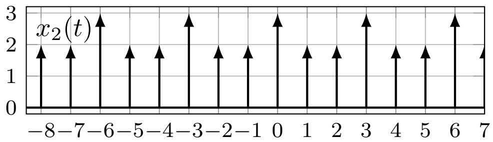
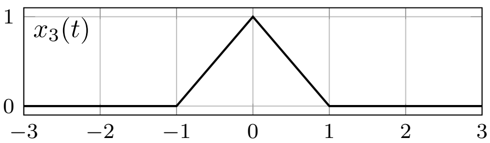
Signals and Systems

Homework 8 — Due : May 4, 2024

Problem 1 (42 pts, 14 pts each)**.** Compute the Fourier transform of each of the following signals:

(a) $x_1(t) = [e^{-\alpha t}\cos(3t)]u(t)$. $\alpha > 0$

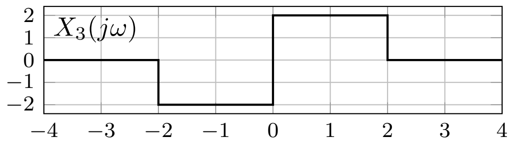
(b) 

(c) 

Problem 2 (42 pts, 14 pts each)**.** Determine the inverse Fourier transform of the following signals:

(a) $X_1(j\omega) = 2\pi\delta(\omega) + \frac{\pi}{j}\delta(\omega + 4\pi) - \frac{\pi}{j}\delta(\omega - 4\pi)$.

(b) $X_2(j\omega) = \cos\left(4\omega + \frac{\pi}{3}\right) + j\sin\left(2\omega + \pi\right)$.

(c) 

Problem 3 (16 pts, 8 pts each)**.** Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5).$$

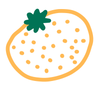
and let

$$h(t) = u(t) - u(t-2).$$

(a) Is $x(t)$ periodic?

(b) Is $x(t) * h(t)$ periodic?

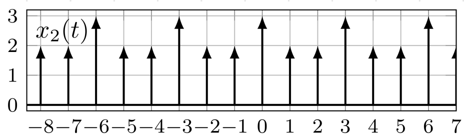
Problem 4 (extra 50 pts)**.** Write down your comment and feedback for the course.



Problem 1 (42 pts, 14 pts each). Compute the Fourier transform of each of the following signals:

(a) $x_1(t) = [e^{-\alpha t}\cos(3t)]u(t)$. $\alpha > 0$

$$X_1(j\omega) = \int_0^{\infty} e^{-\alpha t}\cos(3t)e^{-j\omega t}dt$$
$$= \int_0^{\infty} \frac{e^{-\alpha t}}{2}(e^{j3t}+e^{-j3t})\, e^{-j\omega t}\, dt$$
$$= \frac{1}{2}\int_0^{\infty}[e^{-(\alpha+j\omega-3j)t} + e^{-(\alpha+3j+j\omega)t}]\, dt$$
$$= \frac{1}{2}\left(\frac{1}{\alpha+j\omega-3j} + \frac{1}{\alpha+j\omega+3j}\right)$$

(b) 

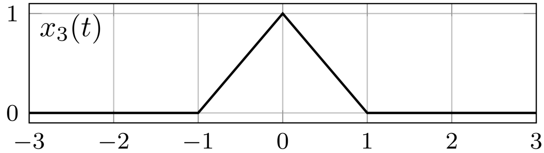
$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[2\pi\,\delta(\omega-\omega_0)]e^{j\omega t}d\omega$$
$$= e^{j\omega_0 t}$$

$$x_2(t) = \sum_{k=-\infty}^{\infty} 2\cdot\delta(t-k) + \sum_{k=-\infty}^{\infty}\delta(t-3k)$$

$$F\left\{\sum_{k=-\infty}^{\infty} 2\delta(t-k)\right\} = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty} 2\delta(t-k)e^{-j\omega t}dt$$
$$= \sum_{k=-\infty}^{\infty} 2\cdot e^{-j\omega k}$$

$$F\left\{\sum_{k=-\infty}^{\infty}\delta(t-3k)\right\} = \sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}\delta(t-3k)\, e^{-j\omega t}dt$$
$$= \sum_{k=-\infty}^{\infty} e^{-j\omega 3k}$$

$$F\{x_2(t)\} = \sum_{k=-\infty}^{\infty}(2\cdot e^{-j\omega k} + e^{-j\omega 3k})$$

