Chapter 1 Introduction

UNITS

In this course, we adopt the International System of Units (SI Units) or the MKSA unit system.

Length **M**eter (m)

Mass **K**ilogram (kg)

Time **S**econd (s)

Current **A**mpere (A, amount of charges flowing through a cross-sectional area per unit time)

Temperature **K**elvin (K), the water freezing temperature is 273 K

Sometimes it is convenient to use short notations to describe a large or small quantity. For example, 10^{-15} second is often written as femto-second or simply fs. The following is a table for such multiple and sub-multiple prefixes.

Multiple prefix			Sub-multiple prefix		
Prefix	Symbol	Magnitude	Prefix	Symbol	Magnitude
Exa	E	10^{18}	Atto	a	10^{-18}
Peta	P	10^{15}	Femto	f	10^{-15}
Tera	T	10^{12}	Pico	p	10^{-12}
Giga	G	10^{9}	Nano	n	10^{-9}
Mega	M	10^{6}	Micro	μ	10^{-6}
Kilo	k	10^{3}	Milli	m	10^{-3}

MODELS OF ELECTROMAGNETICS

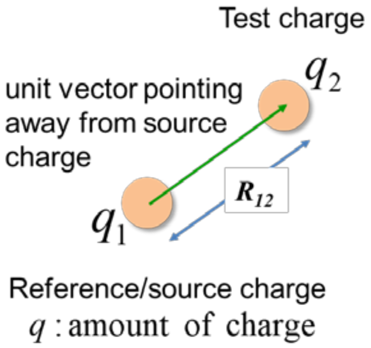
Electromagnetics is a study of the physical phenomena associated with moving and rest charges. A charged particle has an "electric field" associated with it and a moving charge generates a "magnetic field". An accelerated charge produces radiation. A time-varying electromagnetic field produces an electromagnetic wave that propagates at the speed of light or 3×10^8 m/s in vacuum.

Observations for Electric and Magnetic Forces

There are two kinds of charges, positive and negative charges. Cavendish and Coulomb showed the inverse-square law of the electrostatic force over distance,

$$F = \frac{q_1 q_2}{4\pi\varepsilon R_{12}^2} \tag{1-1}$$

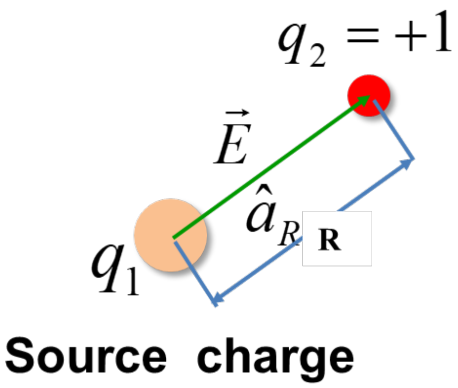
where the subscripts 1, 2 denote two different charges separated by a distance R_{12}, q is the amount of charge, $\varepsilon \equiv \varepsilon_r \varepsilon_0$ is called permittivity with $\varepsilon_0 = 10^{-9}/(36\pi)$ Farad/m being the *vacuum permittivity* (a universal constant) and ε_r being the *relative permittivity* (a material-dependent quantity). In vacuum, ε_r = 1. The electric force is repellent for same charges and is attractive for opposite charges. Like the gravitational force, the electric force obeys the inverse square law for distance (R^2).



Define the "electric field intensity" (Volt/m in SI units) as

$$\vec{E} \equiv \frac{\vec{F}}{q}, \tag{1-2}$$

which is the force "felt" by a unit positive charge in space.



Similarly, one can define the *magnetic force* through the force "felt" by a moving charge *q* in a *magnetic field,*

$$\vec{F} = q\vec{u} \times \vec{B}, \tag{1-3}$$

where $\vec{B}$ is the "magnetic flux density" (Tesla in SI units) and $\vec{u}$ is the velocity of the charge.

Electromagnetic Force on a Charge (Lorentz Force): The sum of the electric and magnetic forces on a charge is given by

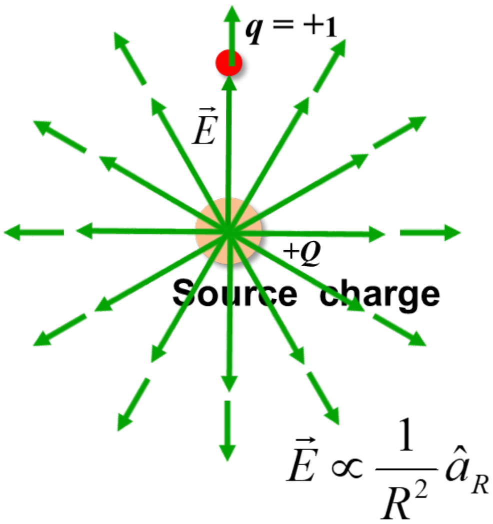
$$\vec{F} = q \cdot (\vec{E} + \vec{u} \times \vec{B}) \tag{1-4}$$

This expression is called the Lorentz force equation. Note that a stationary charge with $u = 0$ does not experience a magnetic force.

