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EE214000 Electromagnetics, Fall 2020

Your name: ________________ ID: __________________ Nov. 15nd, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #10-1, Open books, notes (30 points), due 11 pm, Wednesday, Nov. 18th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. The possible solutions for a 1-D Laplace equation include:

$\sin kx, \cos kx, \sinh kx, \cosh kx, e^{kx}, e^{-kx}$, where k is a positive real number.

(a) Which solution(s) would you choose for a boundary condition $V(x=0)=0$? Explain your choice. (2 points)

(b) Which solution(s) would you choose for a boundary condition $V(x=\infty)=0$? (2 points)

(c) Which solution(s) would you choose for a boundary condition $V(x_0)=V(x_0+ma)$, where m = 1, 2, 3, …is an integer? Calculate k for this case. (4 points)

Ans: (a) $\sin kx, \sinh kx$, because the values go through zero when x = 0.

(b) e^{-kx}, because this is the only solution that gives $V(x=\infty)=0$

(c) $V(x_0)=V(x_0+ma)$ tells a periodic function with period of a for $V(x)$. Possible solutions must be a periodic function, including $\sin kx, \cos kx$. To have a period of a, k must be $k=\dfrac{2m\pi}{a}$.

2. A two dimensional potential problem in the x-y plane satisfies Laplace's equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$$

(a) If the dependence of V in the x direction is $\sin kx$, where k is a nonzero real number, and $V(x, y=0) = 0$, what is(are) the possible solution(s) of $V(x, y)$? (2 points)

(b) If the dependence of V in the x direction is $\sin kx$, where k is a nonzero real positive number, and $V(x, y=\infty) = 0$, what is(are) the possible solution(s) of $V(x, y)$? (2 points)

(c) If the dependence of V in the x direction is e^{kx}, where k is a nonzero real number and $V(x, y=0) = 0$, what is(are) the possible solution(s) of $V(x, y)$? (2 points)

Ans: (a) In the y direction, possible solutions include $\sinh ky, \cosh ky, e^{ky}, e^{-ky}$ but only $\sinh ky$ satisfies the boundary condition $V(x, y=0) = 0$. Therefore the possible solution of the potential is $V(x,y) = V_0 \sin kx \times \sinh ky$.

(b) In the y direction, possible solutions include $\sinh ky, \cosh ky, e^{ky}, e^{-ky}$ but only e^{-ky} satisfies the boundary condition $V(x, y=\infty) = 0$. Therefore the possible solution of the potential is $V(x,y) = V_0 \sin kx \times e^{-ky}$.

(c) In the y direction, it must be a periodic function, such as $\sin ky, \cos ky$, but only $\sin ky$ satisfied the boundary condition $V(x, y=0) = 0$. Therefore the possible solution of the potential is $V(x,y) = V_0 \sin ky \times e^{kx}$.

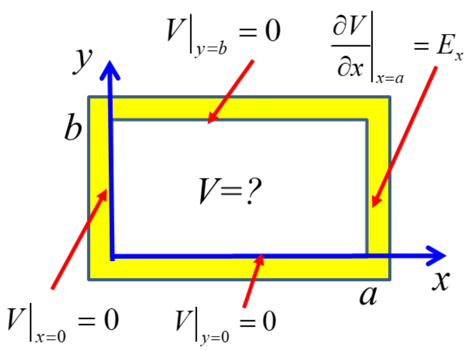
3. Calculate the value of $\int_0^{\pi} \sin(mx)\sin(nx)dx$ for $m \neq n$ with m, n nonzero integers?

Ans: If you calculate it correctly, the value should be 0 (orthogonality property for harmonic functions).

4. What is the value of $\int_0^{2\pi} \cos(mx/2)\cos(nx/2)dx$ for $m = n$ with m, n nonzero integers? (3 points)

Ans: In this case, $a = 2\pi$ and the integration gives $a/2 = \pi$. Do go through the calculation by yourself.

5. Refer to the following 2D figure and find (1) the electric potential, and (2) electric field intensity in the boxed region.



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Ans: (1) Along y, the solution is periodic and therefore that along x has to be monotonic $V(x,y) = A\cosh kx \times \sin ky$. The general solution is then

$$V(x,y) = \sum_{n=1}^{\infty} A_n \cosh\frac{n\pi}{b}x \times \sin\frac{n\pi}{b}y$$

Apply the boundary condition at x = a and use the orthogonal property to obtain

$$\int_0^b E_x \sin\frac{m\pi}{b}y dy = A_m \frac{m\pi}{b}\sinh\frac{m\pi}{b}a \times \frac{b}{2}$$

Insert $A_m = 2\dfrac{\int_0^b E_x \sin\frac{m\pi}{b}y dy}{m\pi \times \sinh\frac{m\pi}{b}a} \rightarrow A_m = \dfrac{4bE_x}{m^2\pi^2 \times \sinh\frac{m\pi}{b}a}, m = \text{odd integer}$ into

$V(x,y) = \sum_{n=1}^{\infty} A_n \cosh\frac{n\pi}{b}x \times \sin\frac{n\pi}{b}y$ to obtain the full solution for $V(x,y)$.

(2) Take the gradient of the electrical potential to obtain the electric field.

$$\vec{E} = -\nabla V(x,y) = -[\sum_{n=1}^{\infty} A_n \frac{n\pi}{b}\sinh\frac{n\pi}{b}x \times \sin\frac{n\pi}{b}y]\hat{a}_x$$

$$-[\sum_{n=1}^{\infty} A_n \frac{n\pi}{b}\cosh\frac{n\pi}{b}x \times \cos\frac{n\pi}{b}y]\hat{a}_y$$