Chapter 10

Exercise

EXERCISE 10.10-1 Find the ratio V_o/V_s for the circuit shown in Figure 10.10-2 when $R_1 = R_2 = 1$ kΩ, $C_2 = 0$, $C_1 = 1$ μF, and $\omega = 1000$ rad/s.

Solution

The circuit of Figure 10.10-2 is an example of the inverting amplifier shown in Figure 10.10-1a. Using Eqs. 10.10-3 and 10.10-6, we obtain

$$\frac{\mathbf{V}_\text{o}}{\mathbf{V}_\text{s}} = -\frac{\mathbf{Z}_1}{\mathbf{Z}_2} = -\frac{\dfrac{R_2}{1 + j\omega C_2 R_2}}{\dfrac{R_1}{1 + j\omega C_1 R_1}} = -\frac{R_2(1 + j\omega C_1 R_1)}{R_1(1 + j\omega C_2 R_2)}$$

Substituting the given values of R_1; R_2; C_1; C_2; and ω gives

$$\frac{\mathbf{V}_\text{o}}{\mathbf{V}_\text{s}} = -\frac{10^3(1 + j10^3(10^{-6})10^3)}{10^3(1 + j10^3(0)10^3)} = -\frac{1 + j}{1} = -1 - j = 45^\circ$$

Section 10.2 Sinusoidal Sources

P10.2-1

Given:

$$v_1(t) = 5\cos\left(150t + 30^\circ\right)\text{ V}$$

$$v_2(t) = 4\cos\left(150t - 60^\circ\right)\text{ V}$$

So,

$$\theta_1 = 30^\circ$$

$$\theta_2 = -60^\circ$$

The period of the sinusoids is given by,

$$150 = \frac{2\pi}{T}$$

$$T = 42\text{ ms}$$

Compare $v_2(t)$ to $v_1(t)$

$$\theta_2 - \theta_1 = -60 - 30 = -90^\circ = -\frac{\pi}{2}\text{rad}$$

The minus sign (–) indicates a delay rather than an advance,
Convert the angle to time.

$$\theta_2 - \theta_1 = 2\pi\frac{t_d}{T}$$

$$t_d = \frac{(\theta_2 - \theta_1)T}{2\pi}$$

$$= \frac{(-\pi/2)\,(0.042)}{2\pi}$$

$$= -0.105s$$

P10.2-2

Given:

$$v_1(t) = 12\cos\left(150t - 102^\circ\right)\text{ V}$$

$$v_2(t) = 12\cos\left(150t - 54^\circ\right)\text{ V}$$

So,

$$\theta_1 = -102^\circ$$

$$\theta_2 = -54^\circ$$

The period of the sinusoids is given by,

$$150 = \frac{2\pi}{T}$$

$$T = 42 \text{ ms}$$

Compare $v_2(t)$ to $v_1(t)$

$$\theta_2 - \theta_1 = -54 + 102 = 48^\circ = 0.838 \text{ rad}$$

The plus sign (+) indicates an advance rather than a delay.

Convert the angle to time.

$$\theta_2 - \theta_1 = 2\pi \frac{t_d}{T}$$

$$t_d = \frac{(\theta_2 - \theta_1)T}{2\pi}$$

$$= \frac{(0.838)(0.042)}{2\pi}$$

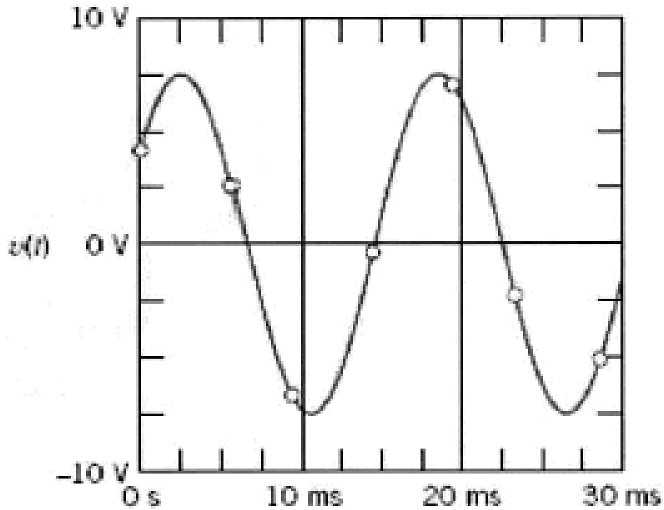
$$= 5.6 \text{ ms}$$

P10.2-3

Solution: The amplitude is *A* = 45 mv and the period is given by $\frac{T}{2} = 60 - 20 = 40 \text{ ms}$ so the period is *T* = 80 ms. The frequency is given by $\omega = \frac{2\pi}{80 \times 10^{-3}} = 78.54 \text{ rad/s}$. Noticing that *v*(*t*) is 0 at time 0 and is increasing at time 0, we can write

$$v(t) = 45\sin(78.54t) = 45\cos(78.54t - 90^\circ) \text{ mV}$$

P 10.2-4
Solution:



$$A = 7.5\text{ V}$$

$$T = 21.5 - 5.5 = 16\text{ ms}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.016} = 392\ \text{rad/s}$$

$$\theta = -\cos^{-1}\left(\frac{4}{7.5}\right) = -58°$$

$$v(t) = 7.5\cos(392t - 58°)\ \text{V}$$

Section 10.3 Phasors and Sinusoids

P10.3-1
Solution:

The current is given as,

$$i(t) = 3\cos(9t - 60°) + 3\sin(9t + 120°)\ \text{mA}$$

Express it in the general form,

$$i(t) = A\cos(\omega t + \theta)\ \text{mA}$$

First, find the Phasor form.

$$\begin{aligned} i(t) &= 3\cos(9t - 60°) + 3\sin(9t + 120°) \\ &= 3\cos(9t - 60°) + 3\cos(9t + 120° - 90) \\ &= 3\cos(9t - 60°) + 3\cos(9t + 30°) \\ &= 3\angle -60° + 3\angle 30° \\ &= 4.25\angle -15° \end{aligned}$$

Now,

$$\begin{aligned} i(t) &= A\cos(\omega t + \theta)\ \text{mA} \\ &= 4.25\cos(9t - 15°)\ \text{mA} \end{aligned}$$

P10.3-2
Solution:

The voltage is given as,

$v(t) = 6\sqrt{2}\cos(10t) + 3\sin(10t + 60^\circ)$ V

Express it in the general form,

$v(t) = A\cos(\omega t + \theta)$ V

First, find the Phasor form.

$$\begin{aligned} v(t) &= 6\sqrt{2}\cos(10t) + 3\sin(10t + 60^\circ) \\ &= 6\sqrt{2}\cos(10t) + 3\cos(10t + 60^\circ - 90^\circ) \\ &= 6\sqrt{2}\cos(10t) + 3\cos(10t - 30^\circ) \\ &= 6\sqrt{2}\angle 0^\circ + 3\angle -30^\circ \\ &= 11.2\angle -77^\circ \end{aligned}$$

Now,

$$\begin{aligned} v(t) &= A\cos(\omega t + \theta) \text{ V} \\ &= \underline{11.2\cos(10t - 7.7^\circ)\text{ V}} \end{aligned}$$

P 10.3-3
Solution:

$$\frac{(25\angle 36.9^\circ)(80\angle -53.1^\circ)}{(4+j8)+(6-j8)} = \frac{25\cdot 80\angle(36.9^\circ - 53.1^\circ)}{(4+6)+j(8-8)} = \frac{2000\angle -16.2^\circ}{10} = 200\angle -16.2^\circ$$

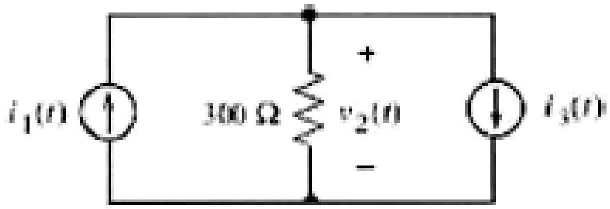
P 10.3-4

Solution:

$$\begin{aligned} \frac{(60\angle 120^\circ)(-16+j12+20\angle 15^\circ)}{5\angle -75^\circ} &= \frac{(60\angle 120^\circ)(-16+j12+19.3185+j5.1764)}{5\angle -75^\circ} \\ &= \frac{(60\angle 120^\circ)(3.3185+j17.1764)}{5\angle -75^\circ} \\ &= \frac{(60\angle 120^\circ)(17.494\angle 79.065^\circ)}{5\angle -75^\circ} \\ &= \frac{1049.6\angle -160.93^\circ}{5\angle -75^\circ} = 139.95\angle 109.07^\circ = 45.714 + j132.28 \end{aligned}$$

P10.3-5
Solution:

The circuit is shown below.



Given

$i_1(t) = 12\cos(30t)$

$i_3(t) = 12\cos(30t + 150^\circ)$

Now, from the circuit,

$i_2(t) = i_1(t) - i_3(t)$

So,

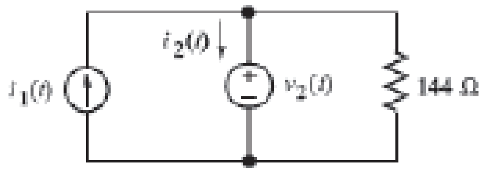
$$
\begin{aligned}
i_2(t) &= 12\cos(30t) - 12\cos(30t + 150^\circ) \\
&= 12\angle 0^\circ - 12\angle 150^\circ \\
&= 23.2\angle -15^\circ \text{ mA}
\end{aligned}
$$

Now,

$R = 300\,\Omega$

$$
\begin{aligned}
v_2(t) &= 300 \times 23.2\angle -15^\circ \times 10^{-3} \\
&= 6.94\angle -15^\circ \\
&= \underline{6.94\cos\left(30t - 15^\circ\right)} \text{ V}
\end{aligned}
$$

P10.3-6
Solution:



Given:

$i_1(t) = 0.18\cos(150t + 60°)$ A

$i_2(t) = 36\cos(150t + 75°)$ A

Now,

$R = 144\ \Omega$

Now, from the circuit, apply the voltage law

$144(i_1(t) - i_2(t)) - v_2(t) = 0$

So,

$$v_2(t) = 144(0.18\cos(150t + 60°)) - 144 i_2(t)$$

$$36\cos(150t - 75°) = 144(0.18\cos(150t + 60°)) - 144 i_2(t)$$

$$36\angle -75° = 25.92\angle 60° - 144 i_2(t)$$

$$i_2(t) = \frac{57.34\angle 86.36°}{144}$$

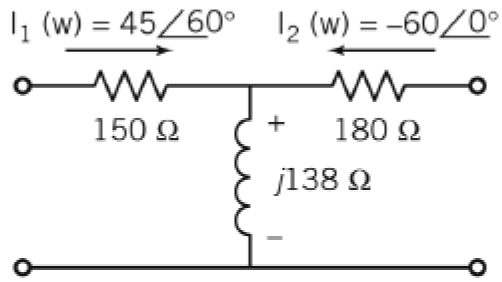
$$i_2(t) = 0.4\angle 86.36° \text{ A}$$

$$i_2(t) = 0.4\cos(150t + 86.36°) \text{ A}$$

P 10.3-7

Solution:

The figure below shows the RL circuit:



The impedance of the inductor is:

$$\mathbf{Z}_L = j\omega L$$

Substitute $6\,\text{rad/s}$ for ω, and $23\,\text{H}$ for L,

$$\begin{aligned}\mathbf{Z}_L &= j(6\,\text{rad/s})(23\,\text{H}) \\ &= j138\,\Omega\end{aligned}$$

The phasor representation of current $i_1(t) = 45\cos(6t + 60°)$ is:

$$\mathbf{I}_1(\omega) = 45\angle 60°$$

The phasor representation of current $i_2(t) = -60\cos(6t)$ is:

$$\mathbf{I}_2(\omega) = -60\angle 0°$$

Apply KCL at node 'a',

$$\begin{aligned}\mathbf{I}(\omega) &= \mathbf{I}_1(\omega) + \mathbf{I}_2(\omega) \\ &= 45\angle 60° - 60\angle 0°\end{aligned}$$

The current through the inductor is given as:

$$\mathbf{I}(\omega) = \frac{\mathbf{V}(\omega)}{\mathbf{Z}_L}$$

Substitute the same in equation $\mathbf{I}(\omega) = 45\angle 60° - 60\angle 0°$,

$$\begin{aligned}\frac{\mathbf{V}(\omega)}{\mathbf{Z}_L} &= 45\angle 60° - 60\angle 0° \\ \mathbf{V}(\omega) &= \mathbf{Z}_L(45\angle 60° - 60\angle 0°) \\ &= (j138\,\Omega)(45\cos 60°\text{ mA} + j45\sin 60°\text{ mA} - 60\cos 0°\text{ mA}) \\ &= (j138\,\Omega)\left((-37.5\text{ mA})\left(\frac{10^{-3}\text{ A}}{1\text{ mA}}\right) + (j38.9\text{ mA})\left(\frac{10^{-3}\text{ A}}{1\text{ mA}}\right)\right) \\ &= -5.17 + j5.36\text{ V}\end{aligned}$$

The phase of voltage $\mathbf{V}(\omega)$ is:

$$\theta = 180° - \tan^{-1}\left(\frac{5.36}{-(-5.17)}\right)$$
$$= 134°$$

The amplitude of the voltage $\mathbf{V}(\omega)$ is:

$$V = \sqrt{(-5.17\text{ V})^2 + (5.36\text{ V})^2}$$
$$= 7.44\text{ V}$$

The voltage $v(t)$ is:

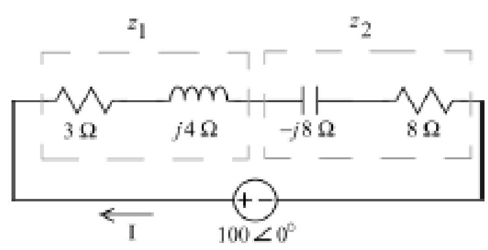
$$v(t) = V\cos(\omega t + \theta)$$

Substitute 7.44 V for V, $134°$ for θ, and 6 rad/s for ω,

$$v(t) = 7.44\text{ V}\cos(6t + 134°)$$

Therefore, the voltage $v(t)$ is $\boxed{7.44\text{ V}\cos(6t+134°)}$.

P 10.3-8
Solution:



(a) $\underline{\mathbf{Z}_1 = 3 + j4 = 5\angle 53.1°\ \Omega}$ and $\underline{\mathbf{Z}_2 = 8 - j8 = 8\sqrt{2}\ \angle -45°\ \Omega}$

(b) Total impedance $= \mathbf{Z}_1 + \mathbf{Z}_2 = 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\angle -20.0°\ \Omega}$

(c) $\mathbf{I} = \dfrac{100\angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2} = \dfrac{100}{11.7\ \angle -20°} = \dfrac{100}{11.7}\angle 20.0° \Rightarrow \underline{i(t) = 8.55\ \cos\ (1250t + 20.0°)\text{ A}}$

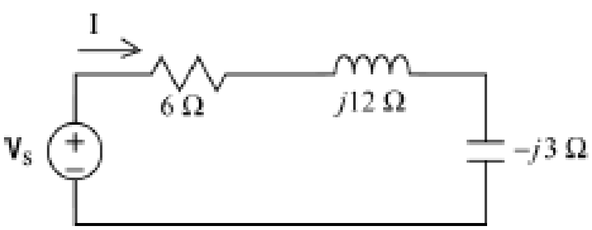
P 10.3-9
Solution:

$$\begin{aligned}\mathbf{V}_1(\omega) = \mathbf{V}_s(\omega) - \mathbf{V}_2(\omega) &= 7.68\angle 47° - 1.59\angle 125° \\ &= (5.23 + j5.62) - (-0.91 + 1.30) \\ &= (5.23 + 0.91) + j(5.62 - 1.30) \\ &= 6.14 + j4.32 \\ &= 7.51\angle 35°\end{aligned}$$

$$v_1(t) = 7.51\cos(2t + 35°) \ \text{V}$$

P 10.3.10

Solution:



$$\mathbf{V}_s = 2\angle 30° \ \text{V}$$

and $\mathbf{I} = \dfrac{2\angle 30°}{6 + j12 + 3/j} = 0.185\angle -26.3° \ \text{A}$

$$\underline{i(t) = 0.185\ \cos\ (4t - 26.3°)\ \text{A}}$$

P 10.3-11

Solution: (a)

$$12 + v(t) - 10(0.02) = 0 \quad \Rightarrow \quad v(t) = 2 - 12 = -10 \ \text{V}$$

(b)

$$\begin{aligned}12 + v(t) - 10\,i(t) = 0 \quad \Rightarrow \quad v(t) &= 10\big(0.020\cos(100t + 15°)\big) - 12 \\ &= 0.2\cos(100t + 15°) - 12\end{aligned}$$

(c)

$$v_C(t) + v(t) - 10\,i(t) = 0$$

$$0.1\cos(100t + 105°) + v(t) - 10\big(0.020\cos(100t + 15°)\big) = 0$$

$$v(t) = 0.2\cos(100t + 15°) - 0.1\cos(100t + 105°) \ \text{V}$$

$$\mathbf{V} = 0.2\angle 15° - 0.1\angle 105° = (0.193 + j\,0.052) - (-0.0259 + j\,0.0966)$$
$$= 0.219 - j0.045 = 0.2236\angle -11.6° \text{ V}$$

$$v(t) = 0.2236\cos(100t - 11.6°) \text{ V}$$

Section 10.4 Impedances

P10.4-1
Solution:

$$\mathbf{Z}_R = 8\ \Omega,\ \mathbf{Z}_C = \frac{1}{j3\frac{1}{12}} = \frac{4}{j} = \frac{j4}{j \times j} = -j\,4\ \Omega,\ \mathbf{Z}_{L1} = j\,3\,(2) = j\,6\ \Omega,$$
$$\mathbf{Z}_{L2} = j\,3\,(4) = j\,12\ \Omega \text{ and } \mathbf{I}_S = 4\angle 15° \text{ A.}$$

P10.4-2
Solution:
The current $i_1(t) = 600\cos(38t - 30°)$ can be represented in phasor form as:

$$\mathbf{I}_1(\omega) = 600\angle -30°$$

The current $i_2(t) = 1125\cos(38t + 75°)$ can be represented in phasor form as:

$$\mathbf{I}_2(\omega) = 1125\angle 75°$$

The impedance of inductor is:

$$\mathbf{Z}_L = j\omega L$$

Substitute 38 rad/s for ω, and 96 mH for L,

$$\mathbf{Z}_L = j(38\text{ rad/s})(96\text{ mH})\left(\frac{10^{-3}\text{ H}}{1\text{ mH}}\right)$$
$$= j3.64\ \Omega$$

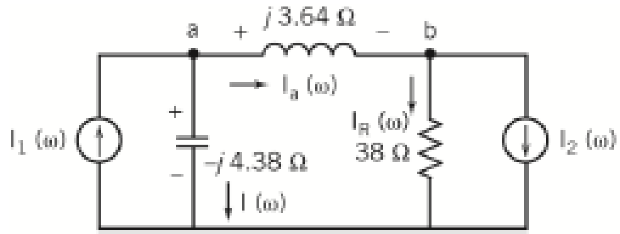
The impedance of the capacitor is:

$$\mathbf{Z}_C = \frac{-j}{\omega C}$$

Substitute 38 rad/s for ω, and 6 mF for C,

$$\mathbf{Z}_C = \frac{-j}{(38\ \text{rad/s})(6\ \text{mF})\left(\frac{10^{-3}\ \text{F}}{1\ \text{mF}}\right)}$$
$$= -j4.38\ \Omega$$

The figure below shows the circuit in frequency domain:



Apply KCL at node 'a':

$$\mathbf{I}_1(\omega) = \mathbf{I}(\omega) + \mathbf{I}_a(\omega) \qquad \text{...... (1)}$$

Apply KCL at node 'b':

$$\mathbf{I}_a(\omega) = \mathbf{I}_R(\omega) + \mathbf{I}_2(\omega) \qquad \text{...... (2)}$$

KVL in middle loop gives,

$$\mathbf{I}(\omega)\mathbf{Z}_C - \mathbf{V}_a(\omega) - \mathbf{I}_R(\omega)R = 0$$
$$\mathbf{I}(\omega)\mathbf{Z}_C - \mathbf{I}_a(\omega)\mathbf{Z}_L - \mathbf{I}_R(\omega)R = 0$$

Substitute $\mathbf{I}_a(\omega) = \mathbf{I}_1(\omega) - \mathbf{I}(\omega)$ (from equation 1),

$$\mathbf{I}(\omega)\mathbf{Z}_C - \left(\mathbf{I}_1(\omega) - \mathbf{I}(\omega)\right)\mathbf{Z}_L - \mathbf{I}_R(\omega)R = 0$$
$$\mathbf{I}(\omega)(\mathbf{Z}_C + \mathbf{Z}_L) - \mathbf{I}_1(\omega)\mathbf{Z}_L = \mathbf{I}_R(\omega)R$$

Substitute $\mathbf{I}_R(\omega)=\mathbf{I}_1(\omega)-\mathbf{I}_2(\omega)-\mathbf{I}(\omega)$ (from equation 3),

$$\begin{aligned}\mathbf{I}(\omega)(\mathbf{Z}_C+\mathbf{Z}_L)-\mathbf{I}_1(\omega)\mathbf{Z}_L &= (\mathbf{I}_1(\omega)-\mathbf{I}_2(\omega)-\mathbf{I}(\omega))R \\ \mathbf{I}(\omega)(\mathbf{Z}_C+\mathbf{Z}_L+R) &= \mathbf{I}_1(\omega)(\mathbf{Z}_L+R)-\mathbf{I}_2(\omega)R \\ \mathbf{I}(\omega) &= \frac{\mathbf{I}_1(\omega)(\mathbf{Z}_L+R)-\mathbf{I}_2(\omega)R}{\mathbf{Z}_C+\mathbf{Z}_L+R}\end{aligned}$$

Substitute $\mathbf{I}_1(\omega)=600\angle-30°$, $\mathbf{I}_2(\omega)=1125\angle75°$, $\mathbf{Z}_L=j3.64\,\Omega$, $\mathbf{Z}_C=-j4.38\,\Omega$, and $R=38\,\Omega$,

$$\begin{aligned}\mathbf{I}(\omega) &= \frac{(600\angle-30°)(j3.64\,\Omega+38\,\Omega)-(1125\angle75°)(38\,\Omega)}{-j4.38\,\Omega+j3.64\,\Omega+38\,\Omega} \\ &= \frac{(600\cos(-30°)+j600\sin(-30°))(j3.64\,\Omega+38\,\Omega)-(1125\cos(75°)+j1125\sin(75°))(38\,\Omega)}{-j4.38\,\Omega+j3.64\,\Omega+38\,\Omega} \\ &= \frac{9772.8-j50802\,\Omega\text{A}}{38-j0.74\,\Omega} \\ &= 231-j1331\text{ A}\end{aligned}$$

The phase of current $\mathbf{I}(\omega)$ is:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-1331}{231}\right) \\ &= -80°\end{aligned}$$

The amplitude of current $\mathbf{I}(\omega)$ is:

$$\begin{aligned}I &= \sqrt{(231\text{ A})^2+(-1331\text{ A})^2} \\ &= 1351\text{ A}\end{aligned}$$

Therefore, the current $\mathbf{I}(\omega)$ in frequency domain is:

$$\begin{aligned}\mathbf{I}(\omega) &= I\angle\theta \\ &= 1351\angle-80°\end{aligned}$$

Substitute $\mathbf{I}(\omega)$ and $\mathbf{I}_1(\omega)$ back in equation $\mathbf{I}_a(\omega)=\mathbf{I}_1(\omega)-\mathbf{I}(\omega)$,

$$\begin{aligned}\mathbf{I}_a(\omega) &= \mathbf{I}_1(\omega) - \mathbf{I}(\omega) \\ &= (600\angle -30°) - (1351\angle -80°) \\ &= 519.6 - j300 - 234.5 + j1330.4 \\ &= 285.1 + j1030.4 \text{ A}\end{aligned}$$

The voltage $\mathbf{V}_a(\omega)$ is:

$$\begin{aligned}\mathbf{V}_a(\omega) &= \mathbf{I}_a(\omega)\mathbf{Z}_L \\ &= (285.1 + j1030.4 \text{ A})(j3.64\ \Omega) \\ &= -3750.6 + j1037.7 \text{ V}\end{aligned}$$

The phase of voltage $\mathbf{V}_a(\omega)$ is:

$$\begin{aligned}\phi &= 180° - \tan^{-1}\left(\frac{1037.7}{3750.6}\right) \\ &= 165°\end{aligned}$$

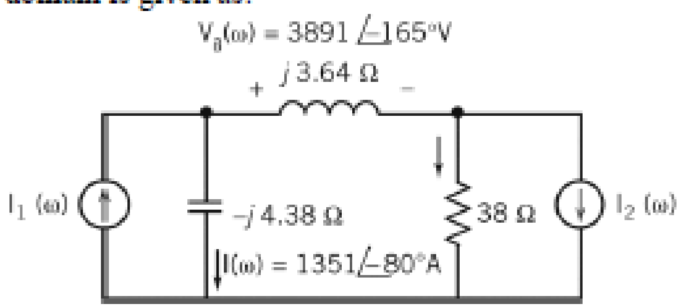
The amplitude of voltage $\mathbf{V}_a(\omega)$ is:

$$\begin{aligned}V &= \sqrt{(-3750.6 \text{ V})^2 + (1037.7 \text{ V})^2} \\ &= 3891 \text{ V}\end{aligned}$$

Therefore, the voltage $\mathbf{V}_a(\omega)$ in frequency domain is:

$$\begin{aligned}\mathbf{V}_a(\omega) &= V\angle\phi \\ &= 3891\angle 165°\end{aligned}$$

The circuit in frequency domain is given as:



P10.4-3
Solution:
In order to show the given circuit in frequency- domain, proceed as follows:

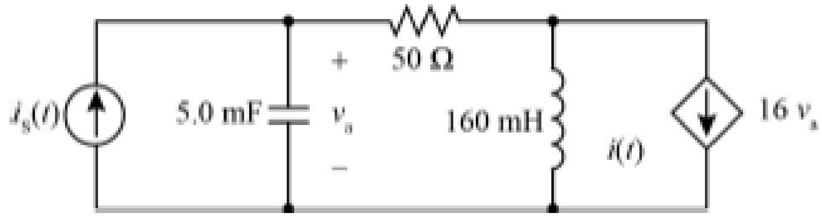


Figure 1: Depicts the circuit

As per Figure 1, the current is $i_s(t) = 2.88(\cos 100t - 48°)$ mA
Resistance $R = 50\,\Omega$
Capacitance $C = 5$ mF
Inductance $L = 160$ mH
Voltage is $16v_A$
Source current is $i_s(t)$

Now the above parameters in s-domain is are as follows:

Current is $i_s(t) = \dfrac{j10^4}{j10^4 + 48}$

Resistance $R = 50\,\Omega$
Capacitance $C = 16/j100$
Inductance $L = j250$
Voltage is $16v_a$
Source current is $i_s(s)$

As per the above s-domain parameters, the circuit diagram shown in Figure 1 is depicted as a s-domain as below:

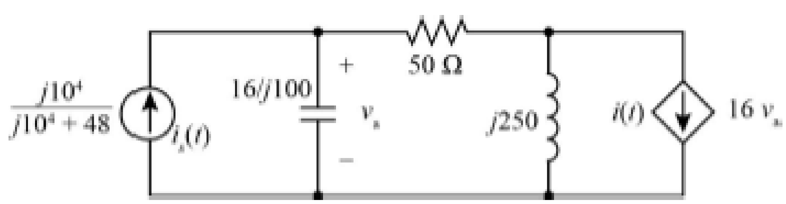
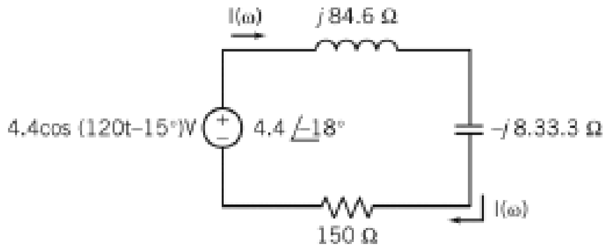


Figure 2: Depicts the s-domain circuit

P10.4-4
Solution:
The figure below shows the series LCR circuit:



The impedance of inductor is:

$$\mathbf{Z}_L = j\omega L$$

Substitute $120\,\text{rad/s}$ for ω, and $288\,\text{mH}$ for L,

$$\begin{aligned}\mathbf{Z}_L &= j(120\,\text{rad/s})(288\,\text{mH})\left(\frac{10^{-3}\,\text{H}}{1\,\text{mH}}\right)\\ &= j34.6\,\Omega\end{aligned}$$

The impedance of the capacitor is:

$$\mathbf{Z}_C = \frac{-j}{\omega C}$$

Substitute $120\,\text{rad/s}$ for ω, and $10\,\mu\text{F}$ for C,

$$\begin{aligned}\mathbf{Z}_C &= \frac{-j}{(120\,\text{rad/s})(10\,\mu\text{F})\left(\frac{10^{-6}\,\text{F}}{1\,\mu\text{F}}\right)}\\ &= -j833.3\,\Omega\end{aligned}$$

The input voltage $v(t) = 4.4\cos(120t - 15°)$ can be written in Cartesian form as:

$$\begin{aligned}\mathbf{V}(\omega) &= 4.4\angle -15°\\ &= 4.4\cos(-15°) + j4.4\sin(-15°)\\ &= 4.25 - j1.13\end{aligned}$$

The inductor, capacitor and the resistor are in series, and the net impedance of the three elements is:

$$\begin{aligned}\mathbf{Z} &= \mathbf{Z}_C + \mathbf{Z}_L + R \\ &= -j833.3\,\Omega + j34.6\,\Omega + 150\,\Omega \\ &= 150\,\Omega - j799\,\Omega\end{aligned}$$

The current in frequency domain is:

$$\begin{aligned}\mathbf{I}(\omega) &= \frac{\mathbf{V}(\omega)}{\mathbf{Z}} \\ &= \frac{4.25 - j1.13\text{ V}}{150 - j799\,\Omega} \\ &= \left(\frac{4.25 - j1.13}{(150)^2 - (j799)^2}\right)(150 + j799) \\ &= 2.33 - j4.88\text{ mA}\end{aligned}$$

The phase angle of current $\mathbf{I}(\omega)$ is:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-4.88}{2.33}\right) \\ &= -64.4°\end{aligned}$$

The amplitude of current $\mathbf{I}(\omega)$ is:

$$\begin{aligned}I &= \sqrt{(2.33\text{ mA})^2 + (-4.88\text{ mA})^2} \\ &= 5.40\text{ mA}\end{aligned}$$

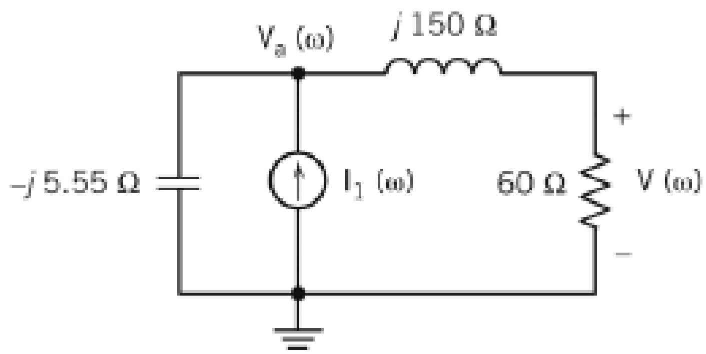
Therefore, the current $\mathbf{I}(\omega)$ in frequency domain is:

$$\begin{aligned}\mathbf{I}(\omega) &= I\angle\theta \\ \mathbf{I}(\omega) &= 5.40\text{ mA}\angle -64.4°\end{aligned}$$

The current $i(t)$ is:

$$\begin{aligned}i(t) &= I\cos(120t - \theta) \\ &= 5.40\text{ mA}\cos(120t - 64.4°)\end{aligned}$$

P10.4-5
Solution:
The figure below shows the given circuit:



The impedance of inductor is:

$$\mathbf{Z}_L = j\omega L$$

Substitute $6000\ \text{rad/s}$ for ω, and $25\ \text{mH}$ for L,

$$\begin{aligned}\mathbf{Z}_L &= j(6000\ \text{rad/s})(25\ \text{mH})\left(\frac{10^{-3}\ \text{H}}{1\ \text{mH}}\right)\\ &= j150\ \Omega\end{aligned}$$

The impedance of the capacitor is:

$$\mathbf{Z}_C = \frac{-j}{\omega C}$$

Substitute $6000\ \text{rad/s}$ for ω, and $30\ \mu\text{F}$ for C,

$$\begin{aligned}\mathbf{Z}_C &= \frac{-j}{(6000\ \text{rad/s})(30\ \mu\text{F})\left(\frac{10^{-6}\ \text{F}}{1\ \mu\text{F}}\right)}\\ &= -j5.55\ \Omega\end{aligned}$$

The phasor representation of current $i(t) = 180\cos(6000t)$ is:

$$\begin{aligned}\mathbf{I}(\omega) &= 180\angle 0^\circ\\ &= 180\cos(0^\circ) - j180\sin(0^\circ)\\ &= 180\ \text{mA}\end{aligned}$$

Apply Nodal analysis at node 'a':

$$\frac{\mathbf{V}_a(\omega)}{\mathbf{Z}_\text{C}}+\frac{\mathbf{V}_a(\omega)}{\mathbf{Z}_\text{L}+R}=\mathbf{I}(\omega)$$

$$\mathbf{V}_a(\omega)=\frac{\mathbf{I}(\omega)}{\left(\frac{1}{\mathbf{Z}_\text{C}}+\frac{1}{\mathbf{Z}_\text{L}+R}\right)}$$

$$=\frac{\mathbf{I}(\omega)(\mathbf{Z}_\text{C})(\mathbf{Z}_\text{L}+R)}{\mathbf{Z}_\text{C}+\mathbf{Z}_\text{L}+R}$$

Substitute the values,

$$\mathbf{V}_a(\omega)=\frac{(180\text{ mA})(-j5.55\,\Omega)(j150\,\Omega+60\,\Omega)}{-j5.55\,\Omega+j150\,\Omega+60\,\Omega}$$

$$=\frac{150-j60\text{ A}\Omega^2}{60\,\Omega+j144.5\,\Omega}$$

The voltage division rule between the inductor and the resistor gives the voltage $\mathbf{V}(\omega)$ as:

$$\mathbf{V}(\omega)=\left(\frac{R}{R+\mathbf{Z}_\text{L}}\right)\mathbf{V}_a(\omega)$$

Substitute the values,

$$\mathbf{V}(\omega)=\left(\frac{60\,\Omega}{60\,\Omega+j150\,\Omega}\right)\left(\frac{150-j60\text{ A}\Omega^2}{60\,\Omega+j144.5\,\Omega}\right)$$

$$=\frac{9000-j3600\text{ A}\Omega^3}{-18075+j17670\,\Omega^2}$$

$$=\frac{1800-j720\text{ A}\Omega^3}{3615+j3534\,\Omega^2}$$

$$=0.51+j0.14\text{ V}$$

The phase angle of the voltage $\mathbf{V}(\omega)$ is:

$$\theta=\tan^{-1}\left(\frac{0.14}{0.51}\right)$$

$$=16^\circ$$

The amplitude of voltage $\mathbf{V}(\omega)$ is:

$$\begin{aligned} V &= \sqrt{(0.14\ \text{V})^2 + (0.51\ \text{V})^2} \\ &= 0.65\ \text{V} \end{aligned}$$

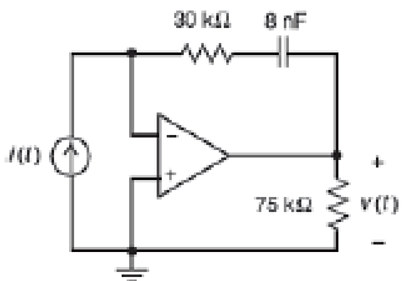
The voltage across the resistor is:

$$\begin{aligned} v(t) &= V\cos(6000t + \theta) \\ &= 0.65\cos(6000t + 16°) \end{aligned}$$

Therefore, the voltage across the resistor is $\boxed{0.65\cos(6000t + 16°)}$.

P10.4-6
Solution:

The figure below shows the op-amp circuit:



The net impedance of the series RL circuit is:

$$\mathbf{Z} = \frac{-j}{\omega C} + R$$

Substitute the given values,

$$\begin{aligned} \mathbf{Z} &= \frac{-j}{(15000\ \text{rad/s})(8\ \text{nF})\left(\frac{10^{-9}\ \text{F}}{1\ \text{nF}}\right)} + 30\ \text{k}\Omega \\ &= -j(8333.3\ \Omega)\left(\frac{10^{-3}\ \text{k}\Omega}{1\ \Omega}\right) + 30\ \text{k}\Omega \\ &= 30\ \text{k}\Omega - j8.33\ \text{k}\Omega \end{aligned}$$

The phasor representation of current $i(t) = 120\cos(15000t)\ \mu\text{A}$ is:

$$\begin{aligned}\mathbf{I}(\omega) &= 120\angle 0^\circ \\ &= 120\ \mu\text{A}\cos(0^\circ) + j120\ \mu\text{A}\sin(0^\circ) \\ &= 120\ \mu\text{A}\end{aligned}$$

KCL at node 'a' gives:

$$\mathbf{I}(\omega) = \frac{\mathbf{V}(\omega) - \mathbf{V}_a(\omega)}{\mathbf{Z}} + \mathbf{I}_-(\omega)$$

For an ideal op-amp, the voltage at the non-inverting terminal is equal to the voltage at the inverting terminal. Therefore, $\mathbf{V}_a(\omega) = 0$. Also, no current passes through these terminal and account for the fact that $\mathbf{I}_-(\omega) = 0$.

Substitute the same in equation $\mathbf{I}(\omega) = \frac{\mathbf{V}(\omega) - \mathbf{V}_a(\omega)}{\mathbf{Z}} + \mathbf{I}_-(\omega)$,

$$\begin{aligned}\mathbf{I}(\omega) &= \frac{\mathbf{V}(\omega) - 0}{\mathbf{Z}} + 0 \\ \mathbf{V}(\omega) &= \mathbf{I}(\omega)\mathbf{Z}\end{aligned}$$

Substitute $120\ \mu\text{A}$ for $\mathbf{I}(\omega)$, and $30\ \text{k}\Omega - j8.33\ \text{k}\Omega$ for $\mathbf{Z}$,

$$\begin{aligned}\mathbf{V}(\omega) &= 120\ \mu\text{A}(30\ \text{k}\Omega - j8.33\ \text{k}\Omega) \\ &= (120\ \mu\text{A})\left(\frac{10^{-6}\ \text{A}}{1\ \mu\text{A}}\right)(30\ \text{k}\Omega)\left(\frac{10^3\ \Omega}{1\ \text{k}\Omega}\right) - j(120\ \mu\text{A})\left(\frac{10^{-6}\ \text{A}}{1\ \mu\text{A}}\right)(8.33\ \text{k}\Omega)\left(\frac{10^3\ \Omega}{1\ \text{k}\Omega}\right) \\ &= 3.6\ \text{V} - j0.99\ \text{V}\end{aligned}$$

The phase of the voltage $\mathbf{V}(\omega)$ is:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-0.99}{3.6}\right) \\ &= -15.6^\circ\end{aligned}$$

The amplitude of the voltage $\mathbf{V}(\omega)$ is:

$$\begin{aligned} V &= \sqrt{(3.6\text{ V})^2 + (-0.99\text{ V})^2} \\ &= 3.73\text{ V} \end{aligned}$$

The voltage in time domain is:

$$\begin{aligned} v(t) &= V\cos(15000t + \theta) \\ &= 3.73\cos(15000t - 15.6^\circ)\text{ V} \end{aligned}$$

Therefore, the voltage across the $75\text{ k}\Omega$ resistor is $\boxed{3.73\cos(15000t - 15.6^\circ)\text{ V}}$.

P 10.4-7
Solution:

(a) $v = 15\cos(400t + 30^\circ)$ V

$i = 3\sin(400t + 30^\circ) = 3\cos(400t - 60^\circ)$ V

v leads i by $90^\circ \Rightarrow$ element is an <u>inductor</u>

$$|\mathbf{Z}_L| = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{15}{3} = 5 = \omega L = 400L \Rightarrow \underline{L = 0.0125\text{ H} = 12.5\text{ mH}}$$

(b) i leads v by $90^\circ \Rightarrow$ the element is a <u>capacitor</u>

$$|\mathbf{Z}_c| = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{8}{2} = 4 = \frac{1}{\omega C} = \frac{1}{900C} \Rightarrow \underline{C = 277.77\ \mu\text{F}}$$

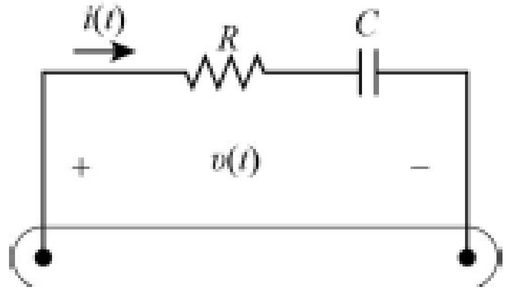
(c) $v = 20\cos(250t + 60^\circ)$ V

$i = 5\sin(250t + 150^\circ) = 5\cos(250t + 60^\circ)$ A

Since v & i are in phase $\Rightarrow$ element is a <u>resistor</u>

$$\therefore R = \frac{v_{\text{peak}}}{i_{\text{peak}}} = \frac{20}{5} = \underline{4\ \Omega}$$

P10.4-8
Solution:
In order to find the values of resistances and capacitances proceed as follows:



Impedance Z is given as follows:

$$Z = \frac{v(t)}{i(t)}$$

Here,

Voltage signal $v(t)$ is $30\cos(30t + 30°)$ V
Current signal $i(t)$ is $2.2\cos(30t + 78°)$ A

Substitute the corresponding values to obtain Z as follows:

$$\begin{aligned} Z &= \frac{v(t)}{i(t)} \\ &= \frac{30\angle 30°}{2.2\angle 78°} \\ &= 9.125 - j0.134 \end{aligned}$$

Impedance Z is also represented as:

$$Z = R - j\omega C$$

Here,

Resistance is R
Capacitance is C
Angular frequency ω is 30

Now resistance R is given as:

$$Z = R - j\omega C$$
$$9.125 - j0.134 = R - j\omega C$$
$$\Rightarrow R = 9.125\,\Omega$$

Hence, resistance R is $\boxed{9.125\,\Omega}$

Capacitance C is given as:

$$Z = R - j\omega C$$
$$9.125 - j0.134 = R - j\omega C$$
$$\Rightarrow -j\omega C = -j0.134$$
$$C = \frac{0.134}{30}$$
$$= 4.467 \text{ mF}$$

Hence, capacitance C is $\boxed{4.467 \text{ mF}}$

P10.4-9

Solution:

The impedance between nodes a and b is given by

$$18 + j(10)(2.5) = 18 + j25 = 30.8\angle 54.2°$$

To find the impedance between nodes b and c we first find the impedance of the capacitor:

$$-j\frac{1}{(10)(0.004)} = -j\frac{1}{0.04} = -j25$$

then

$$\frac{9(-j25)}{9-j25} = \frac{-j225}{26.57\angle -70.2°} = \frac{225\angle -90°}{26.57\angle -70.2°} = 8.47\angle -19.8° \ \Omega$$

The impedance between nodes c and d is given by

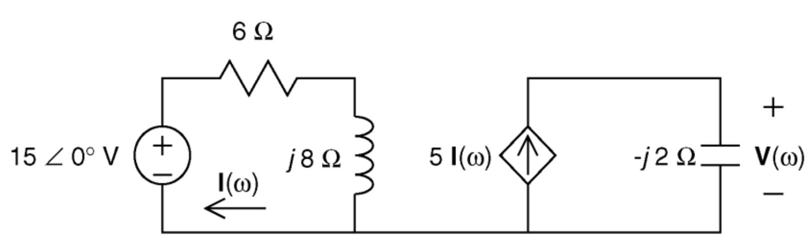
$$\frac{(5)(j(10)(0.88))}{5+j(10)(0.8)} - j\frac{1}{(10)(0.005)} = \frac{j40}{5+j8} - j\frac{1}{0.05} = \frac{j40}{5+j8}\left(\frac{5-j8}{5-j8}\right) - j20$$
$$= \frac{320+j200}{25+64} - j20$$
$$= 3.60 + j2.25 - j20 = 3.60 - j17.75 \ \Omega$$

So

$$A = 30.8 \text{ V}, B = 8.47 \ \Omega, , a = 3.57 \ \Omega \text{ and } b = -17.75 \ \Omega.$$

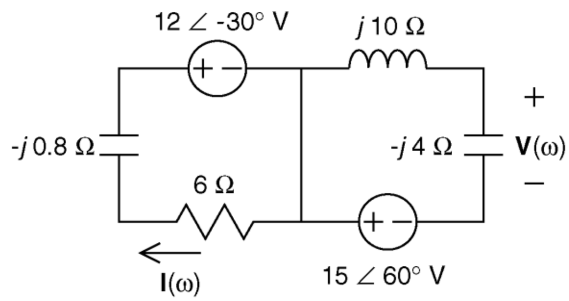
P10.4-10

Solution:



P 10.4-11

Solution:



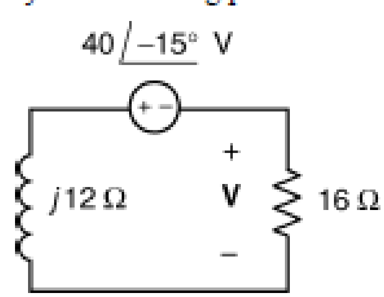
P 10.4-12
Solution:

$$\mathbf{Z}=\frac{\mathbf{V}}{-\mathbf{I}}=\frac{10\ \angle 40^\circ}{-2\times 10^{-3}\ \angle -165^\circ}=\ -5000\angle 205^\circ\Omega\ =4532+j\,2113\ =\ R\ +j\,\omega L$$

$$\text{so } \underline{R=4532\ \ \Omega} \ \text{ and } \ L=\frac{2113}{\omega}=\frac{2113}{2\times 10^6}=\underline{1.057\text{ mH}}$$

Section 10.5 Series and Parallel Impedances

P10.5-1
Solution:
Represent the circuits in the frequency domain using phasors and impedances:



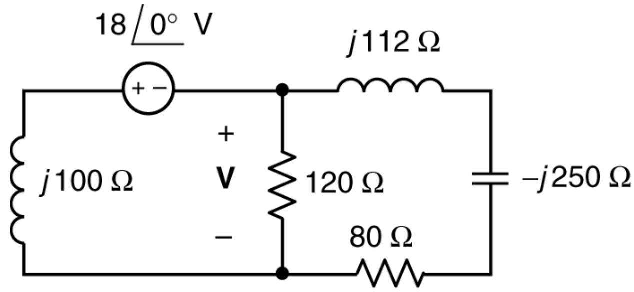
Using voltage division: $\mathbf{V}=-\dfrac{16}{16+j12}(40\angle -15^\circ)=\dfrac{16\angle 180^\circ}{20\angle 36.9^\circ}(40\angle -15^\circ)=32\angle 128.1^\circ\text{ V}$

In the time domain $\quad v(t)=32\cos(250t-57.9^\circ)\text{ V}$

P10.5-2

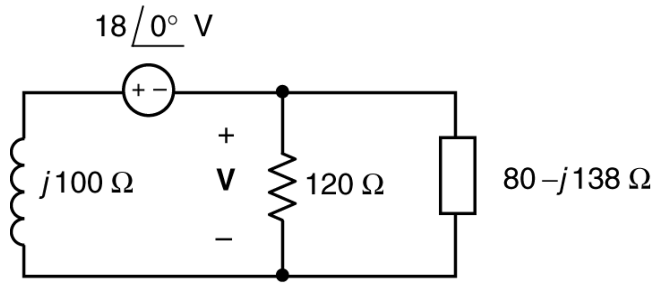
Solution:

Represent the circuit in the frequency domain:



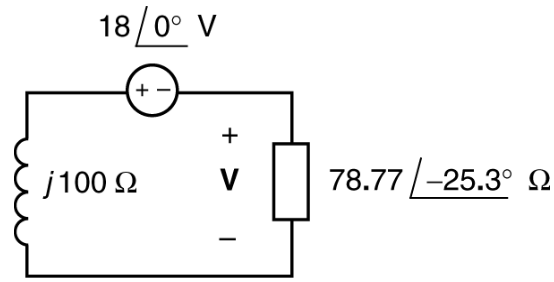
Replace the series impedances at the right of the circuit by an equivalent impedance

$$\mathbf{Z}_S = j112 + (-j250) + 80 = 80 - j138\ \Omega$$



Replace the parallel impedances at right of the circuit by an equivalent impedance

$$\mathbf{Z}_P = \frac{(80 - j138)120}{80 - j138 + 120} = \frac{(80 - j138)120}{200 - j138} = \frac{(159.51\angle -59.9)120}{242.99\angle -34.6} = 78.77\angle 25.3^\circ\ \Omega$$



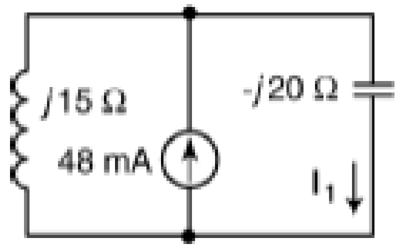
Using voltage division

$$\mathbf{V} = -\frac{78.77\angle -25.3^\circ}{j100 + 78.77\angle -25.3^\circ} 18\angle 0^\circ = -\frac{78.77\angle -25.3^\circ}{97.325\angle 42.97^\circ} 18\angle 0^\circ = 14.57\angle 111.73^\circ\ \text{V}$$

In the time domain $v(t) = 14.57\cos(800t + 111.7^\circ)\ \text{V}$

(checked using LNAP)

P10.5-3
Solution:
Represent the circuits in the frequency domain using phasors and impedances:

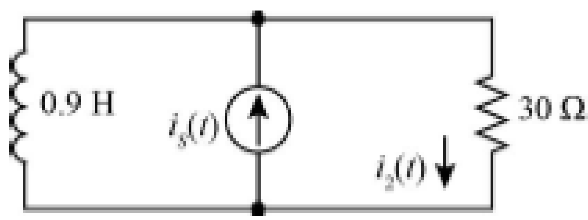


Using current division

$$\mathbf{I}_1 = \frac{j15}{j15 - j20}(48\angle 0°) = \frac{15}{-5}(48\angle 0°) = 144\angle 180° \text{ mA}$$

In the time domain $\quad i_1(t) = 144\cos(25t + 180°)$ mA

P10.5-4
Solution:
In order to find current $i_2(t)$ proceed as follows:



Now current $i_2(t)$ as per Figure 1 is as follows:

$$i_2(t) = \frac{i_s(t)}{R + j34.2}$$

Here,

Source Current $i_s(t)$ is $72\cos(38t)$ mA
Resistance R is $30\,\Omega$
Inductance L is 0.9 H

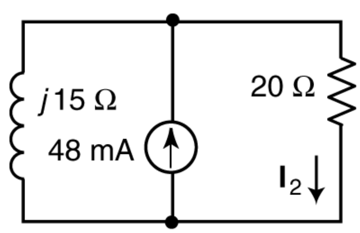
Substitute the corresponding values to obtain $i_2(t)$ as follows:

$$
\begin{aligned}
i_2(t) &= \frac{i_s(t)}{30 + j34.2} \\
&= \frac{72\angle 0^\circ\, j(38)(0.9)}{30 + j34.2} \\
&= \frac{(72\angle 0^\circ)(34.2\angle 90^\circ)}{45.493\angle 48.743^\circ} \\
&= \frac{2462.4\angle 90^\circ}{45.493\angle 48.743^\circ} \\
&= 54.13\angle 41.257^\circ \\
&= 54.13\cos(38t + 41.257)\ \mathrm{A}
\end{aligned}
$$

Hence, current $i_2(t)$ is $\boxed{54.13\cos(38t + 41.257)\ \mathrm{mA}}$

P10.5-5

Solution: Represent the circuits in the frequency domain using phasors and impedances:



Using current division

$$\mathbf{I}_2(\omega) = \frac{j15}{20 + j15}(48\angle 0^\circ) = \frac{15\angle 90^\circ}{25\angle 36.9^\circ}(48\angle 0^\circ) = 28.8\angle 53.1^\circ\ \mathrm{mA}$$

In the time domain $\qquad i_2(t) = 28.8\cos(25t + 53.1^\circ)\ \mathrm{mA}$

P10.5-6
Solution:
Consider $\mathbf{Z}_1$:

$$R_1 - j\frac{1}{20C} = 15.3\angle -24.1° = 14 - j6.25 \quad \Rightarrow \quad R_1 = 14\ \Omega \quad \text{and} \quad C = \frac{1}{20(6.25)} = 0.008\ \text{F} = 8\ \text{mF}$$

Next consider $\mathbf{Z}_2$:

$$\frac{1}{\dfrac{1}{R_2} + \dfrac{1}{j20L}} = 14.4\angle 53.1° \quad \Rightarrow \quad \frac{1}{R_2} + \frac{1}{j20L} = \frac{1}{14.4\angle 53.1°} = \frac{1}{14.4}\angle 53.1 = 0.05556 - j0.04167$$

Equating coefficients gives

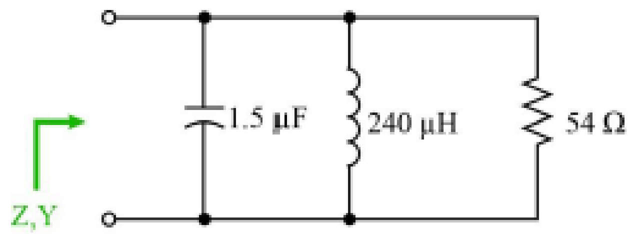
$$R_2 = \frac{1}{0.05556} = 18\ \Omega \quad \text{and} \quad L = \frac{1}{20(0.04167)} = 1.2\ \text{H}$$

Next, consider the voltage divider:

$$\begin{aligned} A\angle 31.5° &= \frac{14.4\angle 36.9°}{15.3\angle -24.1° + 14.4\angle 36.9°}(15\angle 0°) = \frac{(15)(14.4)\angle 36.9°}{(14 - j6.25)(11.52 + j8.64)} \\ &= \frac{216\angle 36.9°}{25.52 + j2.39} \\ &= \frac{216\angle 36.9°}{25.63\angle 5.4°} = 8.43\angle 31.5°\ \text{V} \end{aligned}$$

In the time domain, $\quad v(t) = 8.43\cos(20t + 31.5°)\ \text{V}$.