Field Lines

An electromagnetic force acts at a distance. When talking about, say, the electric force between two charged particles, there is often nothing but vacuum space between the two particles. How do we "visualize" the electric force between them? The answer is to draw lines of force felt by a test positive charge in space and create the concept of "field" to describe action at a distance. Those lines are artificial. Although one cannot see field lines, the space with fields does contain energy.

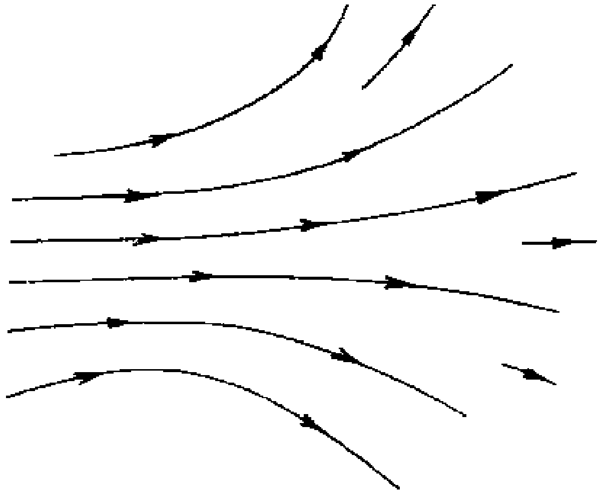
Eg 1. (electric field lines) The following is an example of electric field lines "generated" by a positive point charge.



The field strength is denoted by the discrete *length* and the field direction is denoted by the *arrow* of a field line. Up to this point, you could start to feel that the concept of *vector* ought to be used to describe some electromagnetic quantities. Indeed, we will review vector calculus before we learn to analyze electromagnetics.

Eg. 2. (magnetic field lines)

The concept of a magnetic field line is more subtle, which has to be considered together with the motion of a charged particle due to the cross product in the expression $\vec{F} = q\vec{u} \times \vec{B}$. The direction of the magnetic force felt by a test charge is therefore deduced from the directions of the charge's motion and the magnetic field line.



The magnetic field strength is proportional to the density of the field lines. The field direction is denoted by the arrow of a field line.

Source Quantity: electric charge

Several kinds of charge exist in nature. In electrical engineering, we mostly deal with electrons. The fundamental unit of a charged particle in this study is therefore the electron charge, given by

$$q \equiv e = -1.6 \times 10^{-19} \text{ Coulomb} \qquad (1\text{-}5)$$

A fundamental postulate regarding an electric charge is the *conservation of electric charg*es. No charge can be created or annihilated within the scope of this course, although it is not the case in high energy physics. Since a moving charge generates an electric current (amount of charges moving across an area per unit time), the conservation of electric charge has to be considered together with a current.

In this course, one considers averaged electromagnetic phenomena from aggregate of charges, but not from discrete electrons. For convenience of differential calculation, we define

$$\textit{Volume Charge Density} \quad \rho_V = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v} \qquad (\text{C/m}^3) \quad (1\text{-}6)$$

Surface Charge Density $\rho_S = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s}$ (C/m^2) (1-7)

Line Charge Density $\rho_l = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l}$ (C/m) (1-8)

where $\Delta v, \Delta s, \Delta l$ are differentially volume, surface, and length, respectively. Although the spatial variables $\Delta v, \Delta s, \Delta l$ are taken to approach zero, their dimensions are still considerably larger than the size of an electron.

A flow of electric charges is an electric current. Accordingly, we define

Current (Coulomb/sec = Ampere) $I = \frac{dq}{dt}$ (1-9)

Volume *current density* $\vec{J} = \rho_v \vec{u}$ (A/**m^2**), (1-10)

which can also be understood from $\vec{J} \cdot \vec{S} = I$, where *S* is an area (A current flowing through a unit surface area with its surface normal along the flowing direction of the current)

Surface *current density* $\vec{J}_s$ (A/**m**) (1-11)

understood from $J_s L = I$, where *L* is a length projected along the perpendicular direction of $\vec{J}_s$ (A current flowing through a unit length perpendicular to the flow direction of the current)

