

國立清華大學

Definitions & terminology Cch 1.1 & 1.2)

① If y is a function of t $\xrightarrow[\text{as}]{\text{expressed}}$
 Derivative of y with respect to $t \rightarrow$
 where t : independent variable
 y : dependent variable

② Notations for "derivative"

③ Classification of DEs

— by "type": $\left\{ \begin{array}{l} \text{ODE} \rightarrow \text{one independent variable} \\ \text{ex:} \\ \text{PDE} \rightarrow \text{two or more independent variables} \\ \text{ex:} \end{array} \right.$

— by "order": check out the order of the highest derivative

ex: $y''' - y' + 6 = 0$

$y'' + 5(y')^3 - 4y = e^x$

— by "linearity": check out

if $\left\{ \begin{array}{l} \text{or} \\ \text{power of } y, y', y'', y''', \dots \text{ is more than 1} \end{array} \right. \rightarrow \text{nonlinear}$

otherwise \rightarrow linear

ex: $(1-y)y' + 2y = e^x$, $y'' + \sin y = 0$, $y^{(4)} + y^2 = 0$

國立清華大學

④ Express a DE in "normal form" or "differential form"

ex:

$$4x \frac{dy}{dx} + y = x \quad \begin{cases} \text{"normal form": } \frac{dy}{dx} = \frac{x-y}{4x} \\ \text{"differential form": } \end{cases}$$

⑤ Solution forms of DEs

explicit solution: expressed as $y = \phi(x)$
 (dependent variable is expressed only in terms of independent variable) $= \sqrt{c - \frac{x^2}{2}}$
 $= -\sqrt{c - \frac{x^2}{2}}$

implicit solution: expressed as $\frac{1}{2}x^2 + y^2 = c$

⑥ Problems can be categorized into

Initial-value problem (Ivp) or boundary-value problem (Bvp) depending on the conditions given.

ex: For a DE: $y'' - y = 0$

if given $y(1) = 0$, $y'(1) = 0 \Rightarrow$

if given $y'(0) = 0$, $y(\pi) = 0 \Rightarrow$

⑦ A DE always has more than one solution.

ex: $y'' = 1 \Rightarrow y = \frac{x^2}{2} + bx + c$, where b, c are any constant.