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(2) Physical meaning of Fourier series expansion of a function $f(x)$:

Assume we have a signal

(3) Comparing Fourier series and power series

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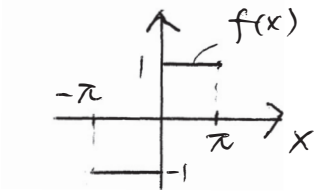
Fourier cosine and sine series

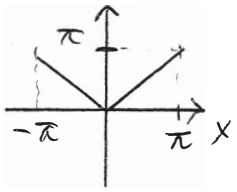
Let's begin our discussion with two examples:

Example 1: Expand the square wave by its Fourier series



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}$$

Example 2: Expand the triangle wave by its Fourier series



$f(x) =$

1) If $f(x)$ is

$$f(x) = \frac{a_0}{2} + a_1 \cos\frac{\pi}{p}x + a_2 \cos\frac{2\pi}{p}x + \cdots \quad :$$

Find a_0: inner product with

Find a_n: inner product with

2) If $f(x)$ is

$$f(x) = b_1 \sin\frac{\pi}{p}x + b_2 \sin\frac{2\pi}{p}x + \cdots \quad :$$

Find b_n: inner product with

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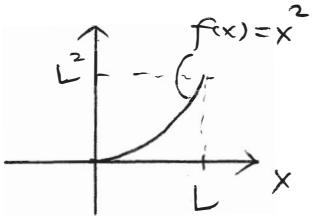
Remarks:

① Fourier $\begin{matrix}\text{cosine}\\ \text{sine}\end{matrix}$ series

② In many physical problems, we may also have a nonperiodic function $f(x)$ defined on $[0, L]$. To express it by the Fourier series, we can

Example: Expand a function $f(x)$ by its Fourier $\begin{matrix}\text{cosine}\\ \text{sine}\end{matrix}$ series.



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Use Fourier series to solve DEs

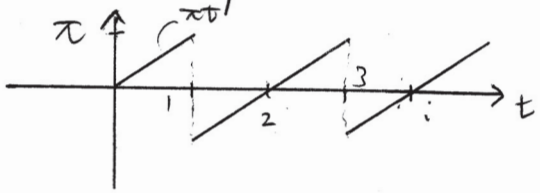
Fourier series is a superposition of , and is itself . Therefore, Fourier series is particularly useful to solve the following two types of problems.

{ I.
{ II.

In the following, we will discuss these two types of problems.

I. Use Fourier series to solve

Example: A spring-mass system is described by $y'' + 6y = f(t)$, where $f(t) = \pi t$ is driven periodically as



Please solve the response y.

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II. Use Fourier series to solve boundary-value problems

Up to now, we mainly deal with "initial-value problems" (IVP). Many physical applications lead to another type of problems, called "boundary-value problems" (BVP). The features of the two types are compared below:

Initial-value problems (IVP) v.s. boundary-value problems (BVP)