Electromagnetic-field Quantities

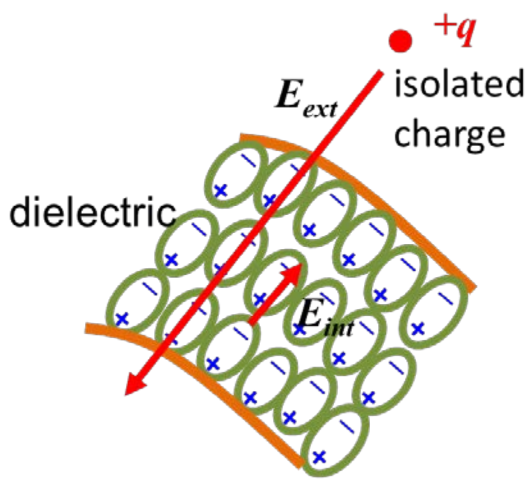
Electric Field Intensity $\vec{E}$ (Volt/m)

Magnetic Flux Density $\vec{B}$ *Tesla (V - sec/m^2)* (1 T = 10^4

Gauss, 1 Gauss ~ the earth magnetic flux density)

$\vec{E}$ and $\vec{B}$ are clearly defined through the Lorentz force equation.

When a material, consisting of free or bonded charges, is present in space, the material can modify the strength of an electric field. For example, a dipole (bonded opposite charges) in a dielectric can realign themselves to cancel part of an external electric field or the free electrons in a metal can flow to completely offset an external electric field. The following shows the dipole re-alignment partially cancels an external electric field in a dielectric.



Similar effects can be found in a magnetic material. For convenience of calculating fields in materials, two more field quantities are defined

Electric Flux Density $\vec{D}$ $(\mathrm{C/m^2})$

Magnetic Field Intensity $\vec{H}$ $(\mathrm{A/m})$

As will be shown later in this course, this definition of *D* will allow us to deal with only excess, isolated, free-moving charges or current in our calculations. In other words, *D* is defined to be proportional to free charges. In free space, the electric flux density and the electric field intensity are related by

$$\vec{D} = \varepsilon_0 \vec{E}\ , \tag{1-12}$$

where the vacuum *permittivity* is given by $\varepsilon_0 \approx \frac{1}{36\pi} \times 10^{-9}$ $(\mathrm{F/m})$ ("F" stands for "Farad").

On the other hand, the definition of *H* will allow us to only deal with free current in our calculation. In other words, *H* is defined to be proportional to a free current in a problem. In free space, the magnetic flux density and the magnetic field intensity are related by

$$\vec{B} = \mu_0 \vec{H}\ , \tag{1-13}$$

where the vacuum *permeability* is given by $\mu_0 \approx 4\pi \times 10^{-7}$ $(\mathrm{H/m})$ (H stands for Henry).

A material contains electric charges and dipoles. For example, in a dielectric, the electric flux density is written as

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_r \varepsilon_0 \vec{E}\ , \tag{1-14}$$

because the induced electric dipoles in a dielectric modify the electric field and introduce an increased permittivity

$$\varepsilon = \varepsilon_r \varepsilon_0\ , \tag{1-15}$$

where ε_r is called *relative permittivity*. Note that for a given amount of excess charges, *D* is fixed and *E* is reduced by the factor of ε_r, a quantity introduced by the polarization of a dielectric under an electric field.

Similarly, in a magnetic material, the magnetic flux density is written as

$$\vec{B} = \mu\vec{H} = \mu_r\mu_0\vec{H}\ , \tag{1-16}$$

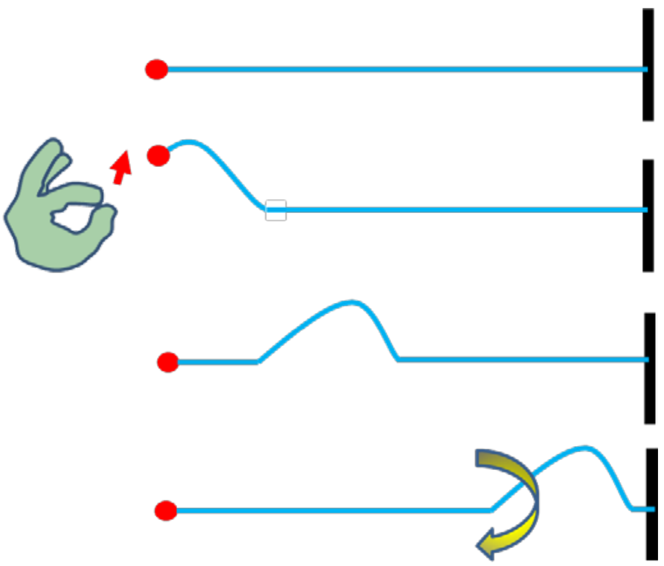
because the induced magnetic dipoles modify the permeability given by

$$\mu = \mu_r\mu_0\,, \tag{1-17}$$

where μ_r is called *relative permeability*. Usually μ_r can be very large for a magnetic material. Unlike a dielectric having a reduced electric field intensity inside, a magnetic material can greatly increase the magnetic flux density nearby a current.

