Assignment 1

#1. Proposition or not ? If it is, decide the truth value of it.

× (a) What time is it ? ⇒ X

○ (b) $4 + x = 5$ ⇒ X

(c) The moon is made of green cheese. ⇒ O, F

(d) $2 + 3 = 5$ ⇒ O, T

(e) $5 + 7 = 10$ ⇒ O, F

(f) This sentence is false. ⇒ X

#2. Negation ?

(a) Mei has an MP3 player.

⇒ Mei doesn't have an MP3 player.

(b) There is no pollution in Taipei.

⇒ There is at least one pollution in Taipei.

(c) $2 + 1 = 3$

⇒ $2 + 1 \neq 3$

(d) The summer in Kenting is Hot and Sunny.

⇒ The summer in Kenting is either not Hot or not Sunny.

#3. Use a truth table to verify the distributive law:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

#4. $(p \wedge q) \vee \neg(p \to q)$

$\equiv (p \wedge q) \vee \neg(\neg p \vee q)$

$\equiv (p \wedge q) \vee (p \wedge \neg q)$

$\equiv p$

p	q	$p \wedge q$	$p \to q$	$\neg(p \to q)$	$(p \wedge q) \vee \neg(p \to q)$
T	T	T	T	F	T
T	F	F	F	T	T
F	T	F	T	F	F
F	F	F	T	F	F

#5. (a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

$\equiv (p \vee \neg q) \wedge \neg p$

$\equiv (p \wedge \neg p) \vee (\neg q \wedge \neg p)$

contradiction

$\Rightarrow$ not satisfiable

(b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee \neg q) \wedge (p \vee \neg q) \wedge (p \vee q)$

$\equiv (\neg p \vee (q \wedge \neg q)) \wedge (p \vee (\neg q \wedge q))$

contradiction

$\Rightarrow$ not satisfiable

#6. $p \uparrow q \equiv \neg(p \wedge q)$

p	q	$p \uparrow q$	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \uparrow \neg q$	$\neg p \vee \neg q$	$p \uparrow p$	$q \uparrow q$	$(p\uparrow p)\uparrow(q\uparrow q)$	$p \uparrow (q \uparrow q)$
T	T	F	T	F	F	T	F	F	F	T	T
T	F	T	F	F	T	T	T	F	T	T	F
F	T	T	F	T	F	T	T	T	F	T	T
F	F	T	F	T	T	F	T	T	T	F	T

(a) Show that $p \uparrow q \equiv \neg(p \wedge q)$

$\Rightarrow$ 如上圖。

(b) Show that $p \uparrow p \equiv \neg p$

$\Rightarrow p \uparrow p \equiv \neg(p \wedge p) \equiv \neg p$ #

(c) Express $p \wedge q$ by using only $\uparrow$:

$\Rightarrow p \wedge q \equiv \neg(\neg p \vee \neg q) \equiv \neg((p\uparrow p) \vee (q \uparrow q))$

$1 < x < y$

$x + y \leq 65$

(d) Express $p \vee q$ by using only $\uparrow$:

$\Rightarrow p \vee q \equiv \neg p \uparrow \neg q \equiv (p \uparrow p) \uparrow (q \uparrow q)$

#7 $H(x) \equiv x$ is happy

Given that $\exists x\, H(x)$, we conclude that $H(Lola)$.

Therefore Lola is happy.

$\forall x\, H(x)$ #

#8 Let $S(x,y)$ be "x is shorter than y."

$$\forall x \exists y\, (S(x,y) \wedge (x \neq y))$$

#9 (a) Alice is a mathematics major. ^{p}

Therefore, Alice is either a mathematics major and a computer science major. ^{q}

$\Rightarrow$ pre: p , con: $p \vee q$

(b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

$\Rightarrow$ pre: $p \wedge q$; con: p

(c) If it is rainy (p), then the pool will be closed (q). It is rainy. Therefore, the pool is closed.

$\Rightarrow$ pre: $p \rightarrow q$. p con: q

(d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

$\Rightarrow$ pre: $p \rightarrow q$ con: $\neg q \rightarrow \neg p$

(e) If I go swimming (p), then I'll stay in the sun too long (q). If I stay in the sun too long, then I will sunburn (r). Therefore, if I go swimming, then I will sunburn.

$\Rightarrow$ pre: $p \rightarrow q$, $q \rightarrow r$, con: $p \rightarrow r$

#10

(3) (5) $\longrightarrow$ Simplification

errors

#11. Determine whether these are valid argument.

(a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

$\Rightarrow$ It's not a valid argument.

$\because p \to q \not\Rightarrow q \to p$

$\therefore$ a can not only $\in \mathbb{R}^+$, but also $\mathbb{R}^-$.

(b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$.
Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

$\Rightarrow$ valid

#12. p = "The food is good."

q = "The service is good."

r = "The rating is three-star."

(a) Either the food is good, or the service is good, or both.

$\Rightarrow p \vee q$

(b) Either the food is good, or the service is good, but not both.

