Chapter 2 Circuit Elements

Exercises

Exercise 2.4-1 Find the power absorbed by a 100-ohm resistor when it is connected directly across a constant 10-V source.
Answer: 1-W

Solution:

$$P=\frac{v^2}{R}=\frac{(10)^2}{100}=\underline{1\ \text{W}}$$

Exercise 2.4-2 A voltage source v = 10 cos t V is connected across a resistor of 10 ohms. Find the power delivered to the resistor.
Answer: 10 $\cos^2 t$ W

Solution:

$$P=\frac{v^2}{R}=\frac{(10\ \cos\ t)^2}{10}=\underline{10\ \cos^2 t\ \text{W}}$$

Exercise 2.7-1 Find the power absorbed by the CCCS in Figure E 2.7-1.

Hint: The controlling element of this dependent source is a short circuit. The voltage across a short circuit is zero. Hence, the power absorbed by the controlling element is zero. How much power is absorbed by the controlled element?

Answer:–115.2 watts are absorbed by the CCCS. (The CCCS delivers +115.2 watts to the rest of the circuit.)

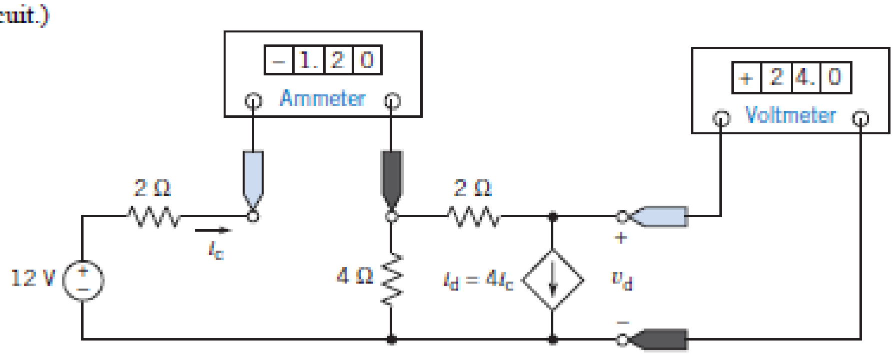


Figure E 2.7-1

Solution:

$$i_c = -1.2 \quad \text{A},\ v_d = 24 \quad \text{V}$$
$$i_d = 4\ (-1.2)\ =\ -4.8 \quad \text{A}$$

i_d and v_d adhere to the passive convention so

$$P = v_d\ i_d = (24)\ (-4.8) = \underline{-115.2 \quad \text{W}}$$

is the power received by the dependent source

Exercise 2.8-1 Consider the potentiometer circuit of Figure 2.8-1. Let R_p = 20 kΩ and I_s =2 mA. As in Example 2.8-1, assume that θ is an angle between 0^0 and 360^0 and that $\alpha = \theta / 360°$. Determine **(a)** the value of θ that causes the voltmeter voltage to be v_m = 5 V and **(b)** the value of v_m that indicates that $\theta = 54°$.

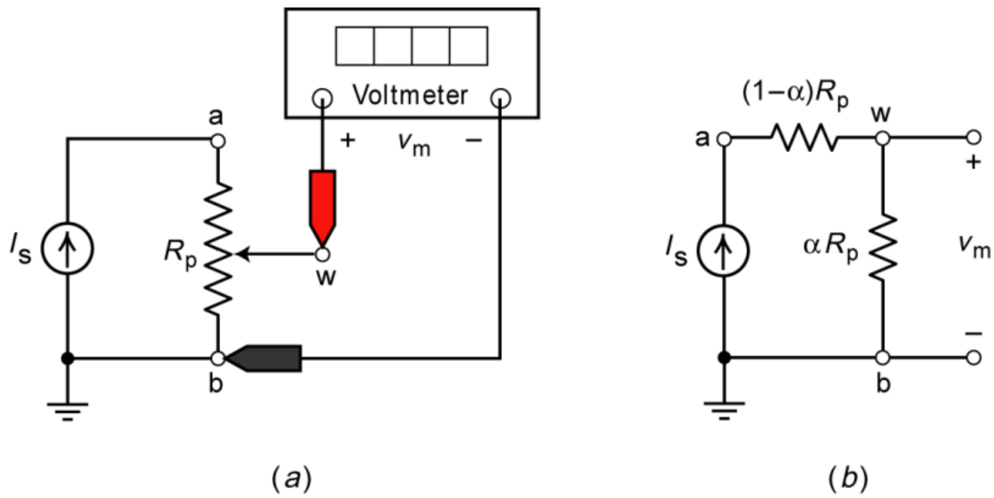


FIGURE E 2.8-1 (*a*) A circuit containing a potentiometer. (*b*) An equivalent circuit containing a model of the potentiometer.

