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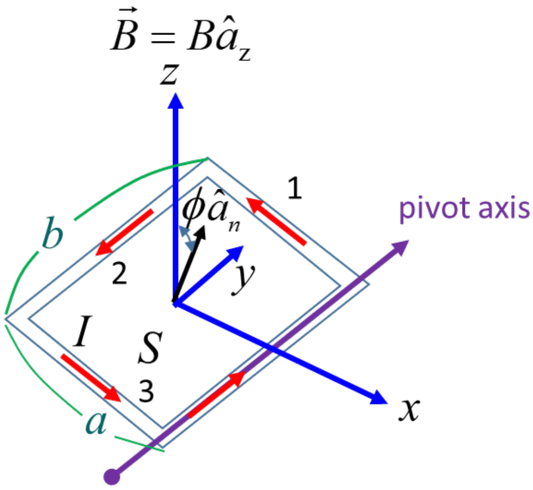
Your name: ________________ ID: __________________ Dec. 28th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #16-1, Open books, notes (22 points), due 11 pm, Wednesday, Dec. 30rd, 2020
(submission through iLMS)

Late submission won't be accepted!

1. Refer to the following coil with a current I in a magnetic field. Calculate the forces on the 1-3 wire segments and determine the torque on the wire loop. (8 points)



Ans: On the wire segment 1, the force is $\vec{F}_1 = I\vec{L} \times \vec{B} = IaB\sin\phi\hat{a}_y$. On the segment 2, the force is $\vec{F}_2 = I\vec{L} \times \vec{B} = -IbB\hat{a}_x$. On the segment 3, $\vec{F}_3 = I\vec{L} \times \vec{B} = -IaB\sin\phi\hat{a}_y$.

$\vec{F}_1, \vec{F}_3$ cancel each other. The torque on the coil is $\vec{T} = \vec{a} \times \vec{F}_2 = -IabB\sin\phi\hat{a}_y$.

Alternatively, the torque is simply $\vec{T} = \vec{m} \times \vec{B}$, where $\vec{m} = \vec{S}I = abI\hat{a}_n$ and $\vec{B} = B\hat{a}_z$.

$\Rightarrow \vec{T} = \vec{m} \times \vec{B} = -\hat{a}_y abIB\sin\phi$, which is the same as the answer in the course slide, except the direction is reversed due to the change of the B field.

2. A piece of n-type semiconductor shown below is known to have a carrier density of 10^{19} electrons/cm^3. When under a magnetic field of 1 kG and applied with V_{ext} = 1 V, a uniform current of 1 A is generated along y. (9 points)

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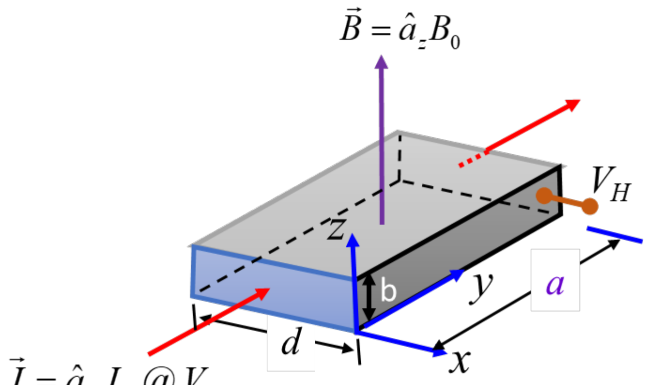
(1) What is the Hall voltage measured from this semiconductor? (2) What is the mobility of the electrons in this semiconductor? (3) what is the conductivity of this material?



a = 1 mm
b = 0.2 mm
d = 0.5 mm

Ans: (1) $\frac{E_H}{JB_0} = \frac{1}{\rho} \rightarrow V_H = E_H d = JB_0 d / ne$, where J = 1 A/(0.5 mm × 0.2 mm), B_0 = 0.1 T, d = 0.5 mm, $n = 10^{19}$/cc, $e = 1.6\times10^{-19}$ C. ⇒ V_H = 0.31 mV.

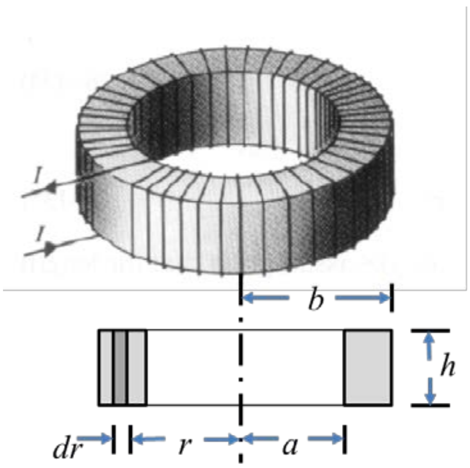
(2) The mobility of the electron in the semiconductor is

$$\mu_e = \frac{u_e}{E_{exr}} = \frac{V_H}{B_0 d E_{ext}} = \frac{V_H a}{B_0 d V_{ext}} = \frac{0.31\times10^{-3}\times10^{-3}}{0.1\times0.5\times10^{-3}\times1} = 6.2\times10^{-3}\ \text{m}^2\text{/s/V}$$

(3) The conductivity of the n-type semiconductor is

$$\sigma = \rho\mu_e = ne\mu_e = 10^{19}/cc\times1.6\times10^{-19}C\times6.2\times10^{-3}m^2/s/V \;=\; 10^4\ \text{S/m}$$

2. Calculate the magnetic energy (3 points) stored in the following N-turn toroid and deduce the inductance of it (2 points). Assume the ferromagnetic material in the toroid has a permeability of μ. (5 points)



Ans: Previously, we have obtained $B_\phi == \frac{\mu NI}{2\pi r}$ for the toroid. The magnetic energy stored in the toroid is the volume integration: $w_m = \frac{1}{2\mu}\int_V B^2 dv$, where $dv = 2\pi rhdr$

$w_m = \frac{1}{2}\int_a^b \frac{\mu N^2 I^2}{4\pi^2 r^2} 2\pi rhdr = \int_a^b \frac{\mu hN^2 I^2}{4\pi r} dr = \frac{\mu hN^2 I^2}{4\pi}\ln(b/a) = \frac{LI^2}{2}$. As a result, the inductance is given by $L = \frac{\mu hN^2}{2\pi}\ln(b/a)$, which is the same as the expression in the course slide, except the permeability is replace by μ.