EE214000 Electromagnetics, Fall, 2020

Homework #2, due in class at 12 pm, noon, Monday, Nov. 2nd, 2020

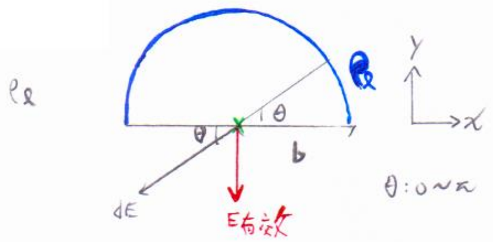
Late submission won't be accepted!

Problems P.3-8, P.3-11, P.3-12, P.3-19, P.3-24, P.3-28, P.3-33, P.3-37 in DK Cheng's textbook

[35]

P.3–8 A line charge of uniform density ρ_ℓ in free space forms a semicircle of radius b. Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

[4%]



$$dE_y = dE \sin\theta\,(-1) \qquad d\ell = b\,d\theta\,,\quad \int_0^\pi \sin\theta\, d\theta = [-\cos\theta]\Big|_0^\pi = 2$$

$$\vec{E}_y = \int_0^\pi \frac{\rho_\ell\, d\theta}{4\pi\varepsilon_0 R^2}\, b\,(-1)\sin\theta \underset{R=b}{=} \frac{\rho_\ell b}{4\pi\varepsilon_0 b^2}(-1) = \hat{a}_y \frac{-2\rho_\ell}{4\pi\varepsilon_0 b} = -\hat{a}_y \frac{\rho_\ell}{2\pi\varepsilon_0 b}$$

P.3–11 A spherical distribution of charge $\rho = \rho_0[1-(R^2/b^2)]$ exists in the region $0 \le R \le b$. This charge distribution is concentrically surrounded by a conducting shell with inner radius $R_i\,(>b)$ and outer radius R_o. Determine **E** everywhere.

[4%]

P. 3-11 解：由球的對稱性 $\vec{E}=\vec{a}_k E_R$. 并應用高斯定律：

(1) $0\leqslant R\leqslant b$ 時 $4\pi R^2 E_{R1} = \frac{\rho_0}{\varepsilon_0}\int_0^R (1-\frac{R^2}{b^2})4\pi R^2 dR = \frac{4\pi\rho_0}{\varepsilon_0}(\frac{R^3}{3} - \frac{R^5}{5b^2})$

則 $E_{R1} = \frac{\rho_0}{\varepsilon_0}R(\frac{1}{3}-\frac{R^2}{5b^2})$

(2) $b\leqslant R< R_i$ 時 $4\pi R^2 E_{R_2} = \frac{\ell_0}{E_0}\int_0^b (1-\frac{R^2}{b^2})4\pi R^2 dR = \frac{8\pi\ell_0}{15t_0}b^3$

則 $E_{R_2} = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$

(3) $R_i\leqslant R\leqslant R_0$ 時 $E_{R3}=0$

(4) $R>R_0$ 時 $E_{R4} = \frac{2\rho_0 b^3}{15\varepsilon_0 R^2}$

P.3–12 Two infinitely long coaxial cylindrical surfaces, $r = a$ and $r = b$ $(b > a)$, carry surface charge densities ρ_{sa} and ρ_{sb}, respectively.

a) Determine **E** everywhere.

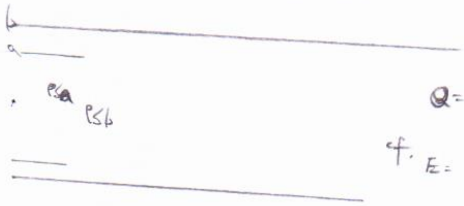
b) What must be the relation between a and b in order that **E** vanishes for $r > b$?

