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1st-order ODEs (Ch2)

In this class, we will introduce three approaches to solve 1st-order ODEs.

I. Qualitative approach

II. Numerical approach

III. Analytical approach

Before we try to "solve the DE" ( ) here are some questions we should ask

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Existence of a unique solution (Theorem 1.2.1)

For a 1st-order ODE $\frac{dy}{dt} = f(t,y)$ with initial value $y(t_0)=y_0$,

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I. Qualitative approach

Use graphical method to solve 1st-order ODEs by plotting the " " (also called "direction field")

<u>Example</u> : Solve $\frac{dy}{dt} = 0.2ty$ by plotting the slope field.

observations:

① "$\frac{dy}{dt}$" means " ". Say, if a function $y(t)$ is a solution of this DE, its slope $(=\frac{dy}{dt})$ at (t_0, y_0) will be

② If we sketch $0.2ty$ in the ty plane, we can obtain a plot of " " →

Let's plot $0.2ty$ in the ty plane:

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If "slopes" of solution look like this, what would the solution curve look like?

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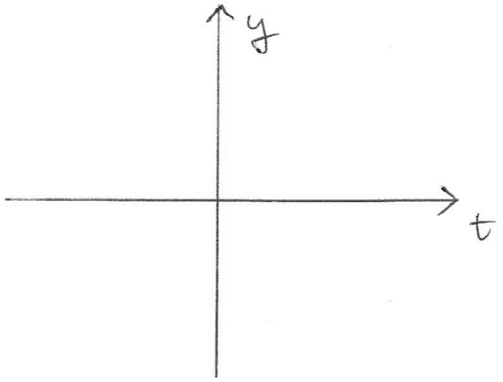
For a 1st-order ODE $\frac{dy}{dt} = f(t, y)$, there are two important special cases: when the right-hand side (RHS) is only a function of t OR only a function of y.

<u>Case I</u>: $\frac{dy}{dt} = f(t) \overset{ex:}{=} 2t$



- for
- geometrically, all the slopes on each

Feature: We can get infinite solutions from one solution curve by translating the curve

This feature can also be supported by its analytical solution: