Chapter 13: Frequency Response

Exercises

Exercise 13.2-1 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose R = 10 kΩ and C = 1 μF. What are the values of the gain and phase shift when the input frequency is ω = 100 rad/s?

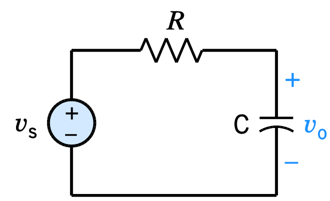
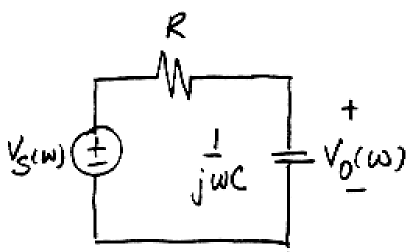


Figure E 13.2-1

Answer: 0.707 and –45°

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$$phase\ shift = -\tan^{-1}(\omega C R)$$

When $R = 10^4\ \Omega$, $\omega = 100$ rad/s, and $C = 10^{-6}$ F, then $gain = \dfrac{1}{\sqrt{2}} = 0.707$ and $phase\ shift = -45^\circ$

Exercise 13.2-2 The input to the circuit shown in Figure E 13.2-2 is the source voltage, v_s, and the response is the resistor voltage, v_o. R = 30 Ω and L = 2 H. Suppose the input frequency is adjusted until the gain is equal to 0.6. What is the value of the frequency?

Answer: 20 rad/s

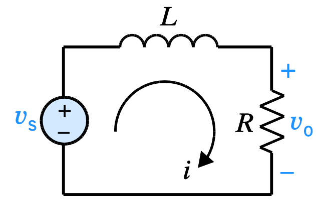
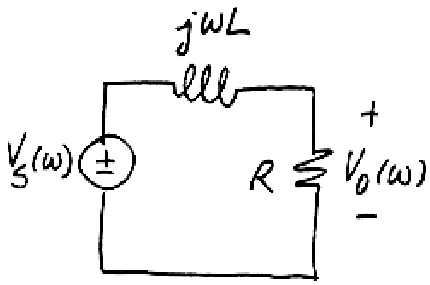


Figure E 13.2-2

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{R}{R + j\omega L}$$

$$gain = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

$$0.6 = \frac{30}{\sqrt{30^2 + (2\omega)^2}} \Rightarrow \omega = \frac{\sqrt{\left(\frac{30}{.6}\right)^2 - 30^2}}{2} = 20 \text{ rad/s}$$

Exercise 13.2-3 The input to the circuit shown in Figure E 13.2-2 is the source voltage, v_s, and the response is the mesh current, i. R = 30 Ω and L = 2 H. What are the values of the gain and phase shift when the input frequency is ω = 20 rad/s?

Answer: 0.02 A/V and –53.1°

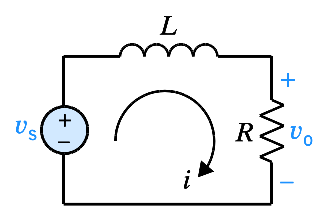
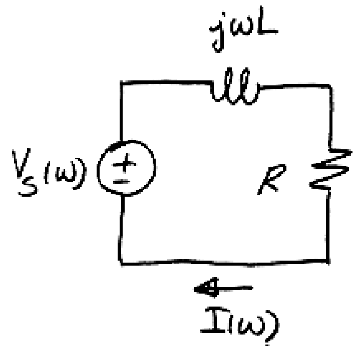


Figure E 13.2-2

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{I}(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{R + j\omega L}$$

$$gain = \frac{1}{\sqrt{R^2 + (\omega L)^2}}$$

$$phase\ shift = -\tan^{-1}\frac{\omega L}{R}$$

When $R = 30\ \Omega$, $L = 2$ H, and $\omega = 20$ rad/s, then

$$gain = \frac{1}{\sqrt{30^2 + 40^2}} = 0.02\ \frac{\text{A}}{\text{V}} \quad \text{and} \quad phase\ shift = -\tan^{-1}\left(\frac{40}{30}\right) = -53.1°$$

Exercise 13.2-4 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose C = 1 μF. What value of R is required to cause a phase shift equal to –45° when the input frequency is ω = 20 rad/s?

Answer: R = 50 kΩ

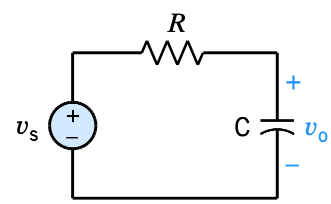
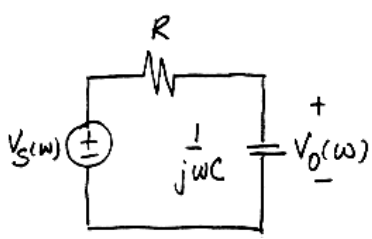


Figure E 13.2-1

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$$phase\ shift = -\tan^{-1}\omega C R$$

$$-45° = -\tan^{-1}(20 \cdot 10^{-6} \cdot R) \quad \Rightarrow \quad R = \frac{\tan(45^\circ)}{20 \cdot 10^{-6}} = 50 \cdot 10^3 = 50\ \text{k}\Omega$$

Exercise 13.2-5 The input to the circuit shown in Figure E 13.2-1 is the source voltage, v_s, and the response is the capacitor voltage, v_o. Suppose C = 1 μF. What value of R is required to cause a gain equal to 1.5 when the input frequency is ω = 20 rad/s?

Answer: No such value of R exists. The gain of this circuit will

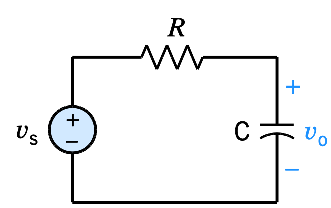
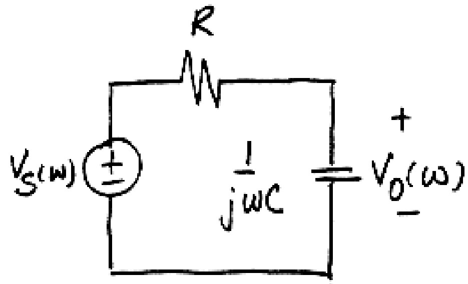


Figure E 13.2-1

never be greater than 1.

Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R}$$

$$gain = \frac{1}{\sqrt{1 + (\omega C R)^2}}$$

$\omega, C,$ and R are all positive, or at least nonnegative, so gain $\le$ 1. These specifications cannot be met.

Exercise 13.3-1 (a) Convert the gain$|\mathbf{V}_o/\mathbf{V}_s| = 2$ to decibels. (b) Suppose $|\mathbf{V}_o/\mathbf{V}_s| = -6.02$ dB. What is the value of this gain "not in dB"?

Answer: (a) + 6.02 dB (b) 0.5

Solution:

(a) $\text{dB} = 20\log(2) = 6.02 \text{ dB}$

(b) $10^{-6.02/20} = 0.5$

Exercise 13.3-2 In a certain frequency range, the magnitude of the network function can be approximated as $H = 1/\omega^2$. What is the slope of the Bode plot in this range, expressed in decibels per decade?

Answer:–40 dB/decade

Solution:

$$20\log|\mathbf{H}| = 20\log\left(\frac{1}{\omega^2}\right) = 20\log(\omega)^{-2} = -40\log\omega$$

$$slope = 20\log|\mathbf{H}(\omega_2)| - 20\log|\mathbf{H}(\omega_1)| = -40\log\omega_2 + 40\log\omega_1 = -40\log\left(\frac{\omega_2}{\omega_1}\right)$$

let $\omega_2 = 10\,\omega_1$ to consider 1 decade, then $slope = \underline{-40\log 10 = -40 \text{ dB/decade}}$

Exercise 13.3-3 Consider the network function

$$\mathbf{H}(\omega) = \frac{j\omega A}{B + j\omega C}$$

Find (a) the corner frequency, (b) the slope of the asymptotic magnitude Bode plot for ω above the corner frequency in decibels per decade, (c) the slope of the magnitude Bode plot below the corner frequency, and (d) the gain for ω above the corner frequency in decibels.

Answer: (a) ω_0 = B/C (b) zero (c) 20 dB/decade (d) 20 $\log_{10}$ = $\dfrac{A}{C}$

Solution:

When $\omega C >> B$, $\mathbf{H}(\omega) \simeq \dfrac{j\omega A}{j\omega C} = \dfrac{A}{C}$

(d) $|\mathbf{H}(\omega)|$ in dB $= 20\log_{10}|\mathbf{H}(\omega)| = 20\log_{10}\left(\dfrac{A}{C}\right)$

(b) $|\mathbf{H}(\omega)|$ does not depend on ω so $slope = 0$

When $\omega C << B$, $\mathbf{H}(\omega) \simeq \dfrac{j\omega A}{B} = j\omega\left(\dfrac{A}{B}\right)$

$$|\mathbf{H}(\omega)| \text{ in dB} = 20 \log_{10} |\mathbf{H}(\omega)| = 20 \log_{10}\omega + 20 \log_{10}\left(\frac{A}{B}\right)$$

(c) The slope is the coefficient of $\log_{10} \omega$, that is, $slope = 20$ dB/decade

(a) The break frequency is the frequency at which $\omega_0 C = B$, that is, $\omega_0 = \dfrac{B}{C}$

Exercise 13.4-1 For the RLC parallel resonant circuit when R = 8 kΩ, L = 40 mH, and C = 0.25 μF, find (a) Q and (b) bandwidth.

Answer: (a) Q = 20 (b) BW = 500 rad/s

Solution:

a) $$Q = \omega_0 RC = R\sqrt{\frac{C}{L}} = 8000\sqrt{\frac{2.5 \times 10^{-7}}{40 \times 10^{-3}}} = \underline{20}$$

b) $$BW = \frac{\omega_o}{Q} = \frac{1}{Q\sqrt{LC}} = \frac{1}{20\sqrt{(40\times10^{-3})\ (2.5\times10^{-7})}} = \underline{500 \text{ rad/s}}$$

Exercise 13.4-2 A high-frequency RLC parallel resonant circuit is required to operate at ω_0 = 10 Mrad/s with a bandwidth of 200 krad/s. Determine the required Q and L when C = 10 pF.

Answer: Q = 50 and L = 1 mH

Solution:

$$Q = \frac{\omega_0}{BW} = \frac{10^7}{2 \times 10^5} = \underline{50}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^{\,2} C} = \frac{1}{(10^7)^2\ (10 \times 10^{-12})} = \underline{1 \text{ mH}}$$

Exercise 13.4-3 A series resonant circuit has L = 1 mH and C = 10 μF. Find the required Q and R when it is desired that the bandwidth be 15.9 Hz.

Answer: Q = 100 and R = 0.1 Ω

Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\left[(10^{-3})(10^{-5})\right]^{1/2}} = 10^4 \ \text{rad/s}$$

$$Q = \frac{\omega_0}{BW} = \frac{10^4}{2\pi\ (15.9)} = \underline{100}$$

$$R = \frac{\omega_0 L}{Q} = \frac{(10^4)(10^{-3})}{100} = \underline{0.1\ \Omega}$$

Exercise 13.4-4 A series resonant circuit has an inductor L = 10 mH. (a) Select C and R so that ω_0 = 10^6 rad/s and the bandwidth is BW = 10^3 rad/s. (b) Find the admittance **Y** of this circuit for a signal at ω = 1.05×10^6 rad/s.

Answer: (a) $C = 100\text{pF}, R = 10\Omega$

(b) $\mathbf{Y} = \dfrac{10}{1+j97.6}$

Solution:

a)

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{(10^6)^2\ (0.01)} = \underline{100\ \text{pF}}$$

$$Q = \frac{\omega_0}{BW} = \frac{1}{\omega_0 RC} \Rightarrow R = \frac{BW}{\omega_0^{\ 2}\ C} = \frac{10^3}{(10^6)^2(10^{-10})} = \underline{10\ \Omega}$$

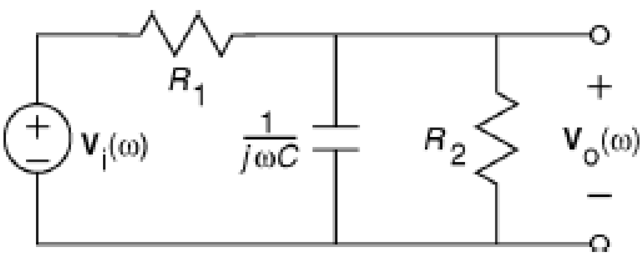
b)

$$Q = \frac{\omega_0}{BW} = \frac{10^6}{10^3} = 1000$$

$$\mathbf{Y} = \frac{1}{1+j\,Q\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)} = \frac{1}{1+j1000\left[\dfrac{1.05\times10^6}{10^6}-\dfrac{10^6}{1.05\times10^6}\right]} = \frac{1}{1+j\ 97.6}$$

Section 13-2: Gain, Phase Shift, and the Network Function

P 13.2-1
Solution:



$$R_2 \parallel \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega C\, R_2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega C\, R_2}}{R_1 + \dfrac{R_2}{1 + j\omega C\, R_2}} = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + j\omega C\, R_p}$$

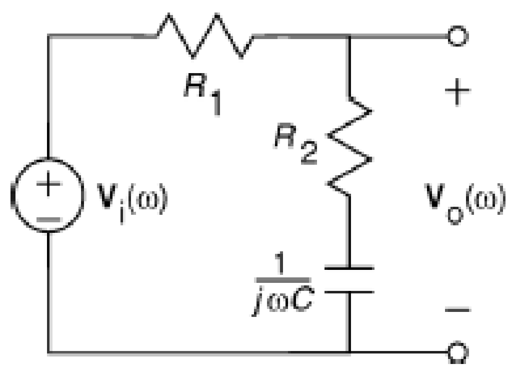
where $R_p = R_1 \parallel R_2$.

When $R_1 = 60\ \Omega$, $R_2 = 15\ \Omega$ and $C = 1$ F

$$\mathbf{H}(\omega) = \frac{0.2}{1 + j12\omega}$$

P 13.2-2
Solution:

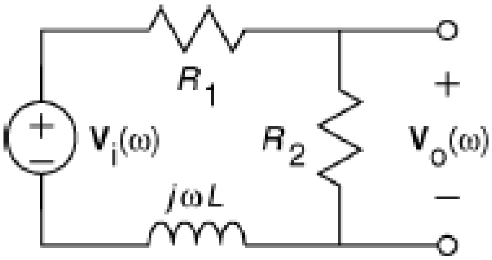
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2 + \dfrac{1}{j\omega C}}{R_1 + R_2 + \dfrac{1}{j\omega C}} = \frac{1 + j\omega C\, R_2}{1 + j\omega C\left(R_1 + R_2\right)}$$



When $R_1 = 50\ \text{k}\Omega$, $R_2 = 200\ \text{k}\Omega$ and $C = 0.025\ \mu\text{F}$

$$\mathbf{H}(\omega) = \frac{1 + j(0.005)\,\omega}{1 + j(0.00625)\,\omega}$$

P 13.2-3
Solution:

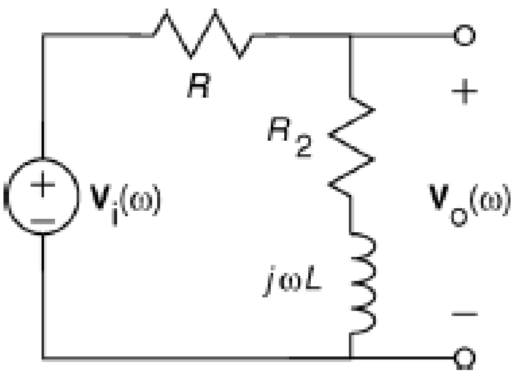


$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2}{R_1 + R_2 + j\omega L} = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + j\omega \dfrac{L}{R_1 + R_2}}$$

When $R_1 = 6\ \Omega$, $R_2 = 8\ \Omega$ and $L = 10$ H

$$\mathbf{H}(\omega) = \frac{0.6}{1 + j(0.7)\omega}$$

P 13.2-4
Solution:



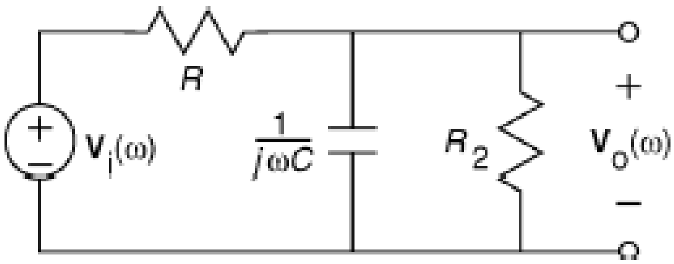
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{R_2 + j\omega L}{R + R_2 + j\omega L} = \left(\frac{R_2}{R + R_2}\right)\left(\frac{1 + j\omega \dfrac{L}{R_2}}{1 + j\omega \dfrac{L}{R + R_2}}\right)$$

Comparing the given and derived network functions, we require

$$\left(\frac{R_2}{R + R_2}\right)\left(\frac{1 + j\omega \dfrac{L}{R_2}}{1 + j\omega \dfrac{L}{R + R_2}}\right) = (0.6)\frac{1 + j\dfrac{\omega}{12}}{1 + j\dfrac{\omega}{20}} \Rightarrow \begin{cases} \dfrac{R_2}{R + R_2} = 0.6 \\ \dfrac{R_2}{L} = 12 \\ \dfrac{R + R_2}{L} = 20 \end{cases}$$

Since $R_2 = 50\ \Omega$, we have $L = \dfrac{50}{12} = 4.2\ \text{H}$, then $R = (20)(4.2) - 50 = 34\ \Omega$.

P 13.2-5
Solution:



$$R_2 \parallel \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega C\, R_2}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{\dfrac{R_2}{1 + j\omega C\, R_2}}{R + \dfrac{R_2}{1 + j\omega C\, R_2}}$$

$$= \frac{\dfrac{R_2}{R + R_2}}{1 + j\omega C\, R_p}$$

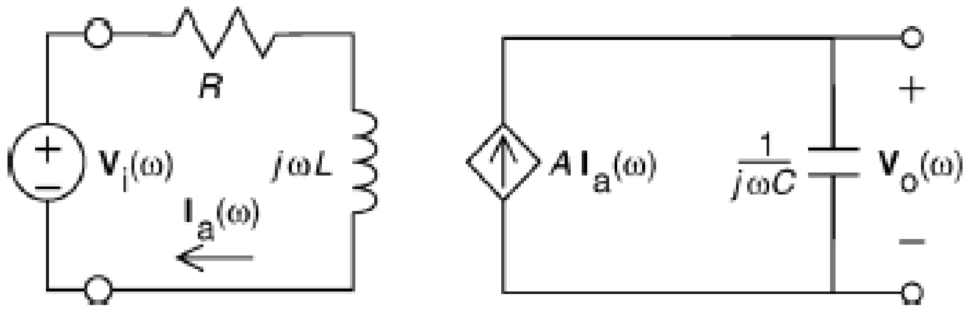
where $R_p = R \parallel R_2$.

Comparing the given and derived network functions, we require

$$\frac{\dfrac{R_2}{R + R_2}}{1 + j\omega C\, R_p} = \frac{0.2}{1 + j4\,\omega} \quad \Rightarrow \quad \begin{cases} \dfrac{R_2}{R + R_2} = 0.2 \\ C\, R_p = 4 \end{cases}$$

Since $R_2 = 3\ \Omega$, we have $\dfrac{3}{R+3} = 0.2 \quad \Rightarrow \quad R = 12\ \Omega$. Then $R_p = \dfrac{(3)(12)}{3+12} = 2.4\ \Omega$. Finally, $C = \dfrac{4}{2.4} = 1.7\ \text{F}$.

P 13.2-6
Solution:

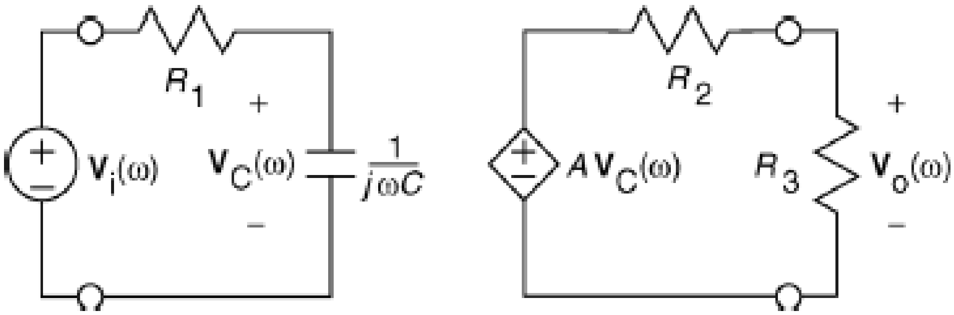


$$\left.\begin{aligned} \mathbf{I}_a(\omega) &= \frac{\mathbf{V}_i(\omega)}{R + j\omega L} \\ \mathbf{V}_o(\omega) &= \frac{1}{j\omega C}\left(A\,\mathbf{I}_a(\omega)\right) \end{aligned}\right\} \Rightarrow \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\frac{A}{CR}}{(j\omega)\left(1 + j\omega\frac{L}{R}\right)}$$

When $R = 30\ \Omega$, $L = 6$ H, $A = 3$ A/A and $C = 0.25$ F

$$\mathbf{H}(\omega) = \frac{0.4}{(j\omega)(1 + j(0.2)\omega)}$$

P 13.2-7
Solution:



In the frequency domain, use voltage division on the left side of the circuit to get:

$$\mathbf{V}_C(\omega) = \frac{\frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}} \mathbf{V}_i(\omega) = \frac{1}{1 + j\omega C R_1} \mathbf{V}_i(\omega)$$

Next, use voltage division on the right side of the circuit to get:

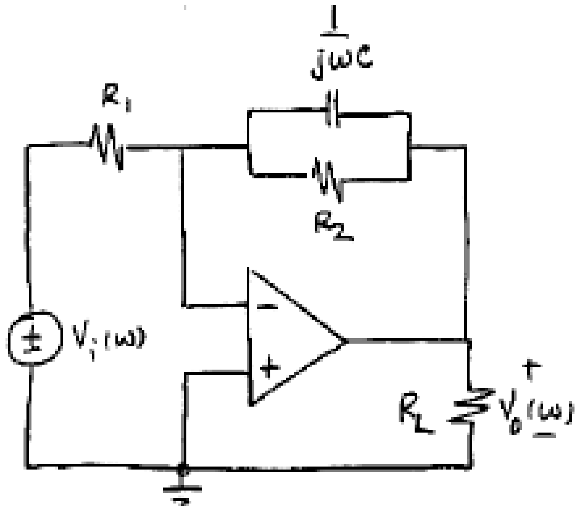
$$\mathbf{V}_o(\omega) = \frac{R_3}{R_2 + R_3} A\,\mathbf{V}_C(\omega) = \frac{2}{3} A\,\mathbf{V}_C(\omega) = \frac{\frac{2}{3}A}{1 + j\omega C R_1} \mathbf{V}_i(\omega)$$

Compare the specified network function to the calculated network function:

$$\frac{4}{1 + j\frac{\omega}{100}} = \frac{\frac{2}{3}A}{1 + j\omega C R_1} = \frac{\frac{2}{3}A}{1 + j\omega C\,4000} \quad \Rightarrow \quad 4 = \frac{2}{3}A \;\text{ and }\; \frac{1}{100} = 4000\,C$$

Thus, C = 2.5 μF and A = 6 V/V.

