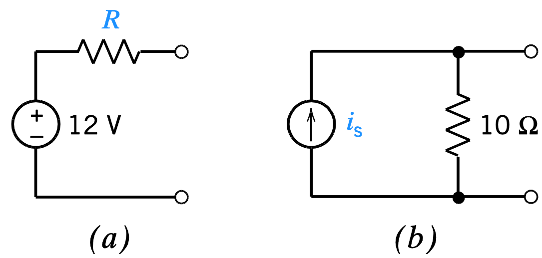
Chapter 5 Circuit Theorems

Exercises

Exercise 5.2-1 Determine values of R and i_s so that the circuits shown in Figures E 5.2-1*a*,*b* are equivalent to each other due to a source transformation.

Answer: $R = 10\ \Omega$ and $i_s = 1.2$ A

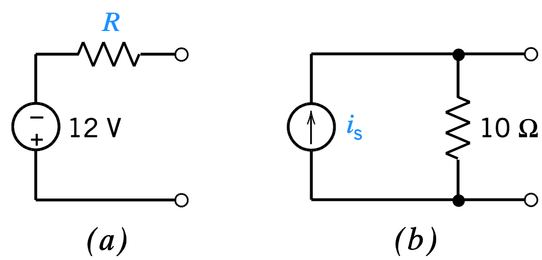


Figures E 5.2-1

Exercise 5.2-2 Determine values of R and i_s so that the circuits shown in Figures E 5.2-2*a*,*b* are equivalent to each other due to a source transformation.

Hint: Notice that the polarity of the voltage source in Figure E 5.2-2*a* is not the same as in Figure E 5.2-1*a*.

Answer: $R = 10\ \Omega$ and $i_s = -1.2$ A



Figures E 5.2-2

Exercise 5.2-3 Determine values of R and v_s so that the circuits shown in Figures E 5.2-3*a*,*b* are equivalent to each other due to a source transformation.

Answer: $R = 8\ \Omega$ and $v_s = 24$ V

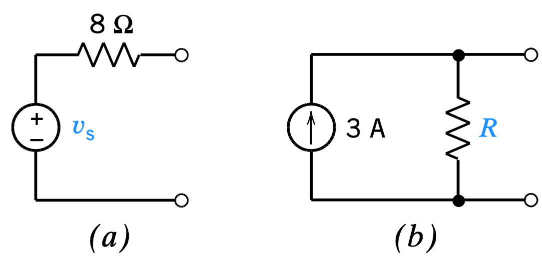


Figure E 5.2-3

Exercise 5.2-4 Determine values of R and v_s so that the circuits shown in Figures E 5.2-4*a*,*b* are equivalent to each other due to a source transformation.

Hint: Notice that the reference direction of the current source in Figure E 5.2-4*b* is not the same as in Figure E 5.2-3*b*.

Answer: $R = 8\ \Omega$ and $v_s = -24$ V

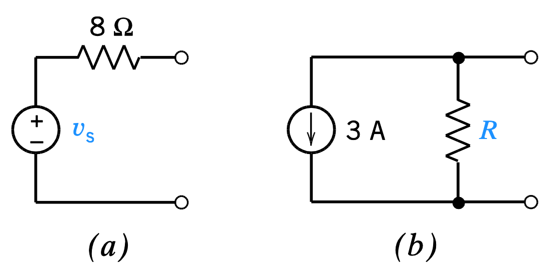


Figure E 5.2-4

Exercise 5.4-1 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-1*a*.

Answer: $R_t = 8\ \Omega$ and $v_{oc} = 2$ V

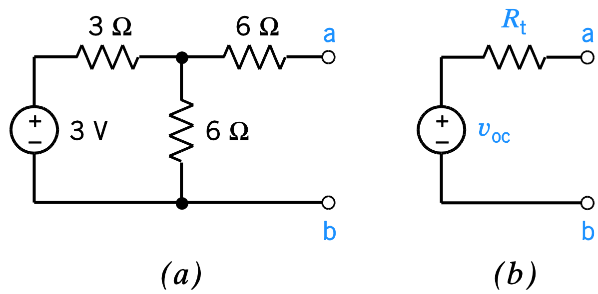
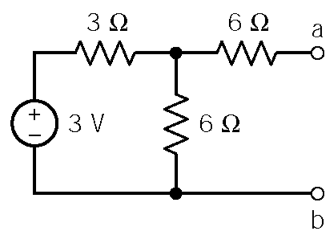
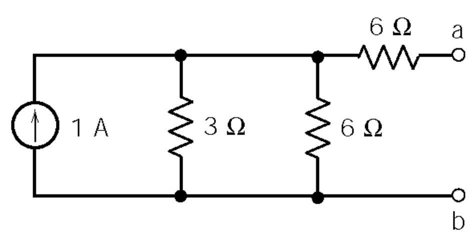
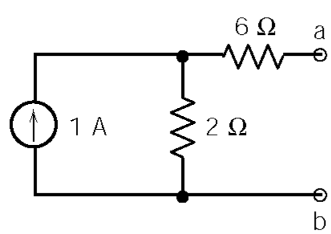


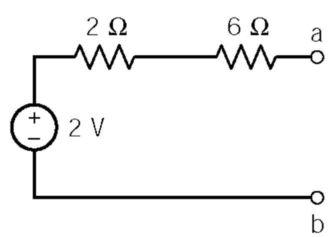
Figure E 5.2-1

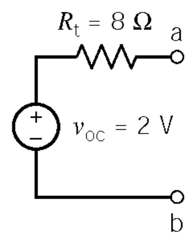
Solution:





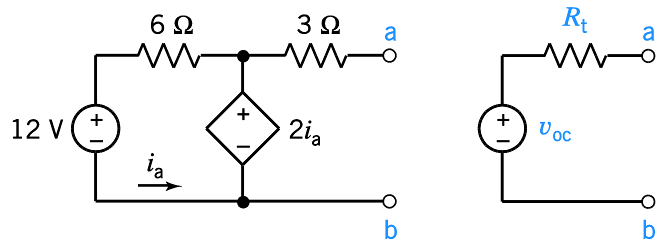






Exercise 5.4-2 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-2*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-2*a*.

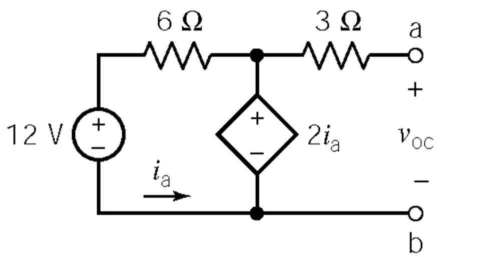
Answer: $R_t = 3\ \Omega$ and $v_{oc} = -6$ V



(a) *(b)*

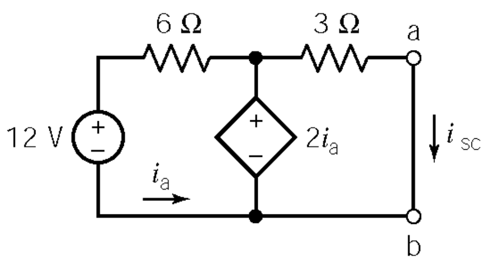
Figure E 5.2-2

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3\ \text{A}$$

$$v_{oc} = 2\,i_a = -6\ \text{V}$$



$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3\ \text{A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2\ \text{A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Exercise 5.5-1 Determine values of R_t and i_{sc} that cause the circuit shown in Figure E 5.5-1*b* to be the Norton equivalent circuit of the circuit in Figure E 5.5-1*a*.

Answer: $R_t = 8\ \Omega$ and $i_{sc} = 0.25$ A

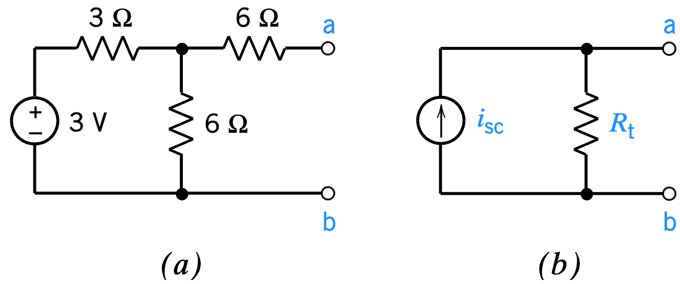
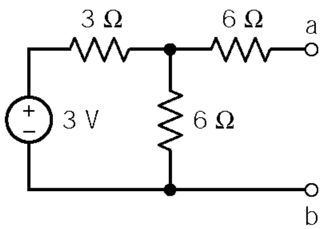
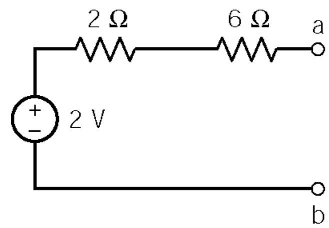
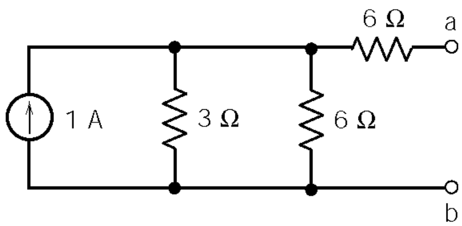


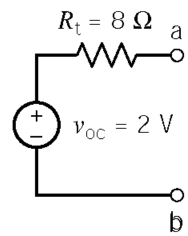
Figure E 5.5-1

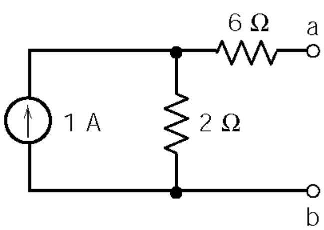
Solution:

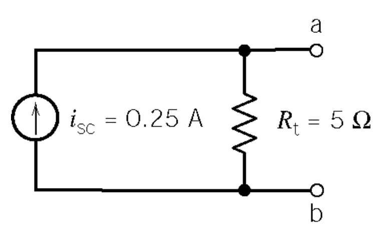












Exercise 5.6-1 Find the maximum power that can be delivered to R_L for the circuit of Figure E 5.6-1 using a Thévenin equivalent circuit.
Answer: 9 W when $R_L = 4\ \Omega$

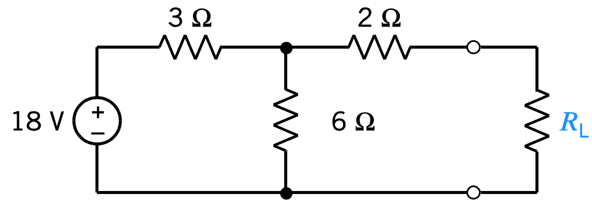
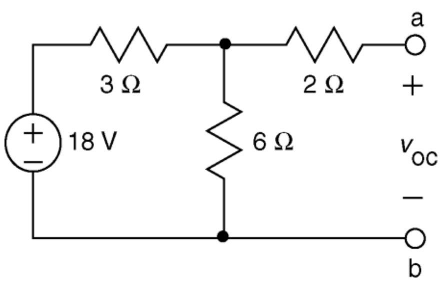
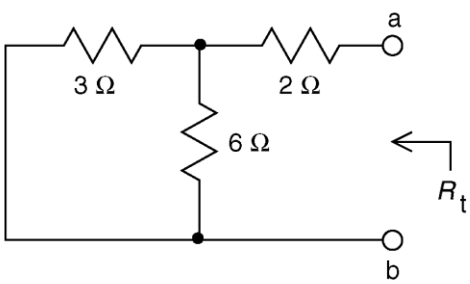


Figure E 5.6-1

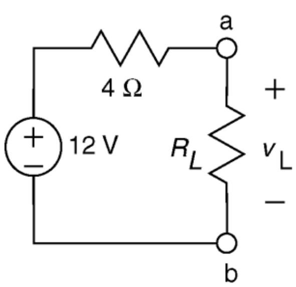
Solution:



$$v_{oc} = \frac{6}{6+3}(18) = 12\ \text{V}$$



$$R_t = 2 + \frac{(3)(6)}{3+6} = 4\ \Omega$$



For maximum power, we require

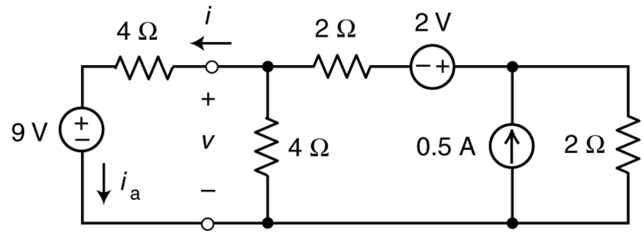
$$R_L = R_t = 4\ \Omega$$

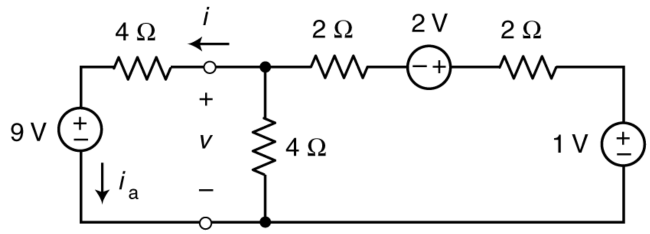
Then

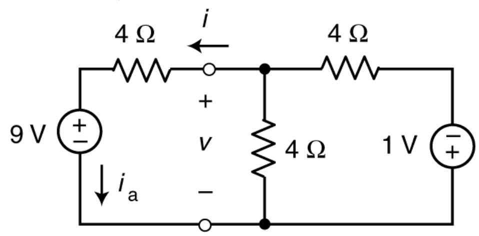
$$p_{\max} = \frac{v_{oc}^{\ 2}}{4\,R_t} = \frac{12^2}{4(4)} = 9\ \text{W}$$

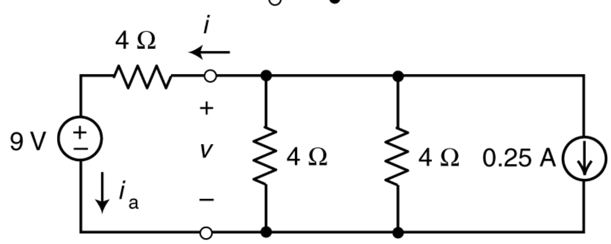
Section 5-2: Source Transformations

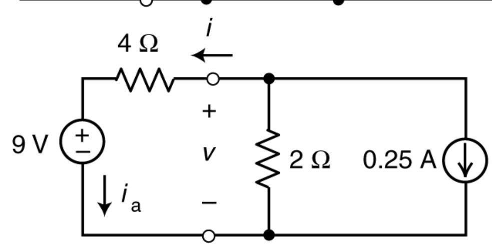
P 5.2-1
Solution:
(a)

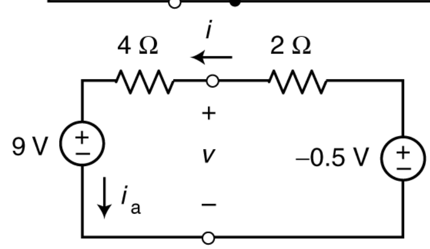












$$\therefore\ R_t = 2\ \Omega$$
$$v_t = -0.5\ \text{V}$$

(b) $$-9 - 4i - 2i + (-0.5) = 0$$

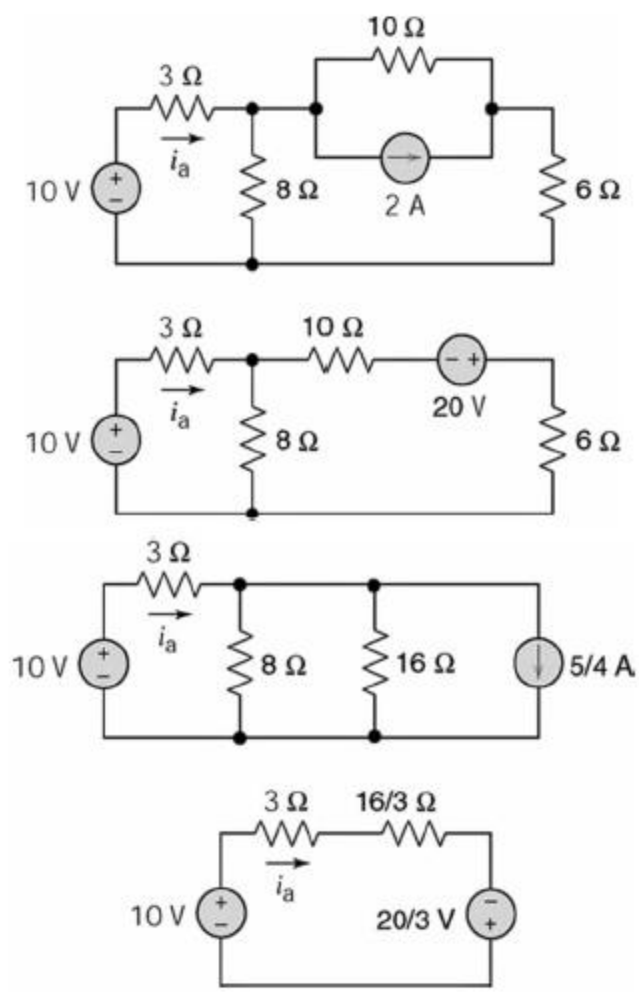
$$i = \frac{-9 + (-0.5)}{4 + 2} = -1.58\ \text{A}$$

$$v = 9 + 4\,i = 9 + 4(-1.58) = 2.67\ \text{V}$$

(c) $i_a = i = -1.58\text{ A}$

(checked using LNAP 8/15/02)

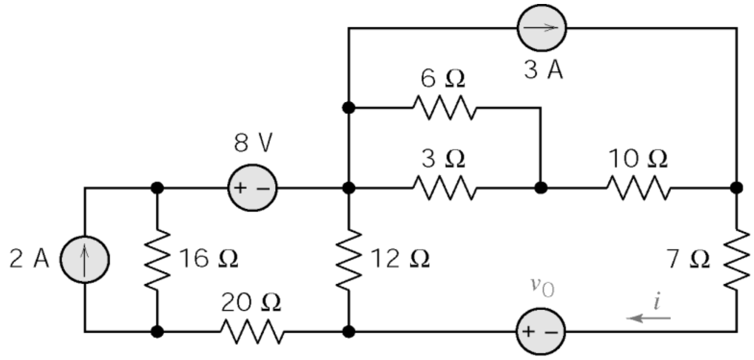
P 5.2-2
Solution:



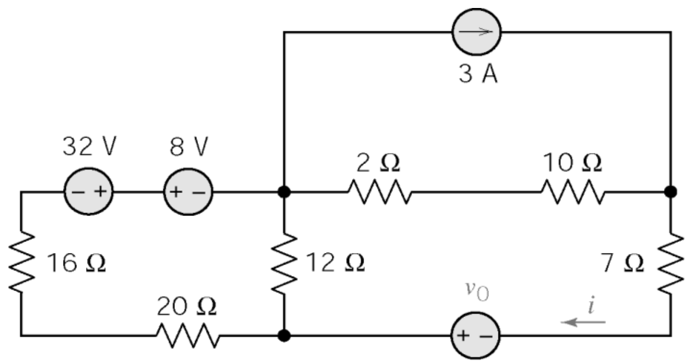
Finally, apply KVL: $-10+3\,i_a+\dfrac{16}{3}\,i_a-\dfrac{20}{3}=0 \quad \therefore \; \underline{i_a = 2\text{A}}$

P 5.2-3

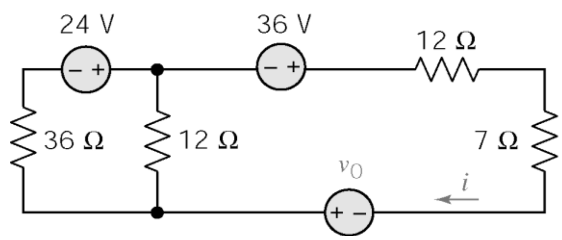
Solution:



