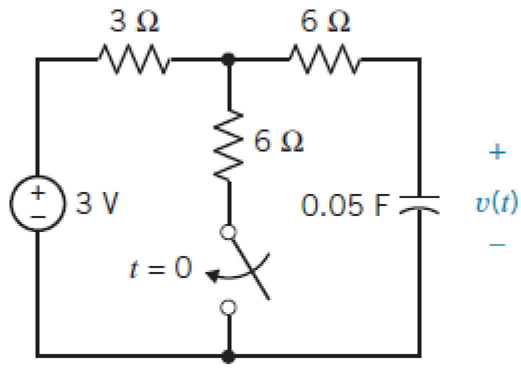
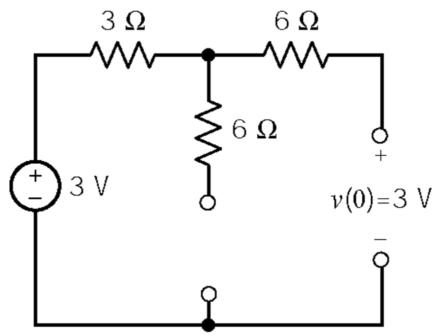
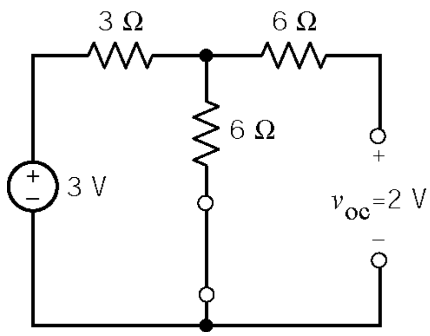


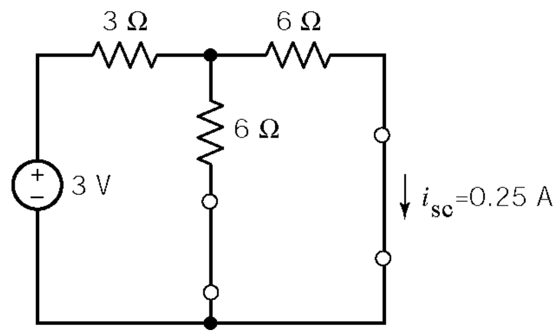
Figure E 8.3-1

Solution: Before the switch closes:



After the switch closes:





Therefore $$R_t = \frac{2}{0.25} = 8\,\Omega \quad \text{so} \quad \tau = 8(0.05) = 0.4 \text{ s}.$$

Finally, $$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 2 + e^{-2.5t} \text{ V} \quad \text{for } t > 0$$

Exercise 8.3-2 The circuit shown in Figure E 8.3-2 is at steady state before the switch closes at time t = 0. Determine the inductor current, $i(t)$, for $t > 0$.

Answer: $i(t)=\dfrac{1}{4}+\dfrac{1}{12}e^{-1.33t}$ A for $t>0$

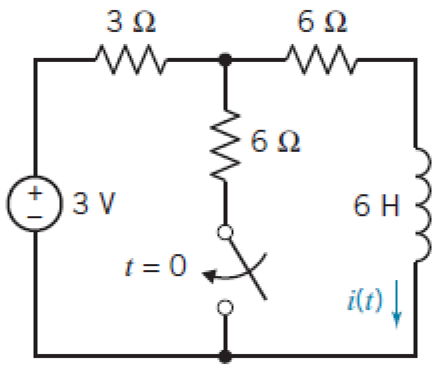
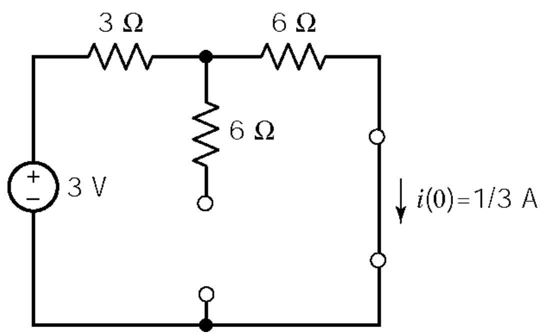
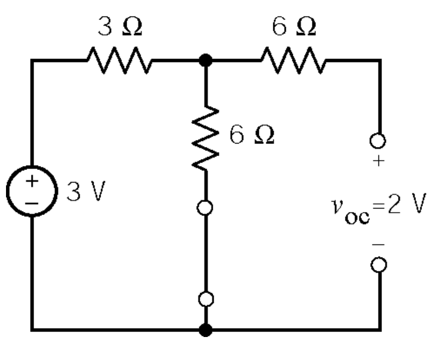


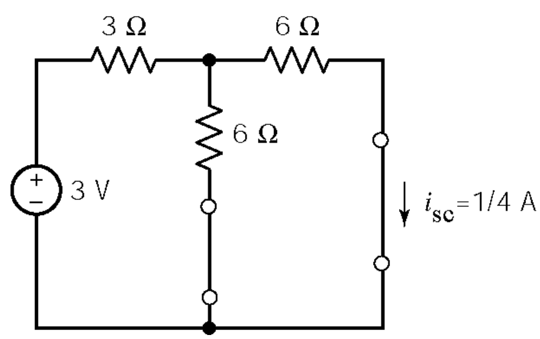
Figure E 8.3-2

Solution: Before the switch closes:



After the switch closes:

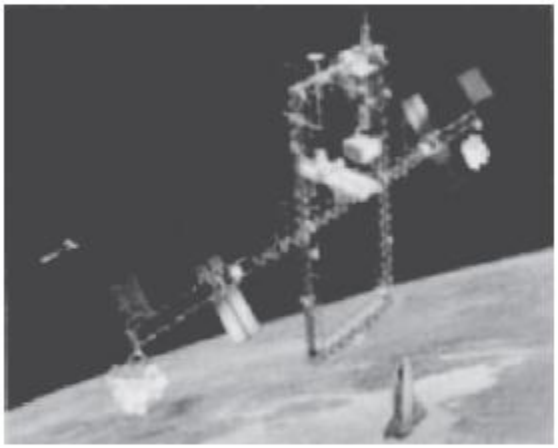




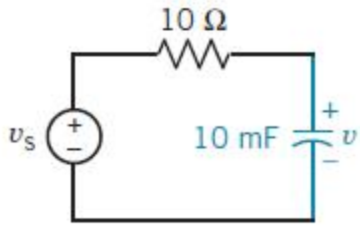
Therefore $$R_t=\frac{2}{0.25}=8\,\Omega \quad \text{so} \quad \tau=\frac{6}{8}=0.75\text{ s}.$$

Finally, $$i(t)=i_{sc}+(i(0)-i_{sc})\,e^{-t/\tau}=\frac{1}{4}+\frac{1}{12}e^{-1.33t}\text{ A} \quad \text{for } t>0$$

Exercise 8.7-1 The electrical power plant for the orbiting space station shown in Figure E 8.7-1*a* uses photovoltaic cells to store energy in batteries. The charging circuit is modeled by the circuit shown in Figure E 8.7-1*b*, where $v_s = 10 \sin 20t$ V. If $v(0^-) = 0$, find $v(t)$ for $t > 0$.
Answer: $v = 4\,e^{-10t} - 4\cos 20t + 2\sin 20t$ V



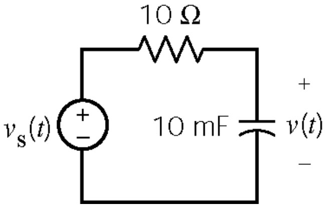
(*a*)



(*b*)

Figure E 8.7-1

Solution:



Apply KVL:

$$-10\sin 20t + 10\left(0.01\frac{d\,v(t)}{dt}\right) + v(t) = 0$$

$$v_s(\text{t}) = 10\sin 20t \text{ V}$$

$$\Rightarrow \frac{d\,v(t)}{dt} + 10\,v(t) = 100\sin 20t$$

Natural Response: $v_n(t) = Ae^{-t/\tau}$ where $\tau = R_t\,C$ $\therefore$ $v_n(t) = Ae^{-10t}$

Forced Response: try $v_f(t) = B_1\cos 20t + B_2\sin 20t$

Plugging $v_f(t)$ into the differential equation and equating like terms yields:

$$-20\,B_1\sin 20t + 20\,B_2\cos 20t + 10\,B_1\cos 20t + 10\,B_2\sin 20t = 100\sin 20t$$
$$20\,B_2 + 10\,B_1 = 0 \quad \text{and} \quad -20\,B_1 + 10\,B_2 = 100$$

$$B_1 = -4 \quad \text{and} \quad B_2 = 2$$

Complete Response: $v(t) = v_n(t) + v_f(t) = Ae^{-10t} - 4\cos 20t + 2\sin 20t$

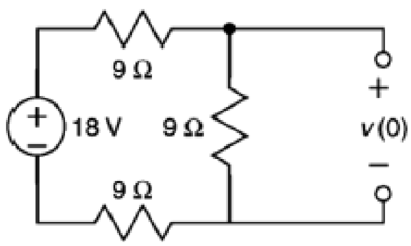
Now $v(0^-) = v(0^+) = 0 = A - 4 \quad \Rightarrow \quad A = 4$

$$\underline{v(t) = 4e^{-10t} - 4\cos 20t + 2\sin 20t \text{ V}}$$

Section 8.3: The Response of a First Order Circuit to a Constant Input

P 8.3-1

Solution:

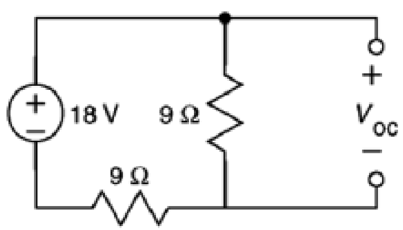


Here is the circuit before t = 0, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit.

A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the initial capacitor voltage, v (0).

By voltage division

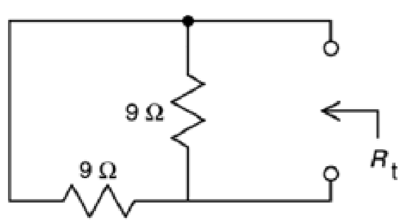
$$v(0) = \frac{9}{9+9+9}(18) = 6 \text{ V}$$



Next, consider the circuit after the switch closes. The closed switch is modeled as a short circuit.

We need to find the Thevenin equivalent of the part of the circuit connected to the capacitor. Here's the circuit used to calculate the open circuit voltage, V_{oc}.

$$V_{oc} = \frac{9}{9+9}(18) = 9 \text{ V}$$



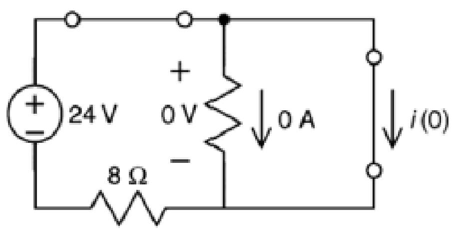
Here is the circuit that is used to determine R_t. A short circuit has replaced the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

$$R_t = \frac{(9)(9)}{9+9} = 4.5 \ \Omega$$

Then $\tau = R_t\, C = 4.5(0.25) = 1.125\ \text{ s}$

Finally, $v(t) = V_{oc} + \left(v(0) - V_{oc}\right) e^{-t/\tau} = 9 - 3\, e^{-0.89\, t}\ \text{ V} \quad \text{for } t > 0$

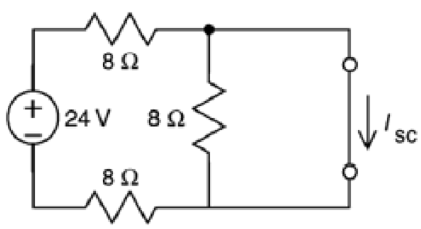
P 8.3-2
Solution:



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit.

An inductor in a steady-state dc circuit acts like an short circuit, so a short circuit replaces the inductor. The current in that short circuit is the initial inductor current, $i(0)$.

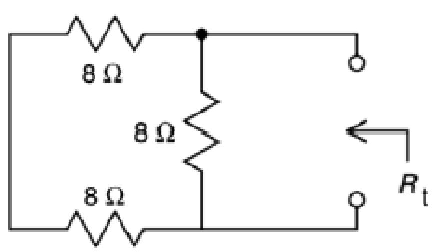
$$i(0) = \frac{24}{8} = 3\ \text{ A}$$



Next, consider the circuit after the switch opens. The open switch is modeled as an open circuit.

We need to find the Norton equivalent of the part of the circuit connected to the inductor. Here's the circuit used to calculate the short circuit current, I_{sc}.

$$I_{sc} = \frac{24}{8+8} = 1.5\ \text{ A}$$



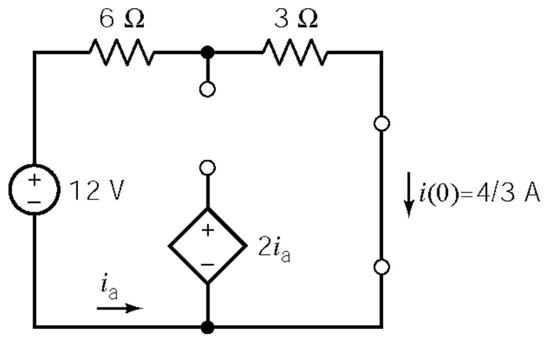
Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = 8 \,\|\, (8+8) = \frac{(8+8)(8)}{(8+8)+8} = 5.33\ \Omega$$

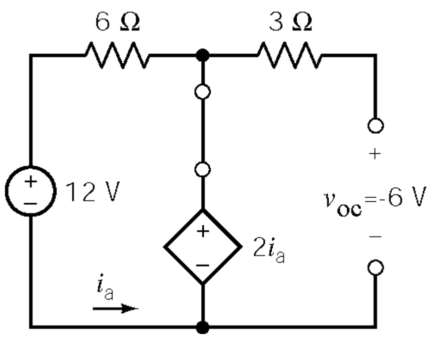
Then $\tau = \dfrac{L}{R_t} = \dfrac{8}{5.33} = 1.5\ \text{ s}$

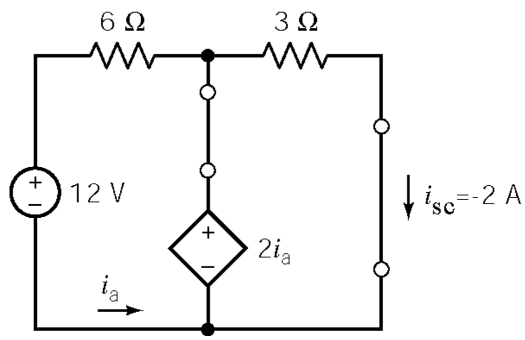
Finally, $i(t) = I_{sc} + \left(i(0) - I_{sc}\right) e^{-t/\tau} = 1.5 + 1.5 e^{-0.67\, t}\ \text{ A} \quad \text{for } t > 0$

P 8.3-3
Solution: Before the switch closes:



After the switch closes:



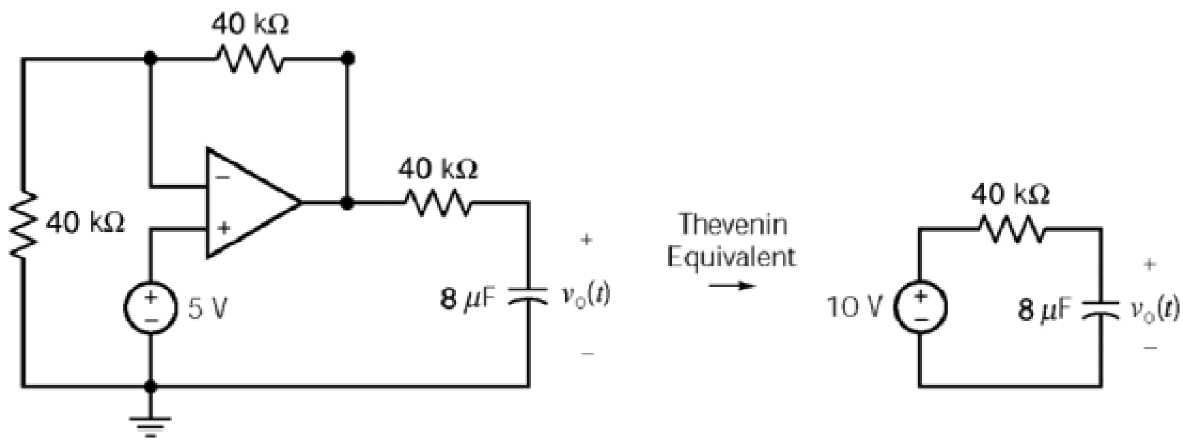


Therefore $R_t = \dfrac{-6}{-2} = 3\,\Omega$ so $\tau = \dfrac{6}{3} = 2$ s .

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \dfrac{10}{3}\, e^{-0.5t}$ A for $t > 0$

P 8.3-4

Solution: Before the switch opens, $v_o(t) = 5$ V $\Rightarrow v_o(0) = 5$ V . After the switch opens the part of the circuit connected to the capacitor can be replaced by it's Thevenin equivalent circuit to get:



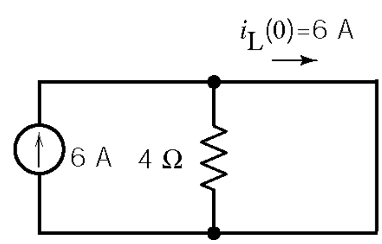
Therefore, $\tau = (40\times10^3)(8\times10^{-6}) = 0.32\text{s}$ $\tau = \left(20\times10^3\right)\left(4\times10^{-6}\right) = 0.08$ s .

Next, $v_C(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 10 - 5\, e^{-3.1t}$ V for $t > 0$

Finally, $v_0(t) = v_C(t) = 10 - 5\, e^{-12.5t}$ V for $t > 0$

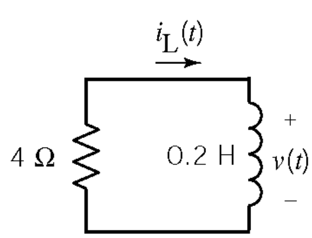
P 8.3-5

Solution: At $t = 0^-$ (steady-state)



Since the input to this circuit is constant, the inductor will act like a short circuit when the circuit is at steady-state:

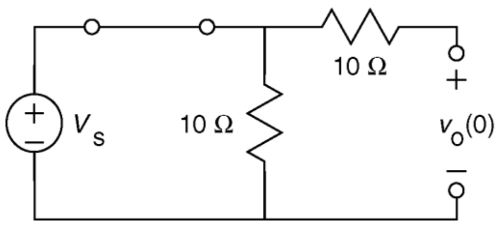
for t > 0



$$\underline{i_L(t) = i_L(0)\, e^{-(R/L)\,t} = 6\, e^{-20t}\ \text{A}}$$

P 8.3-6

Solution: Before the switch opens, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the capacitor voltage, will have constant values. Opening the switch disturbs the circuit. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch opened.



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor *R*. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the capacitor voltage, $v_o(t)$.

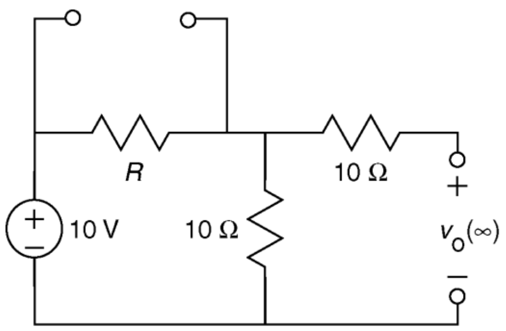
Because the circuit is at steady state, the value of the capacitor voltage will be constant. This constant is the value of the capacitor voltage just before the switch opens. In the absence of unbounded currents, the voltage of a capacitor must be continuous. The value of the capacitor voltage immediately after the switch opens is equal to the value immediately before the switch opens. This value is called the initial condition of the capacitor and has been labeled as $v_o(0)$. There is no current in the horizontal resistor due to the open circuit. Consequently, $v_o(0)$ is equal to the voltage across the vertical resistor, which is equal to the voltage source voltage. Therefore

$$v_o(0) = V_s$$

The value of $v_o(0)$ can also be obtained by setting $t = 0$ in the equation for $v_o(t)$. Doing so gives

$$v_o(0) = 2 + 8\,e^{0} = 10 \text{ V}$$

Consequently, $$V_s = 10 \text{ V}$$



Next, consider the circuit after the switch opens. Eventually (certainly as $t \to \infty$) the circuit will again be at steady state. Here is the circuit at $t = \infty$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the steady-state capacitor voltage, $v_o(\infty)$. There is no current in the horizontal resistor and $v_o(\infty)$ is equal to the voltage across the vertical resistor. Using voltage division,

$$v_o(\infty) = \frac{10}{R+10}(10)$$

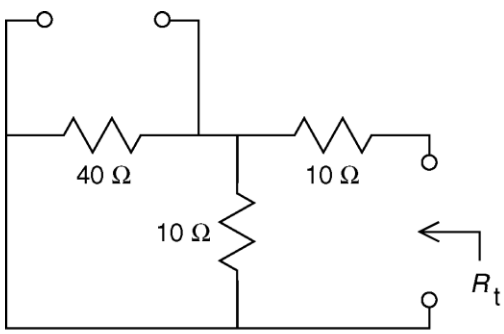
The value of $v_o(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $v_o(t)$. Doing so gives

$$v_o(\infty) = 2 + 8\,e^{-\infty} = 2 \text{ V}$$

Consequently,

$$2 = \frac{10}{R+10}(10) \quad \Rightarrow \quad 2\,R + 20 = 100 \quad \Rightarrow \quad R = 40\,\Omega$$

Finally, the exponential part of $v_o(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = R_t\, C$ and R_t is the Thevenin resistance of the part of the circuit connected to the capacitor.