CONCEPTS OF WAVES

It is a common experience that one can swing a string to propagate some energy over some distance without propagating the material (molecules of the string) carrying the energy, as illustrated below.



This physical quantity that propagates the energy is called a *wave*. Apparently, a wave has an *amplitude*, a *wavefront*, and a *propagation speed*. In general, a $\pm z$-propagating wave can be described by the amplitude function $\Psi(t,z) = f(t \mp z/u)$, which satisfies the so-called wave equation

$$\frac{\partial^2\Psi}{\partial z^2}-\frac{1}{u^2}\frac{d^2\Psi}{dt^2}=0 \tag{1-18}$$

where u is the speed of the wavefront and t is the time variable. This wave equation is very general, applying to all kinds of waves including material waves and electromagnetic waves. Taking into account the variation in all directions
Eq. (1-18) can be re-written as

$$\nabla^2\Psi-\frac{1}{u^2}\frac{d^2\Psi}{dt^2}=0\,, \tag{1-19}$$

where ∇^2 is the Laplacian operator in vector calculus. In the spherical coordinate system, the wave equation has a solution of the form

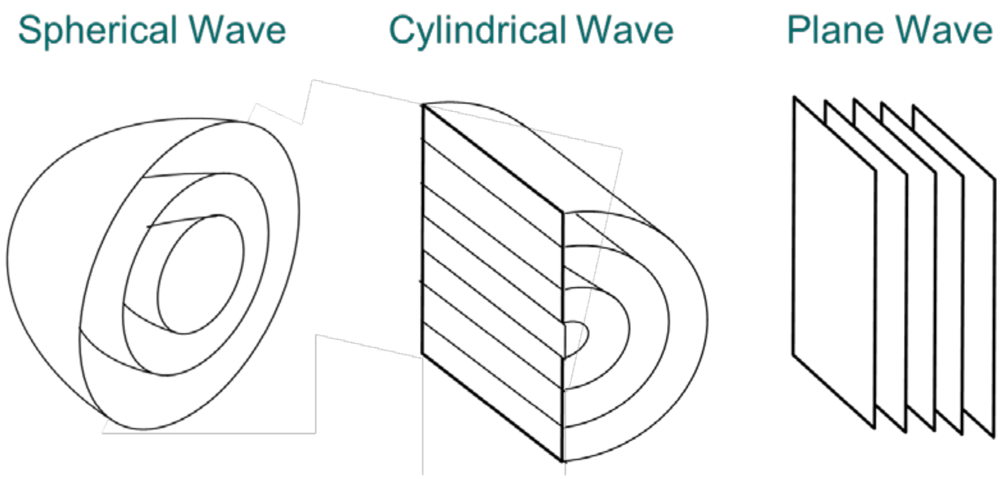
$$\Psi(t,R)=\frac{f(t\pm R/u)}{R}\,, \tag{1-20}$$

where R is the radial distance from the origin. The amplitude of the wave is decreased as the wavefront is expanded over R. Since the intensity of the wave is proportional to the square of the amplitude, this attenuation is consistent with the conservation of power in the wave. (Radiation power is equal to the surface integration of the intensity)

In the cylindrical coordinate system, the wave equation has a solution of the form

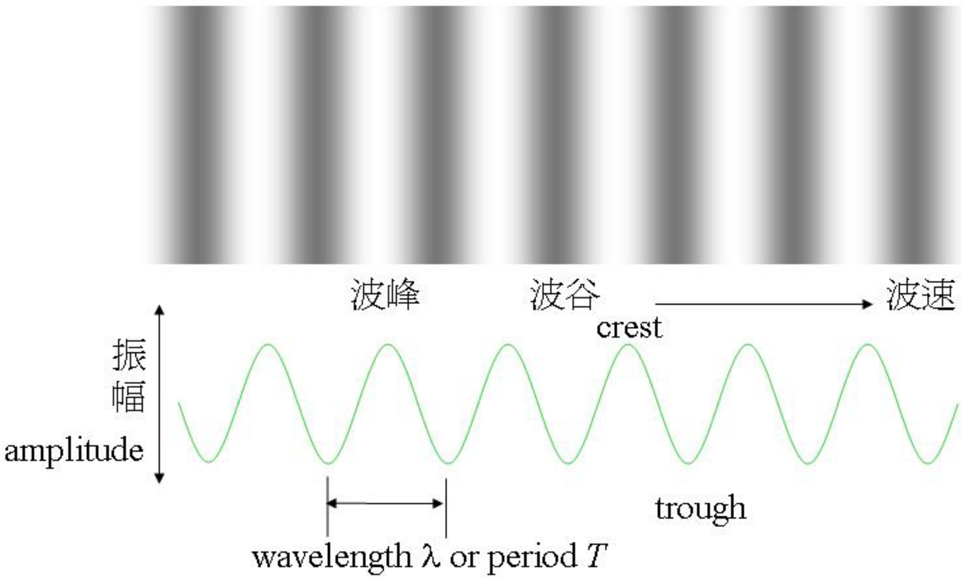
$$\Psi(t,r)=\frac{f(t\pm r\sqrt{\mu\varepsilon})}{\sqrt{r}} \tag{1-21}$$