Solution:

(a) $5\text{ V} = \left(20\times10^3\ \Omega\right)\left(2\times10^{-3}\text{ A}\right)\dfrac{\theta}{360°} \quad \Rightarrow \quad \theta = \dfrac{5\left(360°\right)}{40} = 45°$

(b) $54 = \dfrac{v_m\left(360°\right)}{40} = 9v_m \quad \Rightarrow \quad v_m = \dfrac{54}{9} = 6\text{ V}$

Exercise 2.8-2

The resistor in Figure E2.8-2 is a thermistor that has $\beta = 3500°$K. The thermistor resistance is 400 Ω when the thermistor temperature is 25°C.

(a) Determine the value of thermistor temperature when value of the thermistor voltage is v_T = 8 V.
(b) Determine the value of thermistor voltage, v_T, when value of the thermistor temperature is 60°C.

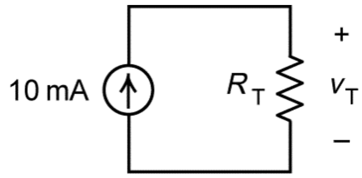


Figure E 2.8-2

Solution:

$$R_0 = 400\ \Omega \ \text{ at } \ T_0 = 25°\text{C}$$

(a)

$$R_T = \frac{v_T}{10\text{ mA}} = \frac{8\text{ V}}{0.01\text{ A}} = 800\ \Omega$$

$$T = \frac{\beta T_0}{\beta + T_0 \ln\left(\dfrac{R_T}{R_0}\right)} = \frac{3500 \times (25+273)}{3500 + (25+273)\ln\left(\dfrac{800}{400}\right)} = 281.4\,°\text{K} = 8.4°\text{C}$$

(b)

$$R_T = 400\, e^{3500\left(\frac{1}{60+273} - \frac{1}{25+273}\right)} = 116.4\ \Omega \quad \text{at} \quad T = 60°\text{C}$$

$$v_T = (116.4)(0.01) = 1.164\text{ V} \quad \text{at} \quad T = 60\,°\text{C}$$

Exercise 2.9-1 What is the value of the current i in Figure E 2.9-1 at time $t = 4$ s?

Answer: $i = 0$ amperes at $t = 4$ s (both switches are open).

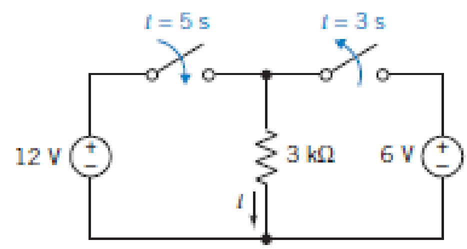


Figure E 2.9-1

Solution:

At $t = 4$ s both switches are open, so $i = 0$ A.

Exercise 2.9-2 What is the value of the voltage v in Figure E 2.9-2 at time $t = 4$ s? At $t = 6$ s?

Answer: $v = 6$ volts at $t = 4$ s, and $v = 0$ volts at $t = 6$ s.

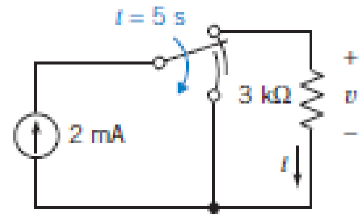


Figure E 2.9-2

Solution:

At $t = 4$ s the switch is in the up position, so $v = i\,R = (2\text{ mA})(3\text{ k}\Omega) = \underline{6\text{V}}$.

At $t = 6$ s the switch is in the down position, so $v = 0$ V.