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

$$R_t = 10 + \frac{(40)(10)}{40+10} = 18 \ \Omega$$

so

$$\tau = R_t\, C = 18\,C$$

From the equation for $v_o(t)$

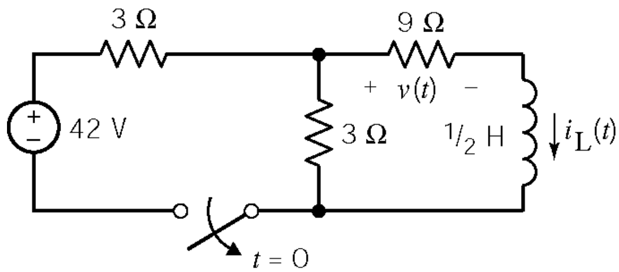
$$-0.5\,t = -\frac{t}{\tau} \quad \Rightarrow \quad \tau = 2 \text{ s}$$

Consequently,

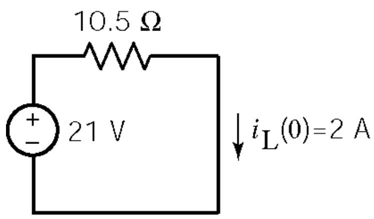
$$2 = 18\,C \quad \Rightarrow \quad C = 0.111 = 111 \text{ mF}$$

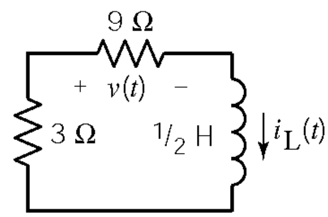
P 8.3-7

Solution: First, use source transformations to obtain the equivalent circuit



for $t < 0$: for $t > 0$:





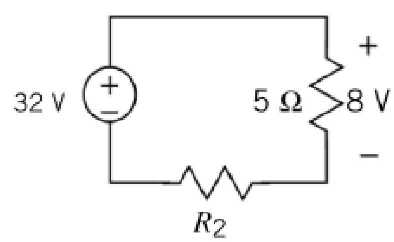
So $i_L(0) = 2$ A, $I_{sc} = 0$, $R_t = 3+9=12\ \Omega$, $\tau = \dfrac{L}{R_t} = \dfrac{\frac{1}{2}}{12} = \dfrac{1}{24}$ s

and $i_L(t) = 2e^{-24t} \quad t > 0$

Finally $v(t) = 9\, i_L(t) = 18\, e^{-24t} \quad t>0$

P 8.3-8

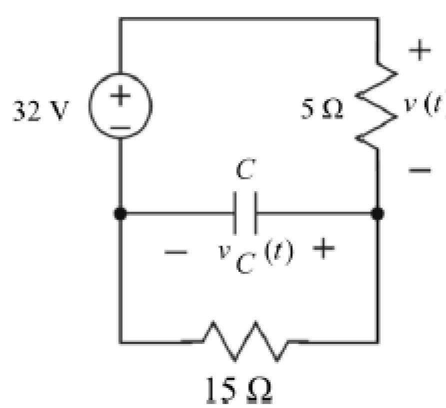
Solution: As $t \to \infty$ the circuit reaches steady state and the capacitor acts like an open circuit. Also, from the given equation, $v(t) \to 8$ V, as labeled on the drawing to the right, then



$$8 = \frac{5}{R_2+5} 32 \qquad \Rightarrow \qquad R_2 = 15\ \Omega$$

After $t = 0$

$$v_C(t) = 32 - v(t) = 24 - 4e^{-2t}$$



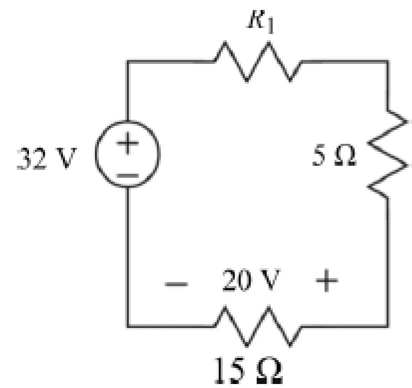
Immediately after $t = 0$

$$v_C(0+) = 24 - 4 = 20 \text{ V}$$

The capacitor voltage cannot change instantaneously so

$$v(0-) = 20 \text{ V}$$

The circuit is at steady state just before the switch closes so the capacitor acts like an open circuit. Then



$$20=\frac{15}{R_1+5+15}32 \quad \Rightarrow \quad R_1=4\ \Omega$$

After t = 0 the Thevenin resistance seen by the capacitor is

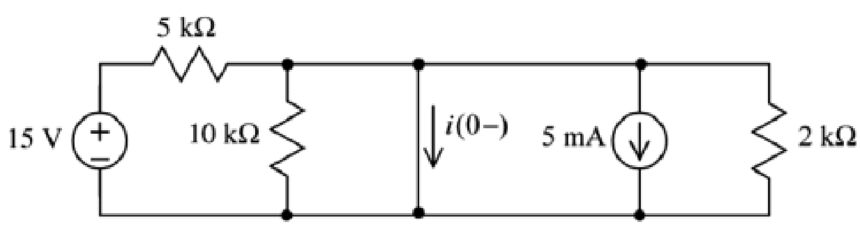
$$R_t=15\parallel 4=\frac{60}{19}\ \Omega$$

so

$$2=\frac{1}{\frac{60}{19}C} \quad \Rightarrow \quad C=\frac{19}{120}\ \text{F}$$

P 8.3-9

Solution: Before t = 0, with the switch closed and the circuit at steady state, the inductor acts like a short circuit so we have

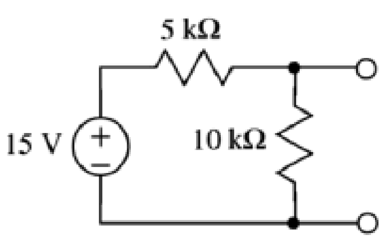


Using superposition

$$i(0-)=\frac{15}{5000}-5\times 10^{-3}=-2\ \text{mA}$$

The inductor current is continuous so $i(0+)=i(0-)=-2\ \text{mA}$.

After t = 0, the switch is open. Determine the Norton equivalent circuit for the part of the circuit connected to the inductor:



$$i_{sc}=\frac{15}{5000}=3\ \text{mA}$$
$$R_t=5000\parallel 10000=3333.3\ \Omega$$

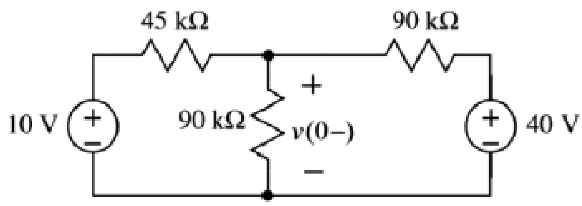
The time constant is given by $\tau=\frac{L}{R_t}=\frac{5}{3333.3}=0.0015$ so $\frac{1}{\tau}=666.67$.

The inductor current is given by

$$i_{\mathrm{L}}(t) = \left(i(0+) - i_{\mathrm{sc}}\right)e^{-t/\tau} + i_{\mathrm{sc}} = (-0.002 - 0.003)e^{-666.67t} + 0.003 = 3 - 5e^{-666.67t} \ \text{ mA} \quad \text{for } t \ge 0$$

P 8.3-10

Solution: Before t = 0, with the switch closed and the circuit at the steady state, the capacitor acts like an open circuit so we have

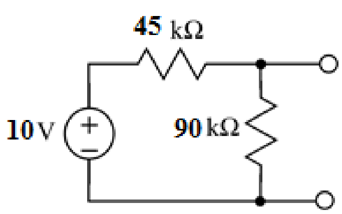


Using superposition

$$v(0-) = \frac{90 \| 90}{45 + (90 \| 90)} 10 + \frac{90 \| 45}{90 + (90 \| 45)} 40 = 15 \text{ V}$$

The capacitor voltage is continuous so $v(0+) = v(0-) = 15 \text{ V}$.

After t = 0 the switch is open. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor:



$$v_{\mathrm{oc}} = \frac{90}{90 + 45} 10 = 6.67 \text{ V}$$

$$R_{\mathrm{t}} = 45 \| 90 = 30 \text{ k}\Omega$$

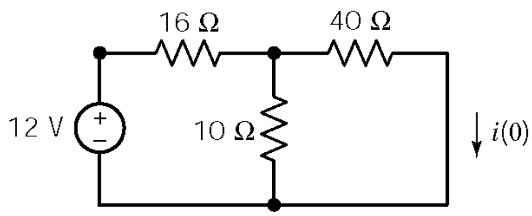
The time constant is $\tau = R_{\mathrm{t}} C = (30 \times 10^3)(5 \times 10^{-6}) = 0.15\text{s}$ s so $\frac{1}{\tau} = 6.67 \ \frac{1}{\text{s}}$.

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{\mathrm{oc}}\right)e^{-t/\tau} + v_{\mathrm{oc}} = (15 - 6.67)e^{-6.67t} + 6.67 = 6.67 + 8.33e^{-6.67t} \ \text{ V} \quad \text{for } t \ge 0$$

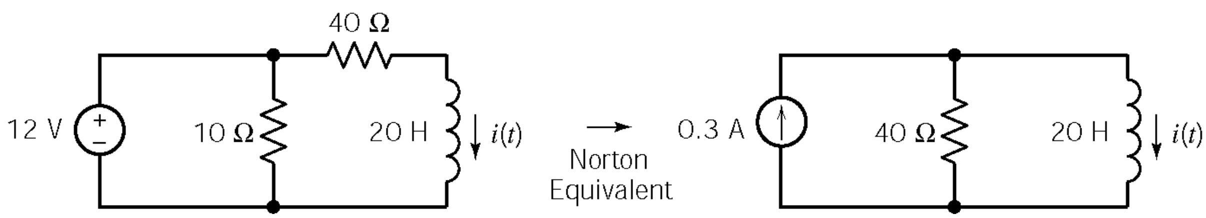
P 8.3-11

Solution: At steady-state, immediately before t = 0:



$$i(0) = \left(\frac{10}{10+40}\right)\left(\frac{12}{16+40\|10}\right) = 0.1 \text{ A}$$

After $t = 0$, the Norton equivalent of the circuit connected to the inductor is found to be



$$\text{so } I_{sc} = 0.3 \text{ A}, \; R_t = 40\,\Omega, \; \tau = \frac{L}{R_t} = \frac{20}{40} = \frac{1}{2} \text{ s}$$

$$\text{Finally:} \qquad i(t) = (0.1-0.3)e^{-2t} + 0.3 = 0.3-0.2e^{-2t} \text{ A}$$

P 8.3-12
Solution:

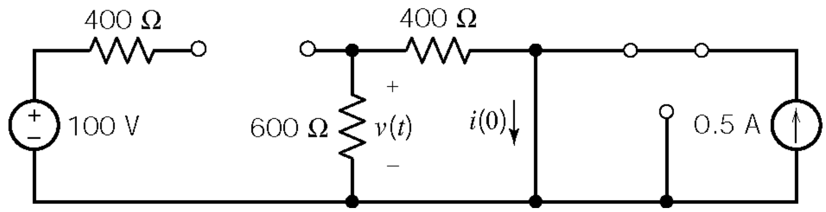
Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.

a.) When the switch is open, $v_{oc} = \left(\frac{150}{75+150}\right)30 = 20 \text{ V}$ and $R_t = 150 \| 75 = 50\ \Omega$. The steady state capacitor voltages is v_{oc} =20 V. The time constant is τ = (50)(0.025) = 1.25 s.

b.) When the switch is closed, $v_{oc} = \left(\frac{150}{37.5+150}\right)30 = 24 \text{ V}$ and $R_t = 150 \| 75 \| 75 = 30\ \Omega$. The steady state capacitor voltages is v_{oc} =24 V. The time constant is τ = (30)(0.025) = 0.75 s.

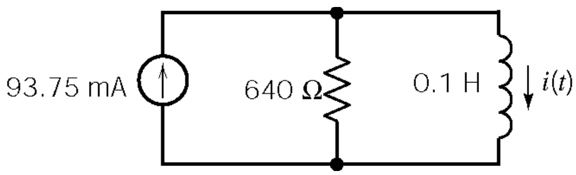
P 8.3-13

Solution: At steady-state, immediately before t = 0



so $i(0) = 0.5$ A.

After t > 0: Replace the circuit connected to the inductor by its Norton equivalent to get

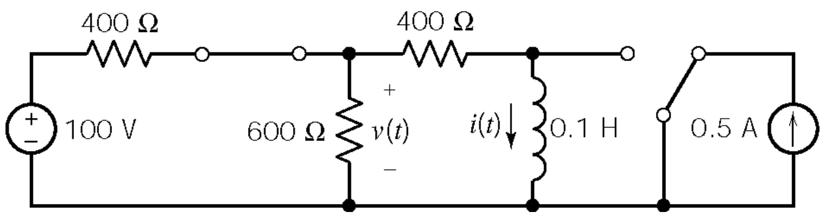


$I_{sc} = 93.75$ mA, $R_t = 640\ \Omega$,

$$\tau = \frac{L}{R_t} = \frac{.1}{640} = \frac{1}{6400}\ \text{s}$$

$$i\ (t) = 406.25\, e^{-6400t} + 93.75\ \text{mA}$$

Finally:



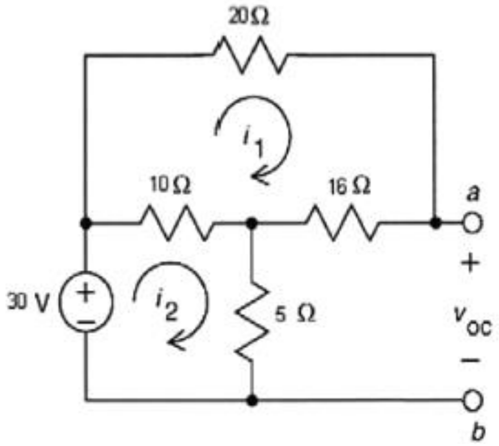
$$v(t) = 400\ i(t) + \ 0.1\ \frac{d}{dt}\ i(t) = 400\ (.40625e^{-6400t} + .09375) + 0.1(-6400)\,(0.40625e^{-6400t})$$
$$= \ 37.5 - 97.5e^{-6400t}\ \text{V}$$

P 8.3-14

Solution: Before the switch closes $v(t) = 0$ so $v(0+) = v(0-) = 0$ V.

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of the terminals $a - b$.

Write mesh equations to find v_{oc}:



Mesh equations:

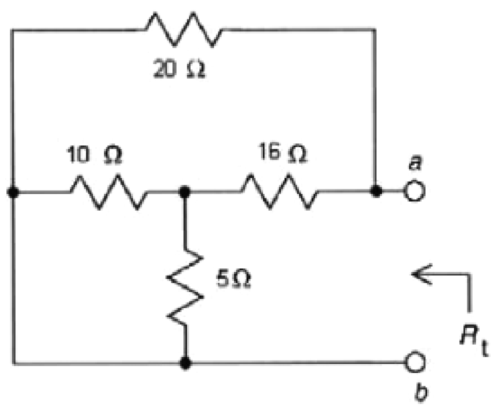
$$20\, i_1 + 16\, i_1 - 10\left(i_2 - i_1\right) = 0$$
$$10\left(i_2 - i_1\right) + 5\, i_2 - 30 = 0$$

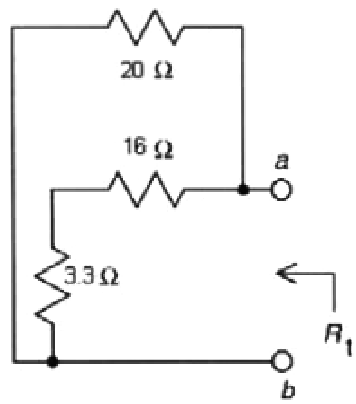
$$46\, i_1 = 10\, i_2$$
$$15\, i_2 - 10\, i_1 = 30$$

$$59\, i_1 = 30 \quad \Rightarrow \quad i_1 = 0.51 \text{ A}$$
$$i_2 = \frac{23}{5}\left(\frac{30}{59}\right) = 2.34 \text{ A}$$

Using KVL, $v_{oc} = 5\, i_2 + 16 i_1 = 5\,(2.34) + 16\,(0.51) = 19.86$ V

Find R_t:





$$R_t = \frac{20\left(16 + 3.3\right)}{20 + \left(16 + 3.3\right)} = 9.8\ \Omega$$

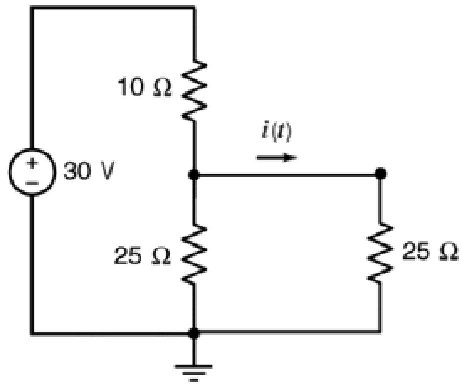
Then $\tau = R_t C = 10\left(\dfrac{1}{40}\right) = \dfrac{1}{4}$ s $\quad \Rightarrow \quad \dfrac{1}{\tau} = 4\ \dfrac{1}{\text{s}}$

and

$$v(t) = \left(v(0+) - v_{oc}\right) e^{-t/\tau} + v_{oc} = \left(0 - 19.86\right) e^{-4t} + 19.86 = 19.86\left(1 - e^{-4t}\right) \text{ V} \quad \text{for } t \ge 0$$

P 8.3-15

Solution: Before the switch closes the circuit is at steady state so the inductor acts like a short circuit. We have

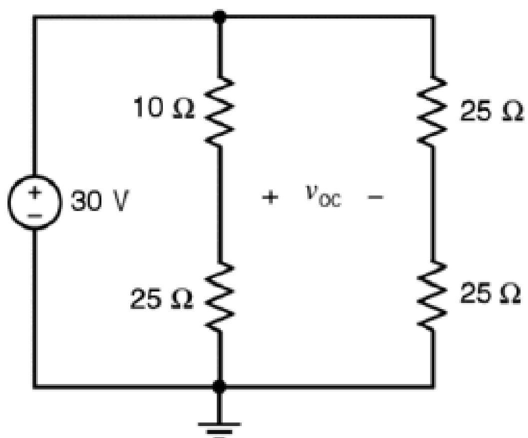


$$i(t)=\frac{1}{2}\left(\frac{30}{10+(25\parallel 25)}\right)=0.67\ \text{A}$$

so

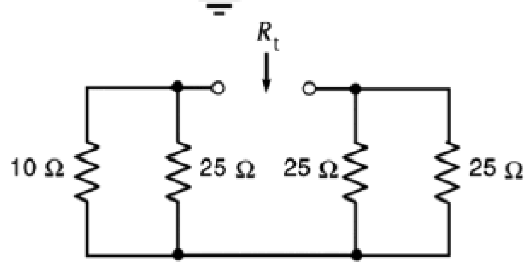
$$i(0+)=i(0-)=0.67\ \text{A}$$

After the switch closes, find the Thevenin equivalent circuit for the part of the circuit connected to the inductor.



Using voltage division twice

$$v_{oc}=\left(\frac{25}{35}-\frac{1}{2}\right)30=6.4\ \text{V}$$



$$R_t=(10\parallel 25)+(25\parallel 25)=19.64\ \Omega$$

$$i_{sc}=\frac{v_{oc}}{R_t}=\frac{6.4}{19.64}=0.33\ \text{A}$$

Then

$$\tau=\frac{L}{R_t}=\frac{3.5}{19.64}=0.18\ \text{s}\quad\Rightarrow\quad\frac{1}{\tau}=5.6\ \frac{1}{\text{s}}$$

and

$$i(t)=\left(i(0+)-i_{sc}\right)e^{-t/\tau}+i_{sc}=(0.67-0.33)e^{-5t}+0.33=0.34e^{-5.6t}+0.33\ \text{A}\quad\text{for }t\geq 0$$

P 8.3-16

Solution

For t < 0, the switch is open and the capacitor acts like an open circuit because the circuit is at steady state. Consequently, the current in the 15 Ω resistor is 0A and so the voltage across this resistor is 0 V. KVL gives $v(t) = 23\text{ V}$. Immediately before the switch opens we have $v(0-) = 23\text{ V}$. The capacitor voltage does not change instantaneously so $v(0+) = v(0-) = 23\text{ V}$.

For t > 0, the Thevenin equivalent of the part of the circuit connected to the capacitor is characterized by

$R_t = 15 \| 60 = 12\ \Omega$ and, using voltage division, $v_{oc} = \dfrac{60}{15+60}(23) = 18.4\text{ V}$.

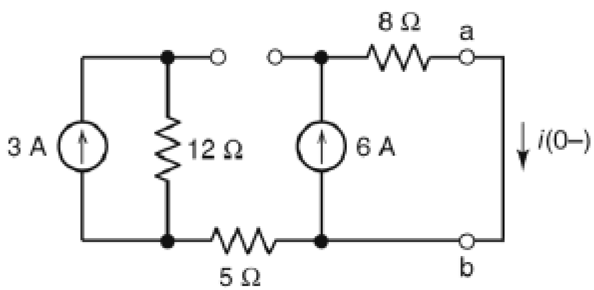
$A = v_{oc} = 18.4\text{ V}$, $B = v(0+) - v_{oc} = 23 - 18.4 = 4.6\text{ V}$ and $a = \dfrac{1}{\tau} = \dfrac{1}{R_t C} = \dfrac{1}{12(0.03)} = 2.78\ \dfrac{1}{s}$

and

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (23 - 18.4)e^{-2.78t} + 18.4 = 18.4 + 4.6\left(e^{-2.78t}\right)\text{ V} \quad \text{for } t \ge 0$$

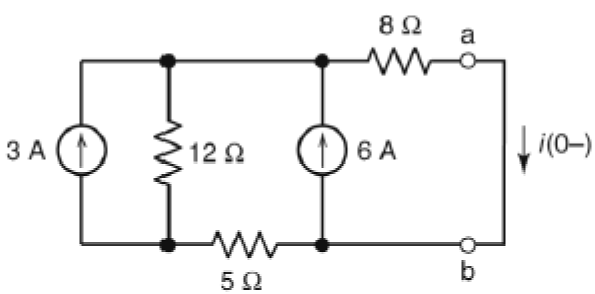
P 8.3-17

Solution: Before t = 0, with the switch open and the circuit at steady state, the inductor acts like a short circuit so we have



$$i(0+) = i(0-) = 6\text{ A}$$

After t = 0, we can replace the part of the circuit connected to the inductor by its Norton equivalent circuit.

