Use LT to solve DEs with $\left\{\begin{array}{l}\text{discontinuous} \\ \text{periodic}\end{array}\right.$ forcing

In many engineering/physics problems, the systems are described by constant-coefficient 2nd-order linear DEs.

Ex:

$$\boxed{\begin{array}{l}\text{system} \\ y'' + p y' + q y = f(t)\end{array}}$$

The input may be "turned on" at a specific time, like a switch.

ex:

LT is particularly useful to solve problems with

In the following, we will discuss

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Some important functions and their LT

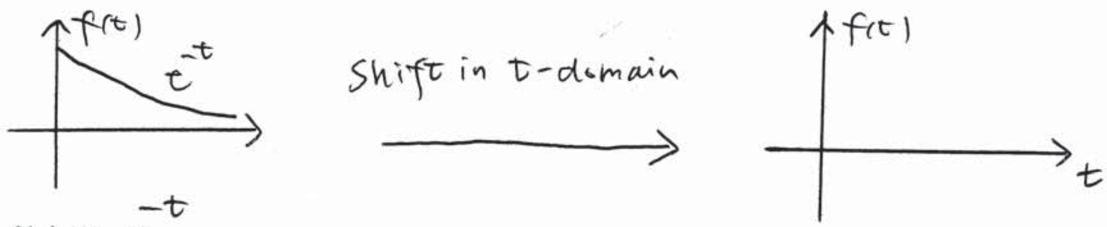
I. Step function

A function turns on at , like a switch.

Def: step function (also called "Heaviside function")

step function is very useful to express a function

ex:



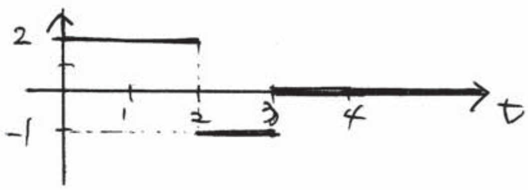
$f(t) = e^{-t}$

In a general form

$f(t) \xrightarrow{\text{shift in t-domain by "a"}}$

Step function is also widely used to express

ex:



LT of step function & piecewise-defined function

* step function : $f(t) = u_a$

$$\mathcal{L}\{u_a\} = \int_0^\infty u_a e^{-st}\, dt =$$

ex: $f(t) = u_2$, $\mathcal{L}\{f(t)\} = ?$

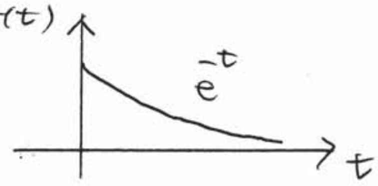
* piecewise-defined function

ex: $f(t) = 2 - 3u_2 + u_3$, $\mathcal{L}\{f(t)\} = ?$

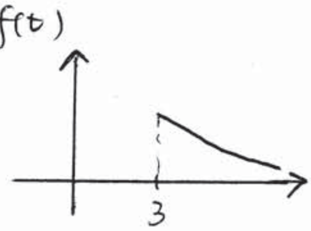
LT of functions shifted in t-domain

We just learn a function shifted in t-domain can be expressed by the step function. Then what's the LT of functions shifted in t-domain?

ex1:



shift in t-domain $\longrightarrow$



$f(t) = e^{-t}$

$\downarrow \mathcal{L}$

$f(t) =$

$\downarrow \mathcal{L}$

General form :

$f(t)$ shift in t-domain $\xrightarrow{\mathcal{L}}$

ex2: Given $F(s) = \frac{1-e^{-2s}}{s^2}$, what's the $\mathcal{L}^{-1}$?

Remark :

shift in t-domain $\longrightarrow$

what about "shift in s-domain" ?

II. Impulse function and delta function

Impulse function describes a very quick push on a system at a specific time.

Def : unit impulse centered at $t = t_0$