

## Midterm Exam 1

Note: All the design of the problems should be with enough information of how you derive the logic circuits (logic diagram) or Boolean functions.

1 (16pt) Number representation:

(a) Convert a decimal number  $104.52_{10}$  to hexadecimal-based and octal number. Leave two digits after the decimal point. Truncate the third digit after the decimal point. (6pt)

(b) What is the decimal value of the code (01000011.10010101) if it is in BCD? (4pt)

(c) What is the decimal value of the code (01000011.10010101) if it is a 2's complement signed binary number? (3pt)

(d) What is the decimal value of the code (01000011.10010101) if it is a sign-magnitude signed binary number? (3pt)

2 (10pt) Design a logic function  $F$  that the input is a 3-bit binary code ( $b_2b_1b_0$ ) from 000, 001, 010, ..., 111 and the output is a 3-bit Gray code ( $g_2g_1g_0$ ). Derive the truth table and draw the logic diagram using NOR and NOT gates only.

3 (24pt) Consider the function  $F$  with the don't care condition  $d$ .  $F(w, x, y, z) = \sum m(1, 3, 5, 7, 9, 15)$ ,  $d(w, x, y, z) = \sum m(4, 11, 12)$ .

(a) Simplify the function  $F$  to sum-of-product and product-of-sum with minimum number of literals (12pt)

(b) Indicate the prime implicants and essential prime implicants of the function  $F$ . (6pt)

(c) Simplify the function  $F$  to a minimum number of literals and implement it using AND-OR-INVERT (AOI) implementation. Derive the Boolean expression and draw the logic diagram. (6pt)

4 (16pt) Perform the subtraction with the following unsigned binary numbers by taking the 2's complement of the subtrahend. (a)  $01001 - 10010$ , (b)  $1100 - 111100$ , (c)  $10101 - 1011$ .

5 (9pt) Majority Indicator: Design a voting machine for three people. When there is more than one person agree on some case (with input 1), the case will be passed (with output 1); otherwise, the case will be rejected (with output 0). Derive the following:

(a) Truth table, (3pt)

(b) Simplified Boolean expression, (3pt)

(c) Logic diagram. (3pt)

6 (25pt) Comparator ( $a > b$ ,  $a < b$ ,  $a = b$ ):

(a) Design a 4-bit binary adder/subtractor with augend ( $a_3a_2a_1a_0$ ), addend ( $b_3b_2b_1b_0$ ), operator  $M$  (0: addition, 1: subtraction), output ( $s_3s_2s_1s_0$ ), and an overflow indicator  $V$  (1: overflow, 0: not overflow). (10pt)

(b) Use the above adder/subtractor as a building block, build the following three kinds of comparators with some additional logics: (1)  $a > b$ , (2)  $a < b$ , (3)  $a = b$ . (15pt)

Hint: You can rearrange the inequality to  $a - b > 0$  or  $a - b < 0$  for further design.