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Ⅲ. Analytical approach

Analytical approach looks for analytical expressions of solutions. The following analytical methods are discussed in Ch 2:

Method of "separation of variables" (Ch 2.2)

Idea:

Example1 : Solve $(1+x)dy - ydx = 0$ by SoV

Example2 : Solve $\frac{dy}{dt} = y^2 - 4$ by SoV

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General procedures of SOV :

① Check out if DE is "separable" with the form of

② Separate the variables.

③ Do the integration

④ Check out if there are

Method of "integrating factor" (Ch 2.3)

⛤ This is a method based on

Idea : Given a DE $\frac{dy}{dx} + P(x)\, y = f(x)$

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Example 1 : Solve $\frac{dy}{dx} - 3y = 6$ by method of integrating factor.

Example 2 : Solve $x\frac{dy}{dx} + 2y = 4x^2$, $y(1) = 2$, by method of integrating factor.

General procedures of method of "integrating factor"

① Express the DE in its

② Write down the product rule :

③ Multiply "M" to the DE

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Remarks:

① Method of "integrating factor" is only useful to solve

② Do not memorize the math form of integrating factor.

Method of "exact (differential) equations"

⛤ This is a method based on

Example 1: Solve $x^2y^3dx + x^3y^2dy = 0$

Example 2: Solve $2xy\,dx + (x^2-1)dy = 0$

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Idea: If a 1st-order ODE : $M(x,y)\,dx + N(x,y)\,dy = 0$ is an "exact equation".

Q_1: How to check if a 1st-order ODE is an
For a DE (in differential form)
$M(x,y)\,dx + N(x,y)\,dy = 0$

Q_2: After we confirm an exact equation,
Redo Example 1 : $x^2y^3\,dx + x^3y^2\,dy = 0$

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Redo Example 2 : $2xy\,dx + (x^2-1)\,dy = 0$

General procedures of method of "exact equations" :

① Express the ODE

② check if

③ If so, find

④ Obtain solution as