Exam 1   109060013 張芯瑜

(1a) 1° For each pair, we can choose 0, 1 or 2, so we can use ternary string.

$\because 3^5 > 100$

$\therefore$ 100 can be written by 5 digits in ternary num.

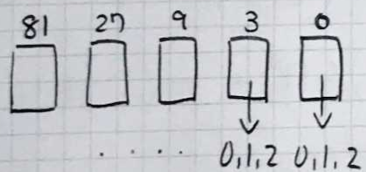
$\Rightarrow$ at least 5 pairs of measuring weights are needed.

(1b) 1° According to (1a), we know that we should choose power of 3 for the pairs.

$\therefore (1,1)\ (3,3)\ (9,9)\ (27,27)\ (81,81)$

2° Justify:

Seeing the 5 pairs above as the ternary string below.



5-digits

We can calculate $1 \sim 100$ by using this 5-digits ternary string uniquely, so it serves our purpose.

(3) $\dfrac{x+1}{(x-2)(1+5x)} = \dfrac{\frac{3}{11}}{x-2} + \dfrac{-\frac{4}{11}}{1+5x} = \dfrac{\frac{3}{-22}}{(1-\frac{1}{2}x)} + \dfrac{-\frac{4}{11}}{(1+5x)}$

$= -\dfrac{3}{22}\sum_{i=0}^{\infty}\binom{-1}{i}(-\tfrac{1}{2}x)^i + (-\tfrac{4}{11})\sum_{j=0}^{\infty}\binom{-1}{j}(5x)^j$

$\therefore$ coeff of $x^n = (-\dfrac{3}{22}) \times \dfrac{(-1)(-2)\cdots(-\cancel{1}-n+1)}{\cancel{n!}} \times (\cancel{-}\tfrac{1}{2})^n$

$+ (-\dfrac{4}{11}) \times \underbrace{\dfrac{(-1)(-2)\cdots(-\cancel{1}-n+1)}{\cancel{n!}}}_{}^{=(-1)^n} \times (5)^n$

$= (-\dfrac{3}{11}) \times (\tfrac{1}{2})^{n+1} + (-\dfrac{4}{11}) \times (-5)^n$

$= \dfrac{(-3)\times 2^{-n-1} + (-4)\times(-5)^n}{11}$ #

(4a) $\underline{1}\ \ \underline{2^{(1)}}\ \underline{3^{(1)}}\ \underline{1}\ \cdots\ \underline{3^{(?)}}$

r

$$\therefore\ EGF_1 = 0 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots = e^x - 1$$

$$EGF_2 = 0 + \frac{1!}{1!}x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 + \cdots = \frac{1}{1-x} - 1$$

$$EGF_3 = 1 + \frac{1!}{1!}x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 + \cdots = \frac{1}{1-x}$$

$$\Rightarrow \underline{EGF = (e^x-1)\left(\frac{x}{1-x}\right)\left(\frac{1}{1-x}\right)}\ \#$$

(4b) $a_6 = [\text{coeff of } x^6] \times 6!$

$$(e^x-1)\frac{x}{(1-x)^2} = (e^x-1)\,x\,(1+x+x^2+\cdots)^2 \rightarrow 找\ x^6$$

$$\Rightarrow \underline{(e^x-1)}\ \underline{(1-x)^{-2}} \rightarrow 找\ x^5$$

$0 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \cdots$ $\qquad$ $\sum_{i=0}^{\infty}\binom{-2}{i}(-x)^i$

$$\Rightarrow \text{coeff} = 0\,\cancel{\binom{-2}{5}(-1)^5} + \frac{1}{1!}\times\binom{-2}{4}(-1)^4 + \frac{1}{2!}\times\binom{-2}{3}(-1)^3$$

$$+ \frac{1}{3!}\binom{-2}{2}(-1)^2 + \frac{1}{4!}\binom{-2}{1}(-1)^1 + \frac{1}{5!}\binom{-2}{0}(-1)^0$$

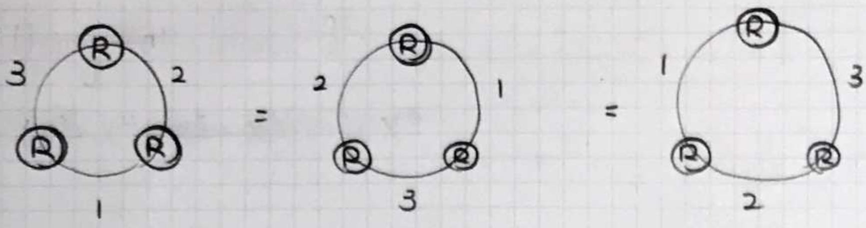
$$= 0 + \frac{\cancel{2\cdot3\cdot4\cdot5}}{\cancel{4!}} + \frac{\cancel{2\cdot3\cdot4}}{\cancel{2!\,3!}}\ 2$$

$$+ \frac{\cancel{2\times3}}{\cancel{3!}\,2!} + \frac{2}{4!\,1!} + \frac{1}{5!} = 5 + 2 + \frac{1}{2!} + \frac{1}{12} + \frac{1}{5!})$$

$$\therefore\ a_6 = 6! \times \text{coeff}$$

$$= 6 + 60 + 6\cdot5\cdot4\cdot3 + 7! = \underline{5466}\ \#$$

(2a) 1°



∴ count 3 times

2° $\dfrac{C^{10+2}_{10}}{3} = \dfrac{C^{12}_{2}}{3} = \underline{22}$ #

3° check :

◡	R	◡	R	◡
10		0		0
9		1		0
9		0		1
8		2		0
8		0		2
8		1		1
7		3		0
7		0		3
7		2		1
7		1		2
6		0		4
6		4		0
6		3		1
6		1		3
6		2		2
5		5		0
5		4		1
5		1		4
5		2		3
5		3		2
4		4		2
4		3		3

⇒ $\underline{22}$ #

(2b) case 1 : ◡ R ◡ R ◡ ⇒ 1

✓ × ×

case 2 : ◡ R ◡ R ◡ ⇒ 99

✓ ✓ ×

case 3 : ◡ R ◡ R ◡

✓ ✓ ✓

(6) $e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$e^x - 1 = \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$

$\Rightarrow \frac{e^x - 1}{x} = 1 + \frac{1}{2!}x + \frac{1}{3!}x^2 + \cdots$

$\Rightarrow \left(\frac{e^x - 1}{x}\right)' = \frac{1}{2!} + \frac{2}{3!}x + \frac{3}{4!}x^2 + \frac{4}{5!}x^3 + \cdots = F(x)$

$\therefore F(x) = \left((e^x - 1)x^{-1}\right)'$

$= e^x \cdot x^{-1} + (e^x - 1)(-x^{-2}) = \frac{e^x}{x} - \frac{e^x - 1}{x^2}$

$\Rightarrow F(1) = \frac{e^1}{1} - \frac{e^1 - 1}{1} = e - (e - 1) = \underline{1}$ #

(5) a_n