Section 2-2 Engineering and Linear Models

P 2.2-1

Solution:
The element is not linear. For example, changing the current from 4 A to 6 A does not change the voltage from 10 V to 15 V. Instead the voltage changes from 10 V to 34 V. Hence, the property of homogeneity is not satisfied.

P 2.2-2

Solution:
(a) The data points do indeed lie on a straight line. The slope of the line is 0.12 V/A and the line passes through the origin so the equation of the line is $v = 0.12\,i$. The element is indeed linear.

(b) When $i = 50$ mA, $v = (0.12\text{ V/A})\times(50\text{ mA}) = (0.12\text{ V/A})\times(0.05\text{ A}) = 6\text{ mV}$

(c) When $v = 6$ V, $i = \dfrac{6}{0.12} = 50\quad\text{A}$

P 2.2-3

Solution:
(a) The data points do indeed lie on a straight line. The slope of the line is 256.5 V/A and the line passes through the origin so the equation of the line is $v = 256.5\,i$. The element is indeed linear.

(b) When $i = 5$ mA, $v = (256.5\text{ V/A})\times(5\text{ mA}) = (256.5\text{ V/A})\times(.005\text{ A}) = 1.282\text{ V}$

(c) When $v = 15$ V, $i = \dfrac{15}{256.5} = .0584\text{ A} = 58.47\text{ mA}$.

P 2.2-4

Solution:

Let $i = 1$ A , then $v = 6i + 10 = 16$ V. Next $2i = 2$A but $16 = 2v \neq 6(2i) + 10 = 22$. Hence, the property of homogeneity is not satisfied. The element is not linear.

P 2.2-5

Solution:

(a)

$$0.4 = \frac{v}{10} + \frac{v}{40} = \frac{v}{8} \quad \Rightarrow \quad v = 3.2 \text{ V}$$

$$i = \frac{v}{40} = 0.08 \text{ A}$$

(b)

$$0.4 = \frac{v}{10} + \frac{v^2}{2} \quad \Rightarrow \quad v^2 + \frac{v}{5} - 0.8 = 0$$

Using the quadratic formula $\quad v = \frac{-0.2 \pm 1.8}{2} = 0.8, \; -1.0 \text{ V}$

When v = 0.8 V then $i = \frac{0.8^2}{2} = 0.32$ A . When v = -1.0 V then $i = \frac{(-1)^2}{2} = 0.5$ A .

(c)

$$0.4 = \frac{v}{10} + 0.8 + \frac{v^2}{2} \quad \Rightarrow \quad v^2 + \frac{v}{5} + 0.8 = 0$$

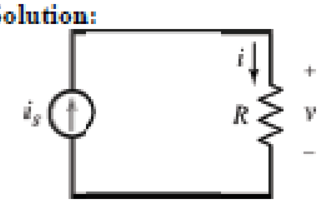
Using the quadratic formula $\quad v = \frac{-0.2 \pm \sqrt{0.04 - 3.2}}{2}$

So there is no real solution to the equation.

Section 2-4 Resistors

P 2.4-1

Solution:



$i = i_s = 5A$

$v = Ri = 10 \times 5 = \underline{50v}$

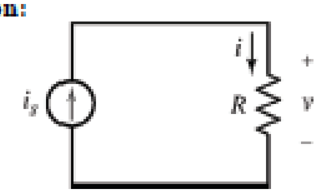
v and i adhere to the passive convention

$\therefore P = vi = 50 \times 5 = \underline{250\ W}$

is the power absorbed by the resistor.

P 2.4-2

Solution:



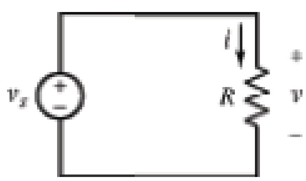
$i = i_s = 5mA$ and $v = 50$ V

$R = \frac{v}{i} = \frac{50}{.005} = 10000 = \underline{10\ K\Omega}$

$P = (5\times10^{-3})\times50 = 250\times10^{-3} = \underline{250\ mW}$

P 2.4-3

Solution:



