Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

Your name: 王昱淳 ID: 107060013 Dec. 7th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #13-1, Open books, notes (20 points), due 11 pm, Wednesday, Dec. 9th, 2020
(submission through iLMS)

Late submission won't be accepted!

1. What are the two fundamental postulates for magnetostatics in vacuum? Define all the symbols in the mathematic expressions. (6 points) State the important physical consequences of the two postulates. (4 points)

Postulate 1 $\nabla \cdot \vec{B} = 0$
Postulate 2 $\nabla \times \vec{B} = \mu_0 \vec{J}$

① $\nabla \cdot \vec{B} = 0$, $\vec{B}$: Magnetic Flux Density in Tesla (Weber/m^2)

Apply the divergence theorem to it.

$\Rightarrow \int_V \nabla \cdot \vec{B}\, dv = \oint_S \vec{B} \cdot d\vec{s} = 0$

$\Rightarrow$ Physical consequence : 1. magnetic field lines always return
2. no manegetic monopole

② $\nabla \times \vec{B} = \mu_0 \vec{J}$, $\vec{B}$: Magnetic Flux Density in Tesla (Weber/m^2)
μ_0 : vacuum permeability , $\mu_0 = 4\pi \cdot 10^{-7}$ (Henry/m)
$\vec{J}$: Currenty Density (A/m^2)

Apply the Stokes theorem to it.

$\Rightarrow \int_S \nabla \times \vec{B} \cdot d\vec{s} = \oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{s} = \mu_0 I$

(By Ampere's circuital law for magnetostotics. $\Rightarrow \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$)

$\Rightarrow$ Physical consequence : The circulation of magnetic fields is proportional to the current bounded by the circular path. Besides, the directional relationship between B and I is understood from the curl operator, which is also the right-hand rule.

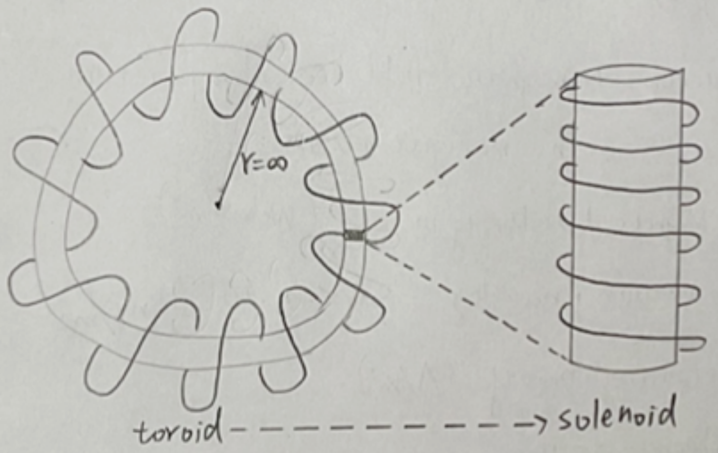
2. Show that the magnetic flux density in a very-large-radius toroid approaches that of a long solenoid. (5 points)

We know that the magnetic flux density of a toroid is $B_\phi = \frac{\mu_0 N I}{2\pi r}$

When $r \to \infty$, $B = \lim_{r\to\infty} \frac{\mu_0 N I}{2\pi r} = \mu_0 I \lim_{r\to\infty} \frac{N}{2\pi r} = \mu_0 I \frac{N}{L} = \mu_0 I \cdot n$

(n : the number of loops per unit length of wires.)

$\Rightarrow$ The solution will be the same as the case of a solenoid.



3. Use the magnetic field derived from the Biot-Savart law for a finite-length current element to solve the magnetic field everywhere generated by an infinite long wire carrying a current of I along +z. The wire is in vacuum and its diameter is negligible. Compare the result with that derived from the Ampere's law. (5 points)

The magnetic field from Biot-Savart Law for a finite-length current element : $\vec{B} = \frac{\mu_0 I L}{2\pi r \sqrt{L^2+r^2}} \hat{a}_\phi$

When $L \to \infty$, $L \gg r \Rightarrow \vec{B} = \frac{\mu_0 I L}{2\pi r \cdot L} \hat{a}_\phi = \frac{\mu_0 I}{2\pi r} \hat{a}_\phi$, $B = \frac{\mu_0 I}{2\pi r}$

$\Rightarrow$ The magnetic field everywhere generated by an infinite long wire . $\frac{\mu_0 I}{2\pi r}$ ———— ①

By Ampere's Law, $B \cdot 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$ ———— ②

$\Rightarrow$ ① has the same result as ②

4. The name and definition of the magnetic vector potential A in $\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$ is somewhat mysterious. The analogy in electrostatics is the scalar potential, V, defined in $\nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla V$. We now know V is the potential energy per unit charge in an electric field, manifested by the electrostatic energy of charges stored in a volume $W_e = \frac{1}{2}\int_{V'} \rho V dv$. Later, we will also derive an expression for the magnetostatic energy of current stored in a volume $W_m = \frac{1}{2}\int_V \vec{A} \cdot \vec{J} dv$. Based on this analogy, what can you say about the magnetic vector potential? (5 points)

$$\begin{cases} \nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla V, \quad W_e = \frac{1}{2}\int_{V_1} \rho V dv \longrightarrow \text{equation set 1} \\ \nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}, \quad W_m = \frac{1}{2}\int_V \vec{A} \cdot \vec{J} dv \longrightarrow \text{equation set 2} \end{cases}$$

Observe the above equation sets are quite analogous, and we can say that $\vec{A} \cdot \vec{J}$ is the total magnetic potential. After integration of $\vec{A} \cdot \vec{J}$, the calculation result becomes the energy. In addition, $\vec{A}$ is a vector, which makes it more easy to calculate the values for the magnetic field.