$\Rightarrow (p \vee q) \wedge \neg(p \wedge q) \equiv (p \vee q) \wedge (\neg p \vee \neg q)$

(c) If both the food and services are good, then the rating will be 3 ★.

$\Rightarrow (p \wedge q) \to r$

(d) It's not true that a 3 ★ rating always means good food and good service.

$\Rightarrow \neg(r \to (p \wedge q)) \equiv \neg(\neg r \vee (p \wedge q)) \equiv r \wedge \neg(p \wedge q)$

$\equiv r \wedge (\neg p \vee \neg q)$

#13 Express each statements

(a) The product of two negative real numbers is positive.

$\Rightarrow \forall x \forall y ((x<0 \wedge y<0) \rightarrow xy>0)$

(b) The difference of a real number and itself is zero.

$\Rightarrow \forall x \ (x \in \mathbb{R} \rightarrow (x-x=0))$

(c) A negative real number doesn't have a square root that is a real number.

$\Rightarrow \forall x (x \in \mathbb{R}^- \longrightarrow \neg(\sqrt{x} \in \mathbb{R}))$

#14 Truth value if the domain for all variables consists of all int.

T (a) $\forall n \exists m \ (n^2 < m)$

T (b) $\exists n \forall m \ (n < m^2)$

T (c) $\forall n \exists m \ (n+m=0)$ ※ $-m+m=0$

T (d) $\exists n \forall m \ (nm=m)$ ※ $1 \times m = m$

T (e) $\exists n \exists m \ (n^2+m^2=5)$ ※ $2^2+1^2=5$

F (f) $\exists n \exists m \ (n^2+m^2=6)$

F (g) $\exists n \exists m \ (n+m=4 \wedge n-m=1)$ ※ $n=\frac{5}{2}$, $m=\frac{3}{2}$ $\Rightarrow n, m \notin \mathbb{Z}$

T (h) $\exists n \exists m \ (n+m=4 \wedge n-m=2)$ ※ $m=1$, $n=3$

o #15 For every $\varepsilon>0$, there exists $\delta>0$, s.t. for every x with $0<|x-x_0|<\delta$, $|f(x)-L|<\varepsilon$. ※ $\lim_{x \to x_0} f(x) = L$

$\Rightarrow \forall \varepsilon (\exists \delta (\delta>0) \wedge \forall x (0<|x-x_0|<\delta \rightarrow |f(x)-L|<\varepsilon))$

$\equiv \forall \varepsilon \exists \delta \forall x (\delta>0 \wedge (0<|x-x_0|<\delta \rightarrow |f(x)-L|<\varepsilon)$

#16

$$\begin{array}{r} SEND \\ +\ MORE \\ \hline MONEY \end{array} \Rightarrow \text{we have } (0\sim 9)\ \overset{9}{S},\ \overset{5}{E},\ \overset{6}{N},\ \overset{7}{D},\ \overset{1}{M},\ \overset{0}{O},\ \overset{8}{R},\ \overset{2}{Y}$$

$S \neq 0$, $M \neq 0$

$O + 10M = S + M + 1$

$$\begin{array}{r} 9\ E\ (E+1)\ 7 \\ +\ 1\ 0\ 8\ E \\ \hline 1\ 0\ (E+1)\ E \end{array} \Rightarrow \begin{array}{r} 9567 \\ +\ 1085 \\ \hline 10652 \end{array}$$ #

#17

$$\begin{array}{r} \boxed{9}\boxed{0}8\boxed{0}\boxed{9} \\ 12\overline{)\boxed{1}\boxed{0}\boxed{8}\boxed{9}\boxed{7}\boxed{0}\boxed{8}} \\ \boxed{1}\boxed{0}\boxed{8} \\ \hline \boxed{9}\boxed{7} \\ \boxed{9}\boxed{6} \\ \hline \boxed{1}\boxed{0}\boxed{8} \\ \boxed{1}\boxed{0}\boxed{8} \\ \hline 0 \end{array}$$ #

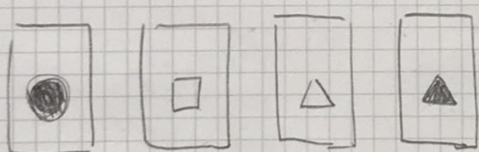
$$\begin{array}{r} 30 \\ 113\overline{)339} \\ 339 \\ \hline \end{array}$$

$8y < 100 < 9y$

$y < \frac{\cancel{100}}{8}$, $\frac{100}{9} < y$

$y < 12.5$, $11.. < y \Rightarrow y = 12$

#18



Peter : $\overset{p}{\boxed{\bullet}} \rightarrow \overset{q}{\boxed{\triangle}} \equiv \cancel{\boxed{\triangle}} \rightarrow \cancel{\boxed{\bullet}}$

(a) check $\boxed{\bullet} \rightarrow \boxed{\triangle}$ $(p \rightarrow q)$ + $\boxed{\square} \not\rightarrow \boxed{\bullet}$ $(\neg q \rightarrow \neg p)$

$\boxed{\triangle}$ 後可以不是 $\boxed{\bullet}$

(b) 2 cards

#19. Answer all Q so that all can be answered correctly.