$v = v_s = 15V$ and $R = 3\Omega$

$i = \frac{v}{R} = \frac{15V}{3\Omega} = \underline{5A}$

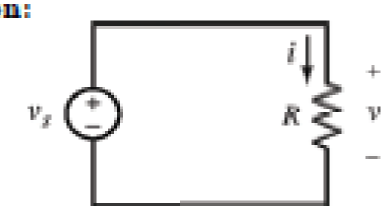
v and i adhere to the passive convention

$\therefore\ p = vi = 15\times5 = \underline{75\ W}$

is the power absorbed by the resistor

P 2.4-4

Solution:



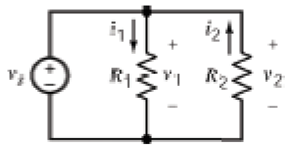
$v = v_s = 25V$ and $i = 5A$

$R = \frac{v}{i} = \frac{25V}{5A} = \underline{5\Omega}$

$p = vi = 25(5) = \underline{125\ W}$

P 2.4-5

Solution:



$v_1 = v_2 = v_s = 200V;\ R_1 = 50\Omega;\ R_2 = 25\Omega$

v_1 and i_1 adhere to the passive convention so

$$i_1 = \frac{v_1}{R_1} = \frac{200}{50} = \underline{4A}$$

v_2 and i_2 do not adhere to the passive convention so $i_2 = -\frac{v_2}{R_2} = -\frac{200}{25}$

The power absorbed by R_1 is $P_1 = v_1 i_1 = 200 \cdot 4$

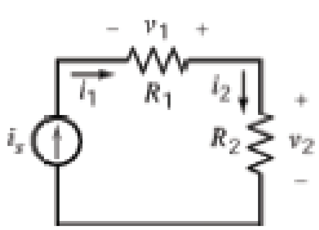
$= \underline{800W}$

The power absorbed by R_2 is $P_2 = -v_2 i_2 = -200(-8)$

$= \underline{1600W}$

P 2.4-6

Solution:



$i_1 - i_2 = i_s = 50$ mA; $R_1 = 8\Omega$ and $R_2 = 16\Omega$

v_1 and i_1 do <u>not</u> adhere to the passive convention so $v_1 = -R_1 i_1 = 8(.050) = -0.4V$.

The power absorbed by R_1 *is* $P_1 = -v_1 i_1 = -(-0.4)(0.050) = 20W$

v_2 and i_2 do adhere to the passive convention so $v_2 = R_2\, i_2 = 16(0.050) = \underline{0.8\ V}$.

The power absorbed by R_2 is $P_2 = v_2\, i_2 = (0.8)(0.050) = \underline{40\ mW}$.

P 2.4-7

Solution:

Model the heater as a resistor, then from $P = \frac{v^2}{R}$

$$\Rightarrow R = \frac{v^2}{P} = \frac{(200)^2}{1000} = \underline{40\Omega}$$

with a 220V source

$$P = \frac{v^2}{R} = \frac{(210)^2}{40} = \underline{1210\ \text{W}}$$

P 2.4-8

Solution:

The current required by the mine lights is: $i = \frac{P}{v} = \frac{5000}{120} = \frac{125}{3}$ A

Power loss in the wire is : $i^2\ R$

Thus the maximum resistance of the copper wire allowed is

$$R = \frac{0.05P}{i^2} = \frac{0.05 \times 5000}{(125/3)^2} = 0.144\ \Omega$$

now since the length of the wire is $L = 2 \times 100 = 200$ m$= 20{,}000$ cm

thus $R = \rho L / A$ with $\rho = 1.7 \times 10^{-6}\,\Omega\cdot\text{cm}$ from Table 2.5–1

$$A = \frac{\rho L}{R} = \frac{1.7 \times 10^{-6} \times 20{,}000}{0.144} = \underline{0.236\ \text{cm}^2}$$

P 2.4-9

Solution:

$$0.7884 = \frac{380}{102+380} \le gain \le \frac{420}{98+420} = 0.8108$$

$$nominal\ gain = \frac{0.7884+0.8108}{2} = 0.7996$$

$$gain\ tolerance = \frac{0.7996-0.7884}{0.7996} \times 100 = \frac{0.8108-0.7996}{0.7996} \times 100 = 1.40\%$$