where r is the radial distance defined in the cylindrical coordinate system. Again, the r dependence in the denominator is consistent with the conservation of the wave energy.



Now it is clear that the name of a wave is related to the shape of the wave front. For example, a wave with a spherical, cylindrical, or a planar wavefront is called a spherical, cylindrical, or a plane wave, respectively. The following plots illustrate the concept.

From Fourier analysis, an arbitrary waveform can be decomposed into superposition of many sinusoidal waves. Therefore we often focus our discussion on a sinusoidal wave.



波長 λ 或者週期 $T = 1/f$, f is the frequency

In the Cartesian coordinate system, a sinusoidal amplitude of a wave propagating along +*z* can be described by the *wave function*

$$\Psi(t,z) = \Psi_0 \cos[\omega(t - z/u) + \phi_0], \qquad (1\text{-}22)$$

where $\omega = 2\pi f$ in radian/s is the angular frequency with f in Hertz (1/s) being the frequency of the wave and ϕ_0 is an arbitrary phase at $t = 0$ and $z = 0$. The wave expressed as (1-22) is sometimes called a *harmonic wave*. For a sinusoidal wave with a temporal period of T, the frequency is the reciprocal of T or $f = 1/T$. The speed of the wave that we previously talked about is the speed of the phase of the sinusoidal amplitude or the speed of the wavefront. To see this, we set $\omega(t - z/u) + \phi_0 =$ constant to derive

$$dz/dt = u \qquad (1\text{-}23)$$

We therefore give it a name, *phase velocity*, denoted by u_p. The spatial period of a wave is called the wavelength, denoted by λ. By setting $\omega(z = \lambda)/u_p = 2\pi$, it is straightforward to obtain the relationship

$$f\lambda = u_p \qquad (1\text{-}24)$$

which means "The multiplication of the frequency and wavelength equals the speed of the wave".

Sometimes it is more convenient to write the wave function into a more general form

$$\Psi(t,z) = \Psi_0 \cos(\omega t - \vec{k} \cdot \vec{R} + \phi_0), \qquad (1\text{-}25)$$

where $k = \omega / u_p = 2\pi / \lambda$ is called the wave number and $\vec{k}$ is called the wavefront vector or simply the wave vector. The wave vector gives the propagation direction of the wavefront. For the special case of a wavefront propagating along z, the wave vector is $\vec{k} = k\hat{a}_z$ and the position vector is $\vec{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$.

For an electromagnetic wave, the wave amplitude Ψ is associated with an electric or magnetic field. In the transmission line circuit, Ψ is associated with voltage and current. The sinusoidal wave function Ψ of an electromagnetic wave is called a *time harmonic field*. As will be shown in this course, the phase velocity of an electromagnetic wave in vacuum can be calculated from

$$u_p = c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \approx 3\times 10^8 \text{ (m/s)}. \qquad (1\text{-}26)$$

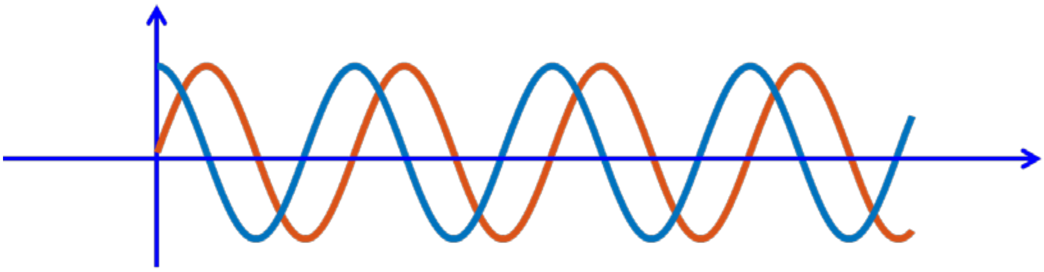
In vacuum, the wave number is denoted in this course handout by a subscript of 0 or $k_0 = \omega / c_0 = 2\pi / \lambda_0$, where λ_0 is the vacuum wavelength. However, the phase velocity of an electromagnetic wave in a material is modified as

