Chapter 4 Exercises

Exercise 4.2-1 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-1.

Answer: $v_a = 3$ V and $v_b = 11$ V

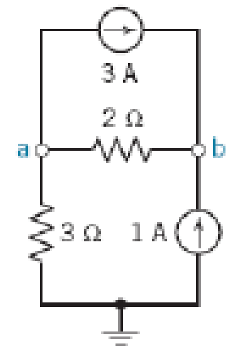


Figure E 4.2-1

Solution:

KCL at a: $$\frac{v_a}{3}+\frac{v_a - v_b}{2}+3=0 \Rightarrow 5\,v_a - 3\,v_b = -18$$

KCL at b: $$\frac{v_b - v_a}{2}-3-1=0 \Rightarrow v_b - v_a = 8$$

Solving these equations gives: $v_a = 3$ V and $v_b = 11$ V

Exercise 4.2-2 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-2.

Answer: $v_a = -4/3$ V and $v_b = 4$ V

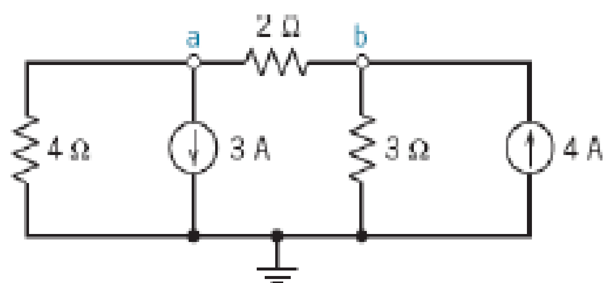


Figure E 4.2-2

Solution:

KCL at a: $$\frac{v_a}{4}+\frac{v_a - v_b}{2}+3=0 \Rightarrow 3\,v_a - 2\,v_b = -12$$

KCL at b: $$\frac{v_b}{3}-\frac{v_a - v_b}{2}-4=0 \Rightarrow -3\,v_a + 5\,v_b = 24$$

Solving: $v_a = -4/3$ V and $v_b = 4$ V

Exercise 4.3-1 Find the node voltages for the circuit of Figure E 4.3-1.

Hint: Write a KCL equation for the supernode corresponding to the 10-V voltage source.

Answer:

$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5 \quad\Rightarrow\quad v_b=30\text{ V} \quad\text{and}\quad v_a=40\text{ V}$$

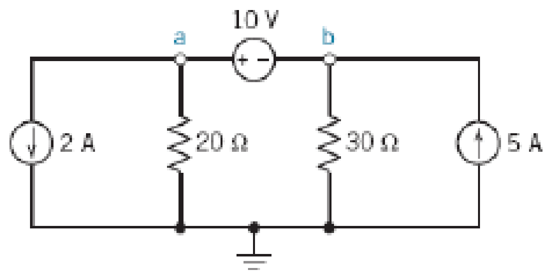
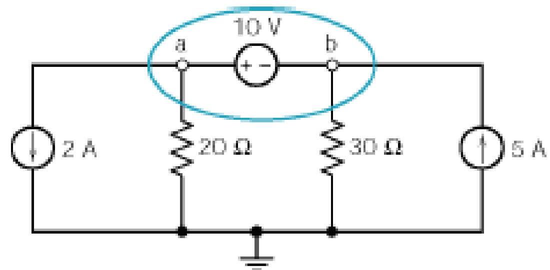


Figure E 4.3-1

Solution:

Apply KCL to the supernode to get

$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5$$



Solving:

$$v_b=30\text{ V and } v_a=v_b+10=40\text{ V}$$

Exercise 4.3-2 Find the voltages v_a and v_b for the circuit of Figure E 4.3-2.

Answer: $$\frac{(v_b+8)-(-12)}{10}+\frac{v_b}{40}=3 \quad\Rightarrow\quad v_b=8\text{ V} \quad\text{and}\quad v_a=16\text{ V}$$

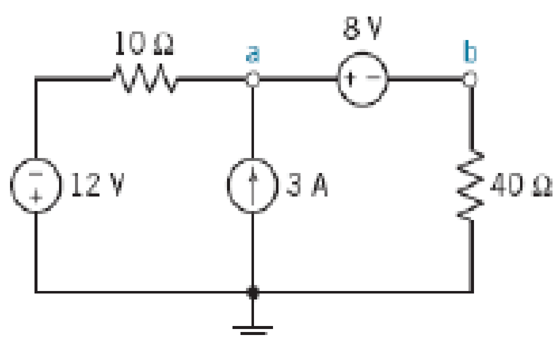


Figure E 4.3-2

Solution:

$$\frac{\left(v_b+8\right)-\left(-12\right)}{10}+\frac{v_b}{40}=3 \quad\Rightarrow\quad v_b=8\text{ V and } v_a=16\text{ V}$$

Exercise 4.4-1 Find the node voltage v_b for the circuit shown in Figure E 4.4-2.

Hint: Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for v_b.

Answer: $-\frac{6}{8}+\frac{v_b}{4}-\frac{v_b}{12}=0 \Rightarrow v_b = 4.5\text{ V}$

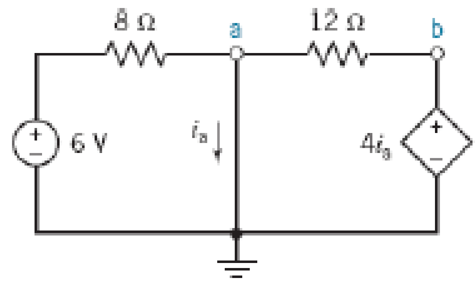


Figure E 4.4-2

Solution:

Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4\,i_a$ and solve for v_b.

$$\frac{6}{8}+\frac{v_b}{12}=i_a \quad \Rightarrow \quad v_b = 4i_a = 4\left(\frac{9+v_b}{12}\right) \quad \Rightarrow \quad v_b = 4.5\text{ V}$$

Exercise 4.4-2 Find the node voltages for the circuit shown in Figure E 4.4-2.

Hint: The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

Answer: $\frac{v_a-6}{20}+\frac{v_a-4v_a}{15}=0 \Rightarrow v_a = -2\text{ V}$

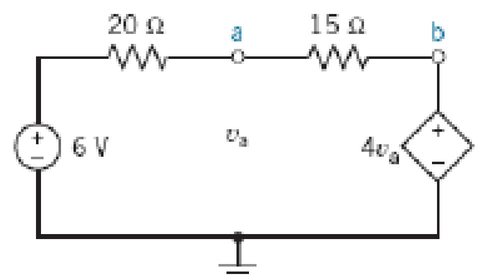


Figure E 4.4-2

Solution:

The controlling voltage of the dependent source is a node voltage so it is already expressed as a function of the node voltages. Apply KCL at node a.

$$\frac{v_a-6}{20}+\frac{v_a-4\,v_a}{15}=0 \quad \Rightarrow \quad v_a = -2\text{ V}$$

Exercise 4.5-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.5-1.

Answer: –1 V

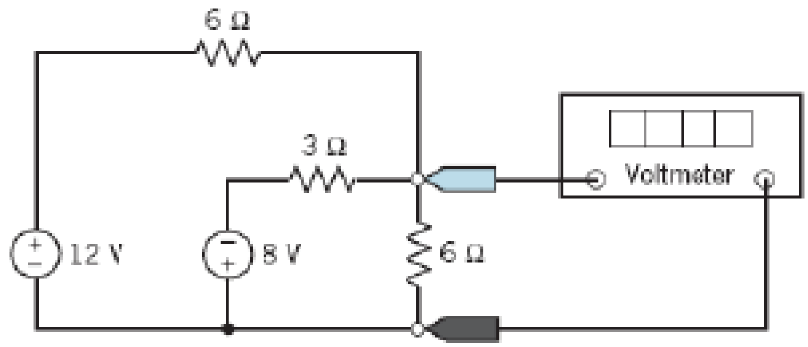
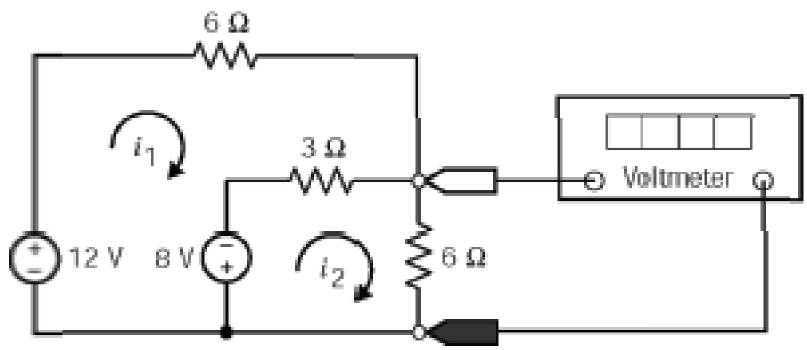


Figure E 4.5-1

Solution:



Mesh equations:

$$-12+6\,i_1+3\left(i_1-i_2\right)-8=0 \;\Rightarrow\; 9\,i_1-3\,i_2=20$$

$$8-3\left(i_1-i_2\right)+6\,i_2=0 \;\Rightarrow\; -3\,i_1+9\,i_2=-8$$

Solving these equations gives:

$$i_1=\frac{13}{6}\text{ A} \text{ and } i_2=-\frac{1}{6}\text{ A}$$

The voltage measured by the meter is $6\,i_2 = -1$ V.

Exercise 4.6-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

Hint: Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

Answer: –4 V

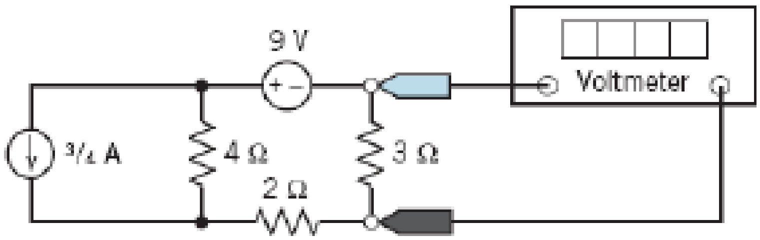
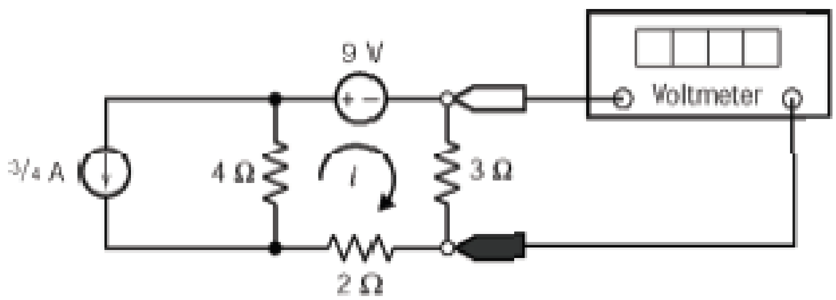


Figure E 4.6-1

Solution:



Mesh equation: $9+3\,i+2\,i+4\left(i+\frac{3}{4}\right)=0 \;\Rightarrow\; (3+2+4)\,i=-9-3 \;\Rightarrow\; i=\frac{-12}{9}$ A

The voltmeter measures $3\,i=-4$ V

Exercise 4.6-2 Determine the value of the current measured by the ammeter in Figure E 4.6-2.

Hint: Write and solve a single mesh equation.

Answer: –3.67 A

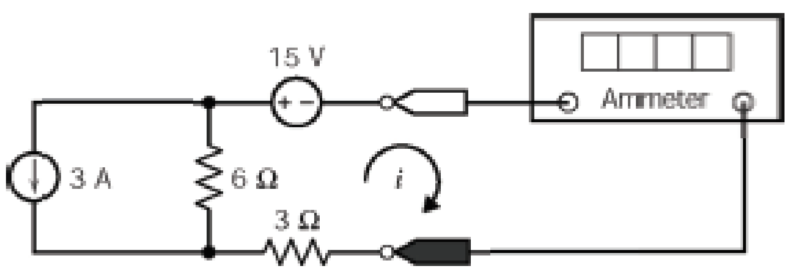


Figure E 4.6-2

Solution:

Mesh equation: $15+3\,i+6\,(i+3)=0 \;\Rightarrow\; (3+6)\,i=-15-6\,(3) \;\Rightarrow\; i=\frac{-33}{9}=-3\frac{2}{3}$ A

Section 4-2 Node Voltage Analysis of Circuits with Current Sources

P 4.2-1

Solution:
KCL at node 1:

$$0=\frac{v_1}{16}+\frac{v_1-v_2}{12}+i=\frac{-8}{16}+\frac{-8-4}{12}+i \Rightarrow -\frac{1}{2}-1+i=0$$
$$\Rightarrow \quad i=1.5 \text{ A}$$

(checked using LNAP 8/13/02)

P 4.2-2

Solution:
KCL at node 1:

$$\frac{v_1-v_2}{40}+\frac{v_1}{10}+1=0 \quad \Rightarrow \quad 5v_1-v_2=-40$$

KCL at node 2:

$$\frac{v_1-v_2}{40}+2=\frac{v_2-v_3}{20} \quad \Rightarrow \quad -v_1+3v_2-2v_3=80$$

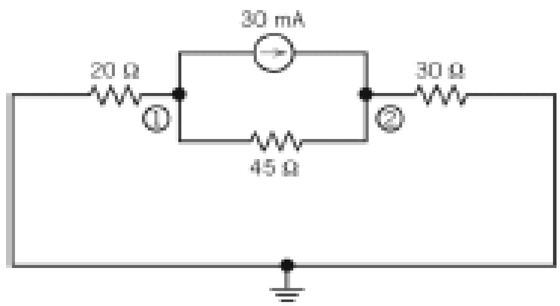
KCL at node 3:

$$\frac{v_2-v_3}{20}+1=\frac{v_3}{30} \quad \Rightarrow \quad -3v_2+5v_3=60$$

Solving gives $v_1 = 4$ V, $v_2 = 60$ V and $v_3 = 48$ V.

P4.2-3

Solution:



KCL at node 1: Expressing resistor currents in terms of node voltages

$$-\frac{v_1}{20}+\frac{v_1-v_2}{45}=-30\text{mA}$$

$$\Rightarrow 5v_1+4v_2=5.4$$

KCL at node 2: Expressing resistor currents in terms of node voltages

$$30\text{mA}=\frac{v_2-v_1}{45}+\frac{v_2}{30}$$

$$\Rightarrow v_1+3v_2=2.7$$

Solving gives $v_1 = 0.49\text{V}$, $v_2 = 0.74$ V.

P 4.2-4

Solution:

Node equations:

$$-.006+\frac{v_1}{R_1}+\frac{v_1-v_2}{1000}=0$$

$$-\frac{v_1-v_2}{1000}+\frac{v_2}{R_2}-0.010 = 0$$

When $v_1 = 2$ V, $v_2 = 4$ V

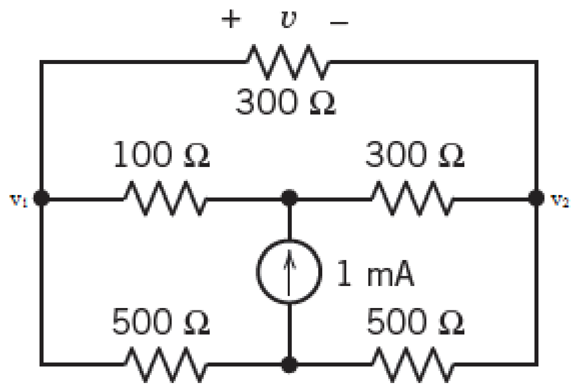
$$-0.006+\frac{2}{R_1}+\frac{-2}{1000} = 0 \Rightarrow R_1 = \frac{2}{0.006+\frac{1}{500}} = \underline{250\,\Omega}$$

$$-\frac{-2}{1000}+\frac{4}{R_2}-0.010=0\Rightarrow R_2=\frac{4}{0.010-\frac{1}{500}}=\underline{500\,\Omega}$$

(checked using LNAP 8/13/02)

P 4.2-5

Solution:



Node equations:

$$\frac{v_1}{500}+\frac{v_1-v_2}{100}+\frac{v_1-v_3}{300}=0$$

$$-\frac{v_1-v_2}{100}-0.001+\frac{v_2-v_3}{300}=0$$

$$-\frac{v_2-v_3}{300}-\frac{v_1-v_3}{300}+\frac{v_3}{500}=0$$

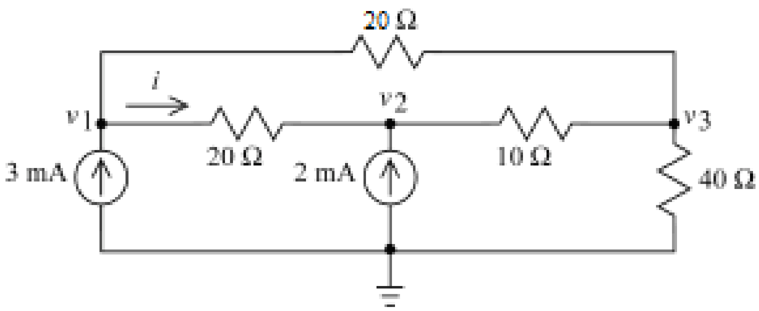
Solving gives:

$$v_1=0.255\text{ V},\quad v_2=0.332\text{ V},\quad v_3=0.223\text{ V}$$

Finally, $v=v_1-v_3=\underline{0.032\text{ V}}$

P 4.2-6

Solution:



$$12\ \Omega + (40\ \Omega \| 10\ \Omega) = 20\ \Omega$$

$$60\ \Omega \| 120\ \Omega = 40\ \Omega$$

The node equations are

$$3\times10^{-3} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{20} \quad \Rightarrow \quad 0.06 = 2v_1 - (v_2 - v_3)$$

$$2\times10^{-3} + \frac{v_1 - v_2}{20} = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad 0.04 = -v_1 + 3v_2 - 2v_3$$

$$\frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{20} = \frac{v_3}{40} \quad \Rightarrow \quad 0 = -(2v_1 + 4v_2) + 7v_3$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -2 \\ -2 & -4 & +7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} .06 \\ .04 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.244 \\ 0.228 \\ 0.200 \end{bmatrix}$$

(a) The power supplied by the 3 mA current source is $(3\times10^{-3})(0.244) = 0.732$ mW. The power supplied by the 2 mA source is $(2\times10^{-3})(0.228) = 0.456$ mW.