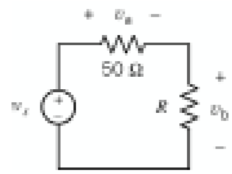
So

$$gain = 0.7996 \pm 1.40\%$$

P 2.4-10

Solution:

Label the current *i* as shown. That current is the element current in both resistors. First



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

$$i = \frac{v_a}{50}$$

Next

$$v_b = R\,i = R\frac{v_a}{50} \;\Rightarrow\; R = 50\frac{v_b}{v_a}$$

For example,

$$R = 50\frac{7.05}{11.75} = 30\ \Omega$$

Section 2-5 Independent Sources

P 2.5-1

Solution:

(a) $i = \frac{v_s}{R} = \frac{10}{5} = \underline{2A}$ and

$P = Ri^2 = 5(2^2) = \underline{20\ \text{W}}$

(b) i and P do not depend on i_s.

The values of i and P are 2A and 20W both when i_s = 2A and when i_s =4A.

P 2.5-2

Solution:

(a) From Ohm's law $v = R\,i_s = 5(5) = \underline{25V}$. (The resistor voltage does not depend on the voltage source voltage.) Next $P = \dfrac{v^2}{R} = \dfrac{25^2}{5} = \underline{125\ \text{W}}$.

(b) Since v and P do not depend on v_s the values of v and P are 25 V and 125 W both when $v_s = 15V$ and when $v_s = 10V$.

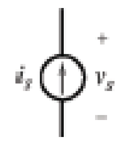
P 2.5-3

Solution:

Consider the current source. i_s and v_s do not adhere to the passive convention, so

$P_{cs} = i_s v_s = 5 \cdot 10 = \underline{50\ \text{W}}$

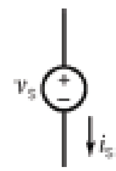
is the power supplied by the current source.



Consider the voltage source. i_s and v_s do adhere to the passive convention, so

$$P_{vs} = i_s\ v_s = 5 \cdot 10 = \underline{50\ \text{W}}$$

is the power absorbed by the voltage source.

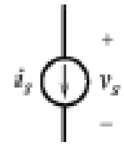


Therefore, $\underline{\text{The voltage source supplies } -50 \text{ W.}}$

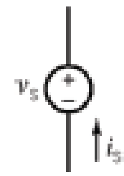
P 2.5-4

Solution:

Consider the current source. i_S and v_S adhere to the passive convention so $P_{CS} = i_S v_S = 3(10) = 30$ W is the received by the current source. The current source supplies $\underline{-30 \text{ W}}$.



Consider the voltages source. i_S and v_S do not adhere to the passive convention so $P_{CS} = i_S v_S = 3(10) = \underline{30 \text{ W}}$ is the supplied by the voltage source.



P 2.5-5

Solution:

(a) $P = vi = (5 \cos t)(20 \cos t) = \underline{100 \cos^2 t \text{ mW}}$

(b) $W = \int_0^1 P\,dt = \int_0^1 100 \cos^2 t\,dt$

$$= 100\left(\frac{1}{2}t + \frac{1}{4}\sin 2t\right)\Big|_0^1 = \underline{50 + 25 \sin 2 \text{ mJ}}$$

P 2.5-6

Solution:

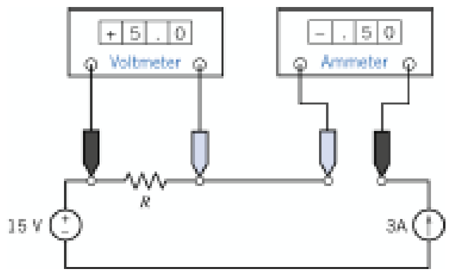
(a) time to discharge $= \dfrac{\text{capacity}}{\text{current}} = \dfrac{800 \text{ mAh}}{25 \text{ mA}} = 32$ hours

(b) energy = (12 V) (0.025 A) (32 * 60*60 seconds) =34.56 kJ

Section 2-6 Voltmeters and Ammeters

P 2.6-1

Solution:



(a) $R = \frac{v}{i} = \frac{5}{3} = 1.7\ \Omega$

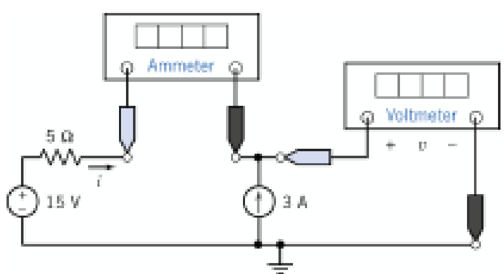
(b) The voltage, 15 V, and the current, 0.3 A, of the voltage source adhere to the passive convention so the power

$$P = 15(3) = 45\ \text{W}$$

is the power received by the source. The voltage source delivers -45 W.