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}} = \frac{\omega}{k_0\sqrt{\mu_r \varepsilon_r}}, \qquad (1\text{-}27)$$

where $k = k_0\sqrt{\mu_r \varepsilon_r}$. For most cases in optics, the material is nonmagnetic $\mu_r = 1$ and $k = k_0\sqrt{\varepsilon_r} = k_0 n$, and the speed of the electromagnetic wave is given by

$$c = \frac{c_0}{\sqrt{\varepsilon_r}} = \frac{c_0}{n}, \qquad (1\text{-}28)$$

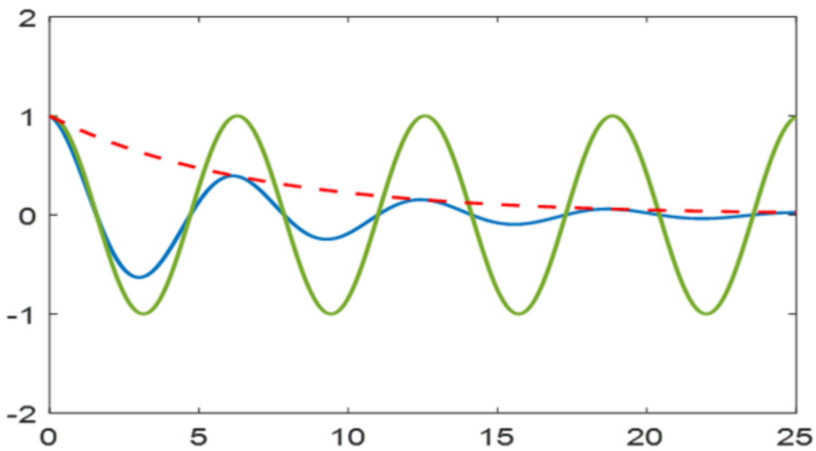
where n is called *refractive index*. The following illustrates the propagation of a sinusoidal wave at different instants of time.



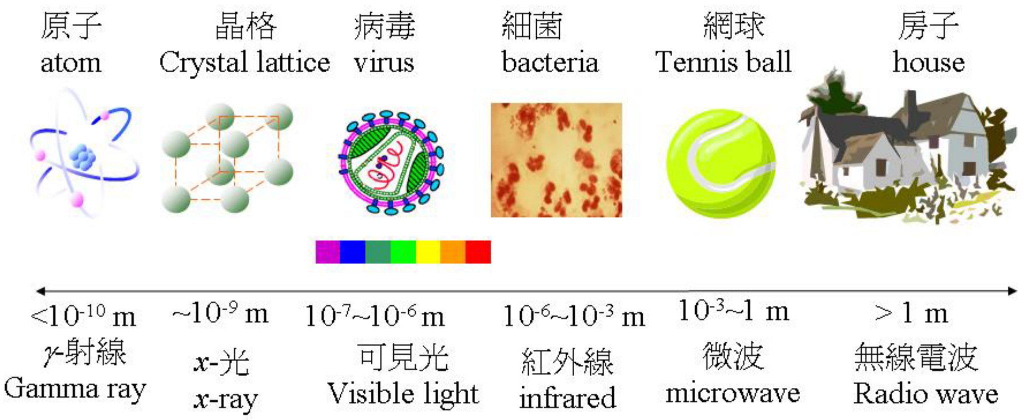
In a lossy medium, the wave amplitude decreases as it moves and the wave function for a wave propagating in z has the form

$$\Psi(t,z) = \Psi_0 e^{-\alpha z}\cos(\omega t - \beta z + \phi_0) \tag{1-29}$$

where the *attenuation coefficient* α in Np/m is responsible for the decay of the wave amplitude and the *propagation constant* (or the *phase constant*) β plays the role of k. The green and blue curves in the following plot represent voltage waves with and without attenuation, respectively.



We give different names to electromagnetic waves according to their wavelengths. The following is a cartoon of the electromagnetic spectrum with the names adopted for most applications. For example, light has a sub-micron wavelength, which is about the dimension of a virus.