(b) The current in the 12 Ω resistor is equal to the current $i = \frac{v_1 - v_2}{20} = \frac{0.244 - 0.228}{20} = 0.8$ mA so the power received by the 12 Ω resistor is $(0.8\times10^{-3})^2(12) = 7.68\times10^{-6} = 7.68\ \mu$W.

(checked: LNAP and MATLAB 5/31/04)

P 4.2-7

Solution
Apply KCL at node a to get

$$2=\frac{v_a}{R}+\frac{v_a}{4}+\frac{v_a-v_b}{2}=\frac{14}{R}+\frac{14}{4}+\frac{14-20}{2}=\frac{14}{R}+\frac{7}{2}-3 \quad\Rightarrow\quad R=9.3\ \Omega$$

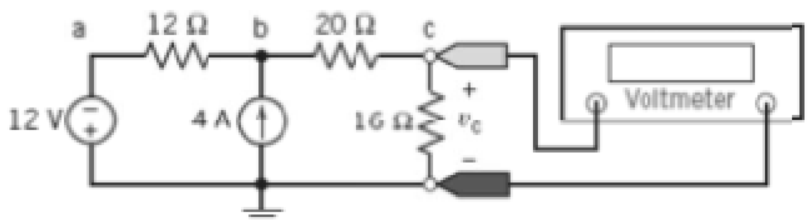
Apply KCL at node b to get

$$i_s+\frac{v_a-v_b}{2}=\frac{v_b}{8}+\frac{v_b}{8}=i_s+\frac{14-20}{2}=\frac{20}{8}+\frac{20}{8} \quad\Rightarrow\quad i_s=8\ \text{A}$$

Section 4-3 Node Voltage Analysis of Circuits with Current and Voltage Sources

P 4.3-1

Solution:



Express the branch voltage of the voltage source in terms of its node voltages:

$$0-v_a=12 \quad\Rightarrow\quad v_a=-12\ \text{V}$$

KCL at node *b*:

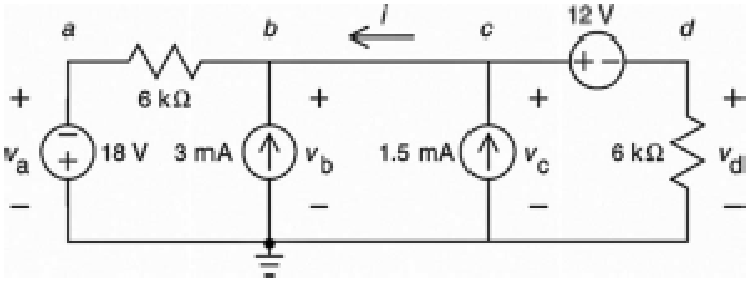
$$\frac{v_a-v_b}{12}+4=\frac{v_b-v_c}{20} \quad\Rightarrow\quad \frac{-12-v_b}{12}+4=\frac{v_b-v_c}{20} \quad\Rightarrow\quad -1-\frac{v_b}{12}+4=\frac{v_b-v_c}{20} \quad\Rightarrow\quad 180=8v_b-3v_c$$

KCL at node *c*: $$\frac{v_b-v_c}{20}=\frac{v_c}{16} \quad\Rightarrow\quad 16v_b-16v_c=20v_c \quad\Rightarrow\quad v_b=\frac{9}{4}v_c$$

Finally: $$180=8\left(\frac{9}{4}v_c\right)-3v_c \quad\Rightarrow\quad v_c=12\ \text{V}$$

P 4.3-2

Solution:



Express the branch voltage of each voltage source in terms of its node voltages to get:

$$v_a = -18 \text{ V},\ v_b = v_c = v_d + 12$$

KCL at node *b*:

$$\frac{v_b - v_a}{6000} = 0.003 + i \quad \Rightarrow \quad \frac{v_b - (-18)}{6000} = 0.003 + i \quad \Rightarrow \quad v_b + 18 = 18 + 6000\, i$$

KCL at the supernode corresponding to the 12 V source:

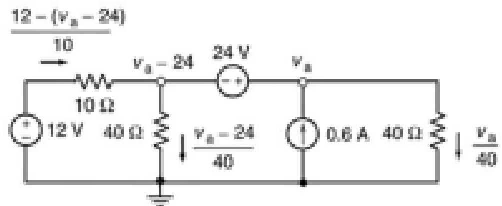
$$0.015 = \frac{v_d}{6000} + i \quad \Rightarrow \quad 9 = v_d + 6000\, i$$

so $\quad v_b = 9 - v_d \quad \Rightarrow \quad (v_d + 12) = 9 - v_d \quad \Rightarrow \quad 2v_d = -3 \;\Rightarrow \text{V=1.5V}$

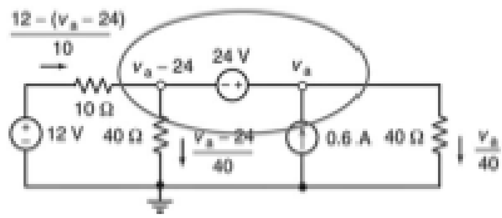
Consequently $v_b = v_c = v_d + 12 = 10.5$ V and $i = \dfrac{9 - v_d}{6000} = 1.75$ mA

P4.3-3.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 24 V source



Apply KCL to the supernode to get

$$\frac{12-(v_a-24)}{10}+0.6=\frac{v_a-24}{40}+\frac{v_a}{40} \Rightarrow 196=6v_a \Rightarrow v_a=32\text{ V}$$

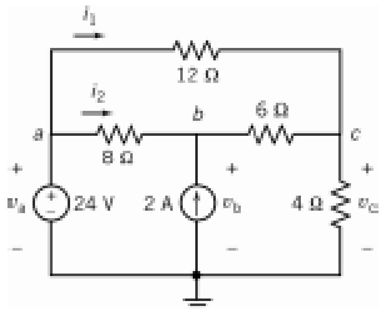
The 12 V source supplies $12\left(\frac{12-(v_a-24)}{10}\right)=12\left(\frac{12-(32-24)}{10}\right)=4.8\text{ W}$

The 24 V source supplies $24\left(-0.6+\frac{v_a}{40}\right)=24\left(-0.6+\frac{32}{40}\right)=4.8\text{ W}$

The current source supplies $0.6v_a=0.6(32)=19.2\text{ W}$

P 4.3-4

Solution:



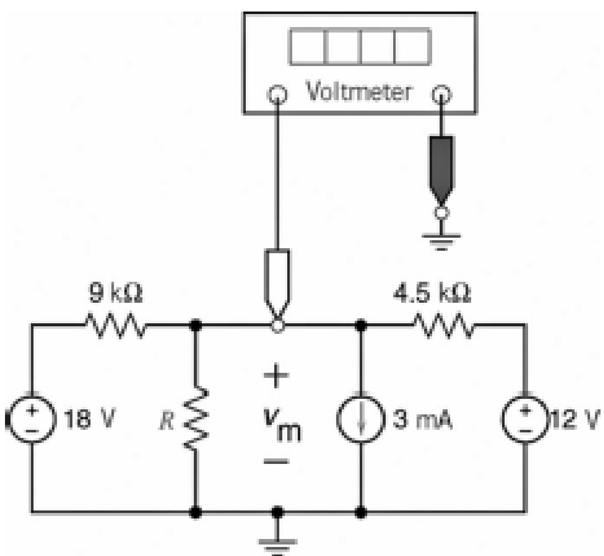
The power supplied by the voltage source is

$$v_a(i_1+i_2)=v_a\left(\frac{v_a-v_b}{8}+\frac{v_a-v_c}{12}\right)=24\left(\frac{24-19.75}{8}+\frac{24-10.588}{12}\right)$$

$$=24(0.53+1.117)=24(1.647)=39.54 \text{ W}$$

P 4.3-5

Solution:
Label the voltage measured by the meter. Notice that this is a node voltage.



Write a node equation at the node at which the node voltage is measured.

$$-\left[\frac{18-v_m}{9000}\right]+\frac{v_m}{R}+0.003+\frac{v_m-12}{4500}=0$$

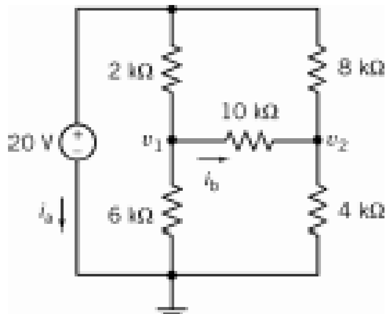
That is

$$R=\frac{3000}{\frac{5}{v_m}-1}$$

(a) The voltage measured by the meter will be 4 volts when R =12kΩ.
(b) The voltage measured by the meter will be 1.7 volts when R = 1.2 kΩ.

P 4.3-6

Solution:



Apply KCL at nodes 1 and 2 to get

$$\frac{20-v_1}{2000}=\frac{v_1}{6000}+\frac{v_1-v_2}{10000} \quad\Rightarrow\quad 23v_1-3v_2=300$$

$$\frac{20-v_2}{8000}+\frac{v_1-v_2}{10000}=\frac{v_2}{4000} \quad\Rightarrow\quad -4v_1+19v_2=100$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 23 & -3 \\ -4 & 19 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 300 \\ 100 \end{bmatrix} \quad\Rightarrow\quad v_1=14.11 \text{ V and } v_2=8.23 \text{ V}$$

Then

$$i_b=\frac{v_1-v_2}{1000}=\frac{14.11-8.23}{10000}=0.588 \text{ mA}$$

Apply KCL at the top node to get

$$i_a=\frac{v_1-20}{2000}+\frac{v_2-20}{8000}=\frac{14.11-20}{2000}+\frac{8.23-20}{8000}=-4.37 \text{ mA}$$

P 4.3-7

Solution:

$$\frac{v_o}{R_3}+\frac{v_o-v_1}{R_1}+\frac{v_o-v_2}{R_2}=0 \quad \Rightarrow \quad v_o=\frac{v_1}{1+\frac{R_1}{R_2}+\frac{R_1}{R_3}}+\frac{v_2}{1+\frac{R_2}{R_1}+\frac{R_2}{R_3}}$$

(a) When $R_1=10\ \Omega, R_2=40\ \Omega$ and $R_3=8\ \Omega$

$$v_o=\frac{v_1}{1+\frac{1}{4}+\frac{5}{4}}+\frac{v_2}{1+4+5}=0.4v_1+0.1v_2$$

So a = 0.4 and b = 0.1.

(b) When $R_1=R_2$ and $R_3=R_1 \,||\, R_2=R_1/2$

$$v_o=\frac{v_1}{1+1+2}+\frac{v_2}{1+1+2}=0.25v_1+0.25v_2$$

So a = 0.25 and b = 0.25.

(checked: LNAP 5/31/04)

P 4.3-8

Solution:

Express the voltage source voltages as functions of the node voltages to get

$$v_2-v_1=10 \ \text{ and } \ v_4=30$$

Apply KCL to the supernode corresponding to the 5 V source to get

$$2.5=\frac{v_1-v_3}{16}+\frac{v_2-30}{40} \quad \Rightarrow \quad 260=5v_1+2v_2-5v_3$$

Apply KCL at node 3 to get

$$\frac{v_1-v_3}{16}=\frac{v_3}{80}+\frac{v_3-30}{24} \quad \Rightarrow \quad -15v_1+28v_3=300$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 2 & -5 \\ -15 & 0 & 28 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 10 \\ 260 \\ 300 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 67.9 \\ 77.9 \\ 47.1 \end{bmatrix}$$

So the node voltages are:

$$v_1=67.9\text{ V}, v_2=77.9\text{V}, v_3=47.1\text{ V, and } v_4=30\text{ V}$$

P 4.3-9

Solution:
Write a node equation to get

$$-\left(\frac{24-9.0}{R_1}\right)+\frac{9}{R_3}+\frac{9-12}{R_2}=0 \quad\Rightarrow\quad -\frac{15}{R_1}+\frac{9}{R_3}-\frac{3}{R_2}=0$$

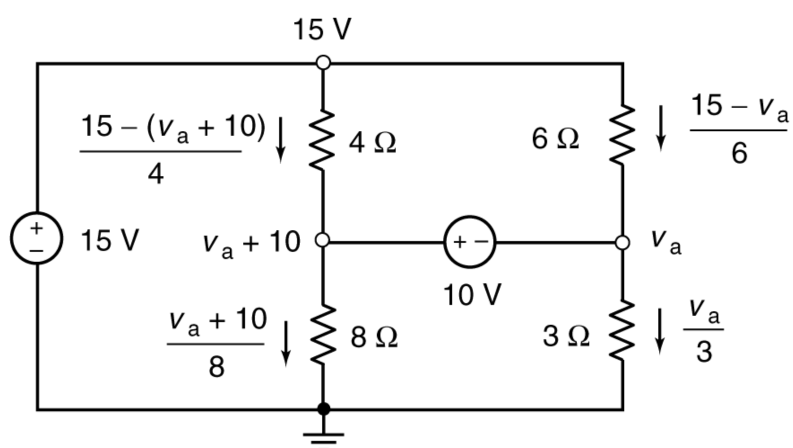
Notice that $\frac{15}{R_1}$ is either 1.5 mA or 3 mA depending on whether R_1 is 10 kΩ or 5 kΩ. Similarly, $\frac{9}{R_3}$ is either 0.9 mA or 1.8 mA and $\frac{3}{R_2}$ is either 0.3 mA or 0.4 mA. Suppose R_1 and R_2 are 10 kΩ resistors and R_3 is a 5 kΩ resistor. Then

$$-\frac{15}{R_1}+\frac{9}{R_3}-\frac{9}{R_2}=-1.5+1.8-0.3=0$$

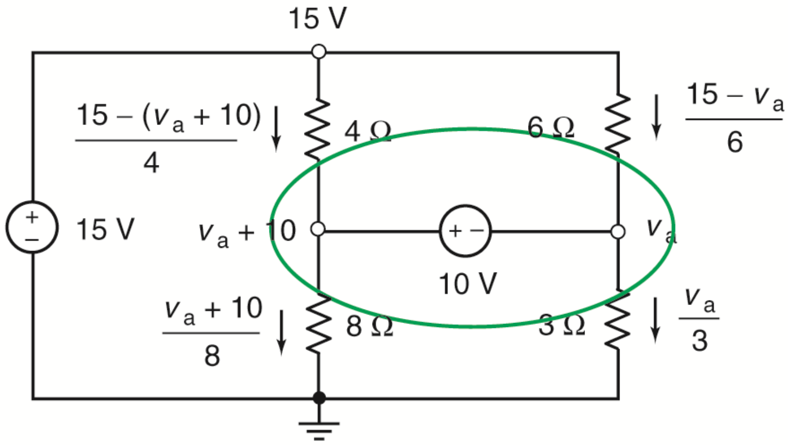
It is possible that two of the resistors are 10 kΩ and the third is 5 kΩ. R_3 is the 5 kΩ resistor.

P4.3-10

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 10 V source



Apply KCL to the supernode to get

$$\frac{15-(v_a+10)}{4}+\frac{15-v_a}{6}=\frac{v_a+10}{8}+\frac{v_a}{3} \quad\Rightarrow\quad 60=21v_a \quad\Rightarrow\quad v_a=2.857\text{ V}$$

The 15 V source supplies

$$15\left(\frac{15-(v_a+10)}{4}+\frac{15-v_a}{6}\right)=15\left(\frac{15-12.857}{4}+\frac{15-2.857}{6}\right)=15(2.56)=38.4\text{ W}$$

The 10 V source supplies $10\left(\frac{15-v_a}{6}+\frac{v_a}{3}\right)=10\left(\frac{15-2.857}{6}+\frac{2.857}{3}\right)=10(1.071)=10.71\text{ W}$

P4.3-11

Solution:

Express the voltage source voltages in terms of the node voltages:

$$v_2-v_1=12 \text{ and } v_3-v_1=18$$

Apply KVL to the supernode to get

$$\frac{v_2}{15}+\frac{v_1}{6}+\frac{v_3}{7.5}=0 \quad\Rightarrow\quad 2v_2+5v_1+4v_3=0$$

So

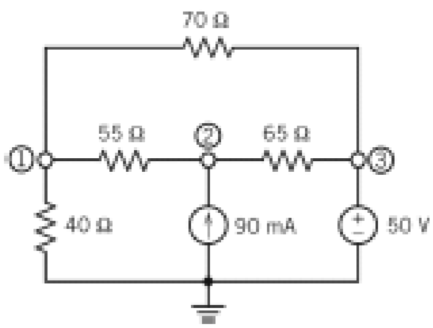
$$2(12+v_1)+5v_1+4(18+v_1)=0 \quad\Rightarrow\quad v_1=-\frac{96}{11}\text{ V}$$

The node voltages are

$$v_1=-8.72\text{ V}$$
$$v_2=3.28\text{ V}$$
$$v_3=9.28\text{ V}$$

P4.3-12

Solution:



KCL at node 1:

$$\frac{v_1 - v_2}{55} + \frac{v_1}{70} + \frac{v_1}{40} = 0 \quad \Rightarrow \quad 0.057v_1 - 0.018v_2 = 0$$

KCL at node 2:

$$\frac{v_2 - v_1}{55} - 0.09 + \frac{v_2 - v_3}{65} = 0 \quad \Rightarrow \quad 0.034v_2 - 0.018v_1 - 0.015v_3 - 0.09 = 0$$

Here $v_3 = 50v$, so the equation becomes

$$0.034v_2 - 0.018v_1 - 0.84 = 0$$

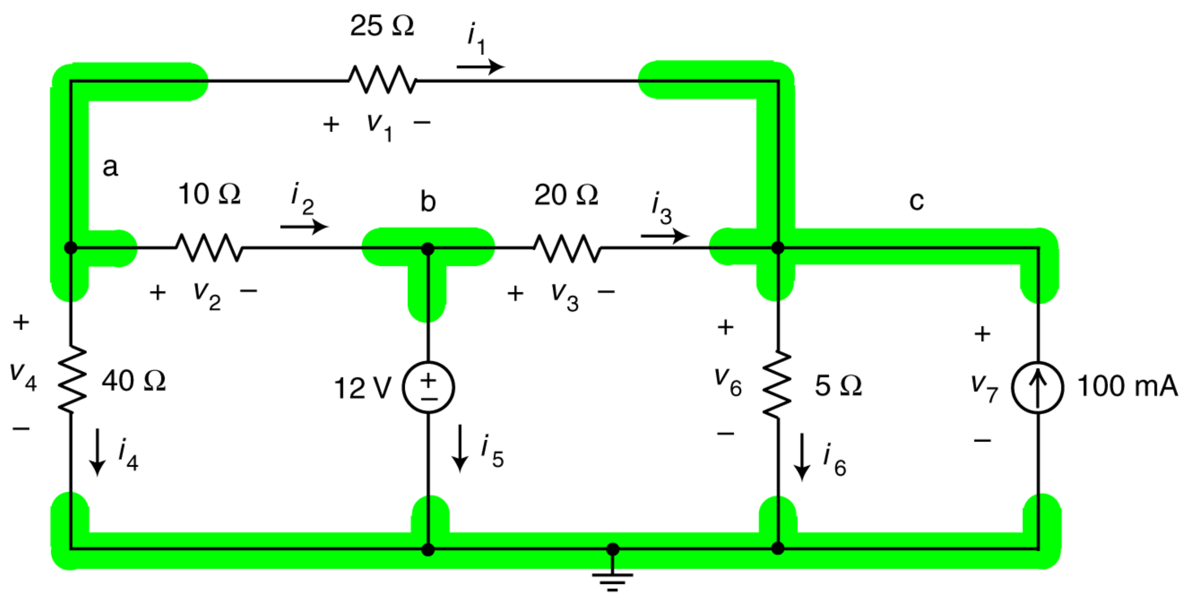
Solving simultaneously, we get

$$v_1 = 9.37 \text{ V}$$

$$v_2 = 29.67 \text{ V}$$

P4.3-13

Solution: Select the bottom node as the reference node. Label and emphasize the nodes. Label the element voltages and currents.