P 2.6-2

Solution:



The voltmeter current is zero so the ammeter current is equal to the current source current except for the reference direction:

$$i = -4\ \text{A}$$

The voltage v is the voltage of the current source. The power supplied by the current source is 40 W so

$$40 = 4v \quad \Rightarrow \quad v = 10\ \text{V}$$

2.6-3

Solution:
(a)

$$i_m = \left(\frac{1000}{1000+20}\right)2 = 1.96\ \text{A}$$

$$\%\ \text{error} = \frac{2-1.96}{2}\times 100 = 2\%$$

(b)

$$0.05 \geq \frac{2-\left(\frac{1000}{1000+R_m}\right)2}{2} \quad \Rightarrow \quad \frac{1000}{1000+R_m} \geq 0.95 \quad \Rightarrow \quad R_m \leq 52.63\ \Omega$$

(checked: LNAP 6/17/04)

P 2.6-4

Solution:
a.)

$$v_R = 30 i_R = 30(-5) = -150\text{V}$$
$$v_m = 15 - v_R = 15 - (-150) = 165\text{V}$$

b.)

Element	**Power supplied**
voltage source	$-15(i_s) = -15(5) = -75$ W
current source	$165(5) = 825$ W
resistor	$-v_R \times i_R = -(-150)(-5) = -750$ W
total	0

P 2.6-5

Solution:

a.)

$$i_R = \frac{v_R}{30} = \frac{15}{30} = 0.5 \text{ A}$$

$$i_m = i_R - 2 = 0.5 - 2 = -1.5 \text{ A}$$

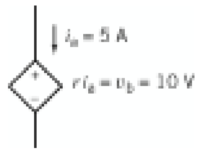
b.)

Element	**Power supplied**
voltage source	$16(i_m) = 16(-1.5) = -24$ W
current source	$v_s(2) = 15(2) = 30$ W
resistor	$-v_R \times i_R = -(15)(0.5) = -6.0$ W
total	0

Section 2-7 Dependent Sources

P 2.7-1

Solution:



$$r = \frac{v_b}{i_a} = \frac{10\text{V}}{5\text{A}} = \underline{2\Omega}$$

P 2.7-2

Solution:

$$v_b = 10\text{V}\,;\; g v_b = i_a = 5\text{A}\,;\; g = \frac{i_a}{v_b} = \frac{5\text{A}}{10\text{V}} = \underline{0.5\frac{\text{A}}{\text{V}}}$$

P 2.7-3

Solution:

$$i_b = 10\text{A}\,;\; d i_b = i_a = 40\text{A}\,;\; d = \frac{i_a}{i_b} = \frac{40\text{A}}{10\text{A}} = \underline{4\frac{\text{A}}{\text{A}}}$$

P 2.7-4

Solution:

$$v_a = 3\text{V}\,;\; b v_a = v_b = 9\text{V}\,;\; b = \frac{v_b}{v_a} = \frac{9\text{V}}{3\text{V}} = \underline{3\frac{\text{V}}{\text{V}}}$$

P 2.7-5

Solution:

$$R = -\frac{4}{-2} = 2\ \Omega \quad \text{and} \quad A = \frac{3}{-0.5} = -6\ \frac{\text{V}}{\text{A}}$$

P 2.7-6

Solution:

$$v_c = -2\ \text{V},\ i_d = 4\,v_c = -8\ \text{A} \text{ and } v_d = 2.2\ \text{V}$$

i_d and v_d adhere to the passive convention so

$$P = v_d\, i_d = (2.2)\,(-8) = -\underline{17.6\ \ \text{W}}$$

is the power received by the dependent source. The power supplied by the dependent source is 17.6 W.

