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EE214000 Electromagnetics, Fall 2020

Your name: 王昱淳 ID: 107060013 Oct. 5th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #5-1, Open books, notes (31 points), due 11 pm, Wednesday, Oct. 7th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. What are the two postulates and their physical meaning for electrostatics? Define all the symbols in your answers (8 points)

① Faraday's law of electrostatics :

$\nabla \times \vec{E} = 0 \Rightarrow$ Physical meaning : A static electric field is irrotational.

Stokes' theorem

$\int \nabla \times \vec{E} \cdot d\vec{s} = \oint_C \vec{E} \cdot d\vec{l} = \oint_C \frac{\vec{F}}{q} \cdot d\vec{l} = 0 \Rightarrow \oint_C \vec{E} \cdot d\vec{l} = 0$

$\Rightarrow$ Physical meaning : The work done on a charge is independent of path.

$\vec{E}$: electric field , $\nabla \times$: curl

② Gauss law :

$\nabla \cdot \vec{D} = \rho \Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \varepsilon_0 (\nabla \cdot \vec{E}) = \rho$

divergence theorem

$\int_V \nabla \cdot \vec{D} \, dV = \varepsilon_0 \oint_S \vec{E} \cdot d\vec{s} = \int_V \rho \, dV = Q \Rightarrow \varepsilon_0 \oint_S \vec{E} \cdot d\vec{s} = Q$

$\Rightarrow$ Physical meaning : The total outward electric flux of V equals total enclosed charge in V.

$\vec{D}$: electric flux density , $\nabla \cdot$: divergence , ρ : volume charge density

ε_0 : vacuum permittivity , Q : total charge

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2. What happens if you define the electric field intensity as $\vec{E} = +\nabla V$, where V is the electric potential ? Is there a way to rescue the definition "work done externally ...." for V? (3 points)

Set $\vec{E} = \pm\nabla V$, $\Rightarrow V(R_0) - V(\infty) = \int_\infty^{R_0} \nabla V \cdot d\vec{l} = \pm\int_\infty^{R_0} \vec{E} \cdot d\vec{l}$

$$V(R_0) - V(\infty) = \pm\int_\infty^{R_0} \frac{q}{4\pi\varepsilon_0 R^2} dR = \mp \frac{q}{4\pi\varepsilon_0 R_0}$$

Then, move on to the definition. We modify it as shown:

Work done externally by moving a unit positive (negative) charge q from infinity to a positive (negative) charge q is "negative".

$\Rightarrow$ Since the work should be negative, $\mp \frac{q}{4\pi\varepsilon_0 R_0}$ will become $-\frac{q}{4\pi\varepsilon_0 R_0}$.

Corresponding to $\vec{E}$, $\vec{E} = \pm\vec{V}$ should become $\vec{E} = +\vec{V}$.

$\Rightarrow$ If we define $\vec{E} = +\nabla V$, it means the work should be negative.

3. Would a lightning rod attract or avoid catastrophic discharge from a high cloud? View the following film to explain it
https://www.youtube.com/watch?v=wGc3q4dVOS0 (5 points)

The lightning rod can avoid catastrophic discharge from a high cloud.
It can provide a conductive pathway of the charge to the Earth, and its tip can ionize the surrounding air to make it more conductive.
That's why the lightning rod can protect the building from being destroyed.

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4. What is the *R* dependence of the field and potential of an electric dipole in the far zone? (2 points) Explain why the field and potential have a weak dependence on *R* in comparison with those of a monopole charge? (3 points)

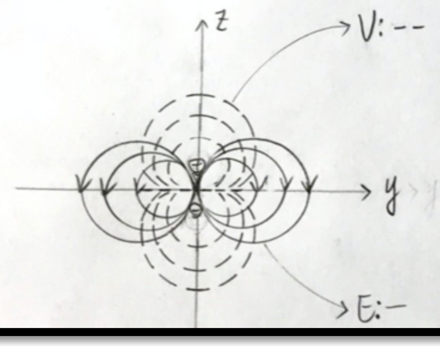
① for electric dipole potential : $V(R,\theta) = \frac{\vec{p}\cdot\hat{a}_R}{4\pi\varepsilon_0 R^2} \propto \frac{1}{R^2}$

for electric dipole field : $\vec{E}(R,\theta) = \frac{p}{4\pi\varepsilon_0 R^3}(\hat{a}_R 2\cos\theta + \hat{a}_\theta \sin\theta) \propto \frac{1}{R^3}$

② Compared with monopole charge ; the electric dipole is always influenced by its positive charge and negative charge, which means that they will cancel out their electric dipole potential and electric dipole field. That's why the field and potential have a week dependence on R.

5. Draw the equipotential lines and electric field lines of an electric dipole on the plane of ϕ = 90°? The two dipole charges are aligned along the z axis. Show the directions of the electric field lines with reference to the polarity of the charges. (5 points)

In spherical coordinate system, the plane on $\phi=90°$ can be corresponding to y-z plane of Cartesian coordinate system.



Two charges are on the z-axis, and the positive charge is above the negative charge. All equipotential lines are from the positive charge toward the negative charge. Besides, each equipotential line is othogonal to each electric field line.

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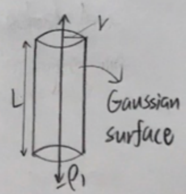
6. What is the Gaussian surface of an infinite straight line charge? (2 points) Use it to derive the electric field (magnitude and direction) intensity at *r*. Assume a line charge density of $-\rho_l$ on the line. (3 points)

① The Gaussian surface of an infinite straight line charge is a cylinder, which has radius r and length L. L is infinite for this infinite straight line.



② $E_r(z=0, r) = \dfrac{-\rho_l}{2\pi\varepsilon_0 r} \dfrac{\frac{L}{2}}{\sqrt{(\frac{L}{2})^2 + r^2}} \xrightarrow{L \to \infty} \dfrac{-\rho_l}{2\pi\varepsilon_0 r}$

By Gauss law : $\varepsilon_0 \oint_s \vec{E} \cdot d\vec{s} = \varepsilon_0 E_r 2\pi r L = L(-\rho_l) \Rightarrow \vec{E} = \dfrac{-\rho_l}{2\pi\varepsilon_0 r} \hat{a}_r$