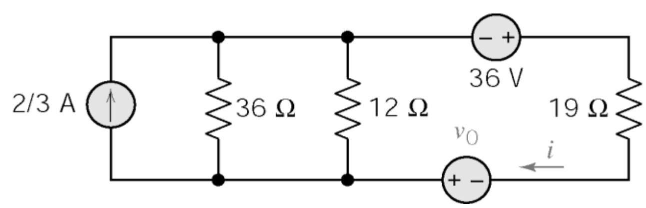
Source transformation at left; equivalent resistor for parallel 6 and 3 Ω resistors:



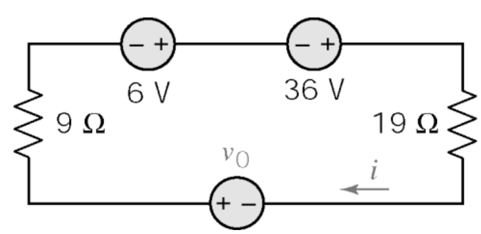
Equivalents for series resistors, series voltage source at left; series resistors, then source transformation at top:



Source transformation at left; series resistors at right:



Parallel resistors, then source transformation at left:

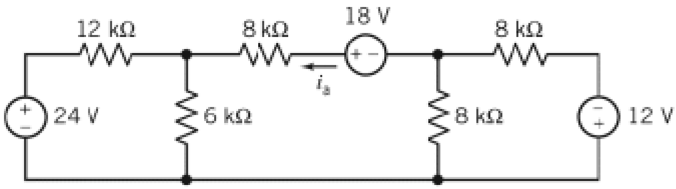


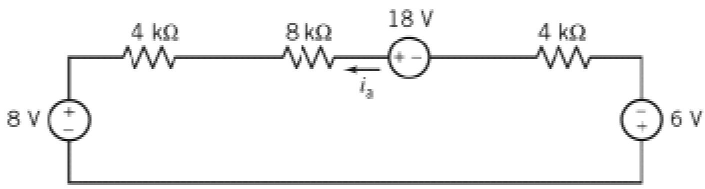
Finally, apply KVL to loop

$$-6+ i\ (9+19)-36-v_o = 0$$
$$i=5/2 \Rightarrow v_o=-42+28\ (5/2)=28\ \text{V}$$

(checked using LNAP 8/15/02)

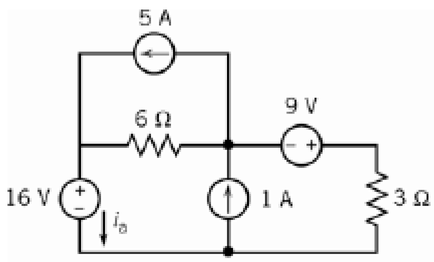
P 5.2-4
Solution:

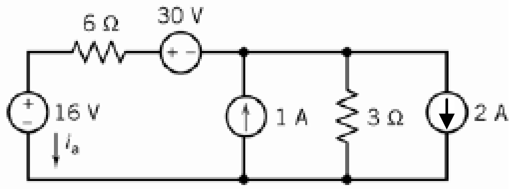


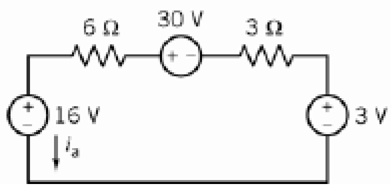


$$-\ 8\ -\ 4000\ i_a -\ 8000\ i_a +18\ -4000\ i_a -6=0$$
$$\therefore\ i_a\ =\ 250\ \mu\text{A}$$

P 5.2-5
Solution:

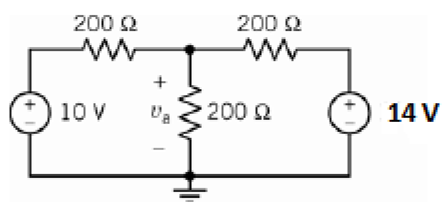




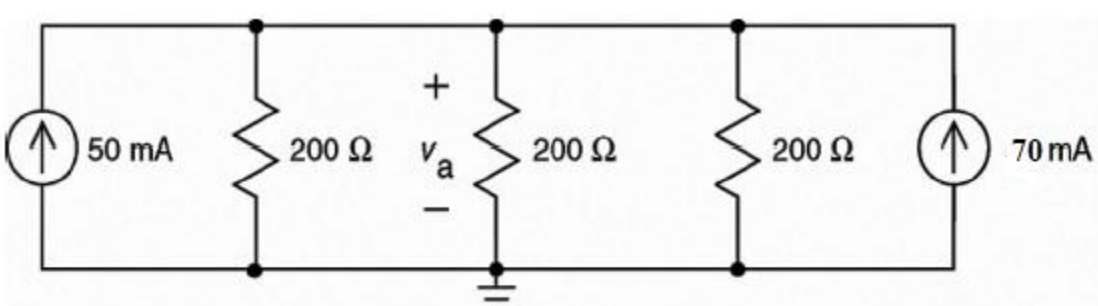


$$-16-6\,i_a \;+\; 30\; -3\,i_a -6 \;=\; 0 \;\Rightarrow\; i_a = 0.89 \text{ A} .$$

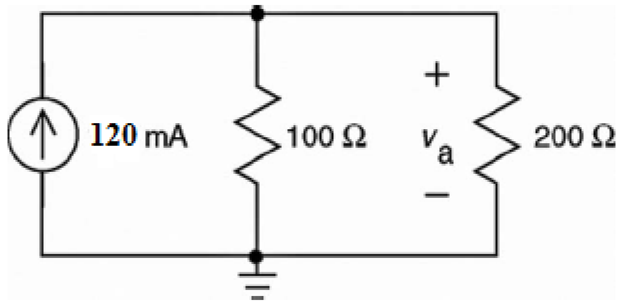
P 5.2-6
Solution:
A source transformation on the right side of the circuit, followed by replacing series resistors with an equivalent resistor:



Source transformations on both the right side and the left side of the circuit:



Replacing parallel resistors with an equivalent resistor and also replacing parallel current sources with an equivalent current source:



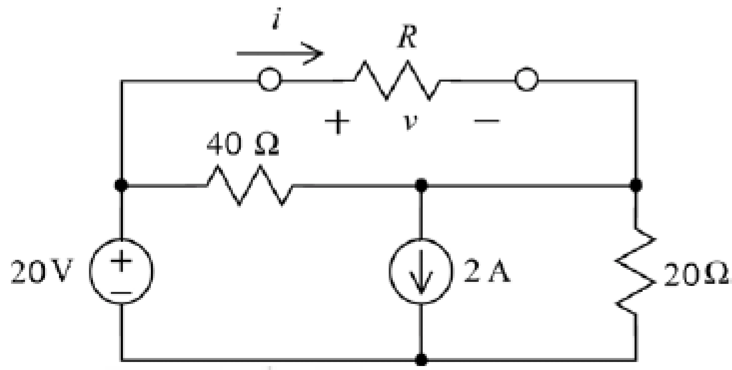
Finally, $$v_a = \frac{100(200)}{100+200}(0.120) = \frac{200}{3}(0.120) = 8 \text{ V}$$

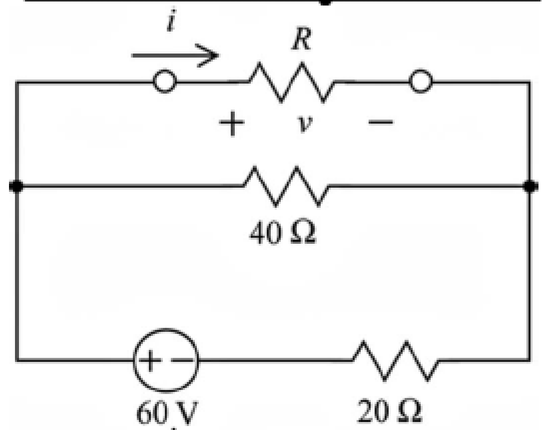
P 5.2-7

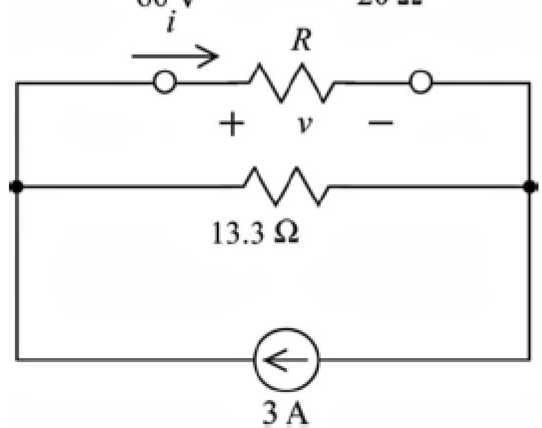
Solution:

Replace series and parallel resistors by an equivalent resistor.

$$30 \| (20+40) = 20\ \Omega$$







Do a source transformation, then replace series voltage sources by an equivalent voltage source.

Do two more source transformations

Now current division gives

$$i = \left(\frac{13.3}{13.3 + R}\right) 3 = \frac{39.9}{13.3 + R}$$

Then Ohm's Law gives

$$v = Ri = \frac{39.9R}{13.3 + R}$$

(a) $\quad i = \dfrac{39.9}{13.3 + 8} = 1.87 \text{ A}$

(b) $\quad v = \dfrac{39.9(16)}{13.3 + 16} = 21.79 \text{ V}$

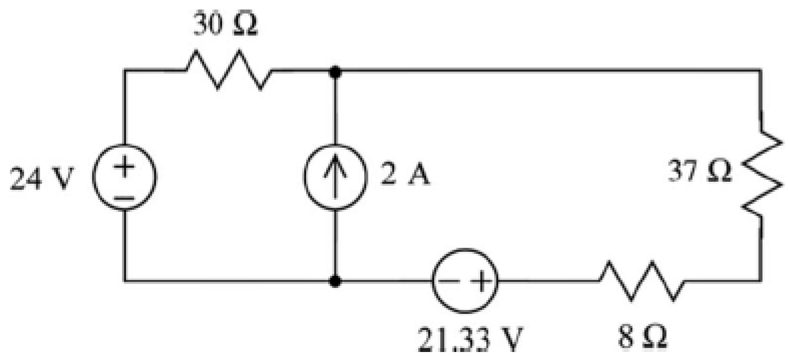
(c) $\quad 2 = \dfrac{39.9}{13.3 + R} \quad \Rightarrow \quad R = 6.65\ \Omega$

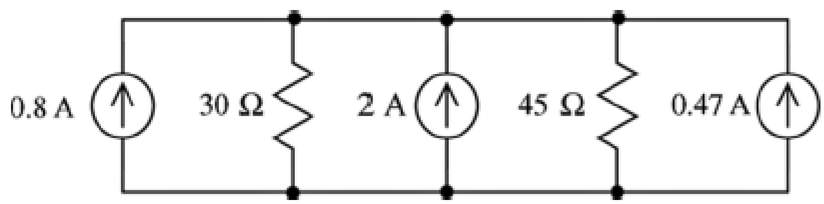
(d) $\quad 32 = \dfrac{39.9R}{13.3 + R} \quad \Rightarrow \quad R = 53.87\ \Omega$

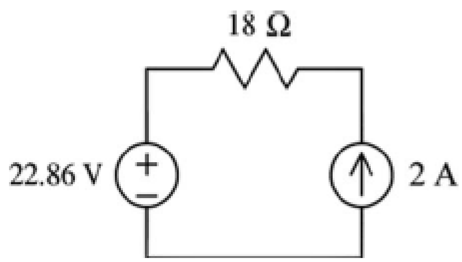
P 5.2-8

Solution:

Use source transformations and equivalent resistances to reduce the circuit as follows







The power supplied by the current source is given by

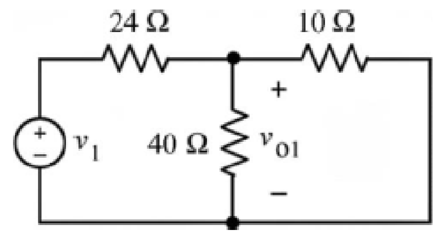
$$p = \left[22.86 + 2(18)\right]2 = 117.72 \text{ W}$$

Section 5-3 Superposition

P5.3-1

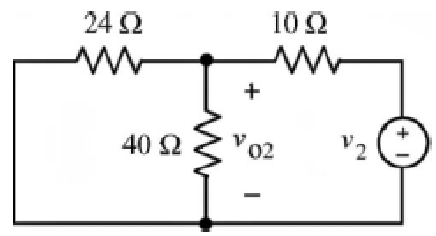
Solution:

Let $v_{o1} = a\, v_1$ be the output when $v_2 = 0$. In this case, the right voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o1} = \frac{40 \| 10}{24 + (40 \| 10)} v_1 = \frac{8}{24+8} v_1 = \frac{1}{4} v_1 \quad \Rightarrow \quad a = \frac{1}{4}$$

Let $v_{o2} = b\, v_2$ be the output when $v_1 = 0$. In this case, the left voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o2} = \frac{40 \| 24}{10 + (40 \| 24)} v_2 = \frac{15}{10+15} v_2 = \frac{15}{25} v_2 = \frac{3}{5} v_2 \quad \Rightarrow \quad b = \frac{3}{5}$$

P5.3-2

Solution:

The output of a linear circuit is a linear combination of the inputs:

$$v_o = a_1 v_1 + a_2 v_2$$

From the first two measurements we have:

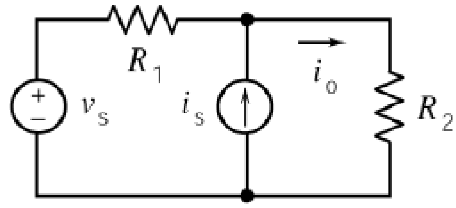
$$\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -1 \end{pmatrix}$$

Now the output of the third measurement can be determine to be

$$v_o = a_1(6) + a_2(6) = \frac{1}{2} \times 6 + (-1)(6) = -3\text{ V}$$

P5.3-3

Solution:



From the 1st fact:

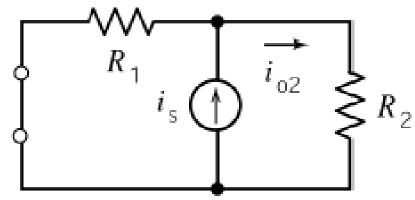
$$0.45 = a(0.25) + b(15)$$

From the 2nd fact:

$$0.30 = a(0.50) + b(0) \quad \Rightarrow \quad a = \frac{0.30}{0.50} = 0.60$$

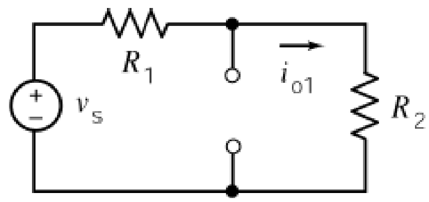
Substituting gives $\quad 0.45 = (0.60)(0.25) + b(15) \quad \Rightarrow \quad b = \dfrac{0.45 - (0.60)(0.25)}{15} = 0.02$

Next, consider the circuit:



$$a\, i_s = i_{o1} = i_o\big|_{v_s=0} = \left(\frac{R_1}{R_1+R_2}\right) i_s$$

so $\quad 0.60 = \dfrac{R_1}{R_1+R_2} \quad \Rightarrow \quad 2R_1 = 3R_2$



and $\quad b\, v_s = i_{o2} = i_o\big|_{i_s=0} = \dfrac{v_s}{R_1+R_2}$

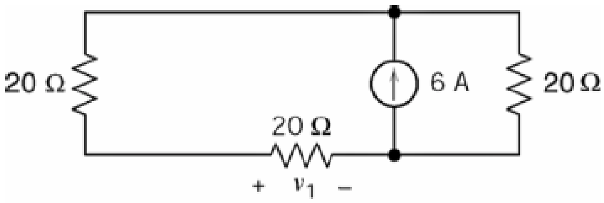
so $\quad 0.02 = \dfrac{1}{R_1+R_2} \quad \Rightarrow \quad R_1 + R_2 = \dfrac{1}{0.02} = 50\ \Omega$

Solving these equations gives $R_1 = 30\ \Omega$ and $R_2 = 20\ \Omega$.

P5.3-4

Solution:

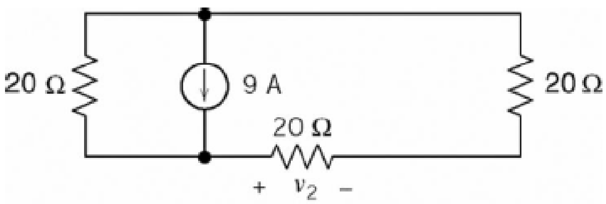
Consider 6 A source only (open 9 A source)



Use current division:

$$\frac{v_1}{20} = 6\left[\frac{20}{20+40}\right] \Rightarrow \underline{v_1 = 40\text{ V}}$$

Consider 9 A source only (open 6 A source)



Use current division:

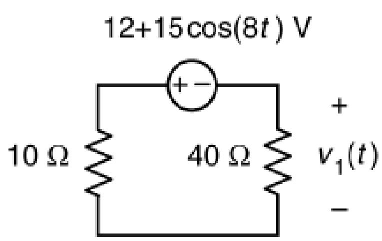
$$\frac{v_2}{20} = 9\left[\frac{20}{20+40}\right] \Rightarrow \underline{v_2 = 60\text{ V}}$$

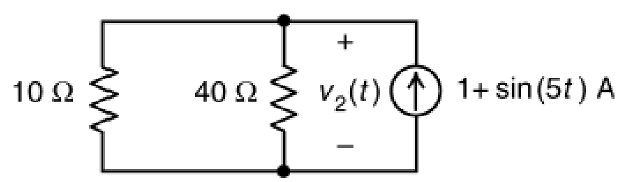
$$\underline{\therefore v = v_1 + v_2 = 40 + 60 = 100\text{ V}}$$

P5.3-5

Solution:

We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.





Using voltage division we calculate

$$v_1(t) = -\frac{40}{10+40}\left(12+15\cos(8t)\right) = -9.6 - 12\cos(8t)$$

Using equivalent resistance we first determine 10||40 = 8 Ω and then calculate

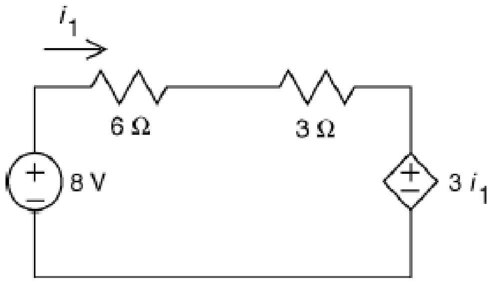
$$v_2(t) = 8\left(1+\sin(5t)\right) = 8 + 8\sin(5t)$$

Using superposition $\quad v(t) = v_1(t) + v_2(t) = -1.6 + 8\sin(5t) - 12\cos(8t)\text{ V}$

P 5.3-6

Solution:

Consider 8 V source only (open the 2 A source)



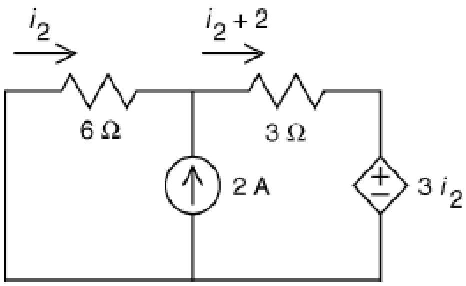
Let i_1 be the part of i_x due to the 8 V voltage source.