Express the element voltages in terms of the node voltages:

$$v_1 = v_a - v_c \ , v_2 = v_a - v_b \ , v_3 = v_b - v_c \ , v_4 = v_a \ , 12 \text{ V} = v_a \ , v_6 = v_c \ , v_7 = v_c$$

Express the element voltages in terms of the node voltages:

$$i_1 = \frac{v_a - v_c}{25} \ , i_2 = \frac{v_a - v_b}{10} \ , i_3 = \frac{v_b - v_c}{20} \ , i_4 = \frac{v_a}{40} \ , 12 \text{ V} = v_b \ , i_6 = \frac{v_c}{5} \ , i_7 = 0.1 \text{ A}$$

Apply KCL at nodes a, c, and c:

$$\text{Node a:} \quad i_1 + i_2 + i_4 = 0 \quad \Rightarrow \quad \frac{v_a - v_c}{25} + \frac{v_a - 12}{10} + \frac{v_a}{40} = 0$$

$$\text{Node b:} \quad i_2 = i_3 + i_5 = 0 \quad \Rightarrow \quad \frac{v_a - 12}{10} = \frac{12 - v_c}{20} + i_5 = 0$$

$$\text{Node c:} \quad i_1 + i_3 + 0.1 = i_6 \quad \Rightarrow \quad \frac{v_a - v_c}{25} + \frac{12 - v_c}{20} + 0.1 = \frac{v_c}{5}$$

The equation at node b involves a variable, i_5, that is not a node voltage. Set this equation aside for the moment and organize the node equations corresponding to nodes a and c into a matrix equation:

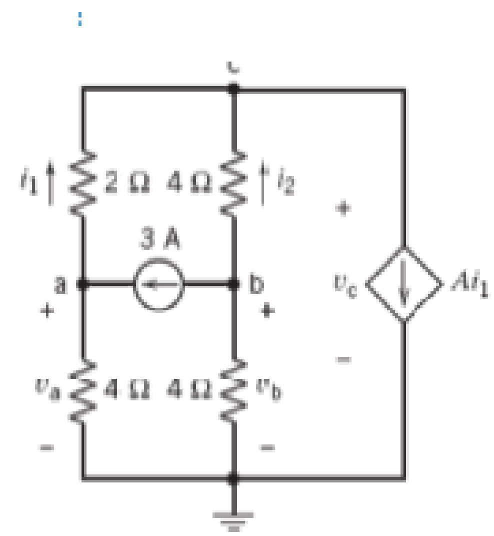
$$\left.\begin{array}{l} \left(\frac{1}{25} + \frac{1}{10} + \frac{1}{40}\right) v_a - \left(\frac{1}{25}\right) v_c = 1.2 \\ -\left(\frac{1}{25}\right) v_a + \left(\frac{1}{25} + \frac{1}{20} + \frac{1}{5}\right) v_c = 0.7 \end{array}\right\} \Rightarrow \begin{bmatrix} \frac{1}{25} + \frac{1}{10} + \frac{1}{40} & -\frac{1}{25} \\ -\frac{1}{25} & \frac{1}{25} + \frac{1}{20} + \frac{1}{5} \end{bmatrix} \begin{bmatrix} v_a \\ v_c \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.7 \end{bmatrix}$$

Solve, for example, using MATLAB to obtain v_a = 8.1297 V and v_c = 3.5351 V. Substitute these voltages into the node equation at node b to obtain i_5 =–0.8103 A.

Finally, the voltage source supplies $-12\,i_5 = 9.7236$ W and the current source supplies $0.1 v_5 = 0.1 v_c = 0.3535$ W.

Section 4-4 Node Voltage Analysis with Dependent Sources

P 4.4-1



Express the resistor currents in terms of the node voltages:

$$i_1 = \frac{v_a - v_c}{2} = \frac{8.667 - 10}{2} = -0.6 \text{ A} \quad \text{and}$$

$$i_2 = \frac{v_b - v_c}{4} = \frac{2 - 10}{4} = -2 \text{ A}$$

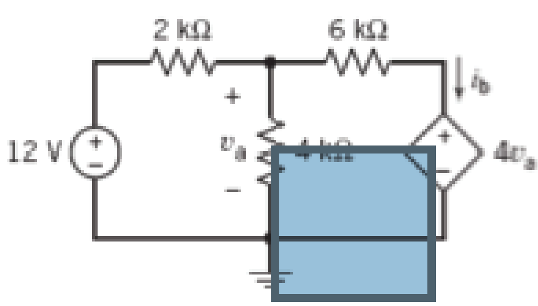
Apply KCL at node *c*:

$$i_1 + i_2 = A\, i_1 \;\Rightarrow\; -0.6 + (-2) = A\,(-0.6)$$

$$\Rightarrow\; A = \frac{-2.6}{-0.6} = 4.3$$

P 4.4-2

Solution:



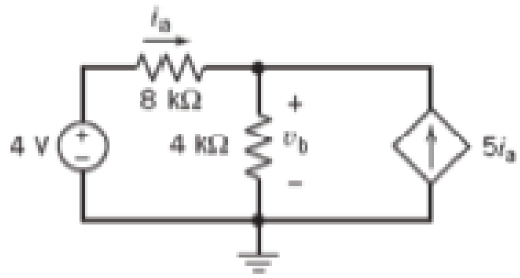
Write and solve a node equation:

$$\frac{v_a - 12}{2000} + \frac{v_a}{4000} + \frac{v_a - 4v_a}{6000} = 0 \;\Rightarrow\; v_a = 24 \text{ V}$$

$$i_b = \frac{v_a - 4v_a}{6000} = \underline{-12 \text{ mA}}$$

P 4.4-3

Solution:



First express the controlling current in terms of the node voltages:

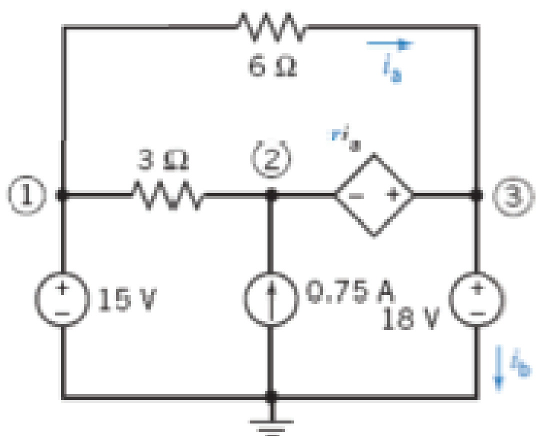
$$i_a = \frac{4 - v_b}{8000}$$

Write and solve a node equation:

$$-\frac{4-v_b}{8000} + \frac{v_b}{4000} - 5\left(\frac{4-v_b}{8000}\right) = 0 \quad \Rightarrow \quad v_b = 3 \text{ V}$$

P 4.4-4

Solution:



Apply KCL to the supernode of the CCVS to get

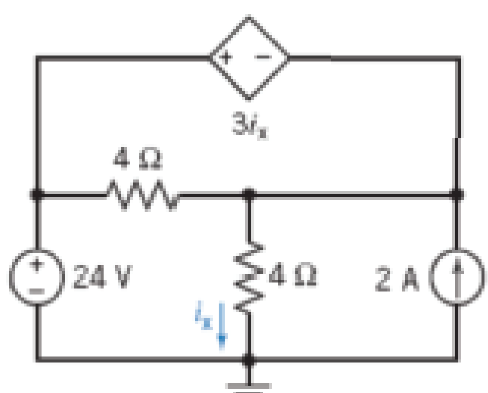
$$\frac{18-15}{6} + \frac{21-15}{3} - \frac{3}{4} + i_b = 0 \quad \Rightarrow \quad i_b = 1.75 \text{ A}$$

Next

$$\left.\begin{array}{l} i_a = \dfrac{15-18}{6} = -1/2 \\ r\, i_a = 18 - 21 \end{array}\right\} \quad \Rightarrow \quad r = \frac{-3}{-1/2} = 6\ \frac{\text{V}}{\text{A}}$$

P 4.4-5

Solution:



First, express the controlling current of the CCVS in terms of the node voltages: $i_x = \dfrac{v_2}{4}$

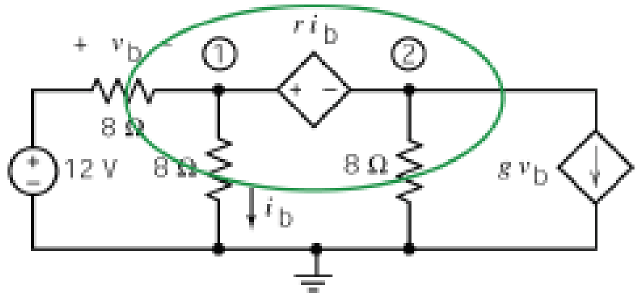
Next, express the controlled voltage in terms of the node voltages:

$$24 - v_2 = 3 i_x = 3\frac{v_2}{4} \quad \Rightarrow v_2 = \frac{96}{7} \text{ V}$$

so i_x = 24/7 A = 3.43 A.

P4.4-6

Solution:



Using Ohm's law, $i_b = \frac{v_1}{8} = \frac{9.74}{8} = 1.2175\text{ A}$. Using KVL, the voltage across the CCVS is

$$r\,i_b = v_1 - v_2 = 9.74 - 6.09 = 3.65\text{ V}$$

Then $$r = \frac{r\,i_b}{i_b} = \frac{3.65}{1.2175} = 2.9979\text{ V/A}$$

Using KVL, $v_b = 12 - v_1 = 12 - 9.74 = 2.26\text{ V}$. Apply KCL to the supernode corresponding to the CCVS to get

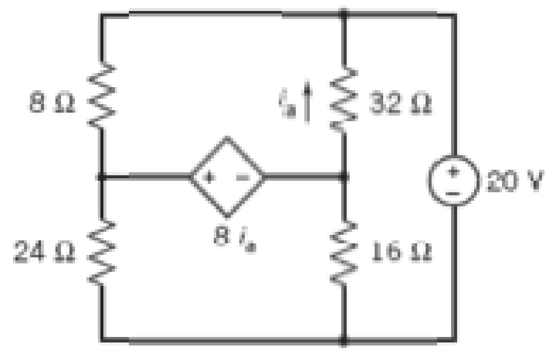
$$\frac{12 - v_1}{8} = \frac{v_1}{8} + \frac{v_2}{8} + g\,v_b \;\Rightarrow\; \frac{12 - 9.74}{8} = \frac{9.74}{8} + \frac{6.09}{8} + g\,v_b \;\Rightarrow\; g\,v_b = -1.6963\text{ A}$$

Then $$g = \frac{g\,v_b}{v_b} = \frac{-1.6963}{2.26} = -0.7506\text{ A/V}$$

P 4.4-7

Solution:

Label the node voltages.



First, v_2 = 20 V due to the independent voltage source. Next, express the controlling current of the dependent source in terms of the node voltages:

$$i_a = \frac{v_3 - v_2}{32} = \frac{v_3 - 20}{32}$$

Now the controlled voltage of the dependent source can be expressed as

$$v_1 - v_3 = 8\,i_a = 8\left(\frac{v_3 - 20}{32}\right) \quad \Rightarrow \quad v_1 = \frac{5}{4}v_3 - 5$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{v_1 - v_2}{8} + \frac{v_1}{24} + \frac{v_3 - v_2}{32} + \frac{v_3}{16} = 0$$

Multiplying by 96 and using v_2 = 20 V gives

$$16v_1 + 9v_3 = 300$$

Substituting the earlier expression for v_1

$$16\left(\frac{5}{4}v_3 - 5\right) + 9v_3 = 300 \quad \Rightarrow \quad v_3 = 13.103 \text{ V}$$

Then v_1 = 11.379 V and i_a = -0.2155 A. Applying KCL at node 2 gives

$$\frac{v_1}{24} = i_b + \frac{20 - v_1}{8} \quad \Rightarrow \quad 24\,i_b = -60 + 4\,v_1 = -60 + 4(11.379)$$

So

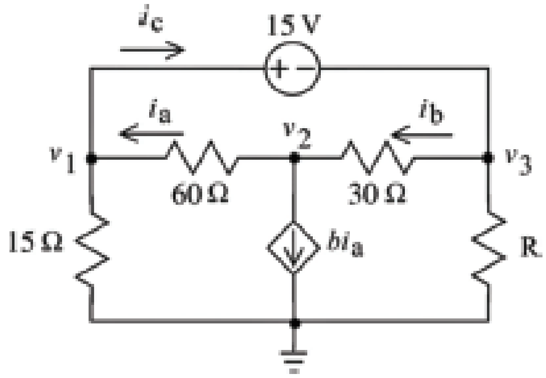
$$i_b = -0.6035.$$

Finally, the power supplied by the dependent source is

$$p = (8\,i_a)\,i_b = 8(-0.2155)(-0.6035) = 1.04 \text{ W}$$

P 4.4-8

Solution:



Apply KCL at node 2:

$$i_a + bi_a = i_b = \frac{v_3 - v_2}{30} = \frac{-9-(0)}{30} = -0.3 \text{ A}$$

but

$$i_a = \frac{v_2 - v_1}{60} = \frac{0-8}{60} = -0.13$$

so

$$(1+b)(-0.13) = (-0.3) \quad \Rightarrow \quad b = 1.308 \ \frac{\text{A}}{\text{A}}$$

Next apply KCL to the supernode corresponding to the voltage source.

$$\frac{v_1}{15} + 1.308\, i_a + \frac{v_3}{R} = 0 \quad \Rightarrow \quad \frac{8}{15} + 1.308(-0.13) + \frac{-12}{R} \quad \Rightarrow \quad R = 33.3\ \Omega$$

P 4.4-9

Solution:

(a) Express the controlling voltage of the dependent source in terms of the node voltages:

$$v_a = 15 - v_b$$

Apply KCL at node b to get

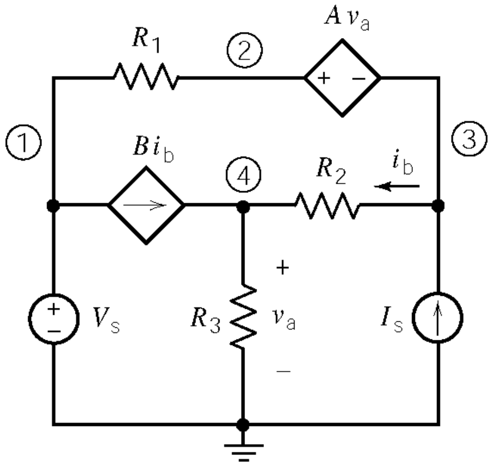
$$\frac{15 - v_b}{150} = A\left(15 - v_b\right) + \frac{v_b}{300} \quad \Rightarrow \quad A = \frac{30 - 3v_b}{300\left(15 - v_b\right)} = 0.026$$

(b) The power supplied by the dependent source is

$$-\left(A\, v_a\right)v_b = -\left(0.026(15-18)\right)(18) = 1.4 \text{ W}$$

P 4.4-10

Solution:



Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 \text{ and } i_b = \frac{v_3 - v_4}{R_2}$$

Express the voltage source voltages in terms of the node voltages:

$$v_1 = V_s \text{ and } v_2 - v_3 = A v_a = A v_4$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \quad \Rightarrow \quad -R_2 v_1 + R_2 v_2 + R_1 v_3 - R_1 v_4 = R_1 R_2 I_s$$

Apply KCL at node 4:

$$B\frac{v_3 - v_4}{R_2} + \frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} \quad \Rightarrow \quad (B+1)v_3 - \left(B + 1 + \frac{R_2}{R_3}\right)v_4 = 0$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -A \\ -R_2 & R_2 & R_1 & -R_1 \\ 0 & 0 & B+1 & -\left(B+1+\dfrac{R_2}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ R_1 R_2 I_s \\ 0 \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -10 \\ -25 & 25 & 15 & -15 \\ 0 & 0 & 6+1 & -7.7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ 1125 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 30 \\ 74.59 \\ 7.39 \\ 6.72 \end{bmatrix}$$

P 4.4-11

Solution:

Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 = 22.5 \text{ V}$$

and
$$i_b = \frac{v_3 - v_4}{R_2} = \frac{-15 - 22.5}{50} = -0.75$$

Express the dependent voltage source voltage in terms of the node voltages:

$$v_2 - v_3 = A v_a = A v_4$$

so
$$A = \frac{v_2 - v_3}{v_4} = \frac{75 - (-15)}{22.5} = 4 \text{ V/V}$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \quad \Rightarrow \quad \frac{75 - 10}{R_1} + \frac{-15 - 22.5}{50} = 2.5 \quad \Rightarrow \quad R_1 = 20 \ \Omega$$

Apply KCL at node 4:

$$\frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} + B\frac{v_3 - v_4}{R_2} \quad \Rightarrow \quad \frac{-15 - 22.5}{50} = \frac{22.5}{20} + B\,\frac{-15 - 22.5}{50} \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

(Checked using LNAP 9/29/04)

P 4.4-12

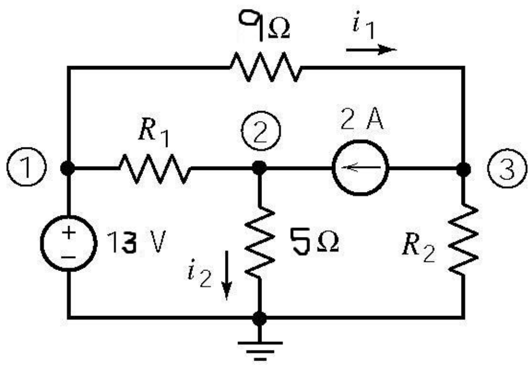
Solution:

(a) $R_1 = \dfrac{v_2 - v_1}{2 - 0.5} = \dfrac{21 - 12}{1.5} = 6 \ \Omega$ and $R_2 = \dfrac{v_2}{1.25 - 2} = \dfrac{-3}{-0.75} = 4 \ \Omega$

(b) The power supplied by the voltage source is $12(0.5 + 1.25 - 2) = -3 \text{ W}$. The power supplied by the 1.25-A current source is $1.25(-3 - 12) = -18.75 \text{ W}$. The power supplied by the 0.5-A current source is $-0.5(21) = -10.5 \text{ W}$. The power supplied by the 2-A current source is $2(21 - (-3)) = 48 \text{ W}$

P 4.4-13

Solution:



$$i_1 = \frac{13-(-2.33)}{9} = 1.703 \text{ A}$$

and

$$i_2 = \frac{10.6}{5} = 2.12 \text{ A}$$

(a) $R_1 = \dfrac{v_2 - v_1}{2 - i_2} = \dfrac{10.6-13}{2-2.12} = 20\ \Omega$ and

$$R_2 = \frac{v_3}{i_1 - 2} = \frac{-2.33}{1.703-2} = 7.84\ \Omega$$

(b) The power supplied by the voltage source is $13(2.12+1.703-2) = 23.7 \text{ W}$. The power supplied by the current source is. $2(10.6-(-2.33)) = 25.86\,\text{W}$

(Checked using LNAP 10/2/04)

P 4.4-14

Solution:

Given the node voltages $v_2 = 24$ V, $v_3 = 12$ V and $v_4 = 9$ V

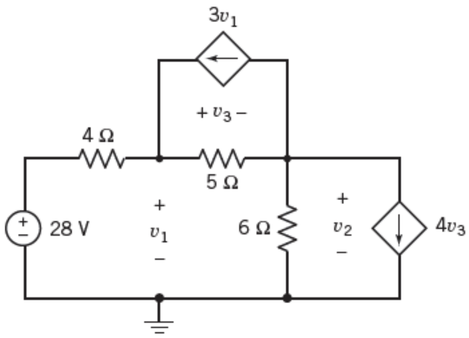
$$A = \frac{A v_a}{v_a} = \frac{24-12}{12-9} = \frac{4\text{V}}{\text{V}}$$

$$R_5\left(\frac{v_3 - v_4}{15}\right) = v_4 \quad \Rightarrow \quad R_5 = \frac{15(9)}{12-9} = 45\ \Omega,$$

$$i_b = \frac{40-36}{12} = 0.33 \text{ A} \text{ and } i_c = \frac{40-24}{12} - \frac{24}{12} = 0.66 \text{ A}$$

$$p_4 = \frac{{v_a}^2}{15} = \frac{3^2}{15} = 0.6 \text{ W}$$

P 4.4-15

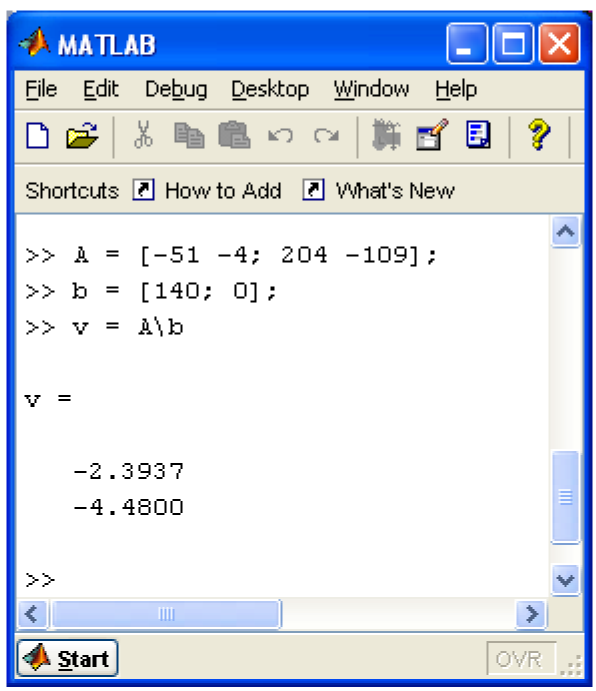


The node equations are

$$\frac{28-v_1}{4}+3v_1=\frac{v_1-v_2}{5} \quad\Rightarrow\quad 5(28-v_1)+20(3v_1)=4(v_1-v_2) \quad\Rightarrow\quad 140=-51v_1-4v_2$$

and
$$\frac{v_1-v_2}{5}=3v_1+\frac{v_2}{6}+4v_3=3v_1+\frac{v_2}{6}+4(v_1-v_2) \quad\Rightarrow\quad 0=204v_1-109v_2$$

Using MATLAB to solve these equations:



Consequently $\qquad v_1 = -2.3937\text{ V}$ and $v_2 = -4.4800\text{ V}$

P 4.4-16

Solution:

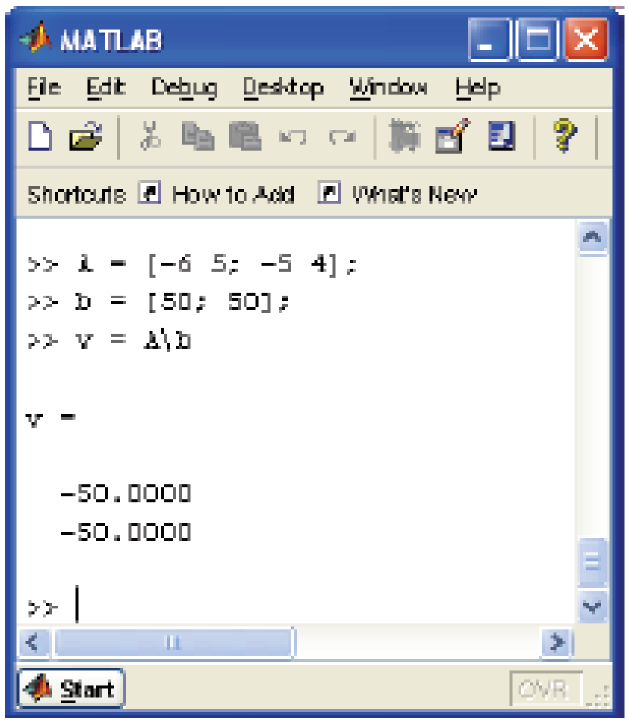
Apply KCL to the supernode corresponding to the horizontal voltage source to get

$$\frac{v_1}{10} = \frac{v_a}{2} = \frac{v_3 - v_2}{2} = \frac{v_3 - (v_1 + 10)}{2} \quad \Rightarrow \quad v_1 = 5\left(v_3 - (v_1 + 10)\right) \quad \Rightarrow \quad 50 = -6v_1 + 5v_3$$

Looking at the dependent source we notice that

$$v_3 = 5v_a = 5(v_3 - v_2) = 5\left(v_3 - (v_1 + 10)\right) \quad \Rightarrow \quad 50 = -5v_1 + 4v_3$$

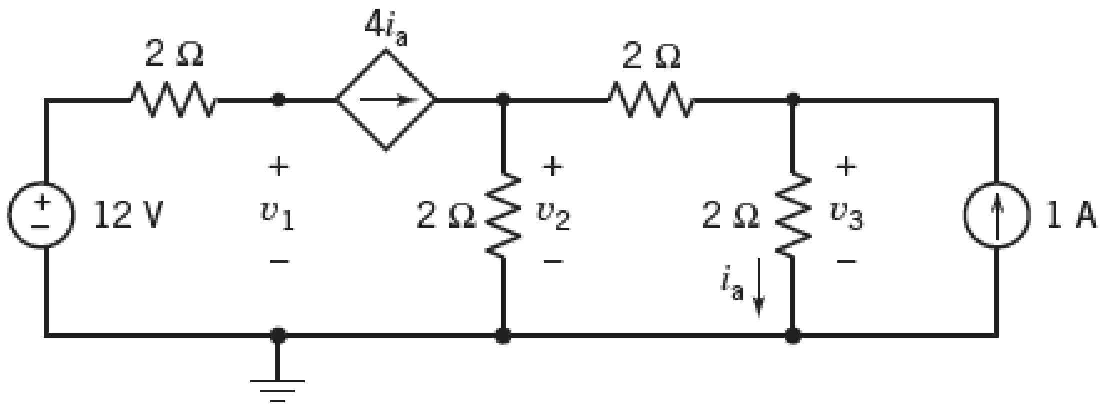
Using MATLAB to solve these equations:



Consequently $\qquad v_1 = -50\text{ V}$ and $v_3 = -50\text{ V}$

Then $\qquad v_2 = v_1 + 10 = -40\text{ V}$

P 4.4-17



Solution:

The node equations are:

$$\frac{12-v_1}{2}=4i_a=4\left(\frac{v_3}{2}\right) \Rightarrow 12-v_1=4v_3 \Rightarrow 12=v_1+4v_3$$

$$4i_a=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 4\left(\frac{v_3}{2}\right)=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 0=2v_2-5v_3$$

$$\frac{v_2-v_3}{2}+1=\frac{v_3}{2} \Rightarrow v_2-v_3+2=v_3 \Rightarrow 2=-v_2+2v_3$$

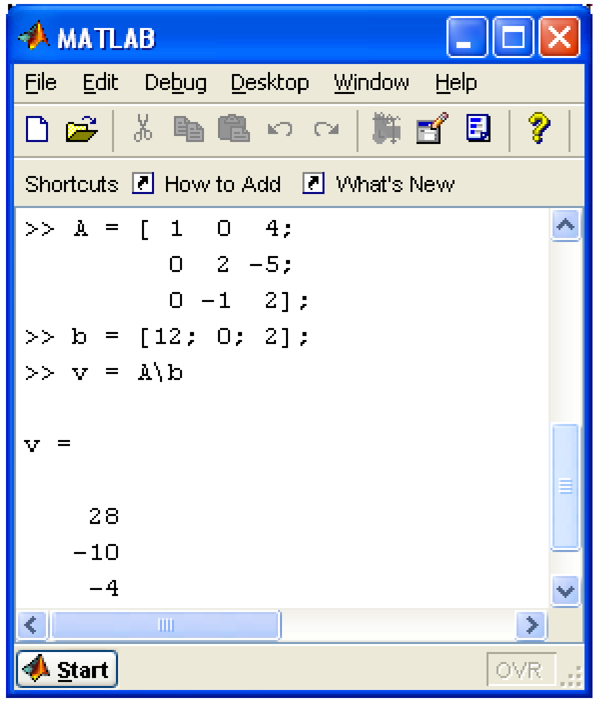
Solving these equations using MATLAB gives

$$v_1 = 28 \text{ V},$$

$$v_2 = -10 \text{ V}$$

and

$$v_3 = -4 \text{ V}$$



Section 4-5 Mesh Current Analysis with Independent Voltage Sources

P 4.5-1

Solution:
The mesh equations are

$$4\, i_1 + 18\,(i_1 - i_3) + 6\,(i_1 - i_2) = 0$$
$$30 - 6\,(i_1 - i_2) + 12\,(i_2 - i_3) = 0$$
$$-12\,(i_2 - i_3) - 18\,(i_1 - i_3) - 42 = 0$$

or

$$28\, i_1 - 6\, i_2 - 18\, i_3 = 0$$
$$-6\, i_1 + 18\, i_2 - 12\, i_3 = -30$$
$$-18\, i_1 - 12\, i_2 + 30\, i_3 = 42$$

so

$$i_1 = 3 \text{ A},\ \ i_2 = 2 \text{ A and } i_3 = 4 \text{ A}.$$

P 4.5-2

Solution:
Top mesh:

$$8(4-6)+R(4)+20(4-8)=0$$

so $R = 24\ \Omega$.

Bottom, right mesh:

$$16(8-6)+20(8-4)+v_2=0$$

so $v_2 = -112$ V.

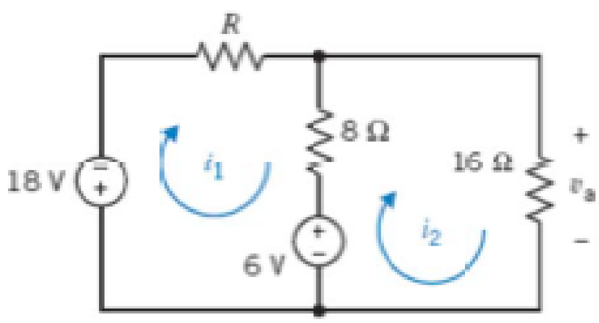
Bottom left mesh

$$-v_1+8(6-4)+16(6-8)=0$$

so $v_1 = -16$ V.

P 4.5-3

Solution:



Ohm's Law: $i_2 = \dfrac{-6}{16} = -0.375$ A

KVL for loop 1:

$$R\,i_1+8(i_1-i_2)+6+18=0$$

KVL for loop 2

$$+(-6)-6-8(i_1-i_2)=0$$

$$\Rightarrow\quad -12-8(i_1-(-0.376))=0$$

$$\Rightarrow\quad i_1 = \ -1.8\text{ A}$$

$$R(-1.9)+8(-1.9-(-0.375))+42=0 \quad\Rightarrow\quad R=15.7\ \Omega$$

P 4.5-4

Solution:
KVL loop 1:

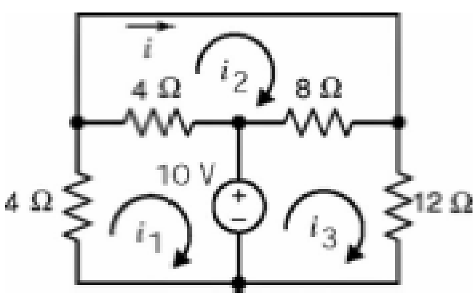
$$50\,i_a - 2 + 275\,i_a + 100\,i_a + 4 + 125\,(i_a - i_b) = 0$$
$$550\,i_a - 125\,i_b = -2$$

KVL loop 2:

$$-125(i_a - i_b) - 4 + 125\,i_b + 125\,i_b + 8 + 225\,i_b = 0$$
$$-125\,i_a + 600\,i_b = -4$$
$$\Rightarrow \underline{i_a = -5.4\text{ mA}},\ \underline{i_b = -7.79\text{ mA}}$$

P 4.5-5

Solution:



Mesh Equations:

$$\text{mesh 1}:\ 4i_1 + 4\,(i_1 - i_2) + 10 = 0$$
$$\text{mesh 2}:\ 4(i_2 - i_1) + 8\,(i_2 - i_3) = 0$$
$$\text{mesh 3}:\ -10 + 8(i_3 - i_2) + 12 i_3 = 0$$

Solving:

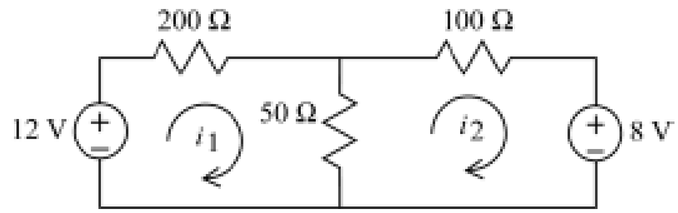
$$i = i_2 \Rightarrow i = -\frac{5}{34} = -0.147\text{ A}$$

P 4.5-6

Solution:
Replace series and parallel resistors with equivalent resistors:

$$60\ \Omega \,||\, 300\ \Omega = 50\ \Omega,\quad 40\ \Omega + 60\ \Omega = 100\ \Omega \quad \text{and} \quad 100\ \Omega + 30\ \Omega + \left(80\ \Omega \,||\, 560\ \Omega\right) = 200\ \Omega$$

so the simplified circuit is



The mesh equations are

$$200\, i_1 + 50\left(i_1 - i_2\right) - 12 = 0$$
$$100\, i_2 + 8 - 50\left(i_1 - i_2\right) = 0$$

or

$$\begin{bmatrix} 250 & -50 \\ -50 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.04 \\ -0.04 \end{bmatrix}$$

The power supplied by the 12 V source is $12\, i_1 = 12\left(0.04\right) = 0.48\ \text{W}$. The power supplied by the 8 V source is $-8 i_2 = -8\left(-0.04\right) = 0.32\ \text{W}$. The power absorbed by the 30 Ω resistor is

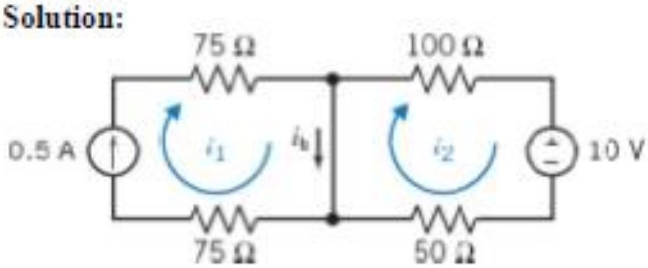
$$i_1^{\,2}\left(30\right) = \left(0.04\right)^2\left(30\right) = 0.048\ \text{W}.$$

