
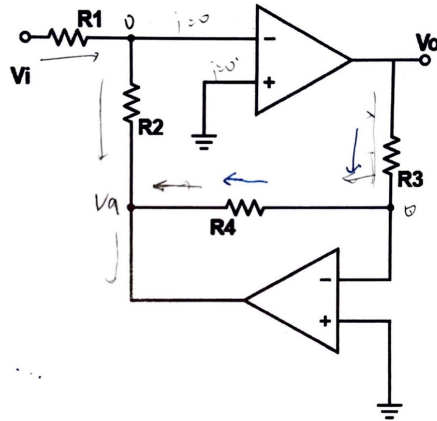


Quiz 3 solution



1. Show the relationship of the output voltage, v_o , to the input voltage, v_i , of the circuit in Figure 1. (15%)



$$V_a = -\frac{R_4}{R_3} V_o$$

$$V_a = -\frac{R_2}{R_1} V_i$$

$$V_o = \frac{R_2 R_3}{R_1 R_4} V_i$$

$$A: V_o = \frac{R_2 R_3}{R_1 R_4} V_i$$

Figure 1.

2. An ideal voltmeter (v_m) is used to measure the output voltage of the circuit in Figure 2. What is the reading of the voltmeter? (15%)

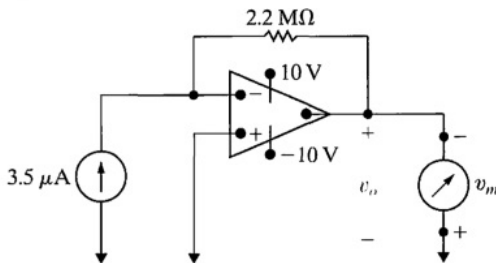


Figure 2.

There is virtual ground at OP input, then

$$V_{op+} = V_{op-} = 0$$

And there is no current entering OP input, so

$$\frac{0 - v_o}{2.2M} = 3.5\mu \rightarrow v_o = -7.7 \rightarrow v_m = -v_o = 7.7 V$$

3.

Solution:

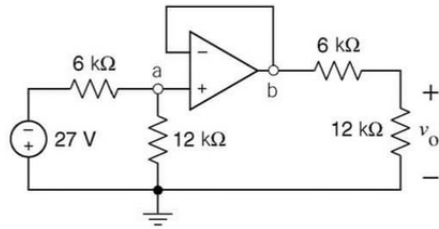
Apply KCL at node a:

$$\frac{v_a - (-27)}{6000} + \frac{v_a}{12000} + 0 = 0 \Rightarrow v_a = -18 \text{ V}$$

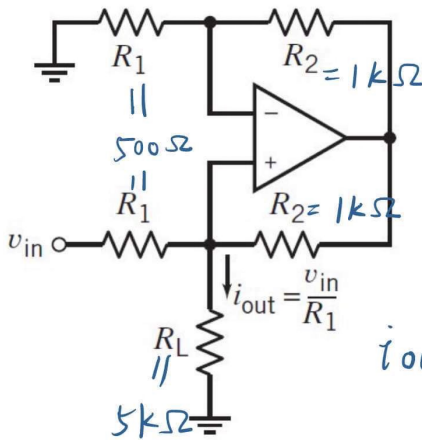
The node voltages at the input nodes of ideal op amps are equal, so $v_b = v_a$.

Using voltage division:

$$v_o = \frac{12000}{6000 + 12000} v_b = -12 \text{ V}$$



4. Use voltage control current source



$$i_{out} = \frac{v_{in}}{500} = 0.002 v_{in}$$

5.

$$V_L = L \frac{di}{dt} \quad \text{for step response:}$$

$$V_L = 8\bar{i} + 2\bar{i} + L \frac{d\bar{i}}{dt} = 10\bar{i} + L \frac{d\bar{i}}{dt}$$

$$\Rightarrow \frac{d\bar{i}}{dt} + \frac{10}{L} \bar{i} = \frac{V_L}{L} \Rightarrow \bar{i}_p = \frac{V_L}{L} \cdot \frac{L}{10} = 10$$

$$\text{for } \bar{i}_h = A e^{st}$$

$$\Rightarrow A s e^{st} + \frac{10}{L} A e^{st} = 0 \Rightarrow (s + \frac{10}{L}) A e^{st} = 0$$

$$\Rightarrow s = -\frac{10}{L} = -25$$

$$\bar{i} = \bar{i}_p + \bar{i}_h = 10 + A e^{-25t}$$

$$\text{for } \bar{i}(0) = 10 + A = 0 \rightarrow A = -10$$

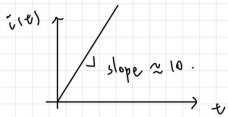
$$\text{step response: } \bar{i}(t) = 10 - 10 e^{-25t}$$

$$\text{ramp response: } \bar{i}(t) = \int_{-\infty}^t 10 - 10 e^{-25\tau} d\tau$$

$$= \bar{i}(0^-) + \int_0^t 10 - 10 e^{-25\tau} d\tau$$

$$= 0 + 10t + \frac{10}{25} e^{-25\tau} \Big|_0^t$$

$$= 10t + 0.4 e^{-25t} - 0.4, \quad t \geq 0.$$



6. Consider the circuit in Figure 6 with the voltage source $v_i = 20u(t-5)$ where $u(t)$ represents a unit step function. Derive and plot v_o for $t \geq 0$. (20%)

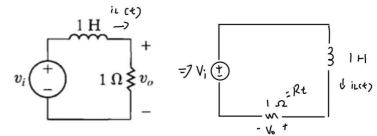


Figure 6.

$$i_L(t) = A + B e^{-at} \quad \text{for } t \geq 5$$

$$i_L(0) = A + B, \quad i_L(\infty) = A$$

$$A + B = 0, \quad A = 20, \quad B = -20$$

$$\Rightarrow i_L(t) = 20 - 20 e^{-at}$$

$$\frac{1}{a} = \tau = \frac{L}{R_L} = 1 \Rightarrow i_L(t) = 20 - 20 e^{-t}$$

$$\begin{cases} V_o(t) = 20 - 20 e^{-(t-5)} & \text{for } t \geq 5 \\ V_o(t) = 0 & \text{for } t < 5 \end{cases}$$

