HW4 Q1

- A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either 10, 26, or 58 hexadecimal digits. How many different WEP keys are there?

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Total = $16^{10} + 16^{26} + 16^{58}$

= 69017463467905637874347558622770254524511089721703865551636237354274888

0-9A-F	…	…	…	…

Length = 10 or 26 or 58

HW4 Q2

- Three integers are selected from the integers {1, 2, . . . , 1000}. In how many ways can these integers be selected such their sum is divisible by 4?

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Note: 題目沒有明確說明是否能重複使用，本解答假設不行，但是很容易推成可重複使用的答案

Hint: make groups

x%4=1	1,5,9,13…	x%4=2	2,6,10,14…
x%4=3	3,7,11,15…	x%4=0	4,8,12,16…

HW4 Q2

- Three integers are selected from the integers {1, 2, . . . , 1000}. In how many ways can these integers be selected such their sum is divisible by 4?

討論如何組成 4 的倍數

x%4=1	1,5,9,13…	x%4=2	2,6,10,14…
x%4=3	3,7,11,15…	x%4=0	4,8,12,16…

- 0+0+0
- 0+1+3
- 0+2+2
- 1+1+2
- 2+3+3

HW4 Q2

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討論如何組成 4 的倍數

x%4=1	1,5,9,13…
x%4=2	2,6,10,14…
x%4=3	3,7,11,15…
x%4=0	4,8,12,16…

- 0+0+0 C(250,3)
- 0+1+3 C(250,1)C(250,1)C(250,1)
- 0+2+2 C(250,1)C(250,2)
- 1+1+2 C(250,1)C(250,2)
- 2+3+3 C(250,1)C(250,2)

上面加一加就是答案

HW4 Q3

- How many arrangements of the letters in MISSISSIPPI have no consecutive S’s ?

HW4 Q3

- How many arrangements of the letters in MISSISSIPPI have no consecutive S’s ?

Sol1. 先決定非 S 的元素，再把 S 插空

_m_i_i_i_p_p_i_

非 S 元素排列 7!/4!/2! = 105
4 個 S 選 8 個空格 C(8,4) = 70
共 105*70 = 7350

HW4 Q3

- How many arrangements of the letters in MISSISSIPPI have no consecutive S's ?

Sol2. 先決定 S 間元素的數量，再排列元素

A個 C+1個 E個

_S_S_S_S_

B+1個 D+1個

A+B+C+D+E+3 = 7 ， ABCDE 整數解有 H(5,4) = C(4+5-1,5-1)=C(8,4) = 70

每種解給定一個填入方法 7!/4!/2! = 105

共 105*70 = 7350

Discrete Mathematics

Homework 4

Question 6, 7, 10

資工所碩士班
李峻丞

Question 10

There are 6 boys and 4 girls. How many ways can they be divided into groups of 2 persons, such that there is no group with two girls?

Question 10

Solution:

They can be divided into 5 groups.

Let the groups be Group1, Group2, Group3, Group4, Group5.

Before grouping, all groups are considered the same group.

Question 10

Solution:

Let the girls be Girl1, Girl2, Girl3, Girl4.

We can assume Girl1 is in Group1, Girl2 is in Group2, Girl3 is in Group3, Girl4 is in Group4.

After allocate all girls, all groups will be considered different groups.

Question 10

Solution:

Now, we split 6 boys into the 5 different groups.

$C(6,2) \times 4! = 360$ ways

Question 6

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes so that each of the boxes contains at least one object?

Question 6

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes so that each of the boxes contains at least one object?

Question 6

Solution1: (Enumeration)

Let $(\cdot , \cdot , \cdot)$ be the number of objects in 3 boxes.

e.g. (1, 2, 2) means there are 2 boxes with 2 objects and 1 box with 1 object.

$(1, 2, 2) = (2, 1, 2) = (2, 2, 1)$

Question 6

Solution1: (Enumeration)

First, suppose that all objects are the same.

Question 6

Solution1: (Enumeration)

First, suppose that all objects are the same.

There are 2 possible groupings:

(3, 1, 1)

(2, 2, 1)

Question 6

Solution1: (Enumeration)

Then, consider that all objects are different.

There are 2 possible groupings:

(3, 1, 1) : $C(5,3)C(2,1)C(1,1)/2!$

(2, 2, 1) : $C(5,2)C(3,2)C(1,1)/2!$

Question 6

Solution1: (Enumeration)

Then, consider that all objects are different.

There are 2 possible groupings:

(3, 1, 1) : $C(5,3)C(2,1)C(1,1)/2! = 10$

(2, 2, 1) : $C(5,2)C(3,2)C(1,1)/2! = 15$

total: $10 + 15 =$ 25 ways

Question 6

Solution2: (Stirling numbers of the second kind)

The Stirling numbers of the second kind, written $S(n,k)$, count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.