P 13.2-8
Solution:



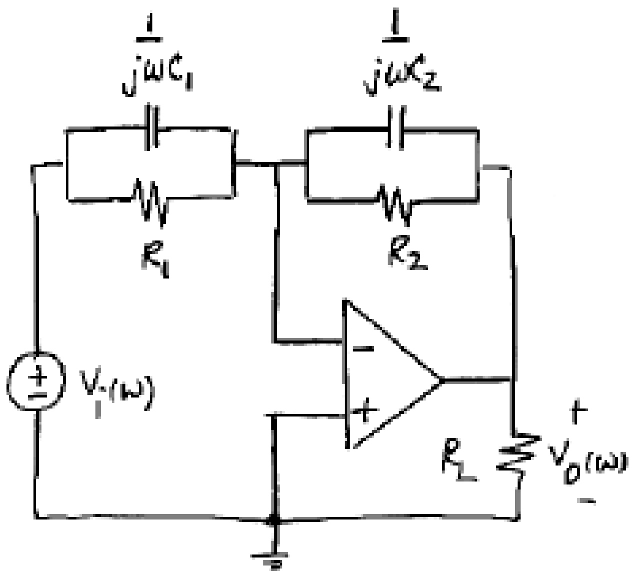
$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=-\frac{R_2 \,\Big\|\, \frac{1}{j\omega C}}{R_1}$$

$$=\frac{-\left(\frac{R_2}{R_1}\right)}{1+j\omega C R_2}$$

When $R_1 = 15$ kΩ, $R_2 = 60$ kΩ, and $C = 2$ μF, then

$\frac{R_2}{R_1} = 4$ and $R_2 C = \frac{3}{25}$ so $\mathbf{H}(\omega)=\frac{-4}{1+j\omega\frac{3}{25}}$

P 13.2-9
Solution:



$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = -\frac{R_2 \left\| \frac{1}{j\omega C_2} \right.}{R_1 \left\| \frac{1}{j\omega C_1} \right.}$$

$$= -\frac{\dfrac{R_2}{1+j\omega C_2R_2}}{\dfrac{R_1}{1+j\omega C_1R_1}}$$

$$\mathbf{H}(\omega) = -\left(\frac{R_2}{R_1}\right)\left(\frac{1+j\omega C_1R_1}{1+j\omega C_2R_2}\right)$$

When $R_1 = 15\text{ k}\Omega$, $R_2 = 60\text{ k}\Omega$, $C_1 = 4\ \mu\text{F}$ and $C_2 = 2\ \mu\text{F}$,

then $\frac{R_2}{R_1} = 4$, $C_1R_1 = \frac{3}{50}$ and $C_2R_2 = \frac{3}{25}$

so

$$\mathbf{H}(\omega) = -4\left(\frac{1+j\omega\frac{3}{50}}{1+j\omega\frac{3}{25}}\right)$$

$$gain = |\mathbf{H}(\omega)| = (4)\frac{\sqrt{1+\frac{9\omega^2}{2500}}}{\sqrt{1+\frac{9\omega^2}{625}}}$$

P 13.2-10
Solution

$$R_3 \left\| \frac{1}{j\omega C} \right. = \frac{R_3 \dfrac{1}{j\omega C}}{R_3 + \dfrac{1}{j\omega C}} = \frac{R_3}{1 + j\omega C\, R_3}$$

$$\mathrm{H}(\omega) = -\frac{R_2 + \dfrac{R_3}{1 + j\omega\, C\, R_3}}{R_1} = -\frac{R_2 + R_3 + j\omega\, R_2 R_3 C}{R_1 + j\omega\, R_1 R_3 C}$$

$$5 = \lim_{\omega \to 0} \left|\mathrm{H}\,(\omega)\right| = \frac{R_2 + R_3}{R_1}$$

$$2 = \lim_{\omega \to \infty} \left|\mathrm{H}\,(\omega)\right| = \frac{R_2}{R_1} \Rightarrow R_2 = 2\, R_1 = 40\ \mathrm{k\Omega}$$

then $\quad R_3 = 5R_1 - R_2 = 60\ \mathrm{k\Omega}$

P 13.2-11
Solution:

$$\mathrm{H}(\omega) = -\frac{R_2 + \dfrac{1}{j\omega\, C}}{R_1} = -\frac{1 + j\omega\, C\, R_2}{j\omega\, C\, R_1}$$

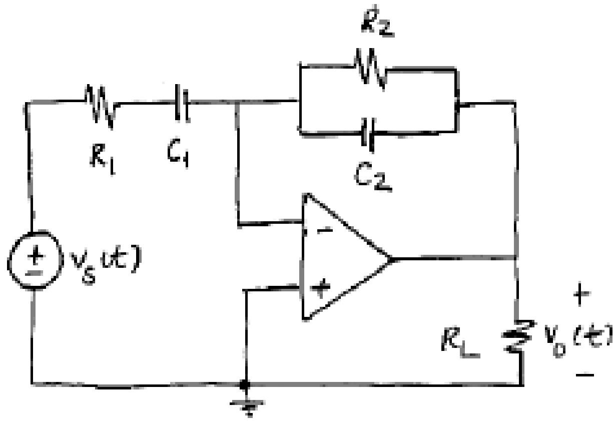
$$\angle \mathrm{H}(\omega) = 180° + \tan^{-1}\left(\omega\, C\, R_2\right) - 90°$$

$$\angle \mathrm{H}(\omega) = 135° \Rightarrow \tan^{-1}\left(\omega C R_2\right) = 45° \Rightarrow \omega\, C\, R_2 = 1$$

$$\Rightarrow R_2 = \frac{1}{\left(2 \times 10^{-7} \times 10^{3}\right)} = 5\ \mathrm{k\Omega}$$

$$10 = \lim_{\omega \to \infty} \left|\mathrm{H}(\omega)\right| = \frac{R_2}{R_1} \Rightarrow R_1 = \frac{R_2}{10} = 0.5\ \mathrm{k\Omega}$$

P 13.2-12
Solution:



$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=-\frac{R_2\left\|\frac{1}{j\omega C_2}\right.}{R_1+\frac{1}{j\omega C_1}}$$

$$=\frac{(-C_1R_2)\,j\omega}{(1+j\omega R_1C_1)\,(1+j\omega R_2C_2)}$$

When $R_1=6\text{ k}\Omega,\ C_1=1.5\ \mu\text{F},$ $R_2=12\text{ k}\Omega$ and $C_2=0.3\ \mu\text{F},$ then

$$\mathbf{H}(\omega)=\frac{(-0.018)\,j\omega}{\left(1+j\dfrac{9\omega}{1000}\right)\left(1+j\dfrac{9\omega}{2500}\right)}$$

so

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
0	0	$-90°$
600	0.82	125.05°
3600	0.15	96.3°

Then

$$v_o(t)=(0)\ 60\ +\ (0.82)(40)\cos(600t+120°+125.05°)-(0.15)(30)\cos(3600t+35°+96.3°)$$
$$=32.8\cos(600t+245.05°)-4.5\cos(3600t+131.3°)\ \text{ mV}$$

When R_1=6 kΩ, C_1=1.5 μF, R_2=12 kΩ and C_2=0.03 μF, then

$$\mathbf{H}(\omega)=-0.018\ \frac{j\omega}{\left(1+j\dfrac{9\omega}{1000}\right)\left(1+j\dfrac{9\omega}{25{,}000}\right)}$$

So

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
0	0	$-90°$
600	1.92	178.2°
3600	1.22	130°

Then

$$v_o(t)=(0)\,60+(1.92)(40)\cos(600t+120°+178.2°)-(1.22)(30)\cos(3600t+35°+130°)$$
$$=76.8\ \cos(600t+298.2°)-36.6\cos(3600t+165°)\ \text{ mV}$$

P 13.2-13
Solution:

(a) $|\mathbf{V}_s| = \dfrac{(8 \text{ div})\left(\dfrac{2\text{ V}}{\text{div}}\right)}{2} = 8\text{ V}$ and $|\mathbf{V}_o| = \dfrac{(6.2 \text{ div})\left(\dfrac{2\text{ V}}{\text{div}}\right)}{2} = 6.2\text{ V}$ so gain $= \dfrac{|\mathbf{V}_o|}{|\mathbf{V}_s|} = \dfrac{6.2}{8} = 0.775$

(b) $\mathbf{H}(\omega) = \dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \dfrac{\dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = \dfrac{1}{1 + j\omega C R}$.

Let $g = |\mathbf{H}(\omega)| = \dfrac{1}{\sqrt{1+\omega^2 C^2 R^2}}$ then $C = \dfrac{1}{\omega R}\sqrt{\left(\dfrac{1}{g}\right)^2 - 1}$

In this case $\omega = 2\pi \cdot 500 = 3142$ rad/s, $|\mathbf{H}(\omega)| = 0.775$ and $R = 1000\ \Omega$ so $C = 0.26\ \mu F$.

(c) $\angle\,\mathbf{H}(\omega) = -\tan^{-1}(\omega R C)$ so $\omega = \dfrac{\tan(-\angle\,\mathbf{H}(\omega))}{RC}$

Recalling that R = 1000 Ω and C=0.26µF, we calculate

ω	$\lvert\mathbf{H}(\omega)\rvert$	$\angle\mathbf{H}(\omega)$
$2\pi(200)$	0.95	−18°
$2\pi(2000)$	0.26	−73°

$$\angle\,\mathbf{H}(\omega) = -45° \text{ requires } \omega = \frac{\tan\left(-\left(-45^\circ\right)\right)}{(1000)\left(.26\times 10^{-6}\right)} = 3846 \text{ rad/s}$$

$$\angle\,\mathbf{H}(\omega) = -135° \text{ requires } \omega = \frac{\tan\left(-(-135°)\right)}{(1000)(0.26\times 10^{-6})} = -3846 \text{ rad/s}$$

A negative frequency is not acceptable. We conclude that this circuit cannot produce a phase shift equal to −135°.

(d)

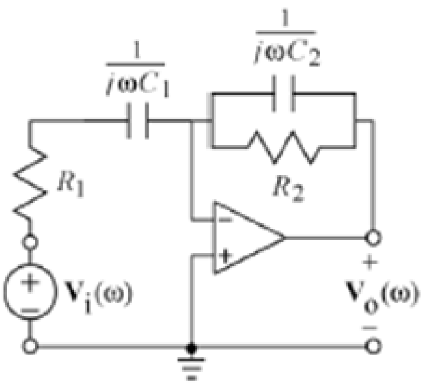
$$C = \frac{\tan\ (-\angle\mathbf{H}(\omega))}{\omega R} \Rightarrow \begin{cases} C = \dfrac{\tan\ (-60°)}{(2\pi\cdot 500)\ (1000)} = \ 0.55\mu\text{F} \\ C = \dfrac{\tan\ (-(-300°))}{(2\pi\cdot 500\ )\ (1000)} = -0.55\mu\text{F} \end{cases}$$

A negative value of capacitance is not acceptable and indicates that this circuit cannot be designed to produce a phase shift at −300° at a frequency of 500 Hz.

(e) $$C = \frac{\tan(-(-120^\circ))}{(2\pi \cdot 500)(1000)} = -0.55\ \mu F$$

This circuit cannot be designed to produce a phase shift of $-120°$ at 500 Hz.

P 13.2-14
Solution:



$$\mathbf{H}(\omega) = -\frac{R_2 \| \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1}} = -\frac{\frac{R_2}{1+j\omega C_2 R_2}}{\frac{j\omega C_1 R_1 + 1}{j\omega C_1}}$$

$$= \frac{(-C_1 R_2) j\omega}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_2)}$$

$$\frac{(-C_1 R_2) j\omega}{(1+j\omega C_1 R_1)(1+j\omega C_2 R_2)} = \frac{(-0.1) j\omega}{\left(1+j\frac{\omega}{p}\right)\left(1+j\frac{\omega}{125}\right)} \Rightarrow \begin{cases} -C_1 R_2 = -0.1 \\ C_1 R_1 = \frac{1}{p} \text{ or } \frac{1}{125} \\ C_2 R_2 = \frac{1}{125} \text{ or } \frac{1}{p} \end{cases}$$

Since $C_1 = 5\mu F, R_1 = 10\ k\Omega$ and $R_2 = 25\ k\Omega$

$$C_1 R_1 = \left(5\times 10^{-6}\right)\left(10\times 10^3\right) = \frac{1}{25} \neq \frac{1}{125} \Rightarrow p = 25 \text{ rad/s}$$

$$\frac{1}{125} = C_2 R_2 \Rightarrow C_2 = \frac{1}{125 R_2} = \frac{1}{125\left(25\times 10^3\right)} = 0.32\times 10^{-6} = 0.32\ \mu F$$

P 13.2-15
Solution:
Mesh equations:

$$\mathbf{V}_s(\omega) = (R_1 + j\omega L_1)\mathbf{I}_1(\omega) + j\omega M\,\mathbf{I}_2(\omega)$$
$$0 = (R_2 + j\omega L_2)\mathbf{I}_2(\omega) + j\omega M\,\mathbf{I}_1(\omega)$$

Solving the mesh equations

$$\mathbf{I}_1(\omega) = -\frac{R_2 + j\omega L_2}{j\omega M}\mathbf{I}_2(\omega)$$
$$\mathbf{V}_s(\omega) = \left[-(R_1 + j\omega L_1)\frac{R_2 + j\omega L_2}{j\omega M} + j\omega M\right]\mathbf{I}_2(\omega)$$

$$\mathbf{I}_2(\omega) = \frac{-j\omega M}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) + \omega^2 M^2}\mathbf{V}_s(\omega)$$
$$\mathbf{V}_o(\omega) = -R_2\,\mathbf{I}_2(\omega) = \frac{j\omega M\,R_2}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) + \omega^2 M^2}\mathbf{V}_s(\omega)$$
$$= \frac{j\omega M\,R_2}{R_1 R_2 + \omega^2\left(M^2 - L_1 L_2\right) + j\omega\left(R_1 L_2 + L_1 R_2\right)}\mathbf{V}_s(\omega)$$
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{M\,R_2}{R_1 R_2 + \omega^2\left(M^2 - L_1 L_2\right)}\,\frac{j\omega}{1 + j\omega\dfrac{R_1 L_2 + L_1 R_2}{R_1 R_2 + \omega^2\left(M^2 - L_1 L_2\right)}}$$

Comparing to the given network function:

$$k = \frac{M\,R_2}{R_1 R_2 + \omega^2\left(M^2 - L_1 L_2\right)} \quad \text{and} \quad p = \frac{R_1 R_2 + \omega^2\left(M^2 - L_1 L_2\right)}{R_1 L_2 + L_1 R_2}$$

P 13.2-16
Solution:
Using voltage division twice gives:

$$\frac{A\mathbf{V}_2(\omega)}{\mathbf{V}_i(\omega)}=\frac{\dfrac{R_2}{1+j\omega C R_2}}{R_1+\dfrac{R_2}{1+j\omega C R_2}}A=\frac{A R_2}{R_1+R_2+j\omega C R_1R_2}=\frac{\dfrac{A R_2}{R_1+R_2}}{1+j\omega\dfrac{C R_1R_2}{R_1+R_2}}$$

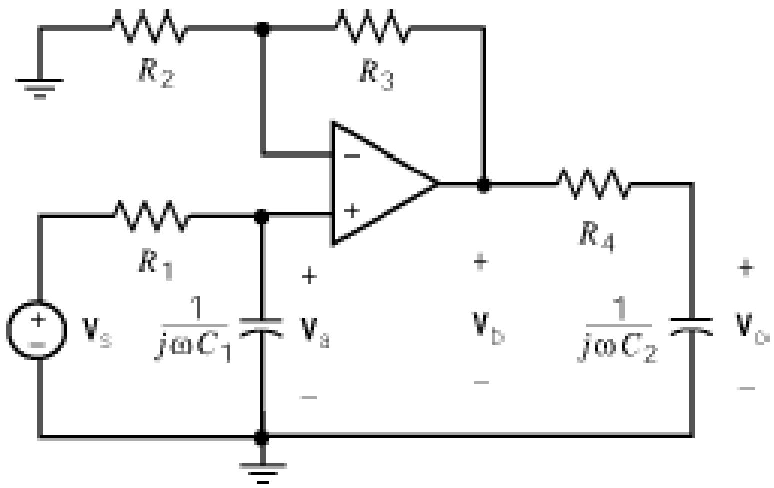
and
$$\frac{\mathbf{V}_o(\omega)}{A\mathbf{V}_2(\omega)}=\frac{\dfrac{j\omega L R_4}{R_4+j\omega L}}{R_3+\dfrac{j\omega L R_4}{R_4+j\omega L}}=\frac{j\omega L R_4}{R_3R_4+j\omega L\left(R_3+R_4\right)}=\frac{L}{R_3}\frac{j\omega}{1+j\omega\dfrac{L\left(R_3+R_4\right)}{R_3R_4}}$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{ALR_2}{R_3\left(R_1+R_2\right)}\frac{j\omega}{\left(1+j\omega\dfrac{L\left(R_3+R_4\right)}{R_3R_4}\right)\left(1+j\omega\dfrac{C R_1R_2}{R_1+R_2}\right)}$$

Comparing to the given network function gives $k=\dfrac{ALR_2}{R_3\left(R_1+R_2\right)}$ and either $p_1=\dfrac{R_3R_4}{L\left(R_3+R_4\right)}$ and $p_2=\dfrac{R_1+R_2}{CR_1R_2}$ or $p_1=\dfrac{R_1+R_2}{CR_1R_2}$ and $p_2=\dfrac{R_3R_4}{L\left(R_3+R_4\right)}$.

P 13.2-17
Solution:
Represent the circuit in the frequency domain.



Apply KCL at the top node of the left capacitor, C_1, to get

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + j\omega C_1 \mathbf{V}_a = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{1}{1 + j\omega C_1 R_1} \mathbf{V}_s$$

The op amp, together with resistors R_2 and R_3, comprise a noninverting amplifier so

$$\mathbf{V}_b = \left(1+\frac{R_3}{R_2}\right)\mathbf{V}_a$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.)
Apply KCL at the top node of the right capacitor, C_2, to get

$$\frac{\mathbf{V}_o-\mathbf{V}_b}{R_4}+j\omega C_2\mathbf{V}_o=0 \;\Rightarrow\; \mathbf{V}_o=\frac{1}{1+j\omega C_2 R_4}\mathbf{V}_b$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{1+\frac{R_3}{R_2}}{\left(1+j\omega C_1 R_1\right)\left(1+j\omega C_2 R_4\right)}$$

Comparing to the specified network function gives

$$\frac{1+\frac{R_3}{R_2}}{\left(1+j\omega C_1 R_1\right)\left(1+j\omega C_2 R_4\right)}=\frac{25}{\left(1+j\frac{\omega}{6}\right)\left(1+j\frac{\omega}{240}\right)}$$

The solution is not unique. For example, we can require

$$1+\frac{R_3}{R_2}=25,\; C_1 R_1=\frac{1}{6}=0.16,\; C_2 R_4=\frac{1}{240}=0.004$$

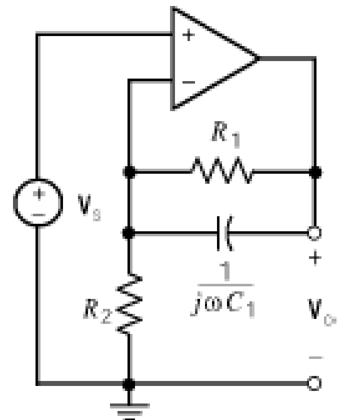
With the given values of capacitance, and choosing R_2 = 10 kΩ, we have

R_1 = 133 kΩ, R_2 = 10 kΩ, R_3 = 240 kΩ and R_4 = 3.3kΩ

P 13.2-18

Solution:

Represent the circuit in the frequency domain. Apply KCL at the inverting input node of the op amp to get



$$\frac{\mathbf{V}_o - \mathbf{V}_s}{R_1} + j\omega C_1(\mathbf{V}_o - \mathbf{V}_s) + \frac{\mathbf{V}_s}{R_2} = 0$$

or

$$(R_1 + R_2 + j\omega C_1 R_1 R_2)\mathbf{V}_s = (R_2 + j\omega C_1 R_1 R_2)\mathbf{V}_o$$

so

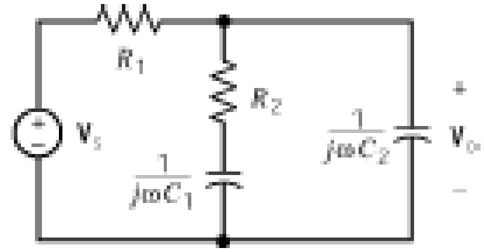
$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{R_1 + R_2 + j\omega C_1 R_1 R_2}{R_2 + j\omega C_1 R_1 R_2} = \frac{R_1 + R_2}{R_2} \times \frac{1 + j\omega C_1 \frac{R_1 R_2}{R_1 + R_2}}{1 + j\omega C_1 R_1}$$

With the given values

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{7}{2} * \frac{1 + \frac{j\omega}{7}}{1 + \frac{j\omega}{2}}$$

P 13.2-19
Solution:

Represent the circuit in the frequency domain. After determining some equivalent impedances, the network function can be determined using voltage division.



$$\frac{1}{j\omega C_2} \| \left(R_2 + \frac{1}{j\omega C_1}\right) = \frac{\frac{1}{j\omega C_2}\left(R_2 + \frac{1}{j\omega C_1}\right)}{R_2 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}$$

Next, using voltage division gives

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}}{R_1 + \frac{1 + j\omega C_1 R_2}{j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2}} = \frac{1 + j\omega C_1 R_2}{R_1\left[j\omega\left(C_1 + C_2\right) - \omega^2 R_2 C_1 C_2\right] + 1 + j\omega C_1 R_2}$$

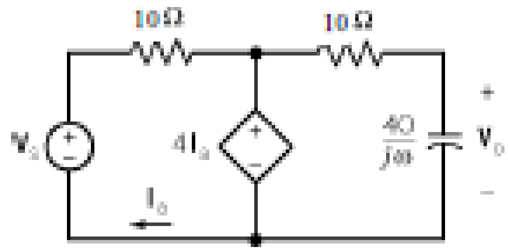
$$= \frac{1 + j\omega C_1 R_2}{1 - \omega^2 C_1 C_2 R_1 R_2 + j\omega\left(C_1 R_1 + C_2 R_1 + R_2 C_1\right)}$$

With the given values

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1 + j\frac{3\omega}{25}}{1 - \frac{9\omega^2}{625} + j\frac{9\omega}{25}} = \frac{625 + j75\omega}{625 - 9\omega^2 + j225\omega}$$

P 13.2-20
Solution:

Represent the circuit in the frequency domain. Apply KVL to the left mesh to get



$$\mathbf{V}_s = 10\mathbf{I}_a + 4\mathbf{I}_a \quad \Rightarrow \quad \mathbf{I}_a = \frac{\mathbf{V}_s}{14}$$

Voltage division gives

$$\mathbf{V}_o = \frac{\frac{40}{j\omega}}{10+\frac{40}{j\omega}} 4\mathbf{I}_a = \frac{4}{1+\frac{j\omega}{4}}\mathbf{I}_a = \frac{4}{1+\frac{j\omega}{4}}\left(\frac{\mathbf{V}_s}{14}\right) = \frac{\frac{2}{7}}{1+\frac{j\omega}{4}}\mathbf{V}_s$$

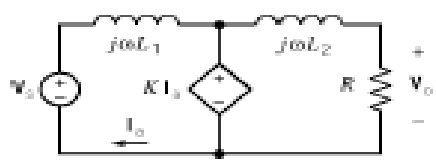
The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\frac{2}{7}}{1+\frac{j\omega}{4}}$$

Comparing this network function to the specified network function gives

$$H_o = \frac{2}{7} \text{ and } p = 4$$

P 13.2-21
Solution:
Represent the circuit in the frequency domain. Apply KVL to the left mesh to get



$$\mathbf{V}_s = j\omega L_1 \mathbf{I}_a + K\mathbf{I}_a \quad \Rightarrow \quad \mathbf{I}_a = \frac{\mathbf{V}_s}{K + j\omega L_1}$$

Voltage division gives

$$\mathbf{V}_o = \frac{R}{R+j\omega L_2} K\mathbf{I}_a = \frac{R}{R+j\omega L_2} K\left(\frac{\mathbf{V}_s}{K+j\omega L_1}\right) = \frac{RK}{(R+j\omega L_2)(K+j\omega L_1)}\mathbf{V}_s$$

The network function of the circuit is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1}{\left(1+j\omega\frac{L_2}{R}\right)\left(1+j\omega\frac{L_1}{K}\right)}$$

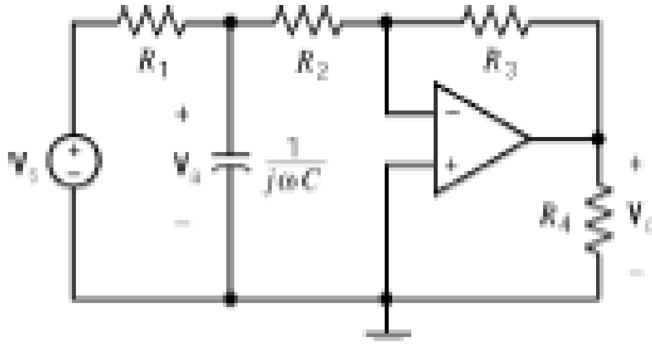
Comparing this network function to the specified network function gives

$$\frac{L_2}{R} = \frac{1}{30} \text{ and } \frac{L_1}{K} = \frac{1}{60} \quad \text{or} \quad \frac{L_2}{R} = \frac{1}{60} \text{ and } \frac{L_1}{K} = \frac{1}{30}$$

These equations do not have a unique solution. One solution is

$$L_1 = 0.07 \text{ H}, \ L_2 = 0.16 \text{ H}, \quad R = 5\ \Omega \text{ and } K = 2 \text{ V/A}$$

P 13.2-22
Solution:
Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_s-\mathbf{V}_a}{R_1}+\frac{\mathbf{V}_a}{\dfrac{1}{j\omega C}}+\frac{\mathbf{V}_a}{R_2}=0 \;\Rightarrow\; \mathbf{V}_a=\frac{R_2}{R_1+R_2+j\omega C R_1 R_2}\mathbf{V}_s$$

and

$$\frac{\mathbf{V}_a}{R_2}+\frac{\mathbf{V}_o}{R_3}=0 \;\Rightarrow\; \mathbf{V}_o=-\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\dfrac{R_3}{R_2}R_2}{R_1+R_2+j\omega C R_1 R_2}=\frac{-\dfrac{R_3}{R_1+R_2}}{1+j\omega C\dfrac{R_1 R_2}{R_1+R_2}}$$

Using the given values for R_1 and R_2 and letting $R_3 = R$ gives

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\dfrac{R}{6\times10^4}}{1+j\omega C\left(1.5\times10^4\right)}$$

Comparing this network function to the specified network function gives

$$C\left(1.5\times10^4\right)=\frac{1}{375} \;\Rightarrow\; C=0.18\ \mu\text{F} \text{ and } \frac{R}{6\times10^4}=12 \;\Rightarrow\; R=720\ \text{k}\Omega$$

P13.2-23
Solution:

When the input to this circuit is $v_s(t) = 5\cos(5t+15°)$ V :

$$\mathbf{H}(5) = \frac{j\,40(5)}{120+j\,20(5)} = 1.2804\angle 50.194° \;\Rightarrow\; v_o(t) = 5(1.2804)\cos(5t+15°+50.194°)$$

$$= 6.4018\cos(5t+65.194°) \text{ V}$$

When the input to this circuit is $v_s(t) = 8\cos(8t-15°)$ V

$$\mathbf{H}(8) = \frac{j\,40(8)}{120+j\,20(8)} = 1.6\angle 36.87° \;\Rightarrow\; v_o(t) = 5(1.6)\cos(8t+15°+36.87°)$$

$$= 8\cos(8t+51.87°) \text{ V}$$

P13.2-24
Solution:

$$\frac{k}{1+j\dfrac{120}{p}} = \frac{k}{\sqrt{1+\left(\dfrac{120}{p}\right)^2}}\angle -\tan^{-1}\left(\frac{120}{p}\right) \text{ and } \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{42.36\angle -48.69°}{12\angle 30°} = 3.53\angle -78.69°$$

so

$$-\tan^{-1}\left(\frac{120}{p}\right) = -78.69° \;\Rightarrow\; \frac{120}{p} = \tan(78.69°) = 5 \;\Rightarrow\; p = \frac{120}{5} = 24 \text{ rad/s}$$

and

$$\frac{k}{\sqrt{1+\left(\dfrac{120}{p}\right)^2}} = \frac{k}{\sqrt{1+(5)^2}} = 3.53 \;\Rightarrow\; k = 3.53\sqrt{26} = 18$$

`

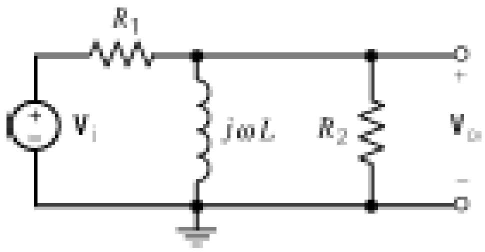
P13.2-25
Solution:

The gain is 2 at the frequency ω_1 so $2 = \dfrac{30}{\sqrt{12^2+\omega_1^{\,2}}}$ and $\omega_1 = \sqrt{\left(\dfrac{30}{2}\right)^2 - 12^2} = 9$ rad/s .

When the frequency is ω_2, the period is $\dfrac{2\pi}{\omega_2}$. Also a delay t_o corresponds to a phase shift $-\omega_2 t_o$. In this case, $t_o = 0.08\left(\dfrac{2\pi}{\omega_2}\right)$ so the phase shift is -0.16 π. Then $-0.16\pi = -\tan^{-1}\left(\dfrac{\omega_2}{12}\right)$ so

$\omega_2 = 12\tan(0.16\pi) = 6.597$ rad/s .

P13.2-26
Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 18° so $\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 18°)}{50} = 0.06$

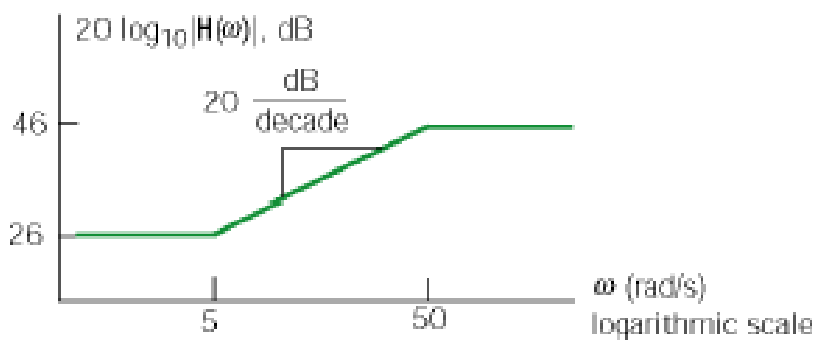
and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{3.1}{10}$ so $\dfrac{50 \dfrac{L}{R_1}}{\sqrt{1+25}} = \dfrac{3.1}{10} \Rightarrow \dfrac{L}{R_1} = 0.0316$.

One set of values that satisfies these two equations is $L = 1.25$ H, $R_1 = 39.5\ \Omega$, $R_2 = 13.39\ \Omega$.

Section 13-3: Bode Plots

P 13.3-1
Solution:

$$\mathbf{H}(\omega)=\frac{20\left(1+j\dfrac{\omega}{5}\right)}{\left(1+j\dfrac{\omega}{50}\right)}\approx\begin{cases}20 & \omega<5\\ 20\left(j\dfrac{\omega}{5}\right) & 5<\omega<50\\ \dfrac{20\left(j\dfrac{\omega}{5}\right)}{\left(j\dfrac{\omega}{50}\right)}=200 & 50<\omega\end{cases}$$

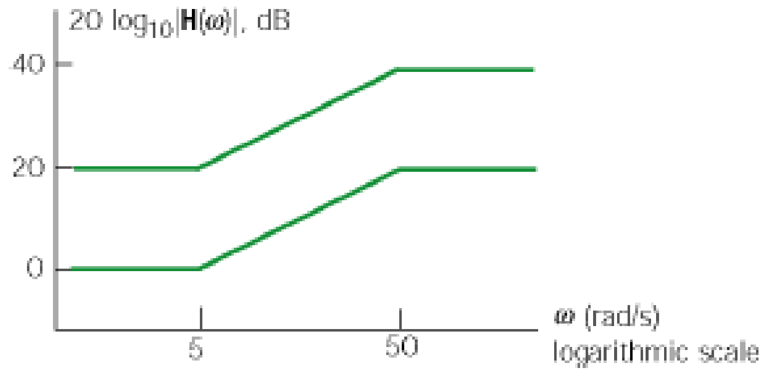


P 13.3-2
Solution:

$$\mathbf{H}_1(\omega) = \frac{10(5+j\omega)}{50+j\omega} = \frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}} \quad \text{and} \quad \mathbf{H}_2(\omega) = \frac{100(5+j\omega)}{50+j\omega} = 10\,\frac{1+j\dfrac{\omega}{5}}{1+j\dfrac{\omega}{50}}$$

Both $\mathbf{H}_1(\omega)$ and $\mathbf{H}_2(\omega)$ have a pole at $\omega = 50$ rad/s and a zero at $\omega = 5$ rad/s. The slopes of both magnitude Bode plots increase by 20 dB/decade at $\omega = 5$ rad/s and decrease by 20 dB/decade at $\omega =$ 50rad/s. The difference is that for $\omega < 5$rad/s

$$|\mathbf{H}_1(\omega)| \simeq 1 = 0 \text{ dB} \quad \text{and} \quad |\mathbf{H}_2(\omega)| \simeq 10 = 20 \text{ dB}$$



P 13.3-3
Solution:

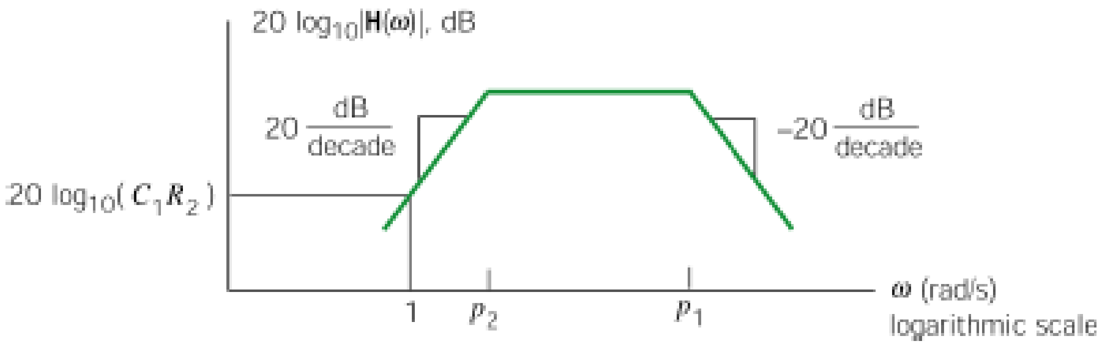
$$\mathbf{H}(\omega) = -\frac{\dfrac{R_2}{1+j\omega C_2 R_2}}{R_1 + \dfrac{1}{j\omega C_1}} = -C_1 R_2 \frac{j\omega}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)}$$

This network function has poles at

$$p_1 = \frac{1}{R_1 C_1} = 1000\,\text{rad/s} \text{ and } p_2 = \frac{1}{R_2 C_2} = 500\,\text{rad/s}$$

so

$$\mathbf{H}(\omega) \simeq \begin{cases} -(C_1 R_2)\, j\omega & \omega < p_2 \\ -(C_1 R_2)\dfrac{j\omega}{j\omega C_1 R_1} = -\dfrac{R_2}{R_1} = -2 & p_2 < \omega < p_1 \\ -(C_1 R_2)\dfrac{j\omega}{(j\omega C_1 R_1)(j\omega C_2 R_2)} = -\dfrac{1}{j\omega C_2 R_1} & \omega > p_1 \end{cases}$$



P 13.3-4
Solution:
Using voltage division twice gives:

$$\frac{\mathbf{V}_2(\omega)}{\mathbf{V}_i(\omega)}=\frac{\dfrac{j\omega L R_2}{R_2+j\omega L}}{R_1+\dfrac{j\omega L R_2}{R_2+j\omega L}}=\frac{j\omega L R_2}{R_1R_2+j\omega L\left(R_1+R_2\right)}=\frac{L}{R_1}\frac{j\omega}{1+j\omega\dfrac{L\left(R_1+R_2\right)}{R_1R_2}}$$

and

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_2(\omega)}=\frac{\dfrac{R_4}{1+j\omega C R_4}}{R_3+\dfrac{R_4}{1+j\omega C R_4}}A=\frac{A R_4}{R_3+R_4+j\omega C R_3R_4}=\frac{\dfrac{A R_4}{R_3+R_4}}{1+j\omega\dfrac{C R_3R_4}{R_3+R_4}}$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{ALR_4}{R_1\left(R_3+R_4\right)}\frac{j\omega}{\left(1+j\omega\dfrac{L\left(R_1+R_2\right)}{R_1R_2}\right)\left(1+j\omega\dfrac{CR_3R_4}{R_3+R_4}\right)}$$

The Bode plot corresponds to the network function:

$$\mathbf{H}(\omega)=\frac{k\,j\omega}{\left(1+j\dfrac{\omega}{p_1}\right)\left(1+j\dfrac{\omega}{p_2}\right)}=\frac{k\,j\omega}{\left(1+j\dfrac{\omega}{200}\right)\left(1+j\dfrac{\omega}{20000}\right)}$$

$$\mathbf{H}(\omega)\approx\begin{cases}\dfrac{k\,j\omega}{1\cdot 1}=k\,j\omega & \omega\le p_1\\[2ex] \dfrac{k\,j\omega}{\dfrac{j\omega}{p_1}\cdot 1}=k\,p_1 & p_1\le\omega\le p_2\\[2ex] \dfrac{k\,j\omega}{\dfrac{j\omega}{p_1}\cdot\dfrac{j\omega}{p_2}}=\dfrac{k\,p_1\,p_2}{j\omega} & \omega\ge p_2\end{cases}$$

This equation indicates that $|\mathbf{H}(\omega)|=k\,p_1$ when $p_1\le\omega\le p_2$. The Bode plot indicates that $|\mathbf{H}(\omega)|=20$ dB $=10$ when $p_1\le\omega\le p_2$. Consequently

$$k=\frac{10}{p_1}=\frac{10}{200}=0.05$$

Finally, $$\mathbf{H}(\omega)=\frac{0.05\,j\omega}{\left(1+j\frac{\omega}{200}\right)\left(1+j\frac{\omega}{20000}\right)}$$

Comparing the equation for $\mathbf{H}(\omega)$ obtained from the circuit to the equation for $\mathbf{H}(\omega)$obtained from the Bode plot gives:

$$0.05=\frac{ALR_4}{R_1\left(R_3+R_4\right)},\ 200=\frac{R_1R_2}{L\left(R_1+R_2\right)}\text{and } 20000=\frac{R_3+R_4}{C\,R_3R_4}$$

Pick L = 1 H, and $R_1 = R_2$, then $R_1 = R_2$ = 400 Ω. Let C = 0.1 μF and $R_3 = R_4$, then $R_3 = R_4$ = 1000 Ω. Finally, A=40.

(Checked using ELab 3/5/01)

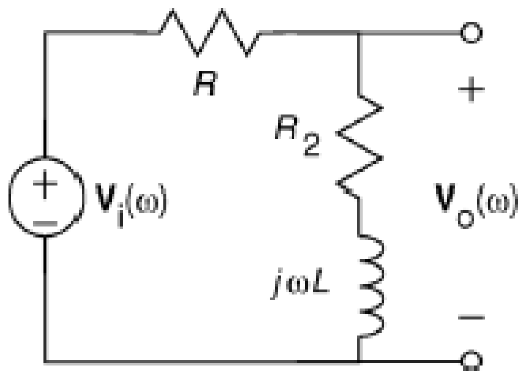
P 13.3-5
Solution:
From Table 13.3-2:

$$\frac{R_2}{R_1}=k=32\text{ dB}=40\quad R_2=40\left(10\times10^3\right)=400\text{ k}\Omega$$

$$\frac{1}{C_2\,R_2}=p=400\text{ rad/s}\quad\Rightarrow\quad C_2=\frac{1}{(400)\left(400\times10^3\right)}=6.25\text{ nF}$$

$$\frac{1}{C_1\,R_1}=z=4000\text{ rad/s}\quad\Rightarrow\quad C_1=\frac{1}{(4000)\left(10\times10^3\right)}=25\text{ nF}$$

P 13.3-6
Solution:



$$\mathbf{H}(\omega)=\frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)}=\frac{R_2+j\omega L}{R+R_2+j\omega L}$$

$$=\left(\frac{R_2}{R+R_2}\right)\left(\frac{1+j\omega\frac{L}{R_2}}{1+j\omega\frac{L}{R+R_2}}\right)$$

$$\mathbf{H}(\omega)=\frac{(0.2)(1+j(0.2)\omega)}{1+j(0.04)\omega} \quad \Rightarrow \quad \begin{cases} k=0.2 \\ z=\frac{1}{0.2}=5 \\ p=\frac{1}{0.04}=25 \end{cases}$$

P 13.3-7
Solution

- The slope is 40dB/decade for low frequencies, so the numerator will include the factor $(j\omega)^2$.
- The slope decreases by 40 dB/decade at ω = 0.7rad/sec. So there is a second order pole at ω_0 = 0.7 rad/s. The damping factor of this pole cannot be determined from the asymptotic Bode plot; call it δ_1. The denominator of the network function will contain the factor

$$1+2\delta_1 j\frac{\omega}{0.7}-\left(\frac{\omega}{0.7}\right)^2$$

- The slope increases by 20 dB/decade at ω = 10 rad/s, indicating a zero at 10 rad/s.
- The slope decreases by 20 dB/decade at ω = 100 rad/s, indicating a pole at 100 rad/s.
- The slope decreases by 40 dB/decade at ω = 600 rad/s, indicating a second order pole at ω_0 = 600rad/s. The damping factor of this pole cannot be determined from an asymptotic Bode plot; call it δ_2. The denominator of the network function will contain the factor

$$1+2\delta_2 j\frac{\omega}{600}-\left(\frac{\omega}{600}\right)^2$$

$$\mathbf{H}(\omega) = \frac{K(1+j\frac{\omega}{10})(j\omega)^2}{\left(1+2\delta_1 j\frac{\omega}{0.7}-\left(\frac{\omega}{0.7}\right)^2\right)\left(1+2\delta_2 j\frac{\omega}{600}-\left(\frac{\omega}{600}\right)^2\right)\left(1+j\frac{\omega}{100}\right)}$$

To determine K, notice that $|\mathbf{H}(\omega)| = 1$ dB=0 when $0.7 < \omega < 10$. That is

$$1=\frac{K(1)\,\omega^2}{-\left(\frac{\omega}{0.7}\right)^2(1)(1)}=K(0.7)^2 \;\Rightarrow\; K=2$$

P 13.3-8
Solution:
(a)

$$H(\omega) = \frac{K\left(1 + j\dfrac{\omega}{z}\right)}{j\omega}$$

$$|H(\omega)| = \frac{K}{\omega}\sqrt{1 + \left(\frac{\omega}{z}\right)^2}$$

$$|H(\omega)|\ dB = 20\ \log_{10} \frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

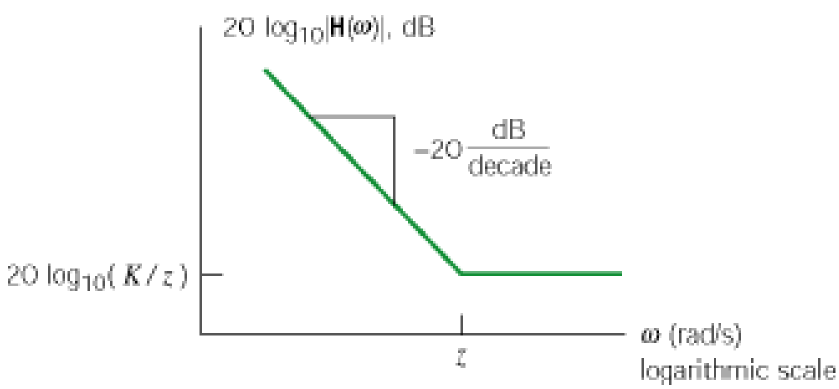
$$= 20\ \log_{10} K - 20\log_{10}\ \omega + 20\ \log_{10}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

Let $|H_L(\omega)|$ dB $= 20\ \log_{10} K - 20\log_{10}\omega$

and $|H_H(\omega)|$ dB $= 20\ \log_{10}\dfrac{K}{z}$

$$\text{Then } |H(\omega)|\text{ dB} \simeq \begin{cases} |H_L(\omega)|\text{ dB} & \omega \ll z \\ |H_H(\omega)|\text{ dB} & \omega \gg z \end{cases}$$

So $|H_L(\omega)|$ dB and $|H_H(\omega)|$ dB are the required low and high-frequency asymptotes.