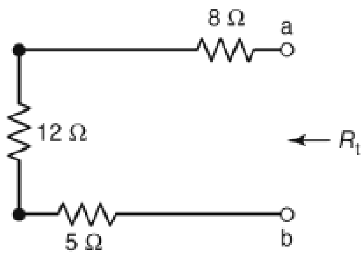


Using superposition, the short circuit current is given by

$$i_{sc} = \left(\frac{12}{12+(8+5)}\right)3 + \left(\frac{5+12}{(5+12)+8}\right)6 = 5.52\text{ A}$$

$$R_t = 12 + 5 + 8 = 25\ \Omega$$

so



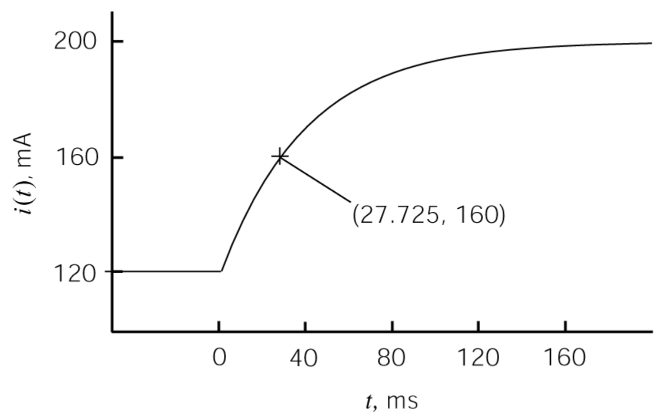
$$\tau = \frac{3}{25} = 0.12 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 8.3 \ \frac{1}{\text{s}}$$

The inductor current is given by

$$i_{\text{L}}(t) = \left(i(0+) - i_{\text{sc}}\right)e^{-t/\tau} + i_{\text{sc}} = (6 - 5.52)e^{-8.3t} + 5.52 = 5.52 - 0.48\, e^{-8.3t} \text{ A} \quad \text{for } t \geq 0$$

P 8.3-18

Solution: From the plot



$D =$ $i(t)$ for $t < 0$ =120 mA = 0.12 A,

$E + F = i(0+) = 120$ mA = 0.12 A

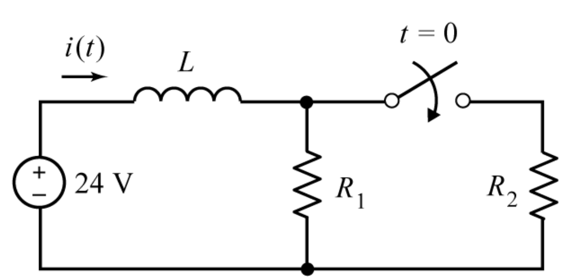
and

$$E = \lim_{t \to \infty} i(t) = 200 \text{ mA} = 0.2 \text{ A}.$$

The point labeled on the plot indicates that $i(t) =$ 160 mA when $t = 27.725$ ms = 0.027725 s. Consequently

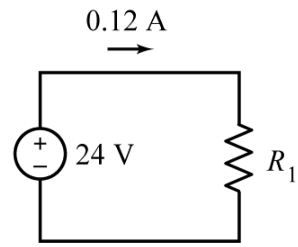
$$160 = 200 - 80\, e^{-a(0.027725)} \quad \Rightarrow \quad a = \frac{\ln\left(\dfrac{160-200}{80}\right)}{-0.027725} = 25 \ \frac{1}{\text{s}}$$

Then



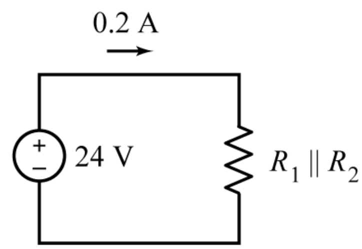
$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t \leq 0 \\ 200 - 80\, e^{-25t} \text{ mA} & \text{for } t \geq 0 \end{cases}$$

When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.



$$R_1 = \frac{24}{0.12} = 200\ \Omega$$

As $t \to \infty$, the circuit is again at steady state so the inductor acts like a short circuit.



$$R_1 \parallel R_2 = \frac{24}{0.2} = 120\ \Omega$$

$$120 = 200 \parallel R_2 \quad \Rightarrow \quad R_2 = 300\ \Omega$$

Next, the inductance can be determined using the time constant:

$$25 = a = \frac{1}{\tau} = \frac{R_1 \parallel R_2}{L} = \frac{120}{L} \quad \Rightarrow \quad L = \frac{120}{25} = 4.8\ \text{H}$$

P 8.3-19

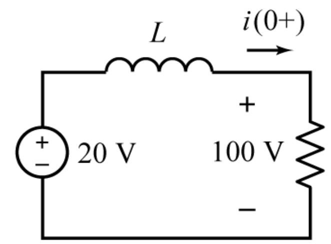
Solution: From the plot $D =$ $v(t)$ for $t < 0$ =20 V, $E + F = v(0+) = 100$ V and $E = \lim_{t\to\infty} v(t) = 20\ \text{V}$. The point labeled on the plot indicates that $v(t) = 60$ V when $t =$ 0.14 s. Consequently

$$60 = 20 + 80e^{-a(0.14)} \quad \Rightarrow \quad a = \frac{\ln\left(\frac{60-20}{80}\right)}{-0.14} = 5\ \frac{1}{\text{s}}$$

Then

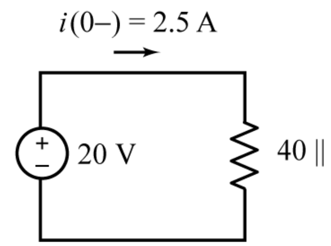
$$v(t) = \begin{cases} 20\ \text{V} & \text{for } t \le 0 \\ 20 + 80e^{-5t}\ \text{V} & \text{for } t \ge 0 \end{cases}$$

At $t = 0+$,



$$i(0+) = \frac{100}{40} = 2.5 \text{ A}$$

When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.



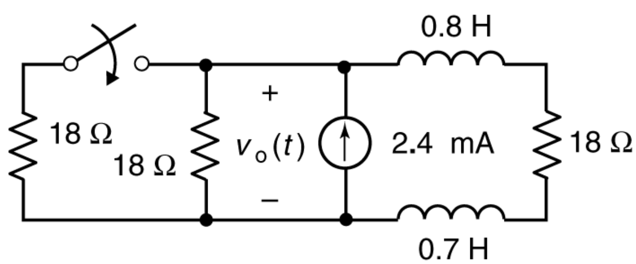
$$40 \| R_2 = \frac{20}{2.5} = 8\ \Omega \Rightarrow \quad R_2 = 10\ \Omega$$

Next, the inductance can be determined using the time constant:

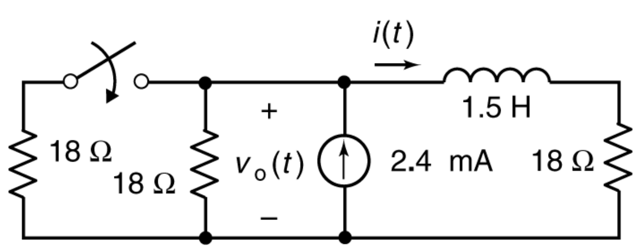
$$5 = a = \frac{1}{\tau} = \frac{40}{L} \quad \Rightarrow \quad L = \frac{40}{5} = 8 \text{ H}$$

P8.3-20

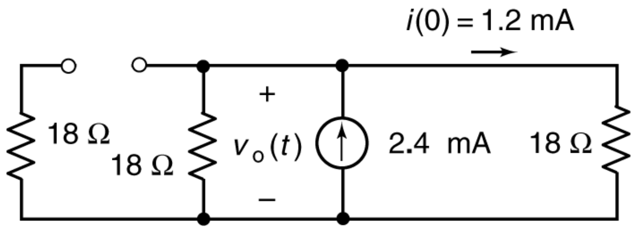
Solution:



Replace the series inductors with an equivalent inductor and label the current in the inductor:



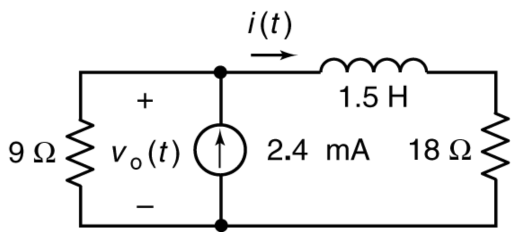
We will determine the inductor current, $i(t)$,first and then use it to determine $v_o(t)$. Determine the initial condition, $i(o)$, by considering the circuit when $t < 0$ and the circuit is at steady state. Since an inductor in a dc circuit acts like a short circuit , we have



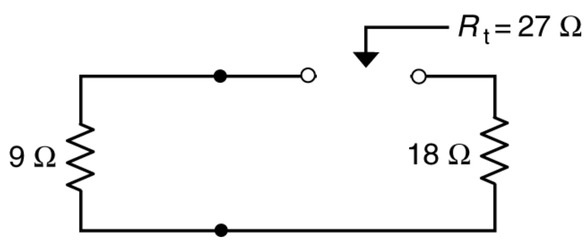
Using current division, we have

$$i(0)=\left(\frac{18}{18+18}\right)2.4=1.2\text{ mA}$$

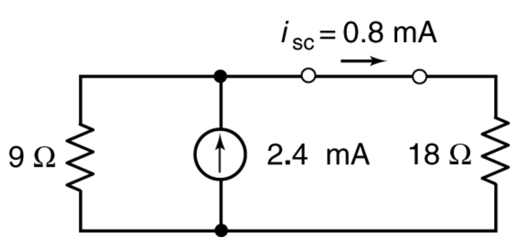
Next, consider the circuit when $t > 0$ and the circuit is not at steady state:



To find the Norton equivalent of the part of the circuit connected to the inductor we determine both the Thevenin resistance and the short circuit current:



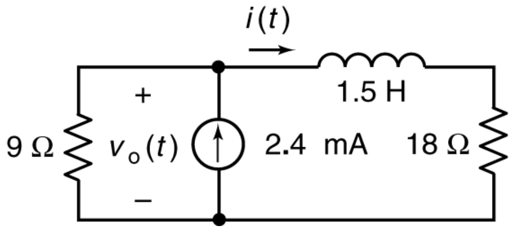
and



The time constant is: $$\tau=\frac{L}{R_t}=\frac{1.5}{27}=\frac{1}{18}=0.0556\text{ second}$$

The inductor current is given by

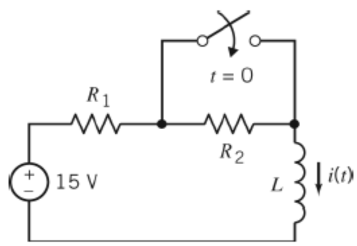
$$i(t)=\left(i(0)-i_{sc}\right)e^{-t/\tau}+i_{sc}=(1.2-0.8)e^{-18t}-0.8\quad\text{mA for } t\geq 0$$



Using KCL $$\frac{v_o(t)}{9}+\left[(1.2-0.8)e^{-18t}-0.8\right]\times10^{-3}=2.4\times10^{-3}$$

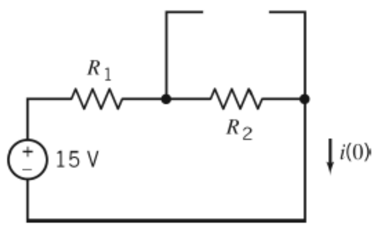
Finally $$v_o(t)=14.4-3.6e^{-18t}\ \text{ mV for } t>0$$

P 8.3-21
Solution:

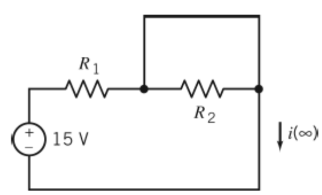


The steady state current before the switch closes is equal to
$i(0)=0.9-0.3e^{-5(0)}=0.6\ \text{ A}$.

The inductor will act like a short circuit when this circuit is at steady state so



$$0.6=i(0)=\frac{15}{R_1+R_2}$$
$$\Rightarrow\quad R_1+R_2=25\Omega$$



After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be
$i(\infty)=0.9-0.3e^{-5(\infty)}=0.9\text{ A}$

The inductor will act like a short circuit when this circuit is at steady state so

$$0.9=i(\infty)=\frac{15}{R_1}\quad\Rightarrow\quad R_1=16.6\Omega$$

Then $R_2 = 8.4\ \Omega$.

After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t = R_1$. Then

$$5 = \frac{1}{\tau} = \frac{R_t}{L} = \frac{R_1}{L} = \frac{16.6}{L} \quad \Rightarrow \quad L = 3.3\ \text{H}$$

P 8.3-22

Solution: The inductor current is given by

$$i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} \quad \text{for } t \geq 0 \text{ where } a = \frac{1}{\tau} = \frac{R_t}{L}.$$

a. Comparing this to the given equation gives

$$21.6 = i_{sc} = \frac{R_1}{R_1 + 4}(36) \quad \Rightarrow \quad R_1 = 6\ \Omega \text{ and}$$

$$4 = \frac{R_t}{2} \quad \Rightarrow \quad R_t = 8\ \Omega\text{. Next}$$

$$8 = R_t = \left(R_1 + 4\right) \| R_3 = 10 \| R_3 \quad \Rightarrow \quad R_3 = 40\ \Omega.$$

b. $R_t = (16+4) \| 20 = 10\ \Omega$ so $a = \dfrac{1}{\tau} = \dfrac{10}{2} = 5\ \text{s}$. also

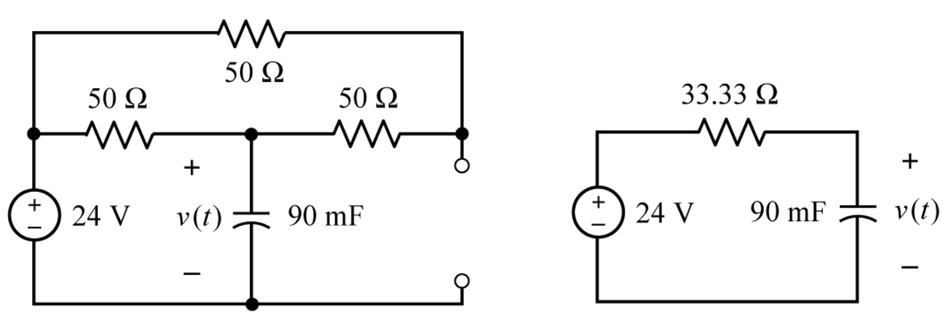
$$i_{sc} = \frac{16}{16+4}(36) = 28.8\ \text{mA}\text{. Then}$$

$$i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = 28.8 + (10 - 28.8)e^{-5t} = 28.2 - 18.8e^{-5t}$$

.

P 8.3-23

Solution: a.) When the switch is open we have

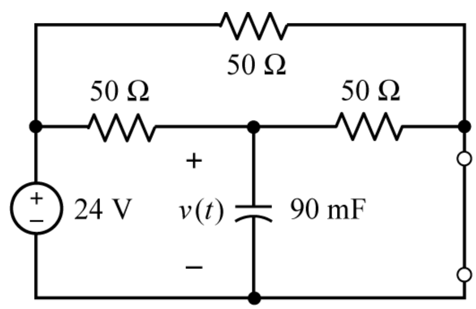


After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is

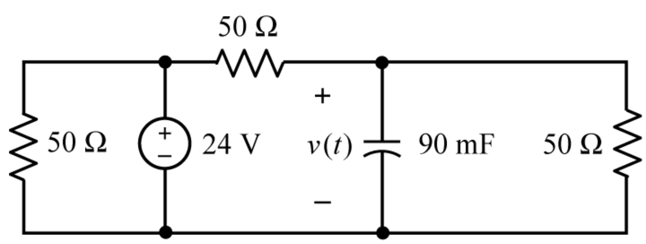
$\tau = R_t C = 33.33(0.090) = 3$ s .

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty) = 24$ V.

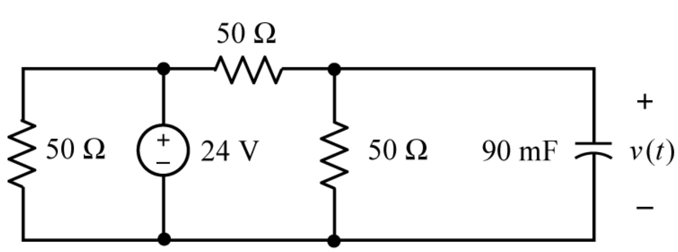
b.) When the switch is closed we have

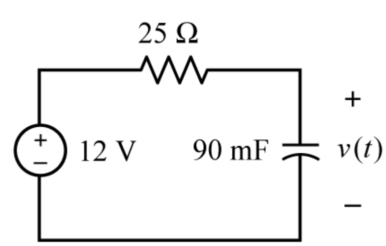


This circuit can be redrawn as



Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:





So $R_t = 25\ \Omega$ and

$$\tau = R_t C = 25(0.090) = 2.25 \text{ s}$$

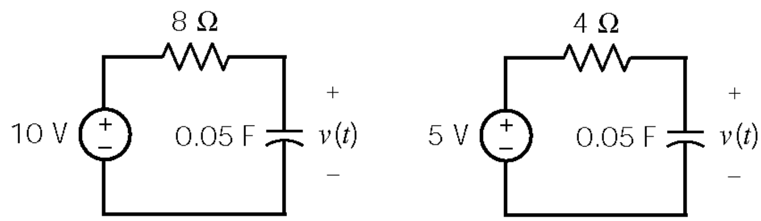
Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.

Section 8-4: Sequential Switching

P 8.4-1

Solution:

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$t < 0$ and $t > 1.5$ s $\qquad\qquad$ $0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $v(0)$ = 10 V. For 0 < t < 1.5s, v_{oc} = 5 V and R_t = 4 Ω so $\tau = 4 \times 0.05 = 0.2$ s . Therefore

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 5 + 5e^{-5t} \text{ V} \quad \text{for } 0 < t < 1.5 \text{ s}$$

At t =1.5 s, $v(1.5) = 5 + 5e^{-0.05(1.5)} = 5$ V . For 1.5 s < t, v_{oc} = 10 V and R_t = 8 Ω so $\tau = 8 \times 0.05 = 0.4$ s . Therefore

$$v(t) = v_{oc} + (v(1.5) - v_{oc})\, e^{-(t-1.5)/\tau} = 10 - 5\, e^{-2.5(t-1.5)} \text{ V} \quad \text{for } 1.5 \text{ s} < t$$

Finally

$$v(t) = \begin{cases} 5 + 5\, e^{-5t} \text{ V} & \text{for } 0 < t < 1.5 \text{ s} \\ 10 - 5\, e^{-2.5(t-1.5)} \text{ V} & \text{for } 1.5 \text{ s} < t \end{cases}$$

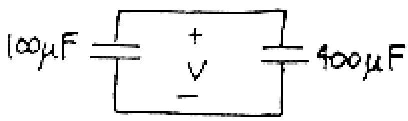
P 8.4-2

Solution:

At $t = 0^-$: Assume that the circuit has reached steady state so that the voltage across the 100 μF capacitor is 3 V. The charge stored by the capacitor is

$$q\left(0^-\right)=\left(100\times10^{-6}\right)(3)=300\times10^{-6}\ \text{ C}$$

0 < t < 10ms: With R negligibly small, the circuit reaches steady state almost immediately (i.e. at $t = 0^+$). The voltage across the parallel capacitors is determined by considering charge conservation:



$$q\left(0^+\right)=(100\ \mu\text{F})\ v\left(0^+\right)+(400\ \mu\text{F})\ v\left(0^+\right)$$

$$v\left(0^+\right)=\frac{q\left(0^+\right)}{100\times10^{-6}+400\times10^{-6}}=\frac{q\left(0^-\right)}{500\times10^{-6}}=\frac{300\times10^{-6}}{500\times10^{-6}}$$

$$\underline{v\left(0^+\right)=0.6\text{ V}}$$

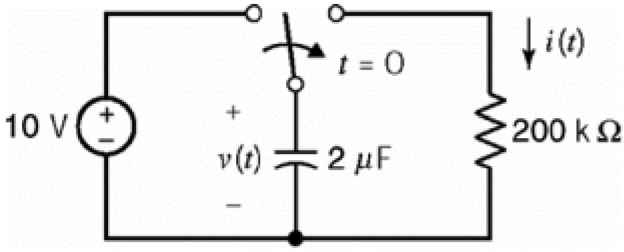
10 ms < t < l s: Combine 100 µF & 400 µF in parallel to obtain

$$v(t)\ =\ v\left(0^+\right)\ e^{-(t-.01)/RC}$$

$$=\ 0.6e^{-(t-.01)/(10^3)\ (5x10^{-4})}$$

$$\underline{v(t)\ =\ 0.6\ e^{-2(t-.01)}\ \text{V}}$$

P 8.4-3
Solution:



$v(0)=5\ \text{V},\ v(\infty)=0$ and

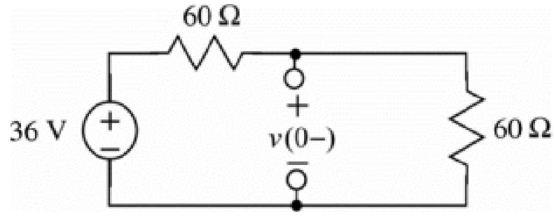
$\tau = 2\times10^{5}\times10^{-6} = 0.25\ \text{s}$

$$\therefore\quad v(t)=10\,e^{-5t}\ \text{V}\quad \text{for } t>0$$

$$5=10e^{-5t_1}\qquad \underline{t_1 = 0.0602\ \text{s}}$$

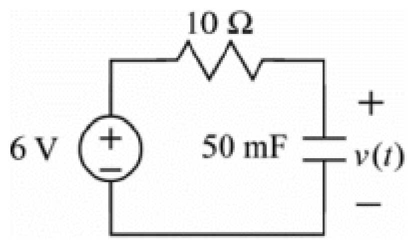
$$i(t_1)=\frac{v(t_1)}{200\times10^{3}}=\frac{5}{200\times10^{3}}=\underline{25\ \mu\text{A}}$$