[3%、2%]

two 無窮長 coaxial cylindrical surface

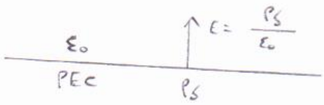
$b > a$

$\vec{E} = \hat{a}_r E_r$



$Q = \rho_s S = \rho_\ell \ell$

cf. $E = \dfrac{\rho_\ell}{2\pi \varepsilon r}$



i. $r < a$, $E_r = 0$

$a < r < b$, $E_r = \dfrac{a\rho_{sa}}{\varepsilon_0 r} = \dfrac{\rho_{sa} 2\pi a}{2\pi \varepsilon_0 r} = \dfrac{\rho_s a}{\varepsilon_0 r}$

$r > b$, $E_r = \dfrac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r}$

$\therefore \vec{E} = \begin{cases} 0 & , r < a \\ \hat{a}_r \dfrac{\rho_s a}{\varepsilon_0 r} & , a < r < b \\ \hat{a}_r \dfrac{a\rho_{sa} + b\rho_{sb}}{\varepsilon_0 r} & , r > b \end{cases}$ #

ii. let $\dfrac{b}{a} = \dfrac{-\rho_{sa}}{\rho_{sb}}$

P.3–19 A charge Q is distributed uniformly over the wall of a circular tube of radius b and height h. Determine V and **E** on its axis

a) at a point outside the tube, then

b) at a point inside the tube.

[2%、2%]

P. 3-19 解:假設圓柱形管位於 xy 平面上且其軸與 z 相交.

表面電荷 $\rho_s = Q/2\pi bh$

$$V = \oint \frac{\rho_s dl}{4\pi\varepsilon_0 R} = \int_0^{2\pi} \frac{\rho_l b d\phi}{4\pi\varepsilon_0 \sqrt{b^2 + (z-z')^2}} = \frac{\rho_l b}{2\varepsilon_0 \sqrt{b^2 + z - z')^2}}$$

其中 $\rho_l = \rho_s dz'$

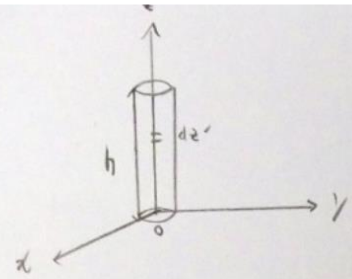
$$(a) dv = \frac{\rho_s b dz'}{2\varepsilon_0 \sqrt{b^2 + (z-z')^2}}$$

管外一點：$V = \int_{z'=0}^{z'=h} dv = \frac{b\rho_s}{2\varepsilon_0} l_n \frac{z+\sqrt{b^2+z^2}}{(z-h)+\sqrt{b^2+(z-h)^2}}$

$$E = -\vec{Q}_Z \frac{dv}{dz} = a_{\vec{z}} \frac{b\rho_s}{2\varepsilon_0}\left[\frac{1}{\sqrt{b^2+(z-h)^2}} - \frac{1}{\sqrt{b^2+z^2}}\right]$$

(b)管內各點 V 和 E 的表達式與管外各點的相同。

微積分:



$\rho_s = \frac{Q}{2\pi b h}$

a. outside

$V_o = \int_0^h dV = \int_0^h \frac{\rho_s b}{2\varepsilon_0\sqrt{b^2+(z-z')^2}} dz' = \frac{\rho_s b}{2\varepsilon_0}\left[-\ln\left[\sqrt{b^2+(z-z')^2}+(z-z')\right]\right]\Big|_0^h$

$= \frac{\rho_s b}{2\varepsilon_0}\ln\left[\frac{\sqrt{b^2+z^2}+z}{\sqrt{b^2+(h-z)^2}+(z-h)}\right]$

$\vec{E}_o = -\nabla V = -\hat{a}_z \frac{dV_o}{dz}$

$\frac{dV_o}{dz} = \frac{\rho_s b}{2\varepsilon_0}\left(\frac{\sqrt{b^2+(h-z)^2}+(z-h)}{\sqrt{b^2+z^2}+z}\right)\left(\frac{\left(\sqrt{b^2+(h-z)^2}+(z-h)\right)\left(\frac{2z}{2\sqrt{b^2+z^2}}+1\right)-\left(\sqrt{b^2+z^2}+z\right)(\Delta)}{\left(\sqrt{b^2+(h-z)^2}+z-h\right)^2}\right)$

