

1. The question asks "inductance per unit-length", which is $\frac{L}{l}$, l is the length of the solenoid. Since $L = \frac{N\Phi}{I}$ (not $\frac{\pi\Phi}{I}$!!) and $N = n l$. We can get

$$\frac{L}{l} = \frac{\pi\Phi}{I}. \text{ Recall that for long solenoid } \Phi = BS = \mu n I S, \text{ thus}$$

$$\frac{L}{l} = \frac{\pi(\mu n I S)}{I} = \mu n^2 S. \quad (\mu \text{ here refers to the permeability of the core of the solenoid, i.e. } \mu = \mu_0 \mu_r, \mu_r \text{ is the relative permeability})$$

2. From the right picture of question 2, you may find the magnetic flux go through R_1, R_2, R_3 is $\Phi_1, \Phi_2, (\Phi_1 + \Phi_2)$, respectively. So we can get

$$\begin{cases} N_1 I_1 = \Phi_1 R_1 + (\Phi_1 + \Phi_2) R_3 & (\text{by KVL for Magnetostatic loop}) \\ N_2 I_2 = \Phi_2 R_2 + (\Phi_1 + \Phi_2) R_3 \end{cases}$$

$$\Rightarrow \begin{cases} N_1 I_1 = (R_1 + R_3) \Phi_1 + R_3 \Phi_2 & \text{①} \\ N_1 I_1 - N_2 I_2 = \Phi_1 R_1 - \Phi_2 R_2 \end{cases} \Rightarrow \Phi_2 = \frac{\Phi_1 R_1 - N_1 I_1 + N_2 I_2}{R_2}, \text{ substitute into ①}$$

$$\Rightarrow N_1 I_1 = (R_1 + R_3) \Phi_1 + \frac{R_3}{R_2} (\Phi_1 R_1 - N_1 I_1 + N_2 I_2) \Rightarrow N_1 I_1 (1 + \frac{R_3}{R_2}) - N_2 I_2 \frac{R_3}{R_2} = (R_1 + R_3 + \frac{R_1 R_3}{R_2}) \Phi_1$$

$$\Rightarrow \Phi_1 = \frac{N_1 I_1 (R_2 + R_3) - N_2 I_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

* You could define the directions of Φ_1, Φ_2 at different ways, but make sure that the directions are consistent. (And better draw it out).

3. Recall that characteristic time constant for RL circuit is $\tau = \frac{L_{\text{total}}}{R'}$, which

L_{total} is the equivalent inductance of the whole circuit. To have a minimum τ_{min} , we should minimize L_{total} and maximize R . Since we only have a resistor with resistance R , $R' = R$ is confined. And to

minimize L_{total} with 10 inductors with inductance L , we should connect them in parallel, which $\frac{1}{L_{\text{total}}} = \underbrace{\frac{1}{L} + \frac{1}{L} + \dots + \frac{1}{L}}_{10} = \frac{10}{L} \Rightarrow L_{\text{total}} = \frac{L}{10}$. In this case,

$$\tau_{\text{min}} = \frac{L}{10R}$$