1) Let $Ev\{x(t)\} = \frac{1}{2}[x(t) + x(-t)] = a_1(t)$. According to the definition of even function, we need to prove $a_1(t) = a_1(-t)$.

$$a_1(-t) = \frac{1}{2}[x(-t) + x(t)] = \frac{1}{2}[x(t) + x(-t)] = a_1(t) \Rightarrow even$$

Similarly, let $Od\{x(t)\} = \frac{1}{2}[x(t) - x(-t)] = a_2(t)$. According to the definition of odd function, we need to prove $a_2(t) = -a_2(-t)$.

$$a_2(-t) = \frac{1}{2}[x(-t) - x(t)] = -\frac{1}{2}[x(t) - x(-t)] = -a_2(t) \Rightarrow odd$$

2.(a) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$

$W_0 = \frac{6\pi}{7}$ $\quad \frac{W_0}{2\pi} = \frac{\frac{6\pi}{7}}{2\pi} = \frac{3}{7}$ is a rational number,

then the fundamental period is 7. #

(b) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$

$W_0 = \frac{1}{8}$ $\quad \frac{W_0}{2\pi} = \frac{1}{16\pi}$ isn't a rational number,

then the signal is not periodic. #

(d) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

By product to sum formula, $x[n] = \frac{1}{2}\left[\cos\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{\pi}{4}n\right)\right]$

Assume $x_1[n] = \cos\left(\frac{3\pi}{4}n\right)$, $W_0 = \frac{3\pi}{4}$, $\frac{W_0}{2\pi} = \frac{3}{8} \Rightarrow N_1 = 8$.

Assume $x_2[n] = \cos\left(\frac{\pi}{4}n\right)$, $W_0 = \frac{\pi}{4}$, $\frac{W_0}{2\pi} = \frac{1}{8} \Rightarrow N_2 = 8$.

The period of $x[n]$ is $LCM(N_1, N_2) = 8$. #

(e) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$

Assume $x_1[n] = \cos\left(\frac{\pi}{4}n\right)$, $W_0 = \frac{\pi}{4}$, $\frac{W_0}{2\pi} = \frac{1}{8} \Rightarrow N_1 = 8$.

Assume $x_2[n] = \sin\left(\frac{\pi}{8}n\right)$, $W_0 = \frac{\pi}{8}$, $\frac{W_0}{2\pi} = \frac{1}{16} \Rightarrow N_2 = 16$.

Assume $x_3[n] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$, $W_0 = \frac{\pi}{2}$, $\frac{W_0}{2\pi} = \frac{1}{4} \Rightarrow N_3 = 4$.

The period of $x[n]$ is $LCM(N_1, N_2, N_3) = 16$. #

3、 $\boxed{1.27}$ (a) (e) (f) (g)

(a) ① $y(t) = x(t-2) + x(2-t)$

$= \underline{x(t-2)} + \underline{x(-t+2)}$

∵ output is not dependent on the input at the same time

⇒ not memoryless

② $y(t) = x(t-2) + \underline{x(-t+2)}$

∵ output depends on future time

∴ not causal

③ $y_1(t) = x_1(t-2) + x_1(2-t)$

$x_2(t) = x_1(t-t_0)$

∴ $y_2(t) = x_1(t-t_0-2) + x_1(2-t+t_0)$

$\neq x_1(t-t_0-2) + x_1(2-t-t_0)$

$\neq y_1(t-t_0)$

∴ not time invariant

④

$|x(t)| < \infty$

∴ $|x(t-2)| < \infty$, $|x(2-t)| < \infty$

∴ $|y(t)| = |x(t-2) + x(2-t)|$

$\leq |x(t-2)| + |x(2-t)| < \infty$

∴ stable

⑤ $y_1(t) = x_1(t-2) + x_1(2-t)$

$y_2(t) = x_2(t-2) + x_2(2-t)$

⇒ $x_3(t) = a x_1(t) + b x_2(t)$

∴ $y_3(t) = x_3(t-2) + x_3(2-t)$

$= a x_1(t-2) + b x_2(t-2)$

$+ a x_1(2-t) + b x_2(2-t)$

$= a[x_1(t-2) + x_1(2-t)]$

$+ b[x_2(t-t) + x_2(2-t)]$

$= a y_1(t) + b y_2(t)$

∴ linear

(e)

① $y(t) = \begin{cases} 0 & , x(t) < 0 \\ x(t) + x(t-2) & , x(t) \geq 0 \end{cases}$

∵ output is not dependent on the input at the same time

∴ not memoryless

②

∵ output depends only on present and past time

∴ causal

③ when $x(t) \geq 0$:

$x_2(t) = x_1(t-t_0)$

∴ $y_2(t) = x_1(t-t_0) + x_1(t-t_0-2)$

$= y_1(t-t_0)$

∴ time invariant

④ when $x(t) \geq 0$:

$|x(t)| < \infty$

∴ $|x(t-2)| < \infty$

⇒ $|y(t)| = |x(t) + x(t-2)|$

$\leq |x(t)| + |x(t-2)|$

$< \infty$

∴ stable

⑤ when $x(t) \geq 0$:

$y_1(t) = x_1(t) + x_1(t-2)$

$y_2(t) = x_2(t) + x_2(t-2)$

⇒ $x_3(t) = a x_1(t) + b x_2(t)$

∴ $y_3(t) = x_3(t) + x_3(t-2)$

$= a x_1(t) + a x_1(t-2)$

$+ b x_2(t) + b x_2(t-2)$

$= a y_1(t) + b y_2(t)$

∴ linear.

(f) $y(t)=x(\frac{t}{3})$

①

ex: $y(3)=x(\frac{3}{3})=x(1)$

$\because$ output is not dependent on the input at the same time

$\Rightarrow$ not memoryless

②

$y(-3)=x(\frac{-3}{3})=x(-1)$

$\because$ output depends on future time

$\therefore$ not causal

③ $y_1(t)=x_1(\frac{t}{3})$

$x_2(t)=x_1(t-t_0)$

$\therefore y_2(t)=x_1(\frac{t-t_0}{3})$

$\neq x_1(\frac{t}{3}-t_0)$

$\neq y_1(t-t_0)$

$\therefore$ not time invariant

④ $|x(t)|<\infty$

$\therefore |x(\frac{t}{3})|<\infty$

$\therefore |y(t)|<\infty$

$\Rightarrow$ stable

⑤ $y_1(t)=x_1(\frac{t}{3})$

$y_2(t)=x_2(\frac{t}{3})$

$\Rightarrow x_3(t)=ax_1(t)+bx_2(t)$

$\therefore y_3(t)=x_3(\frac{t}{3})$

$=ax_1(\frac{t}{3})+bx_2(\frac{t}{3})$

$=ay_1(t)+by_2(t)$

$\therefore$ linear.

(g) $y(t)=\frac{dx(t)}{dt}$

①

$\because y(t)=\frac{dx(t)}{dt}=\lim_{\Delta t\to 0}\frac{x(t)-x(t-\Delta t)}{\Delta t}$