The Bode plot will be within 1% of $|H(\omega)|$ dB both for $\omega << z$ and for $\omega >> z$. The range when $\omega << z$ is characterized by

$$|H_L(\omega)| = 0.99|H(\omega)| \qquad \text{(gains not in dB)}$$

or equivalently

$$20\log_{10}(0.99) = |\mathbf{H}_L(\omega)|\ \text{dB} - |\mathbf{H}(\omega)|\ \text{dB} \qquad \text{(gains in dB)}$$

$$= 20\log_{10}K - 20\log_{10}\omega - 20\log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\log_{10}\sqrt{1+\left(\frac{\omega}{z}\right)^2} = 20\log_{10}\frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}}$$

Therefore

$$0.99 = \frac{1}{\sqrt{1+\left(\frac{\omega}{z}\right)^2}} \Rightarrow \omega = z\sqrt{\left(\frac{1}{.99}\right)^2 - 1} = 0.14z \doteq \frac{z}{7}$$

The range when $\omega >> z$ is characterized by

$$|\mathbf{H}_H(\omega)| = .99|\mathbf{H}(\omega)| \qquad \text{(gains not in dB)}$$

or equivalently

$$20\log_{10}0.99 = |\mathbf{H}_H(\omega)|\ \text{dB} - |\mathbf{H}(\omega)|\ \text{dB} \qquad \text{(gains in dB)}$$

$$= 20\ \log_{10}K - 20\ \log_{10}z - 20\ \log_{10}\frac{K}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2}$$

$$= -20\ \log_{10}\frac{z}{\omega}\sqrt{1+\left(\frac{\omega}{z}\right)^2} = 20\ \log_{10}\frac{1}{\sqrt{\left(\frac{z}{\omega}\right)^2+1}}$$

Therefore

$$\frac{z}{\omega} = \sqrt{\left(\frac{1}{.99}\right)^2 - 1} \Rightarrow \omega = \frac{z}{\sqrt{\left(\frac{1}{.99}\right)^2 - 1}} = \frac{z}{0.14} \doteq 7z$$

The error is less than 1% when $\omega < \frac{z}{7}$ and when $\omega > 7z$.

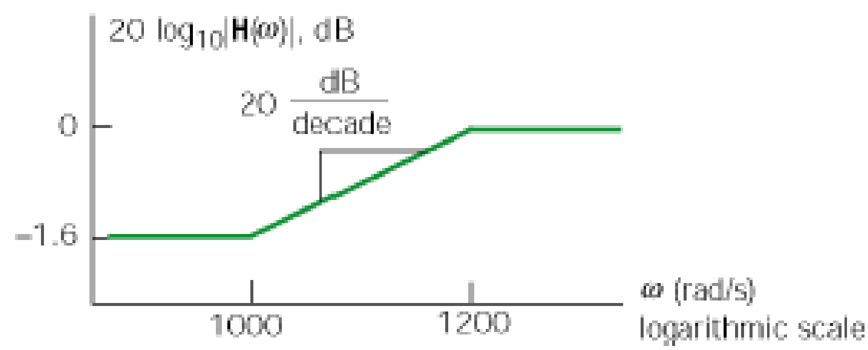
P 13.3-9
Solution:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{R_t}{R_t + R_1 \| \dfrac{1}{j\omega C}} = \frac{R_t}{R_t + \dfrac{R_1}{1+j\omega C R_1}}$$

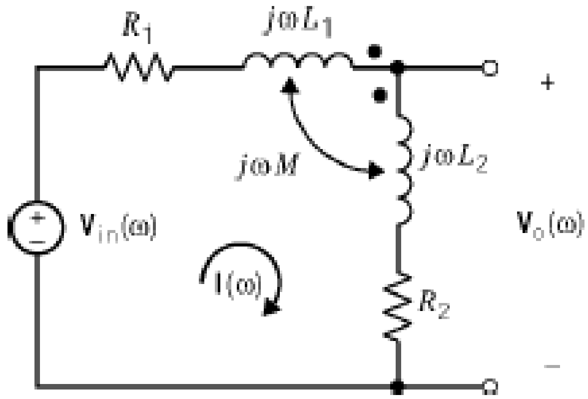
$$= \frac{R_t(1+j\omega C R_1)}{R_1 + R_t + j\omega C R_1 R_t} = \left(\frac{R_t}{R_1+R_t}\right)\frac{1+j\omega C R_1}{1+j\omega\left(\dfrac{C R_1 R_t}{R_1+R_t}\right)}$$

When $R_1 = 2\ \text{k}\Omega$, $C = 2\ \mu F$ and $R_t = 6\ \text{k}\Omega$

$$\mathbf{H}(\omega) = \frac{3}{4}\left(\frac{1+j\dfrac{\omega}{250}}{1+j\dfrac{3\omega}{1000}}\right) \quad \Rightarrow \quad \mathbf{H}(\omega) \cong \begin{cases} \dfrac{3}{4} & \omega < 250 \\ \left(\dfrac{3}{4}\right) j\dfrac{\omega}{250} & 250 < \omega < \dfrac{1000}{3} \\ 1 & \omega < \dfrac{1000}{3} \end{cases}$$



P 13.3-10
Solution:



Mesh equations:

$$\mathbf{V}_{in}(\omega) = \mathbf{I}(\omega)\,[R_1 + (j\omega L_1 - j\omega M) + (-j\omega M + j\omega L_2) + R_2]$$
$$\mathbf{V}_o(\omega) = \mathbf{I}(\omega)\,[(-j\omega M + j\omega L_2) + R_2]$$

Solving yields:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_{in}(\omega)} = \frac{R_2 + j\omega(L_2 - M)}{R_1 + R_2 + j\omega(L_1 + L_2 - 2M)}$$

Comparing to the given Bode plot yields:

$$K_1 = \lim_{\omega\to\infty} |\mathbf{H}(\omega)| = \frac{L_2 - M}{L_1 + L_2 - 2M} = 0.75 \quad \text{and} \quad K_2 = \lim_{\omega\to 0} |\mathbf{H}(\omega)| = \frac{R_2}{R_1 + R_2} = 0.2$$

$$z = \frac{R_2}{L_2 - M} = 333 \text{ rad/s} \quad \text{and} \quad p = \frac{R_1 + R_2}{L_1 + L_2 - 2M} = 1250 \text{ rad/s}$$

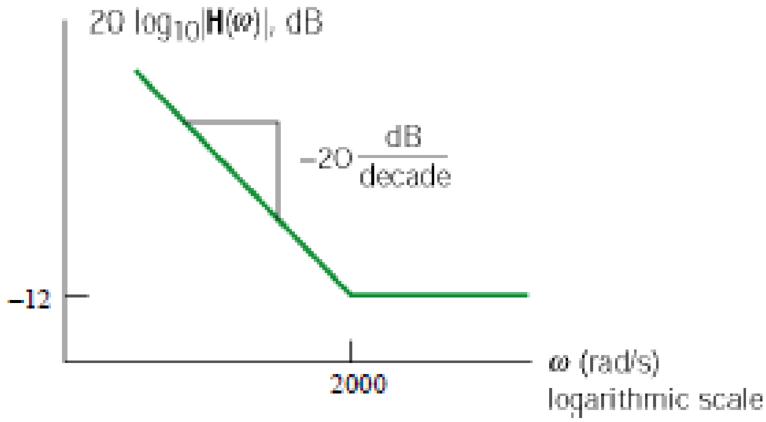
P 13.3-11
Solution:

$$\mathbf{H}(\omega)=-\frac{\dfrac{1}{j\omega C_2}}{R_1 \left\| \dfrac{1}{j\omega C_1}\right.}=-\frac{1+j\omega R_1 C_1}{j\omega R_1 C_2}=-\frac{1}{R_1 C_2}\frac{(1+j\omega R_1 C_1)}{j\omega}$$

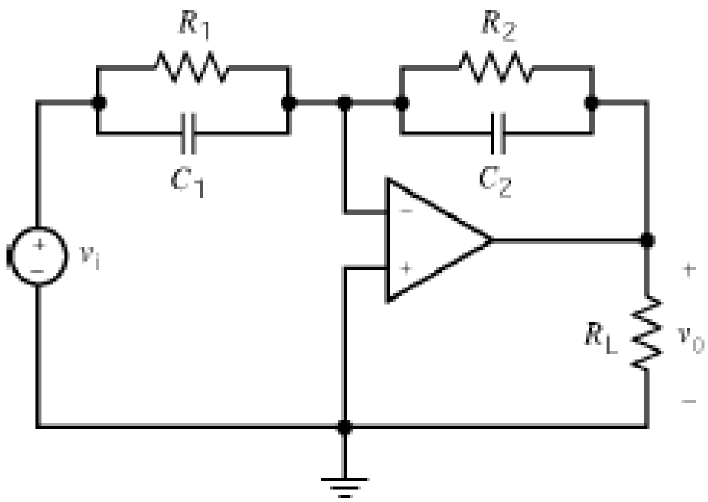
$$\mathbf{H}(\omega)\doteq\begin{cases}-\dfrac{1}{R_1 C_2}\left(\dfrac{1}{j\omega}\right) & \omega<\dfrac{1}{R_1 C_1}\\ -\dfrac{1}{R_1 C_2}(R_1 C_1)=-\dfrac{C_1}{C_2} & \omega>\dfrac{1}{R_1 C_1}\end{cases}$$

With the given values:

$$20\log\left(\frac{C_1}{C_2}\right)=20\log(0.25)=-12\text{ dB},\quad \frac{1}{R_1 C_1}=2000\text{ rad/s}$$



P 13.3-12
Solution:
Pick the appropriate circuit from Table 13.3-2.



$$\mathbf{H}(\omega) = -k\,\frac{1+j\dfrac{\omega}{z}}{1+j\dfrac{\omega}{p}}$$

where

$$k = \frac{R_2}{R_1}$$

$$z = \frac{1}{C_1 R_1}$$

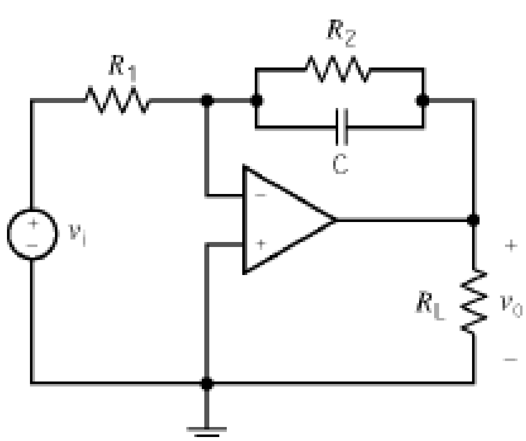
$$p = \frac{1}{C_2 R_2}$$

We require

$$200 = z = \frac{1}{C_1 R_1},\quad 500 = p = \frac{1}{C_2 R_2} \quad \text{and} \quad 14\text{ dB} = 5 = k\frac{p}{z} = \frac{C_1}{C_2}$$

Pick $C_1 = 1\ \mu\text{F}$, then $C_2 = 0.2\ \mu\text{F}$, $R_1 = 5\ \text{k}\Omega$ and $R_2 = 10\ \text{k}\Omega$.

P 13.3-13
Solution:
Pick the appropriate circuit from Table 13.3-2.



$$\mathbf{H}(\omega) = -\frac{k}{1+j\dfrac{\omega}{p}}$$

where $k = \dfrac{R_2}{R_1}$

$$p = \frac{1}{CR_2}$$

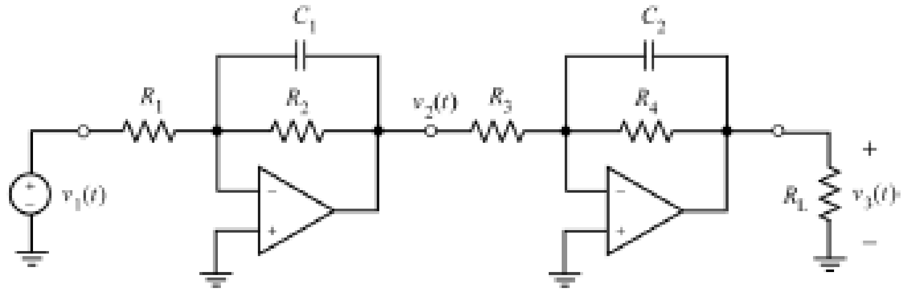
We require

$$500 = p = \frac{1}{CR_2} \quad \text{and} \quad 34\text{ dB} = 50 = \frac{R_2}{R_1}$$

Pick $C = 0.1\ \mu\text{F}$, then $R_2 = 20\ \text{k}\Omega$ and $R_1 = 400\ \Omega$.

P 13.3-14

Solution: Let's try designing the circuit as a cascade circuit using circuits from Table 13.3-2.



A Cascade Circuit

From Table 13.3-2

$$\mathrm{H}_1(\omega)=\frac{\mathrm{V}_2(\omega)}{\mathrm{V}_1(\omega)}=-\frac{k_1}{1+j\dfrac{\omega}{p_1}} \text{ where } k_1=\frac{R_2}{R_1} \text{ and } p_1=\frac{1}{C_1R_2}$$

and
$$\mathrm{H}_2(\omega)=\frac{\mathrm{V}_3(\omega)}{\mathrm{V}_2(\omega)}=-\frac{k_2}{1+j\dfrac{\omega}{p_2}} \text{ where } k_2=\frac{R_4}{R_3} \text{ and } p_2=\frac{1}{C_2R_4}$$

Consequently

$$\mathrm{H}(\omega)=\frac{\mathrm{V}_3(\omega)}{\mathrm{V}_1(\omega)}=\frac{\mathrm{V}_3(\omega)}{\mathrm{V}_2(\omega)}\cdot\frac{\mathrm{V}_2(\omega)}{\mathrm{V}_1(\omega)}=\mathrm{H}_1(\omega)\cdot\mathrm{H}_2(\omega)=\frac{k_1k_2}{\left(1+j\dfrac{\omega}{p_1}\right)\left(1+j\dfrac{\omega}{p_2}\right)}$$

(Perhaps we should verify that this transfer function is correct before proceeding.. Analysis of the cascade circuit, e.g. using node equations, shows that the transfer function is indeed correct.)

Next, from the Bode plot we see that

$$\mathrm{H}(\omega)=\frac{k}{\left(1+j\dfrac{\omega}{p_1}\right)\left(1+j\dfrac{\omega}{p_2}\right)}$$

where

$$200 \text{ rad/s}=p_1,\ \ 500 \text{ rad/s}=p_2 \ \text{ and } \ 34 \text{ dB}=50=k$$

Pick $C_2=C_1=0.1\ \mu\text{F}$. Then

$$200=\frac{1}{(10^{-7})R_2} \ \Rightarrow\ R_2=50\text{k}\Omega \ \text{ and } \ 500=\frac{1}{(10^{-7})R_4} \ \Rightarrow\ R_4=20\text{k}\Omega$$

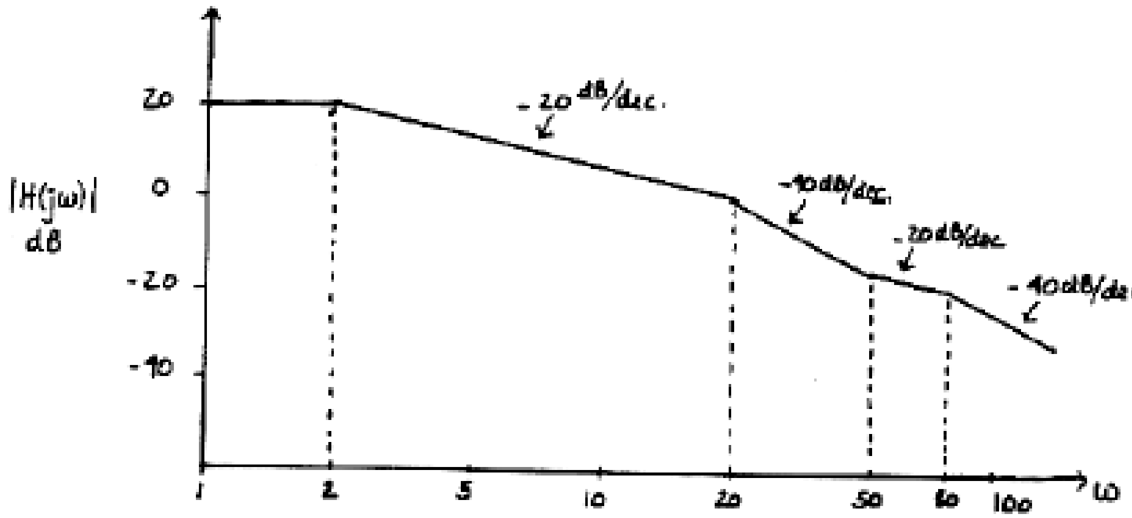
Next

$$50=k=k_1k_2=\frac{R_2}{R_1}\frac{R_4}{R_3}=\frac{(20\text{k}\Omega)(50\text{k}\Omega)}{R_1R_3}$$

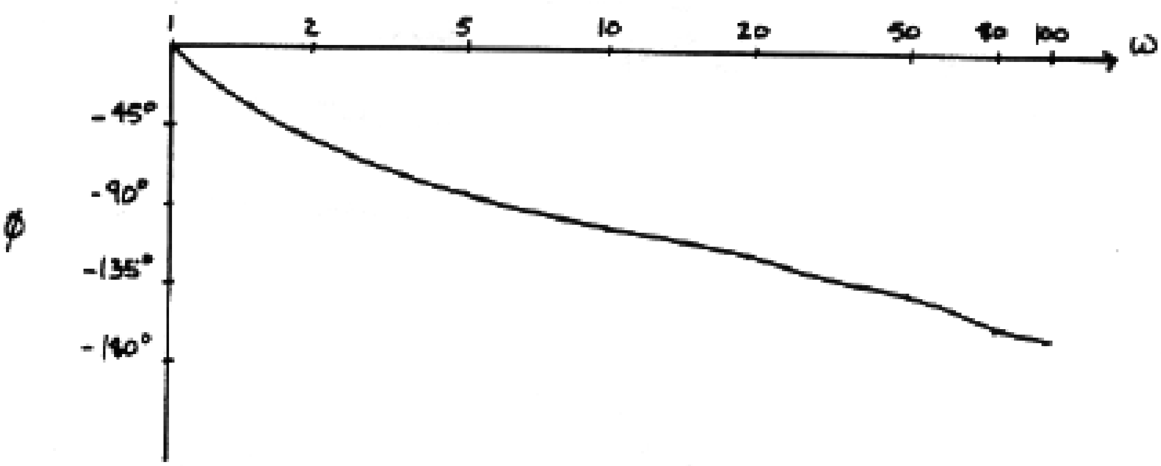
The solution is not unique. For example, we can choose $R_1=4\text{k}\Omega$ and $R_3=5\text{k}\Omega$.