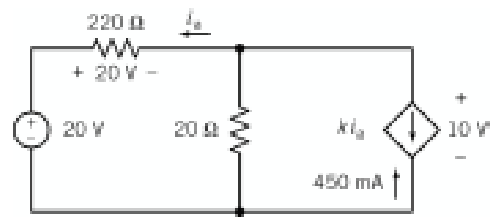
P2.7-7

Solution:

$$i_a = -\frac{20}{220} = -0.09 \text{ A} = -90 \text{ mA}$$

$$k\,i_a = -450 \text{ mA}$$

$$k = \frac{k\,i_a}{i_a} = \frac{-450}{-90} = 5 \;\frac{\text{A}}{\text{A}}$$



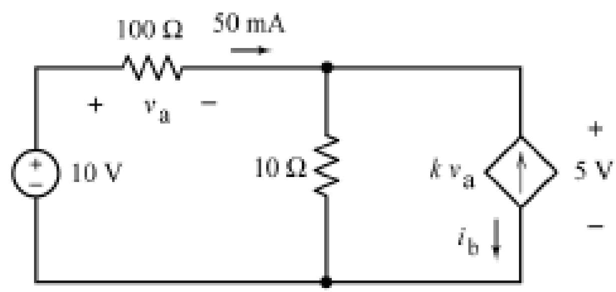
P2.7-8

Solution:

$$v_a = 100(0.05) = 5 \text{ V}$$

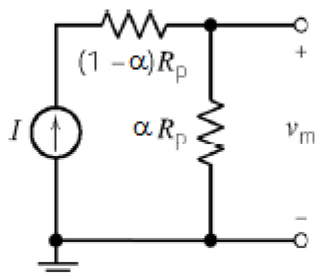
$$k = 100 \;\frac{\text{mA}}{\text{V}} = 0.1 \;\frac{\text{A}}{\text{V}}$$

$$i_b = -(0.1)(5) = -0.5 \text{ A} = -500 \text{ mA}$$



Section 2-8 Transducers

P 2.8-1
Solution:



$$\alpha = \frac{\theta}{360}\ , \quad \theta = \frac{360\ v_m}{R_p i}$$

$$\theta = \frac{(360)(25\text{V})}{(110\text{k}\Omega)(2\text{mA})} = \underline{40.9^\circ}$$

P 2.8-2

Solution:

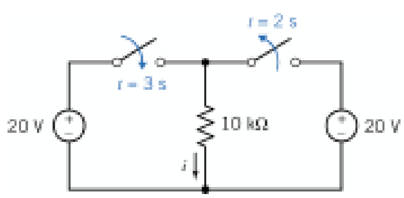
$$\text{AD590}: k = 1\frac{\mu\text{A}}{{}^\circ\text{K}}\ ,$$
$$v = 20\text{ V (voltage condition satisfied)}$$

$$\left.\begin{array}{c} 4\ \mu\text{A} < i < 13\ \mu\text{A} \\ T = \dfrac{i}{k} \end{array}\right\} \Rightarrow \underline{4^\circ\text{K} < T < 13^\circ\text{K}}$$

Section 2-9 Switches

P 2.9-1

Solution:



At $t = 1$ s the left switch is open and the right switch is closed so the voltage across the resistor is 20 V.

$$i = \frac{v}{R} = \frac{20}{10\times10^3} = \underline{2\text{mA}}$$

At $t = 4$ s the left switch is closed and the right switch is open so the voltage across the resistor is 20 V.

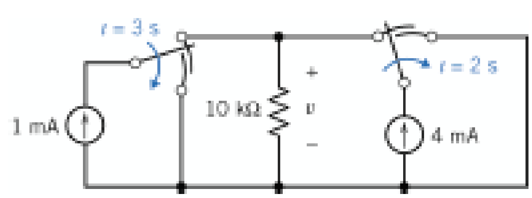
$$i = \frac{v}{R} = \frac{20}{10\times10^3} = \underline{2\text{mA}}$$

P 2.9-2

Solution:

At $t = 1$ s the current in the resistor is 5mA so $\underline{v = 50\text{ V}}$.

At $t = 4$ s the current in the resistor is 0 A so $\underline{v = 0\text{ V}}$.