P 10.5-7
Solution:
In order to determine the impedance and admittance proceed as follows:



As per the figure shown in loop 1 above the impedance Z is given as follows:

$$
\begin{aligned}
Z &= R \parallel X_L \parallel X_C \\
&= R \parallel 2\pi f(L) \parallel 2\pi f(C)
\end{aligned}
$$

Here,

Inductance L is $240\,\mu\text{H}$
Capacitance C is $1.5\,\mu\text{F}$
Frequency f is $15\,\text{kHz}$
Resistance R is $54\,\Omega$

Substitute the corresponding values to obtain Z as follows:

$$
\begin{aligned}
Z &= R \parallel 2\pi f(L) \parallel 2\pi f(C) \\
\frac{1}{Z} &= \frac{1}{R} + j\frac{1}{X_L} - jX_C \\
&= \frac{1}{54\,\Omega} + j\left[\frac{1}{2\pi(15\,\text{kHz})(240\,\mu\text{H})} - 2\pi(15\,\text{kHz})(1.5\,\mu\text{F})\right] \\
&= 0.02 - j7.10 \\
Z &= \frac{1}{0.02 - j7.10} \\
&= 3.97\times10^{-4} + j0.14\,\Omega
\end{aligned}
$$

Admittance Y is given as follows:

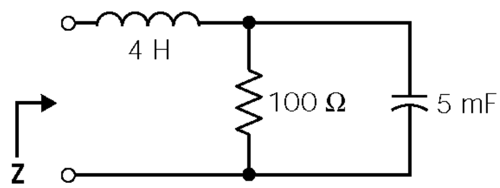
$$
\begin{aligned}
Y &= \frac{1}{Z} \\
&= 0.02 - j7.10 \text{ s}
\end{aligned}
$$

Hence, the admittance Y is $\boxed{0.02 - j7.10 \text{ s}}$

P 10.5-9

Solution:

Replace series and parallel capacitors by an equivalent capacitor and series inductors by an equivalent inductor:

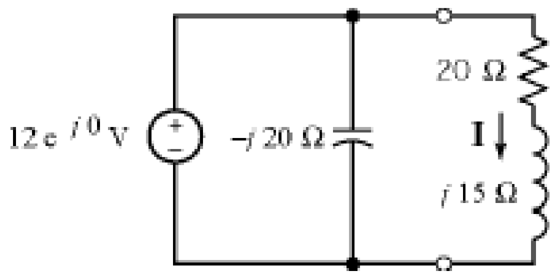


Then

$$
\mathbf{Z} = j\omega 4 + \frac{100 \dfrac{1}{j\omega\left(5\times10^{-3}\right)}}{100 + \dfrac{1}{j\omega\left(5\times10^{-3}\right)}} = j\omega 4 + \frac{100\left(-j\dfrac{200}{\omega}\right)}{100 + \left(-j\dfrac{200}{\omega}\right)} = j\omega 4 + \frac{-j\dfrac{200}{\omega}}{1 - j\dfrac{2}{\omega}} \times \frac{1 + j\dfrac{2}{\omega}}{1 + j\dfrac{2}{\omega}}
$$

$$
\mathbf{Z} = j\omega 4 + 100\,\frac{\dfrac{4}{\omega^2} - j\dfrac{2}{\omega}}{1 + \dfrac{4}{\omega^2}} = j\omega 4 + 100\,\frac{4 - j2\omega}{4 + \omega^2} = \frac{400}{4 + \omega^2} + j\left(4\,\omega - \frac{200\,\omega}{4 + \omega^2}\right)
$$

P 10.5-10
Solution:



$$j(2\pi \cdot 796)(3 \cdot 10^{-3}) = j15\ \Omega$$

$$\mathbf{I} = \frac{12}{20 + j15} = 0.48\ \angle -37°\ \text{A}$$

$$i(t) = 0.48 \cos(2\pi \cdot 796t - 37°)\ \text{A}$$

P 10.5-11
Solution:

$$\mathbf{Z}_1 = R = 8\ \Omega,\quad \mathbf{Z}_2 = j3L,\quad \mathbf{I} = B\angle -51.87° \text{ and } \mathbf{I}_s = 2\ \angle -15°\ \text{A}$$

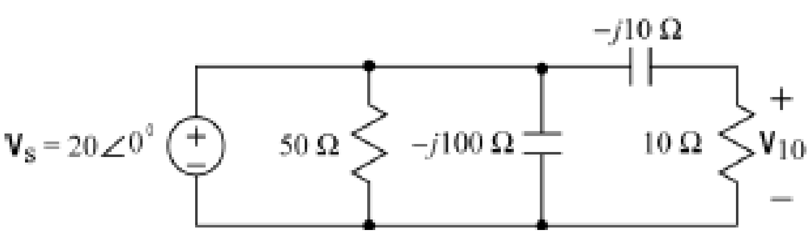
$$\frac{\mathbf{I}}{\mathbf{I}_s} = \frac{B\ \angle -51.87°}{2\ \angle -15°} = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{8}{8 + j3L} = \frac{8\ \angle 0°}{\sqrt{8^2 + (3L)^2}\ \angle \tan^{-1}\left(\frac{3L}{8}\right)}$$

Equate the magnitudes and the angles.

$$\text{angles: } +36.87 = +\tan^{-1}\left(\frac{3L}{8}\right) \Rightarrow \underline{L = 2\ \text{H}}$$

$$\text{magnitudes: } \frac{8}{\sqrt{64 + 9L^2}} = \frac{B}{2} \Rightarrow \underline{B = 1.6}$$

P 10.5-12
Solution:



$$\mathbf{V}_{10} = \mathbf{V}_s\left(\frac{10}{10 - j10}\right)$$

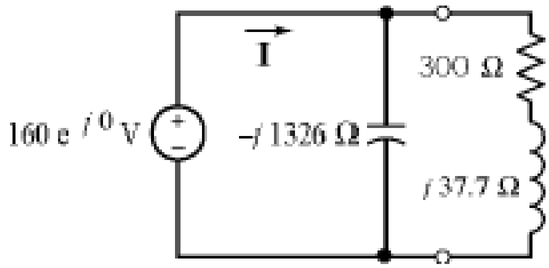
$$= 20\angle 0°\left(\frac{10}{10\sqrt{2}\angle -45°}\right)$$

$$= 10\sqrt{2}\angle 45°$$

$$v_{10}(t) = 10\sqrt{2}\cos(100t + 45°)\ \text{V}$$

P 10.5-13
Solution:

(a)

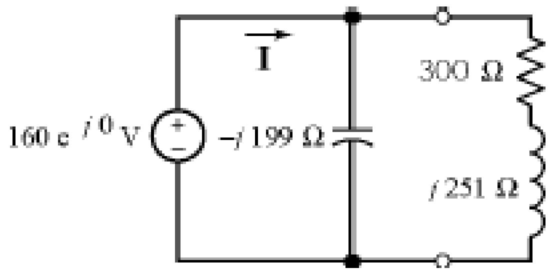


$$\mathbf{I} = \frac{160\ \angle 0^\circ}{\dfrac{(-j1326)\ (300+j37.7)}{-j1326\ +\ 300+j\ 37.7}} = \frac{160\ \angle 0^\circ}{303\ \angle -5.9^\circ}$$

$$= \ 0.53\ \angle 5.9^\circ \text{ A}$$

$$\underline{i(t) = 0.53\cos(120\pi t + 5.9^\circ) \text{ A}}$$

(b)



$$\mathbf{I} = \frac{160\angle 0^\circ}{\dfrac{(-j199)(300+j251)}{-j199+300+j251}} = \frac{160\angle 0^\circ}{256\angle -59.9^\circ}$$

$$= 0.625\angle 59.9^\circ \text{ A}$$

$$\underline{i(t) = 0.625\ \cos\ (800\pi t + 59.9^\circ) \text{ A}}$$

P 10.5-14
Solution:

In order to find the steady state current $i(t)$ proceed as follows:

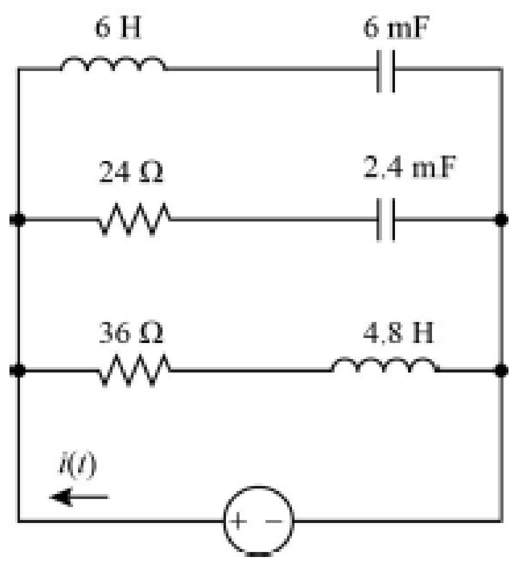


Figure 1: Depicts the circuit

As per Figure 1 the steady state current $i(t)$ is as follows:

$$i(t) = \frac{v(t)}{\text{R} + j(\omega)(L)}$$

Here,

Voltage $v(t)$ is $6\cos(12t + 45°)$ V

Resistance R is 36 Ω

Inductance L is 4.8 H

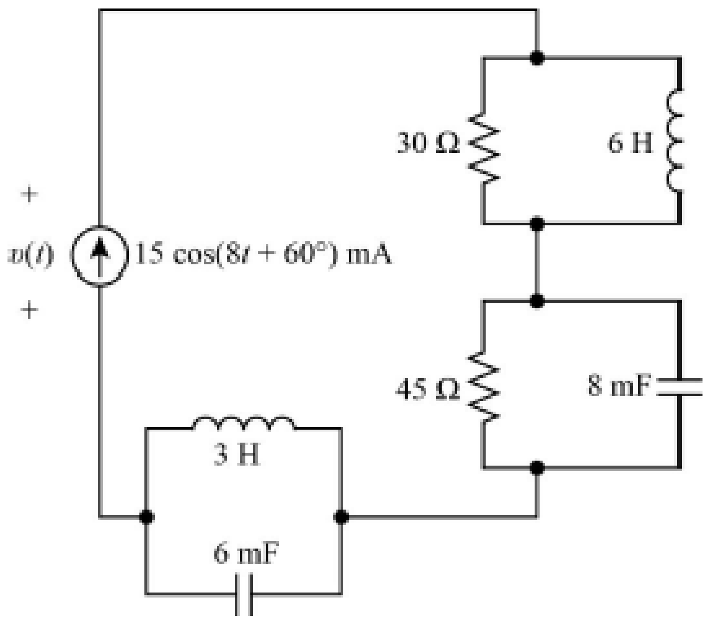
Angular frequency ω is 12 Hz

Substitute the corresponding values to obtain $i(t)$ as follows:

$$\begin{aligned} i(t) &= \frac{v(t)}{36 + j(R)(L)} \\ &= \frac{6\angle 45°}{36 + j(12)(4.8)} \\ &= \frac{6\angle 45°}{36 + j57.6} \\ &= \frac{6\angle 45°}{67.925\angle 57.995°} \\ &= 0.0883\angle -12.995° \\ &= 88.3\cos(12t - 12.995)\,\text{A} \end{aligned}$$

Hence, the steady state current $i(t)$ is $\boxed{88.3\cos(12t - 12.995)\,\text{A}}$

P10.5-15
Solution:
In order to determine the steady state voltage $v(t)$ proceed as follows:



As per the figure above, for the circuit to have a steady state voltage the reactance due to the capacitors and the inductors combined must be zero.

Only the net resistance R_{net} is applicable and is given as:

$$\begin{aligned} R_{net} &= (30+45)\,\Omega \\ &= 75\,\Omega \end{aligned}$$

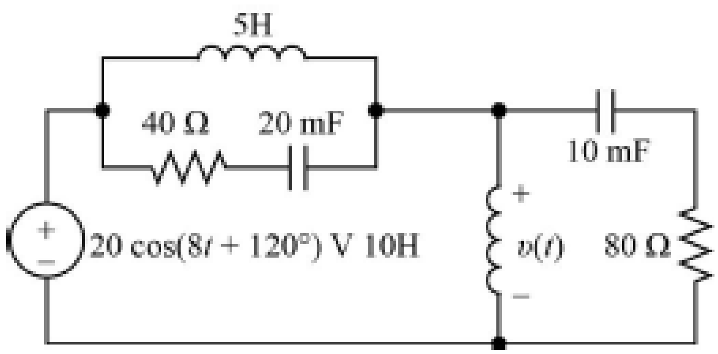
Current $i(t) = 15\cos(8t+60°)$ mA

So the steady state voltage $v(t)$ is given as:

$$\begin{aligned} v(t) &= i(t)R_{net} \\ &= 15\cos(8t+60°)\text{ mA}(75\,\Omega) \\ &= 75(15\cos)(8t+60°)\text{ mV} \end{aligned}$$

Hence, steady state voltage $v(t)$ is $\boxed{75(15\cos\ (8t+60°))\text{mV}}$

P10.5-16
Solution:
In order to determine the steady state voltage $v(t)$ proceed as follows:



As per the figure above, for the circuit has the angular frequency $j\omega = 8$

The reactance for inductance 5H is $\left[j\omega L = j(8)(5\,\mathrm{H})\right] = j40\,\Omega$

The reactance for capacitance 20 mF is $\left[\frac{1}{j\omega C} = \frac{1}{j(8)(20\,\mathrm{mF})}\right] = j6.125\,\Omega$

So the net reactance Z_1 is as follows:

$$\begin{aligned} Z_1 &= 40 \,\|\, 40 - j6.125 \\ &= 23.365 + j20.285\,\Omega \end{aligned}$$

As per the figure above, for the circuit has the angular frequency $\omega = 8$

The reactance for inductance 10H is $\left[j\omega L = j(8)(10\text{ H})\right] = j80\,\Omega$

The reactance for capacitance 10 mF is $\left[\dfrac{1}{j\omega C} = \dfrac{1}{j(8)(10\text{ mF})}\right] = j12.5\,\Omega$

So the net reactance Z_2 is as follows:

$$\begin{aligned} Z_2 &= 80 \parallel 80 - j12.5 \\ &= 46.73 + j40.57\,\Omega \end{aligned}$$

Current $i(t)$ is given as:

$$\begin{aligned} i(t) &= \frac{120}{Z_1 + Z_2} \\ &= \frac{120}{23.365 + j20.285 + 46.73 + j40.57} \\ &= -0.076365 + j0.13227\text{ A} \end{aligned}$$

Voltage $v_2(t)$ across impedance Z_2 is as follows:

$$\begin{aligned} v_2(t) &= i(t)Z_2 \\ &= -0.076365 + j0.13227\text{ A}\left[46.73 + j40.57\,\Omega\right] \\ &= -8.9346 + j3.0827\text{ V} \end{aligned}$$

Now the steady state voltage $v(t)$ is given as follows:

$$\begin{aligned} v(t) &= -8.9346 + j3.0827\text{ V} \\ &= 9.45\text{ V} \end{aligned}$$

Hence, the steady state voltage $v(t)$ is $\boxed{9.45\text{ V}}$

P 10.5-17
Solution:
In order to find the steady state response $i_1(t)$ proceed as follows:

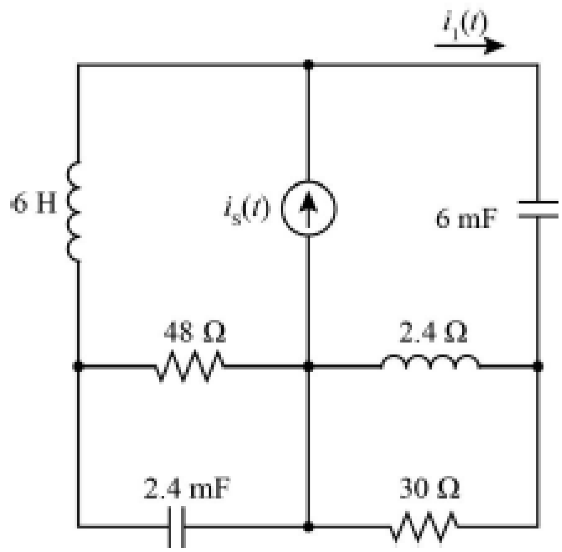


Figure 1: Depicts the circuit

The equivalent circuits are as follows:

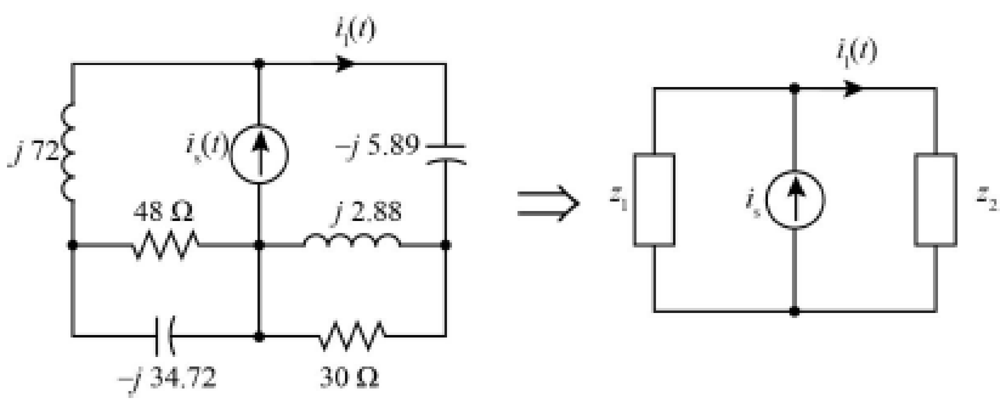


Figure 2: Depicts the equivalent circuit

Source current signal $i_s(t) = 30\cos(12t + 30°)$ mA

Here, the angular frequency $\omega = 12$

Inductance L in Laplace domain is $j\omega L$

Capacitance C in Laplace domain is $\dfrac{1}{j\omega C}$

Impedance Z_1 as per Figure 2 is given as follows:

$$\begin{aligned} Z_1 &= j72 + 48 \parallel -j34.72 \\ &= 16.49 + j49.2 \end{aligned}$$

Impedance Z_2 as per Figure 2 is given as follows:

$$\begin{aligned} Z_2 &= -j3.89 + 30 \parallel -j28.8 \\ &= 14.4 + j11.61 \end{aligned}$$

Now as per the current division rule:

$$\begin{aligned} \overline{i_1} &= \overline{i_s} \frac{Z_1}{Z_1 + Z_2} \\ &= 30\angle 30° \frac{16.49 + j49.2}{14.4 + j11.61} \\ &= 84.15\angle 62.59° \end{aligned}$$

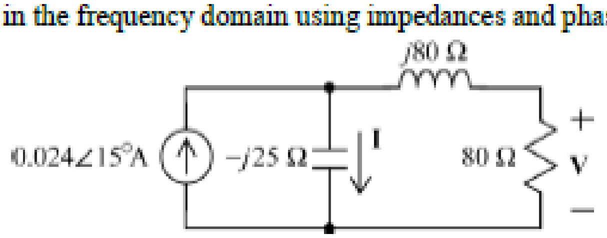
So the steady state response $i_1(t)$ is $\boxed{84.15\cos(12t + 62.59°)\text{ mA}}$

P10.5-18
Solution:

(a) $$i(t) = \frac{80+80}{40+(80+80)} 0.024 = 19.2 \text{ mA}$$

$$v(t) = \frac{80}{80+80} \times \left(40 \Box (80+80)\right) 0.024 = \frac{1}{2}(32)(0.024) = 0.384 \text{ V}$$

(b) Represent the circuit in the frequency domain using impedances and phasors.



$$\mathbf{I} = \frac{80 + j80}{-j25 + (80 + j80)} \times 0.024\angle 15° = 0.028\angle 25.5° \text{ A}$$

$$\mathbf{V} = \frac{80}{80 + j80} \times \left[-j25 \parallel (80 + j80)\right] \times 0.024\angle 15° = 0.494\angle -109.5° \text{ V}$$

So $$i(t) = 28\cos(10t + 25.5°) \text{ mA}$$

and $$v(t) = 0.494\cos(10t - 109.5°) \text{ V}$$

(checked: LNAP 8/1/04)

P 10.5-19

Solution:

Represent the circuit in the frequency domain using phasors and impedances. The impedance capacitor is $\frac{1}{j(100)(0.5\times 10^{-6})} = -j20,000$. When the switch is closed

$$17.89\angle -26.6° = \mathbf{V} = \frac{-j20,000}{R_2 - j20,000} \times 20\angle 0°$$

Equating angels gives

$$-26.6° = -90° - \tan^{-1}\left(\frac{-20,000}{R_2}\right) \quad \Rightarrow \quad R_2 = \frac{-20,000}{\tan(-63.4)} = 10015\ \Omega$$

When the switch is open

$$14.14\angle -45° = \mathbf{V} = \frac{-j20,000}{R_1 + R_2 - j20,000} \times 20\angle 0°$$

Equating angles gives

$$-45° = -90° - \tan^{-1}\left(\frac{-20,000}{R_1 + R_2}\right) \quad \Rightarrow \quad R_1 + R_2 = \frac{-20,000}{\tan(-45°)} = 20,000$$

So

$$R_1 = 20,000 - 10015 = 9985\ \Omega$$

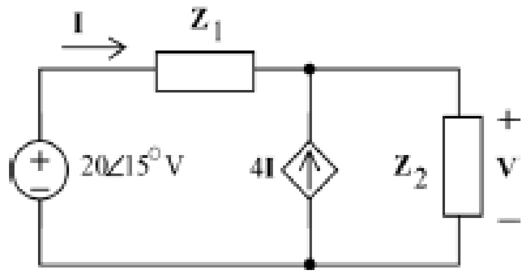
(checked: LNAP 8/2/04)

P10.5-20
Solution:
(a) Using KCL and then KVL gives

$$20 = 50i(t) + 40(5i(t)) \quad \Rightarrow \quad i(t) = \frac{20}{250} = 80 \text{ mA}$$

Then $$v(t) = 40(5i(t)) = 200(0.08) = 16 \text{ V}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



Where $$\mathbf{Z}_1 = 40 + j(10)3 + \frac{1}{j(10)(0.005)} = 40 + j10 = 41.23\angle 26.6° \ \Omega$$

And $$\mathbf{Z}_2 = j(10)2 \parallel 10 = 8 + j4 = 8.944\angle 26.6° \ \Omega$$

Using KCL and then KVL gives

$$20\angle 15° = \mathbf{Z}_1\mathbf{I} + 5\mathbf{Z}_2\mathbf{I} \quad \Rightarrow \quad \mathbf{I} = 0.234\angle -5.6° \text{ A}$$

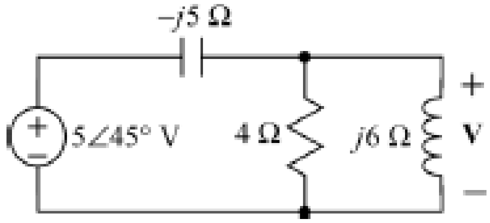
Then $$\mathbf{V} = \mathbf{Z}_2(5\mathbf{I}) = 10.47\angle 21° \text{ A}$$

so $$i(t) = 0.234\cos(10t - 5.6°) \text{ A}$$

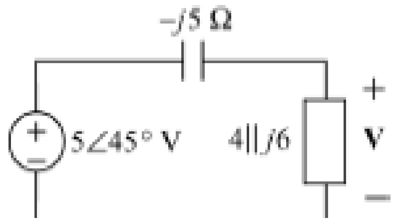
and $$v(t) = 10.47\cos(10t + 21°) \text{ V}$$

(checked: 8/3/04)

P 10.5-21
Solution:
Represent the circuit in the frequency domain using phasors and impedances.



$$4 \| j6 = \frac{4(j6)}{4+j6} = \frac{24\angle 90°}{7.2\angle 56°} = 3.33\angle 34° = 2.76 + j1.86\ \Omega$$



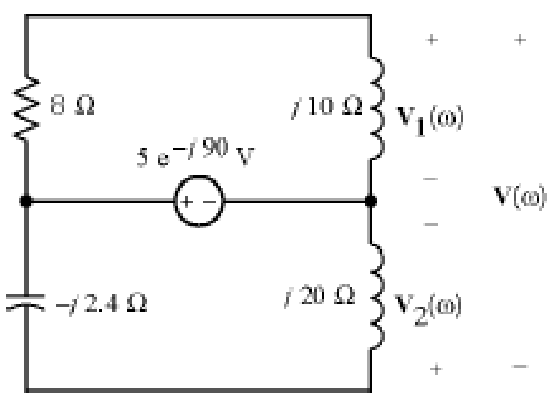
Using voltage division

$$\mathbf{V} = \frac{3.33\angle 34°}{-j5+2.76+j1.86} \times 5\angle 45° = \frac{3.33\angle 34°}{2.76-j3.14} \times 5\angle 45° = \frac{3.33\angle 34°}{4.18\angle -48°} \times 5\angle 45° = 3.98\angle 127°\ \text{V}$$

The corresponding voltage in the time domain is

$$v(t) = 3.98\cos(2t + 127°)\ \text{V}$$

P10.5-22
Solution:



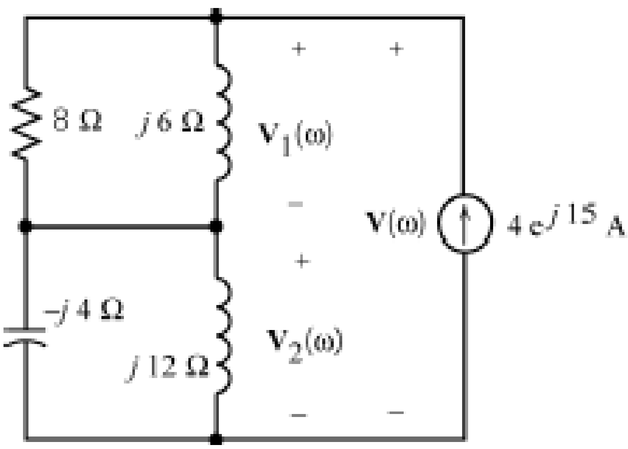
$$\mathbf{V}_1(\omega) = \frac{j10}{8+j10} 5e^{-j90} = 3.9e^{-j51}\ \text{V}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20-j2.4} 5e^{-j90} = 5.68e^{-j90}\ \text{V}$$

$$\begin{aligned}\mathbf{V}(\omega) &= \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90} \\ &= 3.58e^{j47}\ \text{V}\end{aligned}$$

