Homework #05

Problem 1 (CTFT)

Assume $x(t) \overset{CTFT}{\leftrightarrow} X(f)$

Please prove the following CTFT pairs.

$$\Pi(t) \overset{CTFT}{\leftrightarrow} \text{sinc}(f)$$

$$\Lambda(t) \overset{CTFT}{\leftrightarrow} \text{sinc}^2(f)$$

$$x(t-t_0) \overset{CTFT}{\leftrightarrow} e^{-j2\pi f t_0} X(f)$$

$$e^{+j2\pi f_0 t} x(t) \overset{CTFT}{\leftrightarrow} X(f-f_0)$$

$$x(at) \overset{CTFT}{\leftrightarrow} \frac{1}{|a|} X(\frac{f}{a})$$

Problem 2   (DTFT)

$$x[n] = \begin{cases} 1, |n| \leq M \\ 0, else \end{cases} \overset{DTFT}{\leftrightarrow} \qquad X(f) = \frac{\sin((2M+1)\pi f)}{\sin(\pi f)}$$

$$x[n] = \frac{\sin(K\pi n)}{\pi n} \overset{DTFT}{\leftrightarrow} \qquad X(f) = \begin{cases} 1, |f| \leq K/2 \\ 0, else \end{cases}$$

$$\delta[n] \overset{DTFT}{\leftrightarrow} \qquad 1$$

$$1 \overset{DTFT}{\leftrightarrow} \qquad \delta(f)$$

Problem 3   Please prove the following CTFSs

$$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) \quad \overset{CTFS}{\longleftrightarrow} \quad X[k]=1$$

$$e^{+j2\pi\frac{m}{T}t} \quad \overset{CTFS}{\longleftrightarrow} \quad X[k] = \begin{cases} T, k=m \\ 0, else \end{cases}$$

Problem 4   Please prove the following DTFSs

$$x[n] = \sum_{l=-\infty}^{+\infty} \delta[n-lN] \quad \overset{DTFS}{\longleftrightarrow} \quad X[k]=1$$

$$e^{+j2\pi\frac{m}{N}n} \quad \overset{DTFS}{\longleftrightarrow} \quad X[k] = \begin{cases} N, k=m \\ 0, else \end{cases}$$