Apply KVL to the supermesh:

$$6\left(i_1\right)+3\left(i_1\right)+3\left(i_1\right)-8=0$$

$$i_1=\frac{8}{12}=\frac{2}{3}\text{ A}$$

Consider 2 A source only (short the 8 V source)



Let i_2 be the part of i_x due to the 2 A current source.

Apply KVL to the supermesh:

$$6\left(i_2\right)+3\left(i_2+2\right)+3\,i_2=0$$

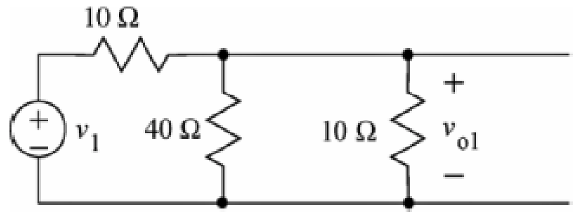
$$i_2=\frac{-6}{12}=-\frac{1}{2}\text{ A}$$

Finally, $\quad i_x=i_1+i_2=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\text{ A}$

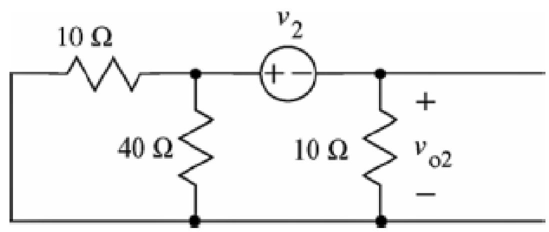
P5.3-7

Solution:

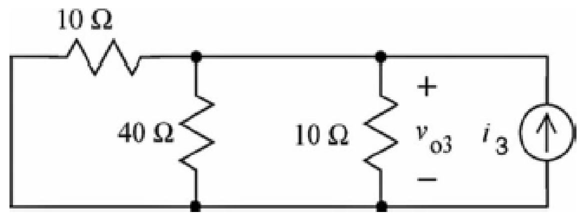
$$v_{o1} = \frac{40 \| 10}{10 + 40 \| 10} v_1 = \frac{4}{9} v_1 \quad \Rightarrow \quad a = \frac{4}{9}$$



$$v_{o2} = -\frac{10}{10 \| 40 + 10} v_2 = -\frac{10}{18} v_2 \quad \Rightarrow \quad b = -\frac{5}{9}$$



$$v_{o3} = (10 \| 10 \| 40) i_3 = \frac{40}{9} i_3 \quad \Rightarrow \quad c = \frac{40}{9}$$



P5.3-8

Solution: Using superposition:

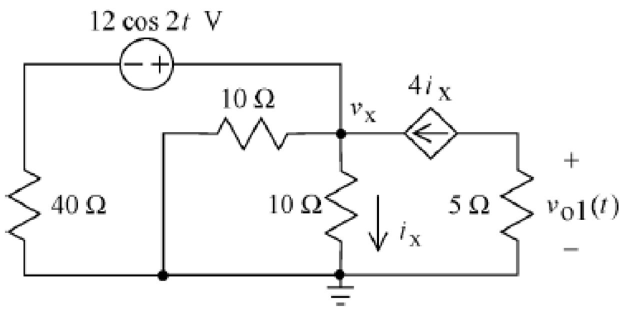
$$v_x = 10\, i_x$$

and

$$\frac{v_x - 12\cos 2t}{40} + \frac{v_x}{10} + \frac{v_x}{10} = 4\, i_x$$

so

$$\frac{10\, i_x - 12\cos 2t}{40} = 2\, i_x \quad \Rightarrow \quad i_x = -\frac{12}{70}\cos 2t$$



Finally,

$$v_{o1} = -5\left(4\, i_x\right) = 3.429\cos 2t \ \text{ V}$$

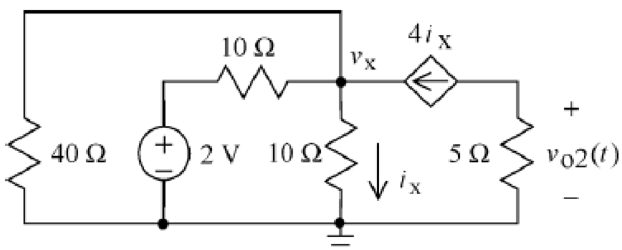
$$v_x = 10\, i_x$$

and

$$\frac{v_x}{40} + \frac{v_x - 2}{10} + \frac{v_x}{10} = 4\, i_x$$

so

$$-0.2 = 1.75\, i_x \quad \Rightarrow \quad i_x = -0.11429 \text{ A}$$



Finally,

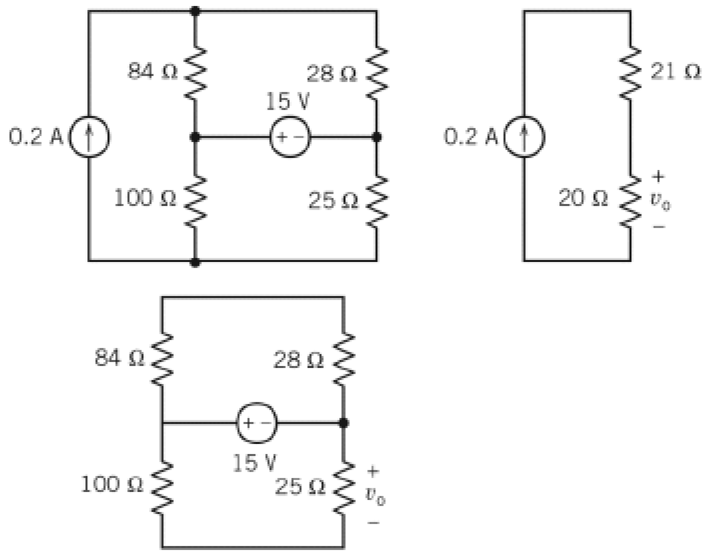
$$v_{o1} = -5\left(4\, i_x\right) = 2.286 \ \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 3.429\cos 2t + 2.286 \ \text{ V}$$

(checked: LNAP 6/22/04)

P 5.3-9

Solution: Using superposition:



$$v_{o1} = 20(0.2) = 4 \text{ V}$$

$$v_{o2} = -\frac{25}{100+25}15 = -3 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 1 \text{ V}$$

P5.3-10

Solution:
Using superposition

$$v_o = -2\left(\frac{R\,\|\,4}{6+(R\,\|\,4)}\right)i_1 + 2\left(\frac{4}{2+(R\,\|\,4)+4}\right)i_2$$

Comparing to $v_o = -0.5\,i_1 + 4$, we require

$$-2\left(\frac{R\,\|\,4}{6+(R\,\|\,4)}\right) = -0.5 \quad \Rightarrow \quad 4(R\,\|\,4) = 6+(R\,\|\,4) \quad \Rightarrow \quad R\,\|\,4 = 2 \quad \Rightarrow \quad R = 4\ \Omega$$

and

$$2\left(\frac{4}{2+(R\,\|\,4)+4}\right)i_2 = 4 \quad \Rightarrow \quad 2\left(\frac{4}{2+(4\,\|\,4)+4}\right)i_2 = 4 \quad \Rightarrow \quad i_2 = 4\text{ A}$$

(checked LNAP 6/12/04)

P5.3-11

Solution:
Use units of mA, kΩ and V.

$$8 + (10\|40) = 16\text{ k}\Omega$$

(a) Using superposition

$$4 = \left(\frac{16}{R+16}\right)14 - \frac{16}{R+16} \quad \Rightarrow \quad 4(R+16) = 208 \quad \Rightarrow \quad R = 36\text{k}\Omega$$

(b) Using superposition again

$$i_a = \left(\frac{10}{10+40}\right)\left[\left(\frac{36}{16+36}\right)14 + \frac{16}{16+36}\right] = \frac{1}{5}\left(\frac{9}{13}\times 14 + \frac{4}{13}\right) = 2\text{ mA}$$

P5.3-12

Solution:

$$i_{o}=\left(-\frac{8}{8+32}\right)\left(\frac{v_{1}}{16+10+(32\parallel 8)}\right)+\left(-\frac{8}{8+32}\right)\left(\frac{16}{16+\left[10+(32\parallel 8)\right]}\right)i_{2}$$
$$+\left(-\frac{16+10}{32+(16+10)}\right)\left(\frac{v_{3}}{8+\left[32\parallel(16+10)\right]}\right)$$

$$i_{o}=\left(-\frac{1}{162}\right)v_{1}+\left(-\frac{8}{81}\right)i_{2}+\left(-\frac{130}{7047}\right)v_{3}$$

So

$$a = 0.006,\ b = \text{-}0.09 \text{ and } c = 0.018$$

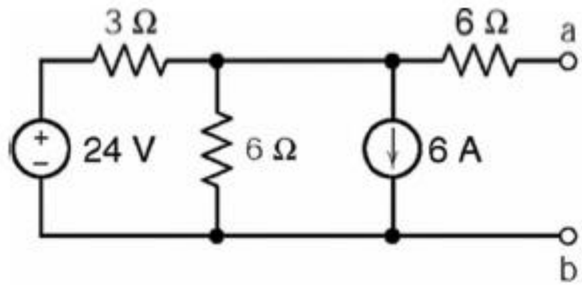
P5.3-13

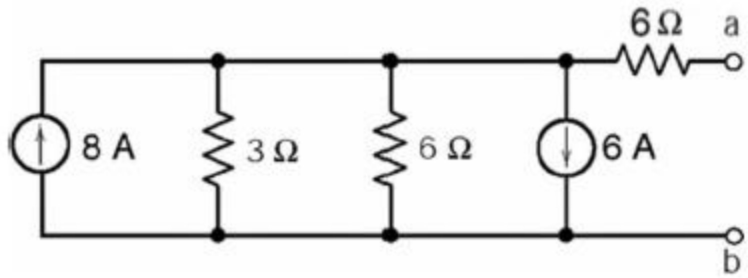
Solution:

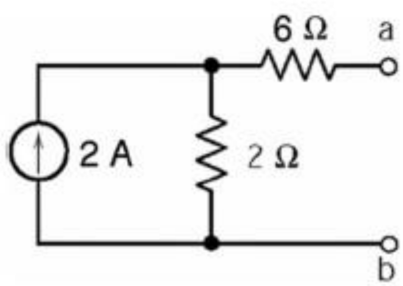
$$i_{m}=\frac{25}{3+2}-\frac{3}{2+3}(5)=5-3=2 \text{ A}$$

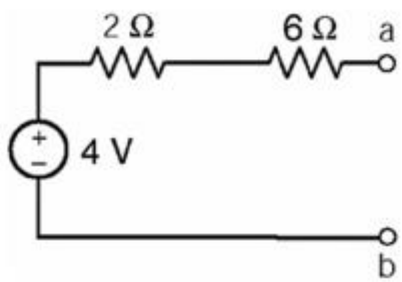
P 5.4-1

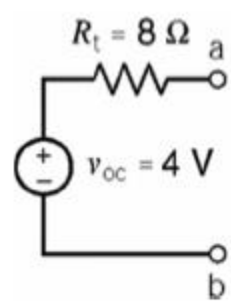
Solution:





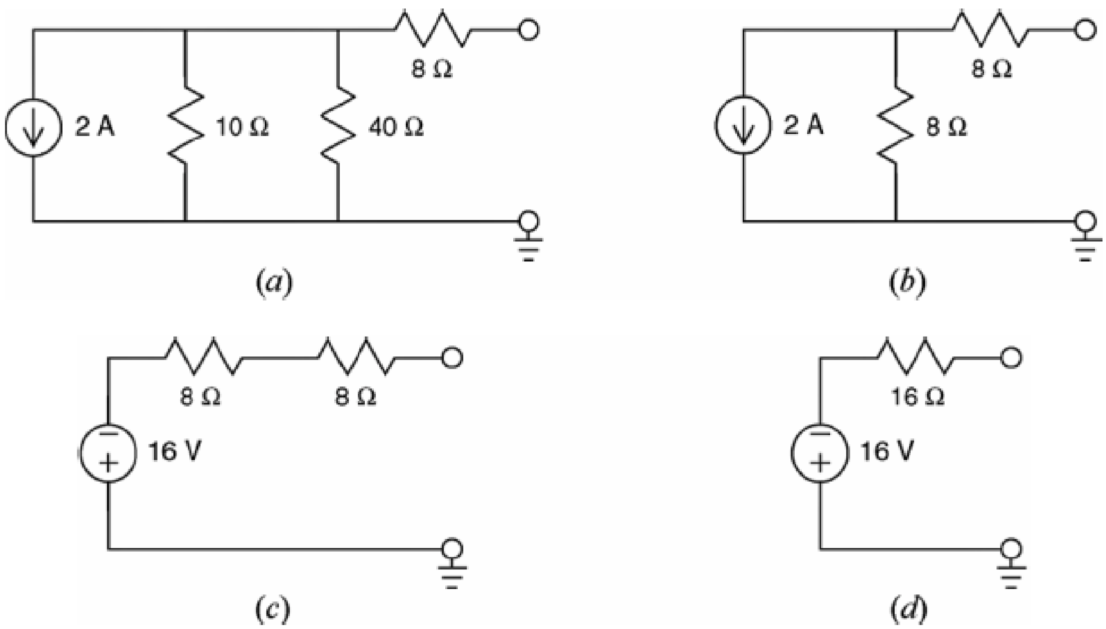






P5.4-2

Solution:
The circuit from Figure P5.4-2*a* can be reduced to its Thevenin equivalent circuit in four steps:

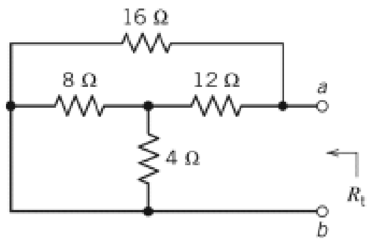


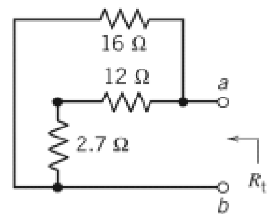
Comparing (*d*) to Figure P5.4-2*b* shows that the Thevenin resistance is $R_t = 16\ \Omega$ and the open circuit voltage, $v_{oc} = -16$ V.

P5.4-3

Solution:

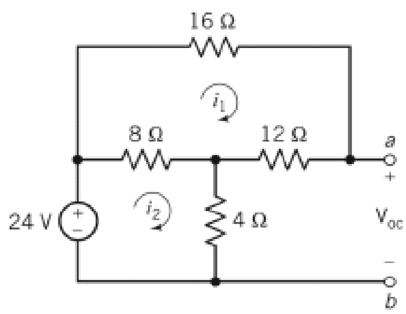
Find R_t:





$$R_t = \frac{16(12+2.7)}{16+(12+2.7)} = 7.7\ \Omega$$

Write mesh equations to find v_{oc}:



Mesh equations:

$$16\,i_1 + 12\,i_1 - 8\left(i_2 - i_1\right) = 0 \qquad (1)$$

$$8\left(i_2 - i_1\right) + 4\,i_2 - 24 = 0 \qquad (2)$$

$$36\,i_1 = 8\,i_2\ \ .\ \ .\ \ . \qquad (1)$$

$$12\,i_2 - 8\,i_1 = 24\ .\ .\ . \qquad (2)$$

$$46\,i_1 = 24 \quad \Rightarrow \quad i_1 = \frac{12}{23}\ \text{A}$$

$$i_2 = \frac{9}{3}\left(\frac{12}{23}\right) = \frac{54}{23}\ \text{A}$$

Finally, $$v_{oc} = 4\,i_2 + 12\,i_1 = 4\left(\frac{54}{23}\right) + 12\left(\frac{12}{23}\right) = 15.65\ \text{V}$$

P5.4-4

Solution:

Find v_{oc}:

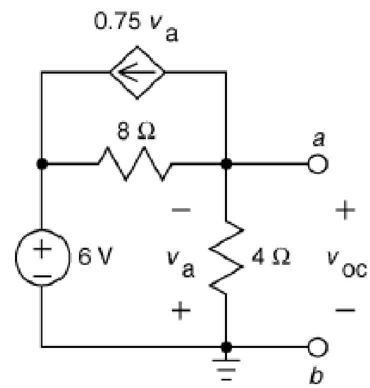
Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = -v_{oc}$$

Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad\Rightarrow\quad v_{oc}=-2\text{ V}$$



Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces

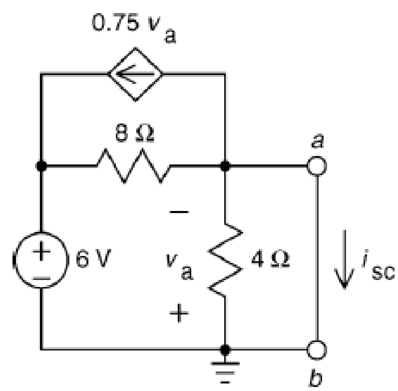
$$v_a = 0$$

Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{6}{8}=\frac{3}{4}\text{ A}$$

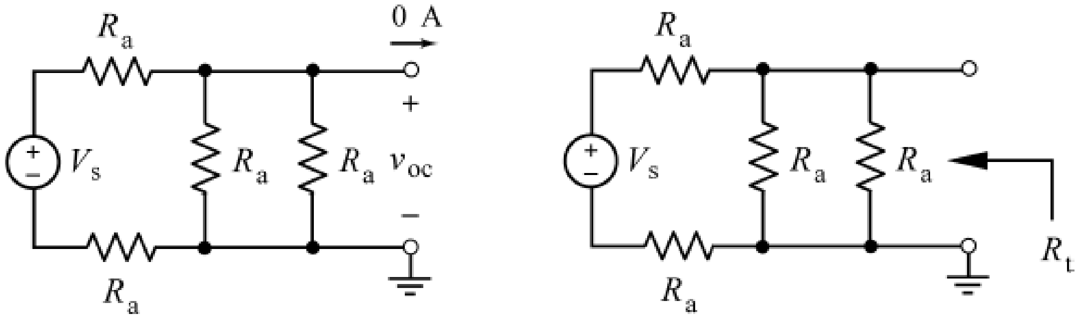
$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\,\Omega$$



(checked using LNAP 8/15/02)

P5.4-5

Solution: a.) From

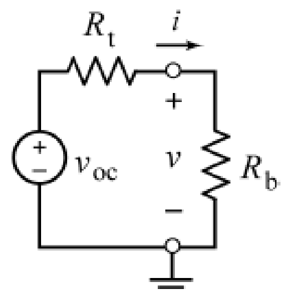


We see that $v_{oc} = \frac{V_s}{5}$ and $R_t = \frac{2}{5} R_a$. With the given values of v_{oc} and R_t we calculate

$$15 = \frac{V_s}{5} \quad \Rightarrow \quad V_s = 75 \text{ V} \quad \text{and } 60 = \frac{2}{5} R_a \quad \Rightarrow \quad R_a = 150\ \Omega .$$

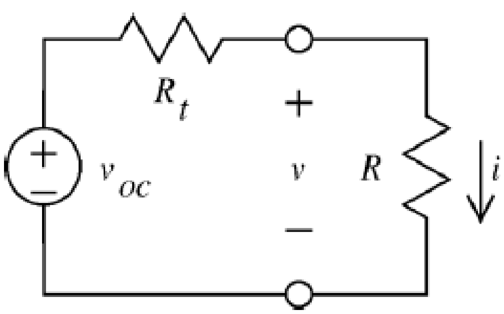
b.) $$i = \frac{v_{oc}}{R_t + R_b} \quad \Rightarrow \quad 0.2 = \frac{15}{60 + R_b} \quad \Rightarrow \quad R_b = 15\ \Omega$$

c.) $$v = \frac{R_b}{R_t + R_b} v_{oc} \quad \Rightarrow \quad 12 = \frac{15 R_b}{60 + R_b} \quad \Rightarrow \quad R_b = 240\ \Omega$$



P5.4-6

Solution:



$$v = \frac{R}{R_t + R} v_{oc}$$

From the given data:

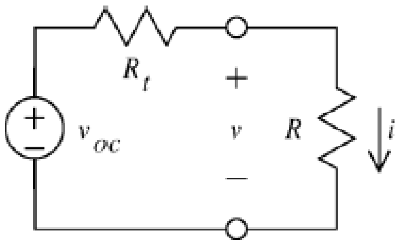
$$\left. \begin{aligned} 9 &= \frac{3000}{R_t + 3000} v_{oc} \\ 2 &= \frac{8000}{R_t + 8000} v_{oc} \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} v_{oc} &= 3 \text{ V} \\ R_t &= -2000\ \Omega \end{aligned} \right.$$

When $R = 15000\ \Omega$,

$$v = \frac{15000}{-2000 + 15000}(3) = 3.46 \text{ V}$$

P5.4-7

Solution:



$$i = \frac{v_{oc}}{R_t + R}$$

From the given data:

$$\left.\begin{aligned} 0.004 &= \frac{v_{oc}}{R_t + 2000} \\ 0.003 &= \frac{v_{oc}}{R_t + 4000} \end{aligned}\right\} \Rightarrow \begin{cases} v_{oc} = 24 \text{ V} \\ R_t = 4000\ \Omega \end{cases}$$

(a) When $i = 0.002$ A:

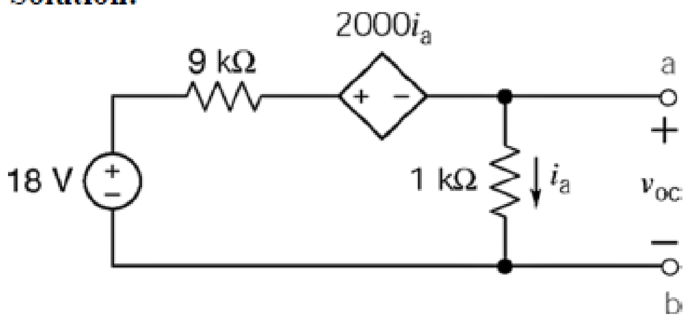
$$0.002 = \frac{24}{4000 + R} \quad \Rightarrow \quad R = 8000\ \Omega$$

(b) Maximum i occurs when R = 0:

$$\frac{24}{4000} = 0.006 = 6 \text{ mA} \quad \Rightarrow \quad i \le 6 \text{ mA}$$

P5.4-8

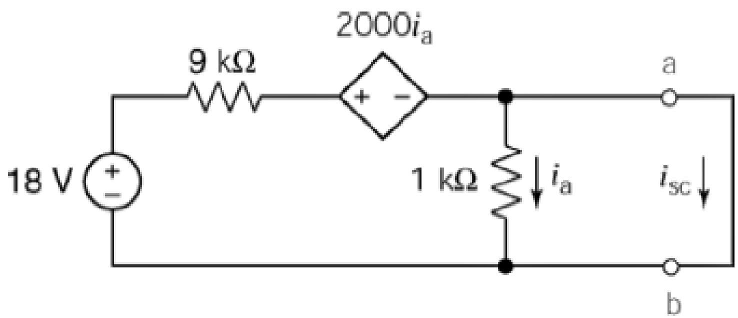
Solution:



$$-18+9000i_a+2000i_a+1000i_a=0$$

$$i_a=3/2000\text{ A}$$

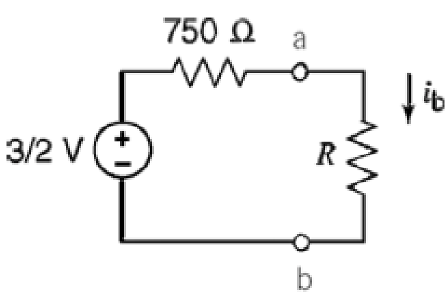
$$v_{oc}=1000i_a=\frac{3}{2}\text{V}$$



$i_a = 0$ due to the short circuit

$$-18+9000i_{sc}=0\Rightarrow i_{sc}=2\text{ mA}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{\frac{3}{2}}{0.002}=750\ \Omega$$



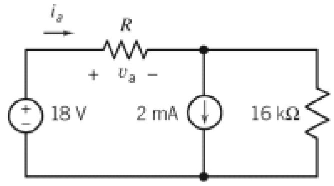
$$i_b=\frac{\frac{3}{2}}{750+R}$$

$i_b = 0.002$ A requires

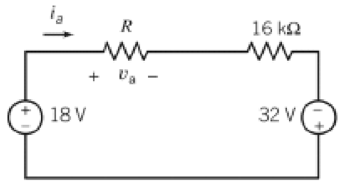
$$R=\frac{\frac{3}{2}}{0.002}-750=0$$

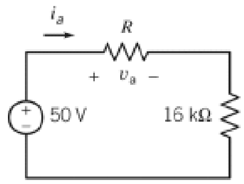
P5.4-9

Solution: Replace the part of the circuit that is connected to the variable resistor by its Thevenin equivalent circuit:



$$24\ \text{k}\Omega \,\|\, (18\ \text{k}\Omega + 30\ \text{k}\Omega) = 24\ \text{k}\Omega \,\|\, 48\ \text{k}\Omega = 16\ \text{k}\Omega$$





$$i_a = \frac{50}{R + 16000} \quad \text{and} \quad v_a = \frac{R}{R+16000} 50$$

$$p = i_a\, v_a = \left(\frac{50}{R+16000}\right)^2 R$$

(a) $i_a = \dfrac{50}{0+16000} = 3.125\ \text{mA}$ when $R = 0\ \Omega$ (a short circuit).

(b) $v_a = \dfrac{150000}{100000+16000} 50 = 45.18$ V when R is as large as possible, i.e. $R = 150$ kΩ.

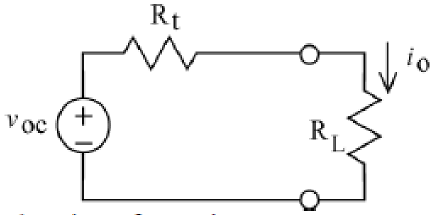
(c) Maximum power is delivered to the adjustable resistor when $R = R_t = 16\ \text{k}\Omega$. Then

$$p = i_a\, v_a = \left(\frac{50}{16000+16000}\right)^2 16000 = 0.039 = 39\ \text{mW}$$

P5.4-10

Solution:

Replace the source by it's Thevenin equivalent circuit to get



$$i_o = \frac{v_{oc}}{R_t + R_L}$$

Using the given formation

$$\left.\begin{aligned} 0.375 &= \frac{v_{oc}}{R_t + 4} \\ 0.300 &= \frac{v_{oc}}{R_t + 8} \end{aligned}\right\} \quad \Rightarrow \quad 0.375(R_t + 4) = 0.300(R_t + 8)$$

So

$$R_t = \frac{(0.300)8 - (0.375)4}{0.075} = 12\ \Omega \text{ and } v_{oc} = 0.3(12+8) = 6\ \text{V}$$

(a) When $R_L = 10\ \Omega,\ i_o = \dfrac{6}{12+10} = 0.27\overline{27}$ A.

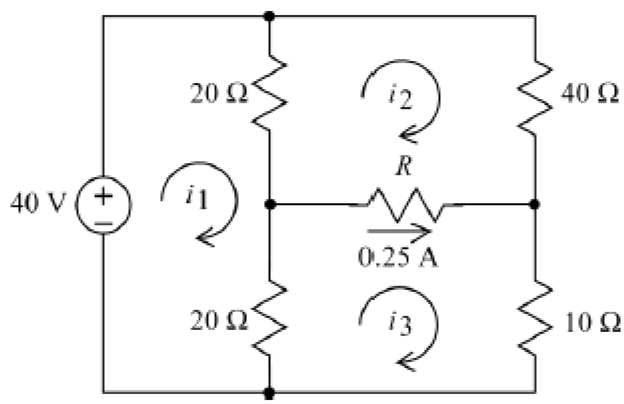
(b) $12\ \Omega = R_t = 48\ 11R \quad \Rightarrow \quad R = 16\ \Omega$.

(checked: LNAP 5/24/04)

P 5.4-11

Solution:

(a)



$$i_3 - i_2 = 0.25\ \text{A}$$

Apply KVL to mesh 1 to get

$$20(i_1 - i_2) + 20(i_1 - i_3) - 40 = 0$$

Apply KVL to the supermesh corresponding to the unspecified resistance to get

$$40i_2 + 10i_3 - 20(i_1 - i_3) - 20(i_1 - i_2) = 0$$

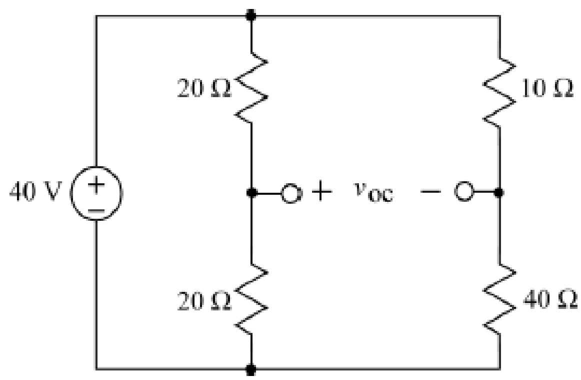
Solving, for example using MATLAB, gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 40 & -20 & -20 \\ -40 & 60 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 40 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 0.750 \\ 1.000 \end{bmatrix}$$

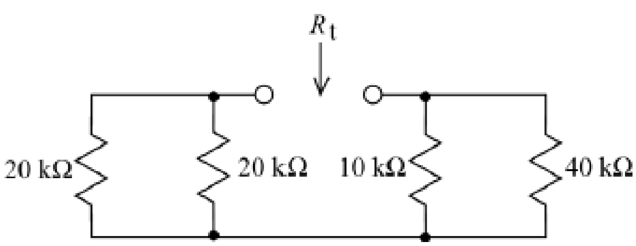
Apply KVL to mesh 2 to get

$$40i_2 + R(i_2 - i_3) - 20(i_1 - i_2) = 0 \quad \Rightarrow \quad R = \frac{20(i_1 - i_2) - 40i_2}{i_2 - i_3} = 30\ \Omega$$

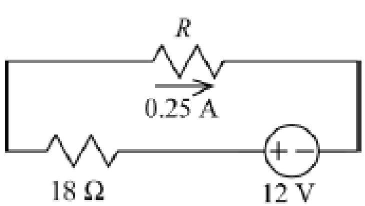
(b)



$$v_{oc} = \left(\frac{20}{20+20}\right)40 - \left(\frac{40}{10+40}\right)40 = -12\ \text{V}$$



$$R_t = 18\ \Omega$$

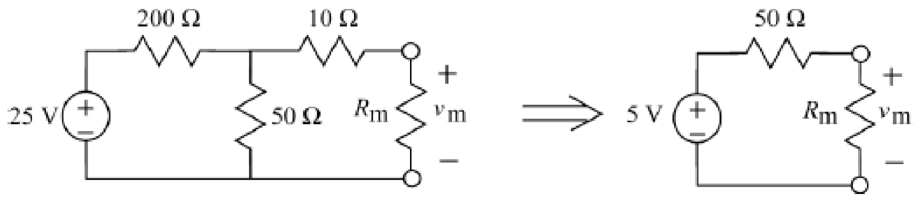


$$0.25 = \frac{12}{18+R} \quad \Rightarrow \quad R = 30\ \Omega$$

(checked: LNAP 5/25/04)

P 5.4-12

Solution: Replace the circuit by its Thevenin equivalent circuit:



$$v_m = \left(\frac{R_m}{R_m + 50}\right)5$$

(a)

$$v_{mi} = \lim_{R_m \to \infty} v_m = 5 \text{ V}$$

(b) When $R_m = 1000\ \Omega$, $v_m = 4.763$ V so

$$\%\ \text{error} = \frac{5 - 4.762}{5} \times 100 = 4.76\%$$

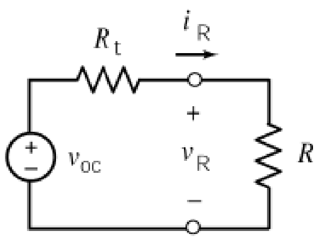
(c)

$$0.02 \ge \frac{5 - \left(\frac{R_m}{R_m + 50}\right)5}{5} \quad \Rightarrow \quad \frac{R_m}{R_m + 50} \ge 0.98 \quad \Rightarrow \quad R_m \ge 2450\ \Omega$$

(checked: LNAP 6/16/04)

P 5.4-13

Solution:
We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:



Using voltage division $v_R = \dfrac{R}{R+R_t} v_{oc}$ and using Ohm's law

$i_R = \dfrac{v_{oc}}{R+R_t}$.

By inspection , $v_R = \dfrac{R}{R+R_t} v_{oc} = \dfrac{v_{oc}}{1+\dfrac{R_t}{R}}$ will be maximum when $R = \infty$. The maximum value of v_R will be v_{oc}. Similarly,

$i_R = \dfrac{v_{oc}}{R+R_t}$ will be maximum when $R = 0$. The maximum value of i_R will be $\dfrac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells us that $p_R = i_R v_R$ will be maximum when $R = R_t$.

Then $p_R = i_R v_R = \left(\dfrac{v_{oc}}{R+R_t}\right)\left(\dfrac{R}{R+R_t} v_{oc}\right) = R\left(\dfrac{v_{oc}}{R+R_t}\right)^2$.

Let's substitute the given data into the equation $i_R = \dfrac{v_{oc}}{R+R_t}$.

When $R = 2\ \Omega$ we get $2 = \dfrac{v_{oc}}{2+R_t} \quad \Rightarrow \quad 4 + 2R_t = v_{oc}$. When $R = 6\ \Omega$ we get

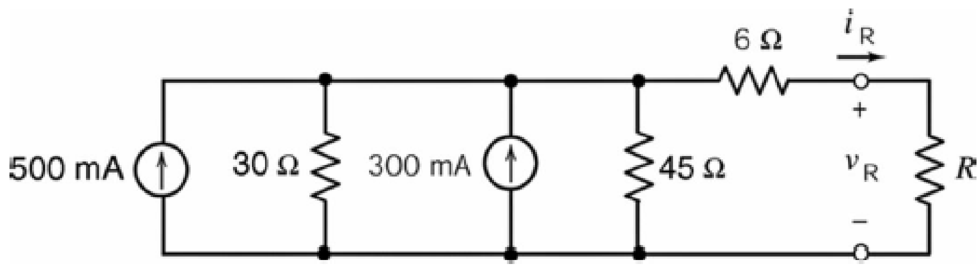
$1 = \dfrac{v_{oc}}{6+R_t} \quad \Rightarrow \quad 6 + R_t = v_{oc}$.

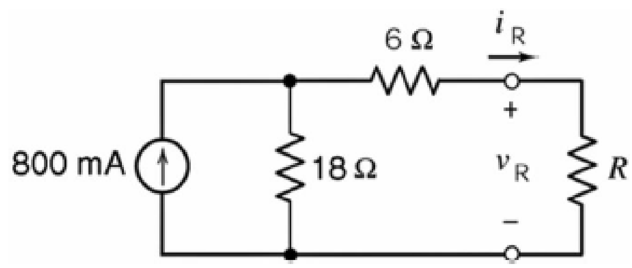
So $6 + R_t = 4 + 2R_t \quad \Rightarrow \quad R_t = 2\ \Omega$ and $v_{oc} = 4 + 2R_t = 8\ \text{V}$. Also $i_{sc} = \dfrac{v_{oc}}{R_t} = \dfrac{8}{2} = 4$ A.

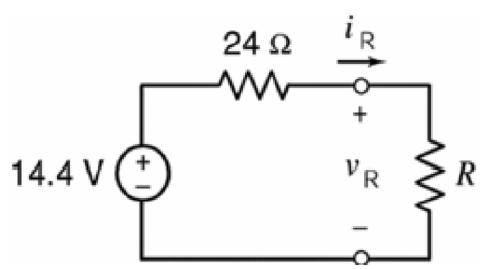
P 5.4-14

Solution:

Reduce this circuit using source transformations and equivalent resistance:







Now $v_R = \left(\dfrac{R}{R+24}\right)14.4$ and $i_R = \dfrac{14.4}{R+24}$ so the questions can be easily answered:

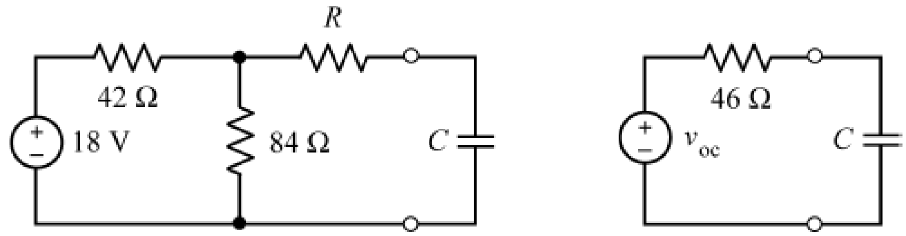
a) When $R = 9\ \Omega$ then $v_R = 3.93$ V.

b) When $R = 27\ \Omega$ then $v_R = 7.62$ V.

c) When $R = 12\ \Omega$ then $i_R = 400$ mA.

P 5.4-15

Solution



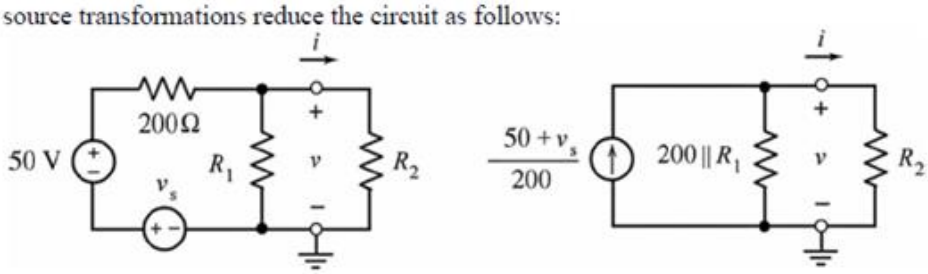
$$46 = R_t = R + (42 \| 84) = R + 28 \quad \Rightarrow \quad R = 18\ \Omega$$

$$v_{oc} = \frac{84}{42+84}(18) = 12\ \text{V}$$

Section 5-5: Norton's Theorem

P5.5-1

Solution:
Two source transformations reduce the circuit as follows:



Recognizing the parameters of the Norton equivalent circuit gives:

$$0.5 = i_{sc} = \frac{50+v_s}{200} \;\Rightarrow\; v_s = 50 \text{ V and } 20 = R_t = 200 \parallel R_1 = \frac{200\,R_1}{200+R_1} \;\Rightarrow\; R_1 = \mathbf{22.2\ \Omega}$$

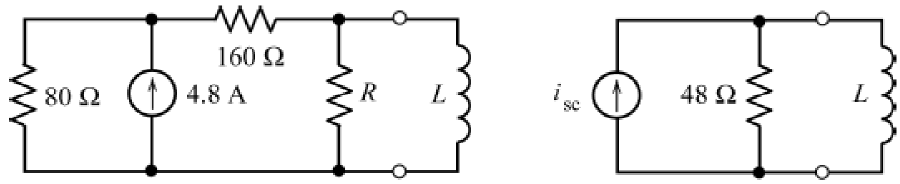
Next, the voltage across resistor R_2 is given by $v = i_{sc}\left(R_t \parallel R_2\right) = \dfrac{R_t\,R_2\,i_{sc}}{R_t+R_2} = \dfrac{R_t\,i_{sc}}{\dfrac{R_t}{R_2}+1}$ so this voltage is maximum when $R_2 = \infty$ and max $v = R_t\,i_{sc}$ =10 V. The power $p = vi$ will be maximum when $R_2 = R_t = 20\ \Omega$. Then $v = \dfrac{R_t\,i_{sc}}{2} = \dfrac{20(0.5)}{2} = 5\text{ V}$, $i = \dfrac{v}{R_2} = \dfrac{5}{20} = 0.25\text{ A}$ and $p = vi = 5(0.25) = 1.25\text{ W}$.