P 2.9-3

Solution:

(a) $v = 12$ V

(b) $v = \left(\frac{100}{105}\right)12 = 11.43$ V

(c) $v = 0$ V

(d) $v = \left(\frac{100}{10100}\right)12 = 0.1188 \simeq 0.12$ V

Section 2-10 How Can We Check…?

P 2.10-1

Solution:

v_o=40 V and $i_s = -(-2) = 2$ A. (Notice that the ammeter measures $-i_s$ rather than i_s.)

So $$\frac{v_o}{i_s} = \frac{40}{2} = 20 \ \frac{\text{V}}{\text{A}}$$

Your lab partner is wrong.

P 2.10-2

Solution:

We expect the resistor current to be $i = \frac{v_s}{R} = \frac{12}{25} = 0.48$ A. The power absorbed by this resistor will be $P = i\ v_s = (0.48)\ (12) = 5.76$ W.

A half watt resistor can't absorb this much power. You should not try another resistor.

Design Problems

DP 2-1 Specify the resistance R in Figure DP 2-1 so that both of the following conditions are satisfied:

1. $i > 40$ mA.

2. The power absorbed by the resistor is less than 0.5 W.

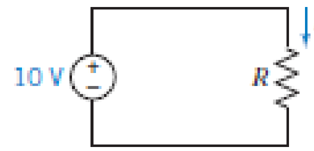


Figure DP 2-1

Solution:

$$1.)\ \frac{10}{R} > 0.04 \quad \Rightarrow \quad R < \frac{10}{0.04} = 250\ \Omega$$

$$2.)\ \frac{10^2}{R} < \frac{1}{2} \quad \Rightarrow \quad R > 200\ \Omega$$

Therefore 200 < R < 250 Ω. For example, R = 225 Ω.

DP 2-2 Specify the resistance R in Figure DP 2-2 so that both of the following conditions are satisfied:

1. $v > 40$ V.

2. The power absorbed by the resistor is less than 15 W.

Hint: There is no guarantee that specifications can always be satisfied.

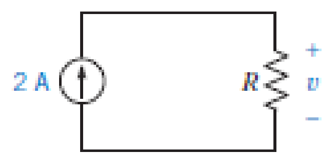


Figure DP 2-2

Solution:

$$1.)\ 2R > 40 \quad \Rightarrow \quad R > 20\ \Omega$$

$$2.)\ 2^2 R < 15 \quad \Rightarrow \quad R < \frac{15}{4} = 3.75\ \Omega$$

Therefore 20 < R < 3.75 Ω. These conditions cannot satisfied simultaneously.

DP 2-3 Resistors are given a power rating. For example, resistors are available with ratings of 1/8 W, 1/4 W, 1/2 W, and 1 W. A 1/2-W resistor is able to safely dissipate 1/2 W of power, indefinitely. Resistors with larger power ratings are more expensive and bulkier than resistors with lower power ratings. Good engineering practice requires that resistor power ratings be specified to be as large as, but not larger than, necessary.

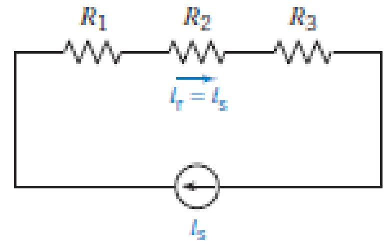


Figure DP 2-3

Consider the circuit shown in Figure DP 2-3. The values of the resistances are

$$R_1 = 1000\,\Omega, R_2 = 2000\,\Omega, \text{ and } R_3 = 4000\,\Omega$$

The value of the current source current is

$$i_s = 30\,\text{mA}$$

Specify the power rating for each resistor.

Solution::

$$P_1 = (30\text{ mA})^2 \cdot (1000\ \Omega) = (.03)^2 (1000) = 0.9\text{ W} < 1\text{ W}$$

$$P_2 = (30\text{ mA})^2 \cdot (2000\ \Omega) = (.03)^2 (2000) = 1.8\text{ W} < 2\text{ W}$$

$$P_3 = (30\text{ mA})^2 \cdot (4000\ \Omega) = (.03)^2 (4000) = 3.6\text{ W} < 4\text{ W}$$