不同電磁波長的電磁波有不同的名稱

Phasor Notations

Complex numbers are very useful for calculations involving sinusoidal phases. A complex number z is defined as

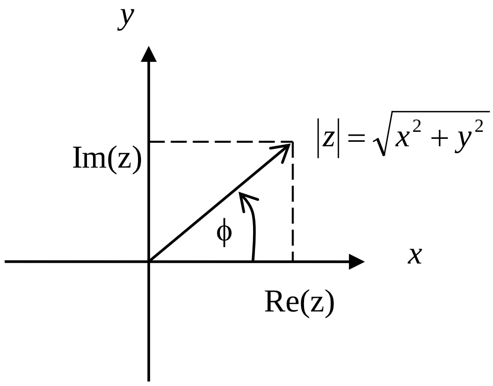
$$z = x + jy$$

where $j \equiv \sqrt{-1}$ is the imaginary unit, x and y are both real numbers. x is called the real part of z, denoted by Re(z) and y is called the imaginary part of z, denoted by Im(z).

Euler's identity tells us $e^{j\theta} = \cos\theta + j\sin\theta$, which further allows us to write

$$z = x + jy = |z|e^{j\phi}, \tag{1-30}$$

with the amplitude $|z| = \sqrt{x^2 + y^2}$ and the phase angle ϕ satisfying $\tan\phi = \dfrac{y}{x}$. So, a complex number can be graphically shown on a rectangular or polar plane, as illustrated below (note the positive sense of the phase angle is along the counter clockwise direction).



To show the powerful complex calculation involving phases, we start from two complex numbers

$z_1 = x_1 + jy_1 = |z_1|e^{j\theta_1}$ and $z_2 = x_2 + jy_2 = |z_2|e^{j\theta_2}$.

The multiplication of the two complex numbers is

$$z_1 z_2 = |z_1 z_2| e^{j(\theta_1 + \theta_2)}, \tag{1-31}$$

which transforms a tedious algebraic multiplication into a summation of the phases in the exponent (summation is certainly easier than multiplication). Similarly, the division of the two complex numbers is

$$z_1 / z_2 = |z_1 / z_2| e^{j(\theta_1 - \theta_2)}, \tag{1-32}$$

which transforms a tedious division into subtraction of the phases in the exponent. When a calculation involves successive multiplications and divisions, this phase addition and subtraction technique can greatly simplify the calculations.

The following are some rules known in complex analysis

i. $\mathrm{Re}(A)+\mathrm{Re}(B)=\mathrm{Re}(A+B)$,

ii. $a\,\mathrm{Re}(A)=\mathrm{Re}(aA)$,

iii. $\frac{\partial}{\partial x}\mathrm{Re}(A)=\mathrm{Re}(\frac{\partial A}{\partial x})$,

iv. $\int \mathrm{Re}(A)dx=\mathrm{Re}(\int A dx)$,

v. $A=B \Rightarrow \mathrm{Re}(A)=\mathrm{Re}(B)$

where a is a real number, A and B are complex functions, and x is an variable. The left hand side (LHS) of "i" is the usual way of calculating a physical quantity by summing two real quantities, but the right hand side (RHS) offers an alternative to the same calculation by summing the complex quantities first and then taking the real part of the sum later. Given the interpretation for "i", YOU please try to interpret ii~v. Note very carefully that the real part of a multiplication of two complex numbers is not equal to the multiplication of the real parts of the two numbers or $\mathrm{Re}(A)\cdot\mathrm{Re}(B)\neq\mathrm{Re}(A\cdot B)$.

In practice, we often deal with time-harmonic fields. From Fourier analysis, an arbitrary temporal function can be synthesized by a superposition of many harmonic functions. By knowing the characteristics of a harmonic field, one can deduce the characteristics of a general wave-form field by performing linear superposition in a linear medium.

For a harmonic field E with an angular frequency ω, the electric field can be expressed by

$$E=E_0\cos(\omega t+\varphi)=\mathrm{Re}(E_0 e^{j\omega t+j\varphi})=\mathrm{Re}(\hat{E}e^{j\omega t}) \quad (1\text{-}33)$$

where $\hat{E}=E_0 e^{j\varphi}$ is called the *phasor* or the complex amplitude of E.

The symbol $\wedge$ denotes a phasor quantity in this handout, but is sometimes dropped in case of no concern on causing confusion. Note that both E_0 and φ could be a function of spatial coordinates, say, $E_0(x, y, z)$ and $\varphi(x, y, z)$.

