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*EE214000 Electromagnetics, Fall 2020*

Your name: 王昱淳 ID: 107060013 Sep. 29, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #4-1, Open books, notes (26 points), due 11 pm, Wednesday, Sep. 30th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. What is the physical meaning of the gradient of a scalar? (3 point)

The physical meaning of the gradient of a scalar is the maximum rate of change of a scalar in space, and a direction along the maximum change. #

2. What is the physical meaning of the divergence of a vector? (3 points)

The physical meaning of the divergence of a vector is a scalar equal to the net outward flux of a vector per unit volume at a "point" in space. #

3. What is the physical meaning of the curl of a vector? (3 points)

The physical meaning of the curl of a vector is the maximum net circulation of a vector per unit area at a "point" in space. The direction of the curl of a vector is chosen to be the surface normal direction of the infinitesimal area with which the net circulation is a maximum. #

4. Verbally describe the meaning of the Stokes theorem? (3 points)

Stokes' Theorem: $\int_S (\nabla \times \vec{A}) \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$

$\Rightarrow$ The surface integral of the curl of a vector over an open surface is equal to the closed line integral of this vector along the path enclosing the surface. #

5. Verbally describe the meaning of the divergence theorem? (3 points)

Divergence Theorem : $\int_V \nabla \cdot \vec{A}\, dv \equiv \oint_S \vec{A} \cdot d\vec{s}$

⇒ The volume integral of the divergence of a vector is equal to the total outward flux of this vector over the enclosed surface. #

6. In the Cartesian coordinate system, what are the mathematic expressions of $\nabla V, \nabla \times \vec{A}, \nabla \cdot \vec{B}, \nabla^2 V, \nabla^2 \vec{A}$? (5 points) *Please get familiar with the expressions, because you will be using them quite often in this class.

① $\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$ #

② $\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$ #

③ $\nabla \cdot \vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$ #

④ $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ #

⑤ $\nabla^2 \vec{A} = \hat{a}_x \nabla^2 A_x + \hat{a}_y \nabla^2 A_y + \hat{a}_z \nabla^2 A_z$ #

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7. Explain intuitively why the two null identities:

$\nabla \times (\nabla V) = 0$ and $\nabla \cdot (\nabla \times \vec{A}) = 0$. (3+3 points)

- Note that proof of the two null identities needs some mathematically skills. As an engineer, you should at least remember the two expression from some intuitive arguments.

① $\nabla \times (\nabla V) = 0$

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla \times (\nabla V) = \left(\frac{\partial^2 V}{\partial y \partial z} - \frac{\partial^2 V}{\partial z \partial y}\right)\hat{a}_x + \left(\frac{\partial^2 V}{\partial z \partial x} - \frac{\partial^2 V}{\partial x \partial z}\right)\hat{a}_y + \left(\frac{\partial^2 V}{\partial x \partial y} - \frac{\partial^2 V}{\partial y \partial x}\right)\hat{a}_z$$

If $V$ is a twice continuously differentiable function, its second derivatives would be independent no matter how to arrange the order of differentiating. $\Rightarrow \nabla \times (\nabla V) = 0$.

Physical meaning: When a scalar field in space is making the fastest and biggest charge, we can't figure out its circulation in this direction, since we have components of vectors in this direction of rotation. #

② $\nabla \cdot (\nabla \times \vec{A}) = 0$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

$$\nabla \cdot (\nabla \times \vec{A}) = \frac{\partial}{\partial x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{a}_x + \frac{\partial}{\partial y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{a}_y + \frac{\partial}{\partial z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{a}_z$$

If we differentiate the unit vector, we can get 1.

$$\nabla \cdot (\nabla \times \vec{A}) = \left(\frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z}\right)\cdot 1 + \left(\frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_z}{\partial y \partial x}\right)\cdot 1 + \left(\frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y}\right)\cdot 1$$

$$= \left(\frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_z}{\partial y \partial x}\right) + \left(\frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_y}{\partial x \partial z}\right) + \left(\frac{\partial^2 A_x}{\partial y \partial z} - \frac{\partial^2 A_x}{\partial z \partial y}\right) = 0$$

If $\vec{A}$ is twice continuously differentiable in $x, y$ and $z$ direction its second derivatives remain the same no matter the order.

$\Rightarrow \nabla \cdot (\nabla \times \vec{A}) = 0$

Physical meaning: When a vector field in space is making the maximum circulation, we can't figure out its components of this flux. #