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Your name: ________________ ID: _________________ Dec. 7th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #13-1, Open books, notes (20 points), due 11 pm, Wednesday, Dec. 9th, 2020
(submission through iLMS)

Late submission won't be accepted!

1. What are the two fundamental postulates for magnetostatics in vacuum? Define all the symbols in the mathematic expressions. (6 points) State the important physical consequences of the two postulates. (4 points)

Ans: The two postulates are $\nabla\cdot\vec{B}=0, \nabla\times\vec{B}=\mu_0\vec{J}$, where B is the magnetic flux density, J is the current density, and μ_0 is the vacuum permeability. The directions of B and J comply with the right hand rule defined in the curl operator.

$\nabla\cdot\vec{B}=0$: the important consequence of this postulate asserts that the magnetic field lines always close upon themselves.

$\nabla\times\vec{B}=\mu_0\vec{J}$: The important consequence of this postulate is that the circulation of the magnetic field is linearly proportional to a current generating the field.

2. Show that the magnetic flux density in a very-large-radius toroid approaches that of a long solenoid. (5 points)

Ans: The toroid field is given by $B_\phi=\frac{\mu_0 NI}{2\pi r}$. You might define $n=\frac{N}{2\pi r}$ and construct the solution $B_z=\mu_0 nI$ for a solenoid but in such a construction, $n(r)$ is a function of r, not the constant in a solenoid. To make it approach a constant, r has to be big and $r >> (b - a)$, so that the field inside the toroid becomes independent of r, and one can write $\frac{N}{2\pi r}\rightarrow n\neq f(r)$, which is the density of the turns of the winding wires. Also, when r is really big, the curved toroid approaches a straight solenoid and the ϕ direction becomes the z direction in the solenoid's coordinate system. The toroid field approaches $B_\phi=\frac{\mu_0 NI}{2\pi r}\rightarrow B_z=\mu_0 nI$, which is the same as the solenoid field.

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3. Use the magnetic field derived from the Biot-Savart law for a finite-length current element to solve the magnetic field everywhere generated by an infinite long wire carrying a current of I along +z. The wire is in vacuum and its diameter is negligible. Compare the result with that derived from the Ampere's law. (5 points)

Ans: For a linear current element of length L and current I, the B field was solved in the lecture, given by $\vec{B} = \hat{a}_\phi \dfrac{\mu_0 IL}{2\pi r\sqrt{L^2+r^2}}$.

Take L in the expression to infinity and obtain $\vec{B} = \hat{a}_\phi \dfrac{\mu_0 IL}{2\pi r\sqrt{L^2+r^2}} \xrightarrow[L=\infty]{} \hat{a}_\phi \dfrac{\mu_0 I}{2\pi r}$.

This result is the same as that derived from the Ampere's law.

4. The name and definition of the magnetic vector potential A in $\nabla\cdot\vec{B}=0 \Rightarrow \vec{B}=\nabla\times\vec{A}$ is somewhat mysterious. The analogy in electrostatics is the scalar potential, V, defined in $\nabla\times\vec{E}=0 \Rightarrow \vec{E}=-\nabla V$. We now know V is the potential energy per unit charge in an electric field, manifested by the electrostatic energy of charges stored in a volume $W_e = \frac{1}{2}\int_{V'} \rho V dv$. Later, we will also derive an expression for the magnetostatic energy of current stored in a volume $W_m = \frac{1}{2}\int_V \vec{A}\cdot\vec{J} dv$. Based on this analogy, what can you say about the magnetic vector potential? (5 points)

Ans: This is a question for you to think about what A could mean in physic. In the textbook, it already tells you one of the physics picture – the circulation of A is the magnetic flux.

In this question, decompose the differential volume into the multiplication of length and surface and combine the multiplication of the current density and surface into a current. Based on the analogy, the vector potential A is the magnetic potential energy per unit current element, where a current element is defined to be a current multiplied with a length.

On the other hand, I urge you to look up some literatures or the internet to learn more about A. For example, in Lagrangian mechanics and quantum physics, the vector potential A has additional meanings.