(1) $\delta(t) \xrightarrow{F} 1$

$\delta(t-b) \to e^{-j2\pi f b}$

$a\delta(t-b) \to a e^{-j2\pi f b}$ #

(2) $F^{-1}\left\{\frac{1}{5+j2\pi f}\right\} = e^{-5t}u(t)$

$F^{-1}\left\{e^{-j2\pi \cdot 2f} \times \frac{1}{5+j2\pi f}\right\}$

$= e^{-5(t-2)}u(t-2)$ #

(3) $F(f) = \int_0^\infty e^{-at} \times e^{-j2\pi f t} dt$

$\frac{-1}{a+j2\pi f} \times \int_0^\infty e^{-(a+j2\pi f)t} d-(a+j2\pi f)t$

$= \frac{1}{a+j2\pi f}$ #

(4) $e^{-a|t|} \Rightarrow a > 0$

$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ e^{at} & t < 0 \end{cases}$

$F(t) = \int_0^\infty e^{-at} \times e^{-j2\pi f t} dt$

$+ \int_{-\infty}^0 e^{at} \times e^{-j2\pi f t} dt$

$= \frac{1}{a+j2\pi f} + \frac{1}{a-j2\pi f}$

$= \frac{2a}{a^2+(2\pi f)^2}$ #

(5) $f(x) = \begin{cases} x^2 e^{-x} & x > 0 \\ 0 & x < 0 \end{cases}$

$\int_0^\infty x^2 e^{-x} e^{-j2\pi f} dx$ $\quad u = x^2 \quad du = 2x dx$

$v = \frac{1}{1+j2\pi f} \quad dv = e^{-xj2\pi f} dx$

$\times e^{(-1+j2\pi f)x}$

$= uv - \int v du$

$= \frac{1}{1+j2\pi f} x^2 e^{-(1+j2\pi f)x} \Big|_\infty^0$

$+ \frac{2}{(1+j2\pi f)^2} x e^{(-1+j2\pi f)x} \Big|_\infty^0 + \frac{2}{(1+j2\pi f)^3} e^{-(1+j2\pi f)x} \Big|_\infty^0$

$= \frac{2}{(1+j2\pi f)^3}$ #

(6) $f(x) = e^{-ax} \quad a > 0$

$\int_{-\infty}^\infty e^{-ax^2} e^{-j2\pi f x} dx = \int_{-\infty}^\infty e^{-ax^2} \cos 2\pi f x dx + \int_{-\infty}^\infty e^{-ax^2} \sin 2\pi f x dx$

$= \int_{-\infty}^\infty e^{-ax^2} \cos 2\pi f x dx = F(f)$

$\frac{dF(f)}{df} = \int_{-\infty}^\infty e^{-ax^2} (-2\pi x) \sin 2\pi f x dx = \frac{\pi}{a} \int_{-\infty}^\infty (-2ax) e^{-ax^2} \sin 2\pi f x dx$

$= \frac{\pi}{a} e^{-ax^2} \sin 2\pi f x \Big|_{-\infty}^\infty - \frac{\pi}{a} \int_{-\infty}^\infty e^{-ax^2} (2\pi f) \cos 2\pi f x dx$

$= -\frac{2\pi^2 f}{a} \times \int_{-\infty}^\infty e^{-ax^2} \cos 2\pi f x dx$

$F'(f) = -\frac{2\pi^2 f}{a} F(f) = C e^{-\frac{\pi^2}{a} f^2}$

$F'(f) + \frac{2\pi^2 f}{a} F(f) = 0 \quad F(f) = C \cdot e^{-\int \frac{2\pi^2 f}{a} df}$

$F(0) = C = \int_{-\infty}^\infty e^{-ax^2} dx$, Let $A = \int_{-\infty}^\infty e^{-ax^2} dx$

$B = \int_{-\infty}^\infty e^{-ay^2} dy$, $A^2 = \int_{-\infty}^\infty \int_{-\infty}^\infty e^{-a(x^2+y^2)} dx dy$

$= \int_0^{2\pi} \int_0^\infty e^{-ar^2} r dr d\theta = \frac{-1}{2a} \int_0^{2\pi} e^{-ar^2} \Big|_0^\infty d\theta - \frac{\pi}{a}$

$\therefore C = \sqrt{\frac{\pi}{a}} \quad F(f) = \sqrt{\frac{\pi}{a}} e^{-\frac{\pi^2}{a} f^2}$ #

(7) $f(x) = x e^{-x^2}$

$F\{e^{-x^2}\} = \sqrt{\pi} e^{-\pi^2 f^2} \quad F\{x e^{-x^2}\} = \frac{-1}{j2\pi} F\{-j2\pi x e^{-x^2}\}$

$= \frac{-1}{j2\pi} F'(f) = \frac{-1}{j2\pi} \sqrt{\pi} (-2\pi^2 f) e^{-\pi^2 f^2} = -j f (\pi)^{\frac{3}{2}} e^{-(\pi f)^2}$ #