

$$1. \quad 4y'' + 36y = \csc 3x$$

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$$y = c_1 \cos 3x + c_2 \sin 3x - \frac{1}{12} x \cos 3x + \frac{1}{36} \ln(\sin 3x) \sin 3x$$

$$\}$$

$$2. \quad y'' + y = (x-1)\cos x$$

$$\{ y = c_1 \cos x + c_2 \sin x + \frac{1}{4} x \cos x + \left(\frac{1}{4} x^2 - \frac{1}{2} x \right) \sin x \}$$

$$3. \quad x^3 y''' - 3x^2 y'' + 6xy' - 6y = x^2 \ln x$$

$$\{ y = c_1 x + c_2 x^2 + c_3 x^3 - \frac{1}{2} x^2 (\ln x)^2 \}$$

$$4. \quad (x^2 + 1)y'' - 2xy' + 2y = 0$$

$$\{ y = c_1 x + c_2 (x^2 - 1) \}$$

$$5. \quad x^2 y'' - 2y = \ln x \quad (x > 0) \quad y(1) = \frac{1}{2} \quad y'(1) = 0$$

$$\{ y = \frac{1}{4} x^2 + \frac{1}{4} - \frac{1}{2} \ln x \}$$

6. $x^2 y'' - 2xy' + 2y = x^2 + 2$

$$\{ y = c_1 x + c_2 x^2 + x^2 \ln x + 1 \}$$

7. $(x^2 - x) y'' - xy' + y = 0$

$$\{ y = c_1 (x \ln x + 1) + c_2 x \}$$

8. Find the inverse Laplace transform of $\frac{2}{s^4} (\frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^6})$

$$\{ \frac{1}{12} t^4 - \frac{1}{20} t^5 + \frac{8}{9!} t^9 \}$$

9. Find the Laplace transform of $4t \sin(2t)$

$$\{ \frac{16s}{(s^2+4)^2} \}$$

10. Find the Laplace transform of $\{\cos 2t + 4\sin(2t)\}$

$$\{ \frac{s+8}{s^2+4} \}$$

11. Find the Laplace transform of $\{2t^2 e^{-3t} - 4t + 1\}$

$$\{ \frac{4}{(s+3)^3} - \frac{4}{s^2} + \frac{1}{s} \}$$

12. Find the Laplace transform of

$$f(t) = \begin{cases} 2t - \sin(t) & ; \text{ for } 0 \leq t < \pi \\ 0 & ; \text{ for } t \geq \pi \end{cases}$$

$$\left\{ \frac{2}{s^2} - \frac{1}{s^2+1} - \frac{2}{s^2} e^{-\pi s} - \frac{2\pi}{s} e^{-\pi s} - \frac{1}{s^2+1} e^{-\pi s} \right\}$$

13. Find the Laplace transform of $e^{-2t} \cos(3t)$

$$\left\{ \frac{s+2}{s^2+4s+13} \right\}$$

14. Find the Laplace transform of $te^{-2t} \cos(3t)$

$$\left\{ \frac{s^2+4s-5}{(s^2+4s+13)^2} \right\}$$

15. Find the Laplace transform of

$$f(t) = \begin{cases} \cos(t) & ; \text{ for } 0 \leq t < 2\pi \\ 2 - \sin(t) & ; \text{ for } t \geq 2\pi \end{cases}$$

$$\left\{ \frac{s}{s^2+1} + \left(\frac{2}{s} - \frac{s}{s^2+1} - \frac{1}{s^2+1} \right) e^{-2\pi s} \right\}$$

16. Find the inverse Laplace transform of $\frac{2s-5}{s^2+16}$

$$\left\{ 2 \cos(4t) - \frac{5}{4} \sin(4t) \right\}$$

17. Find the inverse Laplace transform of $\frac{se^{-3s}}{s^2+4}$

$$\{ u(t-3) \cos(2(t-3)) \}$$

18. Find the Laplace transform of $\{e^{-t}(1-t^2+\sin t)\}$

$$\left\{ \frac{1}{s+1} - \frac{2}{(s+1)^3} + \frac{1}{(s+1)^2+1} \right\}$$

19. Find the inverse Laplace transform of $\frac{5}{(s+7)^2}$

$$\{ 5te^{-7t} \}$$