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EE214000 Electromagnetics, Fall, 2020
Quiz #12-1, Open books, notes (22 points), due 11 pm, Wednesday, Dec. 2nd, 2020
(請交至 iLMS)

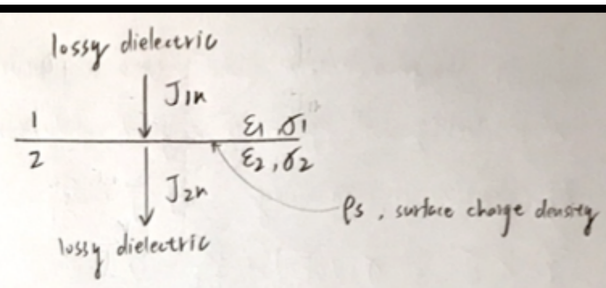
Late submission won't be accepted!

1. On Slide 11 of Lec. 12, the condition to have no charge accumulation at two lossy dielectric is

$$\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}.$$

Give a **thorough explanation** on the consequence of such a condition. (5 points)

Consider the figure :



From the boundary condition for the normal components of J

$\Rightarrow J_{1n} = J_{2n} \Rightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n} \Rightarrow \sigma_1 E_{1n} - \sigma_2 E_{2n} = 0$ —— ①

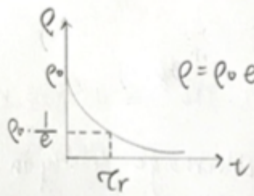
From the boundary condition for the normal components of D

$\Rightarrow D_{1n} - D_{2n} = \rho_s \Rightarrow \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$ —— ②

Combine ① and ②, we can get $\rho_s = \left(\varepsilon_1 \frac{\sigma_2}{\sigma_1} - \varepsilon_2\right)E_{2n} = \left(\varepsilon_1 - \varepsilon_2 \frac{\sigma_1}{\sigma_2}\right)E_{1n}$,

which means that there is no surface charge when $\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}$ —— ③

Besides, $\frac{\varepsilon}{\sigma} = \tau_r$ has the physical meaning that charges relax at an equal rate τ_r.



$\rho = \rho_0 e^{-\frac{t}{\tau_r}}$ here, $\tau_r = \frac{\varepsilon}{\sigma}$ and $\rho_0 = \rho_s$

We can conclude that $\tau_{r1} = \frac{\varepsilon_1}{\sigma_1}$ for region 1 and $\tau_{r2} = \frac{\varepsilon_2}{\sigma_2}$ for region 2.
If $\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}$ as mentioned in ③, we find $\tau_{r1} = \tau_{r2}$ since $\frac{\varepsilon_1}{\sigma_1} = \frac{\varepsilon_2}{\sigma_2}$.
Thus, there will be no surface charge left in the interface, because the relaxation time constant in region 1 is the same as the relaxation time constant in region 2. Since their relaxation time constants are the same, when charges propagate from region 1 to region 2, all of the charges will be injected into region 2.
Thas's why there will be no surface charge left when $\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}$.

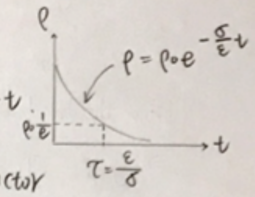
2. Previously, we asserted that there's no electric field in a good conductor. Explain why it takes time for a good conductor to reach the zero-field condition upon some excitation. (3 points) Assume the dielectric constant of silver is equal to the vacuum value. Estimate the time constant, within which the electric in silver is not zero, when an external electric field excite a piece of silver. (3 points)

① Consider charge relaxation time. Upon a perturbation, an excess charge charge density will appear in a good conductor.

By equation of continuity, $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

$\vec{J} = \sigma \vec{E} \Rightarrow \sigma(\nabla \cdot \vec{E}) + \frac{\partial \rho}{\partial t} = 0$

$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \Rightarrow \sigma \cdot \frac{\rho}{\varepsilon} + \frac{\partial \rho}{\partial t} = 0 \Longrightarrow \rho = \rho_0 e^{-\frac{\sigma}{\varepsilon} t}$



Applying an electric field near this good conductor suddenly, there will be some electrons induced, which means the charge distribution will be non-uniform to cancel out the external electric field. Since the charge distribution inside the conductor is uniform without applied electric field in the beginning, it will take time to rearrange the charge distribution in order to cancel out the external electric field, which means that the net electric field is zero inside the conductor.

② We know $\sigma_{Ag} = 6.30 \cdot 10^7$ S/m @ 20°C.

Assume $\varepsilon_{Ag} = 1 \cdot (8.85 \cdot 10^{-12}) = 8.85 \cdot 10^{-12}$ F/m

$\Rightarrow \tau_{Ag} = \frac{\varepsilon_{Ag}}{\sigma_{Ag}} = \frac{8.85 \cdot 10^{-12}}{6.30 \cdot 10^7} = 1.405 \cdot 10^{-19} = 0.1405$ as $\Rightarrow \tau_{Ag} = 0.1405$ as

Thus, we can consider that the charge density in Silver will decay to almost zero at high speed.

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3. An electromagnetic wave has an oscillating electric field. In most electromagnetics textbooks, you are taught that an electromagnetic wave cannot penetrate a metal, because the electric field in a conductor is zero. Would you be able to explain why a hard *x*-ray or gamma ray (wavelength ~ 1 Angstrom or frequency ~ 3×10^{18} Hz) can penetrate metal? (5 points)

When light or any other form of electromagnetic radiation interacts with any metal, one or more of following will take place, including reflection, refraction or scattering.

The extent of each interaction depends on the material and the wavelength of the eletromagnetic radiation. Low energy (long wavelength) radiation gets scattered in a very short layer of the material.

As for reflection and scattering, if the material is transparent to the radiation, scattering is reduced considerably, and there will be portial reflection at the entry surface and refraction through the medium.

In general, the higher the energy of the radiation, the material interact less. When the wavelength is shorter or the frequency is higher, there will be more radiation passing through the material.

X-rays and Gamma rays have much higher energies compared to visible light and hence allow more of the radiation to pass through the matel.

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4. A leaky capacitor consists of two parallel plate electrodes as shown below. The two electrodes are separated by a distance d and biased with a voltage of V0, each having an area of S. The material sandwiched between the two electrodes has a permittivity ε and conductivity σ. Ignore the fringe field near the electrode edges. (1) What is the capacitance of this capacitor? (2 points) (2) What is the resistance between the two electrodes? (2 points) (3) What is the electric power dissipated in this capacitor? (2 points)

(1) $C = \frac{\varepsilon A}{d}$ ⇒ Its capacitance is $C = \frac{\varepsilon S}{d}$

(2) Ignore the fringe field near the electrode edges.

Thus , we can know that $RC = \frac{\varepsilon}{\sigma}$ is valid here.

$$\Rightarrow R = \frac{1}{C}\frac{\varepsilon}{\sigma} = \frac{d}{\varepsilon S}\cdot\frac{\varepsilon}{\sigma} = \frac{d}{\sigma S}$$

(3) We know the electric power $P = CV^2$

$$\Rightarrow P = \frac{\varepsilon S}{d}\cdot V_0^2 = \frac{\varepsilon S V_0^2}{d}$$