國立清華大學

Fourier series and orthogonal functions

So far we have learned some types of series to expand a function. ex: Fourier series is one type of series expanded by

. . .

Fourier series expansion belongs to one kind of "
". So let's first learn something about

Preliminary : About orthogonal function & orthogonal

We know two vectors are "orthogonal" if their is zero. The ideal of "orthogonal" in vectors can also be generally applied

Analogy between vectors & functions

	vectors: $\vec{A}_1, \vec{A}_2$	function: $f_1(x), f_2(x)$
inner product		
norm (magnitude)		
normalized		

國立清華大學

Orthogonal set expansion

General procedures to find coefficients in the expansion

國立清華大學

ex1: Check if $\{1, \cos x, \cos 2x, \cdots, \cos nx\}$ is orthogonal on $[-\pi, \pi]$

ex2: Find the norm in the orthogonal set of ex1.

ex3: Express the orthogonal set of ex1 as an orthonormal set.

Remarks:

① Some commonly used orthogonal

1) $\{1, \cos\frac{\pi}{p}x, \cos\frac{2\pi}{p}x, \cdots, \sin\frac{\pi}{p}x, \sin\frac{2\pi}{p}x, \cdots\}$

2) $\{$ $\}$

3) $\{$ $\}$

4) $\{P_0(x), P_1(x), P_2(x), \cdots\}$:

ex: $f(x) =$

(2) Some functions are orthogonal after "weighted".

⇒