$= \frac{\rho_s b}{2\varepsilon_0}\left(\frac{1}{\sqrt{b^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right)$

$\Delta = \frac{-2(h-z)}{2\sqrt{b^2+(h-z)^2}} + 1$

$\therefore \vec{E}_o = -\hat{a}_z \frac{dV_o}{dz}$

b. inside

$V_i = \int_0^z \frac{\rho_s b}{2\varepsilon_0\sqrt{b^2+(z-z')^2}} dz' + \int_z^h \frac{\rho_s b}{2\varepsilon_0\sqrt{b^2+(z-z')^2}} dz'$

$= \frac{\rho_s b}{2\varepsilon_0}\left[\left(-\ln\left(\sqrt{b^2+(z-z')^2}+(z-z')\right)\right)\Big|_0^z - \ln\left(\sqrt{b^2+(z-z')^2}+(z-z')\right)\Big|_z^h\right]$

$= \frac{\rho_s b}{2\varepsilon_0}\ln\left[\frac{\sqrt{b^2+z^2}+z}{\sqrt{b^2+(h-z)^2}+(z-h)}\right]$

$\therefore \vec{E}_o = \vec{E}_i = \hat{a}_z \frac{\rho_s b}{2\varepsilon_0}\left(\frac{1}{\sqrt{b^2+(h-z)^2}} - \frac{1}{\sqrt{b^2+z^2}}\right)$ #

P.3–24 Solve the following problems:

a) Find the breakdown voltage of a parallel-plate capacitor, assuming that conducting plates are 50 (mm) apart and the medium between them is air.

b) Find the breakdown voltage if the entire space between the conducting plates is filled with plexiglass, which has a dielectric constant 3 and a dielectric strength 20 (kV/mm).

c) If a 10-(mm) thick plexiglass is inserted between the plates, what is the maximum voltage that can be applied to the plates without a breakdown?

[1%、1%、2%]

i. Vbreak down

air ; $d = 50mm$ $E_{ab} = 3\ kV/mm$

$\rightarrow V = 3 \times 50 = 150\ kV$

ii. Vbreak down

$\varepsilon_r = 3$

$E = 20\ kV/mm$ $\rightarrow V = 20 \times 50 = 1000\ kV$

iii.

50mm ; 10mm glass $\rightarrow V = 40 \times 3 + \frac{1}{3} \times 10 \times 3 = 130\ kV$

C法二 3-24 iii

40mm : ε_0, E_1

10mm : $3\varepsilon_0$, E_2

依 B.C., $E_2 = \frac{1}{3} E_1$

因 $E_{B1} < E_{B2}$ ($E_{B1} = 3\ kV/mm$, $E_{B2} = 20\ kV/mm$)

∴ air 先 breakdown

$\rightarrow V_b = \frac{1}{3} \times E_{B1} \times 10 + E_{B1} \times 40 = 130\ kV$ #

materials	dielectric strength (kV/mm)
dry air	~3
mineral oil	~15
glass	~30
mica	~200

P.3–28 Dielectric lenses can be used to collimate electromagnetic fields. In Fig. 3–41 the left surface of the lens is that of a circular cylinder, and the right surface is a plane. If $\mathbf{E}_1$ at point $P(r_o, 45°, z)$ in region 1 is $\mathbf{a}_r 5 - \mathbf{a}_\phi 3$, what must be the dielectric constant of the lens in order that $\mathbf{E}_3$ in region 3 is parallel to the x-axis?