Answer: $v(t) = 3.58\cos(5t + 47.2°)$ V

P10.5-23
Solution:



$$\mathbf{V}_1(\omega)=\frac{8(j6)}{8+j6}4e^{j15}=19.2e^{j68}\text{ V}$$

$$\mathbf{V}_2(\omega)=\frac{j12(-j4)}{j12-j4}4e^{j15}=24e^{-j75}\text{ V}$$

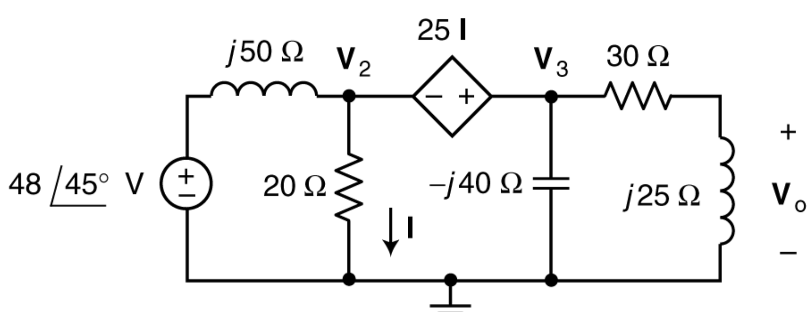
$$\mathbf{V}(\omega)=\mathbf{V}_1(\omega)+\mathbf{V}_2(\omega)=14.4e^{-j22}\text{ V}$$

Answer: $v(t) = 14.4\cos(3t - 22°)$ V

Section 10.6 Mesh and Node Equations

P10.6-1

Solution: Represent the circuit in the frequency domain as



The node voltages are $48\angle 45° = \mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$ and $\mathbf{V}_o$. Express the dependent source voltage in terms of the node voltages:

$$\mathbf{V}_3-\mathbf{V}_2=25\,\mathbf{I}=25\left(\frac{\mathbf{V}_2}{20}\right)\quad\Rightarrow\quad \mathbf{V}_3=2.25\,\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{48\angle 45° - \mathbf{V}_2}{j50} = \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \frac{\mathbf{V}_2}{j50} + \frac{\mathbf{V}_2}{20} + \frac{\mathbf{V}_3 - \mathbf{V}_o}{30} + \frac{\mathbf{V}_3}{-j40}$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)\mathbf{V}_3 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20}\right)\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{-j40}\right)2.25\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

$$\frac{48\angle 45°}{j50} = \left(\frac{1}{j50} + \frac{1}{20} + \frac{2.25}{30} + \frac{2.25}{-j40}\right)\mathbf{V}_2 - \frac{1}{30}\mathbf{V}_o$$

Apply KCL at the right node of the 30 Ω resistor to get

$$\frac{\mathbf{V}_3 - \mathbf{V}_o}{30} = \frac{\mathbf{V}_o}{j25} \quad \Rightarrow \quad 0 = \left(-\frac{1}{30}\right)2.25\,\mathbf{V}_2 + \left(\frac{1}{30} + \frac{1}{j25}\right)\mathbf{V}_o$$

In matrix form

$$\begin{bmatrix} \frac{1}{j50} + \frac{1}{20} + \frac{2.25}{30} + \frac{2.25}{-j40} & -\frac{1}{30} \\ -\frac{2.25}{30} & \frac{1}{30} + \frac{1}{j25} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} \frac{48\angle 45°}{j50} \\ 0 \end{bmatrix}$$

Solving, perhaps using MATLAB,

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} 10.18\angle -44.6° \\ 14.67\angle 5.6° \end{bmatrix} \text{ V}$$

P10.6-2

Solution:

Writing Node equations:

$$\frac{12\angle 45° - \mathbf{V}_b}{j30} = \frac{\mathbf{V}_b}{20 - j25} + \frac{\mathbf{V}_b - \mathbf{V}_c}{15 - j30}$$

$$\frac{\mathbf{V}_b - \mathbf{V}_c}{15 - j30} + \frac{12\angle 45° - \mathbf{V}_c}{40 + j20} = \frac{\mathbf{V}_c}{j40}$$

Rearranging:

$$\frac{12\angle 45^\circ}{j30} = \left(\frac{1}{j30} + \frac{1}{20 - j25} + \frac{1}{15 - j30}\right)\mathbf{V}_\text{b} - \left(\frac{1}{15 - j30}\right)\mathbf{V}_\text{c}$$
$$\frac{12\angle 45^\circ}{40 + j20} = -\left(\frac{1}{15 - j30}\right)\mathbf{V}_\text{b} + \left(\frac{1}{15 - j30} + \frac{1}{40 + j20} + \frac{1}{j40}\right)\mathbf{V}_\text{c}$$

In matrix from:

$$\begin{bmatrix} \frac{1}{j30} + \frac{1}{20 - j25} + \frac{1}{15 - j30} & -\frac{1}{15 - j30} \\ -\frac{1}{15 - j30} & \frac{1}{15 - j30} + \frac{1}{40 + j20} + \frac{1}{j40} \end{bmatrix} \begin{bmatrix} \mathbf{V}_\text{b} \\ \mathbf{V}_\text{c} \end{bmatrix} = \begin{bmatrix} \frac{12\angle 45^\circ}{j30} \\ \frac{12\angle 45^\circ}{40 + j20} \end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{V}_\text{b} = 7.69\angle -19.8^\circ \text{ and } \mathbf{V}_\text{c} = 10.18\angle 7.7^\circ \text{ V}$$

Checked using LNAPAC

P10.6-3

Solution:

Mesh 1:
$$(40 + j15)\mathbf{I}_1 + (25 - j50)(\mathbf{I}_1 - \mathbf{I}_3) - 48\angle 75^\circ = 0$$
$$(65 - j35)\mathbf{I}_1 - (25 - j50)\mathbf{I}_3 = 48\angle 75^\circ$$

Mesh 2:
$$48\angle 75^\circ + (-j50)(\mathbf{I}_2 - \mathbf{I}_3) + (32 + j16)\mathbf{I}_2 = 0$$
$$(32 - j34)\mathbf{I}_2 + j50\mathbf{I}_3 = -48\angle 75^\circ$$

Mesh 3:
$$j40\mathbf{I}_3 - (-j50)(\mathbf{I}_2 - \mathbf{I}_3) - (25 - j50)(\mathbf{I}_1 - \mathbf{I}_3) = 0$$
$$(-25 + j50)\mathbf{I}_1 + j50\mathbf{I}_2 + (25 - j160)\mathbf{I}_3 = 0$$

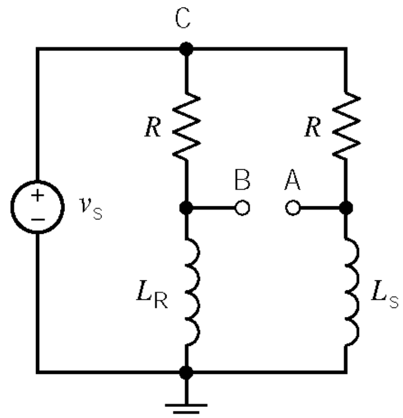
In matrix form:
$$\begin{bmatrix} 65 - j35 & 0 & -25 + j50 \\ 0 & 32 - j34 & +j50 \\ -25 + j50 & +j50 & 25 - j60 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 48\angle 75^\circ \\ -48\angle 75^\circ \\ 0 \end{bmatrix}$$

Solving using MATLAB:

$$\mathbf{I}_1 = 0.794\angle 111^\circ,\ \ \mathbf{I}_2 = 0.790\angle -61.7^\circ \text{ and } \mathbf{I}_3 = 0.229\angle 176^\circ \text{ A}$$

P 10.6-4

Solution:



$v_s = \sin\ (2\pi \cdot 400t)\ \text{V}$

$R = 100\ \Omega$

$L_R = 40\ \text{mH}$

$L_S = \begin{cases} 40\ \text{mH} & \text{door opened} \\ 60\ \text{mH} & \text{door closed} \end{cases}$

With the door open $|\mathbf{V}_A - \mathbf{V}_B| = 0$ since the bridge circuit is balanced.

With the door closed $\mathbf{Z}_{L_R} = j(800\pi)(0.04) = j100.5\ \Omega$ and $\mathbf{Z}_{L_S} = j(800\pi)(0.06) = j150.8\ \Omega$.

The node equations are:

$$\text{KCL at node B:} \quad \frac{\mathbf{V}_B - \mathbf{V}_C}{R} + \frac{\mathbf{V}_B}{\mathbf{Z}_{L_R}} = 0 \quad \Rightarrow \mathbf{V}_B = \frac{j100.5}{j100.5 + 100}\mathbf{V}_C$$

$$\text{KCL at node A :} \quad \frac{\mathbf{V}_A - \mathbf{V}_C}{R} + \frac{\mathbf{V}_A}{\mathbf{Z}_{L_S}} = 0$$

Since $\mathbf{V}_C = |\mathbf{V}_s| = 1\ \text{V}$ $\quad \mathbf{V}_B = 0.709\angle 44.86°\ \text{V}$ and $\mathbf{V}_A = 0.833\angle 33.55\ \text{V}$

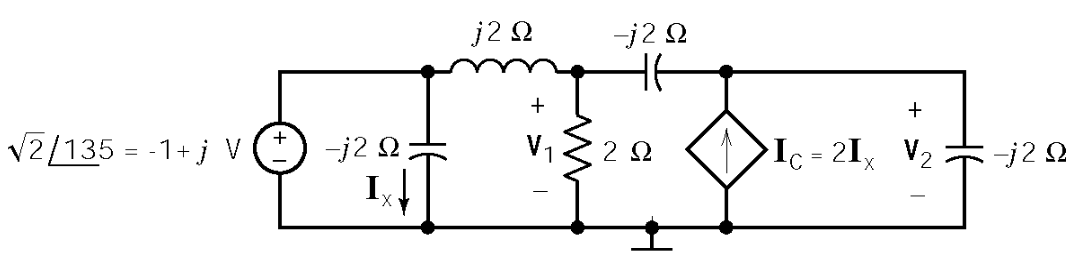
Therefore

$$\mathbf{V}_A - \mathbf{V}_B = 0.833\angle 33.55° - 0.709\angle 44.86° = (0.694 + j.460) - (0.503 + j0.500) = 0.191 - j0.040$$
$$= 0.195\angle -11.83°\ \text{V}$$

P 10.6-5

Solution:

Represent the circuit in the frequency domain



The node equations are: $\quad \dfrac{\mathbf{V}_1 - (-1+j)}{j2} + \dfrac{\mathbf{V}_1}{2} + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j2} = 0$

$$\frac{\mathbf{V}_2-\mathbf{V}_1}{-j2}+\frac{\mathbf{V}_2}{-j2}-\mathbf{I}_C = 0$$

Also, expressing the controlling signal of the dependent source in terms of the node voltages yields

$$\mathbf{I}_x=\frac{-1+j}{-2j} \quad\Rightarrow\quad \mathbf{I}_C=2\mathbf{I}_x = 2\left[\frac{-1+j}{-2j}\right]=-1-j \text{ A}$$

Solving these equations yields

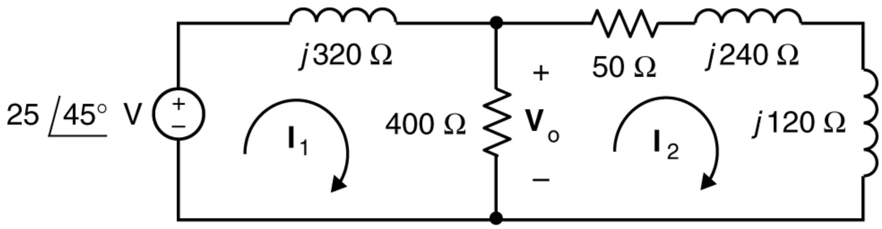
$$\mathbf{V}_2=\frac{-3-j}{1+j2}=\sqrt{2}\ \angle -135° \text{ V} \quad\Rightarrow\quad v(t)=v_2(t)=\sqrt{2}\ \cos\ (40t-135°) \text{ V}$$

(checked: LNAP 7/19/04)

P10.6-6

Solution

Represent the circuit in the frequency domain:



Apply KVL to mesh 1: $j320\mathbf{I}_1+400\left(\mathbf{I}_1-\mathbf{I}_2\right)-25\angle 45°=0$

Apply KVL to mesh 2: $50\mathbf{I}_2+j240\mathbf{I}_2+j120\mathbf{I}_2-400\left(\mathbf{I}_1-\mathbf{I}_2\right)=0$

In matrix form: $$\begin{bmatrix}400+j320 & -400\\ -400 & 450+j360\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}25\angle 45°\\ 0\end{bmatrix}$$

Solving using MATLAB: $$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\end{bmatrix}=\begin{bmatrix}47.5\angle -24.6°\\ 33.0\angle -63.3°\end{bmatrix} \text{ mA}$$

Using Ohm's Law $\mathbf{V}_o=400\left(\mathbf{I}_1-\mathbf{I}_2\right)=12\angle 18.8° \text{ V}$

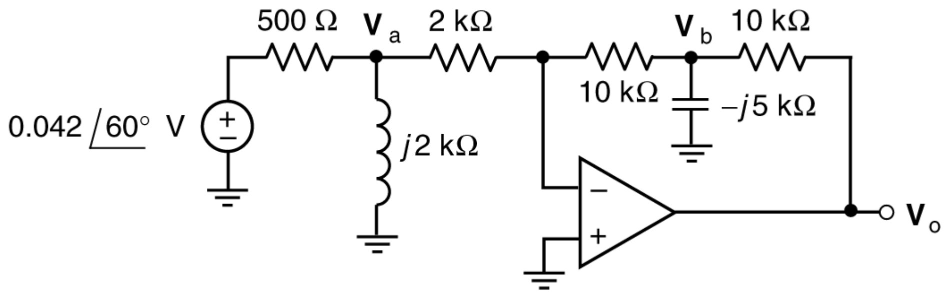
In the time domain $i_1(t)=47.5\cos(40t-24.6°)$ mA , $i_2(t)=33\cos(40t-63.3°)$ mA

and $v_o(t)=12\cos(40t+18.8°)$ V

P10.6-7

Solution

Represent the circuit in the frequency domain:



Apply KCL at the top node of the inductor, node a:

$$\frac{0.042\angle 60° - \mathbf{V}_a}{500} = \frac{\mathbf{V}_a}{j\,2000} + \frac{\mathbf{V}_a}{2000} \quad \Rightarrow \quad \mathbf{V}_a = \frac{4}{5-j}\left(0.042\angle 60°\right)$$

Apply KCL at the inverting input node of the op amp:

$$\frac{\mathbf{V}_a}{2000} + \frac{\mathbf{V}_b}{10,000} = 0 \quad \Rightarrow \quad \mathbf{V}_b = -5\,\mathbf{V}_a$$

Apply KCL at the top node of the capacitor, node b:

$$\frac{\mathbf{V}_b}{10,000} + \frac{\mathbf{V}_b}{-j\,5000} + \frac{\mathbf{V}_b - \mathbf{V}_o}{20,000} = 0 \quad \Rightarrow \quad \mathbf{V}_o = \left(3 + j\,4\right)\mathbf{V}_b$$

Combining these results we get:

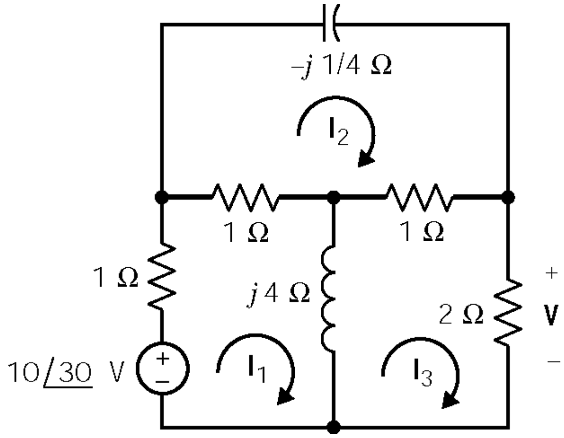
$$\mathbf{V}_o = \left(3 + j\,4\right)\left(-5\right)\frac{4}{5-j}\left(0.042\angle 60°\right) = \frac{\left(5\angle 53.1°\right)\left(20\angle -180°\right)}{5.1\angle -11.3°}\left(0.042\angle 60°\right) = 0.8235\angle -55.6°$$

In the time domain $\quad v_o\left(t\right) = 832.5\cos\left(800\,t - 55.6°\right)\ \text{mV}$

P 10.6-8

Solution:

Represent the circuit in the frequency domain:



The mesh equations are:

$$\begin{bmatrix} (2+j4) & -1 & -j4 \\ -1 & (2+1/\,j4) & -1 \\ -j4 & -1 & (3+j4) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's rule yields

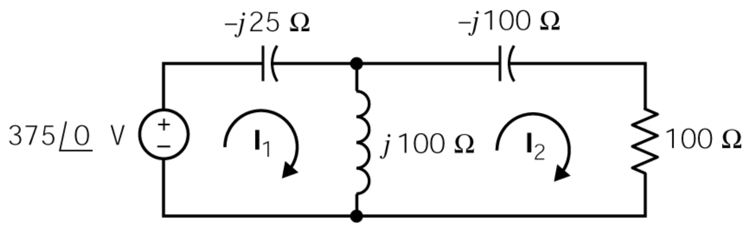
$$\mathbf{I}_3 = \frac{2+j8}{12+j\ 22.5}\left(10\angle 30°\right) = 3.225\angle 44°\ \text{A}$$

Then $\mathbf{V} = 2\ \mathbf{I}_3 = 2\left(3.225\angle 44°\right) = 6.45\angle 44°\ \text{V} \Rightarrow v(t) = 6.45\cos(10^5 t + 44^\circ)\ \text{V}$

(checked: LNAP 7/19/04)

P 10.6-9

Solution: *Represent the circuit in the frequency domain:*



Mesh Equations:
$$\begin{aligned} j\ 75\mathbf{I}_1 - j100\mathbf{I}_2 &= 375 \\ -j100\mathbf{I}_1 + (100+j100)\mathbf{I}_2 &= 0 \end{aligned}$$

Solving for $\mathbf{I}_2$ yields $\mathbf{I}_2 = 4.5 + j1.5 = 3\ \angle 53.1°\ \text{A} \quad \Rightarrow \quad i_2(t) 3\cos\left(400t + 53.1°\right)\ \text{A}$

(checked: LNAP 7/19/04)

P 10.6-10

Solution

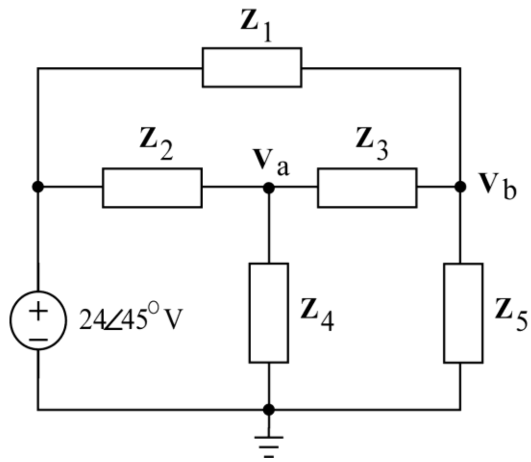
(a)

The node equations are
$$\frac{24-v_a}{40} = \frac{v_a - v_b}{20} + \frac{v_a}{15}$$
$$\frac{24-v_b}{25} + \frac{v_a - v_b}{20} = \frac{v_b}{50}$$

or
$$\begin{bmatrix} \frac{1}{40}+\frac{1}{20}+\frac{1}{15} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{25}+\frac{1}{20}+\frac{1}{50} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \end{bmatrix} = \begin{bmatrix} \frac{24}{40} \\ \frac{24}{25} \end{bmatrix}$$

Solving using MATLAB gives $v_a = 8.713\ \text{V}$ and $v_b = 12.69\ \text{V}$

(b) Use phasors and impedances to represent the circuit in the frequency domain as



where

$$\mathbf{Z}_1 = 25 + j(20)4 = 25 + j80 = 83.82\angle 72.7^\circ\ \Omega$$

$$\mathbf{Z}_2 = \left(40 \parallel \frac{1}{j(20)(0.004)}\right) + j(20)5 = 3.56 + j88.6 = 88.68\angle 87.7^\circ\ \Omega$$

$$\mathbf{Z}_3 = 20\ \Omega$$

$$\mathbf{Z}_4 = 15 + j(20)2 = 15 + j40 = 42.72\angle 69.4^\circ$$

$$\mathbf{Z}_5 = j(20)3 + \frac{1}{j(20)(0.005)} = j50 = 50\angle 90^\circ\ \Omega$$

The node equations are

$$\frac{24\angle 45^\circ - \mathbf{V}_a}{\mathbf{Z}_2} = \frac{\mathbf{V}_a}{\mathbf{Z}_4} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3}$$

$$\frac{24\angle 45^\circ - \mathbf{V}_b}{\mathbf{Z}_1} + \frac{\mathbf{V}_a - \mathbf{V}_b}{\mathbf{Z}_3} = \frac{\mathbf{V}_b}{\mathbf{Z}_5}$$

$$\begin{bmatrix} \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_4} & -\frac{1}{\mathbf{Z}_3} \\ -\frac{1}{\mathbf{Z}_3} & \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_3} + \frac{1}{\mathbf{Z}_5} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix} = \begin{bmatrix} \frac{24\angle 45^\circ}{\mathbf{Z}_2} \\ \frac{24\angle 45^\circ}{\mathbf{Z}_1} \end{bmatrix}$$

Solving using MATLAB gives

$$\mathbf{V}_a = 7.89\angle 44.0^\circ$$

$$\mathbf{V}_b = 8.45\angle 45.1^\circ$$

So

$$v_a(t) = 7.89\cos(20t + 44^\circ)\ \text{V}$$

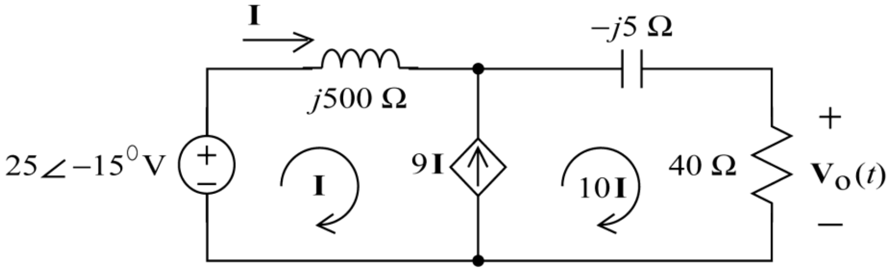
$$v_a(t) = 8.45\cos(20t + 45.1)\ \text{V}$$

(checked: LNAP 8/3/04)

P 10.6-11

Solution:

Represent the circuit in the frequency domain using impedances and phasors.



The mesh currents are **I** and 10**I**. Apply KVL to the supermesh corresponding to the dependant current source to get

$$(j500)\mathbf{I}+(-j5)(10\mathbf{I})+40(10\mathbf{I})-25\angle -15° = 0$$

So $$\mathbf{I}=\frac{25\angle -15°}{400+j450}=0.04152\angle -63.37°\ \text{A}$$

The output voltage is $$\mathbf{V}=40(10\mathbf{I})=16.61\angle -63.37°\ \text{V}$$

So $$v(t)=16.61\cos(100t-63.37°)\ \text{V}$$

(checked: LNAP 8/3/04)

P 10.6-12

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KVL to the center mesh to get

$$0.8394\angle 138.5° = \mathbf{I} = \frac{8\angle 210° - 30\angle -15°}{R+j10L} \quad \Rightarrow \quad R+j10L = 35+j25 = 35+j(10)2.5$$

So $$R = 35\ \Omega \quad \text{and} \quad L = 2.5\ \text{H}$$

(checked: LNAP 8/3/04)

P 10.6-13

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Apply KCL at the top node of *R* and *L* to get

$$\frac{(50\angle-75^\circ)-\mathbf{V}}{j40}+\frac{35\angle100^\circ-\mathbf{V}}{40}=\frac{\mathbf{V}}{R \parallel j\omega L}$$

$$\Rightarrow \qquad \frac{50\angle-75^\circ}{40\angle90^\circ}+\frac{35\angle110^\circ}{40}=\left(\frac{1}{j40}+\frac{1}{40}+\frac{1}{R}-j\frac{1}{20L}\right)\mathbf{V}$$

Using the given equation for $v(t)$ we get

$$21.25\angle-168.8^\circ=\mathbf{V}=\frac{1.587\angle161.7^\circ}{0.025(1-j)+\frac{1}{R}-j\frac{1}{20L}}$$

Then $$\frac{1}{R}-j\frac{1}{20L}=\frac{1.587\angle161.7^\circ}{21.25\angle-168.8^\circ}-0.025(1-j)=0.04-j0.01176$$

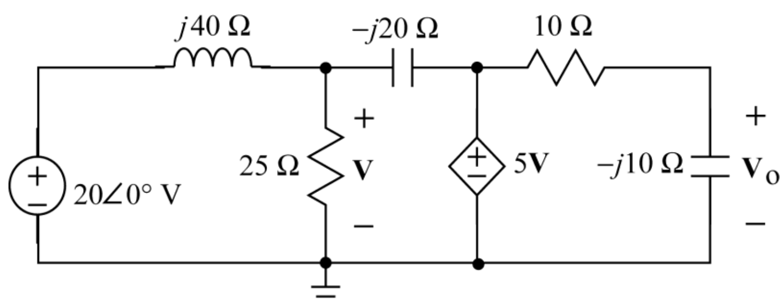
Finally $$R=\frac{1}{0.04}=25\ \Omega \quad \text{and} \quad L=\frac{1}{20(0.01176)}=4.25\ \text{H}$$

(checked: LNAP 8/3/04)

P 10.6-14

Solution:

Represent the circuit in the frequency domain using phasors and impedances.



The node equations are

$$\frac{20\angle0^\circ-\mathbf{V}}{j40}=\frac{\mathbf{V}}{25}+\frac{\mathbf{V}-5\mathbf{V}}{-j20}$$

$$\frac{5\mathbf{V}-\mathbf{V}_o}{10}=\frac{\mathbf{V}_o}{-j10}$$

$$\begin{bmatrix}\frac{1}{25}-j\frac{1}{5}-j\frac{1}{40} & 0\\ -\frac{1}{2} & \frac{1}{10}+j\frac{1}{10}\end{bmatrix}\begin{bmatrix}\mathbf{V}\\ \mathbf{V}_o\end{bmatrix}=\begin{bmatrix}-j0.5\\ 0\end{bmatrix}$$

$$\begin{bmatrix} 0.04 - j0.225 & 0 \\ -0.50 & 0.10 + j0.10 \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{V}_o \end{bmatrix} = \begin{bmatrix} -j0.5 \\ 0 \end{bmatrix}$$

Solving gives $\mathbf{V} = 2.188\angle -10.1° \text{ V}$ and $\mathbf{V}_o = 7.736\angle -55.1° \text{ V}$

So $v_o(t) = 7.736\cos(5t - 55.1°) \text{ V}$

(checked: LNAP 8/4/04)

P 10.6-15

Solution:

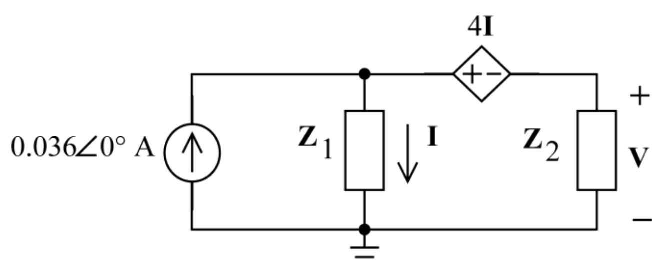
(a) Use KVL to see that the voltage across the 8 Ω resistor is $20i(t) - 4i(t) = 16i(t)$.