(checked: LNAP 5/31/04)

Section 4-6 Mesh Current Analysis with Voltage and Current Sources

P 4.6-1

Solution:



$$\text{mesh 1:}\quad i_1 = \frac{1}{2}\text{ A}$$

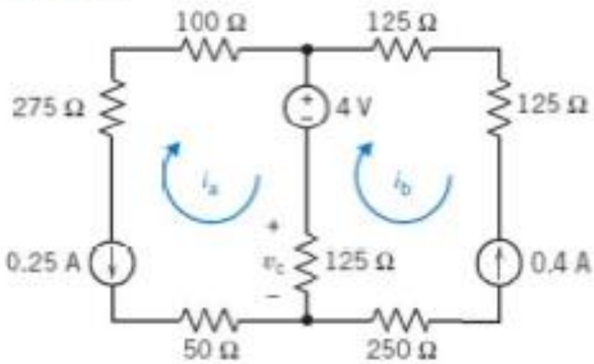
$$\text{mesh 2:}\quad 100\,i_2 + 10 + 50 i_2 = 0$$

$$\Rightarrow i_2 = -0.07\text{ A}$$

$$i_b = i_1 - i_2 = \underline{0.57\text{ A}}$$

P 4.6-2

Solution:



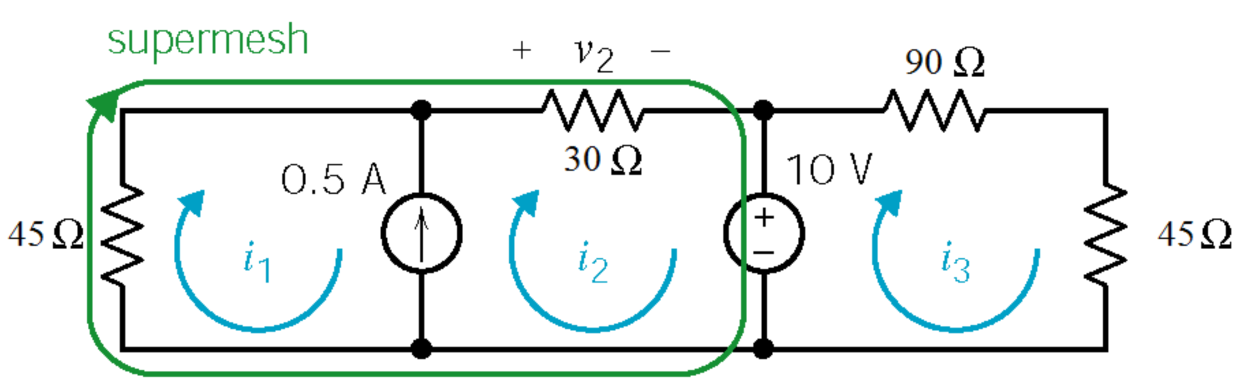
$$\text{mesh a:}\quad i_a = -0.25\text{ A}$$

$$\text{mesh b:}\quad i_b = -0.4\text{ A}$$

$$v_c = 125(i_a - i_b) = 125(0.15) = \underline{18.7\text{ V}}$$

P 4.6-3

Solution:



Express the current source current as a function of the mesh currents:

$$i_1 - i_2 = -0.5 \Rightarrow i_1 = i_2 - 0.5$$

Apply KVL to the supermesh:

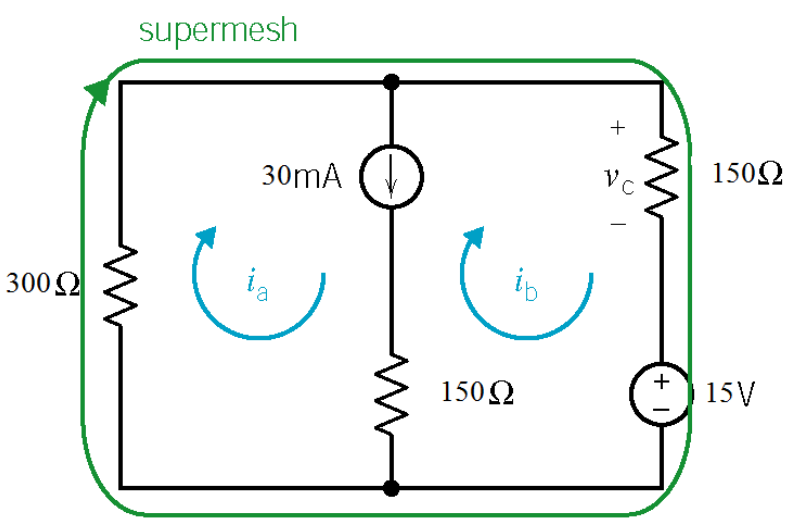
$$45\, i_1 + 30\, i_2 + 10 = 0 \Rightarrow 45\,(i_2 - 0.5) + 30 i_2 = -10$$

$$75\, i_2 - 22.5 = -10 \Rightarrow i_2 = \frac{12.5}{75} = 0.17 \text{ A}$$

$$i_1 = -0.33 \text{ A} \quad \text{and} \quad v_2 = 20\, i_2 = 3.4 \text{ V}$$

P 4.6-4

Solution:



Express the current source current in terms
of the mesh currents:

$$i_b = i_a - 0.03$$

Apply KVL to the supermesh:

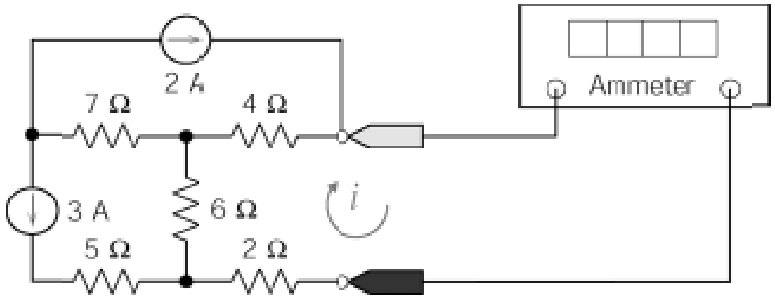
$$300\, i_a + 150\,(i_a - 0.03) + 15 = 0$$

$$\therefore i_a = -0.023 \text{ A} = -23 \text{ mA}$$

$$v_c = 150(i_a - 0.03) = \underline{-7.95 \text{ V}}$$

P 4.6-5

Solution:



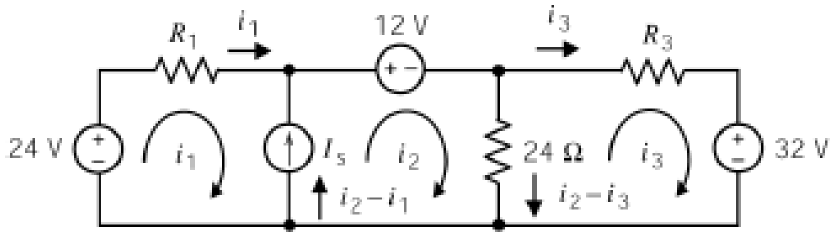
Mesh equation for right mesh:

$$4(i-2)+2i+6(i+3)=0 \quad \Rightarrow \quad 12i-8+18=0 \quad \Rightarrow \quad i=-\frac{10}{12}\text{ A}=-\frac{5}{6}\text{ A}$$

(checked using LNAP 8/14/02)

P4.6-6

Solution: Label the resistor currents and the current source currents in terms of the mesh currents:



a.) Apply KVL to the supermesh corresponding to the current source to get

$$R_1 i_1+12+24(i_2-i_3)-24=0 \quad \Rightarrow \quad R_1=\frac{12-24(i_2-i_3)}{i_1}=\frac{12-24(0.8986-(-0.2899))}{-1.1014}=15\ \Omega$$

Apply KVL to the rightmost mesh to get

$$R_3 i_3+32-24(i_2-i_3)=0 \quad \Rightarrow \quad R_3=\frac{-32+24(i_2-i_3)}{i_3}=\frac{-32+24(0.8986-(-0.2899))}{-0.2899}=12\ \Omega$$

b.)

$$I_s=i_2-i_1=0.8986-(-1.1014)=2\text{ A}$$

c.) Noticing that 12 V and i_2 adhere to the passive convention, the power supplied by the 12 V voltage source is

$$-12i_2=-12(0.8986)=-10.783\text{ W}.$$

P 4.6-7

Solution: Use units of V, mA and kΩ. Express the currents to the supermesh to get

$$i_1 - i_3 = 6$$

Apply KVL to the supermesh to get

$$12\left(i_3 - i_2\right) + (3)\,i_3 - 9 + (3)\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad 3i_1 - 15i_2 + 15i_3 = 9$$

Apply KVL to mesh 2 to get

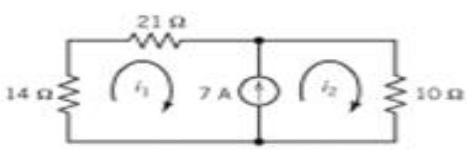
$$6i_2 + 12\left(i_2 - i_3\right) + (3)\left(i_2 - i_1\right) = 0 \quad \Rightarrow \quad (-3)\,i_1 + 21i_2 - 12i_3 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & -15 & 15 \\ -3 & 21 & -12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6.513 \\ 1.23 \\ 1.935 \end{bmatrix}$$

(checked: LNAP 6/21/04)

P4.6-8



Solution: Express the currents to the supermesh to get

$$i_2 - i_1 = 7$$

Apply KVL to the supermesh to get

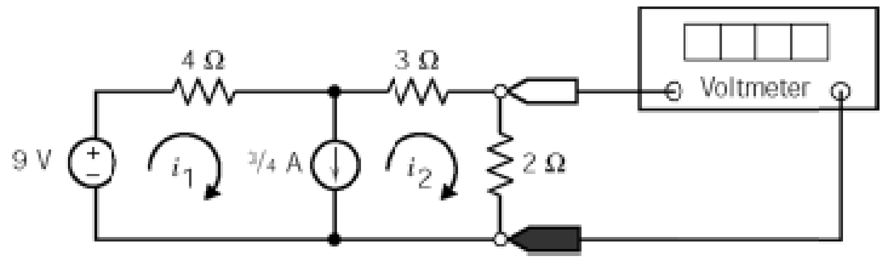
$$(i_1)\,(14 + 21) + 10i_2 = 0$$
$$\Rightarrow \quad (35)i_1 + 10\,(7 + i_1) = 0$$
$$\Rightarrow i_1 = -1.55 \text{ A}$$

Therefore,

$$i_2 = 7 + i_1$$
$$= 7 - 1.55$$
$$= 5.44 \text{ A}$$

P 4.6-9

Solution:



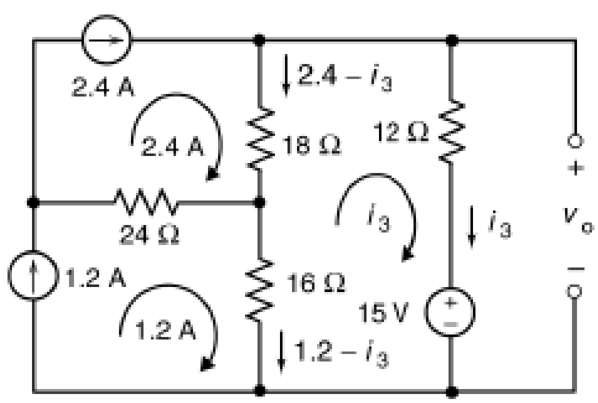
Express the current source current in terms of the mesh currents: $\frac{3}{4} = i_1 - i_2 \;\Rightarrow\; i_1 = \frac{3}{4} + i_2$.

Apply KVL to the supermesh: $-9 + 4i_1 + 3\,i_2 + 2\,i_2 = 0 \;\Rightarrow\; 4\left(\frac{3}{4} + i_2\right) + 5\,i_2 = 9 \;\Rightarrow\; 9\,i_2 = 6$

so $i_2 = \frac{2}{3}$ A and the voltmeter reading is $2\,i_2 = \frac{4}{3}$ V

P4.6-10

Solution: Notice that the current source are each in a single mesh. Consequently, i_1 = 2.4 A and i_2 = 1.2 A. Label the resistor currents in terms of the mesh currents:



Apply KVL to mesh 3 to get

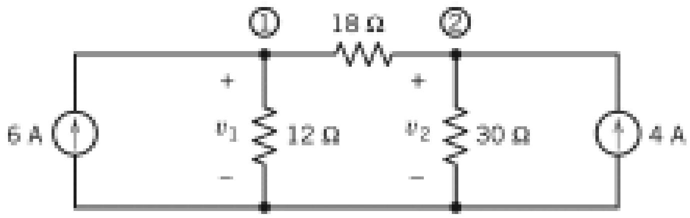
$$12\,i_3 + 15 - 16\left(1.2 - i_3\right) - 18\left(2.4 - i_3\right) = 0 \;\Rightarrow\; 46\,i_3 = 47.4 \;\Rightarrow\; i_3 = 1.0304 \text{ A}$$

Apply KVL to the rightmost mesh to get

$$v_o - 15 - 12\,i_3 = 0 \;\Rightarrow\; v_o = 15 + 12\left(1.0304\right) = 27.3648 \text{ V}$$

P4.6-11

Solution:



Node equations:

Apply KCL at node 1,

$$-6+\frac{v_1}{12}+\frac{v_1-v_2}{18}=0 \;\Rightarrow 5v_1-2v_2=216$$

Apply KCL at node 2,

$$\frac{v_2-v_1}{18}+\frac{v_2}{30}-4=0 \;\Rightarrow 8v_2-5v_1=360$$

Solving gives,

$$v_1=81.6\text{V}$$
$$v_2=96\text{V}$$

The power supplied by 6A current source is,

$$\begin{aligned}P&=(6)\,(81.6)\\&=489.6\text{W}\end{aligned}$$

The power supplied by 4 A current source is,

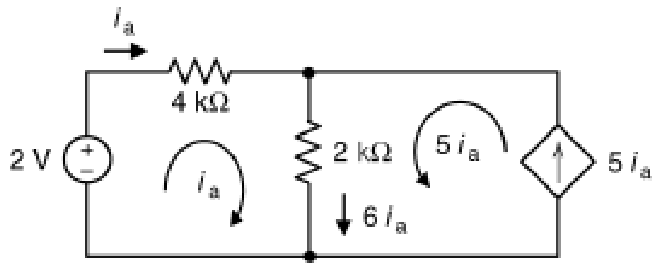
$$\begin{aligned}P&=(4)\,(96)\\&=384\text{W}\end{aligned}$$

Section 4-7 Mesh Current Analysis with Dependent Sources

P4.7-1

Solution:

First, label the mesh currents, taking advantage of the current sources. Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the left mesh: $4000\,i_a + 2000\left(6\,i_a\right) - 2 = 0 \;\Rightarrow\; i_a = \dfrac{1}{8} = 0.125\ \text{mA}$

The 2 A voltage source supplies $2\,i_a = 2\left(0.125\times10^{-3}\right) = 0.25\ \text{mW}$

The CCCS supplies $\left(5\,i_a\right)\left[(2000)\left(6\,i_a\right)\right] = \left(60\times10^{3}\right)\left(0.125\times10^{-3}\right)^{2} = 0.9375\times10^{-3} = 0.9375\ \text{mW}$

P 4.7-2

Solution:

Express the controlling current of the dependent source as a function of the mesh current:

$$i_b = .06 - i_a$$

Apply KVL to the right mesh:

$$-100\ (0.06 - i_a) + 50\ (0.06 - i_a) + 250\ i_a = 0 \quad\Rightarrow\quad i_a = 10\ \text{mA}$$

Finally: $$v_o = 50\,i_b = 50\,(0.06 - 0.01) = 2.5\ \text{V}$$

(checked using LNAP 8/14/02)

P 4.7-3

Solution:

Express the controlling voltage of the dependent source as a function of the mesh current:

$$v_b = 100\ (.006 - i_a)$$

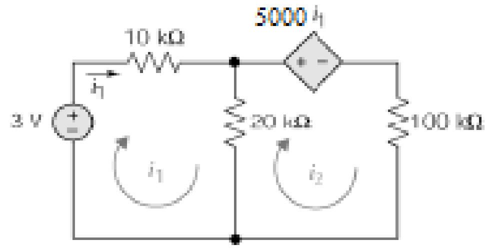
Apply KVL to the right mesh:

$$-100\ (.006 - i_a) + 3\left[100(.006 - i_a)\right] + 250\,i_a = 0 \quad\Rightarrow\quad \underline{i_a = -24\ \text{mA}}$$

(checked using LNAP 8/14/02)

P 4.7-4

Solution:



Apply KVL to left mesh: $-3+10\times10^3\, i_1+20\times10^3\,(i_1-i_2)=0 \Rightarrow \underline{30\times10^3\, i_1-20\times10^3\, i_2=3}$ (1)

Apply KVL to right mesh: $5\times10^3\, i_1+100\times10^3\, i_2+20\times10^3\,(i_2-i_1)=0 \Rightarrow \underline{i_1=8 i_2}$ (2)

Solving (1) & (2) simultaneously $\Rightarrow$ $i_1=\frac{6}{55}$ mA, $i_2=\frac{3}{220}$ mA

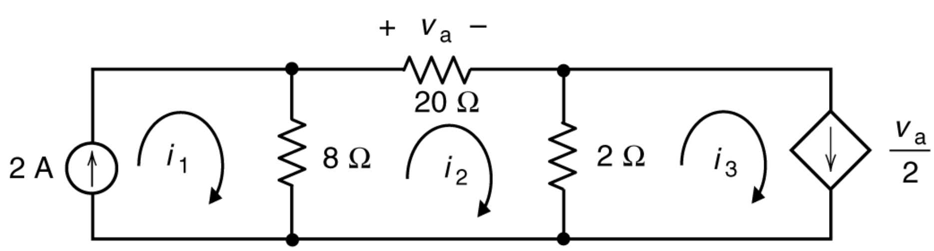
Power delivered to cathode $=(5\,i_1)\,(i_2)+100\,(i_2)^2$

$=5\left(\frac{6}{55}\right)\left(\frac{3}{220}\right)+100\left(\frac{3}{220}\right)^2=0.026$ mW

$\therefore$ Energy in 24 hr. $=\left(2.6\times10^{-5}\text{ W}\right)(24\text{ hr})\left(3600\ \text{s/hr}\right)=2.25$ J

P4.7-5

Solution: First, label the mesh currents.