$\because$ output is not dependent on the input at the same time

$\therefore$ not memoryless

②

$\because$ output depends only on present and past time

$\therefore$ causal

③ $y_1(t)=\frac{dx_1(t)}{dt}$

$x_2(t)=x_1(t-t_0)$

$\therefore y_2(t)=\frac{dx_1(t-t_0)}{dt}$

$=y_1(t-t_0)$

$\therefore$ time invariant

④ let $x(t)=u(t)$

$\therefore y(t)=\frac{dx(t)}{dt}=\frac{du(t)}{dt}=\delta(t)$

$\Rightarrow |y(t)|<\infty$ doesn't hold

$\therefore$ non stable

⑤ $y_1(t)=\frac{dx_1(t)}{dt}$

$y_2(t)=\frac{dx_2(t)}{dt}$

$x_3(t)=ax_1(t)+bx_2(t)$

$\therefore y_3(t)=\frac{dx_3(t)}{dt}$

$=\frac{d[ax_1(t)+bx_2(t)]}{dt}$

$=a\cdot\frac{dx_1(t)}{dt}+b\frac{dx_2(t)}{dt}$

$=ay_1(t)+by_2(t)$

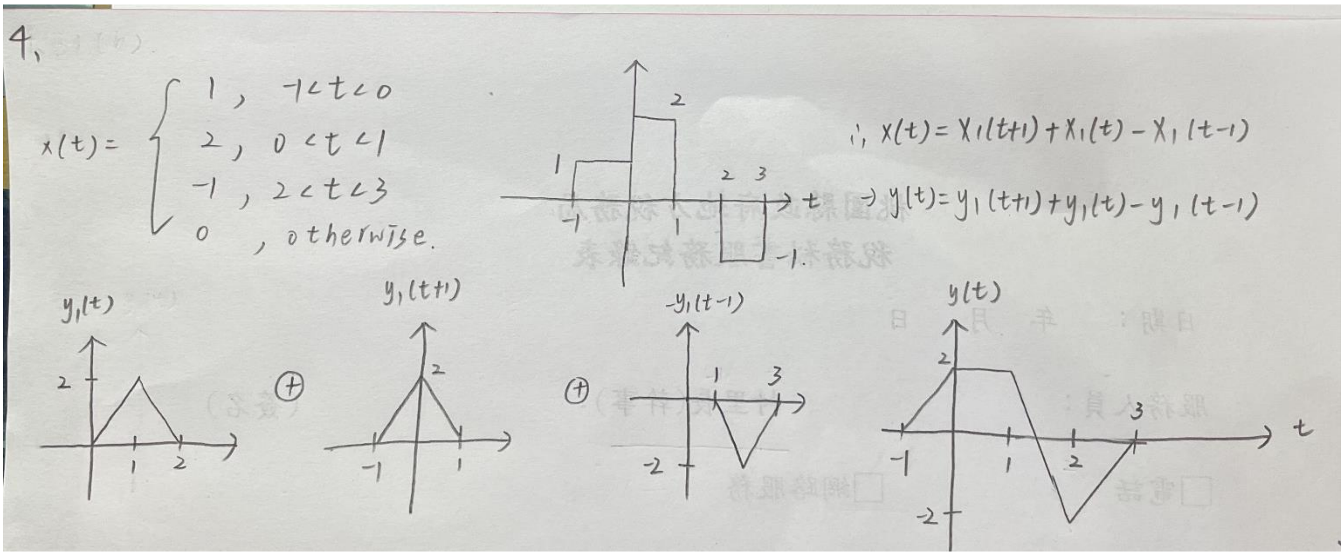
$\therefore$ linear

$\Rightarrow$ (a) linear, stable

(e) time invariant, linear, causal, stable

(f) linear, stable

(g) time invariant, linear, causal ※



5) We need to find the smallest N_0 such that $m(2\pi/N)N_0 = 2\pi k$ or $N_0 = kN/m$, where k is an integer. If N_0 has to be an integer, then N must be a multiple of m/k and m/k must be an integer. This implies that m/k is a divisor of both m and N. Also, if we want the smallest possible N_0, then m/k should be the GCD of m and N. Therefore, $N_0 = N/\gcd(m, N)$.

6) Let us name the output of system 1 as $w[n]$ and the output of system 2 as $z[n]$. Then,

$$\begin{aligned} y[n] = z[2n] &= w[2n] + \frac{1}{2}w[2n-1] + \frac{1}{4}w[2n-2] \\ &= x[n] + \frac{1}{4}x[n-1]. \end{aligned}$$

Linearity: let $y_1[n] = x_1[n] + \frac{1}{4}x_1[n-1]$, $y_2[n] = x_2[n] + \frac{1}{4}x_2[n-1]$ and $x_3[n] = ax_1[n] + bx_2[n]$. Then, we have

$$\begin{aligned} y_3[n] &= x_3[n] + \frac{1}{4}x_3[n-1] \\ &= (ax_1[n] + bx_2[n]) + \frac{1}{4}(ax_1[n-1] + bx_2[n-1]) \\ &= a\left(x_1[n] + \frac{1}{4}x_1[n-1]\right) + b\left(x_2[n] + \frac{1}{4}x_2[n-1]\right) \\ &= ay_1[n] + by_2[n] \Rightarrow \textit{linear}. \end{aligned}$$