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The Stirling numbers of the second kind, written S(n,k), count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.

$$S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$$

Question 6

Solution2: (Stirling numbers of the second kind)

$$S(5,3) = \frac{1}{3!}\sum_{i=0}^{3}(-1)^i\binom{3}{i}(3-i)^5$$

$$= \frac{1}{3!}[\binom{3}{0}3^5 - \binom{3}{1}2^5 + \binom{3}{2}1^5 - \binom{3}{3}0^5]$$

$$= \frac{1}{3!}[243 - 96 + 3 - 0]$$

$$= 25$$

Question 7

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes?

Question 7

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes?

Question 7

Solution1: (Enumeration)

Let $(\cdot, \cdot, \cdot)$ be the number of objects in 3 boxes.

e.g. (0, 2, 3) means there is 1 boxes with 0 objects, 1 box with 2 objects and 1 box with 3 objects.

$$(0, 2, 3) = (0, 3, 2) = (2, 0, 3) = (2, 3, 0) = (3, 0, 2) = (3, 2, 0)$$

Question 7

Solution1: (Enumeration)

First, suppose that all objects are the same.

There are 5 possible groupings:

Question 7

Solution1: (Enumeration)

(5, 0, 0)

(4, 1, 0)

(3, 2, 0)

(3, 1, 1)

(2, 2, 1)

Question 7

Solution1: (Enumeration)

$(5, 0, 0)$: 1

$(4, 1, 0)$: $C(5,4)C(1,1) = 5$

$(3, 2, 0)$: $C(5,3)C(2,2) = 10$

$(3, 1, 1)$: $C(5,3)C(2,1)C(1,1)/2! = 10$

$(2, 2, 1)$: $C(5,2)C(3,2)C(1,1)/2! = 15$

total: $1 + 5 + 10 + 10 + 15 = 41$ ways

Question 7

Solution2: (Stirling numbers of the second kind)

The Stirling numbers of the second kind, written S(n,k), count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.

$$S(n,k) = \frac{1}{k!}\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n$$

Question 7

Solution2: (Stirling numbers of the second kind)

$$S(5,3) + S(5,2) + S(5,1) = 25 + 15 + 1$$
$$= 41$$

S(5,3) : (3, 1, 1) , (2, 2, 1)

S(5,2) : (4, 1, 0) , (3, 2, 0)

S(5,1) : (5, 0, 0)

Question 7

Solution1: (Enumeration)

$(5, 0, 0) : 1$

$(4, 1, 0) : C(5,4)C(1,1) = 5$

$(3, 2, 0) : C(5,3)C(2,2) = 10$

$(3, 1, 1) : C(5,3)C(2,1)C(1,1)/2! = 10$

$(2, 2, 1) : C(5,2)C(3,2)C(1,1)/2! = 15$

total: $1 + 5 + 10 + 10 + 15 =$ 41 ways

More Questions

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes? (Question 7)

More Questions

① 5 distinguishable objects into 3 indistinguishable boxes? (Question 7)

Answer: 41 ways

More Questions

② 5 indistinguishable objects into 3 indistinguishable boxes?

More Questions

② 5 indistinguishable objects into 3 indistinguishable boxes?

Answer: 5 ways

(5, 0, 0)

(4, 1, 0)

(3, 2, 0)

(3, 1, 1)

(2, 2, 1)

More Questions

② 5 indistinguishable objects into 3 indistinguishable boxes?

Answer: 5 ways

(5, 0, 0)	$5 = 5$	
(4, 1, 0)	$5 = 4 + 1$	
(3, 2, 0)	$5 = 3 + 2$	Partition
(3, 1, 1)	$5 = 3 + 1 + 1$	
(2, 2, 1)	$5 = 2 + 2 + 1$	

More Questions

③ 5 distinguishable objects into 3 distinguishable boxes?

More Questions

③ 5 distinguishable objects into 3 distinguishable boxes?

Answer: 3^5 ways

More Questions

④ 5 indistinguishable objects into 3 distinguishable boxes?

More Questions

④ 5 indistinguishable objects into 3 distinguishable boxes?

Answer: 21 ways

$$x_1 + x_2 + x_3 = 5$$

x_1, x_2, x_3: nonnegative integers

$$\binom{3+5-1}{5}$$

More Questions

④ 5 indistinguishable objects into 3 distinguishable boxes?

Answer: 21 ways

$x_1 + x_2 + \ldots + x_n = r$

$x_1, x_2, \ldots, x_n$: nonnegative integers

$$\binom{n+r-1}{r}$$

More Questions

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes? (Question 7)

More Questions

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes so that each of the boxes contains at least one object? (Question 6)

More Questions

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes so that each of the boxes contains at least one object? (Question 6)

Exercise

Exam 2

Date: 12/2, 2019 (Monday)

Time: 10:10–12:30

Scope: All topics covered in HW3 and HW4 (Mathematical Induction, Pigeonhole Principle, Counting)