P 8.4-4
Solution:
The circuit is at steady state before the switch closes. The capacitor acts like an open circuit. The initial condition is



$$v(0+) = v(0-) = \left(\frac{60}{60+60}\right)36 = 18 \text{ V}$$

After the switch closes, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 10\ \Omega$ and $v_{oc} = 6$ V

The time constant is

$$\tau = R_t C = (10)(0.05) = 0.55 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 2\ \frac{1}{\text{s}}$$

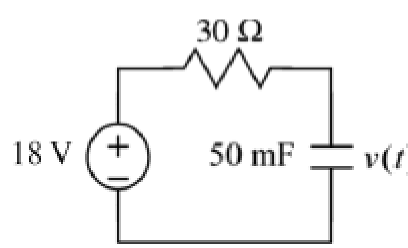
The capacitor voltage is

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (18-6)e^{-2t} + 6 = 6 + 12e^{-2t} \text{ V} \quad \text{for } 0 \geq t \geq 0.5 \text{ s}$$

When the switch opens again at time t = 0.5 the capacitor voltage is

$$v(0.5+) = v(0.5-) = 6 + 12e^{-2(0.5)} = 10.41 \text{ V}$$

After time t = 0.5 s, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 30\ \Omega$ and $v_{oc} = 18$ V

The time constant is

$$\tau = R_{\mathrm{t}} C = 30(0.05) = 1.5 \quad \Rightarrow \quad \frac{1}{\tau} = \frac{2}{3} \ \frac{1}{\mathrm{s}}$$

The capacitor voltage is

$$v(t) = \left(v(0.5+) - v_{\mathrm{oc}}\right) e^{-(t-0.5)/\tau} + v_{\mathrm{oc}} = (10.41 - 18) e^{-0.67(t-0.5)} + 18$$
$$= 18 - 7.59 e^{-0.67(t-0.5)} \ \mathrm{V} \quad \text{for } t \geq 0.5 \ \mathrm{s}$$

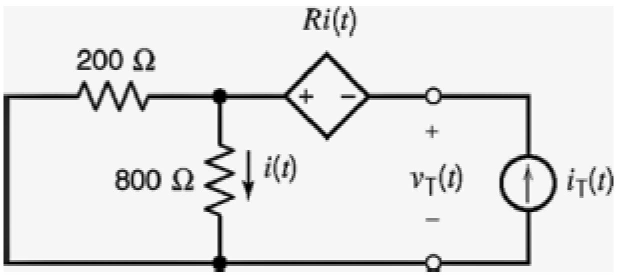
so

$$v(t) = \begin{cases} 18 \ \mathrm{V} & \text{for } t \geq 0 \\ 6 + 12 e^{-2t} \ \mathrm{V} & \text{for } 0 \leq t \leq 0.5 \ \mathrm{s} \\ 18 - 7.59 e^{-0.67(t-0.5)} \ \mathrm{V} & \text{for } t \geq 0.5 \ \mathrm{s} \end{cases}$$

Section 8.5 Stability of First-Order Circuits

P 8.5-1

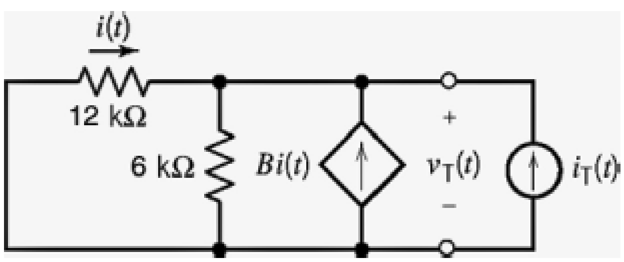
Solution:
This circuit will be stable if the Thèvenin equivalent resistance of the circuit connected to the inductor is positive. The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



$$\left.\begin{aligned} i(t) &= \frac{200}{200+400}\, i_T \\ v_T &= 800\, i(t) - R\, i(t) \end{aligned}\right\} \Rightarrow R_t = \frac{v_T}{i_T} = \frac{(800-R)\,100}{200+800}$$

<u>The circuit is stable when $R < 800\ \Omega$.</u>

P 8.5-2
Solution:
The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



Ohm's law: $i(t) = -\dfrac{v_T(t)}{12000}$

KCL: $i(t) + B\, i(t) + i_T(t) = \dfrac{v_T(t)}{6000}$

$$\therefore i_T(t) = -(B+1)\left(-\frac{v_T(t)}{12000}\right) + \frac{v_T(t)}{6000}$$

$$= \frac{(B+3)\, v_T(t)}{12000}$$

$$R_t = \frac{v_T(t)}{i_T(t)} = \frac{12000}{B+3}$$

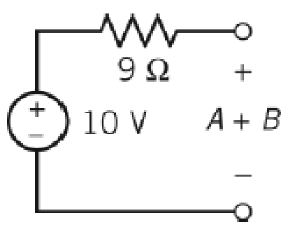
<u>The circuit is stable when $B >$ 3 A/A.</u>

Section 8.6 The Unit Step Source
P 8.6-1
Solution:
The value of the input is one constant, 10 V, before time $t = 0$ and a different constant,   7 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\,e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

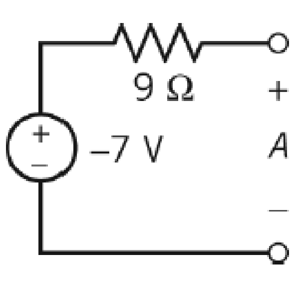
The value of the capacitor voltage at time t = 0, will be equal to the steady state capacitor voltage before the input changes. At time $t = 0$ the output voltage is

$$v_o(0) = A + B\,e^{-a(0)} = A + B$$

Consequently, the capacitor voltage is labeled as $A + B$. Analysis of the circuit gives

$$A + B = 10 \text{ V}$$

The steady-state circuit for t < 0.



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = ∞, will be equal to the steady state capacitor voltage after the input changes. At time $t = \infty$ the output voltage is

$$v_o(\infty) = A + B\,e^{-a(\infty)} = A$$

Consequently, the capacitor voltage is labeled as A.

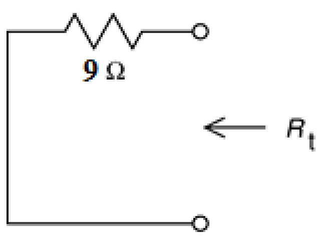
The steady-state circuit for t > 0.

Analysis of the circuit gives

Therefore $A = -7$ V and $B = 17$ V

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the capacitance C and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$$\frac{1}{a} = \tau = R_t\, C$$



Here is the circuit used to calculate R_t.

$$R_t = 9\ \Omega$$

Therefore

$$a = \frac{1}{(9)\left(66.7\times10^{-3}\right)} = 1.6\ \frac{1}{\mathrm{s}}$$

(The time constant is $\tau = (9)\left(66.7\times10^{-3}\right) = 0.6$ s .)

Putting it all together:

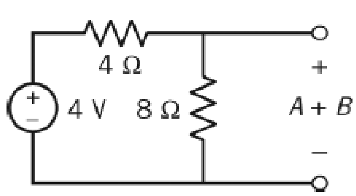
$$v_o(t) = \begin{cases} 10\ \text{V} & \text{for } t \le 0 \\ -7 + 17\, e^{-1.6\,t}\ \text{V} & \text{for } t \ge 0 \end{cases}$$

P 8.6-2
Solution:
The value of the input is one constant, 4 V, before time $t = 0$ and a different constant, 8 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\, e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = 0, will be

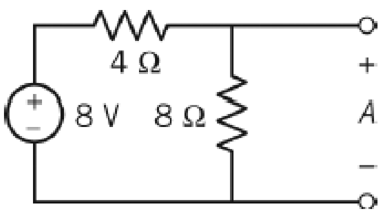
The steady-state circuit for t < 0.

equal to the steady state capacitor voltage before the input changes. At time t = 0 the output voltage is

$$v_o(0) = A + B\, e^{-a(0)} = A + B$$

Consequently, the capacitor voltage is labeled as $A + B$. Analysis of the circuit gives

$$A + B = \frac{8}{4+8}(4) = 2.67 \text{ V}$$



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The steady-state circuit for t > 0.

The value of the capacitor voltage at time t = ∞, will be equal to the steady state capacitor voltage after the input changes. At time $t = \infty$ the output voltage is

$$v_o(\infty) = A + B\, e^{-a(\infty)} = A$$

Consequently, the capacitor voltage is labeled as A.

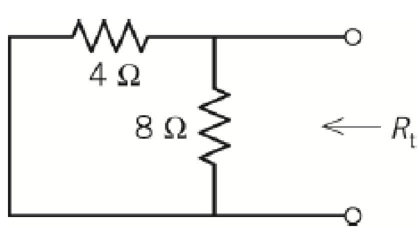
Analysis of the circuit gives

$$A = \frac{8}{4+8}(8) = 5.33 \text{ V}$$

Therefore $B = \;\; 2.67$ V

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the capacitance C and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$$\frac{1}{a} = \tau = R_t\, C$$



Here is the circuit used to calculate R_t.

$$R_t = \frac{(4)(8)}{4+8} = 2.67 \;\; \Omega$$

Therefore

$$a=\frac{1}{(2.67)(0.65)}=0.58\ \frac{1}{\text{s}}$$

(The time constant is $\tau=(2.67)(0.65)=1.74$ s .)

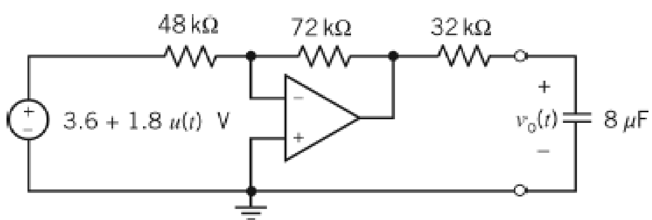
Putting it all together:

$$v_o(t)=\begin{cases} 2.67\ \text{V} & \text{for } t\le 0 \\ 5.33-2.67\,e^{-0.58t}\ \text{V} & \text{for } t\ge 0 \end{cases}$$

P 8.6-3

Solution:

The figure below shows the given op-amp circuit:



The input voltage is given as:

$$v_s(t)=3.6\ \text{V}+1.8u(t)\ \text{V}$$

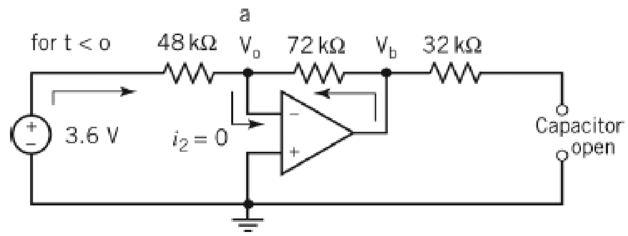
For $t<0$,

$$\begin{aligned} v_s(t)&=3.6\ \text{V}+1.8(0)\ \text{V} \\ &=3.6\ \text{V}\end{aligned}$$

For $t>0$,

$$\begin{aligned} v_s(t)&=3.6\ \text{V}+1.8(1)\ \text{V} \\ &=5.4\ \text{V}\end{aligned}$$

Also, at $t<0$,the capacitor is in steady state and acts as an open circuit as shown in figure below:



KCL at node ‘a’ gives:

$$\frac{v_a - 3.6\text{ V}}{48\text{ k}\Omega} + \frac{v_b - v_a}{72\text{ k}\Omega} = i_2$$

For an ideal op-amp, $i_2 = 0$, and the potential at the inverting terminal is equal to the potential at the non-inverting terminal. The non-inverting terminal being at zero potential highlights that $v_a = 0$. Equate the same in the equation above,

$$\frac{0\text{ V} - 3.6\text{ V}}{48\text{ k}\Omega} + \frac{v_b - 0\text{ V}}{72\text{ k}\Omega} = 0$$

$$\begin{aligned} v_b &= \left(\frac{3.6\text{ V}}{48\text{ k}\Omega}\right)(72\text{ k}\Omega) \\ &= 5.4\text{ V} \end{aligned}$$

Since no current flows through the $32\text{ k}\Omega$ resistor when the capacitor is in steady state, the voltage drop across the capacitor is equal to v_b. Therefore, the voltage drop across the capacitor at $t<0$ is 5.4 V.

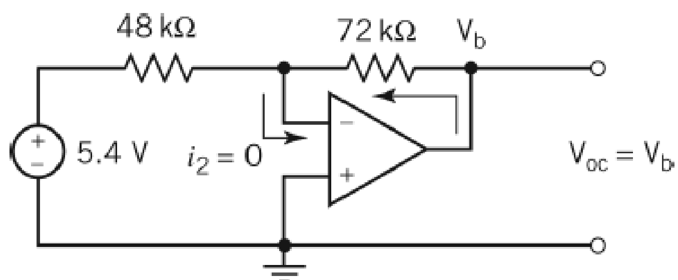
For $t > 0$, the source voltage is $5.4\ \text{V}$, and the Thevenin's voltage across the capacitor is:

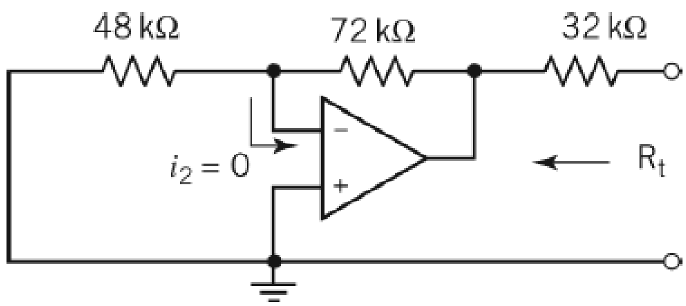
$$V_{oc} = \frac{(5.4\ \text{V})(72\ \text{k}\Omega)}{(48\ \text{k}\Omega)}$$
$$= 8.1\ \text{V}$$

Since $i_2 = 0$, the current through the $48\ \text{k}\Omega$ resistor is equal to the current through $72\ \text{k}\Omega$ resistor, and their combined resistance is $72\ \text{k}\Omega + 48\ \text{k}\Omega = 120\ \text{k}\Omega$. Also, the combined resistance of $120\ \text{k}\Omega$ is parallel to $32\ \text{k}\Omega$ resistor such that the Thevenin's resistance is:

$$R_t = \frac{(120\ \text{k}\Omega)(32\ \text{k}\Omega)}{120\ \text{k}\Omega + 32\ \text{k}\Omega}$$
$$= 25.2\ \text{k}\Omega$$

The Thevenin's equivalent circuit is shown below:





The time constant is given as:

$$\begin{aligned}\tau &= R_t C \\ &= (25.2\ \text{k}\Omega)\left(\frac{10^3\ \Omega}{1\ \text{k}\Omega}\right)(8\ \mu\text{F})\left(\frac{10^{-6}\ \text{F}}{1\ \mu\text{F}}\right) \\ &= 0.20\ \text{s}\end{aligned}$$

From the above Thevenin's equivalent, one can write the voltage across the capacitor as:

$$v_0(t) = V_{oc} + \left[V(0) - V_{oc}\right]e^{-t/R_t C}$$

Substitute the appropriate values,

$$\begin{aligned}v_0(t) &= 8.1\ \text{V} + [5.4\ \text{V} - 8.1\ \text{V}]e^{-t/0.20\ \text{s}} \\ &= 8.1\ \text{V} - 2.7\ \text{V}e^{-t/0.20\ \text{s}}\end{aligned}$$

Therefore, the voltage across the capacitor is $\boxed{v_0(t) = 8.1\ \text{V} - 2.7\ \text{V}e^{-t/0.20\ \text{s}}}$.

P 8.6-4

Solution

$$\tau = R\,C = (5\times10^5)(2\times10^{-6}) = 1\ \text{s}$$

Assume that the circuit is at steady state at t = 1 . Then

$$v(t) = 4 - 4\,e^{-(t-1)}\ \text{V} \quad \text{for } 1 \le t \le 2$$

so

$$v(2) = 4 - 4\,e^{-(2-1)} = 2.53\ \text{V}$$

and

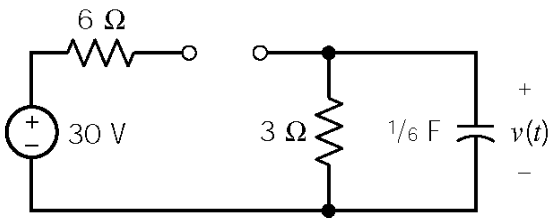
$$v(t) = 2.53\,e^{-(t-2)}\ \text{V} \quad \text{for } t \ge 2$$

Finally $$v(t)=\begin{cases}0 & t\le 1\\ 4-4e^{-(t-1)} & 1\le t\le 2\\ 2.53e^{-(t-2)} & t\ge 2\end{cases}$$

P 8.6-5

Solution:
The capacitor voltage is $v(0^-)$ = 10 V immediately before the switch opens at t = 0.

For 0 < t < 0.5 s the switch is open:

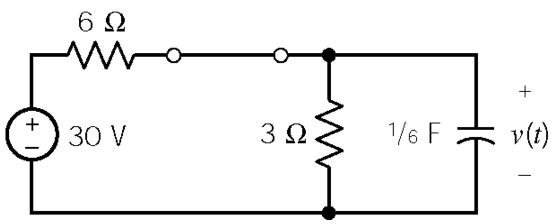


$$v(0)=10\ \text{V},\ v(\infty)=0\ \text{V},\ \tau=3\times\frac{1}{6}=\frac{1}{2}\ \text{s}$$

so $v(t)=10\,e^{-2t}$ V

In particular, $v(0.5)=10\,e^{-2(0.5)}=3.679$ V

For t > 0.5 s the switch is closed:



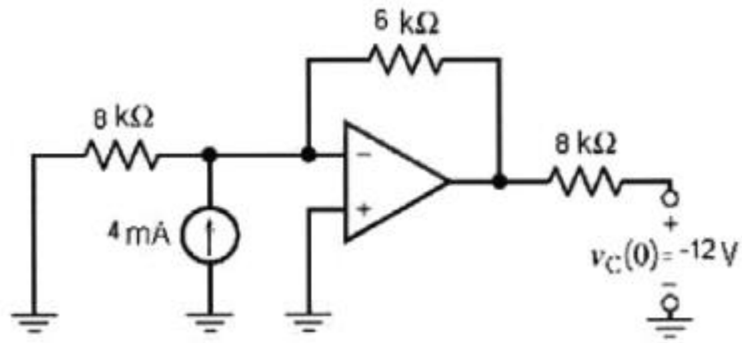
$$v(0)=3.679\ \text{V},\ v(\infty)=10\ \text{V},\ R_t=6\,||\,3=2\ \Omega,$$

$$\tau=2\times\frac{1}{6}=\frac{1}{3}\ \text{s}$$

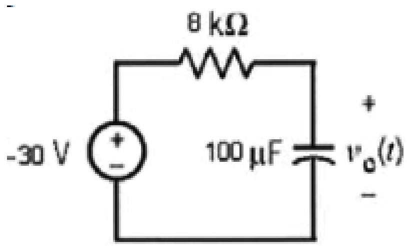
so

$$\begin{aligned}v(t)&=10+(3.679-10)e^{-3(t-0.5)}\ \text{V}\\&=10-6.321\,e^{-3(t-0.5)}\ \text{V}\end{aligned}$$

P 8.6-6
Solution:
For t < 0, the circuit is:



After t = 0, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:

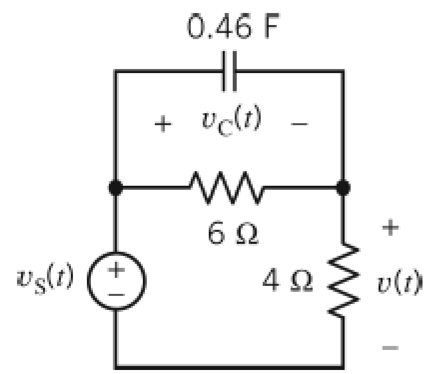


$$v_c(t) = -30 + \left(-12 - (-30)\right) e^{-t/(8000 \times 0.00001)}$$
$$= -30 + 18\, e^{-1.25t} \quad \text{V}$$

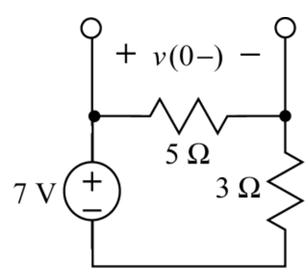
P 8.6-7

Solution:

The input changes abruptly at time $t = 0$. The voltage $v(t)$ may not be continuous at $t = 0$, but the capacitor voltage, $v_C(t)$ will be continuous. We will find $v_C(t)$ first and then use KVL to find $v(t)$.