Apply KCL to the supernode corresponding to the dependent voltage source to get

$$0.036 = i(t) + \frac{16i(t)}{8} = 3i(t)$$

so $$i(t) = 12 \text{ mA}$$

(b) Represent the circuit in the frequency domain using phasors and impedances.



$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20 \ \Omega$$

where

$$\mathbf{Z}_2 = j50 + \left(15 \parallel \frac{1}{j(25)(0.004)}\right) = 43.3\angle 83.9° \ \Omega$$

Use KVL to get $$\mathbf{V} = \mathbf{Z}_1\mathbf{I} - 4\mathbf{I} = (\mathbf{Z}_1 - 4)\mathbf{I}$$

Then apply KCL to the supernode corresponding to the dependent source to get

$$0.036\angle 0° = \mathbf{I} + \frac{(\mathbf{Z}_1 - 4)\mathbf{I}}{\mathbf{Z}_2} = \left(\frac{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}{\mathbf{Z}_2}\right)\mathbf{I}$$

so $$\mathbf{I}=\frac{\mathbf{Z}_2\left(0.036\angle 0^\circ\right)}{\mathbf{Z}_1+\mathbf{Z}_2-4}=50.4\angle 35.7^\circ \text{ mA}$$

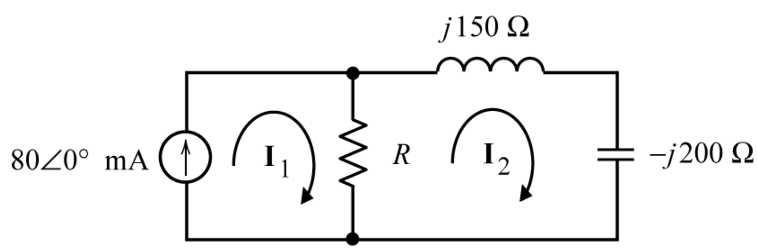
so $$i(t)=50.4\cos\left(25t+35.7^\circ\right)\text{ mA}$$

(checked: LNAP 8/4/04)

10.6-16

Solution:

Represent the circuit in the frequency domain using phasors and impedances. The mesh currents are



$$\mathbf{I}_1=0.080\angle 0^\circ \text{ A}$$

and

$$\mathbf{I}_2=0.06656\angle 33.7^\circ \text{ A}$$

Apply KVL to the right to get

$$\left(j150-j200\right)\left(0.06656\angle 33.7^\circ\right)+R\left(0.06656\angle 33.7^\circ-0.080\angle 0^\circ\right)=0$$

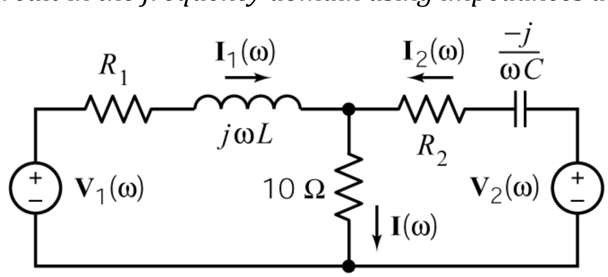
$$\left(-j50\right)\left(0.06656\angle 33.7^\circ\right)+R\left(0.044376\angle 123.7^\circ\right)=0$$

$$R=\frac{\left(50\angle 90^\circ\right)\left(0.06656\angle 33.7^\circ\right)}{0.044376\angle 123.7^\circ}=74.9955\cong 75\ \Omega$$

MATLAB, 11/20/09

P10.6-17

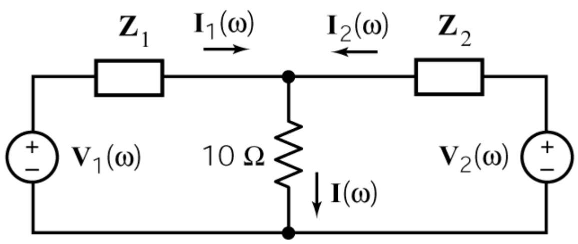
Solution: *Represent the circuit in the frequency domain using impedances and phasors:*



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.744\angle -118° + 0.5405\angle 100 = (-0.349 - j\,0.657) + (-0.094 + j0.532)$$
$$= (-0.349 - 0.094) + j(-0.657 + 0.532)$$
$$= -0.443 - j\,0.125$$
$$= 0.460\angle -164°$$

In the time domain $\qquad i(t) = 460\cos(2t - 164°)\ \text{mA}$

Replacing series impedances by equivalent impedances gives



$$\mathbf{Z}_1 = R_1 + j\,\omega\, L$$

and

$$\mathbf{Z}_2 = R_2 - j\frac{1}{\omega\, C}$$

From KVL

$$\mathbf{Z}_1\mathbf{I}_1 + 10\mathbf{I} - \mathbf{V}_1 = 0 \quad\Rightarrow\quad \mathbf{Z}_1 = \frac{\mathbf{V}_1 - 10\mathbf{I}}{\mathbf{I}_1} = \frac{12\angle -90° - 10(0.460\angle -164°)}{0.744\angle -118°}$$
$$= \frac{-j12 - 10(-0.443 - j\,0.125)}{0.744\angle -118°}$$
$$= \frac{4.43 - j10.75}{0.744\angle -118°} = \frac{11.63\angle -67.6°}{0.744\angle -118°}$$
$$= 15.63\angle 50.4°$$
$$= 10 + j12\ \Omega$$

and

$$-\mathbf{Z}_2\mathbf{I}_2 + \mathbf{V}_2 - 10\mathbf{I} = 0 \quad\Rightarrow\quad \mathbf{Z}_2 = \frac{\mathbf{V}_2 - 10\mathbf{I}}{\mathbf{I}_2} = \frac{5\angle 90° - 10(0.460\angle -164°)}{0.5405\angle 100°}$$
$$= \frac{j5 - 10(-0.443 - j\,0.125)}{0.5405\angle 100°}$$
$$= \frac{4.43 + j\,6.25}{0.5405\angle 100°} = \frac{7.66\angle 54.7°}{0.5405\angle 100°}$$
$$= 14.14\angle -55.3°$$
$$= 10 - j10\ \Omega$$

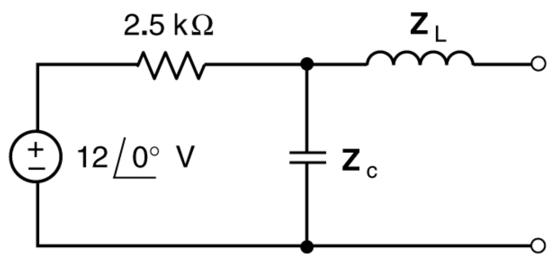
Next $\qquad 10 + j12 = R_1 + j\,\omega\, L = R_1 + j\,2\,L \quad\Rightarrow\quad R_1 = 10\ \Omega \quad \text{and} \quad L = \frac{12}{2} = 6\ \text{H}$

and $$10 - j10 = R_2 - j\frac{1}{\omega C} = R_2 - j\frac{1}{2\,C} \quad \Rightarrow \quad R_2 = 10\ \Omega \quad \text{and} \quad C = \frac{1}{2(10)} = 0.05\ \text{F}$$

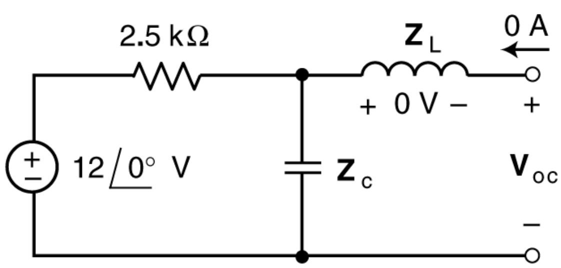
Section 10.7 Thevenin and Norton Equivalent Circuits

P10.7-1

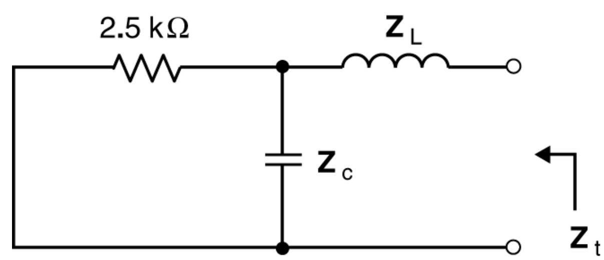
Solution: Represent the circuit in the frequency domain as



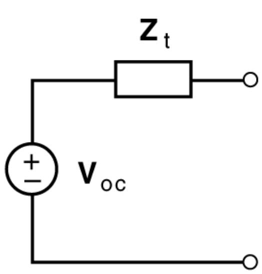
Determine the open circuit voltage and Thevenin impedance:



$$\mathbf{V}_{oc} = \frac{\mathbf{Z}_C}{2500 + \mathbf{Z}_C}(12\angle 0°)$$



$$\mathbf{Z}_t = \frac{2500\,\mathbf{Z}_C}{2500 + \mathbf{Z}_C} + \mathbf{Z}_L$$



a. $\mathbf{V}_{oc} = 2.3534\angle -78.69°\ \text{V}$ and $\mathbf{Z}_t = 775.22\angle 82.875°\ \Omega$

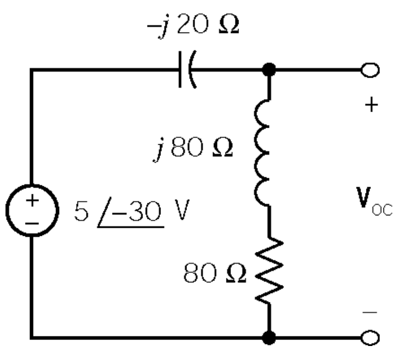
b. $\mathbf{V}_{oc} = 1.194\angle -84.29°\ \text{V}$ and $\mathbf{Z}_t = 2252.6\angle 89.37°\ \Omega$

c. $\mathbf{V}_{oc} = 0.59925\angle -87.14°\ \text{V}$ and $\mathbf{Z}_t = 4875.3\angle 89.93°\ \Omega$

The Thevenin Equivalent Circuit changes whenever the input frequency changes.

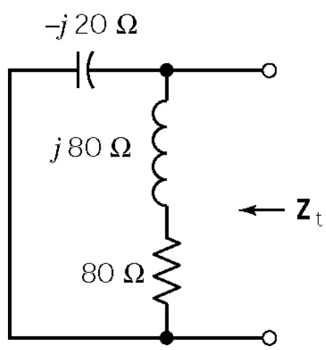
P 10.7-2

Solution:



Find $\mathbf{V}_{oc}$:

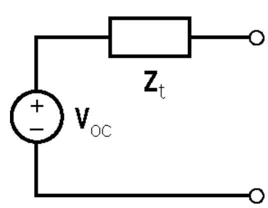
$$\mathbf{V}_{oc} = (5\ \angle{-30°})\left(\frac{80\ +j80}{80\ +j80-j20}\right)$$
$$= (5\ \angle{-30°})\left(\frac{80\sqrt{2}\angle{-45°}}{100\angle 36.90°}\right)$$
$$=\ 4\sqrt{2}\angle -21.9°\ \text{V}$$



Find $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \frac{(-j20)(80+j80)}{-j20+80+j80} = 23\ \angle -81.9°\ \ \Omega$$

The Thevenin equivalent is



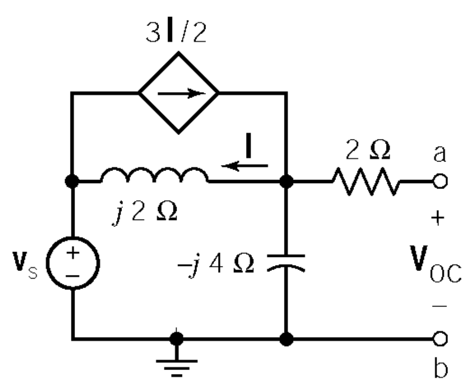
P 10.7-3

Solution:

First, determine $\mathbf{V}_{oc}$:

The node equation is:

$$\frac{\mathbf{V}_{oc}}{-j4}+\frac{\mathbf{V}_{oc}-(6+j8)}{j2}-\frac{3}{2}\left(\frac{\mathbf{V}_{oc}-(6+j8)}{j2}\right)=0$$



$$\mathbf{V}_{oc} = 3 + j4 = 5\angle 53.1^\circ \text{ V}$$

$$\mathbf{V}_s = 10\angle 53^\circ = 6 + j8 \text{ V}$$

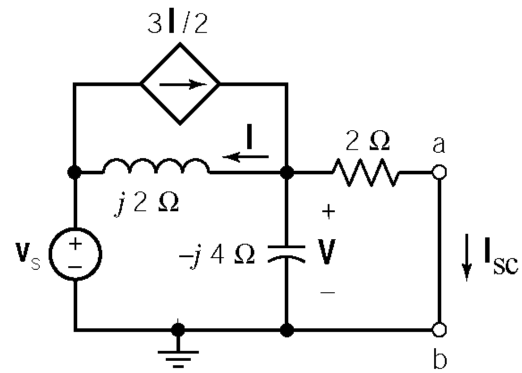
Next, determine $\mathbf{I}_{sc}$:

The node equation is:

$$\frac{\mathbf{V}}{2} + \frac{\mathbf{V}}{-j4} + \frac{\mathbf{V}-(6+j8)}{j2} - \frac{3}{2}\left[\frac{\mathbf{V}-(6+j8)}{j2}\right] = 0$$

$$\mathbf{V} = \frac{3+j4}{1-j}$$

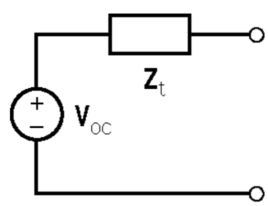
$$\mathbf{I}_{sc} = \frac{\mathbf{V}}{2} = \frac{3+j4}{2-j2}$$



$$\mathbf{V}_s = 10\angle 53^\circ = 6 + j8 \text{ V}$$

The Thevenin impedance is $\mathbf{Z}_T = \dfrac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = 3 + j4\left(\dfrac{2-j2}{3+j4}\right) = 2 - j2\ \Omega$

The Thevenin equivalent is



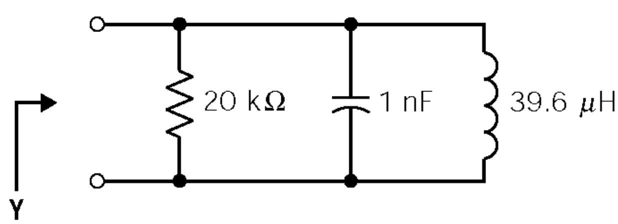
(checked: LNAP 7/18/04)

P10.7-4

Solution:

$$\mathbf{Y} = G + \mathbf{Y}_L + \mathbf{Y}_C$$

$$\mathbf{Y} = G \text{ when } \mathbf{Y}_L + \mathbf{Y}_C = 0 \text{ or } \frac{1}{j\omega L} + j\omega C = 0$$



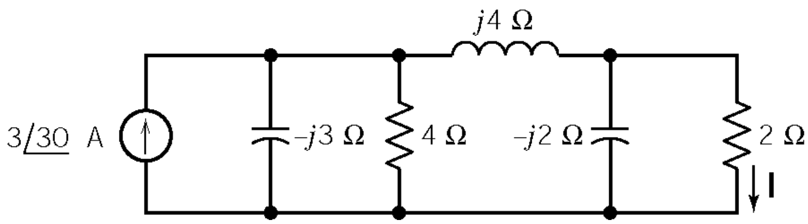
$$\omega_O = \frac{1}{\sqrt{LC}},\ f_O = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.6\times 10^{-15}}}$$

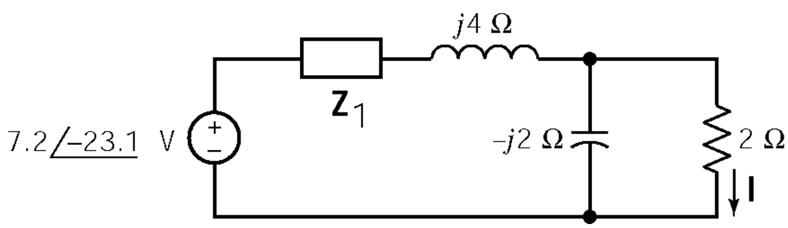
$$= 0.07998\times 10^7 \text{ Hz} = 800 \text{ kHz}$$

(80 on the dial of the radio)

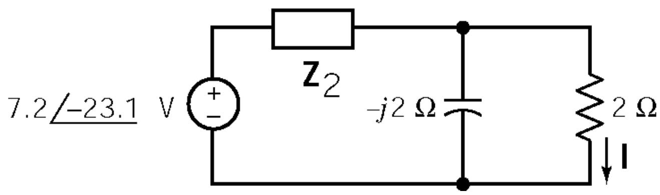
P 10.7-5

Solution:

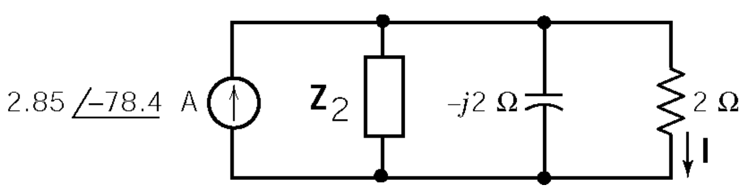




$$\mathbf{Z}_1 = \frac{(-j3)(4)}{-j3+4} = 2.4\angle -53.1°\ \Omega$$
$$= 1.44 - j1.92\ \Omega$$



$$\mathbf{Z}_2 = \mathbf{Z}_1 + j4$$
$$= 1.44 + j2.08$$
$$= 2.53\angle 55.3°\ \Omega$$



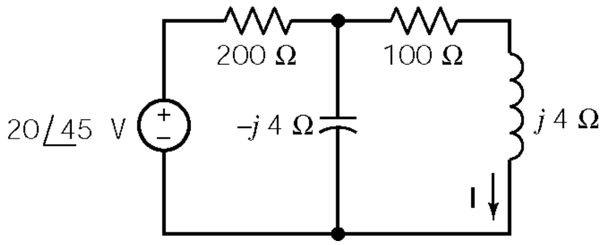
$$\mathbf{Z}_3 = 3.51\angle -37.9°\ \Omega$$
$$= 2.77 - j2.16\ \Omega$$

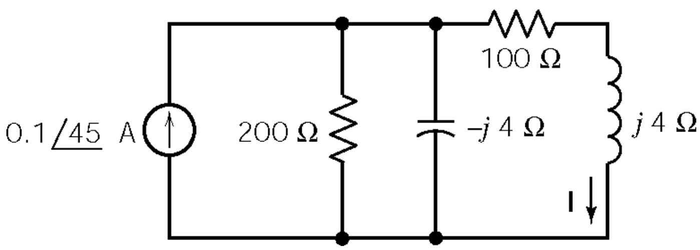
$$\mathbf{I} = (2.85\angle -78.4°)\left(\frac{3.51\angle -37.9°}{2.77 - j2.16 + 2}\right) = (2.85\angle -78.4°)\frac{(3.51\angle -37.9°)}{(5.24\angle -24.4°)} = 1.9\angle -92°\ \text{A}$$

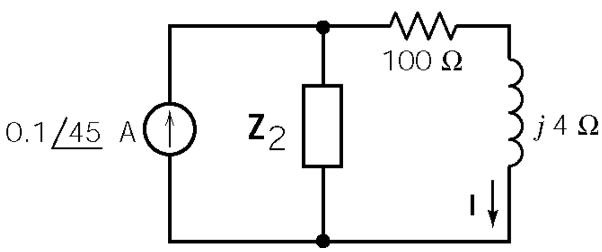
(checked: LNAP 7/18/04)

P 10.7-6

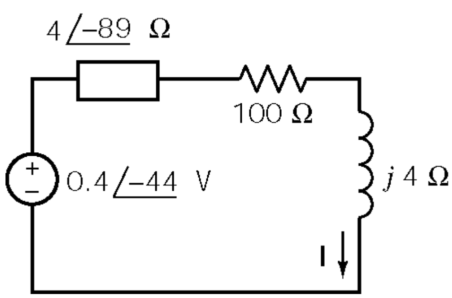
Solution:







$$\mathbf{Z}_2 = \frac{(200)(-j4)}{200-j4} = 4\angle -88.8°\ \Omega$$

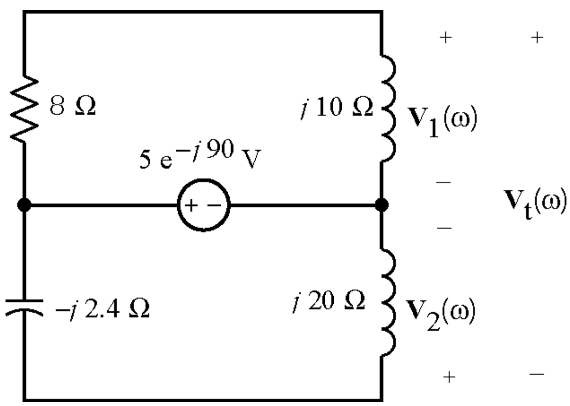


$$\mathbf{I} = \frac{0.4\angle -44°}{-4j+100+j4} = 4\angle -44°\ \text{ mA}$$

$$i(t) = 4\cos(25000t - 44°)\ \text{mA}$$

P10.7-7

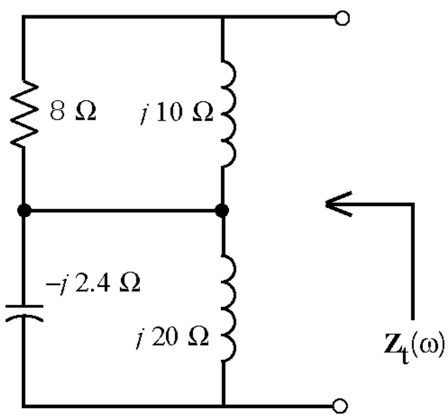
Solution:



$$\mathbf{V}_1 = \frac{j10}{8+j10}5e^{-j90} = 3.9e^{-j51}$$

$$\mathbf{V}_2 = \frac{j20}{j20-j2.4}5e^{-j90} = 5.68e^{-j90}$$

$$\mathbf{V}_t = \mathbf{V}_1 - \mathbf{V}_2 = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47}$$



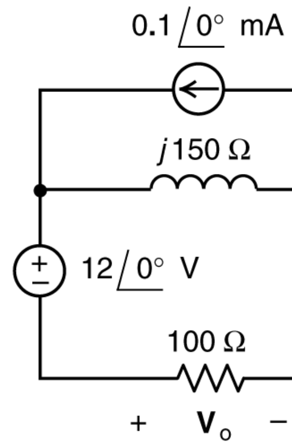
$$\mathbf{Z}_t = \frac{8(j10)}{8+j10} + \frac{-j2.4(j20)}{-j2.4+j20} = 4.9 + j\,1.2$$

Section 10.8 Superposition

P 10.8-1

Solution:

(***a***) Represent the circuit in the frequency domain as

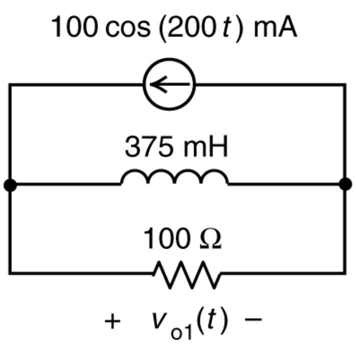


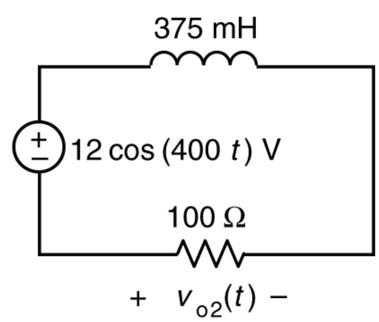
Use superposition *in the frequency domain* to write

$$\mathbf{V}_\mathrm{o} = -\frac{100}{100+j150}(12\angle 0°) + 100\frac{j150}{100+j150}(0.1\angle 0°)$$
$$= \frac{-1200+j1500}{100+j150} = 10.66\angle 72.35°$$

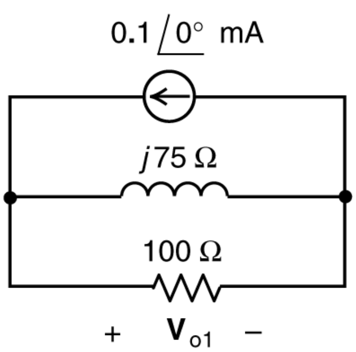
In the time domain $v_\mathrm{o}(t) = 10.66\cos(400t + 72.35)\ \mathrm{V}$

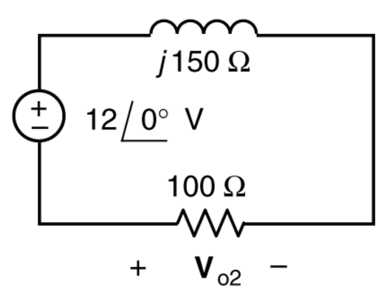
(***b***) Use superposition *in the time domain*. These circuits can be used to find the part of v_o caused by the current source and the part of v_o caused by the voltage source.