P 5.5-2

Solution:
When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current, and dissipate no power. Then Box A will warm up and Box B will cool off.

P5.5-3

Solution:

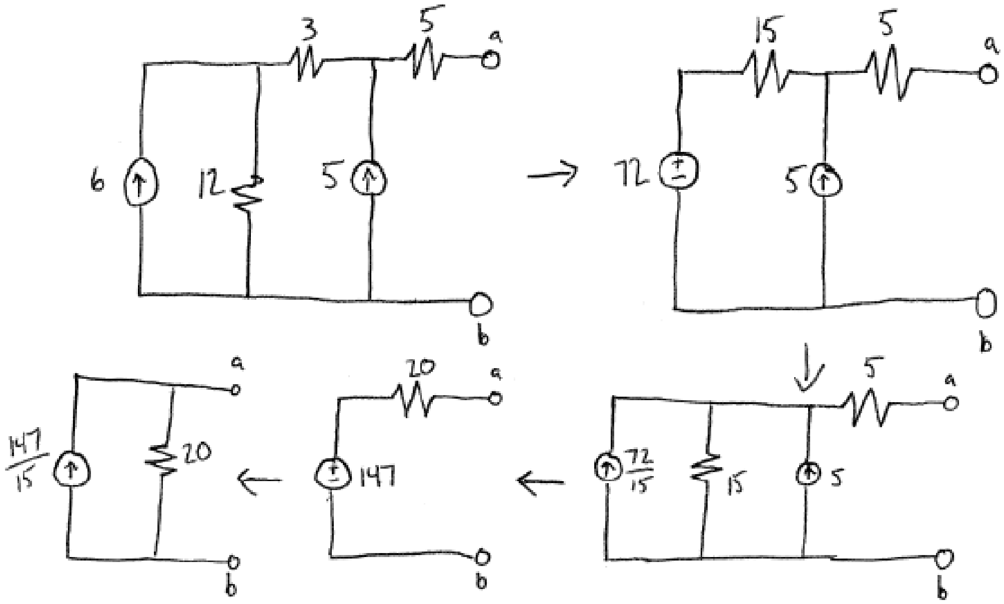


$$48 = R_t = R \,||\, (80+160) = \frac{240\,R}{R+240} \quad \Rightarrow \quad R = 60\ \Omega$$

$$i_{sc} = \frac{80}{80+160}(4.8) = 1.6\ \text{A}$$

P 5.5-4

Solution:



P 5.5-5

Solution:

To determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 4.6-5a after adding the short circuit and labeling the short circuit current. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (a), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The voltage at node 3 is equal to the voltage across a short, $v_3 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across a short, i.e. $v_3 = 0$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 = 3\,v_a \quad \Rightarrow \quad v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}v_2 = i_{sc} \quad \Rightarrow \quad \frac{9}{6}v_a = i_{sc} \quad \Rightarrow \quad i_{sc} = \frac{9}{6}(-16) = -24 \text{ A}$$

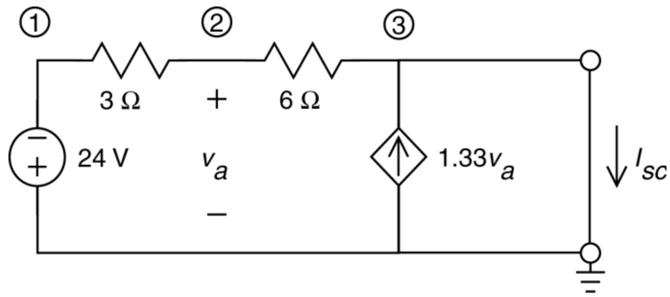


Figure (a) Calculating the short circuit current, I_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 24 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source circuit across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (b), node voltage v_1 is equal to the across a short circuit, i.e. $v_1 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across the current source, i.e. $v_3 = v_T$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}v_2 + i_T = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 + 6\,i_T = 0$$

$$\Rightarrow \quad 9\,v_a - v_T + 6\,i_T = 0$$

$$\Rightarrow \quad 3\,v_T - v_T + 6\,i_T = 0 \quad \Rightarrow \quad 2\,v_T = -6\,i_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = -3\,\Omega$$

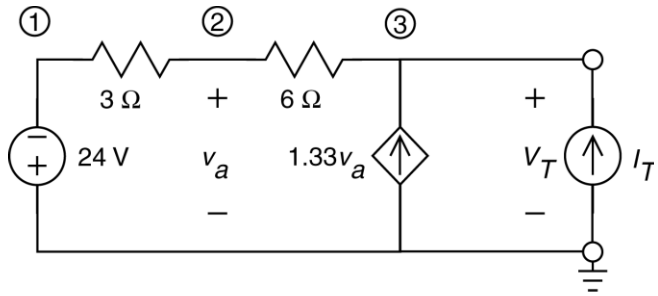


Figure (b) Calculating the Thevenin resistance, $R_{th} = \dfrac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure P 4.6-5a after adding the open circuit and labeling the open circuit voltage. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (c), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the open circuit voltage, i.e. $v_3 = v_{oc}$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 + v_{oc} = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}\,v_2 = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 = 0 \quad \Rightarrow \quad 9\,v_a = v_{oc}$$

Combining these equations gives

$$3\left(-48 + v_{oc}\right) = 9\,v_a = v_{oc} \quad \Rightarrow \quad v_{oc} = 72 \text{ V}$$

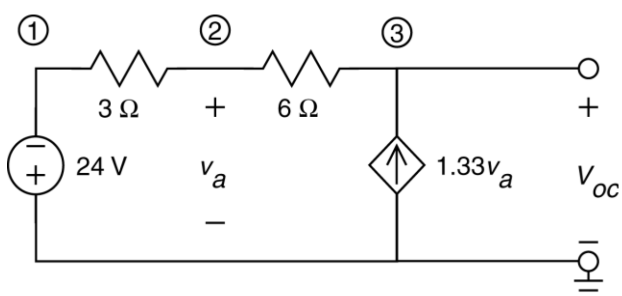


Figure (c) Calculating the open circuit voltage, v_{oc}, using node equations.

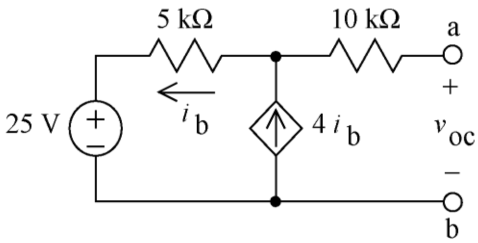
As a check, notice that

$$R_{th}\,I_{sc} = (-3)(-24) = 72 = V_{oc}$$

(checked using LNAP 8/16/02)

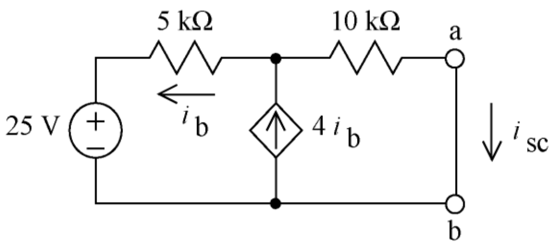
P 5.5-6

Solution: (a) Replace the part of the circuit that is connected to the left of terminals a-b by its Norton equivalent circuit:



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A . Then

$$v_{oc} = 25 + 5000(i_b) = 25 \text{ V}$$

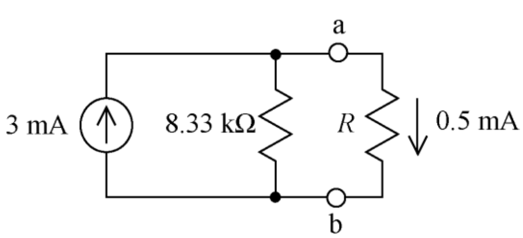


Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_b + 10000(3\, i_b) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$



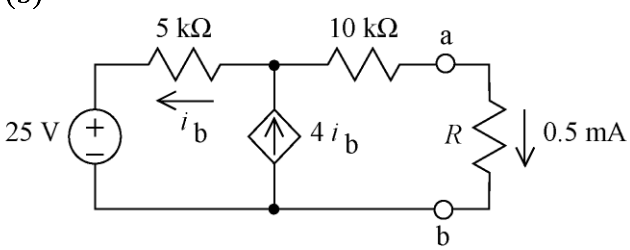
Then

$$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\bar{3} \text{ k}\Omega$$

Current division gives

$$0.5 = \frac{8333}{R + 8333} 3 \quad \Rightarrow \quad R = 41.67 \text{ k}\Omega$$

(b)



Notice that i_b and 0.5 mA are the mesh currents. Apply KCL at the top node of the dependent source to get

$$i_b + 0.5 \times 10^{-3} = 4 i_b \quad \Rightarrow \quad i_b = \frac{1}{6} \text{ mA}$$

Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_b + (10000 + R)(0.5 \times 10^{-3}) - 25 = 0$$

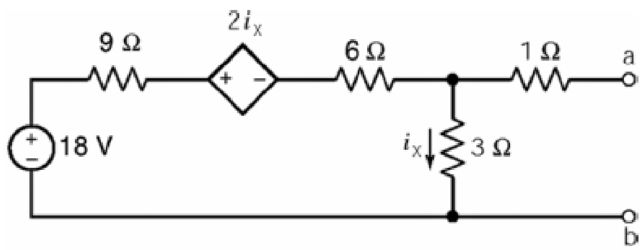
$$-5000\left(\frac{1}{6} \times 10^{-3}\right) + (10000 + R)(0.5 \times 10^{-3}) = 25$$

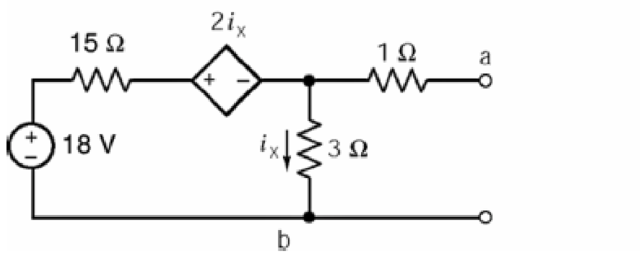
$$R = \frac{\frac{125}{6}}{0.5 \times 10^{-3}} = 41.67 \text{ k}\Omega$$

P 5.5-7

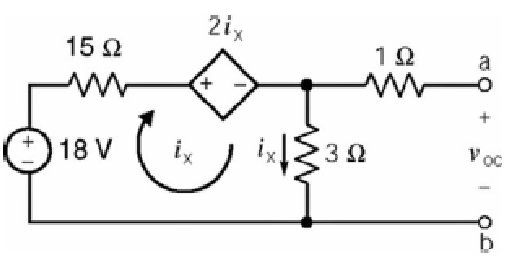
Solution

Simplify the circuit using a source transformation:





Identify the open circuit voltage and short circuit current.

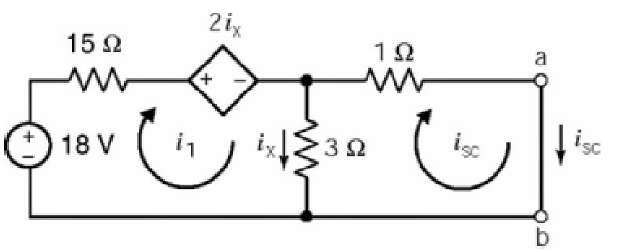


Apply KVL to the mesh to get:

$$(15+2+3)i_x - 18 = 0 \quad \Rightarrow \quad i_x = 0.9 \text{ A}$$

Then

$$v_{oc} = 3i_x = 2.7 \text{ V}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$15i_1 + 2(i_1 - i_{sc}) + 3(i_1 - i_{sc}) - 18 = 0 \quad \Rightarrow \quad 20i_1 - 5i_{sc} = 18$$

and

$$i_{sc} - 3(i_1 - i_{sc}) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3} i_{sc}$$

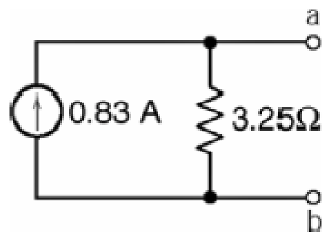
so

$$20\left(\frac{4}{3}i_{sc}\right)-5i_{sc}=18 \quad \Rightarrow \quad i_{sc}=0.83\text{ A}$$

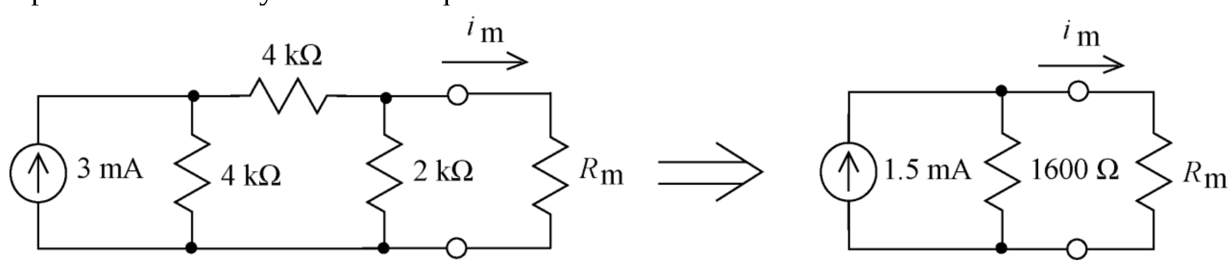
The Thevenin resistance is

$$R_t=\frac{2.7}{0.83}=3.25\ \Omega$$

Finally, the Norton equivalent circuit is



P 5.5-8
Solution:
Replace the circuit by its Norton equivalent circuit:



$$i_m=\left(\frac{1600}{1600+R_m}\right)\left(1.5\times10^{-3}\right)$$

(a)

$$i_{mi}=\lim_{R_m\to 0} i_m=1.5\text{ mA}$$

(b) When R_m = 20 Ω then $i_m=1.48$ mA so

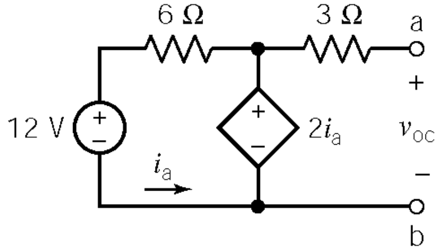
$$\%\text{ error}=\frac{1.5-1.48}{1.5}\times100=1.23\%$$

(c)

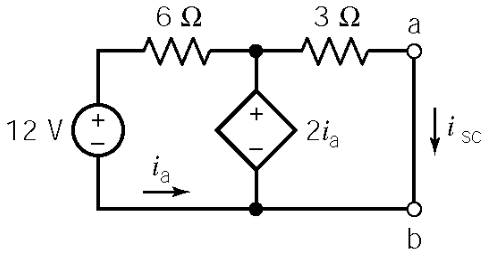
$$0.02\ge\frac{0.015-\left(\frac{1600}{1600+R_m}\right)(0.015)}{0.015} \quad \Rightarrow \quad \frac{1600}{1600+R_m}\ge 0.98 \quad \Rightarrow \quad R_m\le 32.65\ \Omega$$

(checked: LNAP 6/18/04)

P 5.5-9
Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$
$$v_{oc} = 2\,i_a = -6 \text{ V}$$



$$12 + 6i_a = 2i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$
$$3i_{sc} = 2i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

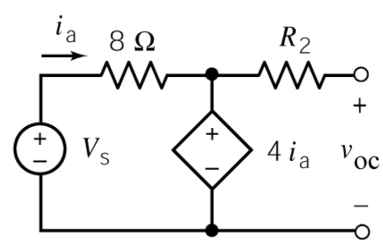
$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Section 5-6: Maximum Power Transfer

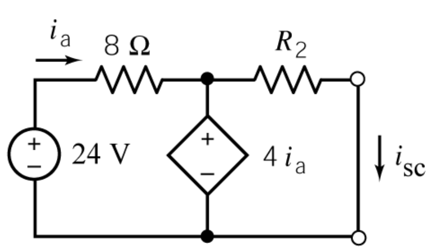
P 5.6-1

Solution:

(a) The value of the current in R_2 is 0 A so $v_{oc} = 4\,i_a$. Then KVL gives



$$8i_a + 4i_a - V_s = 0 \quad \Rightarrow \quad V_s = 12i_a = 3(4i_a) = 3(v_{oc}) = 24 \text{ V}$$



Next, KVL gives

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

and

$$4i_a = R_2\,i_{sc} \quad \Rightarrow \quad 4(2) = R_2(2) \quad \Rightarrow \quad R_2 = 4\,\Omega$$

(b) The power delivered to the resistor to the right of the terminals is maximized by setting R equal to the Thevenin resistance of the part of the circuit to the left of the terminals:

$$R = R_t = \frac{v_{oc}}{i_{sc}} = \frac{8}{2} = 4\ \Omega$$

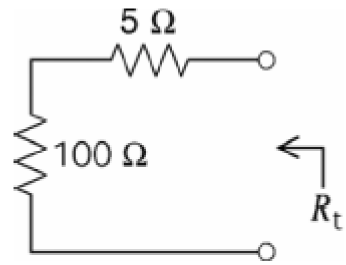
Then $$p_{max} = \frac{{v_{oc}}^2}{4R_t} = \frac{8^2}{4(4)} = 4\ \text{W}$$

P 5.6-2

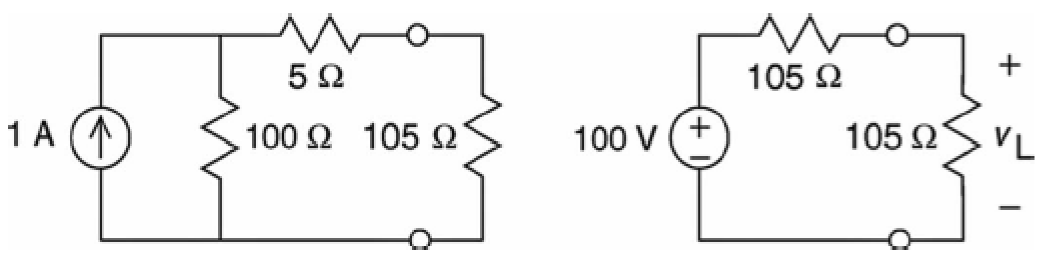
Solution:

a) For maximum power transfer, set R_L equal to the Thevenin resistance:

$$R_L = R_t = 100 + 5 = 105\ \Omega$$



b) To calculate the maximum power, first replace the circuit connected to R_L be its Thevenin equivalent circuit:

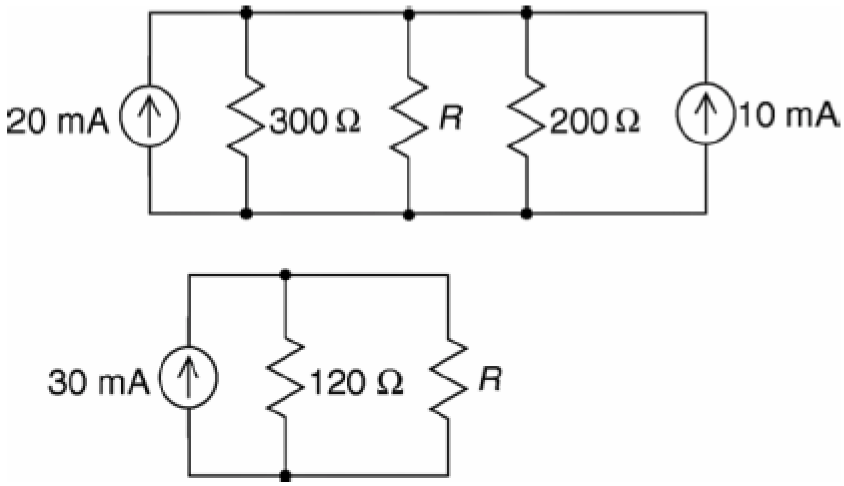


The voltage across R_L is $$v_L = \frac{105}{105+105}(100) = 50\ \text{V}$$

Then $$p_{max} = \frac{{v_L}^2}{R_L} = \frac{50^2}{105} = 23.8\ \text{W}$$

P 5.6-3

Solution:
Reduce the circuit using source transformations:

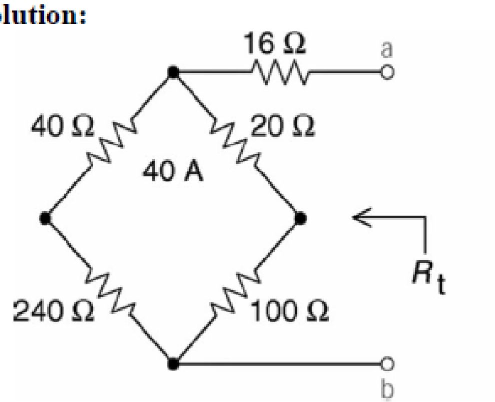


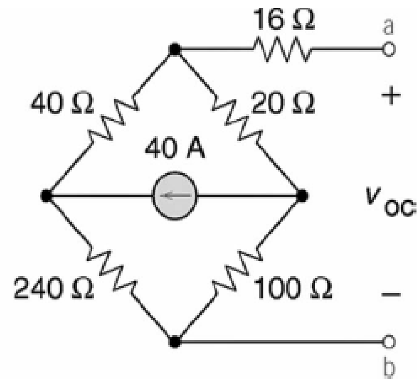
Then (a) maximum power will be dissipated in resistor R when: $R = R_t = 120\ \Omega$ and (b) the value of that maximum power is

$$P_{\max} = i_R^2(R) = (0.015)^2(120) = \underline{27\text{ mW}}$$

P 5.6-4

Solution:





The required value of R is

$$R = R_t = 16 + \frac{(40+240)(20+100)}{(40+240)+(20+100)} = 100\ \Omega$$

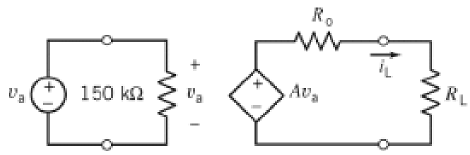
$$v_{oc} = \left[\frac{340}{340+60}(40)\right]20 - \left[\frac{60}{340+60}(40)\right]100$$
$$= \frac{340(40)(20) - 60(40)(100)}{400} = 80\ \text{V}$$

The maximum power is given by

$$p_{\max} = \frac{v_{oc}^2}{4R_t} = \frac{80^2}{4(100)} = 16\ \text{W}$$

P 5.6-5

Solution:



$$i_L = \frac{A}{R_o + R_L} v_s$$

$$P_L = i_L{}^2 R_L = \frac{A^2 v_s{}^2 R_L}{\left(R_o + R_L\right)^2}$$

(a) R_t=R_o so R_L=R_o $= 15\ \Omega$ maximizes the power delivered to the load. The corresponding load power is

$$P_L = \frac{25^2 (0.75)^2 15}{(15+15)^2} = 5.86\ \text{W}.$$

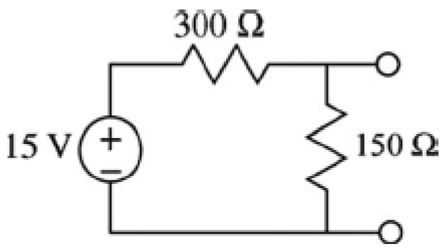
(b) $R_o = 0$ maximizes P_L (The numerator of P_L does not depend on R_o so P_L can be maximized by making the denominator as small as possible.) The corresponding load power is

$$P_L = \frac{A^2 v_s{}^2 R_L}{R_L{}^2} = \frac{A^2 v_s{}^2}{R_L} = \frac{25^2 (0.75)^2}{13} = 27.04\ \text{W}.$$

(c) P_L is proportional to A^2 so the load power continues to increase as A increases. The load can safely receive 32.9 W. This limit corresponds to

$$32.9 = \frac{A^2 (0.75)^2 13}{(28)^2} \quad \Rightarrow \quad A = \frac{28}{0.75}\sqrt{\frac{32.9}{13}} = 56\ \text{V}.$$

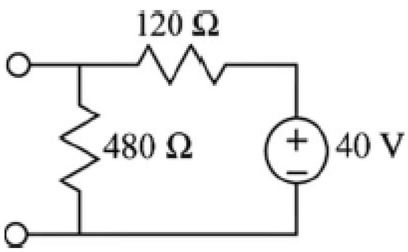
P 5.6-6
Solution:
Replace the part of the circuit connected to the variable resistor by its Thevenin equivalent circuit. First, replace the left part of the circuit by its Thevenin equivalent:



$$v_{oc1} = \left(\frac{150}{150+300}\right)15 = 5\text{ V}$$

$$R_{t1} = 300 \parallel 150 = 100\ \Omega$$

Next, replace the right part of the circuit by its Thevenin equivalent:



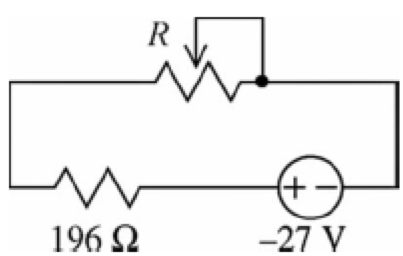
$$v_{oc2} = \left(\frac{480}{480+120}\right)40 = 32\text{ V}$$

$$R_{t2} = 120 \parallel 480 = 96\ \Omega$$

Now, combine the two partial Thevenin equivalents:

$$v_{oc} = v_{oc1} - v_{oc2} = -27\text{ V and } R_t = R_{t1} + R_{t2} = 196\ \Omega$$

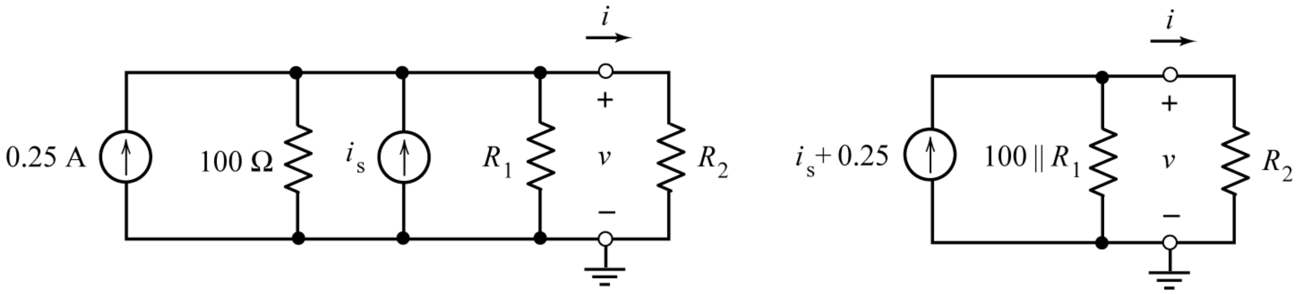
So



The power received by the adjustable resistor will be maximum when $R = R_t = 196\ \Omega$. The maximum power received by the adjustable resistor will be $p = \dfrac{(-27)^2}{4(196\ \Omega)} = 0.93\text{ W}$.

P 5.6-7

Solution: Two source transformations reduce the circuit as follows:



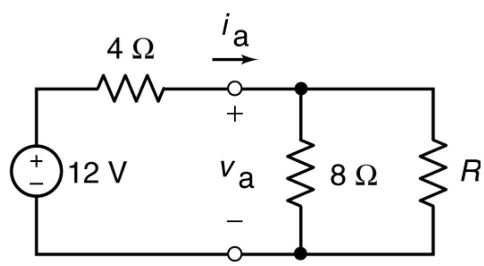
(*a*) Recognizing the parameters of the Norton equivalent circuit gives:

$$1.5 = i_{sc} = i_s + 0.25 \quad \Rightarrow \quad i_s = 1.25 \text{ A} \text{ and } 80 = R_t = 100 \,||\, R_1 = \frac{100\,R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 400 \ \Omega$$

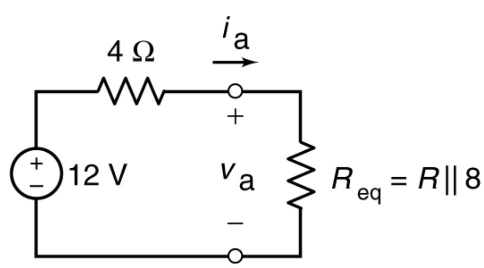
(*b*) The maximum value of the power delivered to R_2 occurs when $R_2 = R_t = 80 \ \Omega$. Then

$$i = \frac{1}{2} i_{sc} = 0.75 \text{ A} \quad \text{and} \quad p = \left(\frac{1}{2} i_{sc} \right)^2 R_t = \left(0.625^2 \right) 80 = 45 \text{ W}$$

P 5.6-8
Solution:



Replace the parallel combination of resistor *R* and the 8 Ω resistor by an equivalent resistance.



Using voltage division $$v_a = \frac{R_{eq}}{4 + R_{eq}} (12) = \frac{1}{\frac{4}{R_{eq}} + 1} (12)$$

Consequently, the maximum value of v_a corresponds to the is obtained by maximizing R_{eq}. The maximum of R_{eq} is obtained by maximizing *R*. Given that $0 \le R \le \infty$, the maximum value of R_{eq} is 8 Ω and the maximum value of v_a is

$$v_{\text{a max}} = \frac{1}{\frac{4}{8}+1}(12) = 8 \text{ V}$$

Using Ohm’s law $$i_{\text{a}} = \frac{12}{4+R_{\text{eq}}}$$

Consequently, the maximum value of i_{a} corresponds to the is obtained by minimizing R_{eq}. The minimum of R_{eq} is obtained by maximizing R. Given that $0 \le R \le \infty$, the minimum value of R_{eq} is 0 Ω and the maximum value of i_{a} is

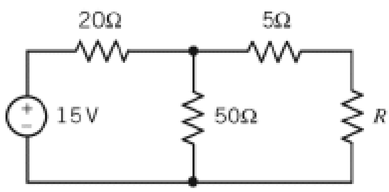
$$i_{\text{a max}} = \frac{12}{4+0} = 3 \text{ A}$$

The maximum power theorem indicates that the maximum value of $p_{\text{a}} = i_{\text{a}} v_{\text{a}}$ occurs when $R_{\text{eq}} = R_{\text{t}}$. In this case, $R_{\text{t}} = 4$ Ω. We require $R_{\text{eq}} = 4$ Ω which is accomplished by making $R = 8$ Ω, an acceptable value since
$0 \le 8 \le \infty$. Then

$$p_{\text{a}} = \frac{\left(\frac{12}{2}\right)^2}{R_{\text{eq}}} = \frac{\left(\frac{12}{2}\right)^2}{4} = 9 \text{ W}$$

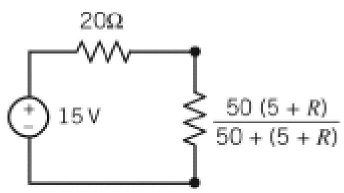
P5.6-9

Solution:



Calculate the resistance in parallel.

$$50 \parallel (5+\mathrm{R}) = \frac{50(5+\mathrm{R})}{50+(5+\mathrm{R})}$$



Therefore,

$$i_a = \frac{15}{20 + \dfrac{50\,(5+R)}{50\,(5+R)}}$$

$$v_a = \frac{\dfrac{50\,(5+R)}{\dfrac{50\,(5+R)}{50\,(5+R)} + 20}}{}15$$

Maximum is delivered when $R = 20\Omega$

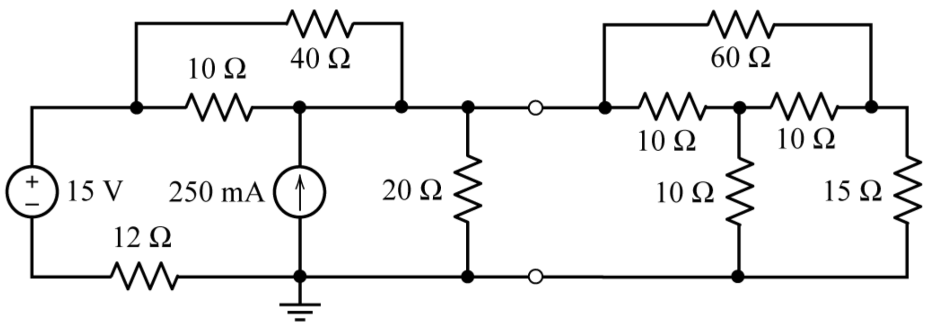
$$\mathrm{P} = i_a v_a = \left(\frac{15}{20 + \dfrac{50\,(5+R)}{50\,(5+R)}}\right)\left(\frac{50\,\dfrac{50\,(5+R)}{50\,(5+R)}}{\dfrac{50\,(5+R)}{50\,(5+R)} + 20}15\right)$$

$$= \left(\frac{15}{20 + \dfrac{50\,(5+20)}{50\,(5+20)}}\right)\left(\frac{50\,\dfrac{50\,(5+20)}{50\,(5+20)}}{\dfrac{50\,(5+20)}{50\,(5+20)} + 20}15\right)$$

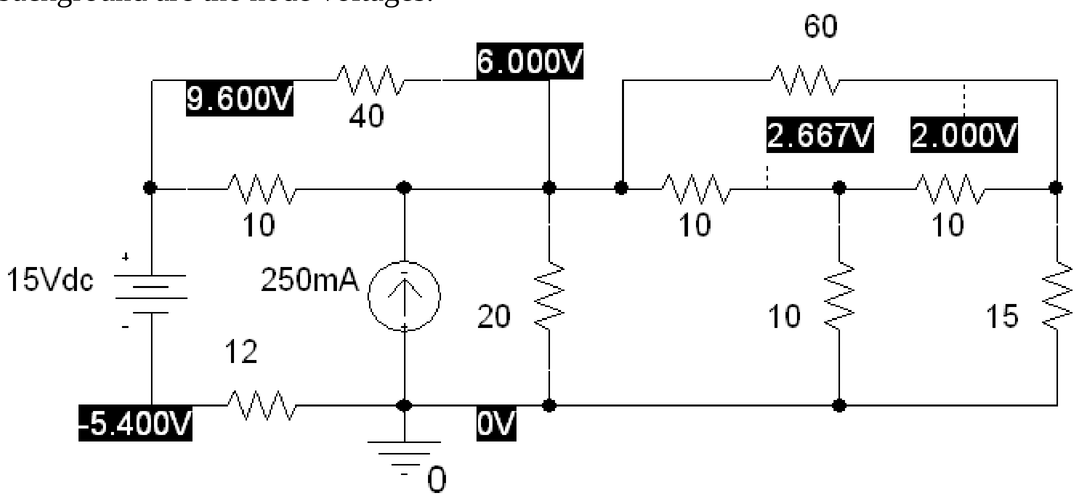
$$= 2.8\mathrm{W}$$

Section 5.8 Using PSpice to Determine the Thevenin Equivalent Circuit

P 5.8-1



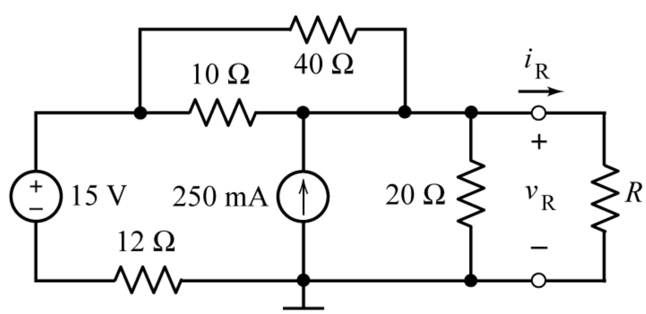
a) Here are the results of simulating the circuit in PSpice. The numbers shown in white on a black background are the node voltages.



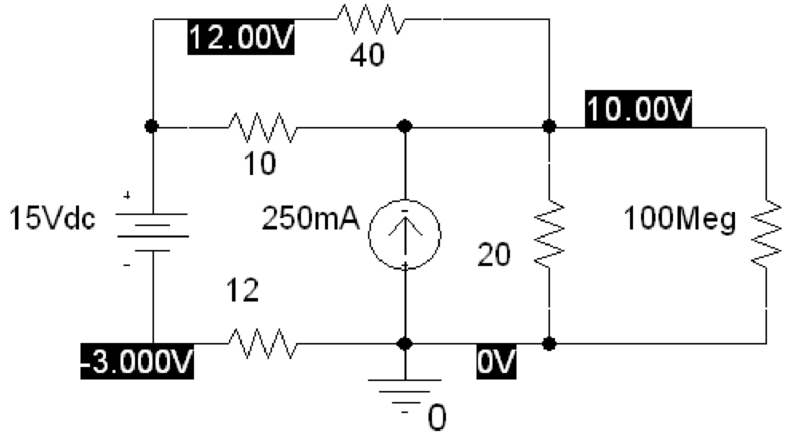
b) Add a resistor across the terminals of Circuit A. Then

$$v_{oc} = v_R \quad \text{when} \quad R \approx \infty$$

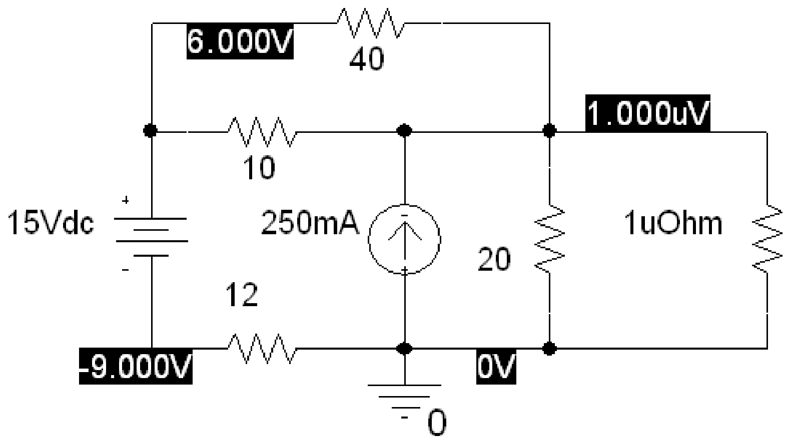
$$i_{sc} = \frac{v_R}{R} \quad \text{when} \quad R \approx 0$$



Here are the PSpice simulation results:



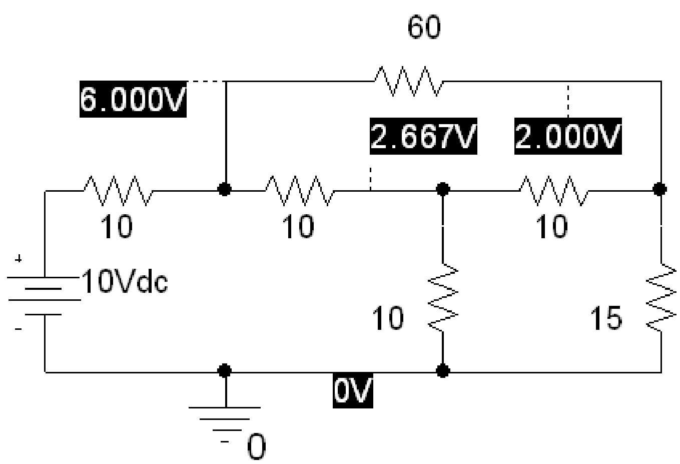
$$v_{oc} = 10 \text{ V}$$



$$i_{sc} = \frac{1\times10^{-6}}{1\times10^{-6}} = 1 \text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{10}{1} = 10\ \Omega$$

c) Here is the result of simulation the circuit after replacing Circuit A by its Thevenin equivalent:

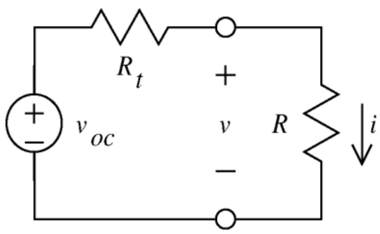


d) The node voltages of Circuit B are the same before and after replacing Circuit A by its Thevenin equivalent circuit.