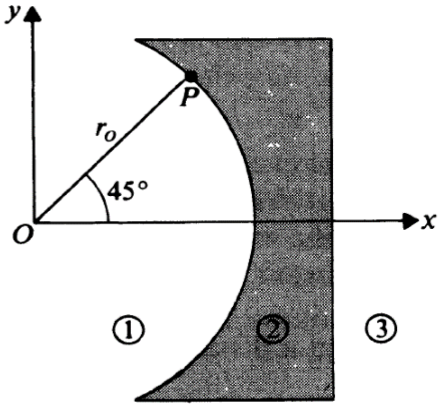


FIGURE 3–41
A dielectric lens (Problem P.3–28).

[4%]

:假設 $\vec{E}_2=\vec{a}_\gamma E_{2\gamma}+\vec{a}_\phi E_{2\phi}$

$B.C: \vec{a}_n \times \vec{E} = \vec{a}_n \times \vec{E}_2 \Rightarrow \vec{E}_{2\phi} = -3$

$\vec{E}$ 與 x 軸平行，$E_{2\phi}=-E_{2\gamma} \Rightarrow E_{2}r=3$

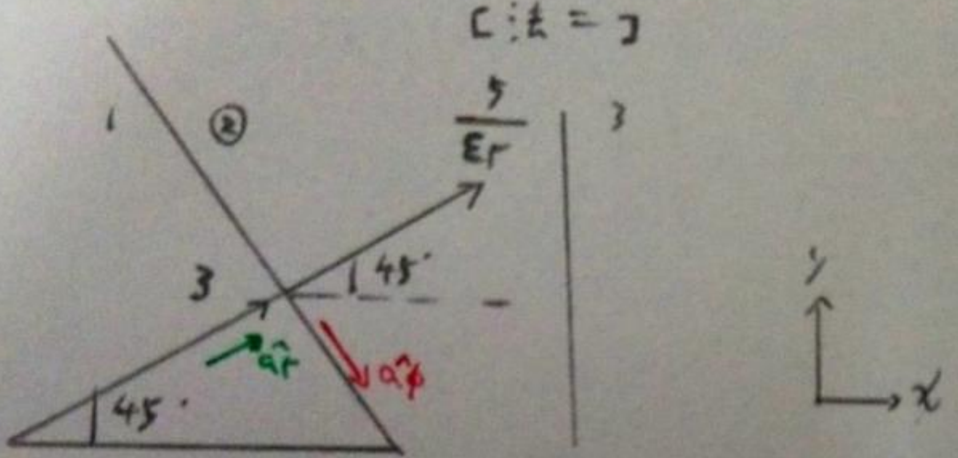
$B.C: \vec{a}_n \cdot \vec{D}_1 = \vec{a}_n \cdot \vec{D}_2$

則 $5=3\varepsilon_{\gamma 2}$

得 $\varepsilon_{\gamma 2}=\dfrac{5}{3}$

即透鏡的介電常數須爲 $\dfrac{5}{3}$，才使得區域 3 中的 $\vec{E}_3$ 平行於 x 軸.

[法二]



∵ let $\vec{E}_3$ 只存在 x 向

∴ 區域②之和電場只存在 x 向

=> let y 向的電場相消

$$3\sin 45° = \frac{5}{\varepsilon_r}\sin 45°$$

$$\varepsilon_r = \frac{5}{3} \quad \#$$

P.3–33 A cylindrical capacitor of length L consists of coaxial conducting surfaces of radii r_i and r_o. Two dielectric media of different dielectric constants ϵ_{r1} and ϵ_{r2} fill the space between the conducting surfaces as shown in Fig. 3–42. Determine its capacitance.