P 13.3-15
Solution:

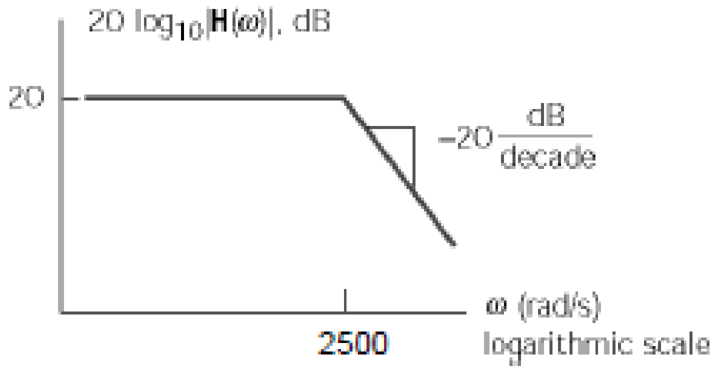
$$\mathbf{H}(\omega)=\frac{10(1+j\omega/50)}{(1+j\omega/2)(1+j\omega/20)(1+j\omega/80)}$$



$$\varphi=\angle\mathbf{H}(\omega)=\tan^{-1}(\omega/50)-\left(\tan^{-1}(\omega/2)+\tan^{-1}(\omega/20)+\tan^{-1}(\omega/80)\right)$$



P 13.3-16
Solution:



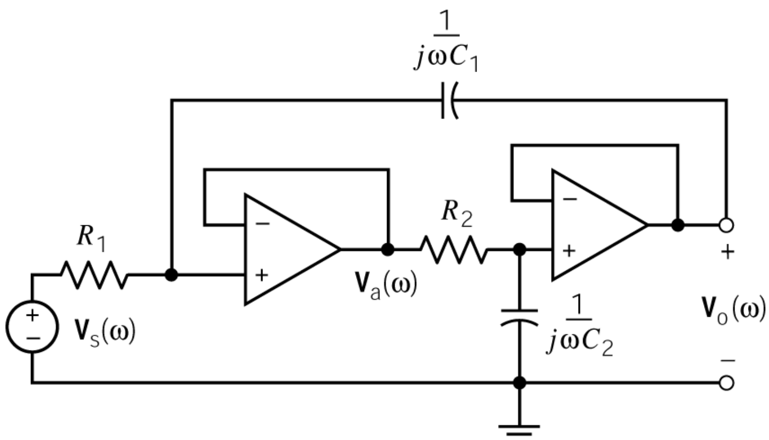
(a) $$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=-\frac{R_2/R_1}{1+j\omega R_2 C}=-\frac{10}{1+j\dfrac{\omega}{2500}}$$

(b) $$\frac{\mathbf{V_o}}{\mathbf{V_s}}=20\text{ dB}$$

(c) 2500 rad/s

P 13.3-17

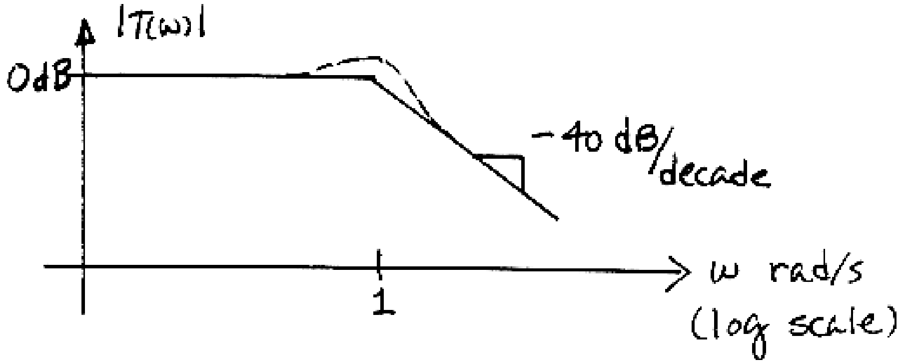
Solution:



$$\left.\begin{aligned}\mathbf{V}_o(\omega) &= \frac{\dfrac{1}{j\omega C_2}}{R+\dfrac{1}{j\omega C_2}}\mathbf{V}_a(\omega)\\ 0 &= \frac{\mathbf{V}_a(\omega)-\mathbf{V}_s(\omega)}{R_1}+j\omega C_1(\mathbf{V}_a(\omega)-\mathbf{V}_o(\omega))\end{aligned}\right\} \Rightarrow \mathbf{V}_o(1+j\omega C_1R_1)(1+j\omega C_2R_2) = j\omega C_1R_1\mathbf{V}_o+\mathbf{V}_s$$

$$\mathbf{T}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{1}{1+C_2R_2j\omega-\omega^2C_1C_2R_1R_2}=\frac{1}{-\omega^2+0.8j\omega+1}$$

This is a second order transfer function with $\omega_o=1$ and $\delta=0.4$.



P 13.3-18
Solution:

$$\mathbf{H}(\omega)=\frac{-3(5+j\omega)}{j\omega(2+j\omega)}=\frac{-\frac{15}{2}\left(1+j\frac{\omega}{5}\right)}{j\omega\left(1+j\frac{\omega}{2}\right)}$$

There is a zero at 5 rad/s and poles at 0 and 2 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right|=\begin{cases}1 & \text{for } \omega<p\\ \dfrac{\omega}{p} & \text{for } \omega>p\end{cases}$$

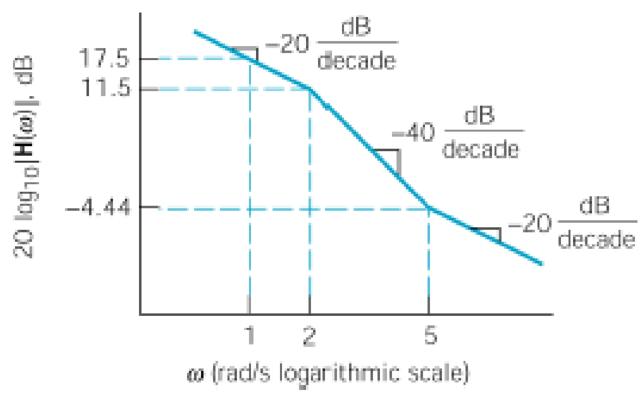
Then

$$H=|\mathbf{H}|=\begin{cases}\dfrac{\frac{15}{2}(1)}{\omega(1)}=\dfrac{7.5}{\omega} & \text{for } \omega<2\\[2ex] \dfrac{\frac{15}{2}(1)}{\omega\left(\frac{\omega}{2}\right)}=\dfrac{15}{\omega^2} & \text{for } 2<\omega<5\\[2ex] \dfrac{\frac{15}{2}\left(\frac{\omega}{5}\right)}{\omega\left(\frac{\omega}{2}\right)}=\dfrac{3}{\omega} & \text{for } \omega>5\end{cases}$$

The slope of the asymptotic magnitude Bode plot is -20 db/decade for $\omega < 2$ and $\omega > 5$ rad/s and is -40 db/decade for $2 < \omega < 5$ rad/s. Also, at $\omega = 1$ rad/s

$$H = \begin{cases} \dfrac{7.5}{1} = 7.5 \quad \text{at } \omega = 1 \text{ rad/s} \\ \dfrac{7.5}{2} = 3.75 \quad \text{at } \omega = 2 \text{ rad/s} \\ \dfrac{3}{5} = 0.6 \quad \text{at } \omega = 5 \text{ rad/s} \end{cases} \Rightarrow \quad 20\log_{10} H = \begin{cases} 20\log_{10}(7.5) = 17.5 \text{ dB} \quad \text{at } \omega = 1 \text{ rad/s} \\ 20\log_{10}(3.75) = 11.5 \text{ dB} \quad \text{at } \omega = 2 \text{ rad/s} \\ 20\log_{10}(0.6) = -4.44 \text{ dB} \quad \text{at } \omega = 5 \text{ rad/s} \end{cases}$$

The asymptotic magnitude Bode plot for **H** is



P 13.3-19
Solution:

$$\mathbf{H}(\omega)=\frac{(j\omega)^3}{(4+j2\omega)}=\frac{\frac{1}{4}(j\omega)^3}{1+j\frac{\omega}{2}}$$

There is a pole at 2 rad/s and three zeros at 0 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right|=\begin{cases}1 & \text{for } \omega<p\\ \frac{\omega}{p} & \text{for } \omega>p\end{cases}$$

Then

$$H=|\mathbf{H}|=\begin{cases}\dfrac{\frac{1}{4}\omega^3}{(1)}=\dfrac{1}{4}\omega^3 & \text{for } \omega<2\\ \dfrac{\frac{1}{4}\omega^3}{\left(\frac{\omega}{2}\right)}=\dfrac{1}{2}\omega^2 & \text{for } \omega>2\end{cases}$$

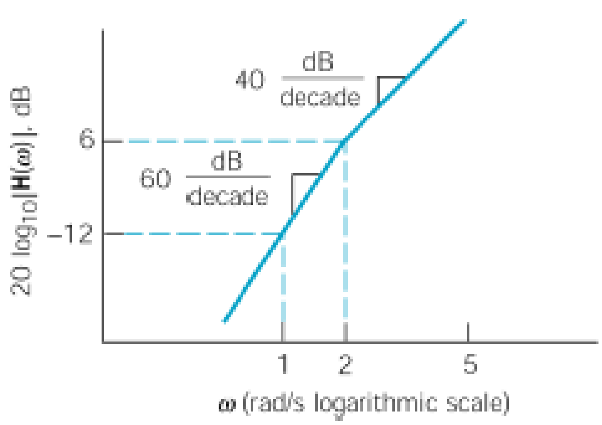
$$20\log_{10}H=\begin{cases}20\log_{10}(0.25)+3\left[20\log_{10}(\omega)\right] & \text{for } \omega<2\\ 20\log_{10}(0.50)+2\left[20\log_{10}(\omega)\right] & \text{for } \omega>2\end{cases}$$

The slope of the asymptotic magnitude Bode plot is 60 db/decade for $\omega<2$ rad/s and is 40 db/decade for $\omega>2$ rad/s. Also,

$$20\log_{10}H=20\log_{10}(0.25)+3\left[20\log_{10}(1)\right]=-12\text{ dB} \quad \text{at } \omega=1\text{ rad/s}$$

$$20\log_{10}H=20\log_{10}(0.25)+3\left[20\log_{10}(2)\right]=6\text{ dB} \quad \text{at } \omega=2\text{ rad/s}$$

The asymptotic magnitude Bode plot for **H** is



P 13.3-20
Solution:

$$\mathrm{H}(\omega)=\frac{4\,(20+j\,\omega)\,(20{,}000+j\,\omega)}{(200+j\,\omega)\,(2000+j\,\omega)}=\frac{4\left(1+j\dfrac{\omega}{20}\right)\left(1+j\dfrac{\omega}{20{,}000}\right)}{\left(1+j\dfrac{\omega}{200}\right)\left(1+j\dfrac{\omega}{2000}\right)}$$

There are zeros at 20 and 20,000 rad/s and poles at 200 and 2000 rad/s. To obtain the asymptotic magnitude Bode plot, use

$$\left|1+j\frac{\omega}{p}\right|=\begin{cases}1 & \text{for } \omega<p\\ \dfrac{\omega}{p} & \text{for } \omega>p\end{cases}$$

Then

$$H=\left|\mathrm{H}\right|=\begin{cases}\dfrac{4(1)(1)}{(1)(1)}=4 & \text{for } \omega<20 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{(1)(1)}=\dfrac{\omega}{5} & \text{for } 20<\omega<200 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{\left(\dfrac{\omega}{200}\right)(1)}=40 & \text{for } 200<\omega<2000 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)(1)}{\left(\dfrac{\omega}{200}\right)\left(\dfrac{\omega}{2000}\right)}=\dfrac{80000}{\omega} & \text{for } 2000<\omega<20{,}000 \text{ rad/s}\\[2ex] \dfrac{4\left(\dfrac{\omega}{20}\right)\left(\dfrac{\omega}{20{,}000}\right)}{\left(\dfrac{\omega}{200}\right)\left(\dfrac{\omega}{2000}\right)}=4 & \text{for } \omega>2000 \text{ rad/s}\end{cases}$$

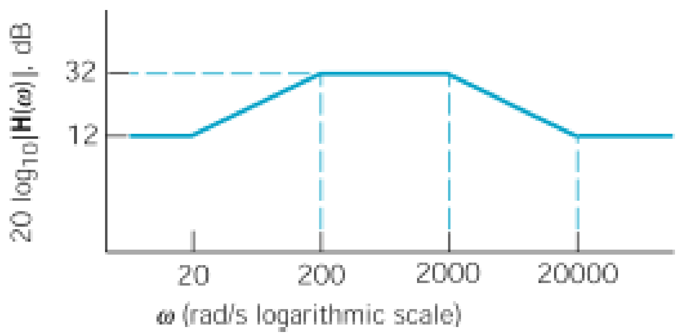
$$20\log_{10} H = \begin{cases} 20\log_{10}(4) & \text{for } \omega < 20 \\ 20\log_{10}(\omega) - 20\log_{10}(5) & \text{for } 20 < \omega < 200 \\ 20\log_{10}(40) & \text{for } 200 < \omega < 2000 \\ 20\log_{10}(80000) - 20\log_{10}(\omega) & \text{for } 2000 < \omega < 20{,}000 \\ 20\log_{10}(4) & \text{for } \omega > 20{,}000 \end{cases}$$

The slope of the asymptotic magnitude Bode plot is 20 db/decade for $20 < \omega < 200$ rad/s and is -20 db/decade for $2000 < \omega < 20{,}000$ rad/s and is 0 db/decade for $\omega < 20$ and $200 < \omega < 2000$ rad/s, and $\omega > 20{,}000$ rad/s. Also,

$$20\log_{10} H = 20\log_{10}(4) = 12 \text{ dB} \quad \text{for } \omega \le 20 \text{ and } \omega \ge 20{,}000 \text{ rad/s}$$

$$20\log_{10} H = 20\log_{10}(40) = 32 \text{ dB} \quad \text{for } 200 \le \omega \le 2000 \text{ rad/s}$$

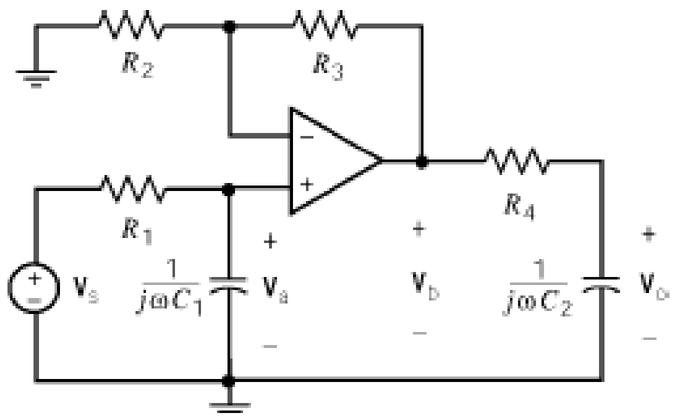
The asymptotic magnitude Bode plot for H is



P 13.3-21
Solution:
From Figure P13.3-24*b*, $\mathbf{H}(\omega)$ has poles at 8 and 320 rad/s and has a low frequency gain equal to 32 dB = 40. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega)=\frac{\pm 40}{\left(1+j\frac{\omega}{8}\right)\left(1+j\frac{\omega}{320}\right)}$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain.



Apply KCL at the top node of the left capacitor, C_1, to get

$$\frac{\mathbf{V}_a-\mathbf{V}_s}{R_1}+j\omega C_1\mathbf{V}_a=0 \;\Rightarrow\; \mathbf{V}_a=\frac{1}{1+j\omega C_1 R_1}\mathbf{V}_s$$

The op amp, together with resistors R_2 and R_3, comprise a noninverting amplifier so

$$\mathbf{V}_b=\left(1+\frac{R_3}{R_2}\right)\mathbf{V}_a$$

(Alternately, this equation can be obtained by applying KCL at the inverting input node of the op amp.)
Apply KCL at the top node of the right capacitor, C_2, to get

$$\frac{\mathbf{V}_o-\mathbf{V}_b}{R_4}+j\omega C_2\mathbf{V}_o=0 \;\Rightarrow\; \mathbf{V}_o=\frac{1}{1+j\omega C_2 R_4}\mathbf{V}_b$$

Combining these equations gives

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{1+\frac{R_3}{R_2}}{\left(1+j\omega C_1 R_1\right)\left(1+j\omega C_2 R_4\right)}$$

Comparing to the specified network function gives

$$\frac{1+\dfrac{R_3}{R_2}}{\left(1+j\omega C_1 R_1\right)\left(1+j\omega C_2 R_4\right)}=\frac{\pm 40}{\left(1+j\dfrac{\omega}{8}\right)\left(1+j\dfrac{\omega}{320}\right)}$$

The solution is not unique. For example, we can require

$$1+\frac{R_3}{R_2}=40,\ C_1 R_1=\frac{1}{8}=0.125,\ C_2 R_4=\frac{1}{320}=0.00758$$

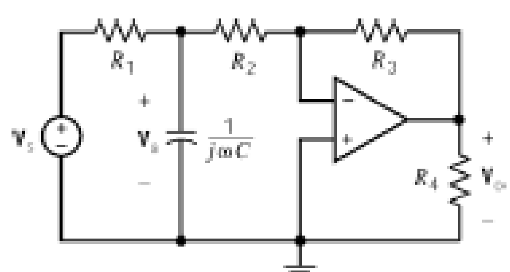
With the given values of capacitance, and choosing R_2 = 12 kΩ, we have

R_1 = 125 kΩ, R_2 = 12 kΩ, R_3 = 468 kΩ and R_4 = 7.58 kΩ

P 13.3-22
Solution:
From Figure P13.3-25*b*, $\mathbf{H}(\omega)$ has a pole at 500 rad/s and a low frequency gain of 18 dB = 8. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega)=\frac{\pm 8}{\left(1+j\dfrac{\omega}{500}\right)}$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_a-\mathbf{V}_s}{R_1}+\frac{\mathbf{V}_a}{\dfrac{1}{j\omega C}}+\frac{\mathbf{V}_a}{R_2}=0 \;\Rightarrow\; \mathbf{V}_a=\frac{R_2}{R_1+R_2+j\omega C R_1 R_2}\mathbf{V}_s$$

and

$$\frac{\mathbf{V}_a}{R_2}+\frac{\mathbf{V}_o}{R_3}=0 \;\Rightarrow\; \mathbf{V}_o=-\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{-\dfrac{R_3}{R_2}R_2}{R_1+R_2+j\omega C R_1 R_2}=\frac{-\dfrac{R_3}{R_1+R_2}}{1+j\omega C\dfrac{R_1 R_2}{R_1+R_2}}$$

Comparing to the specified network function gives

$$\frac{-\dfrac{R_3}{R_1+R_2}}{1+j\omega C\dfrac{R_1 R_2}{R_1+R_2}}=\frac{\pm 8}{\left(1+j\dfrac{\omega}{500}\right)}$$

We require

$$\frac{R_3}{R_1+R_2}=8 \quad \text{and} \quad C\frac{R_1 R_2}{R_1+R_2}=\frac{1}{500}=0.002$$

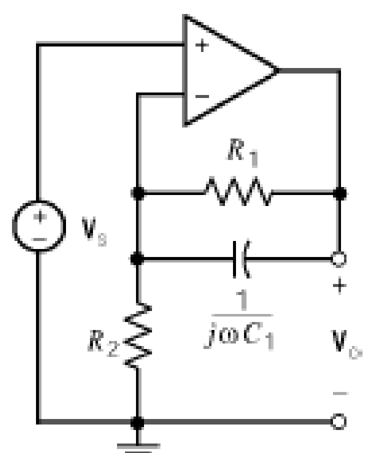
The solution is not unique. With the given values of capacitance, and choosing $R_1 = R_2$, we have

$$R_1 = R_2 = 13.3 \text{ k}\Omega \text{ and } R_3 = 212.8 \text{ k}\Omega$$

P 13.3-23
Solution:
From Figure P13.3-26*b*, $\mathbf{H}(\omega)$ has a pole at 20 rad/s and a zero at 500 rad/s. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega)=\pm K\frac{\left(1+j\dfrac{\omega}{500}\right)}{\left(1+j\dfrac{\omega}{20}\right)}.$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain. Apply KCL at the inverting input node of the op amp to get



$$\frac{\mathbf{V}_o-\mathbf{V}_s}{R_1}+j\omega C_1\left(\mathbf{V}_o-\mathbf{V}_s\right)+\frac{\mathbf{V}_s}{R_2}=0$$

or $\quad \left(R_1+R_2+j\omega C_1 R_1 R_2\right)\mathbf{V}_s=\left(R_2+j\omega C_1 R_1 R_2\right)\mathbf{V}_o$

so

$$\mathbf{H}=\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{R_1+R_2+j\omega C_1 R_1 R_2}{R_2+j\omega C_1 R_1 R_2}=\frac{R_1+R_2}{R_2}\times\frac{1+j\omega C_1\dfrac{R_1 R_2}{R_1+R_2}}{1+j\omega C_1 R_1}$$

a. Comparing to the specified network function gives

$$\frac{R_1+R_2}{R_2}\times\frac{1+j\,\omega\,C_1\dfrac{R_1\,R_2}{R_1+R_2}}{1+j\,\omega\,C_1\,R_1}=K\,\frac{1+j\dfrac{\omega}{500}}{1+j\dfrac{\omega}{20}}$$

We require $$C_1 R_1=\frac{1}{20}=.05 \quad\text{and}\quad C\frac{R_1 R_2}{R_1+R_2}=\frac{1}{500}=0.002$$

Notice that $$K=\frac{R_1+R_2}{R_2}\times\frac{C_1\,R_1}{C_1\dfrac{R_1\,R_2}{R_1+R_2}}=\frac{\dfrac{1}{20}}{\dfrac{1}{500}}=25$$

The solution is not unique. For example, choosing $C = 1\ \mu F$

$R_1 = 50\ k\Omega$ and $R_2 = 2.083\ k\Omega$

b. The network function is

$$\mathbf{H}(\omega)=25\frac{\left(1+j\dfrac{\omega}{500}\right)}{\left(1+j\dfrac{\omega}{20}\right)}$$

so

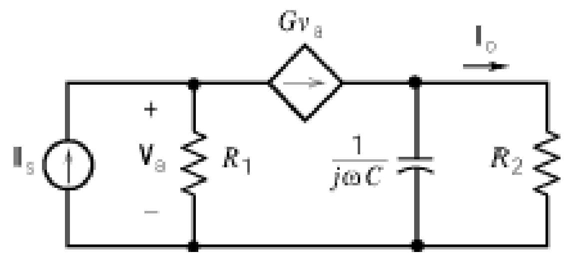
$$K_1=20\log_{10}(25)=28\text{ dB} \quad\text{and}\quad K_2=20\log_{10}\left(25\times\frac{20}{500}\right)=0\text{ dB}$$

(checked using LNAP 10/1/04)

P 13.3-24
Solution:
From Figure P13.3-28*b*, $\mathbf{H}(\omega)$ has a pole at 400 rad/s and a low frequency gain equal to $-8\text{ dB}\left(=20\log \mathbf{H}(\omega)\text{, which gives } \mathbf{H}(\omega)=0.4\right)$. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega)=\frac{\pm 0.4}{1+j\dfrac{\omega}{400}}.$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain. Apply KCL at the top node of R_1 to get



$$\mathbf{I}_s=\frac{\mathbf{V}_a}{R_1}+G\mathbf{V}_a \quad\Rightarrow\quad \mathbf{V}_a=\frac{R_1}{1+GR_1}\mathbf{I}_s$$

Current division gives

$$\mathbf{I}_o=\frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C}+R_2}G\mathbf{V}_a=\frac{G}{1+j\omega C R_2}\mathbf{V}_a=\frac{G}{1+j\omega C R_2}\left(\frac{R_1}{1+GR_1}\mathbf{I}_s\right)$$