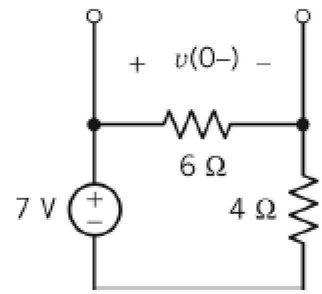


The circuit will be at steady state before $t = 0$ so the capacitor will act like an open circuit.



$$v(0+) = v(0-) = \frac{6}{6+4} 7 = 4.2 \text{ V}$$

After t = 0, we replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



$$v_{oc} = \frac{6}{10}(7-14) = \frac{6}{10}(-7) = -4.2 \text{ V}$$
$$R_t = 6 \| 4 = 2.4\ \Omega$$

The time constant is $\tau = R_t C = 1.1\ s$

$$\frac{1}{\tau} = 0.9\ \frac{1}{s}$$

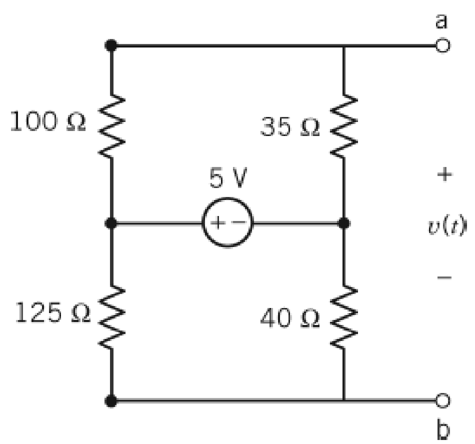
So $v_C(t) = [4.2 - (-4.2)] e^{-0.9t} + (-4.2) = -4.2 + 8.4 e^{-0.9t} \text{ V} \quad \text{for } t \geq 0$

Using KVL

$$v(t) = v_s(t) - v_C(t) = -7 - \left[-4.2 + 8.4 e^{-0.9t}\right] = -2.8 - 8.4 e^{-0.9t} \text{ V} \quad \text{for } t > 0$$

P 8.6-8

Solution:
For t < 0



Using voltage division twice

$$v(t) = \frac{35}{35+100} 5 - \frac{40}{125+40} 5 = 0.084 \text{ V}$$

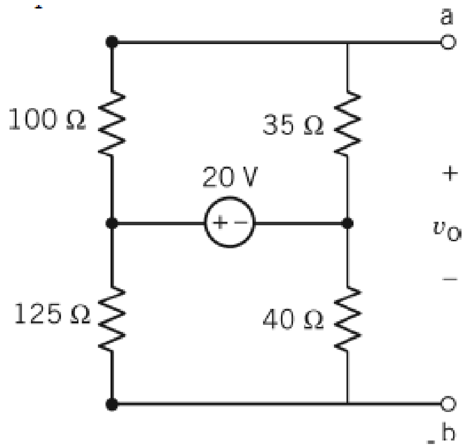
so

$$v(0-) = 0.084 \text{ V}$$

and

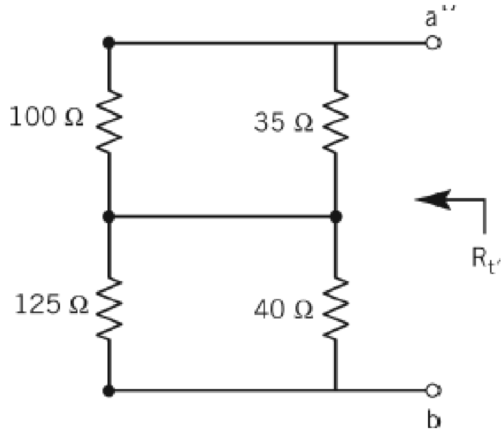
$$v(0+) = v(0-) = 0.084 \text{ V}$$

For t > 0, find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



Using voltage division twice

$$v_{oc} = \frac{35}{35+100}20 - \frac{40}{125+40}20 = 0.35 \text{ V}$$



$$R_t = (100 \,||\, 35) + (125 \,||\, 40) = 56.2 \ \Omega$$

then

$$\tau = 56.2 \times 0.0125 = 0.7 \text{ s}$$

so

$$\frac{1}{\tau} = 1.43 \ \frac{1}{\text{s}}$$

Now

$$v(t) = [0.084 - 0.35]e^{-1.43t} + 1 = 1 - 0.27e^{-1.43t} \ \text{ V} \quad \text{for } t \ge 0$$

P 8.6-9
Solution:
For $t > 0$, the circuit is at steady state so the capacitor acts like an open circuit. We have the following situation.

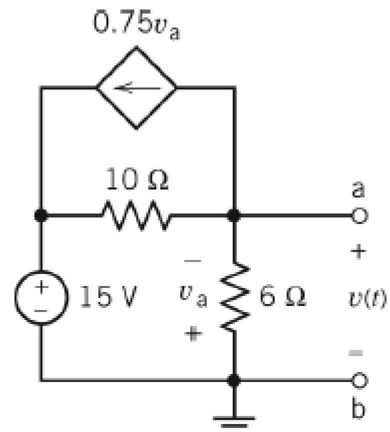
Notice that $v(t)$ is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = \quad v(t)$$

Apply KCL at node a:

$$-\left(\frac{15-v(t)}{10}\right)+\frac{v(t)}{6}+\left(-\frac{3}{4}v(t)\right)=0$$

$$-90+6v(t)+10\,v(t)-45\,v(t)=0 \quad \Rightarrow \quad v(t)=-3.1\text{ V}$$

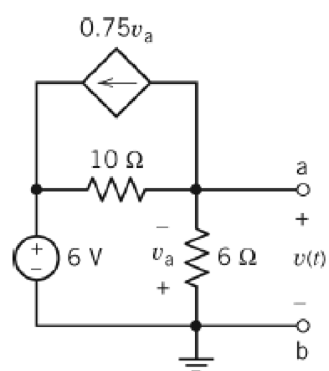


So $\qquad v(0+)=v(0-)=-3.1\text{ V}$

For $t > 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor, i.e. the part of the circuit to the left of terminals $a - b$.

Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = \quad v_{oc}$$



Apply KCL at node a:

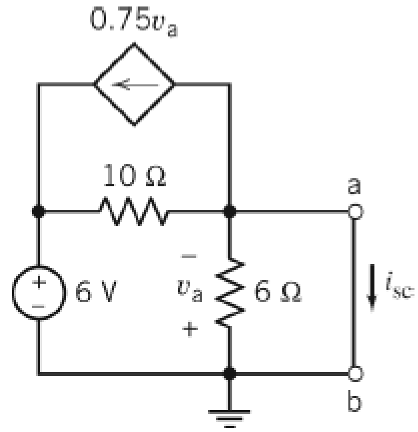
$$-\left(\frac{9-v_{oc}}{10}\right)+\frac{v_{oc}}{6}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-54+6v_{oc}+10\,v_{oc}-45\,v_{oc}=0 \quad \Rightarrow \quad v_{oc}=-1.86\text{ V}$$

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces

$v_a = 0$



Apply KCL at node a:

$$-\left(\frac{9-0}{10}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{9}{10}=0.9\text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-1.86}{0.9}=-\frac{31}{15}\,\Omega$$

Then

$$\tau = R_t C = \left(-\frac{31}{15}\right)\left(\frac{3}{40}\right) = -0.15\text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = -6.67\ \frac{1}{\text{s}}$$

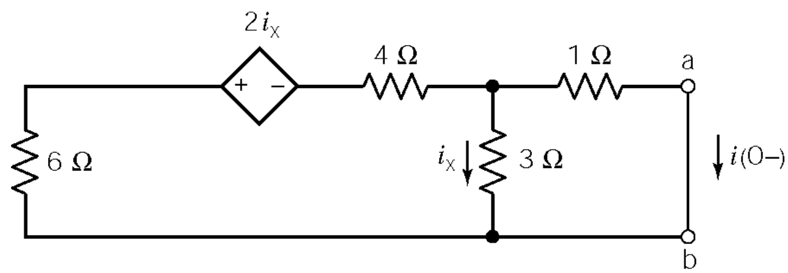
and

$$v(t)=\left(v(0+)-v_{\text{oc}}\right)e^{-t/\tau}+v_{\text{oc}}=\left(-3.1-(-1.86)\right)e^{6.67t}+(-1.86)=-1.24e^{6.67t}-1.86\text{ V}\quad\text{for } t\ge 0$$

Notice that $v(t)$ grows exponentially as t increases.

P 8.6-10

Solution: When $t<0$ and the circuit is at steady state, the inductor acts like a short circuit.



The mesh equations are

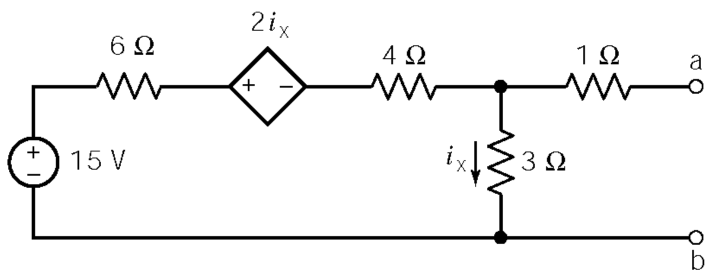
$$2i_{\text{x}}+4\left(i_{\text{x}}+i(0-)\right)+3i_{\text{x}}+6\left(i_{\text{x}}+i(0-)\right)=0$$

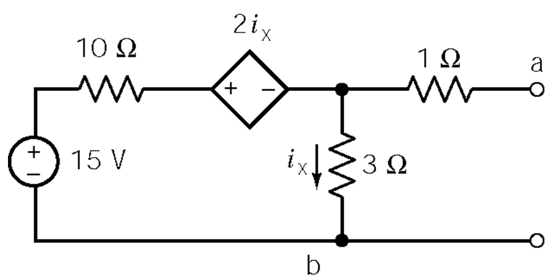
$$1i(0-)-3i_{\text{x}}=0$$

so

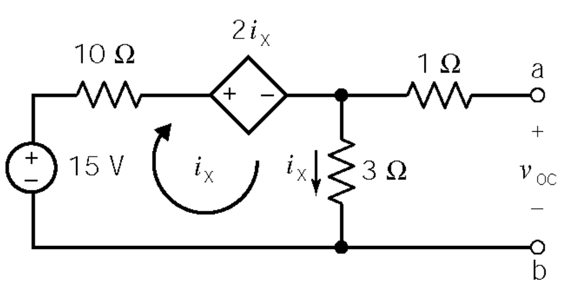
$$i(0+)=i(0-)=0$$

For $t \geq 0$, we find the Norton equivalent circuit for the part of the circuit connected to the inductor. First, simplify the circuit using a source transformation:





Identify the open circuit voltage and short circuit current.

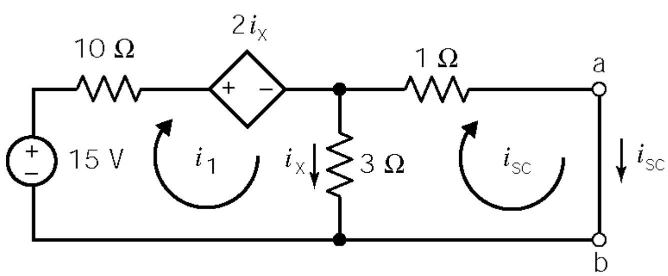


Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\, i_x = 3 \text{ V}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1+2\left(i_1-i_{sc}\right)+3\left(i_1-i_{sc}\right)-15=0 \quad\Rightarrow\quad 15\,i_1-5\,i_{sc}=15$$

and

$$i_{sc}-3\left(i_1-i_{sc}\right)=0 \quad\Rightarrow\quad i_1=\frac{4}{3}\,i_{sc}$$

so

$$15\left(\frac{4}{3}i_{sc}\right)-5\,i_{sc}=15 \quad\Rightarrow\quad i_{sc}=1\text{ A}$$

The Thevenin resistance is

$$R_t=\frac{3}{1}=3\ \Omega$$

The time constant is given by $\tau=\frac{L}{R_t}=\frac{5}{3}=1.67\text{ s}$ so $\frac{1}{\tau}=0.6\ \frac{1}{\text{s}}$.

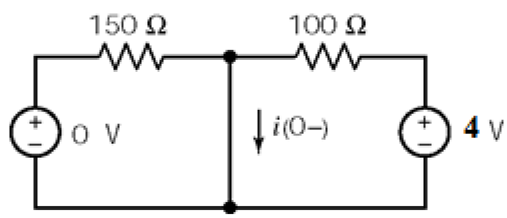
The inductor current is given by

$$i_L(t)=\left(i(0+)-i_{sc}\right)e^{-t/\tau}+i_{sc}=(0-1)e^{-0.6t}+1=1-e^{-0.6t}\text{ A} \quad \text{for } t\ge 0$$

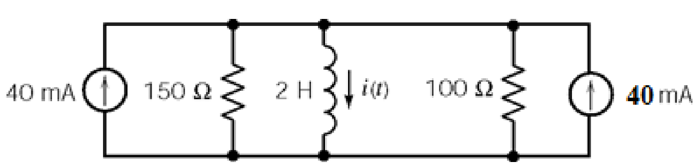
P 8.6-11
Solution:

When $t<0$ and the circuit is at steady state, the inductor acts like a short circuit. The initial condition is

$$i(0+)=i(0-)=\frac{0}{150}+\frac{4}{100}=0.04\text{ A}$$



For $t\ge 0$, we find the Norton equivalent circuit for the part of the circuit connected to the inductor. First, simplify the circuit using source transformations:



$$i_{sc}=40+40=80\text{ mA}$$

$$R_t=100\,||\,150=\frac{100\times 150}{100+150}=60\ \Omega$$

The time constant is given by $\tau = \frac{L}{R_t} = \frac{4}{60} = 0.0666 \text{ s}$ so $\frac{1}{\tau} = 15 \ \frac{1}{\text{s}}$.

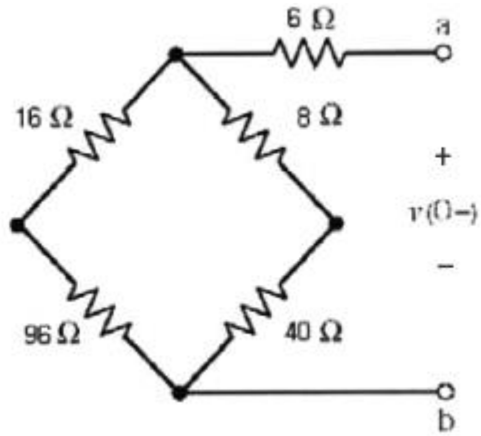
The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (40-80)e^{-15t} + 80 = 80 - 40\,e^{-15t} \text{ mA} \quad \text{for } t \ge 0$$

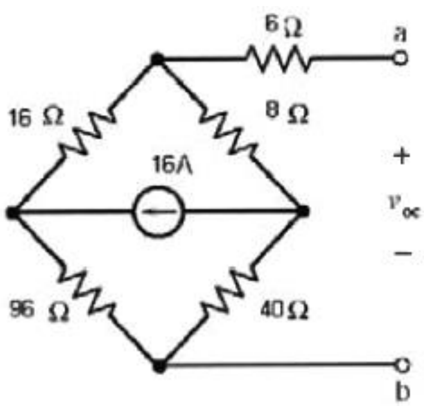
P 8.6-12
Solution:

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. The 0 A current source also acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = 0 \text{ V}$$

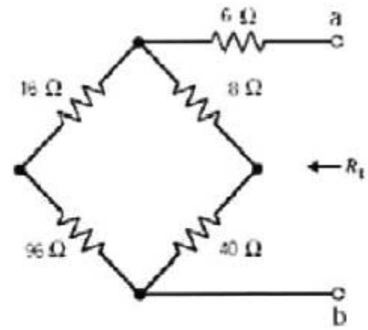


For $t \ge 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



$$\begin{aligned} v_{oc} &= \left[\frac{136}{136+24}(16)\right]8 - \left[\frac{24}{136+24}(16)\right]40 \\ &= \frac{64}{5} = 12.8 \text{ V} \end{aligned}$$

$$R_t = 6 + \frac{(16+96)(8+40)}{(16+96)+(8+40)} = 39.6\ \Omega$$

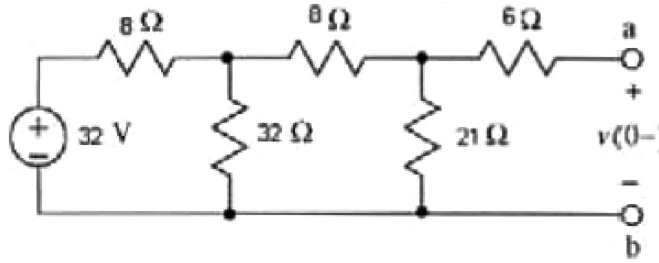


The time constant is $\tau = R_t C = (39.6)(0.8\times10^{-3}) = 0.03\ \text{s}$ s so $\frac{1}{\tau} = 33.3\ \frac{1}{\text{s}}$.

The capacitor voltage is given by

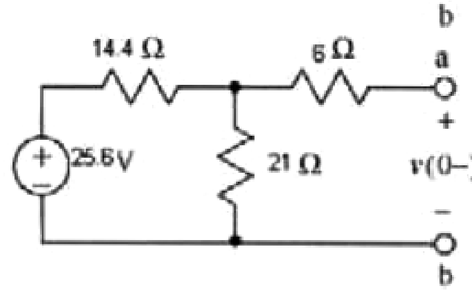
$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (0-12.8)e^{-33.3t} + 12.8 = 12.8\left(1-e^{-33.3t}\right)\ \text{V} \quad \text{for } t \ge 0$$

P 8.6-13
Solution:
When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. The 0 A current source also acts like an open circuit.

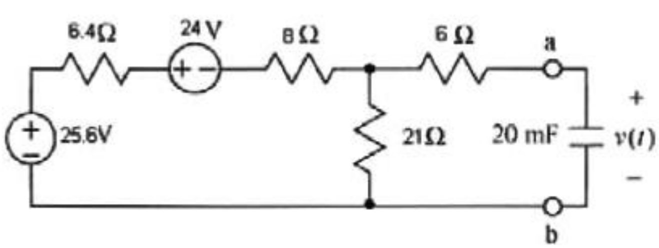


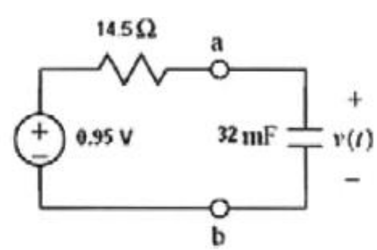
After a couple of source transformations, the initial condition is calculated as

$$v(0+) = v(0-) = \frac{21}{14.4+21}\,25.6 = 15.186\ \text{V}$$



For $t \ge 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. Using source transformations reduce the circuit as follows.





Now recognize $R_t = 14.5\ \Omega$ and $v_{oc} = 0.95\ \text{V}$.

The time constant is $\tau = R_t C = (14.5)(32\times10^{-3}) = 0.5\ \text{s}$ s so $\frac{1}{\tau} = 2\ \frac{1}{\text{s}}$.

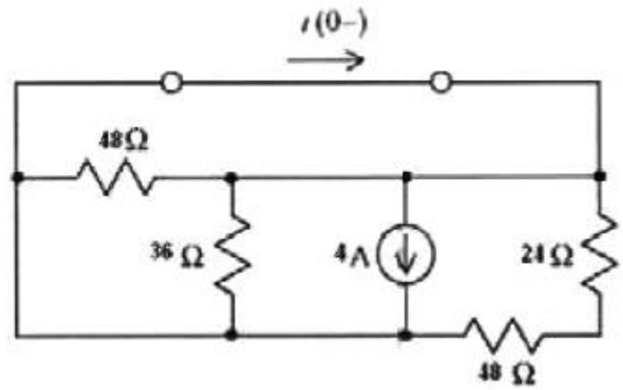
The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_{\text{oc}}\right)e^{-t/\tau} + v_{\text{oc}} = (15.186 - 0.95)e^{-2t} + 0.95 = 0.95 + 14.24\, e^{-2t} \ \text{ V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/15/04)

P 8.6-14
Solution:

When $t < 0$ and the circuit is at steady state, the inductor acts like a short circuit. The 0 V voltage source also acts like a short circuit.



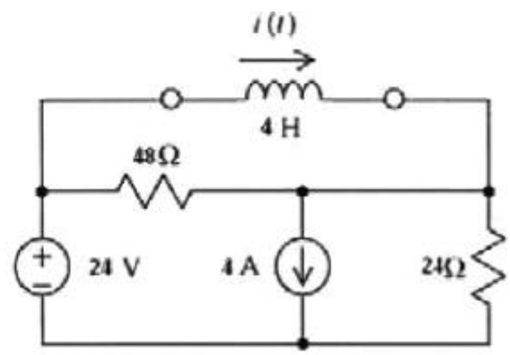
After a replacing series and parallel resistors by equivalent resistors, the equivalent resistors, current source and short circuit are all connected in parallel. Consequently

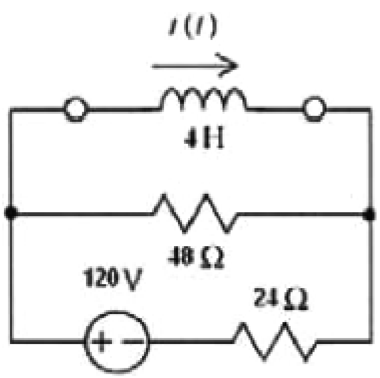
$$i(0+) = i(0-) = 4 \text{ A}$$

For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.

Replace series and parallel resistors by an equivalent resistor.

$$36 \parallel (24 + 48) = 24\ \Omega$$

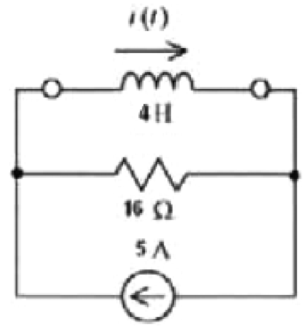




Do a source transformation, then replace series voltage sources by an equivalent voltage source.

Do two more source transformations.

Now recognize $R_t = 16\ \Omega$ and $i_{sc} = 5\ \text{A}$.



The time constant is given by

$$\tau = \frac{L}{R_t} = \frac{4}{16} = 0.25\ \text{s} \ \text{ so } \ \frac{1}{\tau} = 4\ \frac{1}{\text{s}}.$$

The inductor current is given by

$$i_L(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (4-5)e^{-4t} + 5 = 5 - e^{-4t}\ \text{A} \quad \text{for } t \ge 0$$

(checked: LNAP 7/15/04)

P 8.6-15

Solution: This is a first order circuit containing a capacitor. First, determine $v_C(t)$.