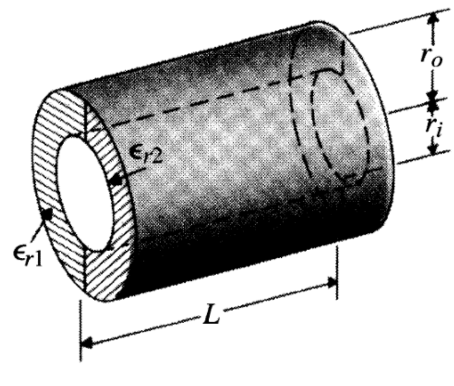
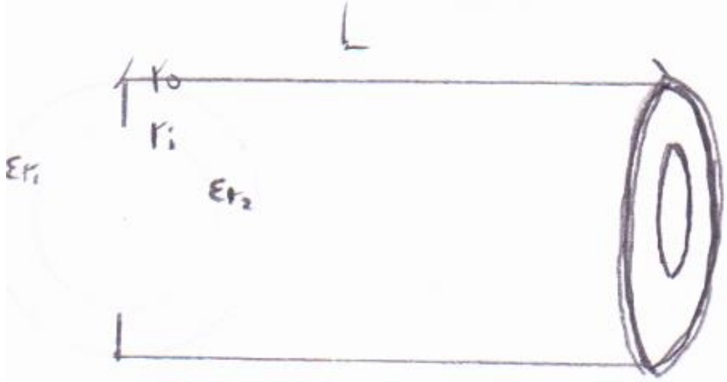


FIGURE 3–42
A cylindrical capacitor with two dielectric media (Problem P.3–33).

[4%]

cylinder capacitor



[法一]

Gauss. $\oint_S \vec{D}\cdot d\vec{S} = Q$

$$\varepsilon_0\varepsilon_{r1}E\pi r L + \varepsilon_0\varepsilon_{r2}E\pi r L = \rho_\ell L$$

$$E = \frac{\rho_\ell}{\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})\pi r}$$

$$V = \int_{r_i}^{r_o} E\,dr = \frac{\rho_\ell}{\pi\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})}\ln\left(\frac{r_o}{r_i}\right)$$

$$\therefore C = \frac{Q}{V} = L\,\frac{\pi\varepsilon_0(\varepsilon_{r1}+\varepsilon_{r2})}{\ln\frac{r_o}{r_i}}$$

[法二]

circuit

並聯, $C = C_1 + C_2 = \dfrac{\varepsilon_0\varepsilon_{r1}\pi L}{\ln\frac{r_o}{r_i}} + \dfrac{\varepsilon_0\varepsilon_{r2}\pi L}{\ln\frac{r_o}{r_i}}$

P.3–37 A capacitor consists of two concentric spherical shells of radii R_i and R_o. The space between them is filled with a dielectric of relative permittivity ϵ_r from R_i to $b(R_i < b < R_o)$ and another dielectric of relative permittivity $2\epsilon_r$ from b to R_o.

a) Determine **E** and **D** everywhere in terms of an applied voltage V.

b) Determine the capacitance.

[3%、3%]

假定電荷 Q 在內殼上，$-Q$ 在外殼上

$R_i<R<R_0$, $\vec{D}=\vec{a}_R\dfrac{Q}{4\pi R^2}$

$R_i<R<b$, $\vec{E}_1=\dfrac{\vec{D}}{\varepsilon_0\varepsilon_\gamma}$;

$b<R<R_0$, $\vec{E}_2=\dfrac{\vec{D}}{2\varepsilon_0\varepsilon_\gamma}$

$$V=-\int_{R_0}^{R_i}\vec{E}d\vec{R}=-\int_{b}^{R_i}E_1\,dR-\int_{R_0}^{b}\vec{E}_2 dR=\frac{Q}{4\pi\varepsilon_0\varepsilon_\gamma}\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_0}\right)$$

(a) $$\vec{D}=\vec{a}_R\frac{\varepsilon_0\varepsilon_\gamma V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_0}\right)}, R_i<R<R_0$$

$\vec{D}=0, \vec{E}=0, R<R$;及 $R>R_0$

$$\vec{E}_1=\vec{a}_R\frac{V}{R^2\left(\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_0}\right)};\ \vec{E}_2=\vec{a}_R\frac{V}{R^2\left(\frac{2}{R_i}-\frac{1}{b}-\frac{1}{R_0}\right)}$$

(b) $$C=\frac{Q}{V}=\frac{4\pi\varepsilon_0\varepsilon_\gamma}{\frac{1}{R_i}-\frac{1}{2b}-\frac{1}{2R_0}}$$