In the frequency domain:





$$\mathbf{V}_{\mathrm{o}1} = 100\frac{j75}{100+j75}(0.1\angle 0°) = 6\angle 53.1°\ \mathrm{V}$$

$$\mathbf{V}_{\mathrm{o}2} = -\frac{100}{100+j150}(12\angle 0°) = 6.656\angle 123.7°\ \mathrm{V}$$

In the time domain

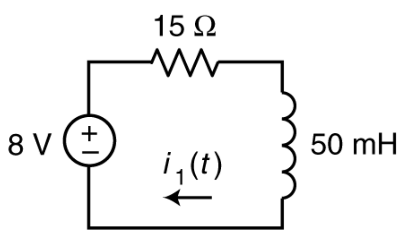
$$v_{o1}(t)=6\cos(200t+53.1°)\ \text{V} \text{ and } v_{o2}(t)=6.656\cos(400t+123.7°)\ \text{V}$$

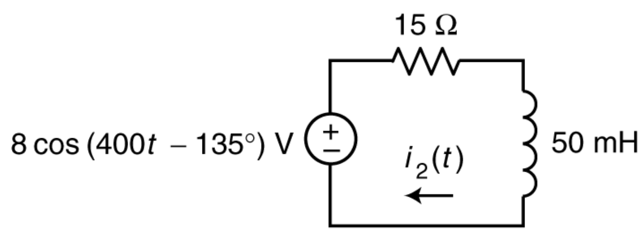
and

$$v_o(t)=v_{o1}(t)+v_{o2}(t)=6\cos(200t+53.1°)+6.656\cos(400t+123.7°)\ \text{V}$$

P 10.8-2

Use superposition *in the time domain*:

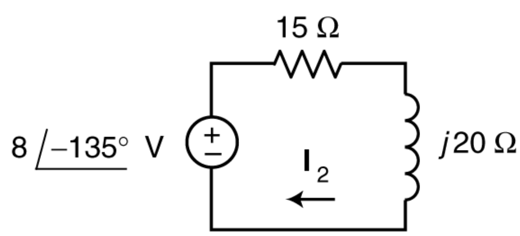




An inductor in a dc circuit acts like a short circuit so:

$$i_1(t)=\frac{8}{15}=0.533\ \text{A}$$

Represent the right circuit the frequency domain:



$$\mathbf{I}_2=\frac{8\angle-135°}{15+j20}=0.32\angle-188°\ \text{A}$$

In the time domain $\quad i_2(t)=0.32\cos(400t-188°)\ \text{A}$

and

$$i(t)=i_1(t)+i_2(t)=0.533+0.32\cos(400t-188°)\ \text{A}$$

P 10.8-3

Solution:

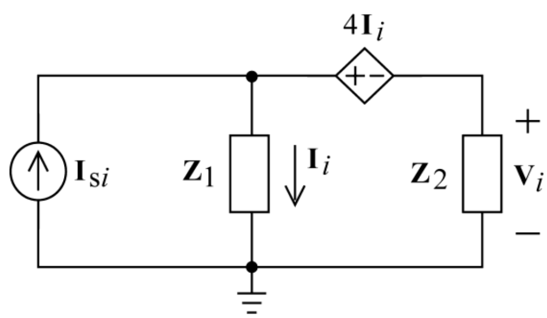
Use superposition in the time domain. Let

$$i_{s1}(t) = 36\cos(25t) \text{ mA} \quad \text{and} \quad i_{s2}(t) = 48\cos(50t + 45°) \text{ mA}$$

We will find the response to each of these inputs separately. Let $i_i(t)$ denote the response to $i_{si}(t)$ for i = 1,2. The sum of the two responses will be $i(t)$, i.e.

$$i(t) = i_1(t) + i_2(t)$$

Represent the circuit in the frequency domain as



Use KVL to get

$$\mathbf{V}_i = \mathbf{Z}_i \mathbf{I}_i - 4\mathbf{I}_i$$

Apply KCL to the supernode corresponding to the dependent voltage source.

$$\mathbf{I}_{si} = \mathbf{I}_i + \frac{\mathbf{V}_i}{\mathbf{Z}_2} = \frac{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}{\mathbf{Z}_2}\mathbf{I}_i$$

or

$$\mathbf{I}_i = \frac{\mathbf{Z}_2 \mathbf{I}_{si}}{\mathbf{Z}_1 + \mathbf{Z}_2 - 4}$$

Consider the case i = 1 : $i_{s1}(t)$ = 26cos(25t) mA.

Here ω = 25 rad/s and

$$\mathbf{I}_{si} = 36\angle 0° \text{ mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(25)(0.002)} = 20 - j20\ \Omega$$

$$\mathbf{Z}_2 = j50 + \left(15 \parallel \frac{1}{j(25)(0.004)}\right) = 43.3\angle 83.9°\ \Omega$$

and

$$\mathbf{I}_1 = 50.4\angle 35.7° \text{ mA}$$

so

$$i(t) = 50.4\cos(25t + 35.7°)\ \text{mA}$$

Next consider $i = 2$: i_{s2} = 48cos(50t + 45°) mA.

Here ω = 50 rad/s and

$$\mathbf{I}_{s2} = 48\angle 45°\ \text{mA}$$

$$\mathbf{Z}_1 = 20 + \frac{1}{j(50)(0.002)} = 20 - j10\ \Omega$$

$$\mathbf{Z}_2 = j100 + \left(15 \| \frac{1}{j(50)(0.004)}\right) = 95.5\angle 89.1°\ \Omega$$

(Notice that $\mathbf{Z}_1$ and $\mathbf{Z}_2$ change when ω changes.)

$$\mathbf{I}_2 = 52.5\angle 55.7°\ \text{mA}$$

so

$$i_2(t) = 52.5\cos(50t + 55.7°)\ \text{mA}$$

Finally, using superposition <u>in the time</u> domain gives

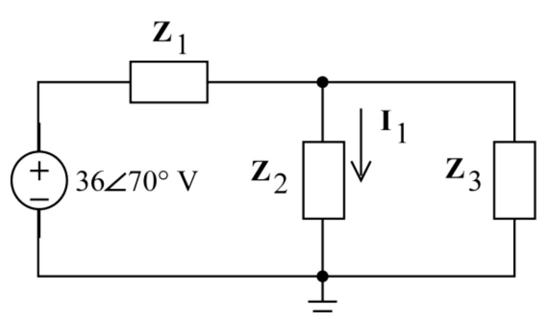
$$i(t) = 50.4\cos(25t + 35.7°) + 52.5\cos(50t + 55.7°)\ \text{mA}$$

(checked: LNAP 8/7/04)

P 10.8-4

Solution:

Use superposition in the time domain. Let $i_1(t)$ be the part of $i(t)$ due to $v_{s1}(t)$ and $i_2(t)$ be the part of $i(t)$ due to $v_{s2}(t)$. To determine $i_1(t)$, set $v_{s2}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_1 = 20 + j80 = 82.46\angle 76°\ \Omega$$

$$\mathbf{Z}_2 = 10 + \left(j40 \parallel 15 \right) = 23.15 + j4.93 = 23.67\angle 12°\ \Omega$$

$$\mathbf{Z}_3 = 20 + \frac{1}{j(20)(0.005)} = 20 - j10 = 22.36\angle -26.6°\ \Omega$$

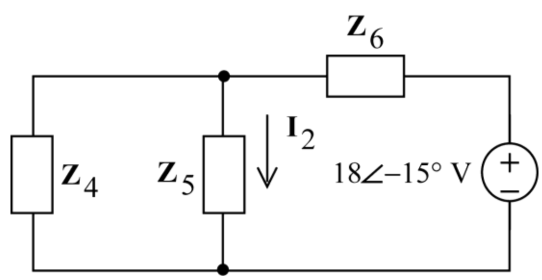
Next, using Ohm's law and current division gives

$$\mathbf{I}_1 = \frac{30\angle 70°}{\mathbf{Z}_1 + \left(\mathbf{Z}_2 \parallel \mathbf{Z}_3\right)} \times \frac{\mathbf{Z}_3}{\mathbf{Z}_2 + \mathbf{Z}_3} = \frac{\mathbf{Z}_3\left(30\angle 70°\right)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.182\angle -17.6°\ \text{A}$$

so

$$i(t) = 0.182\cos\left(20t - 17.6°\right)\ \text{A}$$

To determine $i_2(t)$, set $v_{s1}(t) = 0$. Represent the resulting circuit in the frequency domain to get



where

$$\mathbf{Z}_4 = 20 + j40 = 44.72\angle 63.4°\ \Omega$$

$$\mathbf{Z}_5 = 10 + \left(j20 \parallel 15 \right) = 19.6 + j7.2 = 20.88\angle 20.2°\ \Omega$$

$$\mathbf{Z}_6 = 20 + \frac{1}{j(10)(0.005)} = 20 - j20 = 28.28\angle -45°\ \Omega$$

Next, using Ohm's law and current division gives

$$\mathbf{I}_2 = \frac{18\angle -15°}{\mathbf{Z}_6 + \left(\mathbf{Z}_4 \parallel \mathbf{Z}_5\right)} \times \frac{\mathbf{Z}_4}{\mathbf{Z}_4 + \mathbf{Z}_5} = \frac{\mathbf{Z}_1\left(18\angle -15°\right)}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3} = 0.377\angle 18°\ \text{A}$$

so

$$i_2(t) = 0.377\cos\left(10t + 18°\right)\ \text{A}$$

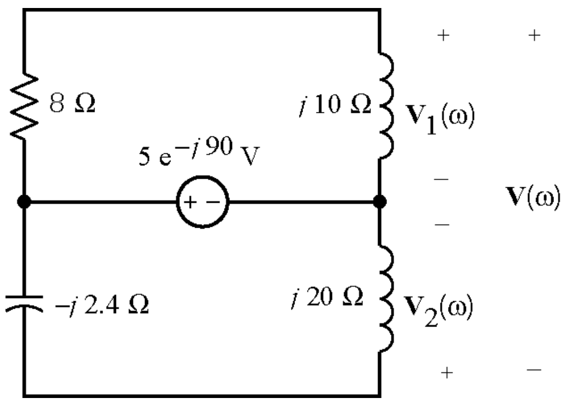
Using superposition,

$$i(t) = i_1(t) + i_2(t) = 0.182\cos\left(20t - 17.6°\right) + 0.377\cos\left(10t + 18°\right)\ \text{A}$$

(checked: LNAP 8/8/04)

P 10.8-5

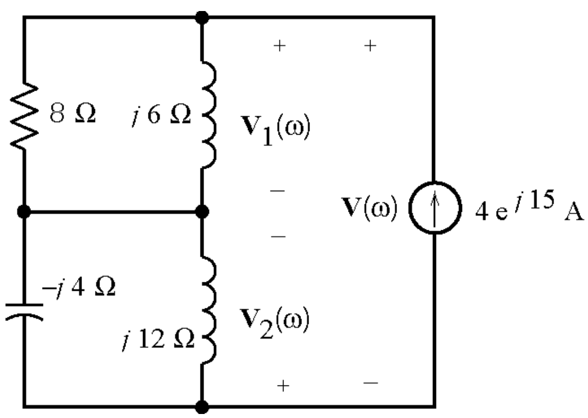
Solution:



$$\mathbf{V}_1(\omega) = \frac{j10}{8 + j10} 5e^{-j90} = 3.9e^{-j51}$$

$$\mathbf{V}_2(\omega) = \frac{j20}{j20 - j2.4} 5e^{-j90} = 5.68e^{-j90}$$

$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) - \mathbf{V}_2(\omega) = 3.9e^{-j51} - 5.68e^{-j90}$$
$$= 3.58e^{j47}$$



$$\mathbf{V}_1(\omega) = \frac{8(j6)}{8 + j6} 4e^{j15} = 19.2e^{j68}$$

$$\mathbf{V}_2(\omega) = \frac{j12(-j4)}{j12 - j4} 4e^{j15} = 24e^{-j75}$$

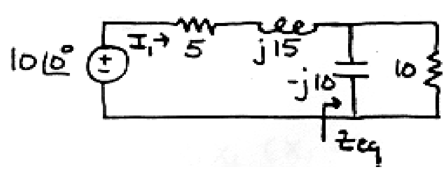
$$\mathbf{V}(\omega) = \mathbf{V}_1(\omega) + \mathbf{V}_2(\omega) = 14.4e^{-j22}$$

Using superposition: $v(t)$ = 3.58 cos (5t + 47°) + 14.4 cos (3t - 22°) V.

P 10.8-6

Solution:

Use superposition. First, find the response to the voltage source acting alone:



$$\mathbf{Z}_{\text{eq}} = \frac{-j10 \cdot 10}{10 - j10} = 5(1 - j)\ \Omega$$

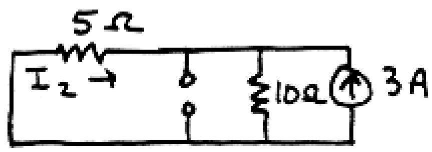
Replacing the parallel elements by the equivalent impedance. The write a mesh equation :

$$-10+5\,\mathbf{I}_1+j15\,\mathbf{I}_1+5(1-j)\,\mathbf{I}_1=0 \quad\Rightarrow\quad \mathbf{I}_1 = \frac{10}{10+j10} = 0.707\angle -45°\ \text{A}$$

Therefore:

$$i_1(t) = 0.707\cos(10t-45^\circ)\ \text{A}$$

Next, find the response to the dc current source acting alone:



Current division: $I_2=-\dfrac{10}{15}\times 3 = -2\ \text{A}$

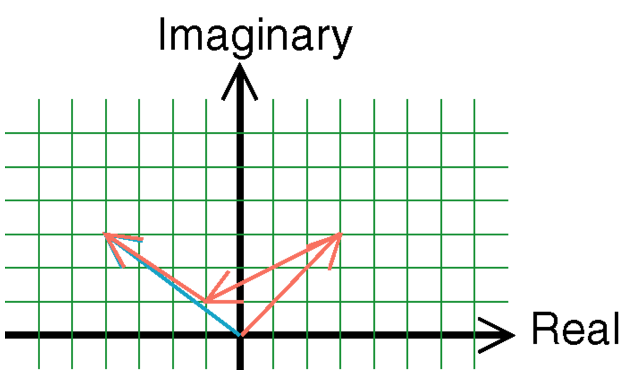
Using superposition:

$$i(t) = 0.707\cos(10t-45°)-2\ \ \text{A}$$

Section 10-9: Phasor Diagrams

P 10.9-1

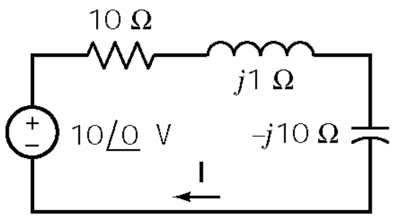
Solution:



$$\mathbf{V}=\mathbf{V}_1-\mathbf{V}_2+\mathbf{V}_3^{\,*}=(3+j3)-(4+j2)+(-3-j2)^{*}=-4+j3$$

P 10.9-2

Solution:



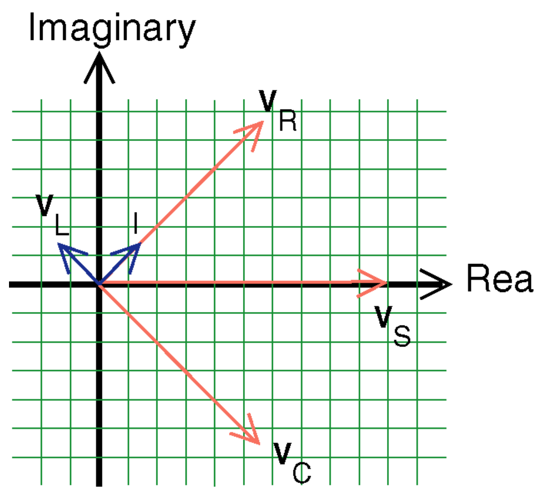
$$\mathbf{I} = \frac{10\angle 0°}{10 + j1 - j10} = 0.74\angle 42°\ \text{A}$$

$\mathbf{V}_R = R\mathbf{I} = 7.4\angle 42°\ \text{V}$

$\mathbf{V}_L = \mathbf{Z}_L\mathbf{I} = (1\angle 90°)(0.74\angle 42°) = 0.74\angle 132°\ \text{V}$

$\mathbf{V}_C = \mathbf{Z}_C\mathbf{I} = (10\angle{-90°})(0.74\angle 42°) = 7.4\angle{-48°}\ \text{V}$

$\mathbf{V}_S = 10\angle 0°\ \text{V}$

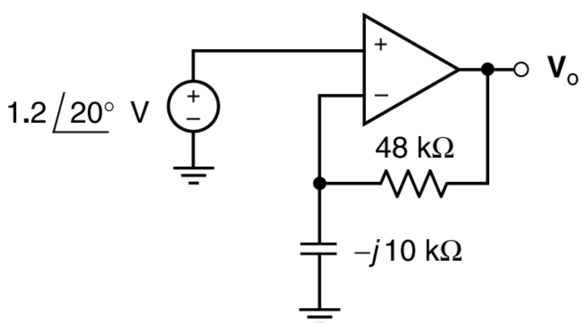


Section 10.10 Op Amps in AC Circuits

P 10.10-1

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as a noninverting amplifier, we can write

$$\mathbf{V}_0 = \left(1 + \frac{48}{-j10}\right)(1.2\angle 20°) = (1 + j\,4.8)(1.2\angle 20°) = 5.88\angle 98°\ \text{V}$$

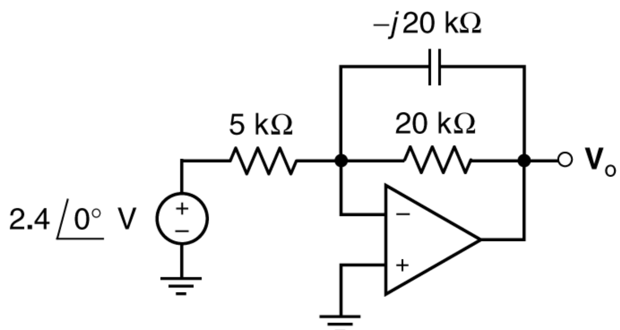
In the time domain $v_o(t) = 5.88\cos(400t + 98°)$ V

(Checked using LNAPAC 3/15/12)

P 10.10-2

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as an inverting amplifier, we can write

$$\mathbf{V}_o = \left(-\frac{20 \parallel -j20}{5}\right)(2.4\angle 0) = \left((1\angle 180°)\frac{14.14\angle -45°}{5}\right)(2.4\angle 0) = 6.788\angle 135° \text{ V}$$

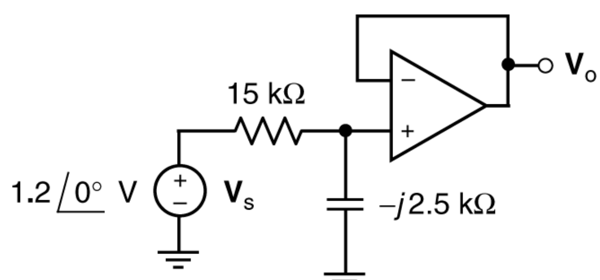
In the time domain $v_o(t) = 6.788\cos(500t + 135°)$ V

(Checked using LNAPAC 3/15/12)

P 10.10-3

Solution:

Represent the circuit in the frequency domain as



Recognizing this circuit as a voltage divider followed by a voltage follower, we can write

$$\mathbf{V}_o = \left(\frac{-j2.5}{15 - j2.5}\right)(1.2\angle 0°) = \left(\frac{2.5\angle -90°}{15.2\angle -9.46°}\right)(1.2\angle 0°) = 0.1974\angle -80.54° \text{ V}$$

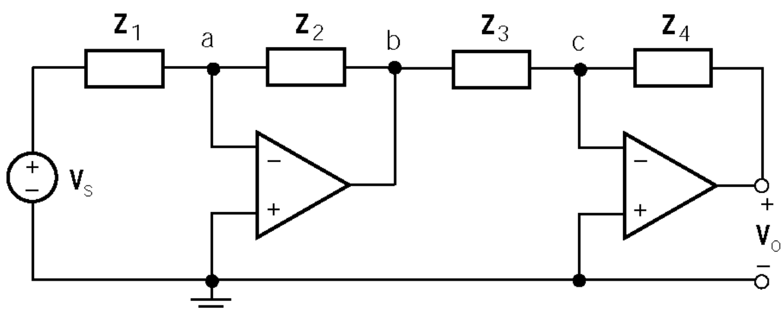
In the time domain $v_o(t) = 0.1974\cos(400t - 80.54°)$ V

(Checked using LNAPAC 3/15/12)

P 10.10-4

Solution:

Label the nodes:



The ideal op amps force $\mathbf{V}_a = 0$ and $\mathbf{V}_c = 0$.

Apply KCL at node a to get $$\mathbf{V}_b = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s$$

Apply KCL at node c to get $$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_b$$

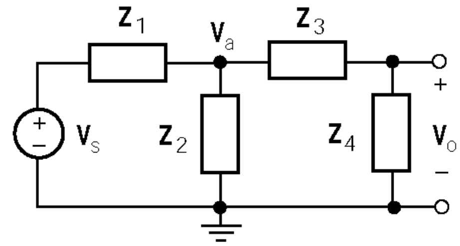
Therefore

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

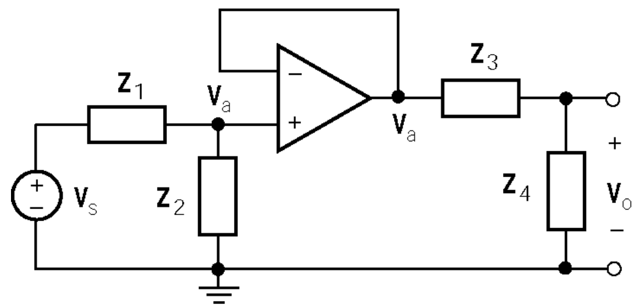
P 10.10-5

Solution:

Label a node voltage as $\mathbf{V}_a$ in each of the circuits.



(a)



(b)

In both circuits, we can apply KCL at the node between $\mathbf{Z}_3$ and $\mathbf{Z}_4$ to get

$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \mathbf{V}_a$$

In (*a*)

$$\mathbf{V}_a = \frac{\mathbf{Z}_2 \,\|\, (\mathbf{Z}_3 + \mathbf{Z}_4)}{\mathbf{Z}_1 + \mathbf{Z}_2 \,\|\, (\mathbf{Z}_3 + \mathbf{Z}_4)} \mathbf{V}_s$$
$$= \frac{\mathbf{Z}_2 (\mathbf{Z}_3 + \mathbf{Z}_4)}{\mathbf{Z}_1 (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) + \mathbf{Z}_2 (\mathbf{Z}_3 + \mathbf{Z}_4)} \mathbf{V}_s$$

so

$$\frac{\mathbf{V}_a}{\mathbf{V}_s} = \frac{\mathbf{Z}_2 \, \mathbf{Z}_4}{\mathbf{Z}_1 (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4) + \mathbf{Z}_2 (\mathbf{Z}_3 + \mathbf{Z}_4)}$$

In (*b*)

$$\mathbf{V}_a = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \mathbf{V}_s$$

so

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4} \times \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

P 10.10-6

Solution:

The network function of the circuit is

$$\frac{\mathbf{V}_o}{\mathbf{V}_s}=\left(1+\frac{R_2}{1000}\right)\frac{\frac{1}{j\omega C}}{R_1+\frac{1}{j\omega C}}=\frac{1+\frac{R_2}{1000}}{1+j\omega C R_1}=\frac{1+\frac{R_2}{1000}}{1+j10^{-3}R_1}$$

Converting the given input and output sinusoids to phasors gives

$$\frac{\mathbf{V}_o}{\mathbf{V}_s}=\frac{5\angle 71.6°}{2}$$

Consequently

$$\frac{5\angle 71.6°}{2}=\frac{1+\frac{R_2}{1000}}{1+j10^{-3}R_1}$$

Equating angles gives

$$71.6° = -\tan^{-1}\left(10^{-3}R_1\right) \quad \Rightarrow \quad R_1=\tan\left(71.6°\right)\times 10^3 = 3006\ \Omega$$

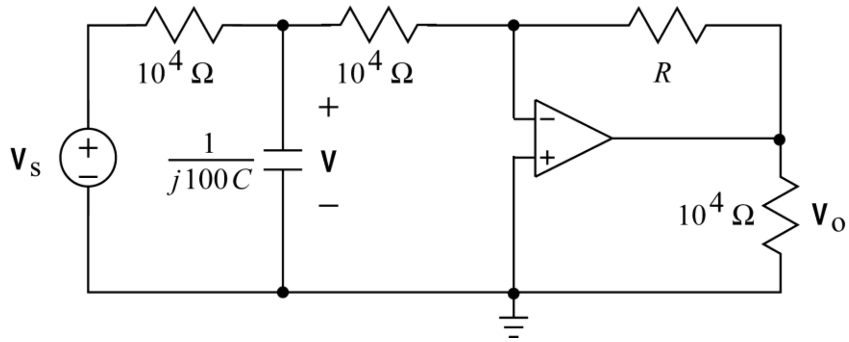
Equating magnitudes gives

$$\frac{5}{2}=\frac{1+\frac{R_2}{1000}}{\sqrt{1+\left(10^{-3}R_1\right)^2}}=\frac{1+\frac{R_2}{1000}}{\sqrt{1+\left(10^{-3}\times 3006\right)^2}} \quad \Rightarrow \quad R_2=\left(\frac{5}{2}\sqrt{10}-1\right)\times 10^3 = 6906\ \Omega$$

P 10.10-7

Solution:

Represent the circuit in the frequency domain as



Apply KCL at the top node of the impedance of the capacitor to get

$$\frac{\mathbf{V}_s - \mathbf{V}}{10^4} = \frac{\mathbf{V}}{\dfrac{1}{j100C}} + \frac{\mathbf{V}}{10^4} \quad \Rightarrow \quad \frac{1}{2}\mathbf{V}_s = \left(1 + j\left(5\times10^5\right)C\right)\mathbf{V}$$

Apply KCL at the inverting node of the op amp to get

$$\frac{\mathbf{V}}{10^4} + \frac{\mathbf{V}_o}{R} = 0 \quad \Rightarrow \quad \mathbf{V}_o = -\frac{R}{10^4}\mathbf{V}$$

so

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\dfrac{R}{2\times10^4}}{1 + j\left(5\times10^5\right)C}$$

Converting the input and output sinusoids to phasors gives

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{8\angle 135°}{4\angle 0°} = 2\angle 135°$$

so

$$2\angle 135° = \frac{-\dfrac{R}{2\times10^4}}{1 + j\left(5\times10^5\right)C} = \frac{\dfrac{R}{2\times10^4}}{\sqrt{1+\left[\left(5\times10^5\right)C\right]^2}}\angle 180° - \tan^{-1}\left(\left(5\times10^5\right)C\right)$$

Equating angles gives

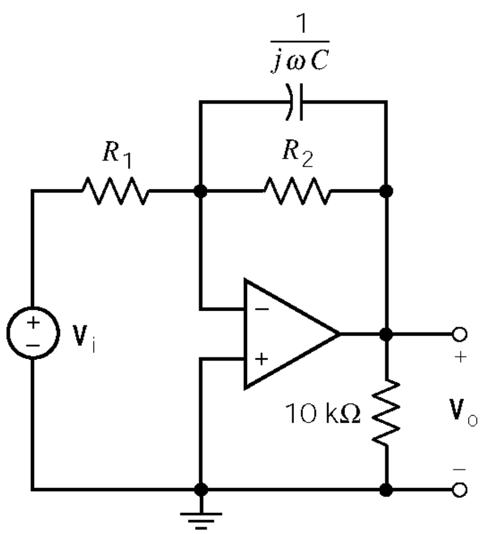
$$135° = 180° - \tan^{-1}\left(\left(5\times10^5\right)C\right) \quad \Rightarrow \quad C = \frac{\tan\left(45°\right)}{5\times10^5} = 2\times10^{-6} = 2\ \mu\text{F}$$

Next, equating magnitudes gives

$$2 = \frac{\dfrac{R}{2\times10^4}}{\sqrt{1+\left(5\times10^5\right)\left(2\times10^{-6}\right)}} = \frac{\dfrac{R}{2\times10^4}}{\sqrt{2}} \quad \Rightarrow \quad R = 10^4 = 10\ \text{k}\Omega$$

P 10.10-8

Solution:



$$R_2 \,\Big\|\, \frac{1}{j\omega C} = \frac{R_2}{1+j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\dfrac{R_2}{1+j\omega CR_2}}{R_1} = -\frac{\dfrac{R_2}{R_1}}{1+j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{R_1}}{\sqrt{1+(\omega CR_2)^2}} e^{j\left(180-\tan^{-1}\omega CR_2\right)}$$

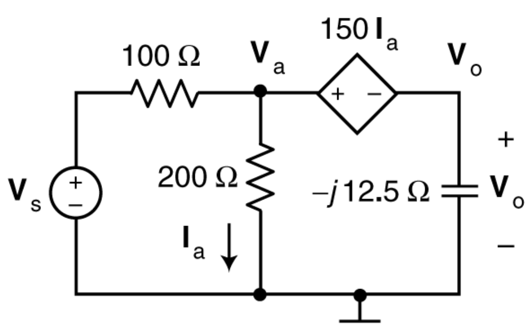
In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180°-104°)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\dfrac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \quad \Rightarrow \quad \dfrac{R_2}{R_1} = 13.2$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 1515\ \Omega,\ R_2 = 20\ \text{k}\Omega$.