The network function of the circuit is

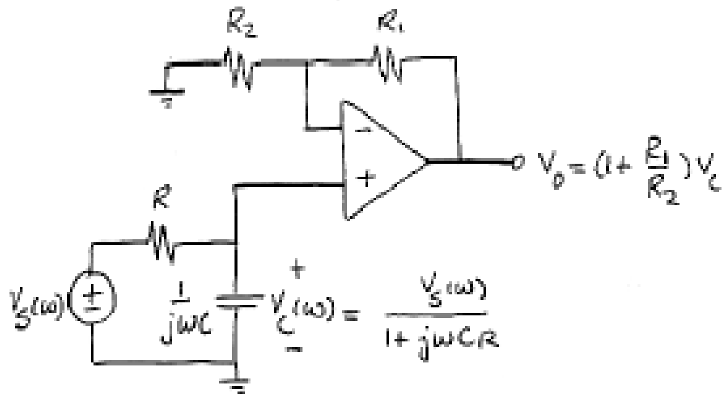
$$\mathbf{H}=\frac{\mathbf{I}_o}{\mathbf{I}_s}=\frac{\dfrac{GR_1}{1+GR_1}}{1+j\omega C R_2}$$

Comparing this network function to the specified network function gives

$$\frac{GR_1}{1+GR_1}=0.4 \text{ and } CR_2=\frac{1}{400}$$

The solution is not unique. Choosing G = 0.01 A/V and C = 10 μF gives R_1 = 66.7 Ω and R_2 = 250 Ω

P 13.3-25
Solution:



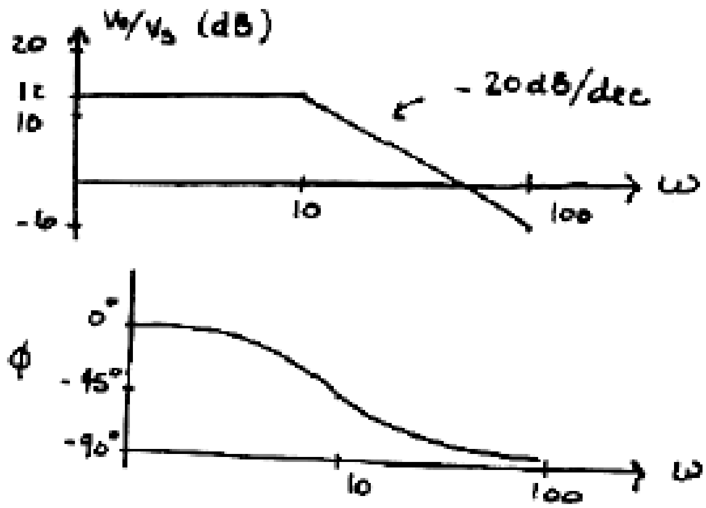
$$\mathbf{V}_o(\omega) = \left(1+\frac{R_1}{R_2}\right)\mathbf{V}_c(\omega)$$

$$= \left(1+\frac{R_1}{R_2}\right)\left(\frac{1}{1+j\omega C R}\right)\mathbf{V}_s(\omega)$$

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\left(1+\frac{R_1}{R_2}\right)\left(\frac{1}{1+j\omega C R}\right)$$

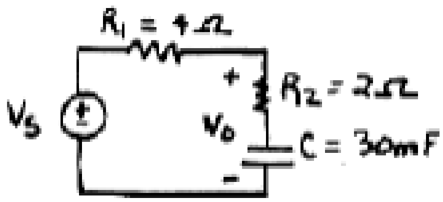
When $R\,C = 0.1$ and $\frac{R_1}{R_2} = 3$,

then $\mathbf{H}(\omega) = \dfrac{4}{1+j\dfrac{\omega}{10}}$



P 13.3-26
Solution:

a)



$$\mathbf{Z_o} = R_2 + \frac{1}{j\omega C}$$

$$\frac{\mathbf{V_o}}{\mathbf{V_s}} = \frac{\mathbf{Z_o}}{\mathrm{R_1} + \mathbf{Z_o}} = \frac{R_2 + \dfrac{1}{j\omega C}}{R_1 + R_2 + \dfrac{1}{j\omega C}} = \frac{1 + j\dfrac{\omega}{\omega_1}}{1 + j\dfrac{\omega}{\omega_2}}$$

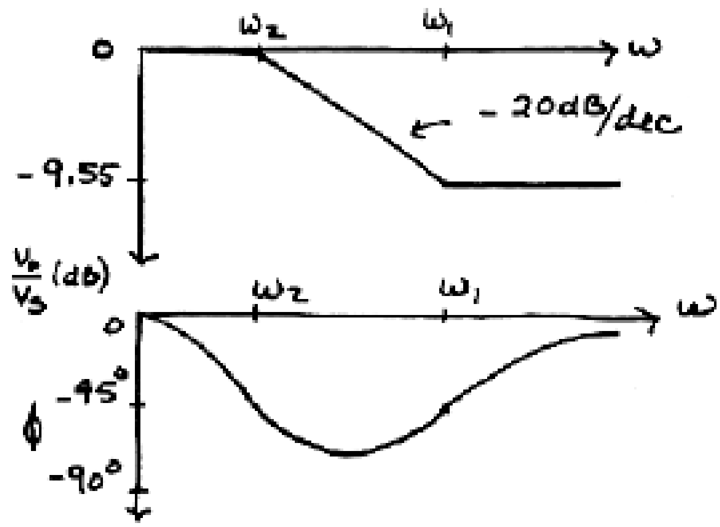
where $\omega_1 = \dfrac{1}{R_2 C} = 16.7$ rad/s

and $\omega_2 = \dfrac{1}{(R_1 + R_2)C} = 5.56$ rad/s

$v_s(t) = 10\cos 20t$ or $\mathbf{V_s} = 10\angle 0°$

$$\therefore \frac{\mathbf{V_o}}{\mathbf{V_s}} = \frac{1 + j\left(20/16.7\right)}{1 + j\left(20/5.56\right)}$$

$$= \frac{1 + j\ 1.20}{1 + j\ 3.60} = 0.418\ \angle{-24.3°}$$

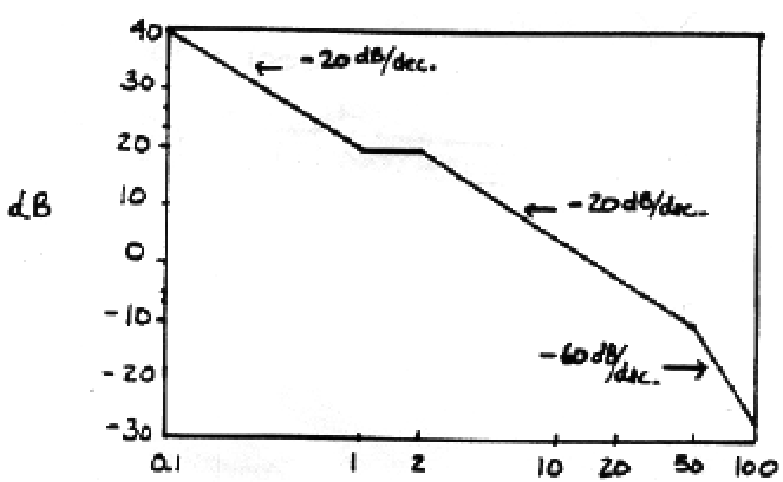


b) So

$$\mathbf{V_o} = 4.18\ \angle{-24.3°}$$

$$\underline{v_o(t) = 4.18\cos(20t - 24.3°)\ \mathrm{V}}$$

P 13.3-27
Solution:



Section 13-4: Resonant Circuits

P 13.4-1
Solution:
For the parallel resonant *RLC* circuit with $R = 20$ kΩ, $L = 1/120$ H, and $C = 1/40$ μF we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(\frac{1}{120}\right)\left(\frac{1}{40}\times 10^{-6}\right)}} = 69 \text{ k rad/sec}$$

$$Q = R\sqrt{\frac{C}{L}} = 20000\sqrt{\frac{\frac{1}{40}\times 10^{-6}}{\frac{1}{120}}} = 35$$

$$\omega_1 = -\frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^{\,2}} = 68.02 \text{ krad/s} \quad \text{and} \quad \omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^{\,2}} = 69.99 \text{ krad/s}$$

$$BW = \frac{1}{RC} = \frac{1}{(20000)\left(\frac{1}{40}\times 10^{-6}\right)} = 2 \text{ k rad/s}$$

Notice that $BW = \omega_2 - \omega_1 = \dfrac{\omega_0}{Q}$.

P 13.4-2

Solution: For the parallel resonant *RLC* circuit we have

$$|\mathbf{H}(\omega)| = \frac{k}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

so

$$R = k = |\mathbf{H}(\omega_0)| = \frac{8}{20\text{·}10^{-3}} = 400\ \Omega \ \text{ and } \ \omega_0 = 1000 \ \text{rad/s}$$

At $\omega = 897.6$ rad/s, $|\mathbf{H}(\omega)| = \frac{4}{20.10^{-3}} = 200$, so

$$200 = \frac{400}{\sqrt{1+Q^2\left(\frac{897.6}{1000}-\frac{1000}{897.6}\right)^2}} \quad \Rightarrow \quad Q = 8$$

Then

$$\left.\begin{aligned} \frac{1}{\sqrt{LC}} &= \omega_0 = 1000 \\ 400\sqrt{\frac{C}{L}} &= Q = 8 \end{aligned}\right\} \Rightarrow \begin{aligned} C &= 20\ \mu\text{F} \\ L &= 50\ \text{mH} \end{aligned}$$

P 13.4-3

Solution:

For the series resonant *RLC* circuit with $R = 100\ \Omega$, $L = 10$ mH, and $C = 0.02\ \mu$F we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 0.7\times10^5 \text{ rad/s}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 7, \quad BW = \frac{R}{L} = 10^4 \text{ rad/s}$$

P 13.4-4

Solution:

For the series resonant *RLC* circuit with $R = 1\ \Omega$, $L = 1$ mH, and $C = 10\ \mu$F we have

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^4 \text{ rad/s}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 10, \quad BW = \frac{R}{L} = 10^3 \text{ rad/s}$$

P 13.4-5

Solution:

For the parallel resonant *RLC* circuit we have

$$R = \mathbf{Z}(\omega_0) = 200\ \Omega$$

$$\frac{1}{200C} = BW = 500 \text{ rad/s} \quad \Rightarrow \quad C = 10\ \mu\text{F}$$

$$\frac{1}{\sqrt{(10\times10^{-6})L}} = \omega_0 = 2500 \text{ rad/s} \quad \Rightarrow \quad L = 16 \text{ mH}$$

P 13.4-6

Solution:

For the series resonant *RLC* circuit we have

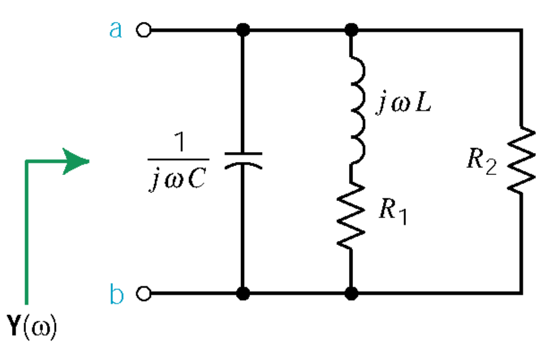
$$R = \frac{1}{\mathbf{Y}(\omega_0)} = 100\ \Omega$$

$$\frac{100}{L} = BW = 600 \text{ rad/s} \quad \Rightarrow \quad L = 0.17 \text{ H}$$

$$\frac{1}{\sqrt{(0.17)C}} = \omega_0 = 2500 \text{ rad/s} \Rightarrow C = 0.9\ \mu\text{F}$$

P 13.4-7

Solution:



C = 600 pF

L = 10 μH

$R_1 = 1.8\ \Omega$

$R_2 = 22\ \text{k}\Omega$

$$\mathbf{Y}(\omega) = j\omega C + \frac{1}{R_1 + j\omega L} + \frac{1}{R_2}$$

$$= \frac{\left(R_1 + R_2 - \omega^2 C L R_2\right) + j\omega\left(L + C R_1 R_2\right)}{R_2\left(R_1 + j\omega L\right)} \times \frac{R_1 - j\omega L}{R_1 - j\omega L}$$

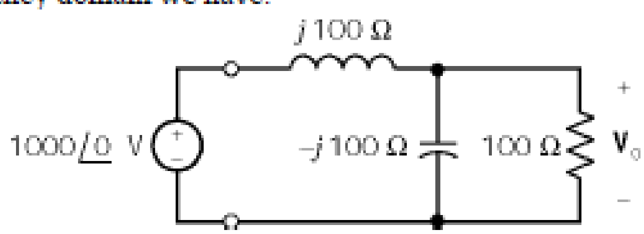
$$= \frac{R_1\left(R_1 + R_2 - \omega^2 C L R_2\right) + \omega^2 L\left(L + C R_1 R_2\right) + j\omega R_1 (L + C R_1 R_2) - j\omega L\left(R_1 + R_2 - \omega^2 C L R_2\right)}{R_2 (R_1 - \omega^2 L^2)}$$

$\omega = \omega_0$ is the frequency at which the imaginary part of $\mathbf{Y}(\omega)$ is zero:

$$R_1\left(L + C R_1 R_2\right) - L\left(R_1 + R_2 - \omega_0^2 C L R_2\right) = 0 \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{L R_2 - C R_1^2 R_2}{C L^2 R_2}} = 12.9 \text{ Mrad/sec}$$

P 13.4-8

Solution: In the frequency domain we have:

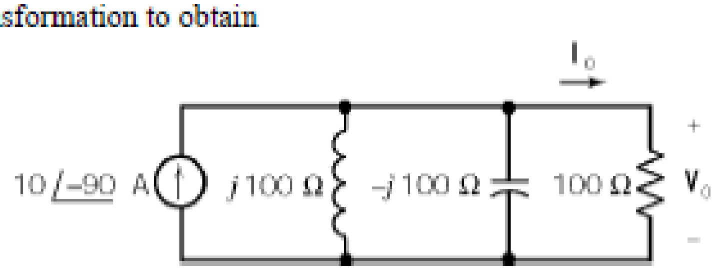


(a) Using voltage division yields

$$\mathbf{V}_o = \left(1000\angle 0^\circ\right) \frac{\dfrac{(100)(-j100)}{100-j100}}{\dfrac{(100)(-j100)}{100-j100} + j100} = \left(1000\angle 0^\circ\right) \frac{\dfrac{100}{\sqrt{2}}\angle -45^\circ}{\dfrac{100}{\sqrt{2}}\angle -45^\circ + j100} = \frac{\dfrac{10^5}{\sqrt{2}}\angle -45^\circ}{50\sqrt{2}\angle 45^\circ} = 1000\angle -90^\circ \text{ V}$$

$$\therefore |\mathbf{V_o}| = 1000 \text{ V}$$

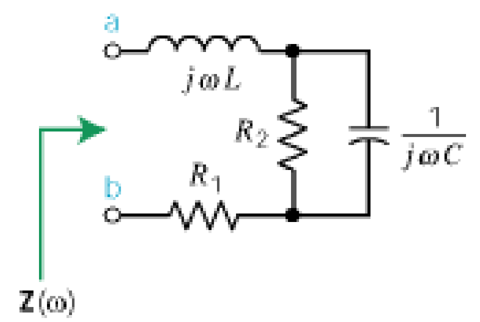
(b) Do a source transformation to obtain



This is a resonant circuit with $\omega_0 = 1/\sqrt{LC}$=400 rad/s. Since this also happens to be the frequency of the input, so this circuit is being operated at resonance. At resonance the admittances of the capacitor and inductor cancel each other, leaving the impedance of the resistor. Increasing the resistance by a factor of 10 will increase the voltage $\mathbf{V}_o$ by a factor of 10. This increased voltage will cause increased currents in both the inductance and the capacitance, causing the sparks and smoke.

P 13.4-9
Solution:

Let $G_2 = \dfrac{1}{R_2}$. Then



$$\mathbf{Z} = R_1 + j\omega L + \frac{1}{G_2 + j\omega C}$$
$$= \frac{\left(R_1 G_2 + 1 - \omega^2 L C\right) + j\left(\omega L G_2 + \omega C R_1\right)}{G_2 + j\omega C}$$

At resonance, $\angle \mathbf{Z} = 0°$ so

$$\tan^{-1}\frac{\omega L G_2 + \omega C R_1}{\left(R_1 G_2 + 1 - \omega^2 L C\right)} = \tan^{-1}\frac{\omega C}{G_2}$$

so

$$\frac{\omega L G_2 + \omega C R_1}{\left(R_1 G_2 + 1 - \omega^2 L C\right)} = \frac{\omega C}{G_2} \quad \Rightarrow \quad \omega^2 = \frac{C - L G_2^2}{L C^2} \quad \text{and} \quad C > G_2^2 L$$

With $R_1 = R_2 = 2\ \Omega$ and $\omega_0 = 150$ rad/s, $\omega_0^{\ 2} = 22500 = \dfrac{C - L}{L C^2}$. Then choose C and calculate L:

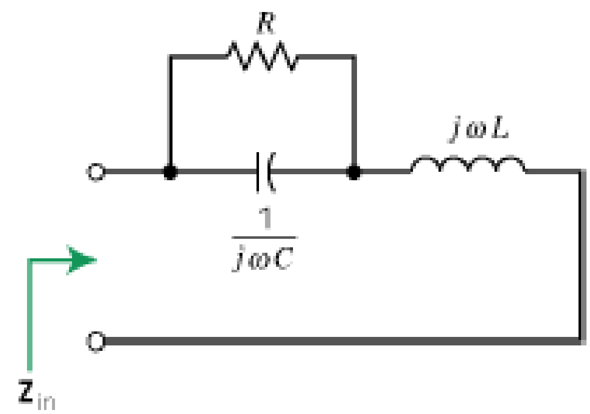
$$C = 10\text{ mF} \Rightarrow L = 3\text{ mH}$$

Since $C > G_2^2 L$, we are done.

P 13.4-10

Solution:

(a)

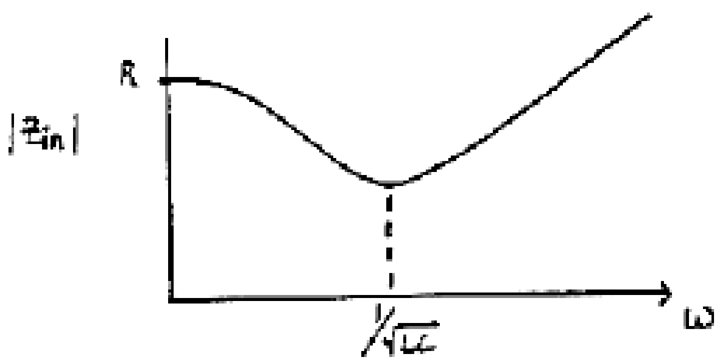


$$\mathbf{Z}_{in} = j\omega L + \frac{\dfrac{R}{j\omega C}}{R+\dfrac{1}{j\omega C}} = \frac{\left(R-\omega^2 RLC\right)+j\omega L}{1+j\omega RC}$$

Consequently,

$$|\mathbf{Z}_{in}| = \sqrt{\frac{\left(R-\omega^2 RLC\right)^2+\left(\omega L\right)^2}{1+\left(\omega RC\right)^2}}$$

(b)



(c)

$$\omega = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad |\mathbf{Z}_{in}| = \frac{1}{\sqrt{\dfrac{C}{L}\left(1+\dfrac{R^2 C}{L}\right)}}$$

P 13.4-11

Solution:

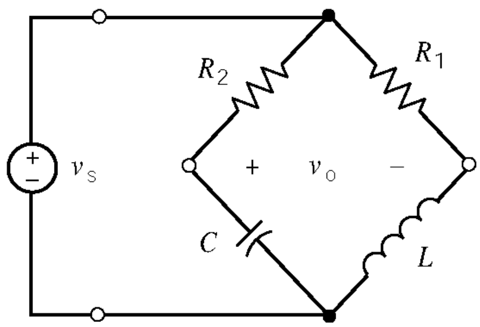
Let $\mathbf{V}(\omega) = A\angle 0$ and $\mathbf{V}_2(\omega) = B\angle\theta$. Then

$$\mathbf{Y}(\omega) = \frac{\mathbf{I}(\omega)}{\mathbf{V}(\omega)} = \frac{\dfrac{\mathbf{V}(\omega)-\mathbf{V}_2(\omega)}{R}}{\mathbf{V}(\omega)} = \frac{A-B\angle\theta}{AR} = \frac{A-B\cos\theta-j\,B\sin\theta}{AR}$$

$$|\mathbf{Y}(\omega)| = \frac{\sqrt{\left(A-B\cos\theta\right)^2+\left(B\sin\theta\right)^2}}{AR}$$

P 13.6-1

Solution:



Using voltage division twice gives

$$\mathbf{V}_o(\omega) = \frac{\frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \mathbf{V}_s(\omega) - \frac{j\omega L}{R_1 + j\omega L} \mathbf{V}_s(\omega)$$

so

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1}{1 + j\omega C R_2} - \frac{j\omega L}{R_1 + j\omega L}$$