(c) 

$$x_3(t) = (t+1)\cdot u(t+1) - 2t\,u(t) + (t-1)\,u(t-1)$$

$$F\{(t+1)\cdot u(t+1)\} = \int_{-1}^{\infty}(t+1)e^{-j\omega t}dt = \frac{1}{-j\omega}(t+1)e^{-j\omega t}\Big|_{-1}^{\infty} + \frac{1}{j\omega}\int_{-1}^{\infty} e^{-j\omega t}dt$$

$$= \frac{e^{-j\omega t}}{-(j\omega)^2}\Big|_{-1}^{\infty} = 0 + \frac{e^{j\omega}}{(j\omega)^2} = \frac{e^{j\omega}}{(j\omega)^2}$$

$$F\{2t\,u(t)\} = \int_{0}^{\infty} 2t\,e^{-j\omega t}dt = \frac{1}{-j\omega}2te^{-j\omega t}\Big|_{0}^{\infty} + \frac{2}{j\omega}\int_{0}^{\infty} e^{-j\omega t}dt$$

$$= \frac{2}{-(j\omega)^2}e^{-j\omega t}\Big|_{0}^{\infty} = \frac{2}{(j\omega)^2}$$

$$F\{(t-1)\,u(t-1)\} = \int_{1}^{\infty}(t-1)e^{-j\omega t}dt = \frac{1}{-j\omega}(t-1)e^{-j\omega t}\Big|_{1}^{\infty} + \frac{1}{j\omega}\int_{1}^{\infty} e^{-j\omega t}dt$$

$$= \frac{1}{-(j\omega)^2}e^{-j\omega t}\Big|_{1}^{\infty} = \frac{e^{-j\omega}}{(j\omega)^2}$$

$$F\{x_3(t)\} = \frac{e^{j\omega} - 2 + e^{-j\omega}}{(j\omega)^2} \quad \#$$

Problem 2 (42 pts, 14 pts each)**.** Determine the inverse Fourier transform of the following signals:

(a) $X_1(j\omega) = 2\pi\delta(\omega) + \frac{\pi}{j}\delta(\omega + 4\pi) - \frac{\pi}{j}\delta(\omega - 4\pi)$.

$$F^{-1}\{2\pi\,\delta(\omega)\} = \int_{-\infty}^{\infty}\frac{1}{2\pi}\cdot 2\pi\,\delta(\omega)\,e^{j\omega t}\,d\omega = 1$$

$$F^{-1}\left\{\frac{\pi}{j}\delta(\omega+4\pi)\right\} = \int_{-\infty}^{\infty}\frac{1}{2\pi}\cdot\frac{\pi}{j}\delta(\omega+4\pi)\,e^{j\omega t}\,d\omega$$

$$= \frac{1}{2j}\int_{-\infty}^{\infty}\delta(\omega+4\pi)e^{j\omega t}d\omega = \frac{e^{-j4\pi t}}{2j}$$

$$F^{-1}\left\{\frac{\pi}{j}\delta(\omega-4\pi)\right\} = \int_{-\infty}^{\infty}\frac{1}{2\pi}\cdot\frac{\pi}{j}\delta(\omega-4\pi)e^{j\omega t}d\omega = \frac{e^{j4\pi t}}{2j}$$

$$F^{-1}\{X_1(j\omega)\} = 1 + \frac{e^{-j4\pi t} - e^{j4\pi t}}{2j} = 1 - \sin(4\pi t) \quad \#$$

(b) $X_2(j\omega) = \cos\left(4\omega + \frac{\pi}{3}\right) + j\sin(2\omega + \pi)$.

$$\int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-j\omega t}\, dt = e^{-jt_0\omega}$$

$$\cos\left(4\omega+\frac{\pi}{3}\right) = \frac{1}{2}\cdot e^{\frac{\pi}{3}j}\cdot e^{j4\omega} + \frac{1}{2}\cdot e^{-\frac{\pi}{3}j}\cdot e^{j(-4)\omega}$$

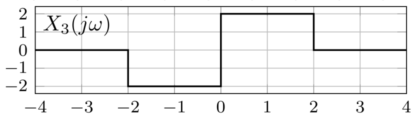
$$F^{-1}\left\{\cos\left(4\omega+\frac{\pi}{3}\right)\right\} = \frac{1}{2}e^{\frac{\pi}{3}j}\cdot\delta(t+4) + \frac{1}{2}\cdot e^{-\frac{\pi}{3}j}\cdot\delta(t-4)$$

$$j\sin(2\omega+\pi) = \frac{1}{2}e^{j\pi}\cdot e^{j2\omega} - \frac{1}{2}e^{j\pi}\cdot e^{-j2\omega}$$

$$F^{-1}\left\{j\sin(2\omega+\pi)\right\} = \frac{1}{2}e^{j\pi}\cdot\delta(t+2) - \frac{1}{2}e^{j\pi}\cdot\delta(t-2)$$

$$F^{-1}\left\{X_2(j\omega)\right\} = \frac{e^{j\frac{\pi}{3}}}{2}\cdot\delta(t+4) + \frac{e^{-j\frac{\pi}{3}}}{2}\cdot\delta(t-4) + \frac{e^{j\pi}}{2}\delta(t+2) - \frac{e^{j\pi}}{2}\delta(t-2) \quad \#$$

(c)



$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X_3(j\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\left(\int_{-2}^{0}(-2)e^{j\omega t}d\omega + \int_{0}^{2} 2\cdot e^{j\omega t}d\omega\right)$$

$$= \frac{1}{2\pi}\left(\frac{-2}{jt}\cdot e^{j\omega t}\Big|_{-2}^{0} + \frac{2}{jt}\cdot e^{j\omega t}\Big|_{0}^{2}\right)$$

$$= \frac{1}{2\pi}\left[\frac{2\cdot(e^{-2jt}-1)}{jt} + \frac{2(e^{2jt}-1)}{jt}\right]$$

$$= \frac{e^{2jt}+e^{-2jt}-2}{j\pi t} \quad \#$$

Problem 3 (16 pts, 8 pts each). Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5).$$

and let

$$h(t) = u(t) - u(t-2).$$

(a) Is $x(t)$ periodic?

(b) Is $x(t) * h(t)$ periodic?

(a) $\frac{1}{2\pi}\int_{-\infty}^{\infty}[\delta(\omega)+\delta(\omega-\pi)+\delta(\omega-5)]e^{j\omega t}d\omega$

$= \frac{1}{2\pi}\left(1+e^{j\pi t}+e^{j5t}\right)$

$\frac{2\pi}{\pi}=2$, $\frac{2\pi}{5}$ $\Rightarrow$ not periodic #

(b) $F\{u(t)-u(t-2)\} = \int_0^{\infty} e^{-j\omega t}dt - \int_2^{\infty} e^{-j\omega t}dt$

$= \frac{1-e^{-2j\omega}}{j\omega}$

$F^{-1}\{\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega)\} = \int_{-\infty}^{\infty}\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega)e^{j\omega t}d\omega$

$= \lim_{\omega\to 0}\frac{e^{j\omega t}-e^{j\omega(t-2)}}{j\omega} \overset{\text{L}}{=} \frac{jt-j(t-2)}{j} = 2$

$F^{-1}\{\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega-\pi)\} = \int_{-\infty}^{\infty}\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega-\pi)e^{j\omega t}d\omega$

$= \frac{1-e^{-2j\pi}}{j\pi}e^{j\pi t}$

$F^{-1}\{\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega-5)\} = \int_{-\infty}^{\infty}\frac{1-e^{-2j\omega}}{j\omega}\delta(\omega-5)e^{j\omega t}d\omega$

$= \frac{1-e^{-10j}}{5j}e^{j5t}$

$x(t) * h(t) = 2 + \frac{1-e^{-2j\pi}}{j\pi}e^{j\pi t} + \frac{1-e^{-10j}}{5j}e^{j5t}$ #

Problem 4 (extra 50 pts)**.** Write down your comment and feedback for the course.

希望作業能標注對應的章節，不然卡住的時候會不知道應該要去複習哪個部分。