Section 10.11 The Complete Response

P10.11-1

Solution:

Before the switch closes the circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I}_a = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so $$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

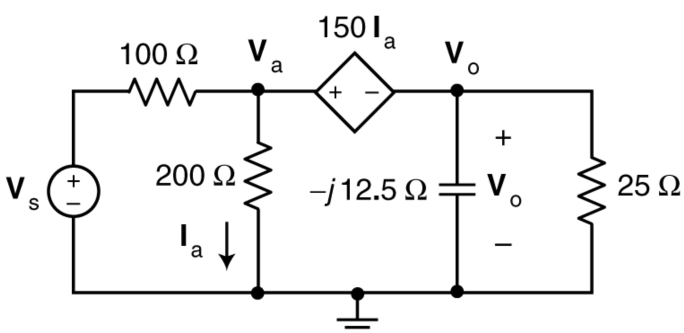
$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} \quad \Rightarrow \quad \mathbf{V}_o = \frac{-j12.5}{100}\mathbf{V}_s - \left(\frac{-j12.5}{100} + \frac{-j12.5}{200}\right)\mathbf{V}_a$$

$$\mathbf{V}_o = -j\,0.125\left(12\angle 0°\right) + j\,0.1875\left(4\,\mathbf{V}_o\right)$$

$$\mathbf{V}_o\left(1 - j0.75\right) = -j\,0.125\left(12\angle 0°\right) \quad \Rightarrow \quad \mathbf{V}_o = \frac{-j\,0.125\left(12\angle 0°\right)}{1 - j0.75} = 1.2\angle -53.1° \text{ V}$$

The corresponding sinusoid is $1.2\cos\left(4000t - 53.1°\right)$ V. The initial capacitor voltage is $v_o\left(0\right) = 1.2\cos\left(-53.1°\right) = 0.7205$ V.

The steady state response ***after the switch closes*** is the forced response. The circuit is represented in the frequency domain as



Express the dependent source voltage in terms of the node voltages:

$$150\,\mathrm{I}_a = \mathbf{V}_a - \mathbf{V}_o$$

Using Ohm's law

$$150\frac{\mathbf{V}_a}{200} = \mathbf{V}_a - \mathbf{V}_o$$

so $$\mathbf{V}_o = \frac{1}{4}\mathbf{V}_a$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{100} = \frac{\mathbf{V}_a}{200} + \frac{\mathbf{V}_o}{-j12.5} + \frac{\mathbf{V}_o}{25} \Rightarrow \quad \mathbf{V}_o\left(\frac{1}{-j12.5} + \frac{1}{25}\right) = \frac{1}{100}\mathbf{V}_s - \left(\frac{1}{100} + \frac{1}{200}\right)\mathbf{V}_a$$

Multiply by 200 to get

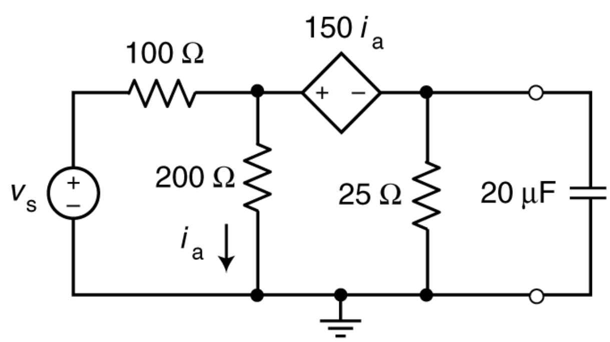
$$\mathbf{V}_o(8+j16)=2(12\angle 0°)-3(4\mathbf{V}_o) \quad \Rightarrow \quad \mathbf{V}_o=\frac{24}{20+j16}=0.937\angle -38.7° \text{ V}$$

The corresponding sinusoid is the forced response:

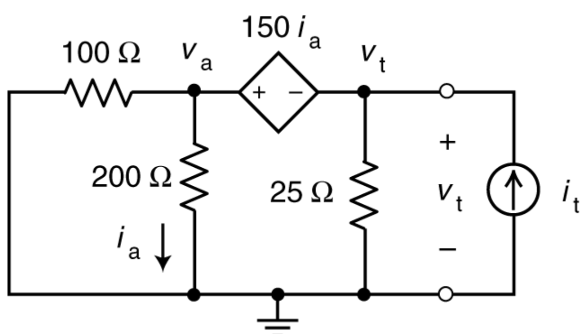
$$v_f(t)=0.937\cos(4000t-38.7°) \text{ V}$$

The natural response is $$v_n(t)=k\,e^{-t/\tau} \text{ V}$$

To determine the time constant τ we need find to find the Thevenin resistance of the part of the circuit connected to the capacitor after the switch closes. Here's the circuit:



The terminals separate the capacitor from the part of the circuit connected to the capacitor. Now (1) remove the capacitor, (2) replace the voltage source by a short circuit to set the input to zero and (3) connect a current source to the terminals to get



The Thevenin resistance is given by

$$R_t=\frac{v_t}{i_t}$$

Express the dependent source voltage in terms of the node voltages to get

$$v_a-v_t=150\,i_a=150\frac{v_a}{200} \quad \Rightarrow \quad v_a=4v_t$$

Apply KCL to the supernode corresponding to the dependent source to get

$$i_t=\frac{v_a}{100}+\frac{v_a}{200}+\frac{v_t}{25}=\frac{4v_t}{100}+\frac{4v_t}{200}+\frac{v_t}{25}=\frac{v_t}{10} \quad \Rightarrow \quad R_t=\frac{v_t}{i_t}=10\ \Omega$$

The time constant is $\tau = R_t\, C = 10\left(20\times10^{-6}\right) = 0.2\times10^{-3} = 0.2 \text{ ms}$

The natural response is $v_n\left(t\right) = k\, e^{-t/\tau} = k\, e^{-5000t} \text{ V}$

The complete response is

$$v_o\left(t\right) = 0.937\cos\left(4000\,t - 38.7°\right) + k\, e^{-5000t} \text{ V for } t \ge 0$$

Using the initial condition we calculate

$$0.7205 = v_o\left(0\right) = 0.937\cos\left(-38.7°\right) + k \quad \Rightarrow \quad k = -0.0108$$

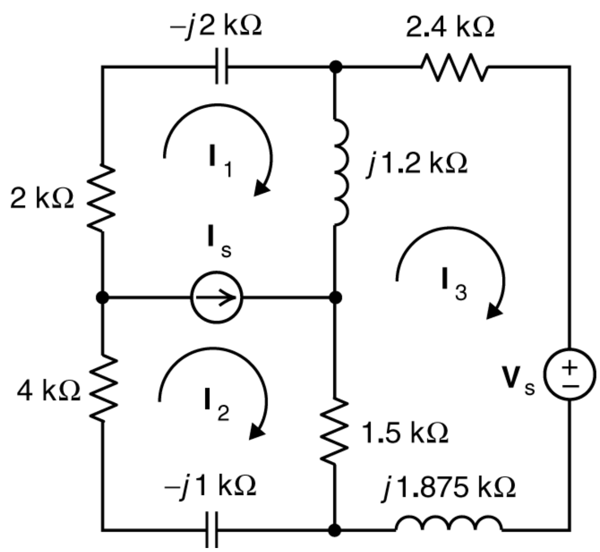
Finally $$v_o\left(t\right) = 0.937\cos\left(4000\,t - 38.8°\right) - 0.0108\,e^{-5000t} \text{ V for } t \ge 0$$

Section 10.12 Using MATLAB to Analyze Electric Circuits

10.12-1

Solution:

Represent the circuit in the frequency domain:



Represent the source current in terms of the mesh currents: $\mathbf{I}_2 - \mathbf{I}_1 = \mathbf{I}_s = 0.002\angle -15° \text{ A}$

Apply KVL to the supermesh corresponding to the current source:

$$\left(2000 - j\,2000\right)\mathbf{I}_1 + j1200\left(\mathbf{I}_1 - \mathbf{I}_3\right) + 1500\left(\mathbf{I}_2 - \mathbf{I}_3\right) + \left(4000 - j1000\right)\mathbf{I}_2 = 0$$

Apply KVL to mesh 3:

$$\left(2400 + j1875\right)\mathbf{I}_3 + 1500\left(\mathbf{I}_3 - \mathbf{I}_2\right) + j1200\left(\mathbf{I}_3 - \mathbf{I}_1\right) = -\mathbf{V}_s = -12\angle 60°$$

In matrix form: $\begin{bmatrix} -1 & 1 & 0 \\ 2000-j800 & 5500-j1000 & -1500-j1200 \\ -j1200 & -1500 & 3900+j3075 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0.002\angle -15° \\ 0 \\ -12\angle 60° \end{bmatrix}$

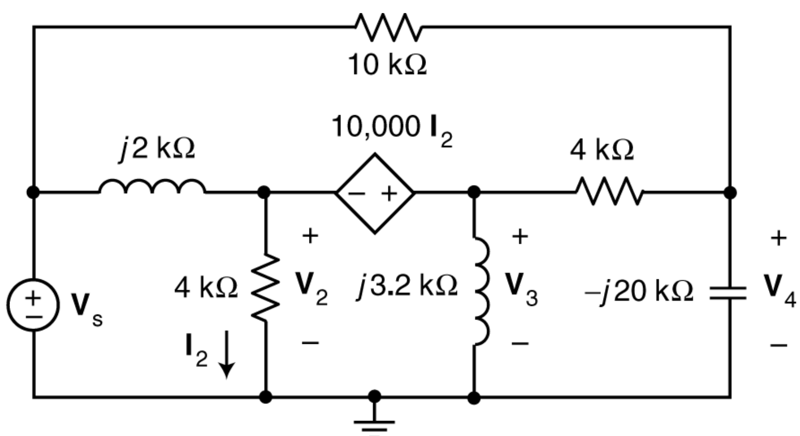
Solving, using MATLAB, gives $\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 1.549\angle -164° \\ 1.039\angle -65° \\ 2.904\angle -148° \end{bmatrix}$ mA

In the time domain: $\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.549\cos(2500t-164°) \\ 1.039\cos(2500t-65°) \\ 2.904\cos(2500t-148°) \end{bmatrix}$ mA

P 10.12-2

Solution:

Represent the circuit in the frequency domain:



Represent the dependent source voltage in terms of the node voltages currents:

$$\mathbf{V}_3 - \mathbf{V}_2 = 10000\frac{\mathbf{V}_2}{4000} \quad \Rightarrow \quad \mathbf{V}_3 = 3.5\mathbf{V}_2$$

Apply KCL to the supernode corresponding to the dependent voltage source:

$$\frac{\mathbf{V}_s - \mathbf{V}_2}{j2000} = \frac{\mathbf{V}_2}{4000} + \frac{\mathbf{V}_3}{j3200} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{4000}$$

Rearranging: $\frac{\mathbf{V}_s}{j2000} = \left(\frac{1}{4000} + \frac{1}{j2000}\right)\mathbf{V}_2 + \left(\frac{1}{4000} + \frac{1}{j3200}\right)\mathbf{V}_3 - \left(\frac{1}{4000}\right)\mathbf{V}_4$

Apply KCL at node 3: $\frac{\mathbf{V}_s - \mathbf{V}_4}{10,000} + \frac{\mathbf{V}_3 - \mathbf{V}_4}{4000} = \frac{\mathbf{V}_4}{-j20,000}$

Rearranging: $$\frac{\mathbf{V}_s}{10,000} = -\frac{\mathbf{V}_3}{4000} + \left(\frac{1}{4000} + \frac{1}{10,000} + \frac{1}{-j\,20,000}\right)\mathbf{V}_4$$

In matrix form: $$\begin{bmatrix} 3.5 & -1 & 0 \\ \frac{1}{4000} + \frac{1}{j\,2000} & \frac{1}{4000} + \frac{1}{j\,3200} & -\frac{1}{4000} \\ 0 & -\frac{1}{4000} & \frac{1}{4000} + \frac{1}{10,000} + \frac{1}{-j\,20,000} \end{bmatrix} \begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\mathbf{V}_s}{j\,2000} \\ \frac{\mathbf{V}_s}{10,000} \end{bmatrix}$$

Solving, using MATLAB, gives $$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{V}_3 \\ \mathbf{V}_4 \end{bmatrix} = \begin{bmatrix} 3.236\angle 34° \\ 11.324\angle 34° \\ 10.798\angle 29° \end{bmatrix} \text{ V}$$

In the time domain: $$\begin{bmatrix} v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3.236\cos(400t + 34°) \\ 11.324\cos(400t + 34°) \\ 10.798\cos(400t + 29°) \end{bmatrix} \text{ V}$$

P 10.12-3

Solution:

Represent the source current in terms of the mesh currents: $\mathbf{I}_1 - \mathbf{I}_2 = 4.2\angle 30° \text{ A}$

Apply KVL to the supermesh corresponding to the current source:

$$j8(\mathbf{I}_1 - \mathbf{I}_o) + 5\left[j8(\mathbf{I}_1 - \mathbf{I}_o)\right] + (4 + j5)\mathbf{I}_2 + (3 - j8)\mathbf{I}_1 = 0$$

Apply KVL to mesh 3: $$5\mathbf{I}_o + 6(j8)(\mathbf{I}_1 - \mathbf{I}_o) = 0$$

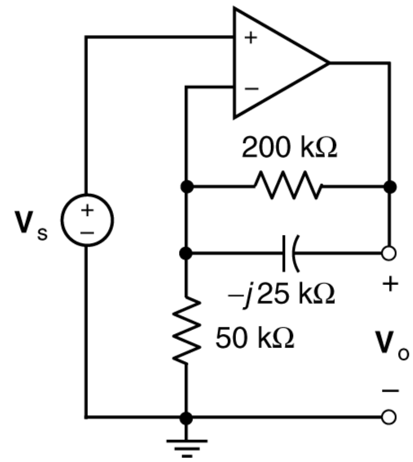
In matrix form: $$\begin{bmatrix} 1 & -1 & 0 \\ 3 + j40 & 4 + j5 & -j48 \\ -j48 & 0 & 5 + j48 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_o \end{bmatrix} = \begin{bmatrix} 4.2\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

Solving, using MATLAB, gives $$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_o \end{bmatrix} = \begin{bmatrix} 2.204\angle 93.1° \\ 3.758\angle 178° \\ 2.192\angle 99° \end{bmatrix} \text{ A}$$

P 10.12-4

Solution:

Represent the circuit in the frequency domain:



Write a node equation: $$\frac{12\angle 60° - \mathbf{V}_o}{200,000} + \frac{12\angle 60° - \mathbf{V}_o}{-j\,25,000} + \frac{12\angle 60°}{20,000} = 0$$

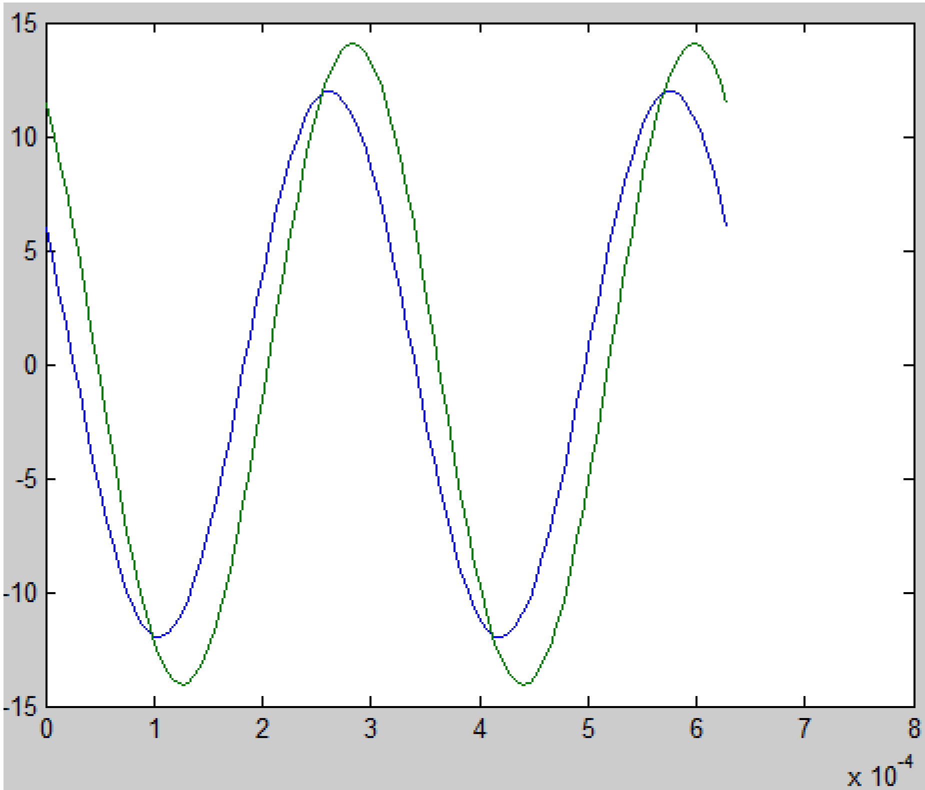
Rearrange: $$\left(\frac{1}{200,000} + \frac{1}{-j\,25,000} + \frac{1}{20,000}\right) 12\angle 60° = \left(\frac{1}{200,000} + \frac{1}{-j\,25,000}\right)\mathbf{V}_o$$

Modify the MATLAB script given in the textbook (and posted on the Student Companion Site for *Introduction to Electric Circuits*):

```
%----------------------------------------------

%        Describe the input voltage source.

%----------------------------------------------

w = 20000;

A = 12;

theta = (pi/180)*60;

Vs = A*exp(j*theta);

%-----------------------------------------------

%             Describe the impedances.

%-----------------------------------------------

R1=50e3; R2=200e3; ZC=-j*25e3;

%-----------------------------------------------

%    Calculate the phasor corresponding to the

%                   output voltage.

%-----------------------------------------------
```

```
Vo=(1/R2 + 1/ZC + 1/R1)*Vs/(1/R2 + 1/ZC);

B = abs(Vo)

phi = angle(Vo)

%-----------------------------------------------

%

%-----------------------------------------------

T = 2*pi/w;

tf = 2*T; N = 100; dt = tf/N;

t = 0 : dt : tf;

%-----------------------------------------------

%     Plot the input and output voltages.

%-----------------------------------------------

for k = 1 : 101

   vs(k) = A * cos(w * t(k) + theta);

   vo(k) = B * cos(w * t(k) + phi);

end

plot (t, vs, t, vo)
```

to get the plot:



Section 10.14 How Can We Check…?

P 10.14-1

Solution:

Generally, it is more convenient to divide complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the division in rectangular form.

Express **V**$_1$ and **V**$_2$ as: $\mathbf{V}_1 = -j\,20$ and $\mathbf{V}_2 = 20 - j\,40$

KCL at node 1:

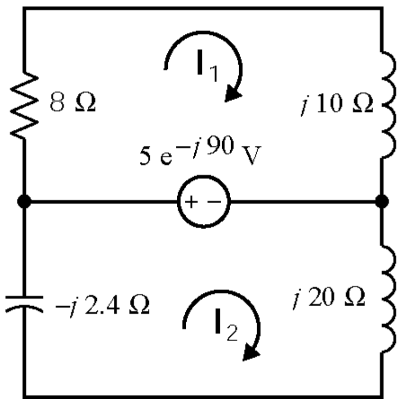
$$2 - \frac{\mathbf{V}_1}{10} - \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 2 - \frac{-j\,20}{10} - \frac{-j\,20 - (20 - j\,40)}{j10} = 2 + j\,2 - 2 - j\,2 = 0$$

KCL at node 2:

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} - \frac{\mathbf{V}_2}{10} + 3\left(\frac{\mathbf{V}_1}{10}\right) = \frac{-j\,20 - (20 - j\,40)}{j10} - \frac{20 - j\,40}{10} + 3\left(\frac{-j\,20}{10}\right) = (2 + j\,2) - (2 - j4) - j\,6 = 0$$

The currents calculated from $\mathbf{V}_1$ and $\mathbf{V}_2$ satisfy KCL at both nodes, so it is very likely that the $\mathbf{V}_1$ and $\mathbf{V}_2$ are correct.

P 10.14-2
Solution:



$\mathbf{I}_1 = 0.390\angle 39°$ and $\mathbf{I}_2 = 0.284\angle 180°$

Generally, it is more convenient to multiply complex numbers in polar form. Sometimes, as in this case, it is more convenient to do the multiplication in rectangular form.

Express $\mathbf{I}_1$ and $\mathbf{I}_2$ as: $\mathbf{I}_1 = 0.305 + j\,0.244$ and $\mathbf{I}_2 = -0.284$

KVL for mesh 1:

$$8\left(0.305 + j\,0.244\right) + j10\left(0.305 + j\,0.244\right) - (-j\,5) = j\,10 \neq 0$$

Since KVL is not satisfied for mesh 1, the mesh currents are not correct.

Here is a MATLAB file for this problem:

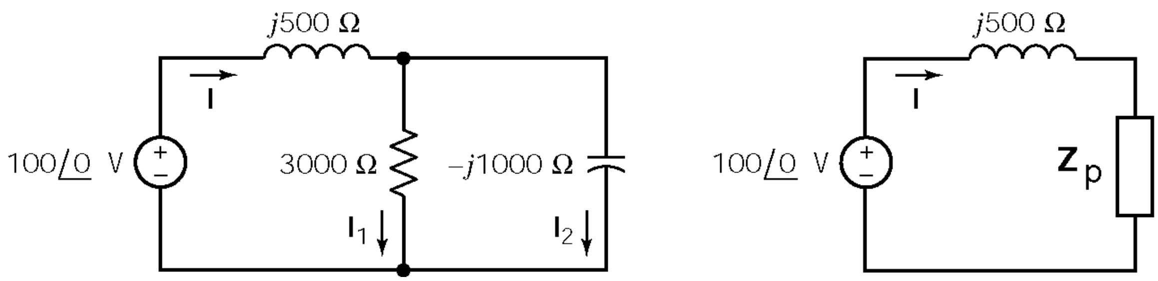
```
Vs = -j*5;

Z1 = 8;

Z2 = j*10;

Z3 = -j*2.4;

Z4 = j*20;

% Mesh equations in matrix form

Z = [  Z1+Z2      0;

          0      Z3+Z4 ];

V = [ Vs;

     -Vs  ];

I = Z\V

abs(I)

angle(I)*180/3.14159

% Verify solution by obtaining the algebraic sum of voltages for

% each mesh. KVL requires that both M1 and M2 be zero.
```

```
M1 = -Vs + Z1*I(1) +Z2*I(1)

M2 = Vs + Z3*I(2) + Z4*I(2)
```

P 10.14-3

Solution: First, replace the parallel resistor and capacitor by an equivalent impedance

$$\mathbf{Z}_P = \frac{(3000)(-j1000)}{3000-j1000} = 949\ \angle -72° = 300 - j900\ \Omega$$



The current is given by

$$\mathbf{I} = \frac{\mathbf{V}_S}{j500+\mathbf{Z}_P} = \frac{100\ \angle 0^\circ}{j500+300-j900} = 0.2\angle 53°\ \text{A}$$

Current division yields

$$\mathbf{I}_1 = \left(\frac{-j1000}{3000-j1000}\right)(0.2\ \angle 53°) = \ 63.3\ \angle -18.5°\ \text{mA}$$

$$\mathbf{I}_2 = \left(\frac{3000}{3000-j1000}\right)(0.2\ \angle 53°) = 190\angle 71.4°\ \text{mA}$$

The reported value of $\mathbf{I}_1$ is off by an order of magnitude.

P 10.14-4
Solution:

Represent the circuit in the frequency domain using phasors and impedances. Use voltage division to get

$$18.3\angle -24° = \frac{\dfrac{1}{j200C}}{R+\dfrac{1}{j200C}} \times 20\angle 0°$$

So $$0.915\angle -24° = \frac{1}{1+j200CR} = \frac{1}{\sqrt{1+(200CR)^2}}\angle -\tan^{-1}(200CR)$$

Equating angles gives

$$-24° = -\tan^{-1}(200CR) \quad \Rightarrow \quad 200CR = \tan(24°) = 0.4452$$

The nominal component values cause 200*CR* = 0.5. So we expect that the actual component values are smaller than the nominal values.

Try $$C = 5(1-0.10)\times 10^{-6} = 4.5\ \mu\text{F}$$

Then $$R = \frac{0.4452}{200\times 4.5\times 10^{-6}} = 494.67\ \Omega$$

Since $\dfrac{500-494.67}{500} = 0.01066 = 1.066\%$ this resistance is within 2% of 500 Ω. We conclude that the measured angle could have been caused by a capacitance that is within 10% of 5 μF and the resistance is within 2% of 500 Ω. Let's check the amplitude. We require

$$\frac{1}{\sqrt{1+(0.4452)^2}} = 0.9136 \approx 0.915$$

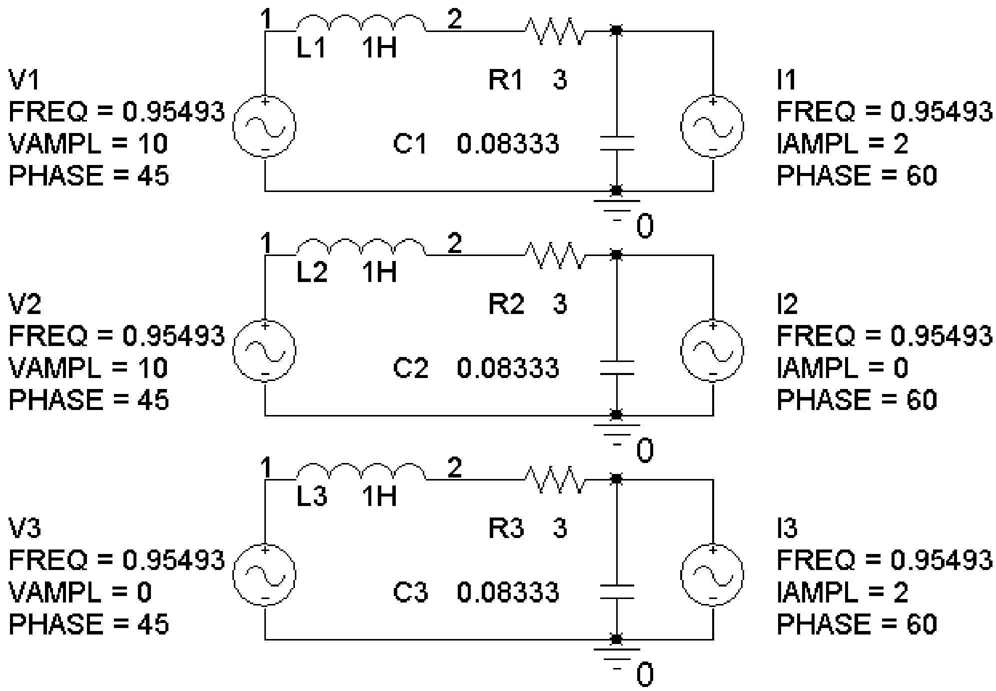
So the measured amplitude could also have been caused by the given circuit with *C* = 4.5 μF and *R* = 494.67 Ω.