(d) Q_1: Which is the first question where (c) is V.

(a) Q3 (b) Q4 (c) Q1 (d) Q2

(c) Q_2: Which is the first question where (a) is V.

(a) Q4 (b) Q2 (c) Q3 (d) Q1

(a) Q_3: Which is the first Q where (d) is V.

(a) Q_1 (b) Q_2 (c) Q4 (d) Q3

(b) Q_4: ________________ (b) ______

(a) Q2 (b) Q4 (c) Q3 (d) Q1

<Ans> ∵ all of a, b, c, d will be an answer for at least one

if $Q_1 \rightarrow$ (c), can't

if $Q_3 \rightarrow$ (d), can't

if $Q_4 \rightarrow$ (b), $Q_3 \rightarrow$ (a), $Q_1 \rightarrow$ (d), $Q_2 \rightarrow$ (c) #

#20. _ = "A, B are liars"

_ = "A, C are liars"

① T/T : A, B, C are liars $(\rightarrow\leftarrow)$

② T/F : A, B are liars ✓

③ F/T : A, C are liars ✓

④ F/F : A liar (only one X)

B, C liars ✓

No liar $(\rightarrow\leftarrow)$

⇒ 2 liars #

#21. Sam : $x+y$ $(1<x<y\ ,\ x+y\le 65)$

Peter : xy $\Downarrow$ $5\le$ 32.5

$x\le 32$

x	y
2 3 4 5	33 34 35
6 7 8 9 10	36 37 38 39 4
11 12 13 14 15	41 42 43 44 4
16 17 18 19 20	46 47 48 49 5
21 22 23 24 25	51 52 53 54 5
26 27 28 29 30	56 57 58 59 6
31 32	61 62 63

o 1° P : I don't know the number x, y.

$\Rightarrow$ y won't be prime that >32

$\Rightarrow$ x, y won't both be prime

$\Rightarrow xy \ne a^3$ (a is prime)

3 4 5
6 7 8 9 10
11 12 13 14 15
16 17 18 19 20
21 22 23 24 25
26 27 28 29 3
31 32

2° S : I already knew U don't know.

$\Rightarrow$ x+y can't be prime + prime

5 = 2+3
✓ 6 =
7 = 2+5
8 = 3+5
9 = 2+7
10 = 3+7
✓ 11 =
12 = 5+7
13 = 2+11
14 = 3+11
15 = 2+13
16 = 3+13
✓ 17 =
18 = 5+13
19 = 2+17
20 = 7+13
21 = 2+19
22 = 3+19
✓ 23 =
24 = 5+19
25 = 2+23
26 = 3+23
✓ 27

28 = 5+23
✓ 29 =
30 = 7+23
31 = 2+29
32 = 3+29
33 = 2+31
34 = 3+31
✓ 35 =
36 = 5+31
✓ 37 =
38 = 7+31
39 = 2+37
40 = 3+37
~~✓ 41 = 4+37~~
42 = 5+37
43 = 2+41
44 = 3+41
45 = 2+43
46 = 3+43
~~✓ 47 = 4+43~~
48 = 5+43
49 = 2+47
50 = 3+47
~~✓ 51 = 4+47~~
52 = 5+47

~~✓ 53 = 6+47~~
54 = 7+47
55 = 2+53
56 = 3+53
~~✓ 57 = 4+53~~
58 = 5+53
~~✓ 59 = 6+53~~
60 = 7+53
61 = 2+59
62 = 3+59
63 = 2+61
64 = 5+59
~~✓ 65 = 4+61~~

$\Rightarrow$ x+y maybe :

6, 11, 17, 23, ~~27~~, 29,
35, 37, ~~41, 47, 51, 53, 57~~,

3° P: Oh, now I know the x, y.

6 = 2+4 (8)

$2 \times 9 = 3 \times 6$ = 9

11 = 2+9 (18) = 3+8 (24) = 4+7 (28) = 5+6 (30)

☆ 17 = 2+15 (30) = 3+14 (42) = 4+13 (52) = 5+12 (60) = 6+11 (66) = 33×2 = 7+10 (70) = 2×35 = 8+9 (72) = 3×24

23 = 2+21 (42) = 3+20 (60) = 4+19 (76) = 5+18 (90) = 6+17 (102) = 7+16 (112) = 8+15 (120)

= 9+14 (126) = 10+13 (130) = 11+12 (132)

29 = 2+27 (54) = 3+26 (78) = 4+25 (100) = 5+24 (120) = 6+23 (138) = 7+22 (154) = 8+21 (168)

= 9+20 (180) = 10+19 (190) = 11+18 (198) = 12+17 (

27

35

37 = 2 * 35 = (70)

$\Rightarrow$ $x+y=17$, $xy=52$ $\Rightarrow$ $x=4$, $y=13$ //

set P get 18

$\downarrow$

2 × 9 = 11

3 × 6 = 9