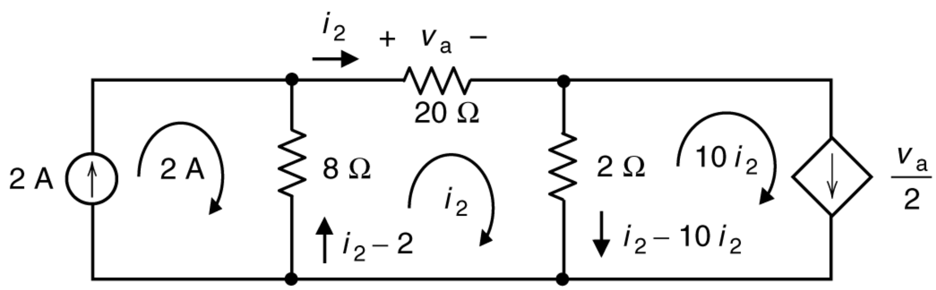


Next, express the controlling voltage of the VCCS in terms of the mesh currents:

$$v_a = 20\, i_2$$

Notice that $i_1 = 2$ A and $i_3 = \frac{v_a}{2} = 10\, i_2$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $20 i_2 + 2\left(i_2 - 10 i_2\right) + 8\left(i_2 - 2\right) = 0 \quad \Rightarrow \quad i_2 = 1.6 \text{ A}$

Consequently $v_a = 20 i_2 = 20(1.6) = 32 \text{ V}$ and $i_3 = \frac{v_a}{2} = \frac{32}{2} = 16 \text{ A}$

The VCCS supplies $\frac{v_a}{2}\left[2\left(i_3 - i_2\right)\right] = \frac{32}{2}(2)(16 - 1.6) = 460.8 \text{ W}$

P 4.7-6

Solution:

Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5\left(i_2 - i_3\right) \text{ and } i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\, i_b = -20\, i_2 \text{ and } 3\, v_a = 15\left(i_2 - i_3\right)$$

Apply KVL to the meshes

$$-15\left(i_2 - i_3\right) + \left(-20\, i_2\right) + 10\, i_1 = 0$$
$$-\left(-20\, i_2\right) + 5\left(i_2 - i_3\right) + 20\, i_2 = 0$$
$$10 - 5\left(i_2 - i_3\right) + 15\,\left(i_2 - i_3\right) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

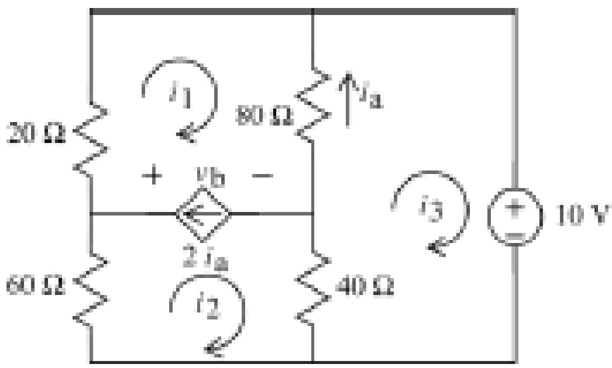
Solving, e.g. using MATLAB, gives

$$i_1 = -1.25 \text{ A}, \; i_2 = +0.125 \text{ A, and } i_3 = +1.125 \text{ A}$$

(checked: MATLAB & LNAP 5/19/04)

P 4.7-7

Solution:
Label the mesh currents:



Express i_a, the controlling current of the CCCS, in terms of the mesh currents

$$i_a = i_3 - i_1$$

Express 2 i_a, the controlled current of the CCCS, in terms of the mesh currents:

$$i_1 - i_2 = 2\,i_a = 2(i_3 - i_1) \;\Rightarrow\; 3\,i_1 - i_2 - 2\,i_3 = 0$$

Apply KVL to the supermesh corresponding to the CCCS:

$$80(i_1 - i_3) + 40(i_2 - i_3) + 60\,i_2 + 20\,i_1 = 0 \quad\Rightarrow\quad 100 i_1 + 100 i_2 - 120 i_3 = 0$$

Apply KVL to mesh 3

$$10 + 40(i_3 - i_2) + 80(i_3 - i_1) = 0 \quad\Rightarrow\quad -80\,i_1 - 40\,i_2 + 120\,i_3 = -10$$

These three equations can be written in matrix form

$$\begin{bmatrix} 3 & -1 & -2 \\ 100 & 100 & -120 \\ -80 & -40 & 120 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -0.2\text{ A},\; i_2 = -0.1\text{ A and } i_3 = -0.25\text{ A}$$

Apply KVL to mesh 2 to get

$$v_b + 40(i_2 - i_3) + 60 i_2 = 0 \;\Rightarrow\; v_b = -40(-0.1-(-0.25)) - 60(-0.1) = 0\text{ V}$$

So the power supplied by the dependent source is $p = v_b(2 i_a) = 0\text{ W}$.

(checked: LNAP 6/7/04)

P 4.7-8

Solution:
The controlling and controlled currents of the CCCS, i_b and $40\,i_b$, are the mesh currents. Apply KVL to the left mesh to get

$$1000\,i_b + 2000\,i_b + 300\left(i_b + 40 i_b\right) - v_s = 0 \quad \Rightarrow \quad 15300 i_b = v_s$$

The output is given by
$$v_o = -3000\left(40\,i_b\right) = -120000\,i_b$$

(a) The gain is
$$\frac{v_o}{v_s} = -\frac{120000}{15300} = -7.84 \text{ V/V}$$

(b) The input resistance is
$$\frac{v_s}{i_b} = 15300\ \Omega$$

(checked: LNAP 5/24/04)

P 4.7-9

Solution:
Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = 20\left(i_1 - i_2\right) = 20\left(-1.375 - (-2.5)\right) = 22.5$$

and

$$i_b = i_3 - i_2 = -3.25 - (-2.5) = -0.75 \text{ A}$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -2.5 \text{ A}$$

and

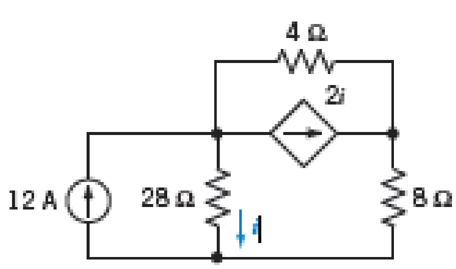
$$i_2 - i_1 = B i_b \quad \Rightarrow \quad -1.375 - (-2.5) = B(-0.75) \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

Apply KVL to the supermesh corresponding to the dependent current source

$$0 = 20\,i_3 + A v_a + 50\,i_b + v_a - 10 = 20(-3.25) + A(22.5) + 50(-0.75) + 22.5 - 10 \quad \Rightarrow \quad A = 4 \text{ V/V}$$

(Checked using LNAP 9/29/04)

P 4.7-11



Solution:

Label the node voltages as shown. The controlling currents of the CCCS is expressed as $i = \dfrac{v_a}{28}$.

The node equations are
$$12 = \frac{v_a}{28} + \frac{v_a - v_b}{4} + \frac{v_a}{14}$$

and
$$\frac{v_a - v_b}{4} + \frac{v_a}{14} = \frac{v_b}{8}$$

Solving the node equations gives $v_a = 84$ V and $v_b = 72$ V. Then $i = \dfrac{v_a}{28} = \dfrac{84}{28} = 3$ A.

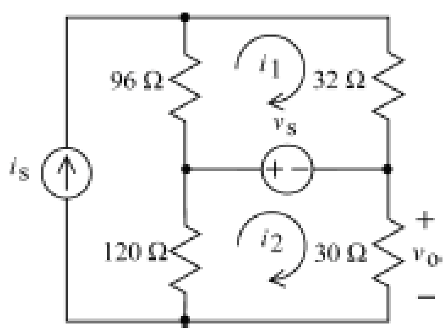
(checked using LNAP 6/16/05)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-1

Solution:

(a)

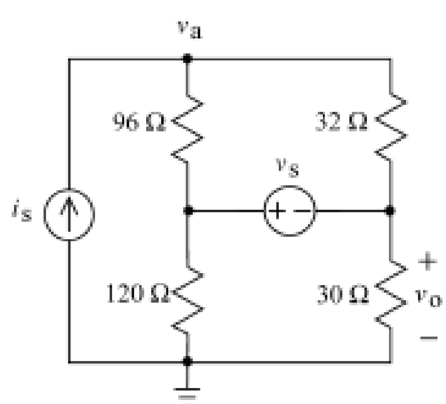


Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96(i_1 - i_s) = 0$$
$$v_s + 30 i_2 + 120(i_2 - i_s) = 0$$
$$150 i_2 = +120 i_s - v_s$$
$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$
$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So $a = 24$ and $b = -.02$.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - (v_s + v_o)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So
$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then
$$v_o = 24 i_s - \frac{1}{5} v_s$$

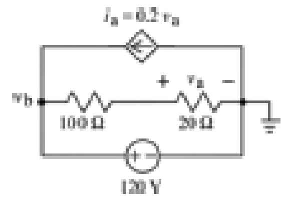
So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-2

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120 \text{ V}$$

due to the voltage source.

Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{100} = \frac{v_a}{20} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

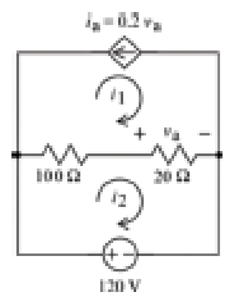
Then

$$i_a = 0.2(20) = 4 \text{ A}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)(4) = 480 \text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 20(i_2 - i_1)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[20(i_2 - i_1)\right] = 4i_2 - 4i_1 \quad \Rightarrow \quad i_1 = 4/3 i_2$$

Apply KVL to the bottom mesh to get

$$100(i_2 - i_1) + 20(i_2 - i_1) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 1$$

So

$$i_2 - 4/3 i_2 = 1 \quad \Rightarrow \quad i_2 = -3 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$$

Then

$$v_a = 20(-3 - (-4)) = 20 \text{ V and } i_a = 0.2(20) = 4 \text{ A}$$

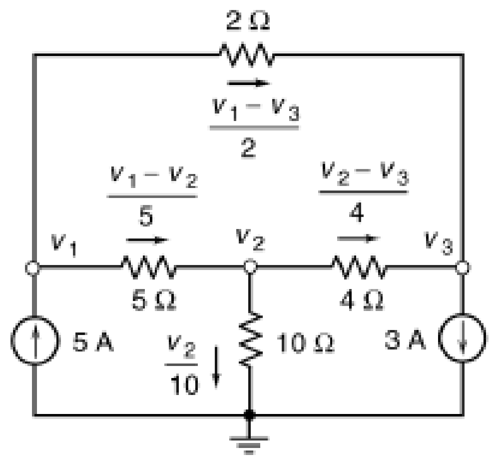
The power supplied by the dependent source is

$$p = 120(i_a) = 120(4) = 480 \text{ W}$$

Section 4.9 Circuit Analysis Using MATLAB

P4.9-1

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $5 = \dfrac{v_1 - v_2}{5} + \dfrac{v_1 - v_3}{2} \Rightarrow 0.7v_1 - 0.2v_2 - 0.5v_3 = 5$

Apply KCL at node 2 to get $\dfrac{v_1 - v_2}{5} = \dfrac{v_2}{10} + \dfrac{v_2 - v_3}{4} \Rightarrow -0.2v_1 + 0.55v_2 - 0.25v_3 = 0$

Apply KCL at node 3 to get $\dfrac{v_2 - v_3}{4} + \dfrac{v_1 - v_3}{2} = 3 \Rightarrow -0.5v_1 - 0.25v_2 + 0.75v_3 = -3$

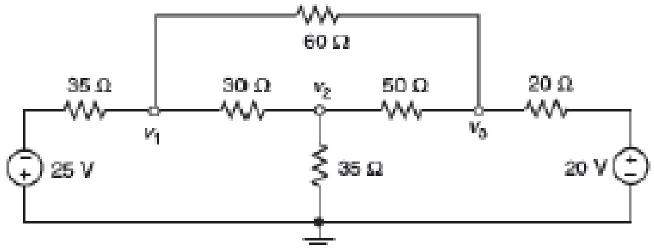
In matrix form:
$$\begin{bmatrix} 0.7 & -0.2 & -0.5 \\ -0.2 & 0.55 & -0.25 \\ -0.5 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

Solving using MATLAB: $v_1 = 28.1818 \text{ V}, \; v_2 = 20 \text{ V}$ and $v_3 = 21.4545$

P4.9-2

Solution:

The figure below shows the assumed direction of current entering or leaving nodes 1, 2 and 3 respectively.



KCL at node 1 gives:

$$\frac{v_1 + 25\text{ V}}{35\,\Omega} + \frac{v_1 - v_3}{60\,\Omega} + \frac{v_1 - v_2}{30\,\Omega} = 0$$

$$\frac{1}{5}\left(\frac{v_1 + 25\text{ V}}{7\,\Omega} + \frac{v_1 - v_3}{12\,\Omega} + \frac{v_1 - v_2}{6\,\Omega}\right) = 0$$

$$\frac{v_1 + 25\text{ V}}{7\,\Omega} + \frac{v_1 - v_2}{6\,\Omega} + \frac{v_1 - v_3}{12\,\Omega} = 0$$

$$72v_1 + 1800\text{ V} + 84v_1 - 84v_2 + 42v_1 - 42v_3 = 0$$

$$198v_1 - 84v_2 - 42v_3 + 1800 = 0$$

$$99v_1 - 42v_2 - 21v_3 + 900 = 0 \qquad \ldots\ldots (1)$$

KCL at node 2 gives:

$$\frac{v_1 - v_2}{30\,\Omega} = \frac{v_2}{35\,\Omega} + \frac{v_2 - v_3}{50\,\Omega}$$

$$\frac{1}{5}\left(\frac{v_2}{7\,\Omega} + \frac{v_2 - v_3}{10\,\Omega} - \left(\frac{v_1 - v_2}{6\,\Omega}\right)\right) = 0$$

$$\frac{v_2}{7\,\Omega} + \frac{v_2 - v_3}{10\,\Omega} - \left(\frac{v_1 - v_2}{6\,\Omega}\right) = 0$$

$$60v_2 + 42v_2 - 42v_3 - 70v_1 + 70v_2 = 0$$

$$172v_2 - 42v_3 - 70v_1 = 0$$

$$86v_2 - 21v_3 - 35v_1 = 0 \qquad \ldots\ldots (2)$$

KCL at node 3 gives:

$$\frac{v_1 - v_3}{60\,\Omega} + \frac{v_2 - v_3}{50\,\Omega} = \frac{v_3 - 20\text{ V}}{20\,\Omega}$$

$$\frac{1}{5}\left(\frac{v_1 - v_3}{12\,\Omega} + \frac{v_2 - v_3}{10\,\Omega} - \left(\frac{v_3 - 20\text{ V}}{4\,\Omega}\right)\right) = 0$$

$$\frac{v_1 - v_3}{12\,\Omega} + \frac{v_2 - v_3}{10\,\Omega} - \left(\frac{v_3 - 20\text{ V}}{4\,\Omega}\right) = 0$$

$$40v_1 - 40v_3 + 48v_2 - 48v_3 - 120v_3 + 2400 = 0$$

$$40v_1 - 208v_3 + 48v_2 + 2400 = 0$$

$$20v_1 - 104v_3 + 24v_2 + 1200 = 0 \qquad \ldots\ldots (3)$$

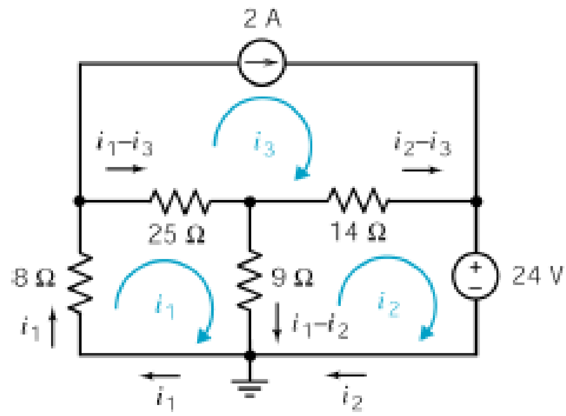
Equations (1), (2), (3) fpr a system of linear equations, and can be solved to obtain the solution for v_1, v_2, and v_3 as:

$$\begin{aligned} v_1 &= -\frac{6070}{849}\text{ V} \\ &= -7.14\text{ V} \\ v_2 &= -\frac{385}{849}\text{ V} \\ &= -0.45\text{ V} \\ v_3 &= \frac{8540}{849}\text{ V} \\ &= 10.0\text{ V} \end{aligned}$$

The negative sign highlights the fact that the direction of the associated current is to be reversed. Therefore, the voltages at node 1, node 2 and node 3, with the assumed directions is, $\boxed{-7.14\text{ V}}$, $\boxed{-0.45\text{ V}}$, and $\boxed{10.0\text{ V}}$ respectively.

