④ Replace the LHS by , and integrate both sides

Remarks:

① Method of integrating factor is useful to solve

② Do NOT memorize the math form of integrating factor.

Method of "exact (differential) equations"

Preliminary: This is a method based on

There are some 1st-order ODEs that can be categorized as "exact (differential) equations", and can be solved by this technique.

Example 1: Solve $x^2y^3dx + x^3y^2dy = 0$

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Example 2: Solve $2xy\,dx+(x^2-1)\,dy=0$

Idea: If a 1st-order ODE: $M(x,y)dx+N(x,y)dy=0$ is an "exact equation"

Q: How to check if a 1st-order ODE is an "exact equation"?

For an ODE (in differential form)

$M(x,y)\,dx+N(x,y)\,dy=0 \Rightarrow$

$(df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy)$

$\frac{\partial}{\partial y}M=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)= \Rightarrow$

General procedures of method of "exact equations":

① Express the ODE in "differential form".

② check if

③ If so, find $f(x,y)$ such that

④ Obtain solution as

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Then, how to find $f(x,y)$?

Redo Example 1: $x^2y^3\,dx + x^3y^2\,dy = 0$

Redo Example 2: $2xy\,dx + (x^2-1)\,dy = 0$

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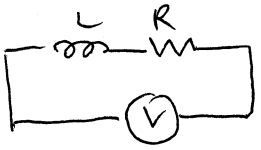
Examples of systems modeled by 1st-order ODEs (Ch 3)

Examples :

Example 1 : Series circuit

LR-series circuit



Given $V=12$, $R=10\Omega$, $L=\frac{1}{2}H$, $i(0)=0$

Example 2 : Population dynamics

Growth of population :

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But, due to limited resources, at some point the growth rate will decrease. →

For simplicity, let's assume f(p) and P are related

✡ DE with a form of $\frac{dy}{dt} =$ is called a " " and has been used as a good model to mathematically formulate

✡✡ Features of the " " :