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Some important functions and their LT

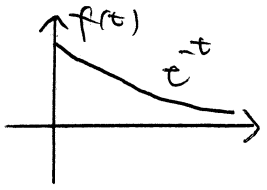
I. Step function

A function turns on at , like a switch.

Def : step function (also called "Heaviside function")

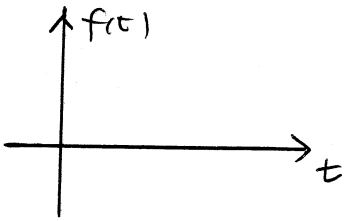
Step function is very useful to express a function

ex :



$f(t) = e^{-t}$

Shift in t-domain →

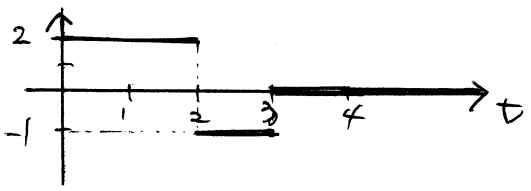


In a general form

$f(t)$ → shift in t-domain by "a"

Step function is also widely used to express

ex:



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LT of step function & piecewise-defined function

✱ step function: $f(t) = u_a$

$$\mathcal{L}\{u_a\} = \int_0^\infty u_a e^{-st}\,dt =$$

ex: $f(t) = u_2$. $\mathcal{L}\{f(t)\} = ?$

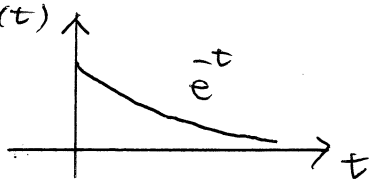
✱ piecewise-defined function

ex: $f(t) = 2 - 3u_2 + u_3$. $\mathcal{L}\{f(t)\} = ?$

LT of functions shifted in t-domain

We just learn a function shifted in t-domain can be expressed by the step function. Then what's the LT of functions shifted in t-domain?

ex1: $f(t)$

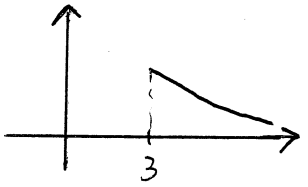


$f(t) = e^{-t}$

↓ $\mathcal{L}$

Shift in t-domain →

$f(t)$



$f(t) =$

↓ $\mathcal{L}$

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General form :

$f(t)$ shift in t-domain $\xrightarrow{\mathcal{L}}$

ex2: Given $F(s) = \frac{1-e^{-2s}}{s^2}$, what's the $\mathcal{L}^{-1}$?

Remark :

shift in t-domain $\longrightarrow$

what about "shift in s-domain" ?

II. Impulse function and delta function

Impulse function describes a very quick push on a system at a specific time.

Def : unit impulse centered at $t = t_0$

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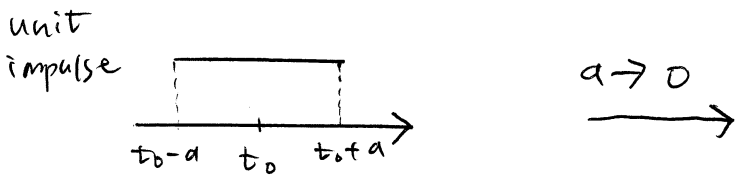
when the impulse is getting shorter:



as $a\to 0$, the unit impulse is called

L T of impulse function & delta function

$$\mathcal{L}\{\delta_a(t-t_0)\} = \int_0^\infty \delta_a(t-t_0)\, e^{-st}\, dt =$$

ex: $\mathcal{L}\{2\delta_4\} =$

III. Periodic function

ex:

Def: periodic function with period T