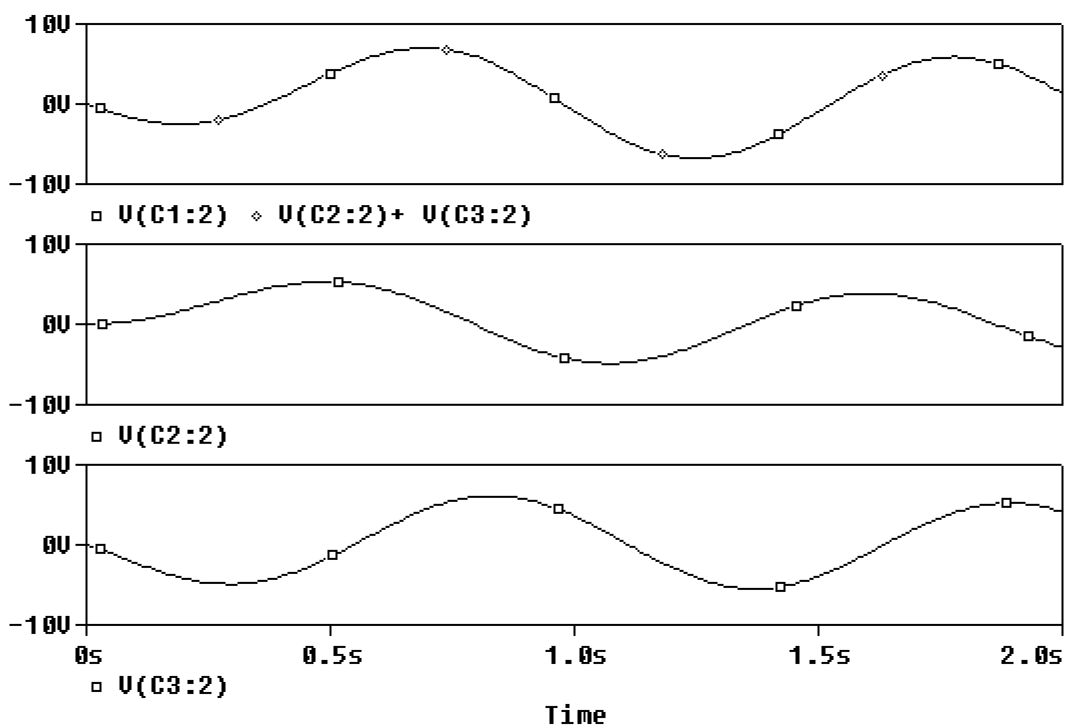
We conclude that he measured capacitor voltage could indeed have been produced by the given circuit with a resistance that is within 2 % of 500 Ω and a capacitance that is within 10% of 5 μF.

PSpice Problems

SP 10-1

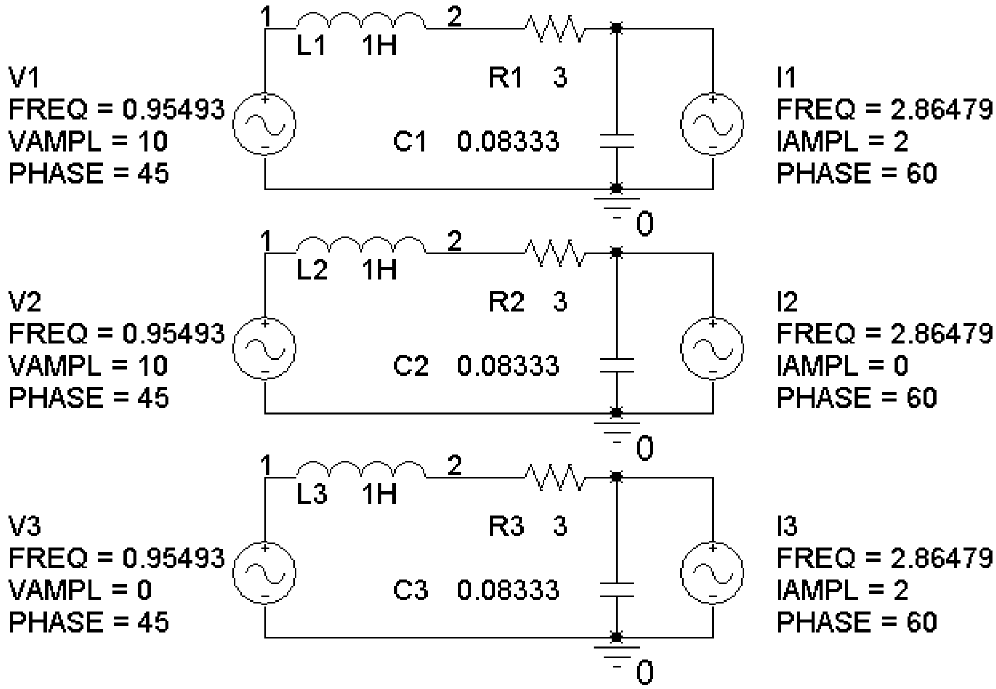
Solution:

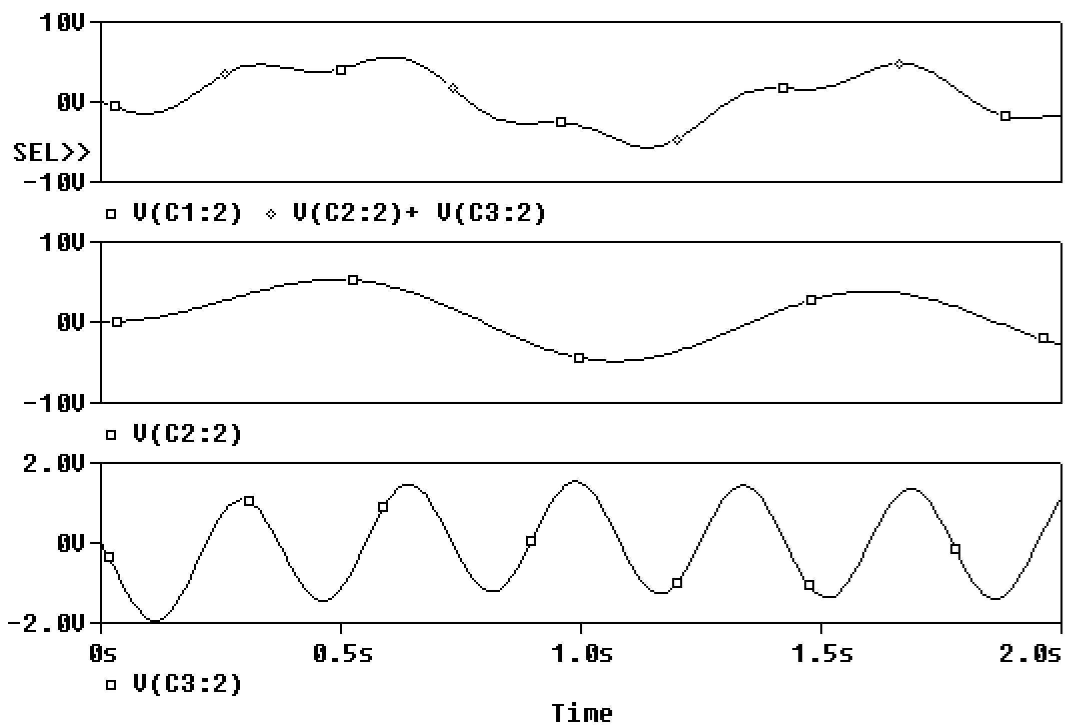




SP 10-2

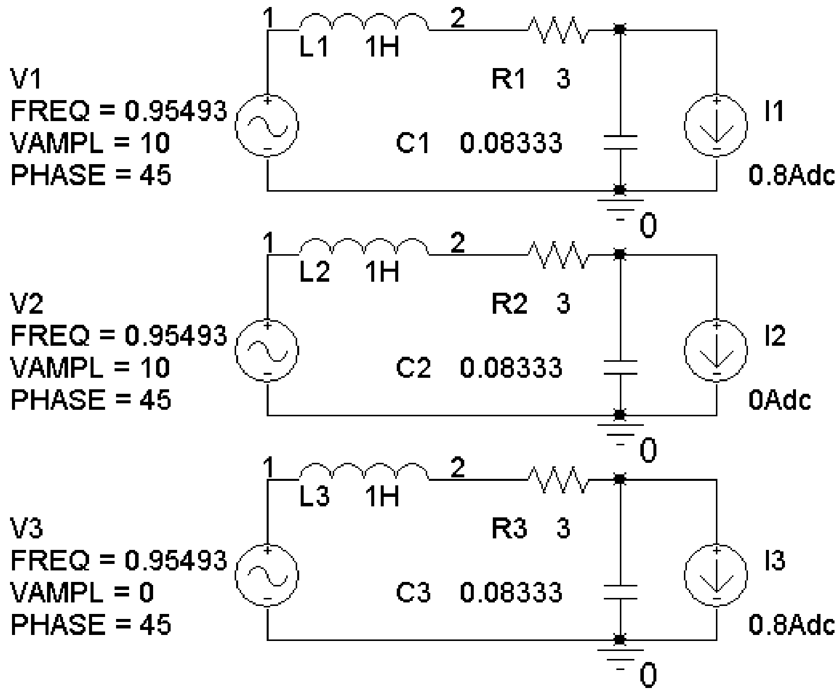
Solution:

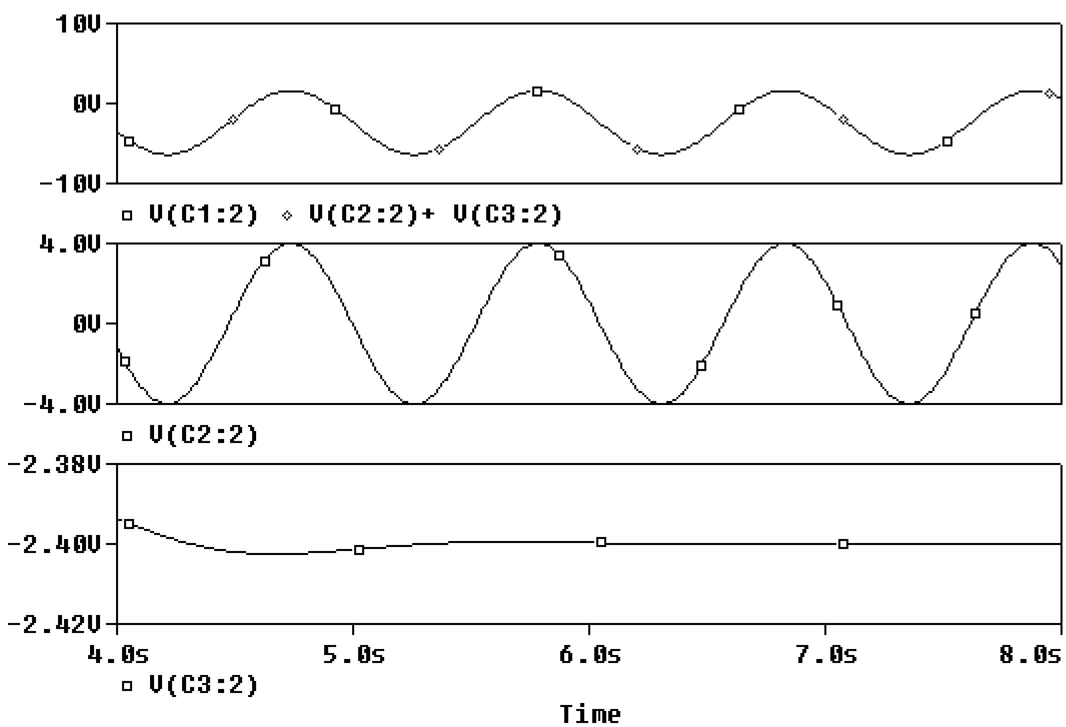




SP 10-3

Solution:

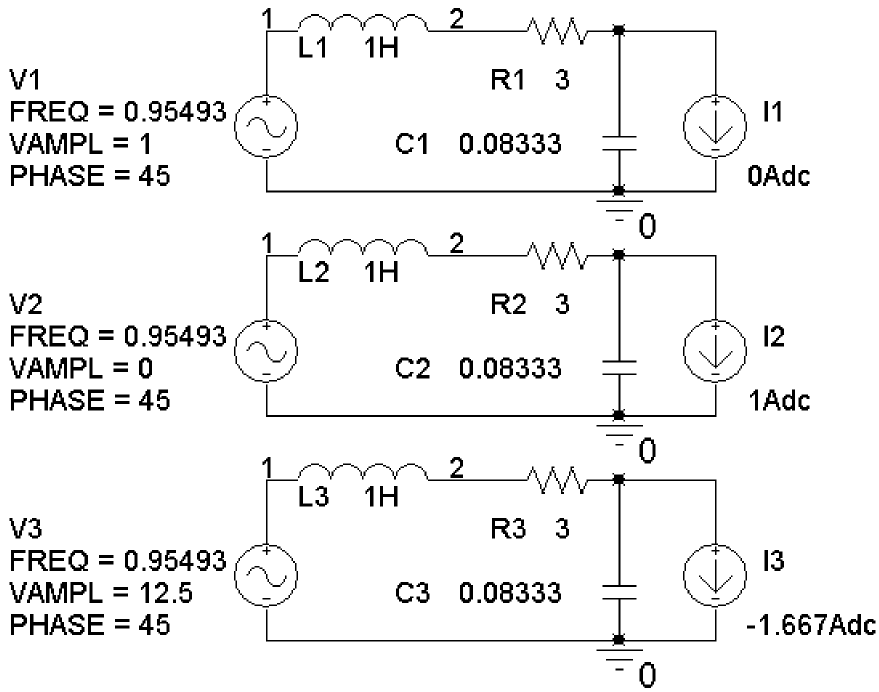


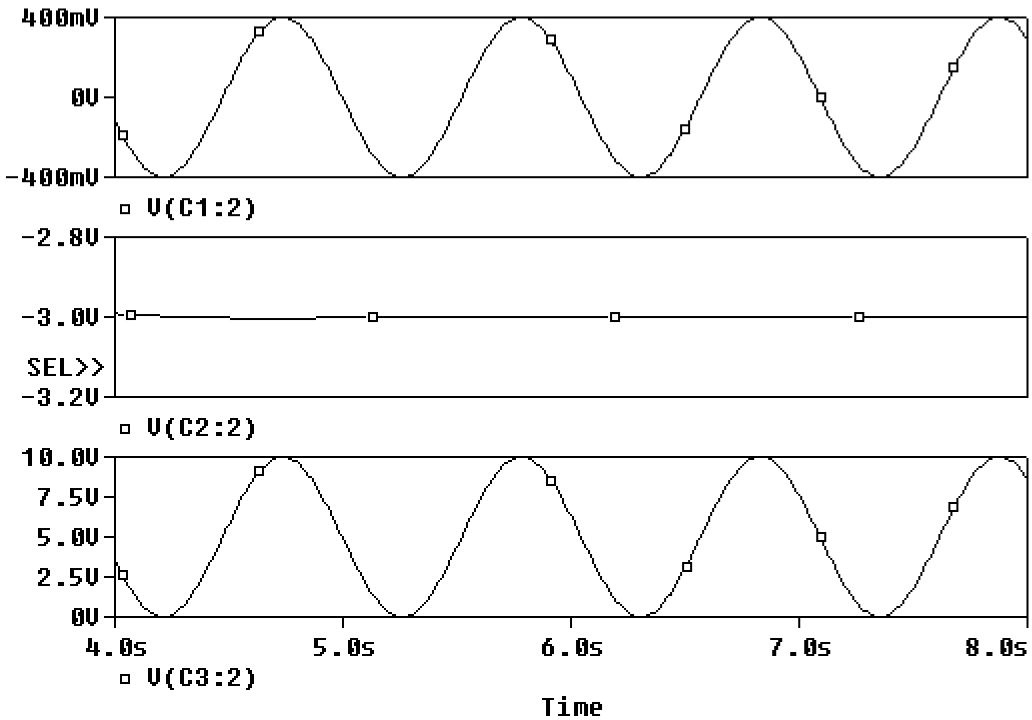


SP 10-4

Solution:

The following simulation shows that k_1 = 0.4and k_2 = -3 V/A. The required values of V_m and I_m are V_m = 12.5 V and I_m = -1.667 A.

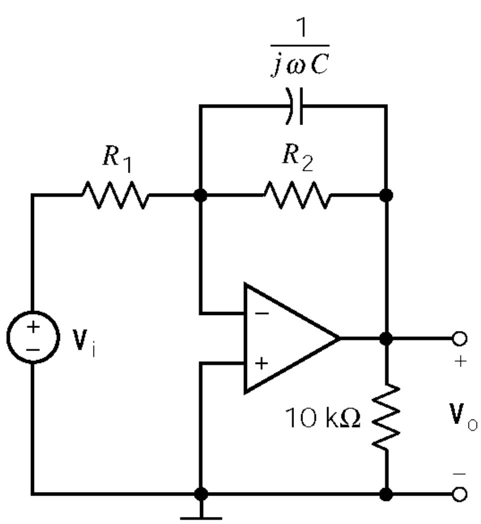




Design Problems

DP 10-1

Solution:



$$R_2 \left\| \frac{1}{j\omega C} = \frac{R_2}{1 + j\omega CR_2} \right.$$

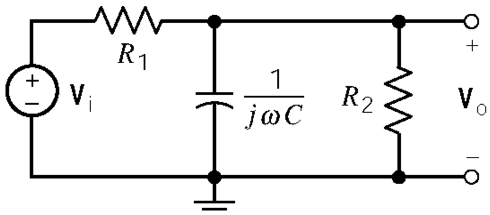
$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = -\frac{\frac{R_2}{1 + j\omega CR_2}}{R_1} = -\frac{\frac{R_2}{R_1}}{1 + j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\frac{R_2}{R_1}}{\sqrt{1 + (\omega CR_2)^2}} e^{j\left(180 - \tan^{-1} \omega CR_2\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 104° so $CR_2 = \dfrac{\tan(180° - 104°)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{8}{2.5}$ so $\dfrac{\frac{R_2}{R_1}}{\sqrt{1+16}} = \dfrac{8}{2.5} \quad \Rightarrow \quad \dfrac{R_2}{R_1} = 13.2$. One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 1515\ \Omega,\ R_2 = 20\ \text{k}\Omega$.

DP 10-2

Solution:



$$R_2 \left\| \frac{1}{j\omega C} \right. = \frac{R_2}{1 + j\omega CR_2}$$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{R_2}{1 + j\omega CR_2}}{R_1 + \dfrac{R_2}{1 + j\omega CR_2}} = \frac{K}{1 + j\omega CR_p}$$

$$\text{where } K = \frac{R_1}{R_1 + R_2} \text{ and } R_p = \frac{R_1 R_2}{R_1 + R_2}$$

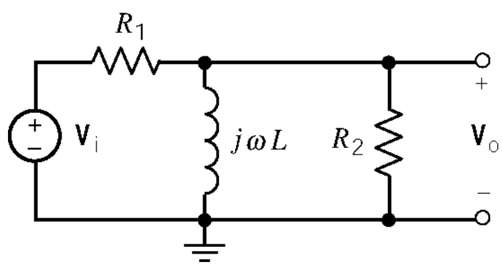
$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{K}{\sqrt{1 + (\omega CR_p)^2}} e^{-j\tan^{-1}\omega CR_p}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be -76° so $CR_p = C\dfrac{R_1 R_2}{R_1 + R_2} = -\dfrac{\tan(-76)}{1000} = 0.004$ and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{12}$ so $\dfrac{K}{\sqrt{1+16}} = \dfrac{2.5}{12} \quad \Rightarrow \quad 0.859 = K = \dfrac{R_2}{R_1 + R_2}$.

One set of values that satisfies these two equations is $C = 0.2\ \mu\text{F},\ R_1 = 23.3\ \text{k}\Omega,\ R_2 = 142\ \text{k}\Omega$.

DP 10-3

Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

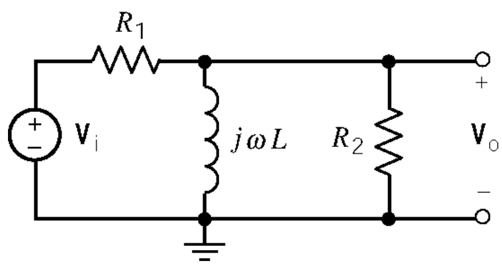
$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}}\, e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be 14° so $\dfrac{L}{R_p} = \dfrac{L(R_1 + R_2)}{R_1 R_2} = \dfrac{\tan(90° - 14°)}{40} = 0.1$

and the magnitude of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be $\dfrac{2.5}{8}$ so $\dfrac{40 \dfrac{L}{R_1}}{\sqrt{1+16}} = \dfrac{2.5}{8} \quad \Rightarrow \quad \dfrac{L}{R_1} = 0.0322$. One set of values that satisfies these two equations is $L = 1\,\text{H},\ R_1 = 31\,\Omega,\ R_2 = 14.76\,\Omega$.

DP 10-4

Solution:



$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\dfrac{j\omega L R_2}{R_2 + j\omega L}}{R_1 + \dfrac{j\omega L R_2}{R_2 + j\omega L}} = \frac{j\omega \dfrac{L}{R_1}}{1 + j\omega \dfrac{L}{R_p}}$$

where $R_p = \dfrac{R_1 R_2}{R_1 + R_2}$

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)} = \frac{\omega \dfrac{L}{R_1}}{\sqrt{1 + \left(\omega \dfrac{L}{R_p}\right)^2}} e^{j\left(90 - \tan^{-1} \omega \frac{L}{R_p}\right)}$$

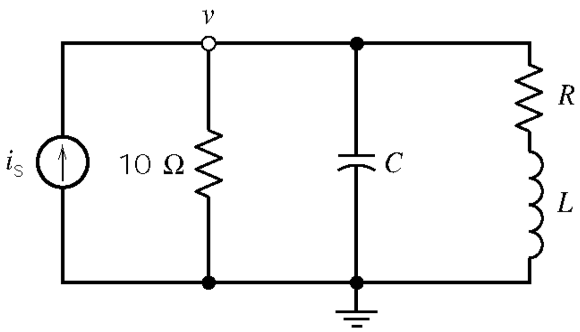
In this case the angle of $\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$ is specified to be –14°. This requires

$$\frac{L}{R_p} = \frac{L(R_1 + R_2)}{R_1 R_2} = \frac{\tan(90 + 14)}{40} = -0.1$$

This condition cannot be satisfied with positive element waves.

DP 10-5

Solution:



$\mathbf{Z}_1=10\ \Omega \qquad \mathbf{Y}_1=\frac{1}{10}\ \text{S}$

$\mathbf{Z}_2=\frac{1}{j\omega C} \qquad \mathbf{Y}_2=j\omega C$

$\mathbf{Z}_3=R+j\omega L \qquad \mathbf{Y}_3=\frac{1}{R+j\omega L}$

$$v(t)=\ 80\ \cos\ (1000t-\theta)\ \text{V} \quad \Rightarrow \quad \mathbf{V}=80\angle-\theta\ \text{V}$$
$$i_S(t)\ =10\ \cos\ 1000t\ \text{A} \quad \Rightarrow \quad \mathbf{I}_s=10\angle 0°\ \text{A}$$

try $\theta=0°$. Then

$$(80\angle-\infty)\left[\frac{1}{10}+\frac{1}{R+j\omega L}+j\omega C\right]=10\angle 0° \quad \Rightarrow \quad R+10-10\,\omega^2 LC+j(\omega L+10\,\omega RC)=1.25\,R+j1.25\,\omega L$$

Equate real part: $40-40\omega^2\ LC=R$ where $\omega=1000$ rad/sec

Equate imaginary part: $40\,RC=L$

Solving yields $R=40(1-4\times10^7\,RC^2)$

Now try $\underline{R=20\ \Omega} \Rightarrow\ 1-2(1-4\times10^7(20)C^2)$

which yields $C=2.5\times10^{-5}\text{F}=\underline{25\ \mu\text{F}}$ so $L=40\ RC=0.02\ \text{H}=\underline{20\ \text{mH}}$

Now check the angle of the voltage. First

$$\mathbf{Y}_1\ =1/10\ =\ 0.1\ \text{S}$$
$$\mathbf{Y}_2\ =j0.25\ \text{S}$$
$$\mathbf{Y}_3\ =1/(20+j20)\ =.025-j.025\ \text{S}$$

then

$$\mathbf{Y}=\mathbf{Y}_1+\mathbf{Y}_2+\mathbf{Y}_3\ =0.125\text{ , so }\mathbf{V}=\mathbf{Y}\mathbf{I}_s=(0.125\angle 0°)(10\angle 0°)=1.25\angle 0°\ \text{V}$$

So the angle of the voltage is $\theta=0°$, which satisfies the specifications.

DP 10-6

Solution:

Represent the circuit in the frequency domain using phasors and impedances. Using voltage division gives

$$A\angle\theta = \frac{\dfrac{1}{j1000C}}{R+\dfrac{1}{j1000C}} \times 10\angle 0° = \frac{10}{1+j10^3 RC}$$

Equating magnitudes and angles gives

$$A = \frac{10}{\sqrt{1+10^6 R^2 C^2}} \qquad \Rightarrow \qquad RC = \frac{\sqrt{\left(\dfrac{10}{A}\right)^2 -1}}{1+j10^3 RC}$$

and

$$\theta = -\tan^{-1}\left(10^3 RC\right) \qquad \Rightarrow \qquad RC = \frac{\tan\left(-\theta\right)}{10^3}$$

(a) $$\theta = -30° \qquad \Rightarrow \qquad RC = \frac{\tan\left(30°\right)}{10^3} = \frac{0.577}{10^3}.$$

Pick C = 1 μF, then $R = \dfrac{0.577}{10^6 \times 10^3} = 577\ \Omega$ and A = 8.66 V.

(b) $$A = 5\text{ V} \qquad \Rightarrow \qquad RC = \frac{\sqrt{\left(\dfrac{10}{5}\right)^2 -1}}{10^3} = \frac{\sqrt{3}}{10^3}.$$

Pick C = 1 μF, then $R = \dfrac{\sqrt{3}}{10^{-6} \times 10^3} = 1732\ \Omega$ and $\theta = -60°$.

(c) $$A = 4 \qquad \Rightarrow \qquad RC = \frac{\sqrt{\left(\dfrac{10}{4}\right)^2 -1}}{10^3} = \frac{2.29}{10^3}$$

$$\theta = -60^\circ \quad \Rightarrow \quad RC = \frac{\tan(60^\circ)}{10^3} = \frac{1.73}{10^3}$$

Since *RC* cannot be both 0.00229 and 0.00173 simultaneously, the specifications cannot be satisfied using this circuit.

(d) $$A = 7.07 \quad \Rightarrow \quad RC = \frac{\sqrt{\left(\frac{10}{7.07}\right)^2 - 1}}{10^3} = 10^{-3}$$

$$\theta = -45^\circ \quad \Rightarrow \quad RC = \frac{\tan(45^\circ)}{10^3} = 10^{-3}$$

Both specifications can be satisfied by taking R = 1000 Ω and C = 1 μF.