Consider the circuit for time *t* < 0.

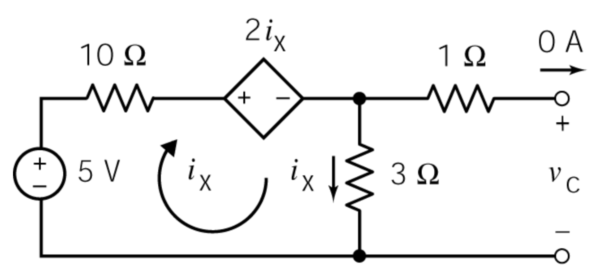
Step 1: Determine the initial capacitor voltage.

The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 5 V, a constant.

The capacitor will act like an open circuit.

$t < 0$, at steady state:



Apply KVL to the mesh to get:

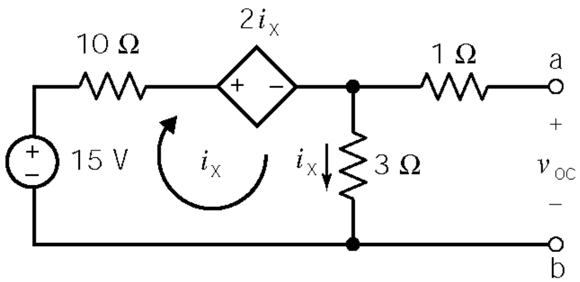
$$(10+2+3)i_x - 5 = 0 \quad \Rightarrow \quad i_x = \frac{1}{3}\ \text{A}$$

Then

$$v_C(0) = 3\,i_x = 1\ \text{V}$$

Consider the circuit for time *t* > 0.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. First, determine the open circuit voltage, v_{oc}:



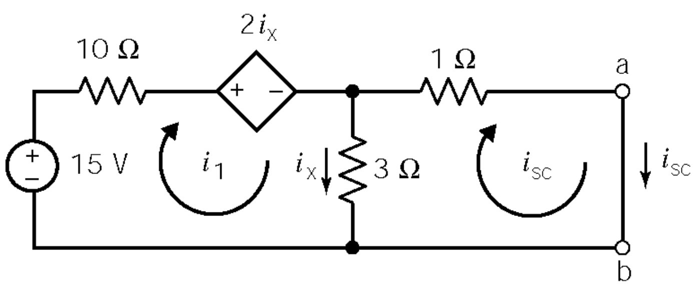
Apply KVL to the mesh to get:

$$(10+2+3)\,i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\,i_x = 3 \text{ V}$$

Next, determine the short circuit current, i_{sc}:



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\,i_1 - 5\,i_{sc} = 15$$

And
$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\,i_{sc}$$

so
$$15\left(\frac{4}{3} i_{sc}\right) - 5\,i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1 \text{ A}$$

The Thevenin resistance is

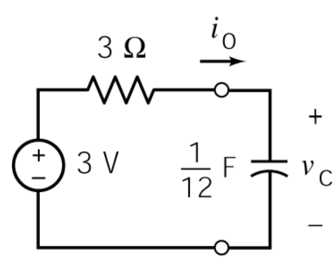
$$R_t = \frac{3}{1} = 3\ \Omega$$

Step 3. The time constant of a first order circuit containing an capacitor is given by

$$\tau = R_t\,C$$

Consequently

$$\tau = R_t\,C = 3\left(\frac{1}{12}\right) = 0.25 \text{ s and } a = \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$



Step 4. The capacitor voltage is given by:

$$v_C(t) = v_{oc} + \left(v_C(0) - v_{oc}\right)e^{-at} = 3 + (1-3)e^{-4t} = 3 - 2e^{-4t} \quad \text{for } t \geq 0$$

Step 5. Express the output current as a function of the source voltage and the capacitor voltage.

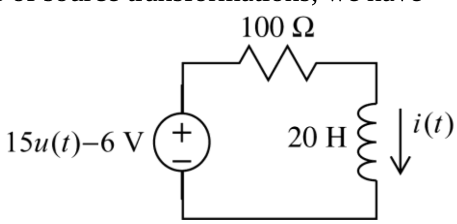
$$i_o(t) = C\frac{d}{dt}v_C(t) = \frac{1}{12}\frac{d}{dt}v_C(t)$$

Step 6. The output current is given by

$$i_o(t) = \frac{1}{12}\frac{d}{dt}\left(3 - 2e^{-4t}\right) = \frac{1}{12}(-2)(-4)e^{-4t} = \frac{2}{3}e^{-4t} \quad \text{for } t \geq 0$$

P 8.6-16

Solution: Simplify the circuit by replacing the series inductors by an equivalent inductor. Then, after a couple of source transformations, we have



For $t < 0$ the circuit is at steady state and so the inductor acts like a short circuit. The voltage source voltage is 6 V so

$$i(0+) = i(0-) = -60 \text{ mA}$$

For $t > 0$ we find the Norton equivalent circuit for the part of the circuit connected to the inductor. In this case we recognize v_{oc} = 9V and R_t = 100   so i_{sc} = 90 mA.

The time constant is $\tau = \frac{L}{R_t} = \frac{20}{100} = 0.2 \text{ s} \quad \Rightarrow \quad \frac{1}{\tau} = 5 \ \frac{1}{\text{s}}$

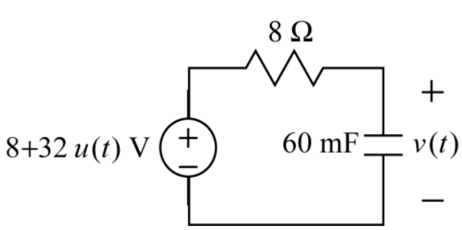
Then

$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (-60-90)e^{-5t} + 90 = 90 - 150e^{-5t} \text{ mA} \quad \text{for } t \geq 0$$

(checked: LNAP 7/12/04)

P 8.6-17

Solution: Simplify the circuit by replacing the series capacitors by an equivalent capacitor. Then, after doing some source transformations, we have



For $t < 0$ the circuit is at steady state so the capacitor acts like an open circuit. The voltage source voltage is 8 V so

$$v(0+) = v(0-) = 8 \text{ V}$$

For $t > 0$ we find the Thevenin equivalent circuit of the part of the circuit connected to the capacitor. In this case we recognize v_{oc} = 40 V and R_t = 8 .

The time constant is

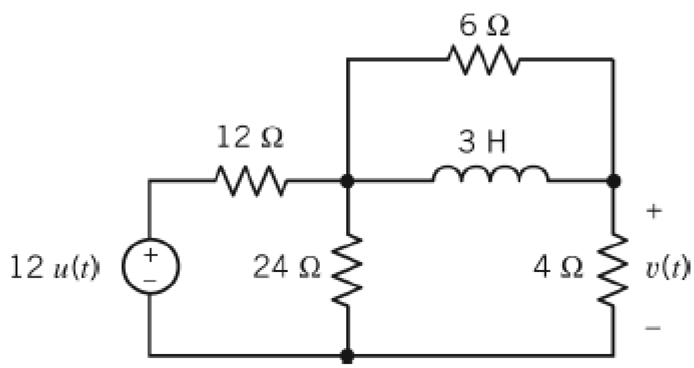
$$\tau = R_t C = (8)\left(60\times10^{-3}\right) = 0.48 \quad \Rightarrow \quad \frac{1}{\tau} = 2.08 \ \frac{1}{\text{s}}$$

Then

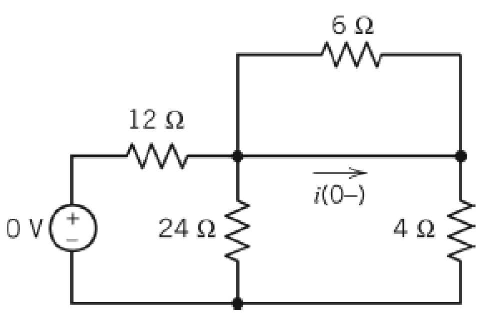
$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (8-40)e^{-2.08t} + 40 = 40 - 32e^{-2.08t} \ \text{ V} \quad \text{for } t \ge 0$$

(checked: LNAP 7/13/04)

P 8.6-18



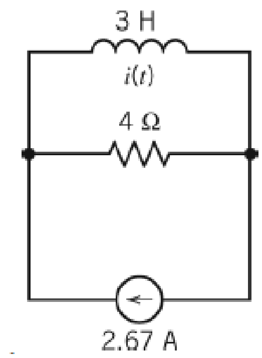
Solution: The resistor voltage, $v(t)$, may not be continuous at time t = 0. The inductor will be continuous. We will find the inductor current first and then find $v(t)$. Label the inductor current as $i(t)$.



For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = 0 \text{ A}$$

For $t > 0$ use source transformations to simplify the part of the circuit connected to the inductor until is a Norton equivalent circuit.



Recognize that

$$R_t = 4\ \Omega \quad \text{and} \quad i_{sc} = 2.67\ \text{A}$$

The time constant is $$\tau = \frac{L}{R_t} = \frac{3}{4} \quad \Rightarrow \quad \frac{1}{\tau} = 1.333\ \frac{1}{\text{s}}$$

Then $$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = 2.67\left(1 - e^{-1.333t}\right)\ \text{A} \quad \text{for}\ \ t \geq 0$$

Returning to the original circuit we see that

$$\frac{v(t)}{2} = i(t) + \frac{3\frac{d}{dt}i(t)}{3} = i(t) + \frac{d}{dt}i(t)$$
$$= 2.67\left(1 - e^{-1.333t}\right) + (-1.333)(2.67)\left(-e^{-1.333t}\right) = 2.67 - 0.889e^{-1.333t}$$

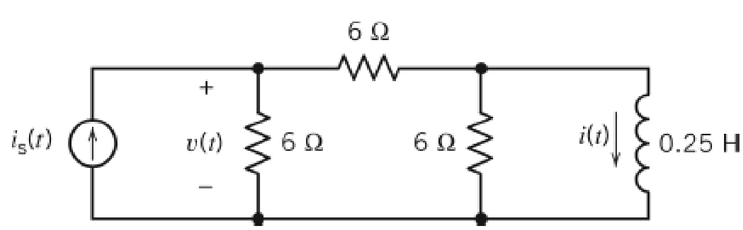
Finally,

$$v(t) = 5.34 - 1.778e^{1.333t}\ \ \text{V} \quad \text{for}\ \ t > 0$$

P 8.6-19

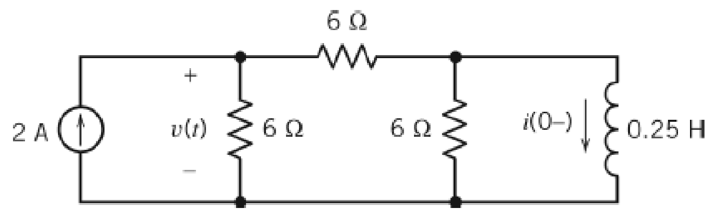
Solution:

Label the inductor current, $i(t)$. We will find $i(t)$ first, then find $v(t)$.



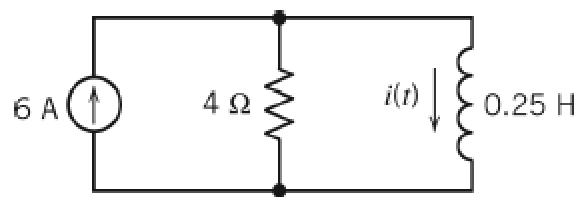
For $t < 0$ the circuit is at steady state and the inductor acts like a short circuit. The initial condition is

$$i(0+) = i(0-) = \left(\frac{6}{6+6}\right)4 = 2\ \text{A}$$



For $t > 0$ use source transformations to simplify the part of the circuit connected to the inductor until it is a Norton equivalent circuit.

Recognize that $R_{\text{t}} = 4\ \Omega$ and $i_{\text{sc}} = 6\ \text{A}$



The time constant is $\tau = \dfrac{L}{R_{\text{t}}} = \dfrac{0.25}{4} = 0.0625\ \text{s} \quad \Rightarrow \quad \dfrac{1}{\tau} = 16\ \dfrac{1}{\text{s}}$

Then $i(t) = \left(i(0+) - i_{\text{sc}}\right)e^{-t/\tau} + i_{\text{sc}} = (2-6)e^{-16t} + 6 = 6 - 4e^{-16t}\ \text{A} \quad \text{for } t \geq 0$

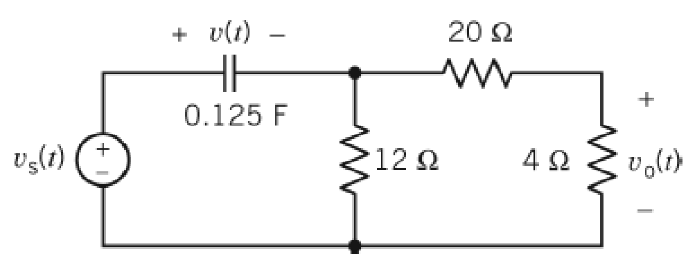
Returning to the original circuit

$$v(t) = 6\left(i(t) + \frac{0.25\frac{d}{dt}i(t)}{6}\right) + 0.25\frac{d}{dt}i(t) = 6i(t) + 0.5\frac{d}{dt}i(t) = 6\left(6 - 4e^{-16t}\right) + 0.5\left(64e^{-16t}\right)$$

$$= 36 + 8\,e^{-16t}\ \text{V} \quad \text{for } t > 0$$

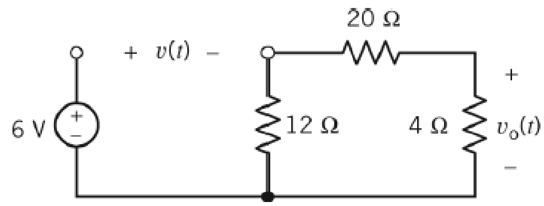
P 8.6-20

Solution:



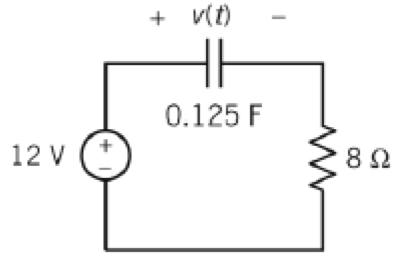
Label the capacitor voltage, $v(t)$. W will find $v(t)$ first then find $v_{\text{o}}(t)$.

For $t < 0$ the circuit is at steady state and the capacitor acts like an open circuit. The initial condition is



$$v(0+) = v(0-) = 6 \text{ V}$$

For $t > 0$ we replace series and then parallel resistors by equivalent resistors in order to replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



We recognize $R_\text{t} = 8\ \Omega$ and $v_\text{oc} = 12$ V

The time constant is $\tau = R_\text{t}C = 8(0.125) = 1\text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 1\ \frac{1}{\text{s}}$

The capacitor voltage is given by

$$v(t) = \left(v(0+) - v_\text{oc}\right)e^{\frac{-t}{\tau}} + v_\text{oc} = (6-12)e^{-t} + 12 = 12 - 6e^{-t} \text{ V} \quad \text{for } t \geq 0$$

Returning to the original circuit and applying KCL we see

$$0.125\frac{d}{dt}v(t) = \frac{12 - v(t)}{12} + \frac{v_\text{o}(t)}{4}$$

so $$v_\text{o}(t) = 0.5\frac{d}{dt}v(t) - 4 + \frac{v(t)}{3} = 0.5\left(6e^{-t}\right) - 4 + 4 - 2e^{-2t} = e^{-t} \text{ V} \quad \text{for } t > 0$$

P 8.6-21

Solution: Apply KCL at the inverting input of the op amp to get

$$\frac{v_\text{o}(t) - v(t)}{R_2} = \frac{v(t)}{1000} \quad \Rightarrow \quad v_\text{o}(t) = \left(1 + \frac{R_2}{1000}\right)v(t)$$

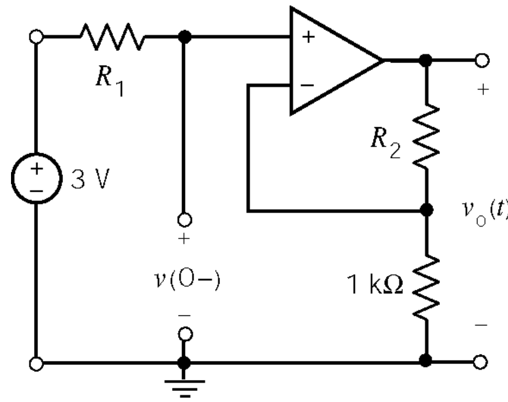
We will determine the capacitor voltage first and then use it to determine the output voltage.

When $t < 0$ and the circuit is at steady state, the capacitor acts like an open circuit. Apply KCL at the noninverting input of the op amp to get

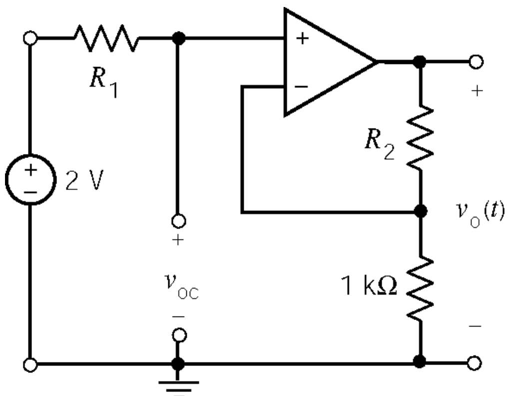
$$\frac{3 - v(0-)}{R_1} = 0 \quad \Rightarrow \quad v(0-) = 3 \text{ V}$$

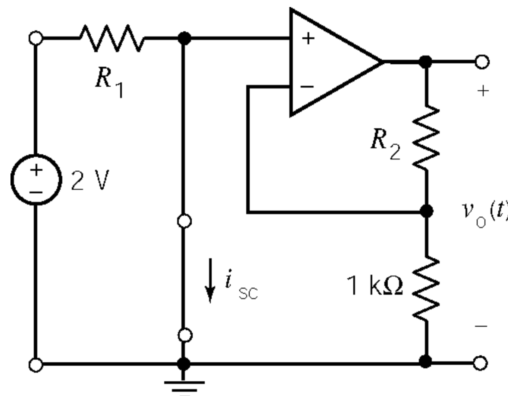
The initial condition is

$$v(0+) = v(0-) = 3 \text{ V}$$



For $t \geq 0$, we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.





$$\frac{2 - v_{\text{oc}}}{R_1} = 0 \quad \Rightarrow \quad v_{\text{oc}} = 2 \text{ V}$$

$$\frac{2}{R_1} = i_{\text{sc}} \quad \Rightarrow \quad R_{\text{t}} = \frac{v_{\text{oc}}}{i_{\text{sc}}} = R_1$$

The time constant is $\tau = R_{\text{t}} C = R_{\text{t}} \left(10^{-6}\right)$. From the given equation for $v_{\text{o}}(t)$, $\dfrac{1}{\tau} = 50 \ \dfrac{1}{\text{s}}$, so

$$R_{\text{t}} \left(10^{-6}\right) = \frac{1}{50} \quad \Rightarrow \quad R_1 = R_{\text{t}} = \frac{10^6}{50} = 20 \text{ k}\Omega$$

The capacitor voltage is given by

$$v(t) = \left(v(0) - v_{\text{oc}}\right) e^{-t/\tau} + v_{\text{oc}} = (3-2) e^{-50t} + 2 = 2 + e^{-50t} \ \text{ V} \quad \text{for } \ t \geq 0$$

So

$$v_{\text{o}}(t) = 5\, v(t) \quad \Rightarrow \quad 5 = 1 + \frac{R_2}{1000} \quad \Rightarrow \quad R_2 = 4 \text{ k}\Omega$$

(checked LNAPTR 7/31/04)

P 8.6-22
Solution:

The capacitor voltage can be represented by the equation $v(t) = A + B e^{-4t}$ for t ≥ 0. Given that

$A + B = v(0) =$ 2.5 V and $A = v(\infty) =$ V we determine $v(t) = 4.2 - 6.7\,e^{-4t}$.

Let t_1 be the time at which $v(t_1) =$ 2.0 V. Then $t_1 = \dfrac{\ln\left(\dfrac{-2-4.2}{-6.7}\right)}{-4} = 0.01939\text{ s}$.

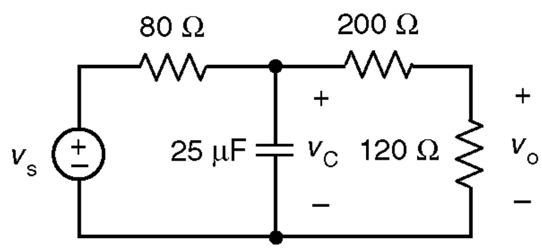
Let t_2 be the time at which $v(t_2) = 2.0$ V. Then $t_2 = \dfrac{\ln\left(\dfrac{2-4.2}{-6.7}\right)}{-4} = 0.27841\text{ s}$.

The transition requires 0.27841   0.01939 = 0.25902 s.

P 8.6-23

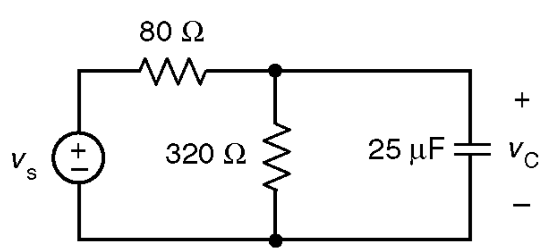
Solution:

Label the capacitor voltage:



Using voltage division $\qquad v_o = \dfrac{120}{120+200} v_c = 0.625 v_c$

Replace the series resistors by an equivalent resistance to redraw the circuit as



Before time t = 10 ms the steady state capacitor voltage is $v_c = \dfrac{320}{80+320} v_s = 0.8(12) = 9.6\text{ V}$.