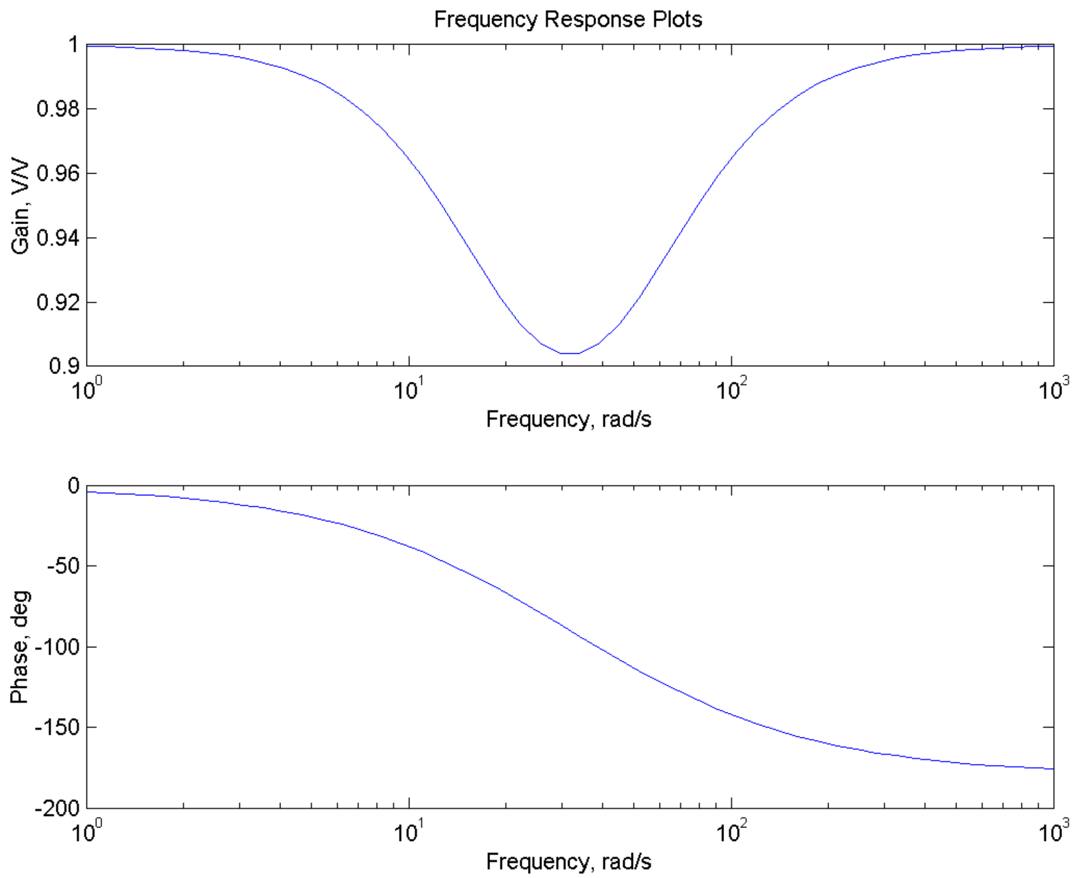
Modify the MATLAB script given in Section 13.7 of the text:

```
% P13_7_1.m - plot the gain and phase shift of a circuit

%-------------------------------------------------------------
%     Create a list of logarithmically spaced frequencies.
%-------------------------------------------------------------

wmin=1;       % starting frequency, rad/s
wmax=1000;    % ending frequency, rad/s

w = logspace(log10(wmin),log10(wmax));

%-------------------------------------------------------------
%   Enter values of the parameters that describe the circuit.
```

```
%-------------------------------------------------------------

R1 = 10;   % Ohms
R2 = 20;   % Ohms
C = 0.001; % Farads
L = 0.5;   % Henries

%-------------------------------------------------------------
% Calculate the value of the network function at each frequency.
%   Calculate the magnitude and angle of the network function.
%-------------------------------------------------------------

for k=1:length(w)
    H(k) = 1/(1+j*R2*C*w(k)) - j*L*w(k)/(R1+j*L*w(k));
    gain(k) = abs(H(k));
    phase(k) = angle(H(k))*180/pi;
end

%-------------------------------------------------------------
%                   Plot the frequency response.
%-------------------------------------------------------------

subplot(2,1,1), semilogx(w, gain)
xlabel('Frequency, rad/s'), ylabel('Gain, V/V')
title('Frequency Response Plots')
```

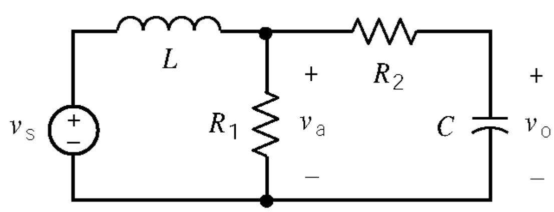
```
subplot(2,1,2), semilogx(w, phase)

xlabel('Frequency, rad/s'), ylabel('Phase, deg')
```

Here are the plots produced by MATLAB:



P 13.6-2

Solution:



Let

$$\mathbf{Z}_s = R_2 + \frac{1}{j\omega C} \text{ and } \mathbf{Z}_p = \frac{R_1 \mathbf{Z}_s}{R_1 + \mathbf{Z}_s}$$

Using voltage division twice gives

$$\mathbf{V}_{\mathrm{a}}(\omega)=\frac{\mathbf{Z}_{\mathrm{p}}}{j\omega L+\mathbf{Z}_{\mathrm{p}}}\mathbf{V}_{\mathrm{s}}(\omega) \text{ and } \mathbf{V}_{\mathrm{o}}(\omega)=\frac{\frac{1}{j\omega C}}{R_2+\frac{1}{j\omega C}}\mathbf{V}_{\mathrm{a}}(\omega)$$

so

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_{\mathrm{o}}(\omega)}{\mathbf{V}_{\mathrm{s}}(\omega)}=\frac{\mathbf{Z}_{\mathrm{p}}}{\left(j\omega L+\mathbf{Z}_{\mathrm{p}}\right)\left(1+j\omega C R_2\right)}$$

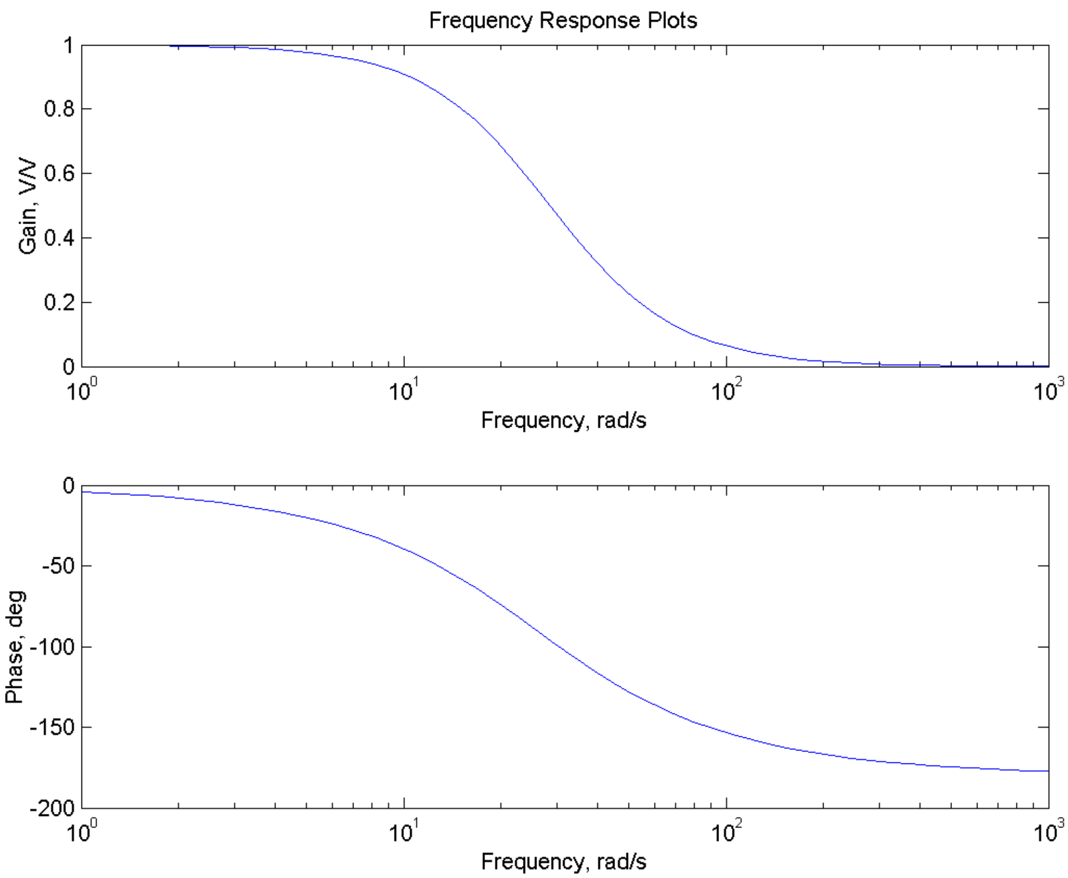
Modify the MATLAB script given in Section 13.7 of the text:

```
% P13_7_2.m - plot the gain and phase shift of a circuit
pi = 3.14159;
%--------------------------------------------------------------
%     Create a list of logarithmically spaced frequencies.
%--------------------------------------------------------------

wmin=1;       % starting frequency, rad/s
wmax=1000;    % ending frequency, rad/s

w = logspace(log10(wmin),log10(wmax));

%--------------------------------------------------------------
%   Enter values of the parameters that describe the circuit.
%--------------------------------------------------------------
```

```
R1 = 10;   % Ohms
R2 = 20;   % Ohms
C = 0.001; % Farads
L = 0.5;   % Henries

%-------------------------------------------------------------
% Calculate the value of the network function at each frequency.
%   Calculate the magnitude and angle of the network function.
%-------------------------------------------------------------

for k=1:length(w)
   Zs(k) = R2+1/(j*w(k)*C);
   Zp(k) = R1*Zs(k)/(R1+Zs(k));
   H(k) = Zp(k)/((j*w(k)*L+Zp(k))*(1+j*w(k)*C*R2));
   gain(k) = abs(H(k));
   phase(k) = angle(H(k))*180/pi;
end

%-------------------------------------------------------------
%                 Plot the frequency response.
%-------------------------------------------------------------

subplot(2,1,1), semilogx(w, gain)
xlabel('Frequency, rad/s'), ylabel('Gain, V/V')
title('Frequency Response Plots')
```

```
subplot(2,1,2), semilogx(w, phase)

xlabel('Frequency, rad/s'), ylabel('Phase, deg')
```

Here are the plots produced by MATLAB:



Section 13.8 How Can We Check…?

P 13.8-1
Solution:
When $\omega < 630$ rad/s, $\mathbf{H}(\omega) \cong 0.1$, which agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 200$ and 400 rad/s.

When $\omega > 6300$ rad/s, $\mathbf{H}(\omega) \cong 1.0$, which agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 12600$, 25000, 50000 and 100000 rad/s.

At $\omega = 6300$ rad/s, we expect $|\mathbf{H}(\omega)| = -3$ dB $= 0.707$. This agrees with the tabulated value of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 6310$ rad/s.

At $\omega = 630$ rad/s, we expect $|\mathbf{H}(\omega)| = -20$ dB $= 0.14$. This agrees with the tabulated values of $|\mathbf{H}(\omega)|$ corresponding to $\omega = 400$ and 795 rad/s.

This data does seem reasonable.

P 13.8-2
Solution:

$$BW = \frac{\omega_0}{Q} = \frac{10{,}000}{70} = 143 \neq 71.4 \text{ rad/s}$$. Consequently, this report is not correct.

P 13.8-3
Solution:

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10 \text{ krad/s} = 1.59 \text{ kHz}, \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 20 \text{ and } BW = \frac{R}{L} = 500 \text{ rad/s} = 79.6 \text{ Hz}$$

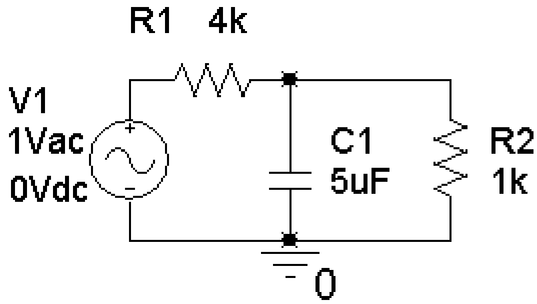
The reported results are correct.

P 13.8-4
Solution:
The network function indicates a zero at 200 rad/s and a pole at 800 rad/s. In contrast, the Bode plot indicates a pole at 200 rad/s and a zero at 800 rad/s. Consequently, the Bode plot and network function don't correspond to each other.

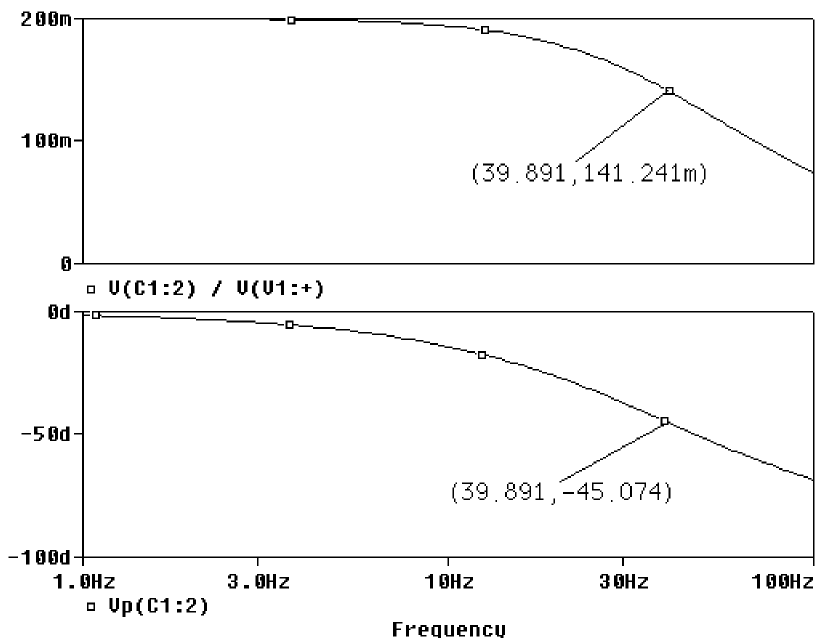
PSpice Problems

SP 13-1

Solution:



Here are the magnitude and phase frequency response plots:

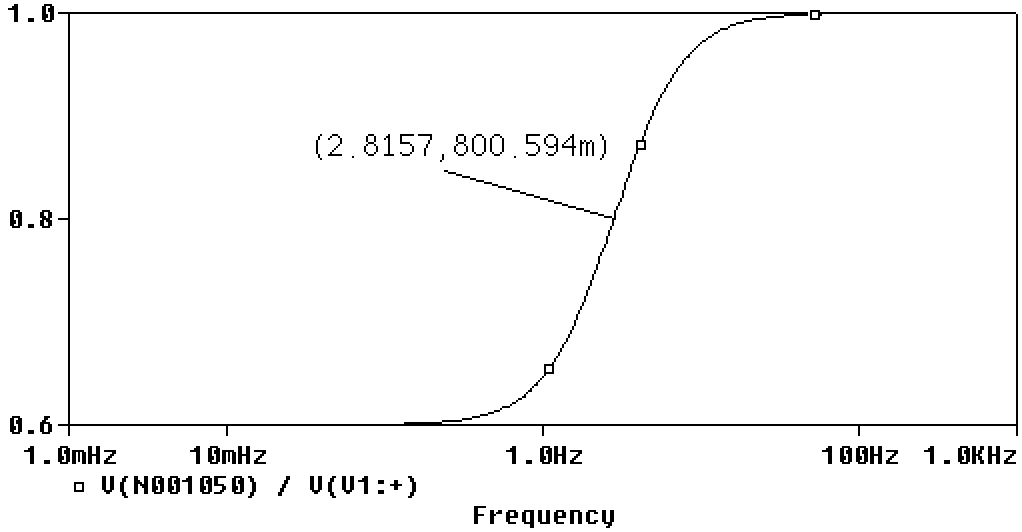


From the magnitude plot, the low frequency gain is k = 200m = 0.2.

From the phase plot, the angle is -45° at $p = 2\pi(39.891) = 251$ rad/s .

SP 13-2

Solution: Here is the magnitude frequency response plot:



The low frequency gain is $0.6 = \lim_{\omega \to 0} \mathbf{H}(\omega) = k \quad \Rightarrow \quad k = 0.6$.

The high frequency gain is $1 = \lim_{\omega \to \infty} \mathbf{H}(\omega) = k\,\dfrac{p}{z} \quad \Rightarrow \quad z = (0.6)\,p$

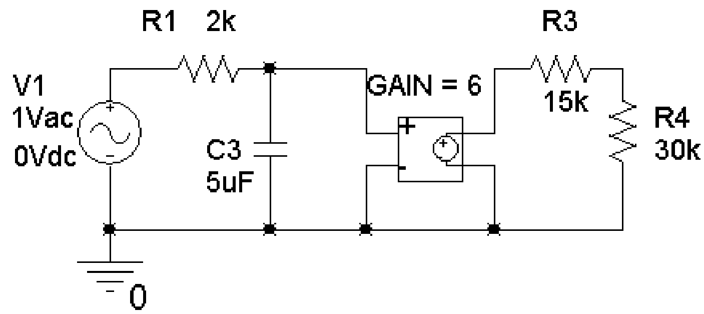
At $\omega = 2\pi(2.8157) = 17.69$ rad/s,

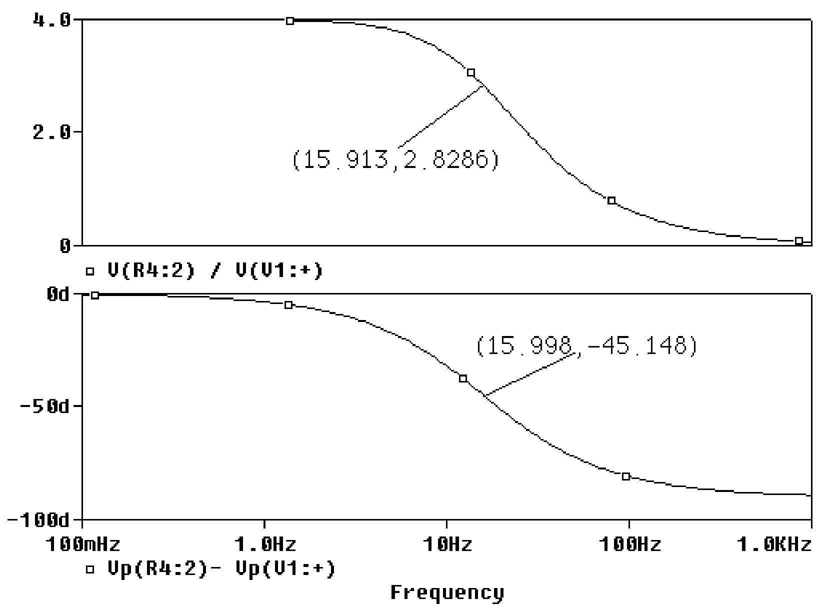
$$0.8 = 0.6\sqrt{\frac{1+\left(\dfrac{17.69}{0.6\,p}\right)^2}{1+\left(\dfrac{17.69}{p}\right)^2}} \quad \Rightarrow \quad \frac{16}{9} = \frac{p^2+869}{p^2+313} \Rightarrow \quad \frac{16}{9}\left(p^2+313\right) = p^2+869 \Rightarrow \quad (0.77778)\,p^2 = 312.56$$

$$\Rightarrow \quad p = 20 \text{ rad/s and } z = 12 \text{ rad/s}$$

SP 13-3

Solution:



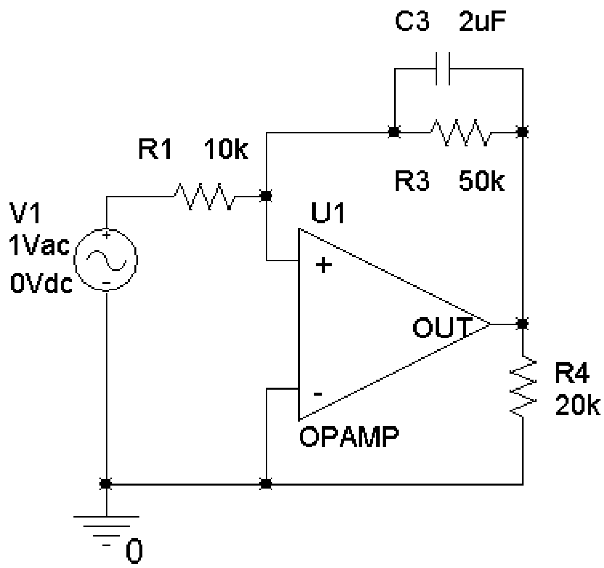


From the magnitude plot, the low frequency gain is k = 4.0. Also, the gain is 4/sqrt(2) = 2.828 at 15.914 hertz.

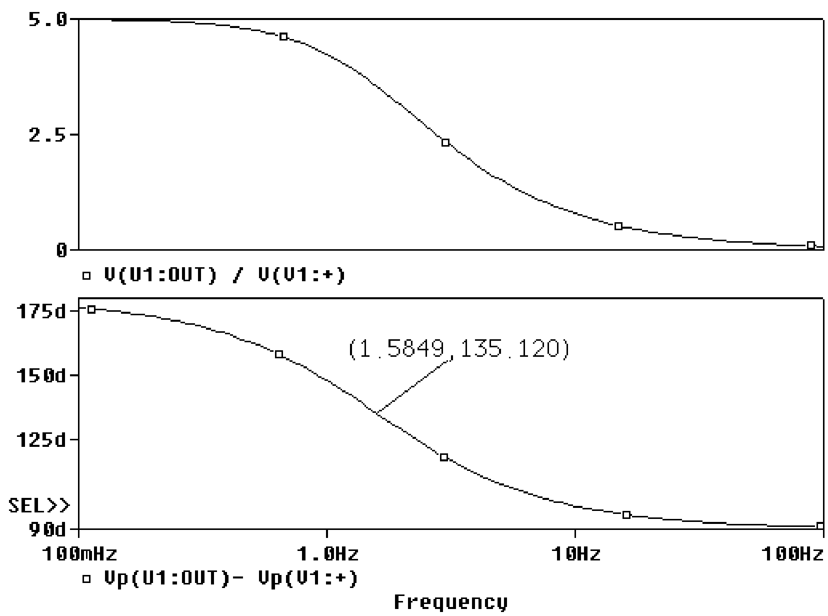
From the phase plot, the angle is -45° at $p = 2\pi\left(15.998\right) = 100.5\ \text{rad/s}$.

SP 13-4

Solution:
Here’s the circuit drawn in the PSpice workspace:



Here are the frequency response plots:



From the magnitude plot, the low frequency gain is k = 5.0.

From the phase plot, the angle is 180°-45°=135° at $p = 2\pi(1.5849) = 9.958$ rad/s .