P4.9-3

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 2 A source on the outside of the circuit is in mesh 3 and that the currents 2 A and i_3 have the same direction. Consequently

$$i_3 = 2 \text{ A}$$

Apply KVL to mesh 1 to get

$$25(i_1 - i_3) + 9(i_1 - i_2) + 8i_1 = 0$$

In this equation $25(i_1 - i_3)$ is the voltage across the 25 Ω resistor (+ on the left), $9(i_1 - i_2)$ is the voltage across the 9 Ω resistor (+ on top) and $8i_1$ is the voltage across the 8 Ω resistor (+ on bottom). Substituting $i_3 = 2$ A and doing a little algebra gives

$$42i_1 - 9i_2 = 50$$

Next, apply KVL to mesh 2 to get

$$14(i_2 - i_3) + 24 - 9(i_1 - i_2) = 0$$

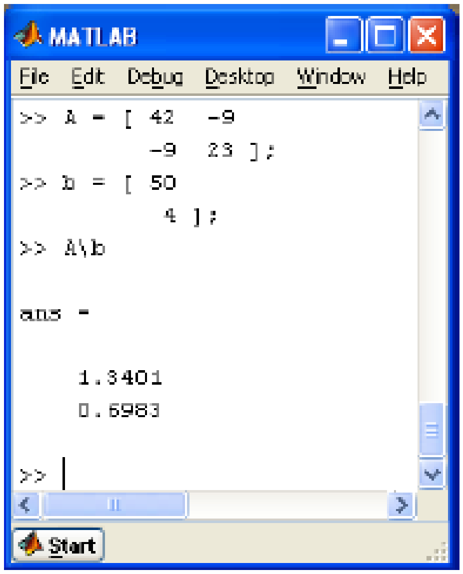
In this equation $14(i_2 - i_3)$ is the voltage across the 14 Ω resistor (+ on the left), 24 is the voltage source voltage and $9(i_1 - i_2)$ is the voltage across the 9 Ω resistor (+ on top). Substituting $i_3 = 2$ A and doing a little algebra gives

$$-9i_1 + 23i_2 = -24 + 14(2) = 4$$

The simultaneous equations can be written in matrix form

$$\begin{aligned} 42i_1 - 9i_2 &= 50 \\ -9i_1 + 23i_2 &= 4 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 42 & -9 \\ -9 & 23 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 4 \end{bmatrix}$$

We can use MATLAB to solve the matrix equation:

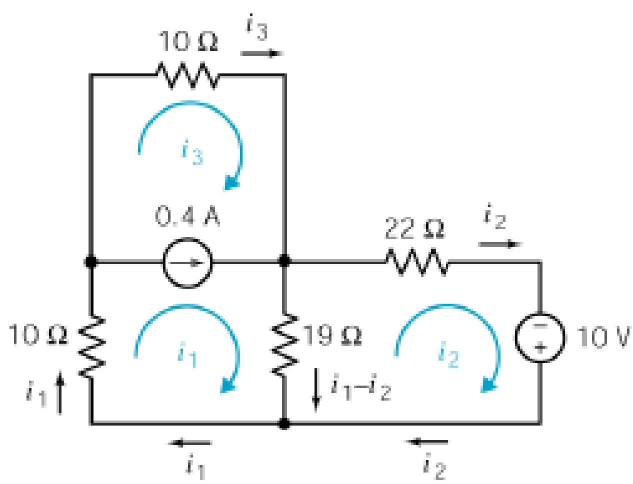


Then $$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1.3401 \\ 0.6983 \end{bmatrix}$$

That is, the mesh currents are $i_1 = 1.3401$ A and $i_2 = 0.6983$ A.

P4.9-4

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 0.4 A source on the inside of the circuit is in both mesh 1 and mesh 3. Mesh current i_1 is directed in the same way as current source current but the mesh current i_3 is directed opposite to the current source current. Consequently

$$i_1 - i_3 = 0.4 \text{ A}$$

The current source is in both mesh 1 and mesh 3 so we apply KVL to the supermesh corresponding to the current source (i.e. the perimeter of meshes 1 and 3). The result is

$$10 i_3 + 19(i_1 - i_2) + 10 i_1 = 0$$

In this equation $10 i_3$ is the voltage across the horizontal 10 Ω resistor (+ on the left), $19(i_1 - i_2)$ is the voltage across the 19 Ω resistor (+ on top) and $10 i_1$ is the voltage across the vertical 10 Ω resistor (+ on bottom). Substituting $i_3 = i_1 - 0.4$ and doing a little algebra gives

$$39 i_1 - 19 i_2 = 4$$

Next, apply KVL to mesh 2 to get

$$22 i_2 - 10 - 19(i_1 - i_2) = 0$$

In this equation $22 i_2$ is the voltage across the 22 Ω resistor (+ on the left), 10 is the voltage source voltage and $19(i_1 - i_2)$ is the voltage across the 19 Ω resistor (+ on top). Doing a little algebra gives

$$-19 i_1 + 41 i_2 = 10$$

To summarize, the circuit is represented by the simultaneous equations:

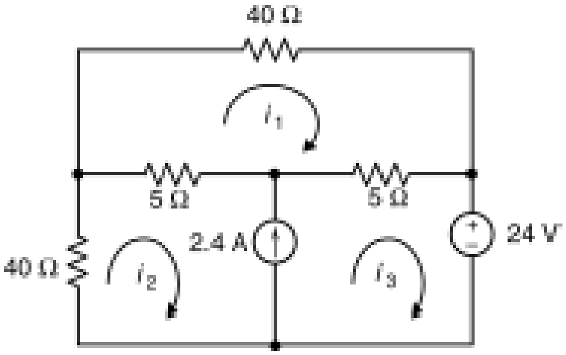
$$\begin{array}{r} 39 i_1 - 19 i_2 = 4 \\ -19 i_1 + 41 i_2 = 10 \end{array} \Rightarrow \begin{bmatrix} 39 & -19 \\ -19 & 41 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

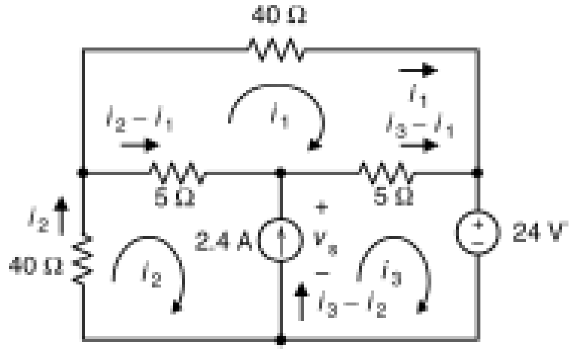
Comparing these equations to the given equations shows

$$a_{11} = 39,\ a_{12} = -19,\ a_{21} = -19 \text{ and } a_{22} = 41.$$

P4.9-5

Solution: First, label the mesh currents and then label the element currents:





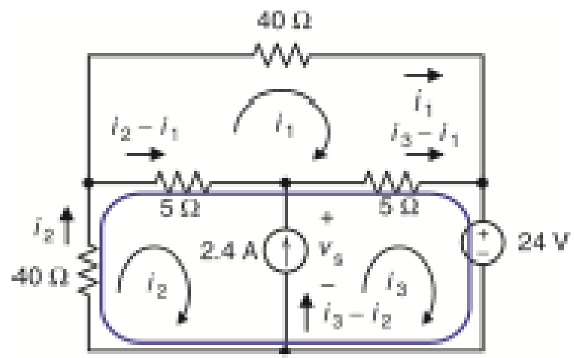
Notice the 2.4 A source in both mesh 2 and mesh 3. We have

$$i_3 - i_2 = 2.4 \text{ A}$$

Apply KVL to mesh 1 to get

$$40 i_1 - 5(i_3 - i_1) - 5(i_2 - i_1) = 0 \quad \Rightarrow \quad 50 i_1 - 5 i_2 - 5 i_3 = 0$$

Identify the supermesh corresponding to the 2.4 A current source:



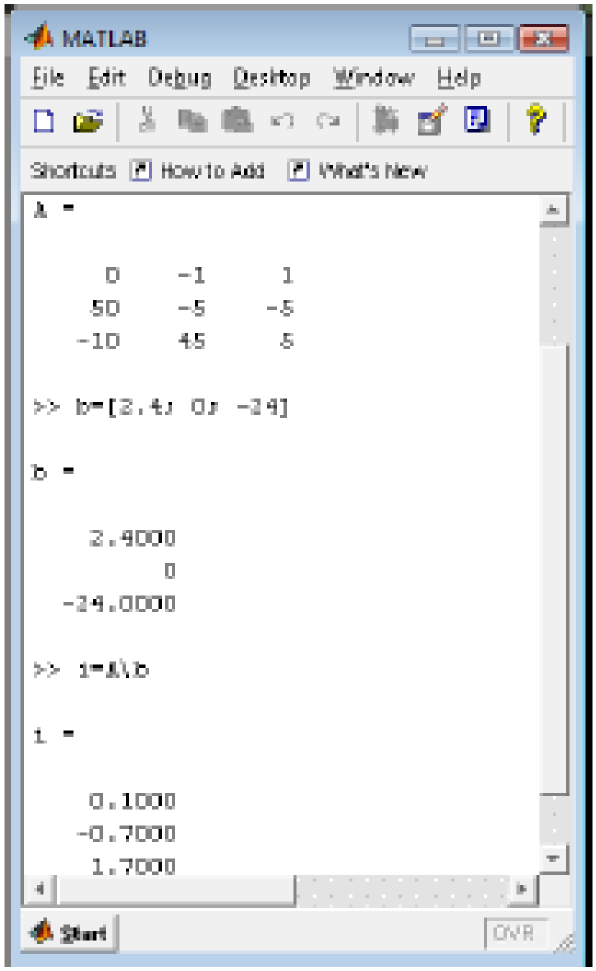
Apply KVL to the supermesh to get

$$5(i_2 - i_1) + 5(i_3 - i_1) + 24 + 40 i_2 = 0 \quad \Rightarrow \quad -10 i_1 + 45 i_2 + 5 i_3 = -24$$

Writing the mesh equations in matrix form gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 50 & -5 & -5 \\ -10 & 45 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 0 \\ -24 \end{bmatrix}$$

Solving using MATLAB:



That is, the mesh currents are $i_1 = 0.1$ A, $i_2 = -0.7$ A and $i_3 = 1.7$ A.

The 24 V source supplies $\quad -24 i_3 = (-24)(1.7) = -40.8$ W

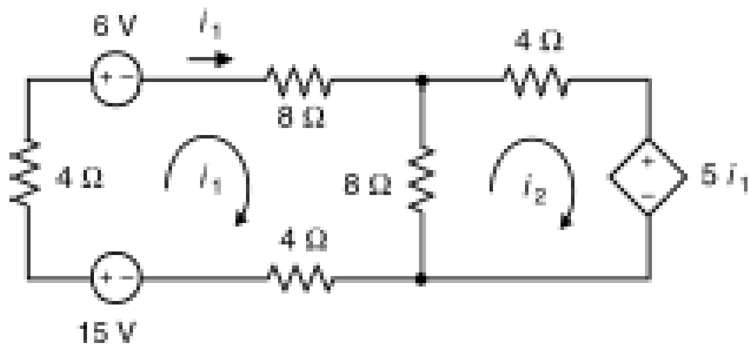
The power supplied by the current source depends on v_x, the voltage across the current source. Apply KVL to mesh 3 to get

$$5(i_3 - i_1) + 24 - v_x = 0 \quad \Rightarrow \quad v_x = 5(1.7 - 0.1) + 24 = 32 \text{ V}$$

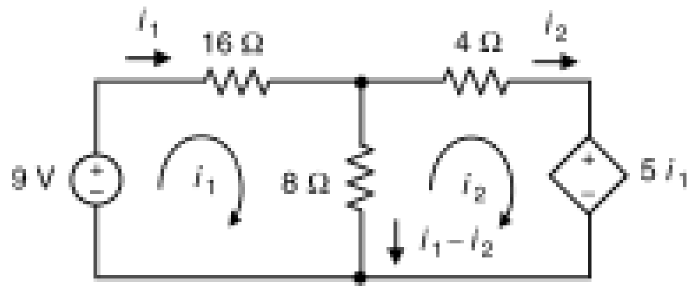
The current source supplies $\quad 2.4 v_x = 2.4(32) = 76.8$ W

P4.9-6

Solution: Determine the value of the mesh currents i_1 and i_2.



Replace series resistors with an equivalent resistor and series voltage sources with and equivalent voltage source to get

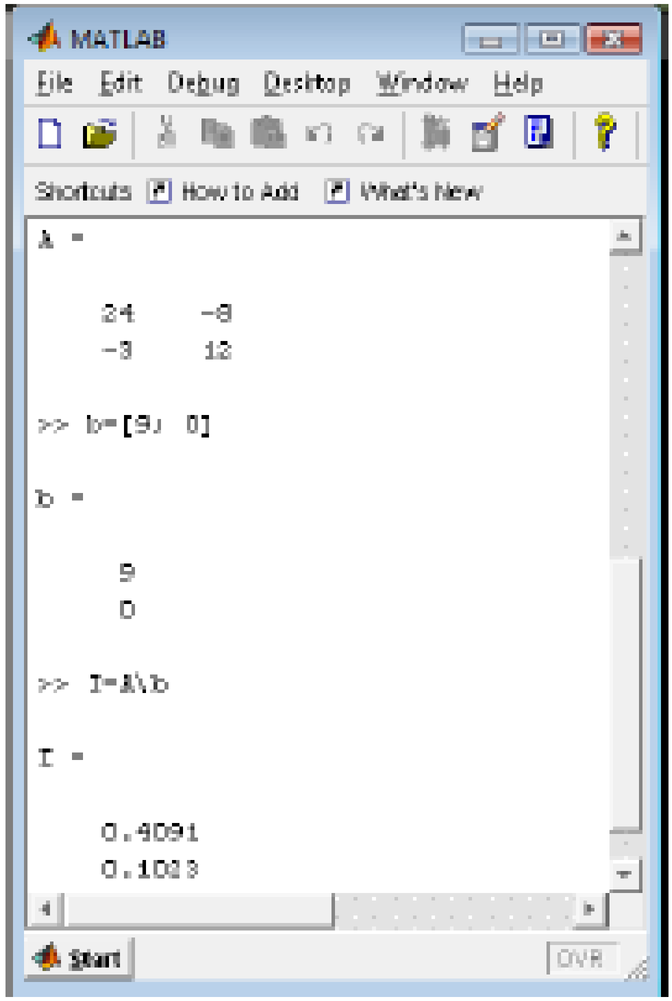


Apply KVL to mesh 1 $\quad 16 i_1 + 8(i_1 - i_2) - 9 = 0 \;\Rightarrow\; 24 i_1 - 8 i_2 = 9$

Apply KVL to mesh 2 $\quad 4 i_2 + 5 i_1 - 8(i_1 - i_2) = 0 \;\Rightarrow\; -3 i_1 + 12 i_2 = 0$

In matrix form
$$\begin{bmatrix} 24 & -8 \\ -3 & 12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

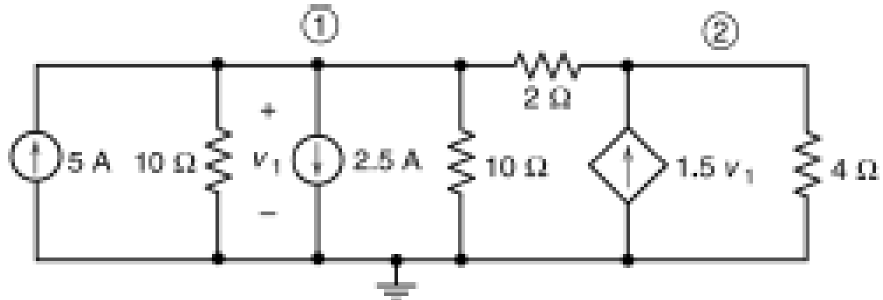
Solving using MATLAB



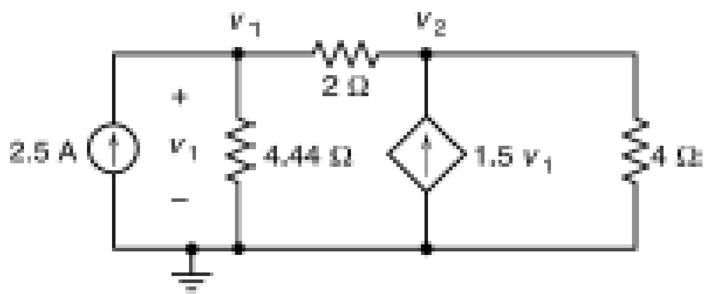
So the mesh currents are $i_1 = 0.4091\ A$ and $i_2 = 0.1023\ A$

P4.9-7

Solution: Determine the value of the node voltages, v_1 and v_2.



Replace parallel resistors with an equivalent resistor and parallel sources with and equivalent current source to get

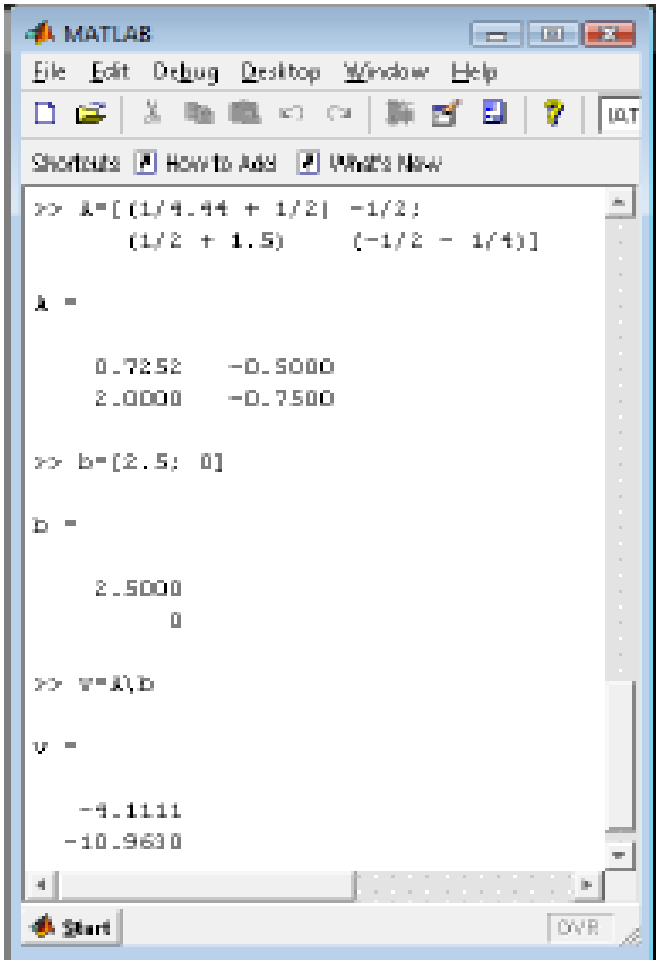


Apply KCL at node 1 $$2.5 = \frac{v_1}{4.44} + \frac{v_1 - v_2}{2} = 0$$

Apply KCL at node 2 $$\frac{v_1 - v_2}{2} + 1.5v_1 = \frac{v_2}{4}$$

In matrix form $$\begin{bmatrix} \frac{1}{4.44} + \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} + 1.5 & -\frac{1}{2} - \frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \end{bmatrix}$$

Solving using MATLAB



So the node voltages are $v_1 = -4.1111$ V and $v_2 = -10.9630$ V

Section 4.11 How Can We Check … ?