After time t = 10 ms the steady state capacitor voltage is $v_c = \dfrac{320}{80+320} v_s = 0.8(36) = 28.8 \text{ V}$.

The Thevenin resistance of the part of the circuit connected to the capacitor is

$$R_t = 80 \,||\, 320 = 64 \ \Omega .$$

The time constant of the circuit is

$$\tau = R_t\, C = 64\left(25\times10^{-6}\right) = 1.6 \text{ ms}$$

Now we can calculate

$$V_1 = 0.625(9.6) = 6 \text{ V}, \quad V_2 = 0.625(28.8) = 18 \text{ V} \quad \text{and} \quad T_s = 4\tau = 4(1.6) = 6.4 \text{ ms}$$

P 8.6-24

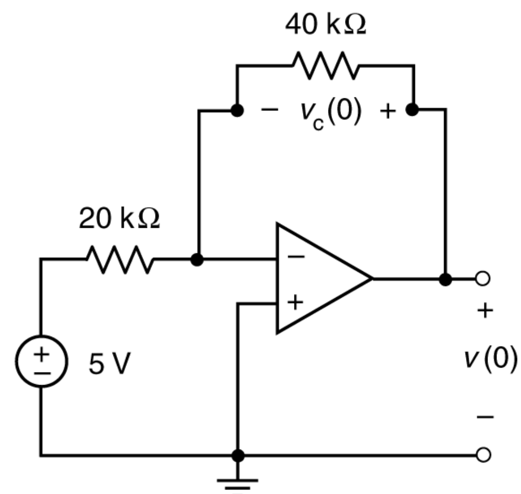
Solution:

(a) Determine the initial voltage across the capacitor, i.e. the capacitor voltage at time t = 0.

$$v_C(0) = v(0) = -10 \text{ V}$$

Similarly, at steady state after t = 0

$$v_C(\infty) = v(\infty) = -6 \text{ V}$$

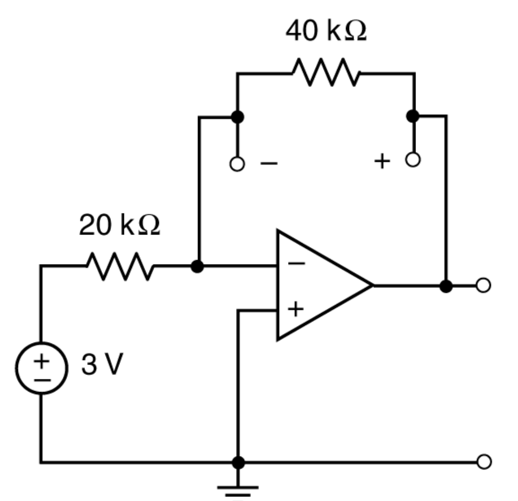


(b) Determine the Thevenin equivalent of the part of the circuit connected to the capacitor for t > 0:

$$v_{oc} = 3 \text{ V},\ i_{sc} = -\frac{3}{20\times10^3} \quad \Rightarrow \quad R_t = 20 \text{ k}\Omega$$

Determine the time constant of the circuit

$$\tau = \left(40\times10^3\right)\left(2.5\times10^{-6}\right) = 10^{-1}$$



(c) For $t \geq 0$, the output voltage is

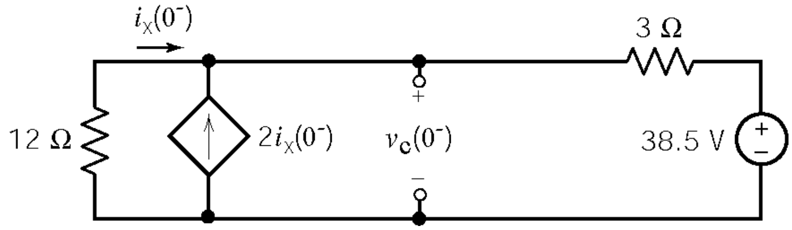
$$v(t) = -6 - 4e^{-10t} \text{ V}$$

(LNAPTR 6/24/13)

Section 8.7 The Response of a First-Order Circuit to a Nonconstant Source

P 8.7-1

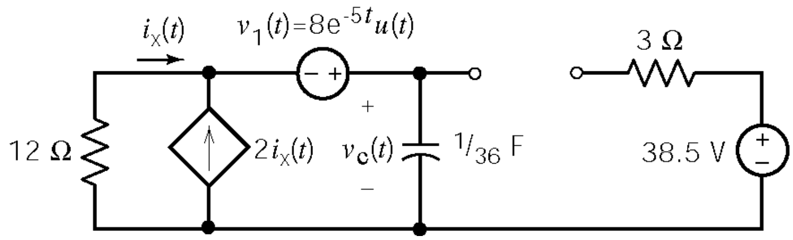
Solution: Assume that the circuit is at steady state before t = 0:



$$\text{KVL}: \; 12i_x + 3(3\,i_x) + 38.5 = 0 \quad \Rightarrow \quad i_x = -1.8\overline{3}\text{ A}$$

$$\text{Then } \underline{v_c(0^-) = -12\,i_x = 22\text{ V} = v_c(0^+)}$$

After t = 0:



$$\text{KVL}: \; 12i_x(t) - 8e^{-5t} + v_c(t) = 0$$

$$\text{KCL}: -i_x(t) - 2i_x(t) + (1/36)\frac{dv_c(t)}{dt} = 0 \quad \Rightarrow \quad i_x(t) = \frac{1}{108}\frac{dv_c(t)}{dt}$$

$$\therefore \quad 12\left[\frac{1}{108}\frac{dv_c(t)}{dt}\right] - 8e^{-5t} + v_c(t) = 0$$

$$\frac{dv_c(t)}{dt} + 9v_c(t) = 72e^{-5t} \quad \Rightarrow v_{cn}(t) = Ae^{-9t}$$

Try $v_{cf}(t) = Be^{-5t}$ & substitute into the differential equation $\Rightarrow B = 18$

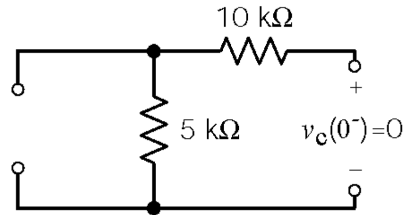
$$\therefore\ v_c(t) = Ae^{-9t} + 18\,e^{-5t}$$

$$v_c(0) = 22 = A + 18 \ \Rightarrow\ A = 4$$

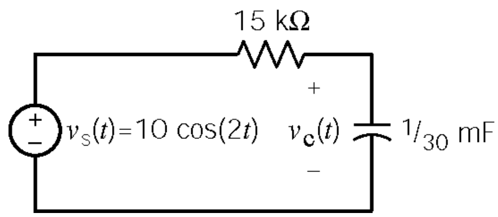
$$\therefore v_c(t) = 4e^{-9t} + 18e^{-5t}\ \text{V}$$

P 8.7-2

Solution: Assume that the circuit is at steady state before t = 0:



Replace the circuit connected to the capacitor by its Thevenin equivalent (after t=0) to get:



$$\text{KVL: } -10\cos 2t + 15\left(\tfrac{1}{30}\frac{dv_c(t)}{dt}\right) + v_c(t) = 0 \quad \Rightarrow \quad \frac{dv_c(t)}{dt} + 2v_c(t) = 20\cos 2t$$

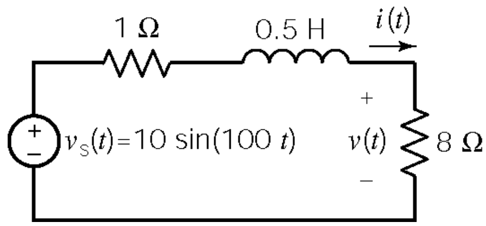
$v_n(t) = Ae^{-2t}$, Try $v_f(t) = B\cos 2t + C\sin 2t$ & substitute into the differential equation to get

$B = C = 5 \quad \Rightarrow \quad v_f(t) = 5\cos 2t + 5\sin 2t. \quad \therefore\ v_c(t) = v_n(t) + v_f(t) = Ae^{-2t} + 5\cos 2t + 5\sin 2t$

Now $v_c(0) = 0 = A + 5 \Rightarrow A = -5 \quad \Rightarrow \quad v_c(t) = -5e^{-2t} + 5\cos 2t + 5\sin 2t$ V

P 8.7-3

Solution: Assume that the circuit is at steady state before t = 0. There are no sources in the circuit so i(0) = 0 A. After t = 0, we have:



$$\text{KVL}: -10\sin 100t + i(t) + 5\frac{di(t)}{dt} + v(t) = 0$$

$$\text{Ohm's law}: i(t) = \frac{v(t)}{8}$$

$$\therefore \quad \underline{\frac{dv(t)}{dt} + 18\,v(t) = 160\sin 100t}$$

$\therefore\ v_n(t) = Ae^{-18t}$, try $v_f(t) = B\cos 100t + C\sin 100t$, substitute into the differential equation and equate like terms $\Rightarrow\ B = -1.55\ \&\ C = 0.279 \Rightarrow\ v_f(t) = -1.55\cos 100t + 0.279\sin 100t$

$\therefore\ v(t) = v_n(t) + v_f(t) = Ae^{-18t} - 1.55\cos 100t + 0.279\sin 100t$

$v(0) = 8\,i(0) = 0 \quad \Rightarrow \quad v(0) = 0 = A - 1.55 \Rightarrow A = 1.55$

so $\underline{v(t) = 1.55e^{-18t} - 1.55\cos 100t + 0.279\ \sin 100t\ \text{ V}}$

P 8.7-4

Solution: Assume that the circuit is at steady state before $t = 0$.

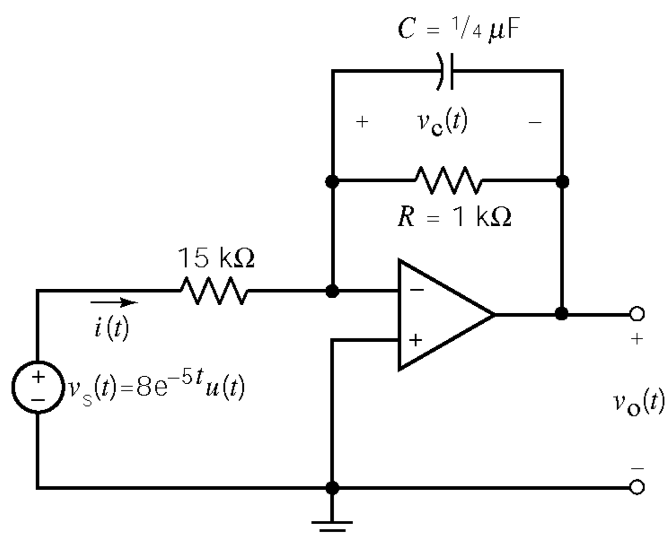
$$v_o(t) = -v_c(t)$$

$$v_C(0^+) = v_C(0^-) = -10\text{ V}$$

After $t = 0$, we have

$$i(t) = \frac{v_s(t)}{15000} = \frac{8\,e^{-5t}}{15000} = 0.533\,e^{-5t}\ \text{ mA}$$

The circuit is represented by the differential equation: $i(t) = C\dfrac{dv_C(t)}{dt} + \dfrac{v_C(t)}{R}$. Then



$$\left(0.533\times10^{-3}\right)e^{-5t} = \left(0.25\times10^{-6}\right)\frac{dv_c(t)}{dt} + \left(10^{-3}\right)v_c(t) \quad \Rightarrow \quad \frac{dv_c(t)}{dt} + 4000\,v_c(t) = 4000\,e^{-5t}$$

Then $v_n(t) = Ae^{-4000t}$. Try $v_f(t) = Be^{-5t}$. Substitute into the differential equation to get

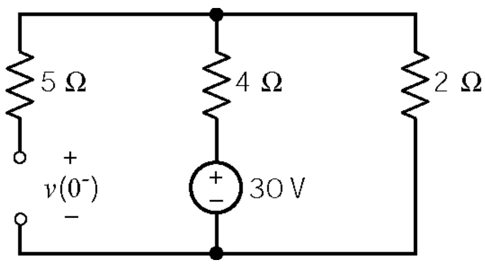
$$\frac{d\left(Be^{-5t}\right)}{dt} + 4000\left(Be^{-5t}\right) = 4000\,e^{-5t} \quad \Rightarrow \quad B = \frac{4000}{-3995} = -1.00125 \cong -1$$

$$v_C(t) = v_f(t) + v_n(t) = e^{-5t} + Ae^{-4000t}$$

$$v_C(0) = -10 = 1 + A \Rightarrow A = -11 \Rightarrow \quad v_C(t) \ = \ 1\,e^{-2t} - 11\,e^{-4000t} \ \text{V}$$

Finally $\qquad \underline{v_o(t) \ = \ -\,v_C(t) \ = 11e^{-4000t} - 1e^{-5t} \ \text{V} \ , \ t \geq 0}$

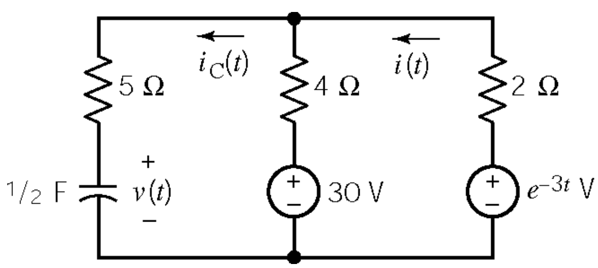
P 8.7-5



Solution: Assume that the circuit is at steady state before t = 0.

$$v(0^+) = v(0^-) = \frac{2}{4+2}30 = 10 \text{ V}$$

After t = 0 we have



$$\text{KVL}: \ \frac{5}{2} \ \frac{d\,v(t)}{dt} + v(t) + 4\left(\frac{1}{2}\frac{d\,v(t)}{dt} \ -i\right) = 30$$

$$2\,i(t) + 4\left(i(t) - \frac{1}{2}\frac{d\,v(t)}{dt}\right) + 30 \ = \ e^{-3t}$$

The circuit is represented by the differential equation

$$\frac{d\,v(t)}{dt} + \ \frac{6}{19}v(t) = \frac{6}{19} \ (10 + \frac{2}{3}e^{-3t})$$

Take $v_n(t) = Ae^{-(6/19)t}$. Try , $v_f(t) = \ B + Ce^{-3t}$, substitute into the differential equation to get

$$-3Ce^{-3t} + \frac{6}{19}(B + Ce^{-3t}) \ = \ \frac{60}{19} + \frac{4}{19}e^{-3t}$$

Equate coefficients to get

$$B = 10 \ , \ C = -\frac{4}{51} \quad \Rightarrow \quad v_f(t) = \frac{4}{51}e^{-3t} + Ae^{-(6/19)t}$$

Then $\qquad v(t) = v_n(t) + v_f(t) = \ 10 - \frac{4}{51}e^{-3t} + Ae^{-(6/19)t}$

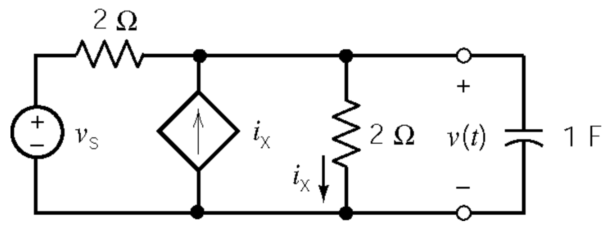
Finally $\quad v_c(0^+) = 10 \text{ V}, \quad \Rightarrow \quad 10 = 10 - \frac{4}{51} + A \quad \Rightarrow \quad A = \frac{4}{51}$

$$\therefore \; \underline{v_c(t) = 10 + \frac{4}{51}\left(e^{-(6/19)t} - e^{-3t}\right) \text{ V}}$$

P 8.7-6

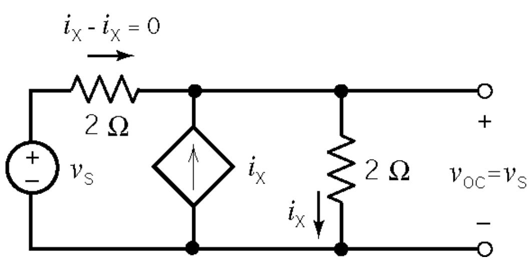
Solution: We are given $v(0) = 0$. From part *b* of the figure:

$$v_s(t) = \begin{cases} 5t & 0 \le t \le 2 \text{ s} \\ 10 & t > 2 \text{ s} \end{cases}$$

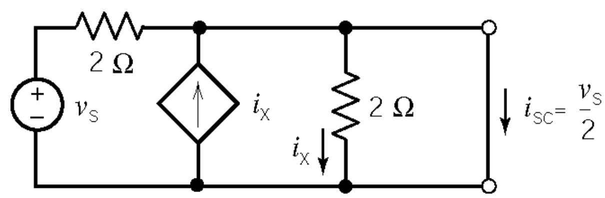


Find the Thevenin equivalent of the part of the circuit that is connected to the capacitor:

The open circuit voltage:

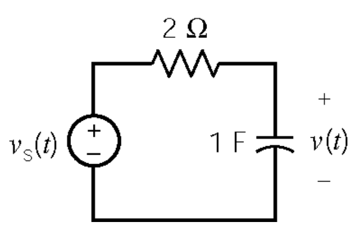


The short circuit current:



(i_x=0 because of the short across the right 2 Ω resistor)

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent:



$$2\frac{dv(t)}{dt} + v(t) - v_s(t) = 0$$

$$\frac{dv(t)}{dt} + \frac{v(t)}{2} = \frac{v_s(t)}{2}$$

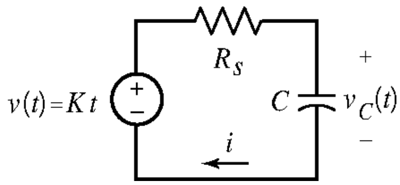
$$v_n(t) = A e^{-0.5t}$$

For $0 < t < 2$ s, $v_s(t) = 5t$. Try $v_f(t) = B + C\,t$. Substituting into the differential equation and equating coefficients gives $B = -10$ and $C = 5$. Therefore, $v(t) = 5t - 10 + A\,e^{-t/2}$. Using $v(0) = 0$, we determine that $A = 10$. Consequently, $\underline{v(t) = 5t + 10(e^{-t/2} - 1)}$.

At $t = 2$ s, $v(2) = 10e^{-1} = 3.68$.

Next, for $t > 2$ s, $v_s(t) = 10$ V. Try $v_f(t) = B$. Substituting into the differential equation and equating coefficients gives $B = 10$. Therefore $v(t) = 10 + Ae^{-(t-2)/2}$. Using $v(2) = 3.68$, we determine that $A = -6.32$. Consequently, $\underline{v(t) = 10 - 6.32\,e^{-(t-2)/2}}$.

P 8.7-7
Solution:



KVL: $-kt + R_s\left[C\,\frac{d\,v_C(t)}{dt}\right] + v_C(t) = 0$

$$\Rightarrow\ \frac{d\,v_C(t)}{dt} + \frac{1}{R_sC}\,v_C(t) = \frac{k}{R_sC}\,t$$

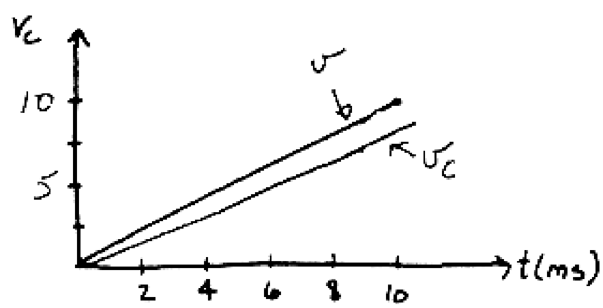
$v_c(t) = v_n(t) + v_f(t)$, where $v_c(t) = Ae^{-t/R_sC}$. Try $v_f(t) = B_0 + B_1\,t$

& plug into D.E. $\Rightarrow\ B_1 + \frac{1}{R_sC}\left[B_0 + B_1 t\right] = \frac{k}{R_sC}t$ thus $B_0 = -kR_sC$, $B_1 = k$.

Now we have $v_c(t) = Ae^{-t/R_sC} + k(t - R_sC)$. Use $v_c(0) = 0$ to get $0 = A - kR_sC \ \Rightarrow\ A = kR_sC$.

$\therefore\ v_c(t) = k[t - R_sC(1 - e^{-t/R_sC})]$. Plugging in k=1000, R_s= 625 kΩ & C=2000 pF get

$\underline{v_c(t) = 1000[t - 1.25\times10^{-3}(1 - e^{-800\,t})]}$



$v(t)$ and $v_C(t)$ track well on a millisecond time scale.

Section 8.10 How Can We Check…?
P 8.10-1
Solution: First look at the circuit. The initial capacitor voltage is $v_c(0) = 8$ V. The steady-state capacitor voltage is $v_c = 4$ V.

We expect an exponential transition from 8 volts to 4 volts. That’s consistent with the plot.

Next, let's check the shape of the exponential transition. The Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = \frac{(2000)(4000)}{2000+4000} = \frac{4}{3}$ kΩ so the time constant is $\tau = R_t\, C = \left(\frac{4}{3}\times 10^3\right)\left(0.5\times 10^{-6}\right) = \frac{2}{3}$ ms . Thus the capacitor voltage is

$$v_c(t) = 4\, e^{-t/0.67} + 4 \text{ V}$$

where t has units of ms. To check the point labeled on the plot, let t_1 = 1.33 ms. Then

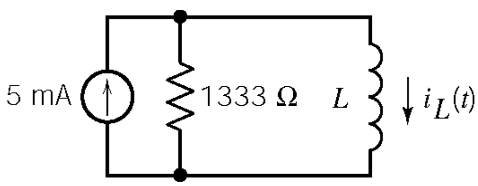
$$v_c(t_1) = 4\, e^{-\left(\frac{1.33}{.67}\right)} + 4 = 4.541 \simeq 4.5398 \text{ V}$$

So the plot is correct.