With rule "iii", the derivative of a time-harmonic field with respect to time *t* can be calculated as

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\mathrm{Re}(E_0 e^{j\omega t + j\varphi}) = \mathrm{Re}(\hat{E}\frac{\partial}{\partial t}e^{j\omega t}) = \mathrm{Re}[(j\omega\hat{E})e^{j\omega t}] \quad (1\text{-}34)$$

The time-integral of a harmonic electric field can also be calculated as

$$\int_a^b E dt = \int_a^b \mathrm{Re}(E_0 e^{j\omega t + j\varphi})dt = \mathrm{Re}[\int_a^b (\hat{E}e^{j\omega t})dt] = \mathrm{Re}[(\hat{E}/j\omega)e^{j\omega t}]\Big|_a^b \quad (1\text{-}35)$$

In terms of the phasor notation, the wave equation of Eq. (1-18) can be written as

$$\frac{\partial^2 \Psi}{\partial z^2} - \frac{1}{u^2}\frac{d^2\Psi}{dt^2} = 0 \Rightarrow \frac{\partial^2\hat{\Psi}}{\partial z^2} - (j\omega)^2\frac{1}{u^2}\hat{\Psi} = 0,$$

yielding

$$\frac{\partial^2\hat{\Psi}}{\partial z^2} + k^2\hat{\Psi} = 0 \quad (1\text{-}36)$$

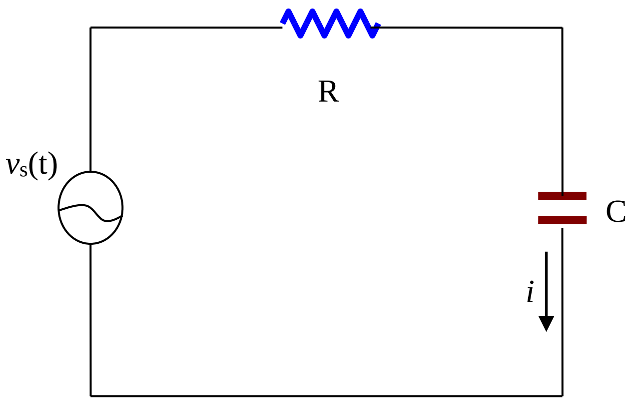
This expression is the starting wave equation of our discussion for a harmonic electromagnetic wave. If $\hat{\Psi}$ represents an electric or magnetic phasor field, Eq. (1-36) is called the *Helmoltz equation*.

The phasor notation has also been used extensively in circuit analysis. For instance, consider the following RC circuit. From Kirchhoff's voltage law, one can write

$$Ri(t)+\frac{q(t)}{C}=v_s(t)\Rightarrow Ri(t)+\frac{1}{C}\int i(t)dt=v_s(t) \tag{1-37}$$

Assume an AC driving voltage $v_s(t)=\mathrm{Re}(\hat{V}_s e^{j\omega t})$, thus the current has the form $i(t)=\mathrm{Re}(\hat{I}e^{j\omega t})$. The phasor of the current is therefore related to that of the voltage source by

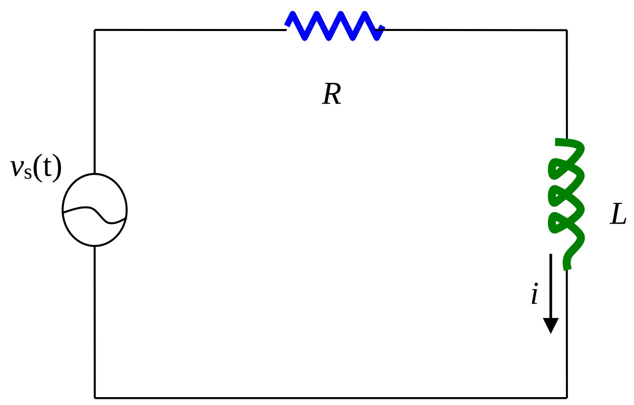
$$\hat{I}(R+\frac{1}{j\omega C})=\hat{V}_s, \tag{1-38}$$



from which $\hat{I}=\dfrac{\hat{V}_s}{R+\dfrac{1}{j\omega C}}$ and the solution for the current i(t) is obtained by performing the calculation

$$i(t)=\mathrm{Re}(\hat{I}e^{j\omega t})=\mathrm{Re}[\frac{\hat{V}_s}{R+\dfrac{1}{j\omega C}}e^{j\omega t}] \tag{1-39}$$

Another popular example is an RL circuit shown below:



Again, using the Kirchhoff's law to write

$$Ri(t) + L\frac{di}{dt} = v_s(t) \qquad (1\text{-}40)$$

For a time harmonic input voltage $v_s(t) = \mathrm{Re}(\hat{V}_s e^{j\omega t})$, one can write the phasor of the current as

$$\hat{I}(R + j\omega L) = \hat{V}_s \qquad (1\text{-}41)$$

and obtain the expression of the current *i*(t) from

$$i(t) = \mathrm{Re}(\hat{I}e^{j\omega t}) = \mathrm{Re}[\frac{\hat{V}_s}{R + j\omega L}e^{j\omega t}] \qquad (1\text{-}42)$$

This phasor algebra is particular useful for calculating sinusoidal variables, because describing a sinusoidal variable involves phases.