Section 5-9 How Can We Check…?

P 5.9-1

Solution:



Use the data in the table to determine v_{oc} and i_{sc}:

$$v_{oc} = 12\text{ V} \qquad \text{(line 1 of the table)}$$

$$i_{sc} = 3\text{ mA} \qquad \text{(line 3 of the table)}$$

so $\quad R_t = \dfrac{v_{oc}}{i_{sc}} = 4\text{ k}\Omega$

Next, check line 2 of the table. When R = 10 kΩ:

$$i = \frac{v_{oc}}{R_t + R} = \frac{12}{10(10^3) + 5(10^3)} = 0.857\text{ mA}$$

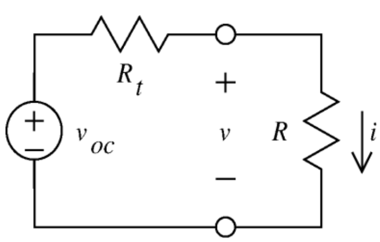
which agrees with the data in the table.

To cause i = 1 mA requires $\quad 0.001 = i = \dfrac{v_{oc}}{R_t + R} = \dfrac{12}{10(10^3) + R} \quad \Rightarrow \quad R = 8000\ \Omega$

I agree with my lab partner's claim that R = 8000 causes i = 1 mA.

P 5.9-2

Solution:



$$\frac{1}{R_t} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{3R} = \frac{11}{6R} \quad \Rightarrow \quad R_t = \frac{6R}{11}$$

and

$$v_{oc} = \left(\frac{2/3}{3+2/3}\right)30 + \left(\frac{3/4}{2+3/4}\right)20 + \left(\frac{6/5}{1+6/5}\right)10 = \frac{180}{11}$$

so the prelab calculation isn't correct.

But then

$$i = \frac{v_{oc}}{R_t + R} = \frac{\frac{180}{11}}{\frac{6}{11}(110) + 40} = \frac{\frac{180}{11}}{60 + 40} = 163\text{ mA} \neq 54.5\text{ mA}$$

so the measurement does not agree with the corrected prelab calculation.

P 5.9-3

Solution:

$$6000 \parallel 3000 \parallel (500 + 1500) = 2000 \parallel 2000 = 1000\ \Omega$$

$$i = \frac{12}{R + 1000} \leq \frac{12}{1000} = 12\text{ mA}$$

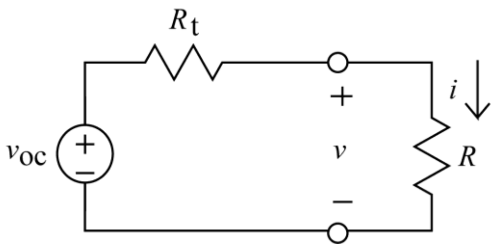
How about that?! Your lab partner is right.

(checked using LNAP 6/21/05)

P 5.9-4

Solution:

(a)



KVL gives

$$v_{oc} = \left(R_t + R\right)i$$

from row 2

$$v_{oc} = \left(R_t + 10\right)\left(1.333\right)$$

from row 3

$$v_{oc} = \left(R_t + 20\right)\left(0.857\right)$$

So

$$\left(R_t + 10\right)\left(1.333\right) = \left(R_t + 20\right)\left(0.857\right)$$

$$28\left(R_t + 10\right) = 18\left(R_t + 20\right)$$

Solving gives

$$10R_t = 360 - 280 = 80 \quad \Rightarrow \quad R_t = 8\ \Omega$$

and

$$v_{oc} = \left(8 + 10\right)\left(1.333\right) = 24\ \text{V}$$

(b)

$$i = \frac{v_{oc}}{R_t + R} = \frac{24}{8 + R} \quad \text{and} \quad v = \frac{R}{R + R_t} v_{oc} = \frac{24R}{R + 8}$$

When $R = 0$, $i = 3$ A, and $v = 0$ V.

When $R = 40\ \Omega$, $i = \frac{1}{2}$ A.

When $R = 80\ \Omega$, $v = \frac{24(80)}{88} = \frac{240}{11} = 21.82$.

These are the values given in the tabulated data so the data is consistent.

(c) When $R = 40\ \Omega$, $v = \frac{24(40)}{48} = 20$ V.

When $R = 80\ \Omega$, $i = \frac{24}{88} = 0.2727$ A.

(d) First

$$8 = R_t = 24 \| 18 \| \left(R_1 + 12\right) \quad \Rightarrow \quad R_1 = 24\ \Omega$$

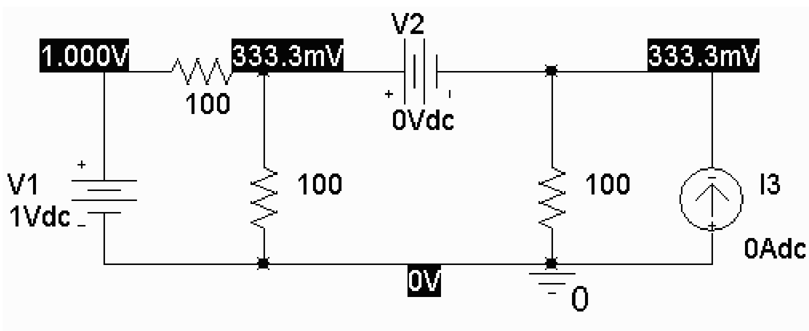
the, using superposition,

$$24 = v_{oc} = \frac{24}{24+\left(18 \parallel \left(R_1+12\right)\right)} 12 + \left(24 \parallel 18\left(R_1+12\right)\right) i_s = 8 + 8 i_s \quad \Rightarrow \quad i_s = 2 \text{ A}$$

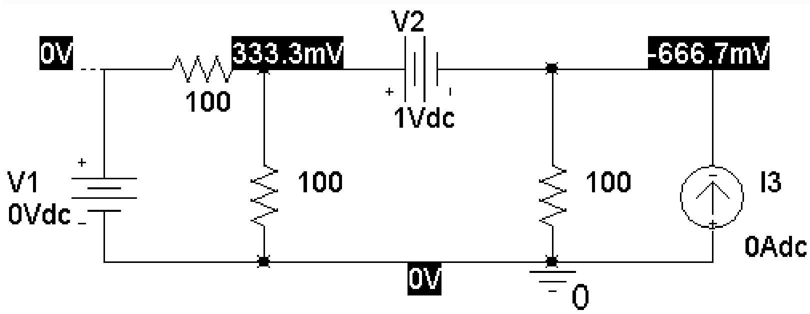
(checked using LNAP 6/21/05)

PSpice Problems

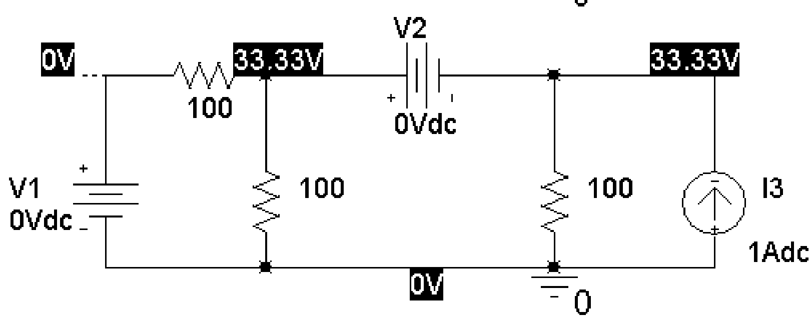
SP 5-1
Solution:



a = 0.3333



b = 0.3333



c =33.33 V/A

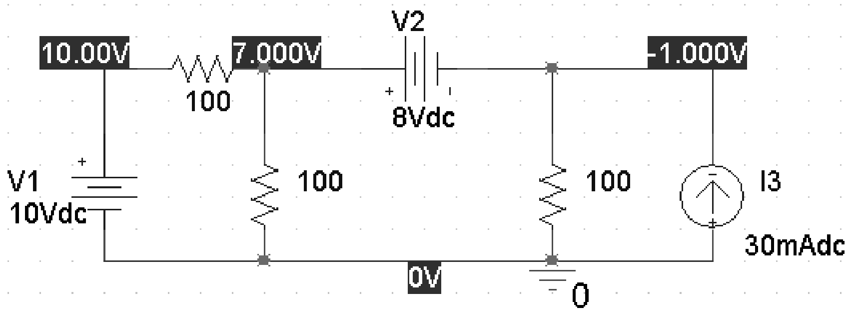
(a) $$v_o = 0.3333\, v_1 + 0.3333\, v_2 + 33.33\, i_3$$

(b) $$7 = 0.3333\left(10\right) + 0.3333\left(8\right) + 33.33\, i_3 \quad \Rightarrow \quad i_3 = \frac{7 - \frac{18}{3}}{\frac{100}{3}} = \frac{3}{100} = 30 \text{ mA}$$

SP 5-2

Solution:

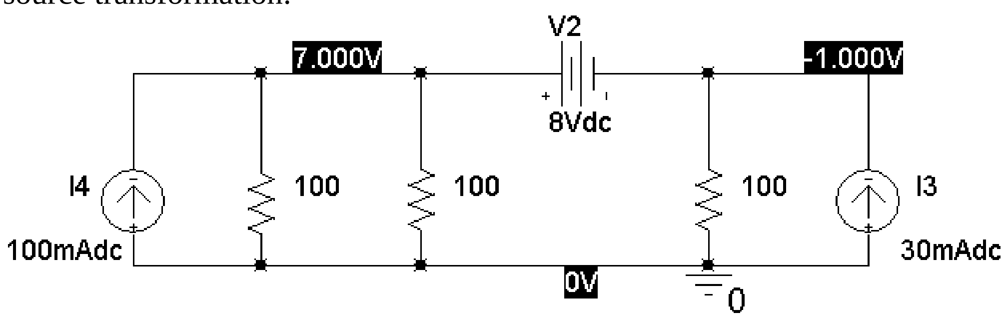
Before the source transformation:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V1	-3.000E-02
V_V2	-4.000E-02

After the source transformation:

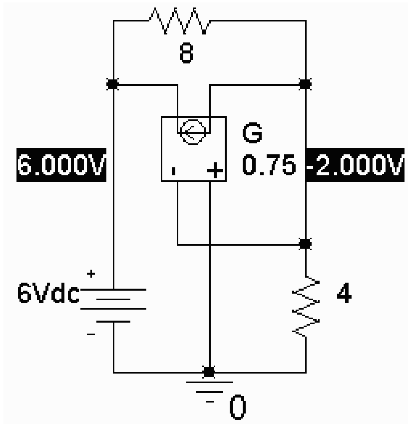


VOLTAGE SOURCE CURRENTS

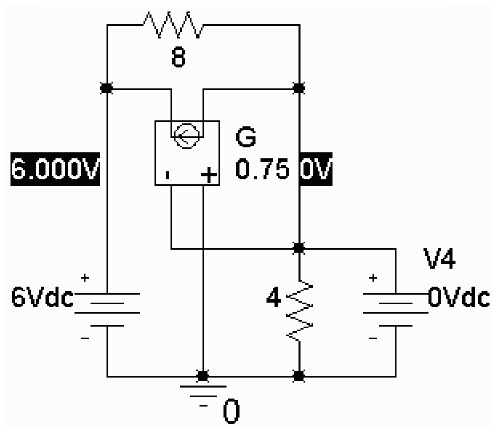
NAME	CURRENT
V_V2	-4.000E-02

SP 5-3

Solution:



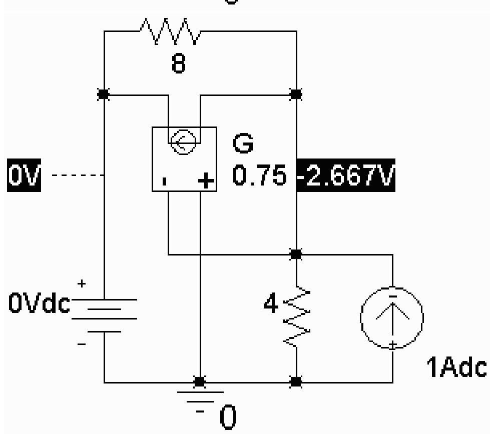
$v_{oc} = -2$ V



VOLTAGE SOURCE CURRENTS

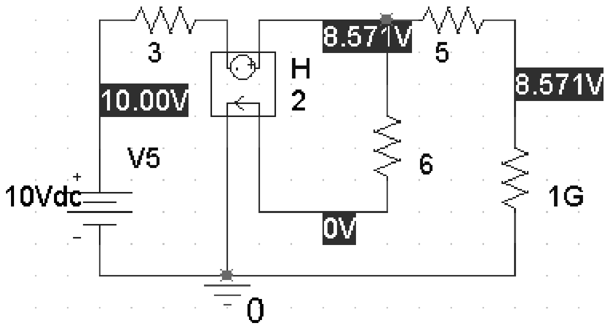
NAME	CURRENT
V_V3	-7.500E-01
V_V4	7.500E-01

$i_{sc} = 0.75$ A

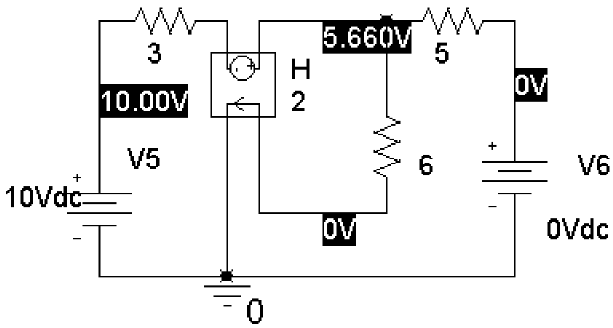


$R_t = -2.66\ \Omega$

SP 5-4
Solution:



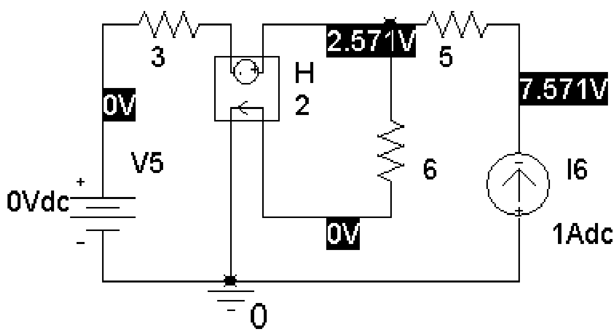
$v_{oc} = 8.571$ V



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V5	-2.075E+00
V_V6	1.132E+00
X_H1.VH_H1	9.434E-01

$i_{sc} = 1.132$ A



$R_t = 7.571\ \Omega$

Design Problems

DP 5-1
Solution:
(a) The open-circuit voltage and Thèvenin resistance of the circuit in Figure DP5-1(b) are

$$v_{oc} = \frac{R_2}{R_1 + R_2}(24) \quad \text{and} \quad R_t = \frac{R_1 R_2}{R_1 + R_2}$$

Consequently specification (a) requires

$$14 = \frac{R_2}{R_1 + R_2}(24) \quad \text{and} \quad \frac{R_1 R_2}{R_1 + R_2} \le 100\ \Omega$$

Consequently $$14(R_1 + R_2) = 24 R_2 \quad \Rightarrow \quad R_2 = 1.4\, R_1$$

and $$100 \ge \frac{R_1 R_2}{R_1 + R_2} = \frac{1.4\, R_1{}^2}{2.4\, R_1} \quad \Rightarrow \quad R_1 \le 171.4\ \Omega$$

(b) The power supplied by the 24-V source is $p = \dfrac{24^2}{R_1 + R_2}$ so specification 2 requires

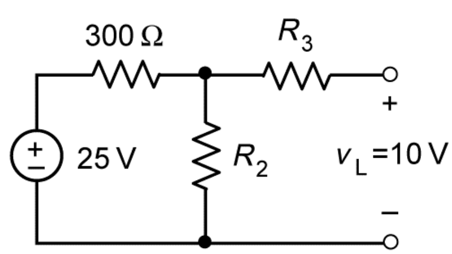
$$2 \ge \frac{24^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \ge 288 \quad \Rightarrow \quad 2.4\, R_1 \ge 288 \quad \Rightarrow \quad R_1 \ge 120\ \Omega$$

Together, specifications (a) and (b) require

$$120\ \Omega \le R_1 \le 171.4\ \Omega$$

The solution is not unique. Pick $R_1 = 150\ \Omega$. Then $R_2 = 1.4(150) = 210\ \Omega$.

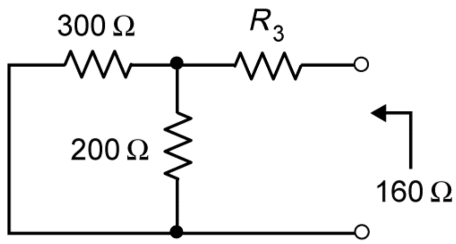
DP 5-2
Solution:
From specification (b) we obtain:



$$10 = v_{\mathrm{L}} = \frac{R_2}{R_2 + 300}(25) \quad \Rightarrow \quad R_2 + 300 = \frac{R_2}{10}(25) = \frac{5}{2} R_2 \quad \Rightarrow \quad 300 = \frac{3}{2} R_2$$

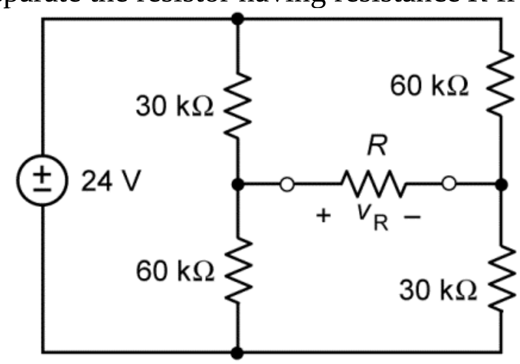
so $R_2 = 200\ \Omega$.

Next, since specifications (a) and (b) together indicate that the Thèvenin resistance of the part of the circuit connected to R_L is 160 Ω, we have

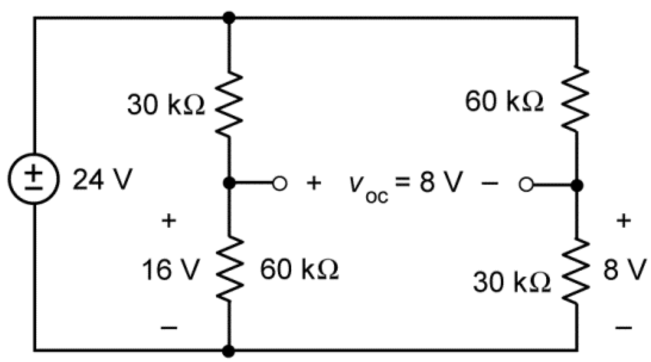


$$160 = R_3 + (200 \| 300) = R_3 + 120 \quad \Rightarrow \quad R_3 = 40\ \Omega$$

DP5-3
Solution:
Add a pair of terminals to separate the resistor having resistance *R* from the rest of the circuit.