P 8.10-2

Solution: Notice that the steady-state inductor current does not depend on the inductance, L. The initial and steady-state inductor currents shown on the plot agree with the values obtained from the circuit.

After $t = 0$



So $I_{sc} = 5$ mA and $\tau = \frac{L}{1333}$

The inductor current is given by $i_L(t) = -2e^{-1333t/L} + 5$ mA , where t has units of seconds and L has units of Henries. Let t_1 = 3.75 ms, then

$$4.836 = i_L(t_1) = -2\, e^{-(1333)\cdot(0.00375)/L} + 5 = -2e^{-5/L} + 5$$

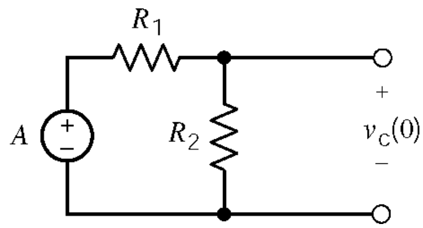
so
$$\frac{4.836-5}{-2} = e^{-5/L}$$

and
$$L = \frac{-5}{\ln\left(\frac{4.836-5}{-2}\right)} = 2 \text{ H}$$

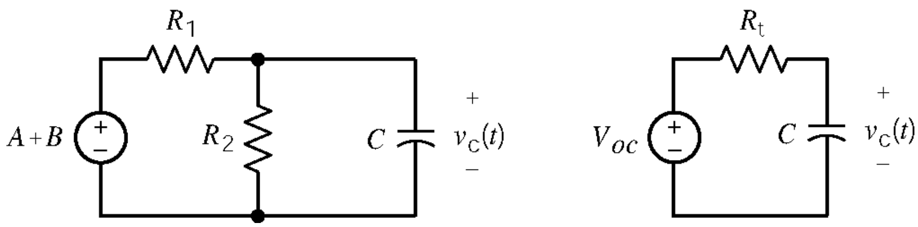
is the required inductance.

P 8.10-3

Solution: First consider the circuit. When t < 0 and the circuit is at steady-state:



For t > 0



So $$V_{oc} = \frac{R_2}{R_1+R_2}(A+B)\ ,\quad R_t = \frac{R_1 R_2}{R_1+R_2} \text{ and } \tau = \frac{R_1 R_2 C}{R_1+R_2}$$

Next, consider the plot. The initial capacitor voltage is (v_c (0)=) –2 and the steady-state capacitor voltage is (V_{oc} =) 4 V, so

$$v_C(t) = -6e^{-t/\tau} + 4$$

At $t_1 = 1.333$ ms $$3.1874 = v_C(t_1) = -6\,e^{-0.001333/\tau} + 4$$

so $$\tau = \frac{-0.001333}{\ln\left(\dfrac{-4+3.1874}{-6}\right)} = 0.67 \text{ ms}$$

Combining the information obtained from the circuit with the information obtained from the plot gives

$$\frac{R_2}{R_1+R_2}A = -2,\quad \frac{R_2}{R_1+R_2}(A+B) = 4,\quad \frac{R_1 R_2 C}{R_1+R_2} = 0.67 \text{ ms}$$

There are many ways that A, B, R_1, R_2, and C can be chosen to satisfy these equations. Here is one convenient way. Pick $R_1 = 3000$ and $R_2 = 6000$. Then

$$\frac{2A}{3} = -2 \;\Rightarrow\; A = -3$$

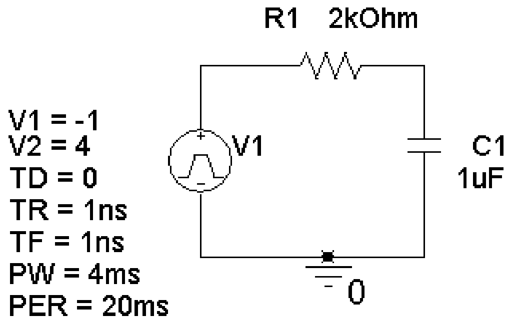
$$\frac{2(A+B)}{3} = 4 \Rightarrow B-3 = 6 \Rightarrow B = 9$$

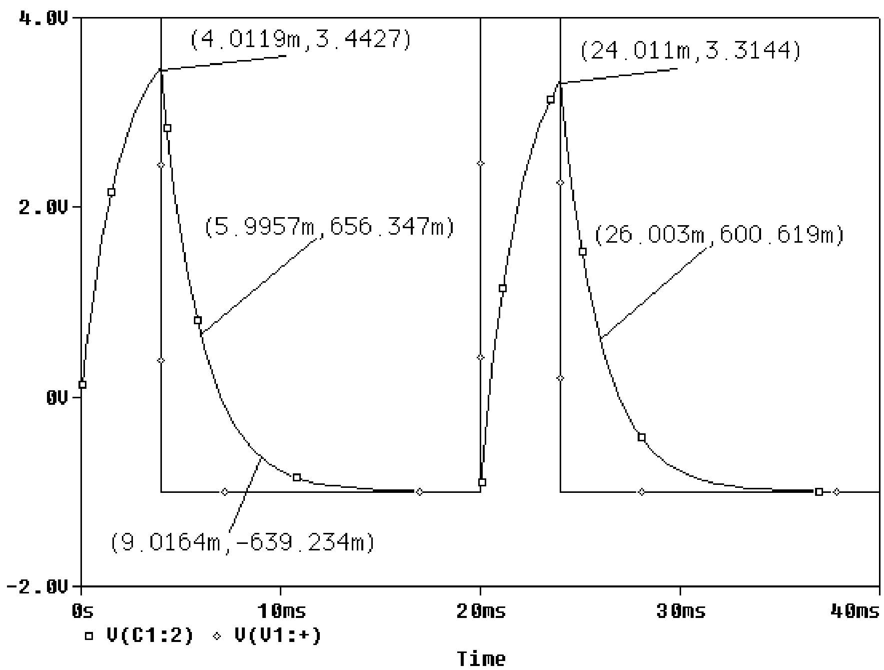
$$2000 \cdot C = \frac{2}{3} \text{ ms} \;\Rightarrow\; \frac{1}{3}\mu\text{F} = C$$

Spice Problems

SP 8-1

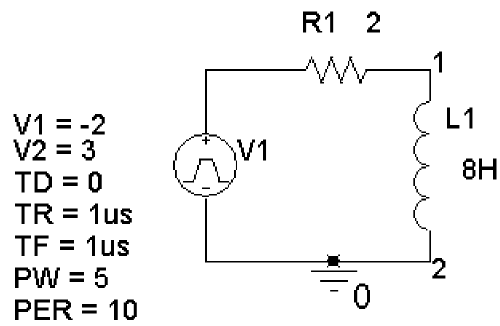
Solution

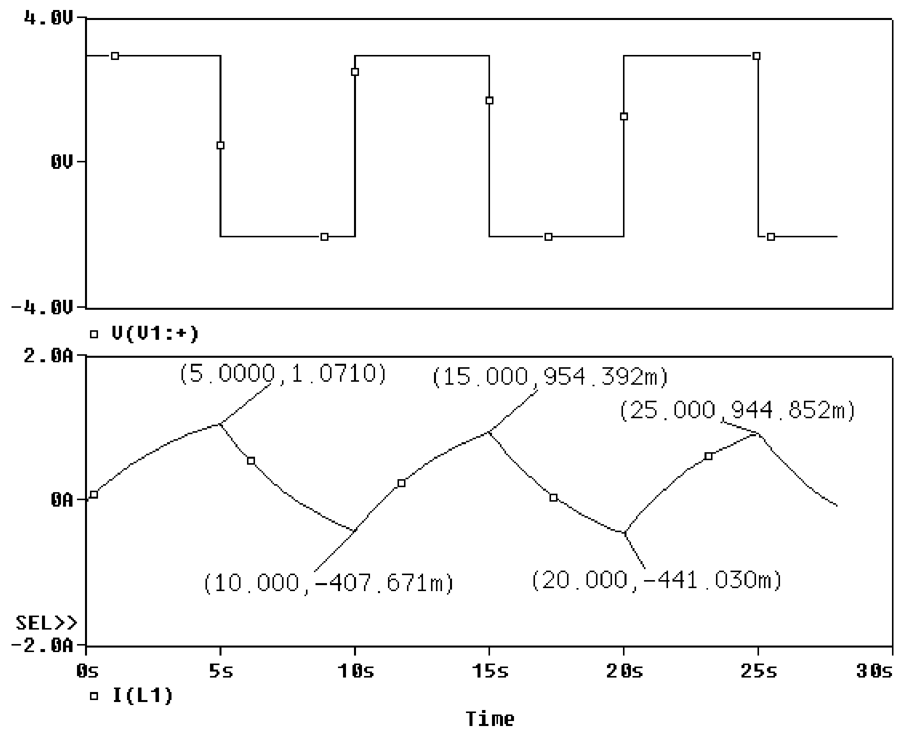




SP 8-2

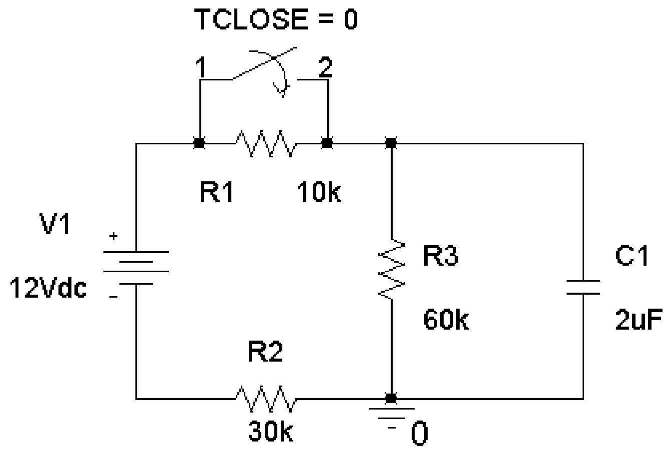
Solution

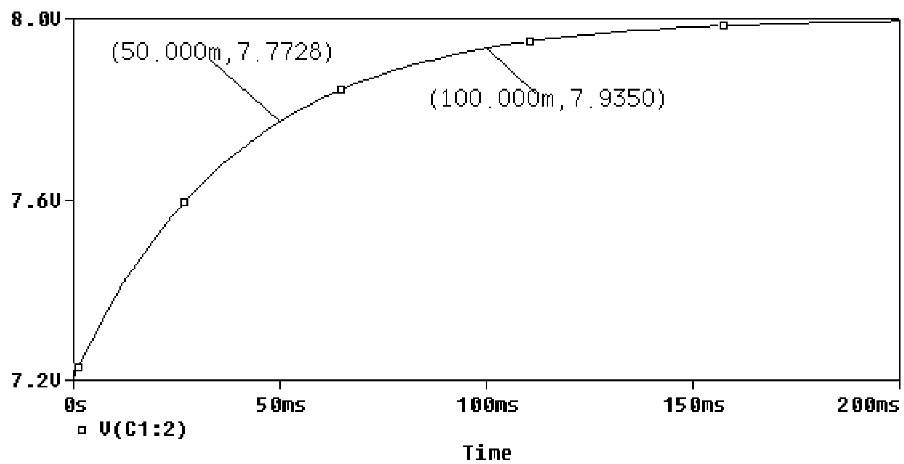




SP 8-3

Solution





$$v(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{l} 7.2 = v(0) = A + B\,e^{0} \quad \Rightarrow \quad 7.2 = A + B \\ 8.0 = v(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 8.0 \text{ V} \end{array}\right\} \Rightarrow \quad B = -0.8 \text{ V}$$

$$7.7728 = v(0.05) = 8 - 0.8\,e^{-0.05/\tau} \quad \Rightarrow \quad -\frac{0.05}{\tau} = \ln\left(\frac{8 - 7.7728}{0.8}\right) = -1.25878$$

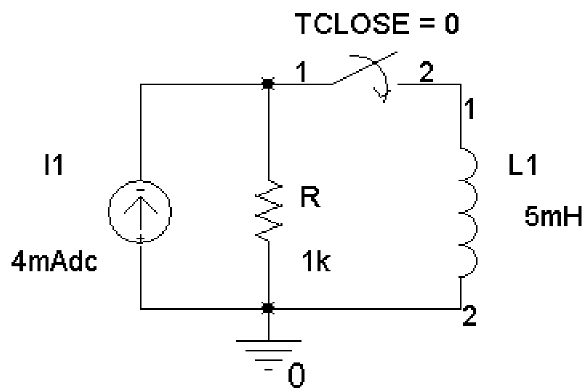
$$\Rightarrow \quad \tau = \frac{0.05}{1.25878} = 39.72 \text{ ms}$$

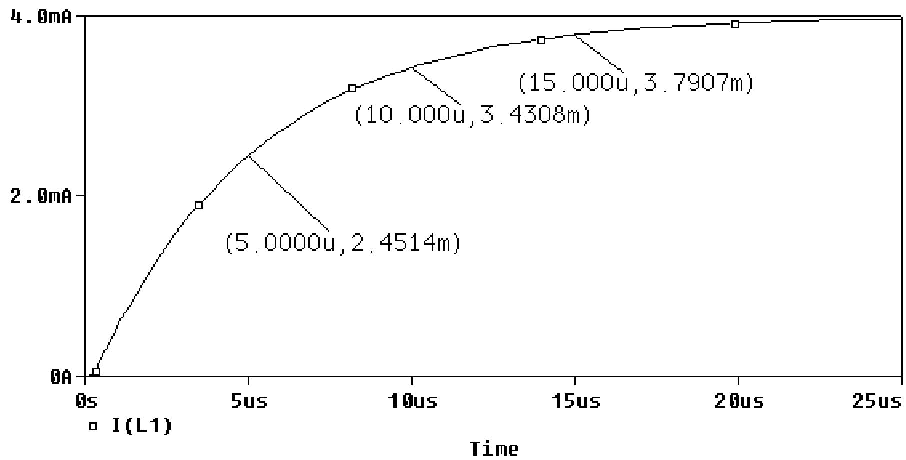
Therefore

$$v(t) = 8 - 0.8\,e^{-t/0.03972} \text{ V} \quad \text{for } t > 0$$

SP 8-4

Solution





$$i(t) = A + B\,e^{-t/\tau} \quad \text{for } t > 0$$

$$\left.\begin{array}{r} 0 = i(0) = A + B\,e^{0} \quad \Rightarrow \quad 0 = A + B \\ 4\times10^{-3} = i(\infty) = A + B\,e^{-\infty} \quad \Rightarrow \quad A = 4\times10^{-3}\ \text{A} \end{array}\right\} \quad \Rightarrow \quad B = -4\times10^{-3}\ \text{A}$$

$$2.4514\times10^{-3} = v(5\times10^{-6}) = \left(4\times10^{-3}\right) - \left(4\times10^{-3}\right) e^{-\left(5\times10^{-6}\right)/\tau}$$

$$\Rightarrow \quad -\frac{5\times10^{-6}}{\tau} = \ln\left(\frac{\left(4-2.4514\right)\times10^{-3}}{4\times10^{-3}}\right) = -0.94894$$

$$\Rightarrow \quad \tau = \frac{5\times10^{-6}}{0.94894} = 5.269\ \mu\text{s}$$

Therefore

$$i(t) = 4 - 4\,e^{-t/5.269\times10^{-6}}\ \text{mA} \quad \text{for } t > 0$$

DESIGN PROBLEMS

DP 8-1

Solution: We will

(a) Specify the value of R_3 to cause A = 16 V.

(b) Specify the value of R_2 to cause B = 16 V.

(c) Specify the value of C to cause the time constant of the circuit to be τ = 4 ms.

(a) Apparently the circuit is at steady sate before the switch opens that the circuit. While the switch is closed we require:

$$16 = \frac{R_3}{30+R_3}(24) \quad \Rightarrow \quad R_3 = 60 \text{ k}\Omega$$

(b) After the switch opens we require:

$$8 = \frac{60}{90+R_2}(24) \quad \Rightarrow \quad R_2 = 90 \text{ k}\Omega$$

(c) When the switch is open the time constant is given by $\tau = R_t C$ where

$$R_t = R_3 \,||\, (30+R_2) = \frac{R_3(30+R_2)}{R_3+(30+R_2)} = \frac{60(30+90)}{60+(30+90)} = 40 \text{ k}\Omega$$

We require

$$4 \text{ ms} = R_t C = 40 \text{ k}\Omega \quad \Rightarrow \quad C = \frac{0.004}{40,000} = 0.1\times 10^{-6} = 0.1 \ \mu\text{F}$$

In summary:

$$R_2 = 60 \text{ k}\Omega, \quad R_3 = 90 \text{ k} \quad \Omega \quad \text{and} \quad C = 0.1 \ \mu\text{F}$$

DP 8-2

Solution:

steady state response when the switch is open: $0.001 = \dfrac{15}{R_1+R_2} \quad \Rightarrow \quad R_1 + R_2 = 15 \text{ k}\Omega$.

steady state response when the switch is closed: $0.006 = \dfrac{15}{R_1} \quad \Rightarrow \quad R_1 = 2.5 \text{ k}\Omega$.

Therefore, R_2 = 12.5 kΩ.

$$10 \text{ ms} = 5\,\tau = 5\left(\frac{L}{R_1+R_2}\right) = \frac{L}{3000} \quad \Rightarrow \quad L = 30 \text{ H}$$

DP 8-3

Solution

R_t = 50 kΩ when the switch is open and R_t = 49 kΩ ≈ 50 kΩ when the switch is closed so use

R_t = 50 kΩ.

(a) $\Delta t = 5 R_t C \quad \Rightarrow \quad C = \dfrac{10^{-6}}{5\left(50\times10^{3}\right)} = 4 \text{ pF}$

(b) $\Delta t = 5\left(50\times10^{3}\right)\left(2\times10^{-6}\right) = 0.5 \text{ s}$

DP 8-4

Solution: R_t = 50 kΩ when the switch is open and R_t = 49 kΩ ≈ 50 kΩ when the switch is closed so use R_t = 50 kΩ.

When the switch is open: $5e^{-\Delta t/\tau} = (1-k)5 \quad \Rightarrow \quad \ln(1-k) = -\dfrac{\Delta t}{\tau} \quad \Rightarrow \quad \Delta t = -\tau \ln(1-k)$

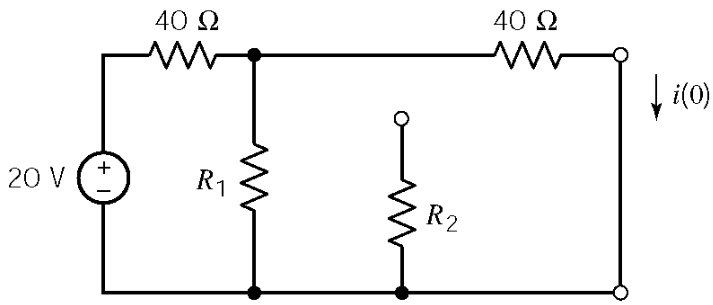
When the switch is open: $5 - 5e^{-\Delta t/\tau} = k5 \quad \Rightarrow \quad \Delta t = -\tau \ln(1-k)$

(a) $C = \dfrac{10^{-6}}{-\ln(1-.95)\left(50\times10^{3}\right)} = 6.67 \text{ pF}$

(b) $\Delta t = -\ln(1-.95)\left(50\times10^{3}\right)\left(2\times10^{-6}\right) = 0.3 \text{ s}$

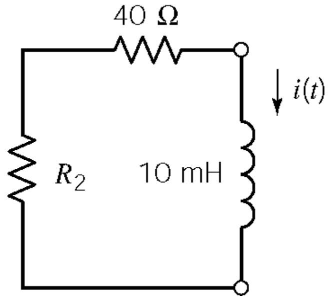
DP 8-5

Solution:



$$i(0) = \frac{20}{40+\dfrac{40R_1}{40+R_1}} \times \frac{R_1}{R_1+40}$$

For $t > 0$:



$$i(t) = i(0)\, e^{-t/\tau} \quad \text{where } \tau = \frac{L}{R_t} = \frac{10^{-2}}{40+R_2}$$

At $t < 200\mu s$ we need $i(t) > 60$ mA and $i(t) < 180$ mA

First let's find a value of R_2 to cause $i(0) < 180$ mA.

Try $\underline{R_2 = 40\,\Omega}$. Then $i(0) = \frac{1}{6}$ A = 166.7 mA so $i(t) = 0.1667\ e^{-t/\tau}$.

Next, we find a value of R_2 to cause $i(0.0002) > 60$ mA.

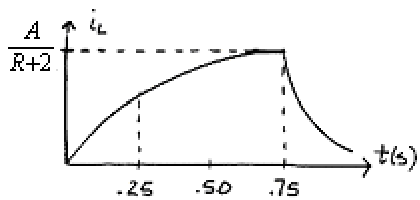
Try $\underline{R_2 = 10\Omega}$, then $\tau = \frac{10^{-2}}{50} = 0.2$ ms $= \frac{1}{5000}$ s.

$i(0.0002) = 166.7\times10^{-3}e^{-5000\times0.0002} = 166.7\times10^{-3}e^{-1} = 61.3$ mA

DP 8-6

Solution:

The current waveform will look like this:



We only need to consider the rise time:

$$i_L(t) = \frac{V_s}{R+2}(1-e^{-t/\tau}) = \frac{A}{R+2}(1-e^{-t/\tau})$$

where

$$\tau = \frac{L}{R_t} = \frac{0.2}{3} = \frac{1}{15} \text{ s}$$

$$\therefore i_L(t) = \frac{A}{3}(1-e^{-15t})$$

Now find A so that $i_L^2 R_{fuse} \geq 10$ W during $0.25 \leq t \leq 0.75$ s

$\therefore$ we want $[i_L^2(0.25)]R_{fuse} = 10$ W $\Rightarrow \frac{A^2}{9}(1-e^{-15(.25)})^2(1)=10 \Rightarrow \underline{A = 9.715 \text{ V}}$