SP 13-5

Solution: From the circuit

$$\mathbf{H}(\omega) = -\frac{\frac{10^4}{R}}{1+j\omega C\,10^4} = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

From the plot, at ω = 200 rad/sec = 31.83 Hertz **H**(ω) is

$$1.8565\angle 158° = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} \angle -\tan^{-1}\left(\omega C\,10^4\right)$$

Equating phase shifts gives

$$\omega C\,10^4 = 10^3 \frac{C\,R\,10^4}{R+10^4} = \tan(22°) = 0.404 \quad \Rightarrow \quad C = 0.2\ \mu\text{F}$$

Equating gains gives

$$1.8565 = \frac{\frac{10^4}{R}}{\sqrt{1+\left(\omega C\,10^4\right)^2}} = \frac{\frac{10^4}{R}}{\sqrt{1+\left(0.404\right)^2}} \quad \Rightarrow \quad R = 5\ \text{k}\Omega$$

SP 13-6

Solution: From the circuit

$$\mathbf{H}(\omega) = \frac{\frac{10^4}{1+j\omega C\,R_2}}{\frac{10^4}{1+j\omega C\,10^4}+R} = \frac{\frac{10^4}{R+10^4}}{1+j\omega\frac{C\,R\,10^4}{R+10^4}} = \frac{\frac{10^4}{R+10^4}}{\sqrt{1+\left(\omega\frac{C\,R\,10^4}{R+10^4}\right)^2}} \angle -\tan^{-1}\left(\omega\frac{C\,R\,10^4}{R+10^4}\right)$$

From the plot, at ω = 1000 rad/sec = 159.1 Hertz **H**(ω) is

$$0.171408\angle -59° = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+\left(\omega \dfrac{C\,R\,10^4}{R+10^4}\right)^2}} \angle -\tan^{-1}\left(\omega \frac{C\,R\,10^4}{R+10^4}\right)$$

Equating phase shifts gives

$$\omega \frac{C\,R\,10^4}{R+10^4} = 10^3 \frac{C\,R\,10^4}{R+10^4} = \tan(59°) = 1.665$$

Equating gains gives

$$0.171408 = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+\left(\omega \dfrac{C\,R\,10^4}{R+10^4}\right)^2}} = \frac{\dfrac{10^4}{R+10^4}}{\sqrt{1+(1.665)^2}} \quad \Rightarrow \quad R = 20\text{ k}\Omega$$

Substitute this value of R into the equation for phase shift to get:

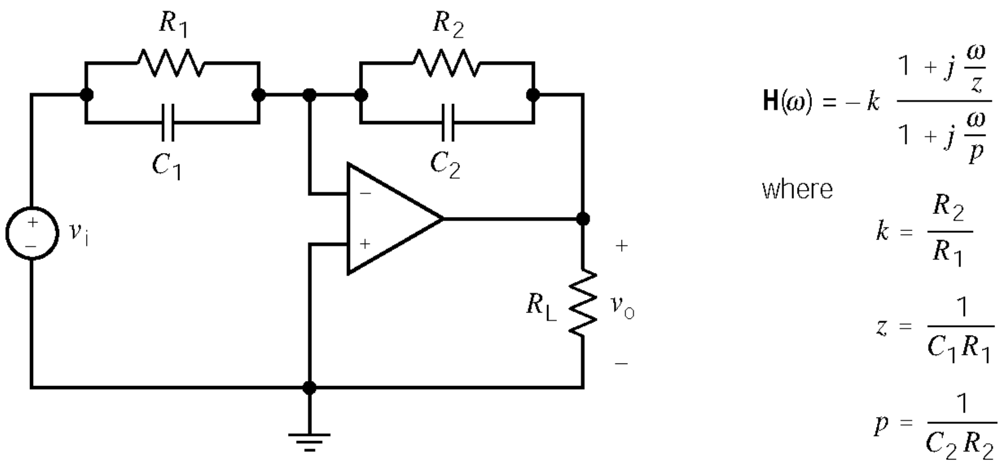
$$1.665 = 10^3 \frac{C\,R\,10^4}{R+10^4} = 10^3 \frac{C\left(20\times10^3\right)10^4}{\left(20\times10^3\right)+10^4} \quad \Rightarrow \quad C = 0.25\ \mu\text{F}$$

Design Problems

DP 13-1

Solution:

Pick the appropriate circuit from Table 13.4-2.



We require

$$2\pi\times1000 < z = \frac{1}{C_1 R_1},\ \ 2\pi\times10000 > p = \frac{1}{C_2 R_2},\ \ 2 = k = \frac{R_2}{R_1}\ \text{ and }\ 5 = k\frac{p}{z} = \frac{C_1}{C_2}$$

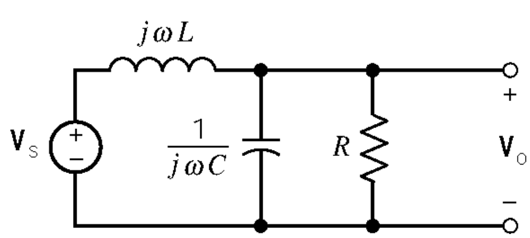
Try $z = 2\pi\times2000$. Pick $C_1 = 0.05\ \mu\text{F}$. Then

$$R_1 = \frac{1}{C_1 z} = 1.592\ \text{k}\Omega,\ R_2 = 2R_1 = 3.183\ \text{k}\Omega\ \text{ and }\ C_2 = \frac{C_1}{k\dfrac{p}{z}} = \frac{C_1}{2} = 0.01\ \mu\text{F}$$

Check: $p = \dfrac{1}{C_2 R_2} = 31.42\ \text{k rad/s} < 2\pi\cdot10{,}000\ \text{rad/s}$.

DP 13-2

Solution:



$$\mathbf{H}(\omega)=\frac{\mathrm{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{\frac{1}{j\omega C}\|R}{j\omega L+\left(\frac{1}{j\omega C}\|R\right)}=\frac{\frac{R}{1+j\omega C R}}{j\omega L+\frac{R}{1+j\omega C R}}=\frac{\frac{1}{LC}}{-\omega^2+j\omega\frac{1}{RC}+\frac{1}{LC}}$$

Pick $\frac{1}{\sqrt{LC}}=\omega_0=2\pi(100\cdot 10^3)$ rad/s . When $\omega=\omega_0$

$$\mathbf{H}_0(\omega)\ =\ \frac{\frac{1}{LC}}{-\frac{1}{LC}+j\frac{1}{\sqrt{LC}}\frac{1}{RC}+\frac{1}{LC}}$$

So $|\mathbf{H}(\omega_0)|\ =\ R\sqrt{\frac{C}{L}}$. We require

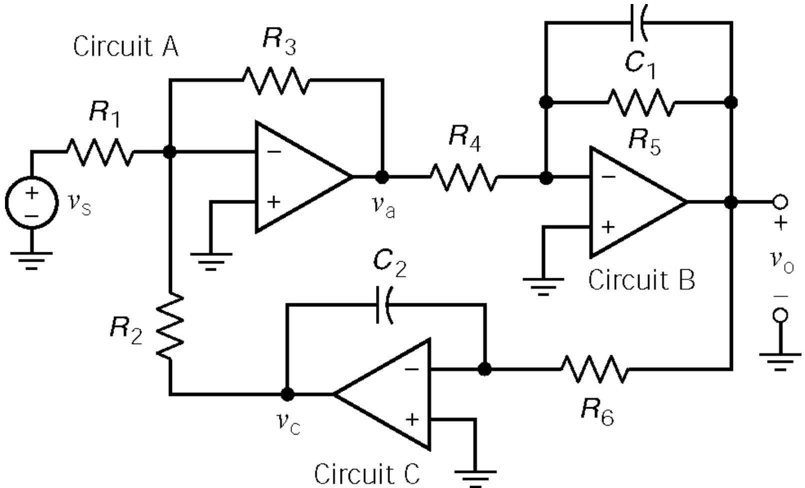
$$-3\text{ dB}\ =\ 0.707\ =\ |\mathbf{H}(\omega_0)|\ =\ R\sqrt{\frac{C}{L}}\ =\ 1000\sqrt{\frac{C}{L}}$$

Finally

$$\left.\begin{array}{l}\frac{1}{\sqrt{LC}}=2\pi(100\cdot 10^3)\\ 0.707=1000\sqrt{\frac{C}{L}}\end{array}\right\}\Rightarrow\begin{array}{l}C=1.13\text{ nF}\\ L=2.26\text{ mH}\end{array}$$

DP 13-3

Solution:



$R_1 = 10\ \text{k}\Omega$

$R_2 = 866\ \text{k}\Omega$

$R_3 = 8.06\ \text{k}\Omega$

$R_4 = 1\ \text{M}\Omega$

$R_5 = 2.37\ \text{M}\Omega$

$R_6 = 499\ \text{k}\Omega$

$C_1 = 0.47\ \mu F$

$C_2 = 0.1\ \mu F$

Circuit A $$\mathbf{V}_a = -\frac{R_3}{R_2}\mathbf{V}_c - \frac{R_3}{R_1}\mathbf{V}_s = -\mathbf{H}_1\,\mathbf{V}_c - \mathbf{H}_2\,\mathbf{V}_s$$

Circuit B $$\mathbf{V}_o = -\frac{\frac{R_5}{R_4}}{1 + j\omega C_1 R_5}\mathbf{V}_a = -\mathbf{H}_3\,\mathbf{V}_a$$

Circuit C $$\mathbf{V}_c = -\frac{1}{j\omega C_2\,R_6}\mathbf{V}_o = -\mathbf{H}_4\,\mathbf{V}_o$$

Then

$$\mathbf{V}_c = \mathbf{H}_3\,\mathbf{H}_4\,\mathbf{V}_a$$

$$\mathbf{V}_a = -\mathbf{H}_2\,\mathbf{V}_s - \mathbf{H}_1\,\mathbf{H}_3\,\mathbf{H}_4\,\mathbf{V}_a \quad \Rightarrow \quad \mathbf{V}_a = \frac{-\mathbf{H}_2}{1 + \mathbf{H}_1\,\mathbf{H}_3\,\mathbf{H}_4}\mathbf{V}_s$$

$$\mathbf{V}_o = -\mathbf{H}_3\,\mathbf{V}_a = \frac{\mathbf{H}_2\,\mathbf{H}_3}{1 + \mathbf{H}_1\,\mathbf{H}_3\,\mathbf{H}_4}\mathbf{V}_s$$

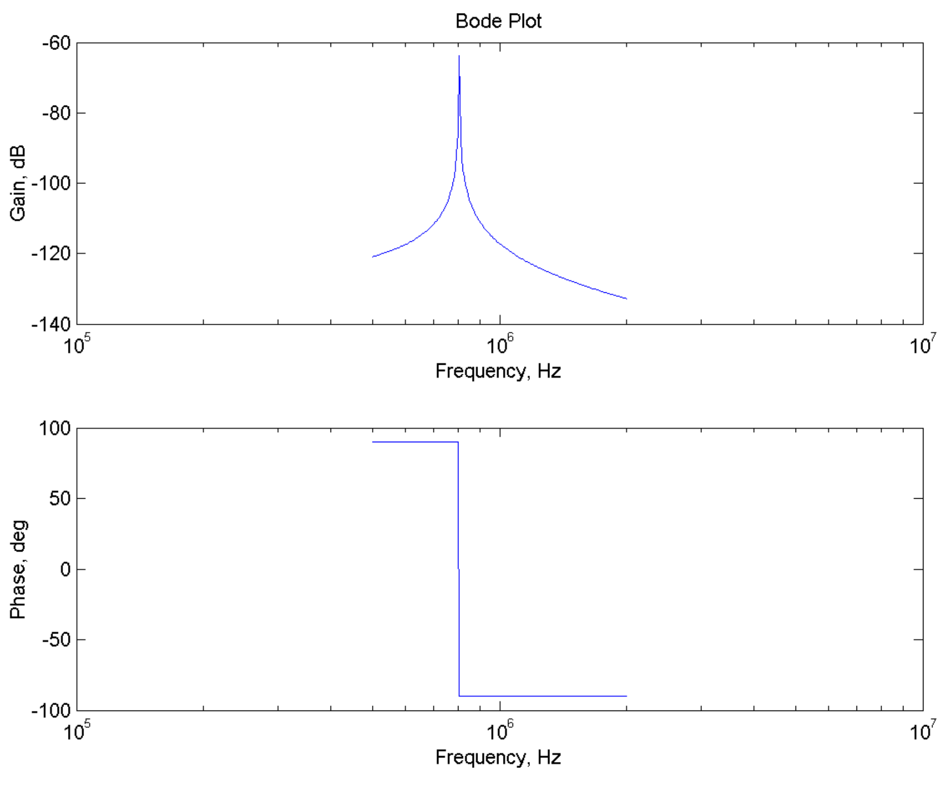
After some algebra

$$\mathbf{V}_o = \frac{j\omega \frac{R_3}{R_1\,R_4\,C_1}}{\frac{R_3}{R_2\,R_4\,R_6\,C_1\,C_2} - \omega^2 + j\frac{\omega}{R_5\,C_1}}\mathbf{V}_s$$

This MATLAB program plots the Bode plot:

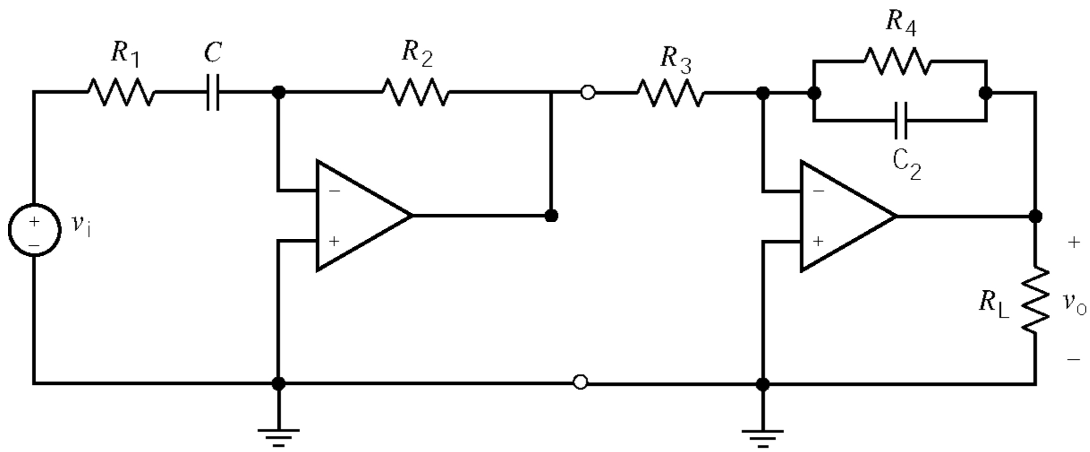
```
R1=10;      % units: kOhms and mF so RC has units of sec
R2=866;
R3=8.060;
R4=1000;
R5=2370;
R6=449;
C1=0.00047;
C2=0.0001;

pi=3.14159;
```

```
fmin=5*10^5;
fmax=2*10^6;
f=logspace(log10(fmin),log10(fmax),200);
w=2*pi*f;

b1=R3/R1/R4/C1;
a0=R3/R2/R4/R6/C1/C2;
a1=R5/C1;

for k=1:length(w)
   H(k)=(j*w(k)*b1)/(a0-w(k)*w(k)+j+w(k)*a1);
   gain(k)=abs(H(k));
   phase(k)=angle(H(k));
end

subplot(2,1,1), semilogx(f, 20*log10(gain))
xlabel('Frequency, Hz'), ylabel('Gain, dB')
title('Bode Plot')
subplot(2,1,2), semilogx(f, phase*180/pi)
xlabel('Frequency, Hz'), ylabel('Phase, deg')
```



DP 13-4

Solution:
Pick the appropriate circuits from Table 13.4-2.



$$\mathbf{H}_1(\omega) = -k_1 \frac{j\omega}{1 + j\frac{\omega}{p_1}}$$

where

$$k_1 = R_2 C$$

$$p_1 = \frac{1}{CR_1}$$

$$\mathbf{H}_2(\omega) = -\frac{k_2}{1 + j\frac{\omega}{p_2}}$$

where

$$k_2 = \frac{R_2}{R_1}$$

$$p_2 = \frac{1}{CR_2}$$

We require

$$10 = -k_1 k_2 = R_2 C_1 \frac{R_4}{R_3},\ \ 200 \ = \ p_1 = \frac{1}{R_1 C_1} \ \text{ and } \ 500 \ = \ p_2 = \frac{1}{C_2 R_4}$$

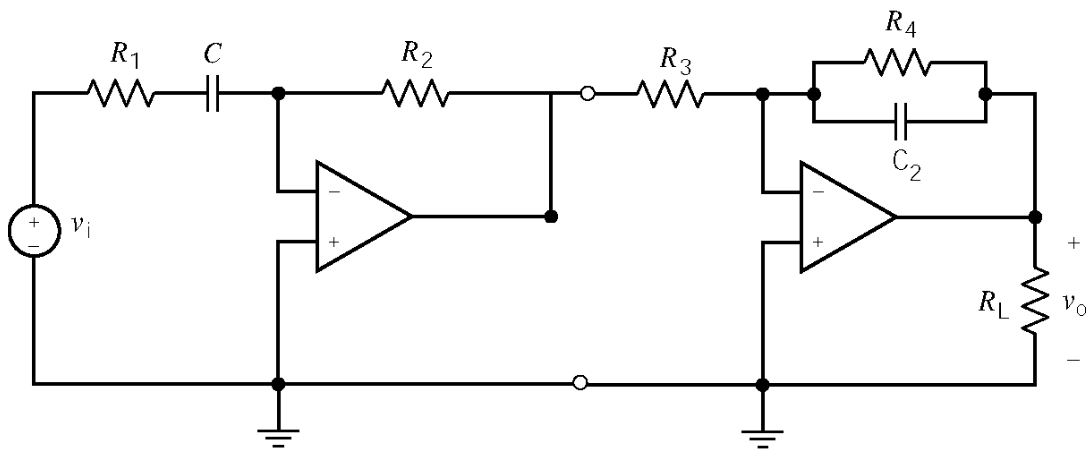
Pick $C_1 = 1\ \mu$F. Then $R_1 = \dfrac{1}{p_1 C_1} =$ 5 kΩ. Pick $C_2 = 0.1\ \mu$F. Then $R_4 = \dfrac{1}{p_2 C_2} = 20$ kΩ.

Next $$10 = \frac{R_2}{R_3}(10^{-6})(20\cdot 10^3) \ \Rightarrow \ \frac{R_2}{R_3} = 500$$

Let $R_2 = 500$ kΩ and $R_3 = 1$ kΩ.

DP 13-5

Solution:
Pick the appropriate circuits from Table 13.4-2.



$$\mathbf{H}_1(\omega) = -k_1 \frac{j\omega}{1 + j\frac{\omega}{p_1}}$$

where

$$k_1 = R_2 C$$

$$p_1 = \frac{1}{CR_1}$$

$$\mathbf{H}_2(\omega) = -\frac{k_2}{1 + j\frac{\omega}{p_2}}$$

where

$$k_2 = \frac{R_2}{R_1}$$

$$p_2 = \frac{1}{CR_2}$$

We require

$$20\text{ dB} = 10 = -k_1 k_2 = R_2 C_1 \frac{R_4}{R_3},\ \ 0.1 \ = \ p_1 = \frac{1}{R_1 C_1} \ \text{ and } \ 100 \ = \ p_2 = \frac{1}{C_2 R_4}$$

Pick $C_1 = 20\ \mu\text{F}$. Then $R_1 = \dfrac{1}{p_1 C_1} =$ 500 kΩ. Pick $C_2 = 1\ \mu\text{F}$. Then $R_4 = \dfrac{1}{p_2 C_2} = 10\text{ k}\Omega$.

Next

$$10 = \frac{R_2}{R_3}(20\cdot 10^{-6})(10\cdot 10^{3}) \ \Rightarrow \ \frac{R_2}{R_3} = 50$$

Let $R_2 = 200\text{ k}\Omega$ and $R_3 = 4\text{ k}\Omega$.

DP 13-6

Solution:

The network function of this circuit is $\mathbf{H}(\omega) = \dfrac{1+\dfrac{R_2}{R_3}}{1+j\omega R_1 C}$

The phase shift of this network function is $\theta = -\tan^{-1}\omega R_1 C$

The gain of this network function is $G = \left|\mathbf{H}(\omega)\right| = \dfrac{1+\dfrac{R_3}{R_2}}{\sqrt{1+(\omega R_1 C)^2}} = \dfrac{1+\dfrac{R_3}{R_2}}{\sqrt{1+\left(\tan\theta\right)^2}}$

Design of this circuit proceeds as follows. Since the frequency and capacitance are known, R_1 is calculated from $R_1 = \dfrac{\tan(-\theta)}{\omega C}$. Next pick R_2 = 10kΩ (a convenient value) and calculated R_3 using $R_3 = (G\cdot\sqrt{1+(\tan\theta)^2} - 1)\cdot R_2$. Finally

$$\theta = -45\ \text{deg},\ G = 2,\ \omega = 1000\ \text{rad/s} \Rightarrow R_1 = 10\text{ k}\Omega,\ R_2 = 10\text{ k}\Omega,\ R_3 = 18.284\text{ k}\Omega,\ C = 0.1\ \mu\text{F}$$

DP 13-7

Solution: From Table 13.4-2 and the Bode plot:

$$800 = z = \frac{1}{R_1(0.5\times10^{-6})} \Rightarrow R_1 = 2.5 \text{ k}\Omega$$

$$32 \text{ dB } = 40 = \frac{R_2}{R_1} \Rightarrow R_2 = 100 \text{ k}\Omega$$

$$200 = p = \frac{1}{R_2 C} \Rightarrow C = \frac{1}{(200)(100\times10^3)} = 0.05\mu\text{F}$$

$$(\text{Check: } 20 \text{ dB } = 10 = k\frac{p}{z} = \frac{0.5\times10^{-6}}{C} = \frac{0.5\times10^{-6}}{0.05\times10^{-6}})$$

DP 13-8

Solution:

$$\mathbf{H}(\omega) = \frac{-R_2}{1+\frac{1}{j\omega C}} = -\frac{j\omega C R_2}{1+j\omega C R_1}$$

$$195° = 180 + 90 - \tan^{-1}\omega C R_1 \Rightarrow R_1 = \frac{\tan(270°–195°)}{(1000)(0.1\times10^{-6})} = 37.3 \text{ k}\Omega$$

$$10 = \lim_{\omega\to\infty} |\mathbf{H}(\omega)| = \frac{R_2}{R_1} \Rightarrow R_2 = 10R_1 = 373 \text{ k}\Omega$$