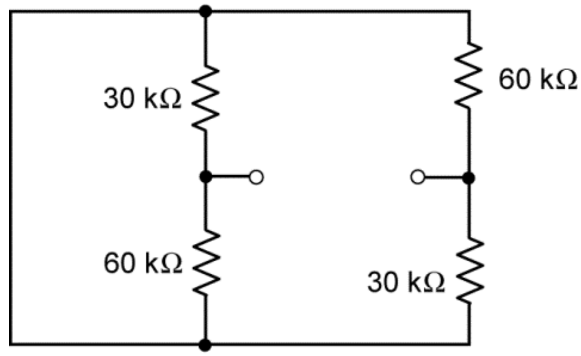
Let's find the Thèvenin equivalent of the part of the circuit connected to the resistor having resistance *R*. First,



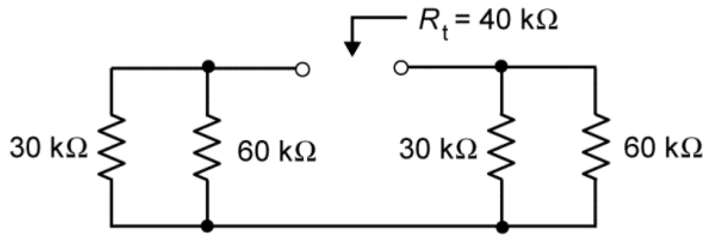
Use voltage division twice and KVL to write

$$v_{oc} = \frac{60}{60+30}24 - \frac{30}{60+30}24 = 16 - 8 = 8\ \text{V}$$

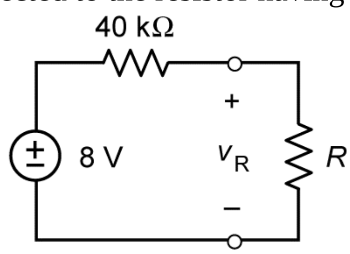
Next, we find the Thèvenin resistance of the part of the circuit connected to the resistor having resistance *R*. Set the voltage source voltage to zero:



Redraw this circuit as



Replace the part of the circuit connected to the resistor having resistance *R* by its



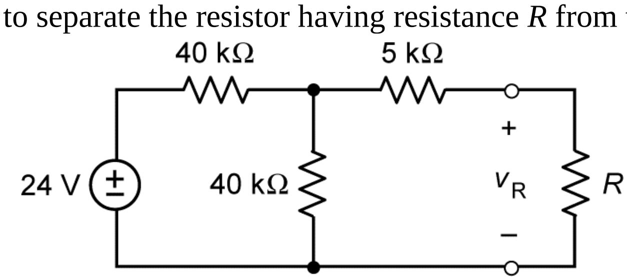
We require: $$6 = \frac{R}{R+40000} 8 \quad \Rightarrow \quad R = 120000 = 120\ \text{k}\Omega$$

Consequently 120 kΩ is the value of the resistance *R* in the circuit shown in Figure DP 5-3 that will cause the voltage v_R to be 6 V.

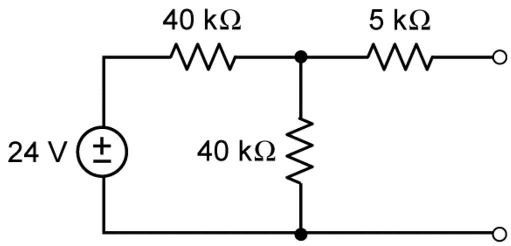
DP5-4

Solution:

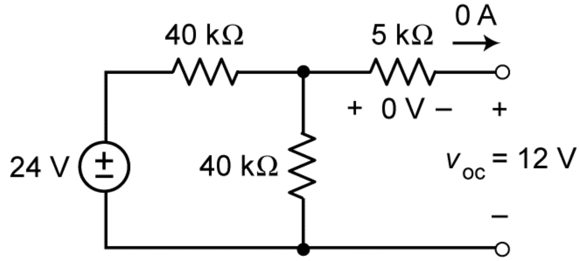
Add a pair of terminals to separate the resistor having resistance *R* from the rest of the circuit.



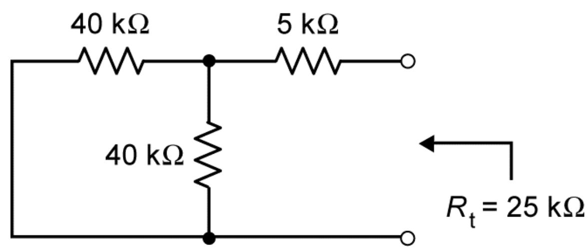
Here's the part of the circuit connected to the resistor having resistance *R*.



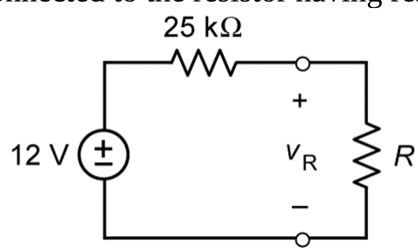
Let's find the Thèvenin equivalent of this part of the circuit. First,



Next,



Replace the part of the circuit connected to the resistor having resistance R by its

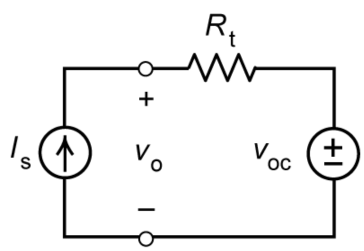


We require: $$8 = \frac{R}{R+25000} 12 \quad \Rightarrow \quad R = 50000$$

Consequently 50 kΩ is the value of the resistance R in the circuit shown in Figure DP 5-4 that will cause the voltage v_R to be 8 V.

DP 5-5

Solution:



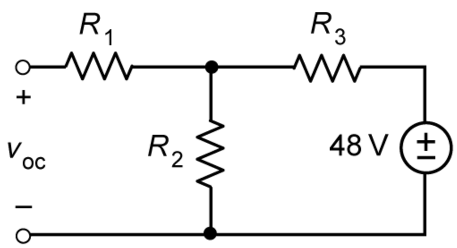
Use Ohm’s Law and KVL to write $v_o = R_t I_s + v_{oc}$

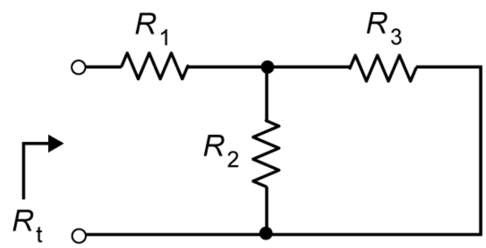
Compare this equation to $v_o = 12 + 20 I_s$

To specify the required values of R_t and v_{oc}:

$$R_t = 20\ \text{k}\Omega \text{ (because } v_o \text{ is in volts and } I_s \text{ is in mA) and } v_{oc} = 12\text{ V.}$$

The open circuit voltage, v_{oc}, and Thèvenin resistance, R_t, of the part of the circuit to the right of the terminals are determined using:





$$v_{oc} = \frac{R_2}{R_2 + R_3} 48 \qquad R_t = R_1 + R_2 \parallel R_3$$

Now use the required values of R_t and v_{oc} to determine that

$$12 = v_{oc} \frac{R_2}{R_2 + R_3} 48 \quad \Rightarrow \quad R_3 = 3R_2 \quad \text{and} \quad 20\ \text{k}\Omega = R_t = R_1 + R_2 \parallel R_3$$

The solution is not unique. Choosing

$$R_2 = 20\ \text{k}\Omega$$

requires

$$R_3 = 3R_2 = 60\ \text{k}\Omega$$

and

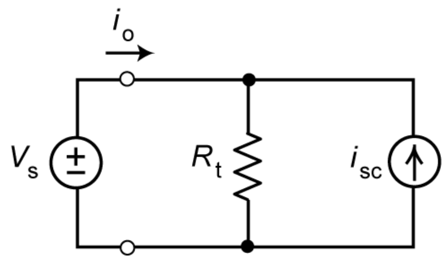
$$R_2 \parallel R_3 = 15\ \text{k}\Omega.$$

Finally,

$$R_1 = 20\ \text{k}\Omega - R_2 \parallel R_3 = 20\ \text{k}\Omega - 15\ \text{k}\Omega = 5\ \text{k}\Omega.$$

DP 5-6

Solution:
Replace the part of the circuit to the right of the terminals by a Norton equivalent circuit to obtain



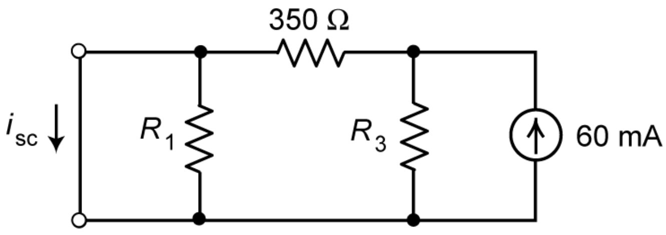
Use Ohm's Law and KVL to write $$i_o = \frac{V_s}{R_t} - i_{sc}$$

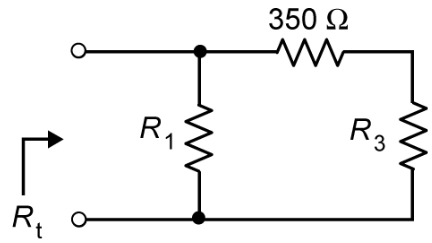
Compare this equation to $$i_o = 6.25V_s - 33.75$$

To specify the required values of R_t and i_{sc}:

$$\frac{1}{R_t} = 6.25 \ \frac{\text{mA}}{\text{V}} \quad \Rightarrow \quad R_t = \frac{1}{6.25} \ \frac{\text{V}}{\text{mA}} = 0.16 \text{ k}\Omega = 160 \ \Omega \quad \text{and} \quad i_{sc} = 33.75 \text{ mA}$$

The short circuit current, i_{sc}, and Thèvenin resistance, R_t, of the part of the circuit to the right of the terminals are determined using:





$$i_{sc} = \frac{R_3}{350 + R_3} 60 \qquad\qquad R_t = R_1 \,||\, \left(R_2 + R_3\right)$$

Now use the required values of R_t and i_{sc} to determine that

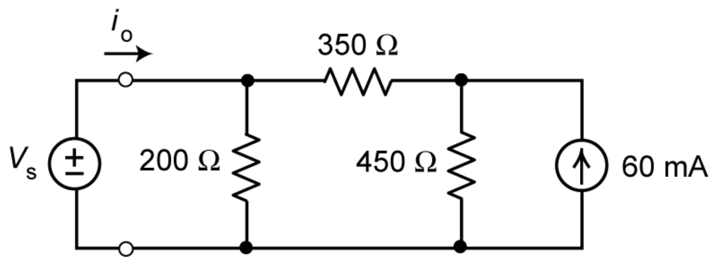
$$33.75 = \frac{R_3}{350 + R_3} 60 \quad \Rightarrow \quad R_3 = 450 \ \Omega \quad \text{and} \quad 160 \ \Omega = R_1 \,||\, \left(350 + 450\right) = R_1 \,||\, 800$$

$$160 = \frac{800\, R_1}{800 + R_1} \quad \Rightarrow \quad R_1 = \frac{160(800)}{640} = 200 \ \Omega$$

The required values of R_1 and R_3 are

$$R_1 = 200 \ \Omega \text{ and } R_3 = 450 \ \Omega$$

The finished circuit is



DP 5-7

Solution:

The open circuit voltage of Prototype #1 is $v_{oc} = \dfrac{8000}{R_1 + 8000} 10 < 10 \text{ V}$. Consequently, Prototype #1 cannot be used to satisfy the given specifications.

The Thèvenin resistance of Prototype #3 is $R_t = 5000 + 2000 \,||\, \left(R_1 + R_2\right) \le 5000 + 2000 < 10 \text{ k}\Omega$. Consequently, Prototype #3 cannot be used to satisfy the given specifications.

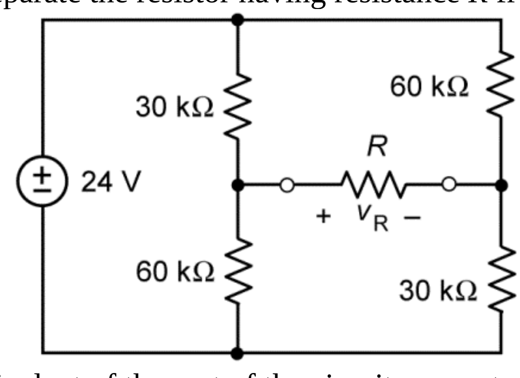
Consider Prototype #2.

$$12 = v_{oc} \quad \Rightarrow \quad 12 = \frac{6000}{R_1 + 6000} 20 \quad \Rightarrow \quad R_1 = 4000 = 4 \text{ k}\Omega$$

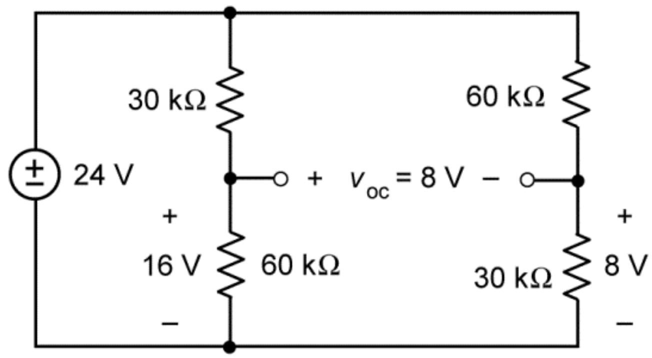
$$10 \text{ k}\Omega = R_t = R_2 + \left(R_1 \,||\, 6000\right) = R_2 + \left(4000 \,||\, 6000\right) = R_2 + 2.4 \text{ k}\Omega = \quad \Rightarrow \quad R_2 = 7.6 \text{ k}\Omega$$

Chose Prototype #2 and specify R_1 = 4 kΩ and R_2 =7.6 kΩ.

DP 5-8
Solution:
Add a pair of terminals to separate the resistor having resistance *R* from the rest of the circuit.



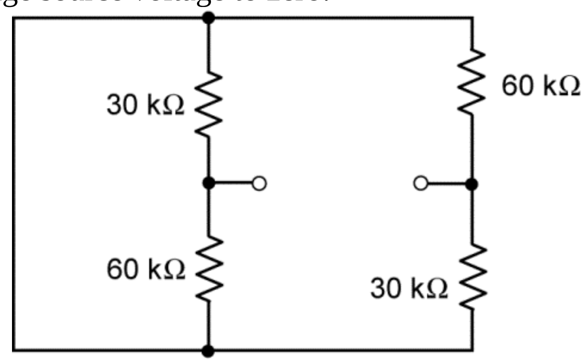
Let's find the Thèvenin equivalent of the part of the circuit connected to the resistor having resistance *R*. First,



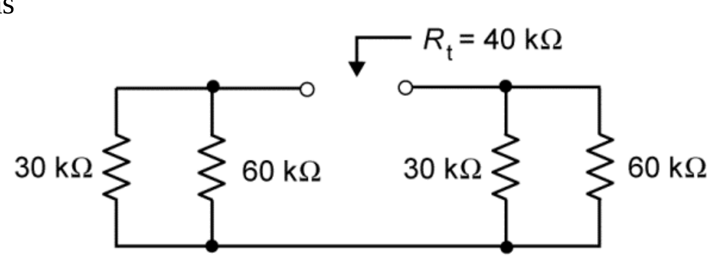
Use voltage division twice and KVL to write

$$v_{oc} = \frac{60}{60+30}24 - \frac{30}{60+30}24 = 16-8 = 8 \text{ V}$$

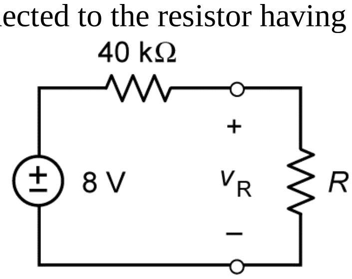
Next, we find the Thèvenin resistance of the part of the circuit connected to the resistor having resistance R. Set the voltage source voltage to zero:



Redraw this circuit as



Replace the part of the circuit connected to the resistor having resistance R by its



We require: $$6 = \frac{R}{R+40000}8 \quad \Rightarrow \quad R = 120000 = 120 \text{ k}\Omega$$

Consequently 120 kΩ is the value of the resistance R in the circuit shown in Figure DP 5-8 that will cause the voltage v_R to be 6 V.

DP 5-9

Solution:

The equation of representing the straight line in Figure DP 5-9b is $v=-R_t\,i+v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t=-\dfrac{0-5}{0.008-0}=625\ \Omega$ and v_{oc} = 5 V.

Try $R_1=R_2=1\ \text{k}\Omega$. ($R_1 \parallel R_2$ must be smaller than R_t = 625 Ω.) Then

$$5=\frac{R_2}{R_1+R_2}v_s=\frac{1}{2}v_s \quad \Rightarrow v_s=10\ \text{V}$$

and

$$625=R_3+\frac{R_1R_2}{R_1+R_2}=R_3+500 \quad \Rightarrow \quad R_3=125\,\Omega$$

Now v_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-10

Solution:

The equation of representing the straight line in Figure DP 5-10b is $v=-R_t\,i+v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t=-\dfrac{0-(-3)}{-0.006-0}=500\ \Omega$ and v_{oc} = –3 V.

From the circuit we calculate

$$R_t=\frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} \text{ and } v_{oc}=-\frac{R_1R_3}{R_1+R_2+R_3}i_s$$

so

$$500\ \Omega=\frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} \text{ and } -3\ \text{V}=-\frac{R_1R_3}{R_1+R_2+R_3}i_s$$

Try $R_3=1\text{k}\Omega$ and $R_1+R_2=1\text{k}\Omega$. Then $R_t=500\ \Omega$ and

$$-3=-\frac{1000R_1}{2000}i_s=-\frac{R_1}{2}i_s \quad \Rightarrow 6=R_1\,i_s$$

This equation can be satisfied by taking R_1 = 600 Ω and i_s = 10 mA. Finally, R_2 = 1 kΩ - 400 Ω = 400 Ω. Now i_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-11

Solution:

The slope of the graph is positive so the Thevenin resistance is negative. This would require $R_3+\dfrac{R_1R_2}{R_1+R_2}<0$, which is not possible since R_1, R_2 and R_3 will all be non-negative.

Is it not possible to specify values of v_s, R_1, R_2 and R_3 that cause the current i and the voltage v in Figure DP 5-11a to satisfy the relationship described by the graph in Figure DP 5-11b.

DP 5-12

Solution:

The equation of representing the straight line in Figure DP 5-12b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to the Thevenin impedance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{-5-0}{0-0.008} = -625\,\Omega$ and v_{oc} = –5 V.

The open circuit voltage, v_{oc}, the short circuit current, i_{sc}, and the Thevenin resistance, R_t, of this circuit are given by

$$v_{oc} = \frac{R_2\left(d+1\right)}{R_1+\left(d+1\right)R_2} v_s ,$$

$$i_{sc} = \frac{\left(d+1\right)}{R_1} v_s$$

and

$$R_t = \frac{R_1 R_2}{R_1+\left(d+1\right)R_2}$$

Let $R_1 = R_2 = 1$ kΩ. Then

$$-625\,\Omega = R_t = \frac{1000}{d+2} \quad\Rightarrow\quad d = \frac{1000}{-625} - 2 = -3.6 \text{ A/A}$$

and

$$-5 = \frac{\left(d+1\right)v_s}{d+2} \quad\Rightarrow\quad v_s = \frac{-3.6+2}{-3.6+1}\left(-5\right) = -3.077 \text{ V}$$

Now v_s, R_1, R_2 and d have all been specified so the design is complete.