P 4.11-1

Solution:

Apply KCL at node b:

$$\frac{v_b - v_a}{4} - \frac{1}{2} + \frac{v_b - v_c}{5} = 0$$

$$\frac{-4.8 - 5.2}{4} - \frac{1}{2} + \frac{-4.8 - 3.0}{5} \neq 0$$

The given voltages do not satisfy the KCL equation at node b. They are not correct.

P 4.11-2

Solution:

Apply KCL at node a:

$$-\left(\frac{v_b - v_a}{4}\right) - 2 + \frac{v_a}{2} = 0$$

$$-\left(\frac{20-4}{4}\right) - 2 + \frac{4}{2} = -4 \neq 0$$

The given voltages do not satisfy the KCL equation at node *a*. They are not correct.

P 4.11-3

Solution:

Writing a node equation:

$$-\left(\frac{12-7.5}{R_1}\right) + \frac{7.5}{R_3} + \frac{7.5-6}{R_2} = 0$$

So

$$-\frac{4.5}{R_1} + \frac{7.5}{R_3} + \frac{1.5}{R_2} = 0$$

There are only three cases to consider. Suppose $R_1 = 5\text{ k}\Omega$ and $R_2 = R_3 = 10\text{ k}\Omega$. Then

$$-\frac{4.5}{R_1} + \frac{7.5}{R_3} + \frac{1.5}{R_2} = \frac{-0.9+0.75+0.15}{1000} = 0$$

This choice of resistance values corresponds to branch currents that satisfy KCL. Therefore, it is indeed possible that two of the resistances are 10 kΩ and the other resistance is 5 kΩ. The 5 kΩ is R_1.

P 4.11-4

Solution: Applying KVL to each mesh:

Top mesh: $10\,(2-4) + 12(2) + 4(2-3) = 0$

Bottom right mesh $8\,(3-4) + 4\,(3-2) + 4 = 0$

Bottom, left mesh: $28 + 10\,(4-2) + 8\,(4-3) \neq 0$

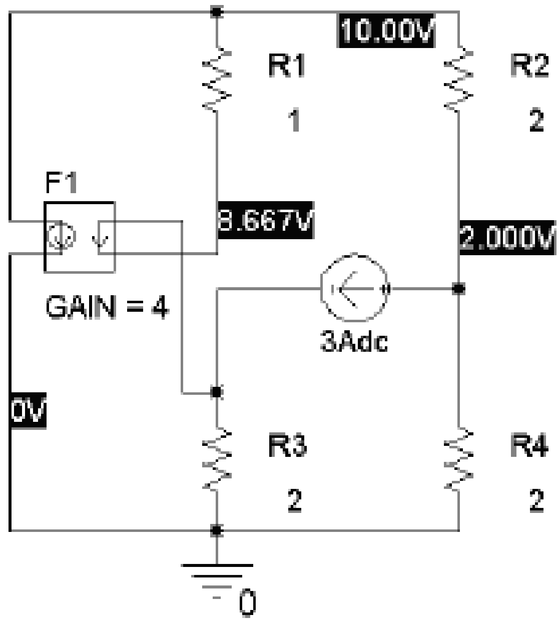
(Perhaps the polarity of the 28 V source was entered incorrectly.)

KVL is not satified for the bottom, left mesh so the computer analysis is **not correct.**

PSpice Problems

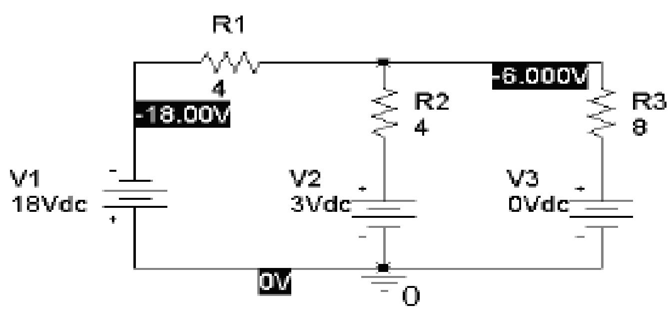
SP 4-1

Solution: The PSpice schematic after running a "Bias Point" simulation:



SP 4-2

Solution: The PSpice schematic after running a "Bias Point" simulation:



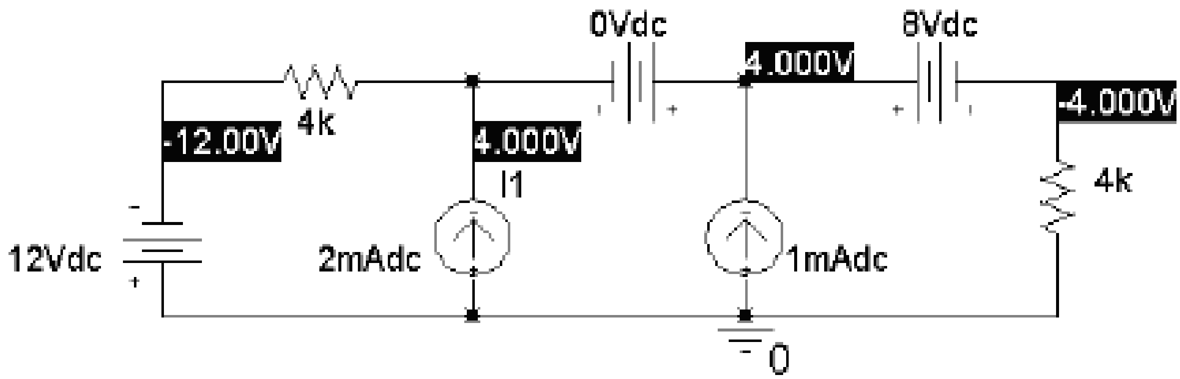
From the PSpice output file:

```
VOLTAGE SOURCE CURRENTS
NAME        CURRENT

V_V1       -3.000E+00
V_V2       -2.250E+00

V_V3       -7.500E-01
```

The voltage source labeled V3 is a short circuit used to measure the mesh current. The mesh currents are $i_1 = -3$ A (the current in the voltage source labeled V1) and $i_2 = -0.75$ A (the current in the voltage source labeled V3).

SP 4-3

Solution: The PSpice schematic after running a "Bias Point" simulation:



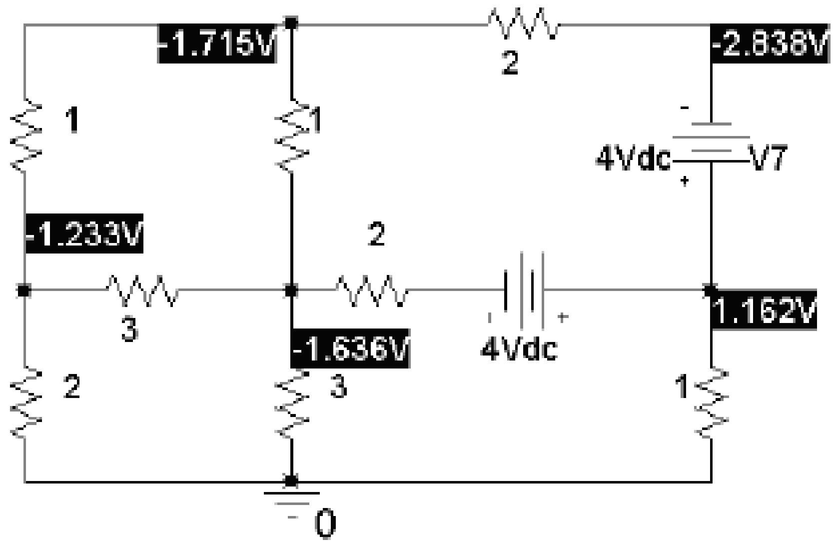
The PSpice output file:

```
**** INCLUDING sp4_2-SCHEMATIC1.net ****
* source SP4_2
V_V4        0 N01588 12Vdc
R_R4        N01588 N01565  4k
V_V5        N01542 N01565 0Vdc
R_R5        0 N01516  4k
V_V6        N01542 N01516 8Vdc
I_I1        0 N01565 DC 2mAdc
I_I2        0 N01542 DC 1mAdc

VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   V_V4       -4.000E-03
   V_V5        2.000E-03
   V_V6       -1.000E-03
```

From the PSpice schematic: $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V. From the output file: $i = 2$ mA.

SP 4-4

Solution: The PSpice schematic after running a “Bias Point” simulation:



The PSpice output file:

VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V7	-5.613E-01
V_V8	-6.008E-01

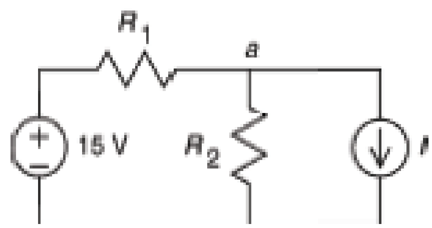
The current of the voltage source labeled V7 is also the current of the 2 Ω resistor at the top of the circuit. However this current is directed from right to left in the 2 Ω resistor while the current *i* is directed from left to right. Consequently, *i* = +5.613 A.

Design Problems

DP 4-1

Solution:

Model the circuit as:



(a) We need to keep v_2 across R_2 in the range $4.8 \le v_2 \le 5.4$

$$\text{For } I = \begin{cases} 0.3 \text{ A} & \text{display is active} \\ 0.1 \text{ A} & \text{display is not active} \end{cases}$$

KCL at a: $\dfrac{v_2 - 15}{R_1} + \dfrac{v_2}{R_2} + I = 0$

Assumed that maximum I results in minimum v_2 and visa-versa.

Then

$$v_2 = \begin{cases} 4.8 \text{ V} & \text{when } I = 0.3 \text{ A} \\ 5.4 \text{ V} & \text{when } I = 0.1 \text{ A} \end{cases}$$

Substitute these corresponding values of v_2 and I into the KCL equation and solve for the resistances

$$\frac{4.8-15}{R_1} + \frac{4.8}{R_2} + 0.3 = 0$$

$$\frac{5.4-15}{R_1} + \frac{5.4}{R_2} + 0.1 = 0$$

$$\Rightarrow \underline{R_1 = 7.89\ \Omega}, \ \underline{R_2 = 4.83\ \Omega}$$

(b)

$$I_{R_{1\max}} = \frac{15-4.8}{7.89} = 1.292 \text{ A} \Rightarrow P_{R_{1\max}} = (1.292)^2(7.89) = 13.17 \text{ W}$$

$$I_{R_{2\max}} = \frac{5.4}{4.83} = 1.118 \text{ A} \Rightarrow P_{R_{2\max}} = \frac{(5.4)^2}{4.83} = 6.03 \text{ W}$$

maximum supply current $= I_{R_{1\max}} = 1.292$ A

(c) No; if the supply voltage (15V) were to rise or drop, the voltage at the display would drop below 4.8V or rise above 5.4V.

The power dissipated in the resistors is excessive. Most of the power from the supply is dissipated in the resistors, not the display.

DP 4-2

Solution:

Express the voltage of the 8 V source in terms of its node voltages to get $v_b - v_a = 8$. Apply KCL to the supernode corresponding to the 8 V source:

$$\frac{v_a - v_1}{R} + \frac{v_a}{R} + \frac{v_b}{R} + \frac{v_b - (-v_2)}{R} = 0 \quad \Rightarrow \quad 2\,v_a - v_1 + 2\,v_b + v_2 = 0$$

$$\Rightarrow \quad 2\,v_a - v_1 + 2\,(v_a + 8) + v_2 = 0$$

$$\Rightarrow \quad 4\,v_a - v_1 + v_2 + 16 = 0$$

$$\Rightarrow \quad v_a = \frac{v_1 - v_2}{4} - 4$$

Next set $v_a = 0$ to get

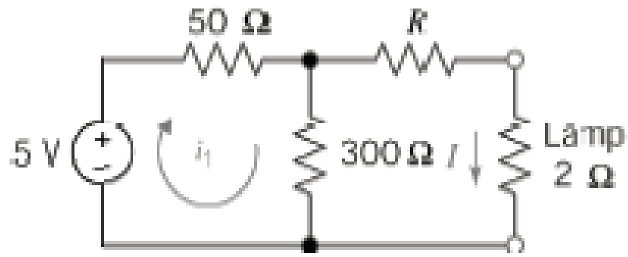
$$0 = \frac{v_1 - v_2}{4} - 4 \quad \Rightarrow \quad v_1 - v_2 = 16 \text{ V}$$

For example, $v_1 = 18$ V and $v_2 = 2$ V.

DP 4-3

Solution:

(a)



Apply KCL to left mesh: $-5+50i_1+300\,(i_1-I)=0$

Apply KCL to right mesh: $(R+2)\,I+300\,(I-i_1)=\ 0$

Solving for I: $$I=\frac{150}{1570\ +\ 35\,R}$$

We desire 50 mA $\le I \le$ 75 mA so if R = 100 Ω, then I = 29.59 m<u>A</u> ⇒ l amp so the lamp will not light.

(b) From the equation for I, we see that decreasing R increases I:

try R = 50 Ω ⇒ I = 45 mA (won't light)

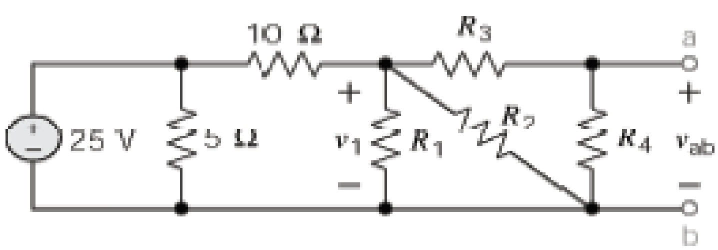
try R = 25Ω ⇒ I = 61 mA ⇒ <u>will light</u>

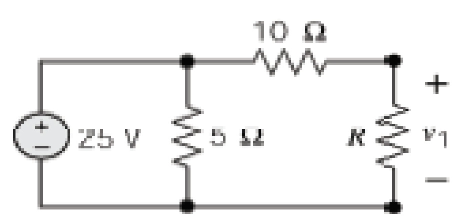
Now check R±10% to see if the lamp will light and not burn out:

$$\left.\begin{array}{l} -10\% \ \rightarrow\ 22.5\Omega\ \rightarrow\ I = 63.63\text{ mA} \\ +10\% \ \rightarrow\ 27.5\Omega\ \rightarrow\ I = 59.23\text{ mA} \end{array}\right\}\begin{array}{l}\text{lamp will}\\ \text{stay on}\end{array}$$

DP 4-4

Solution:





Equivalent resistance: $R=R_1\,||\,R_2\,||\left(R_3+R_4\right)$

Voltage division in the equivalent circuit: $v_1 = \dfrac{R}{10+R}(25)$

We require v_{ab} = 10 V. Apply the voltage division principle in the left circuit to get:

$$10 = \frac{R_4}{R_3+R_4}\ v_1 = \frac{R_4}{R_3+R_4} \times \frac{\left(R_1 \| R_2 \|\ (R_3 + R_4)\right)}{10\ +\ \left(R_1 \| R_2 \|\ (R_3 + R_4)\right)} \times 25$$

This equation does not have a unique solution. Here's one solution:

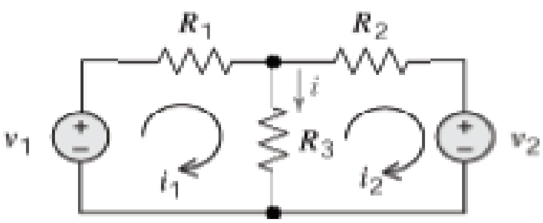
$$\text{choose } R_1 = R_2\ =\ 25\ \Omega \text{ and } R_3 + R_4 = 20\ \Omega$$

$$\text{then } 10 = \frac{R_4}{20} \times \frac{\left(12.5 \| 20\ \right)}{10\ +\ \left(12.5 \| 20\ \right)} \times 25 \ \Rightarrow\ \underline{R_4 = 18.4\Omega}$$

$$\text{and } R_3 + R_4 = 20 \Rightarrow \underline{R_3 = 1.6\,\Omega}$$

DP 4-5

Solution:



Apply KCL to the left mesh:

$$\left(R_1 + R_3\right)\ i_1 -\ R_3\, i_2 - v_1 = 0$$

Apply KCL to the left mesh:

$$-R_3\, i_1 +\ \left(R_2 + R_3\right)\ i_2 + v_2 = 0$$

Solving for the mesh currents using Cramer's rule:

$$i_1 = \frac{\begin{bmatrix} v_1 & -\ R_3 \\ -v_2 & \left(R_2 + R_3\right) \end{bmatrix}}{\Delta} \text{ and } i_2 = \frac{\begin{bmatrix} (R_1 + R_3) & v_1 \\ -R_3 & -v_2 \end{bmatrix}}{\Delta}$$

$$\text{where } \Delta\ =\ \left(R_1 + R_3\right)\ \left(R_2 + R_3\right)\ - R_3^{\,2}$$

Try $R_1 =\ R_2 = R_3$ = 1 kΩ = 1000 Ω. Then Δ = 3 MΩ. The mesh currents will be given by

$$i_1 = \frac{\left[2v_1 - v_2\right]\ 1000}{3\times 10^6} \text{ and } i_2 = \frac{\left[-2v_2 + v_1\right]\ 1000}{3\times 10^6}\ \ \Rightarrow\ i = i_1 - i_2 = \frac{v_1 + v_2}{3000}$$

Now check the extreme values of the source voltages:

$$\text{if } v_1 = v_2\ = 1\text{ V} \Rightarrow\ i\ =\ {}^{2}\!/_{3}\text{ mA} \quad \text{okay}$$

$$\underline{\text{if } v_1 = v_2\ = 2\text{ V} \Rightarrow\ i\ =\ {}^{4}\!/_{3}\text{ mA} \quad \text{okay}}$$