Time-invariance: let $y_1[n] = x_1[n] + \frac{1}{4}x_1[n-1]$, $x_2[n] = x_1[n-n_0]$. Then, we have

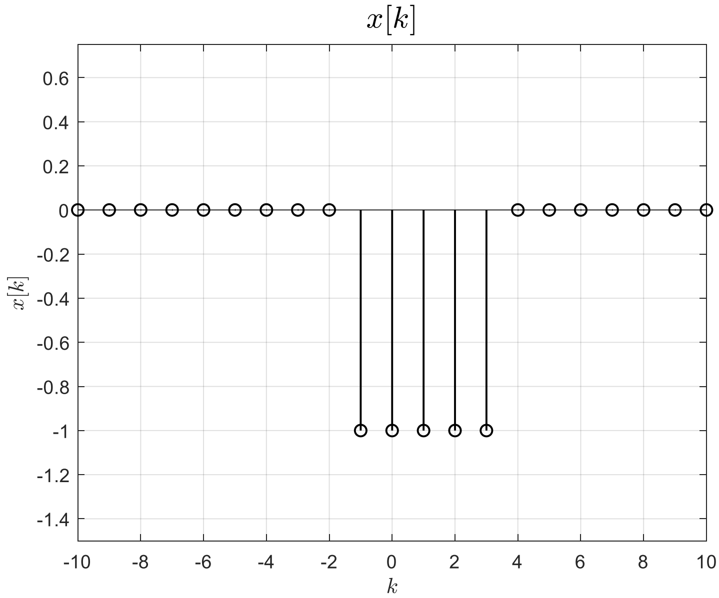
$$\begin{aligned} y_2[n] &= x_2[n] + \frac{1}{4}x_2[n-1] \\ &= x_1[n-n_0] + \frac{1}{4}x_1[n-n_0-1] \\ &= y_1[n-n_0] \Rightarrow \textit{time-invariant}. \end{aligned}$$

The overall system is linear and time-invariant.

7.

(a)

$x[k] = u[k-4] - u[k+1]$

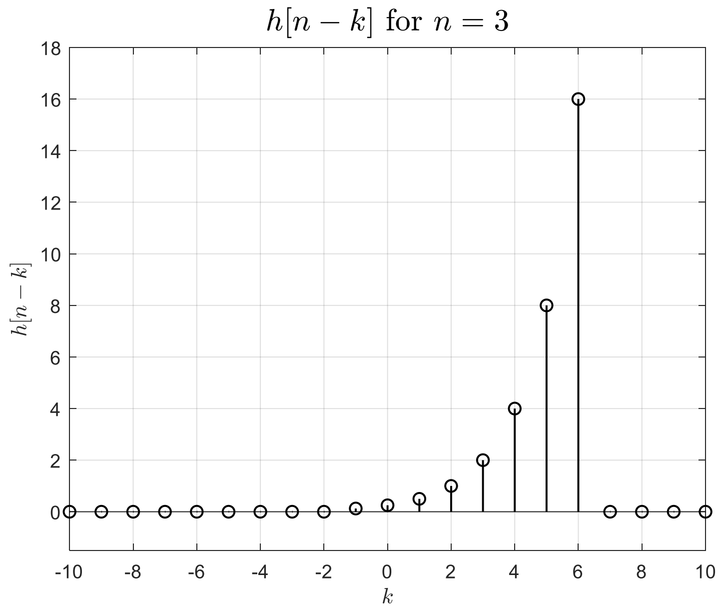


$h[n-k] = (\frac{1}{2})^{(n-k)-1}(u[(n-k)+3] - u[(n-k)-5])$

$h[n-k] = \left(\frac{1}{2}\right)^{-k+(n-1)}(u[-k+(n+3)] - u[-k+(n-5)]$

$Let\ n = 3$ ：

$h[3-k] = \left(\frac{1}{2}\right)^{-k+2}(u[-k+6] - u[-k-2])$



(b)

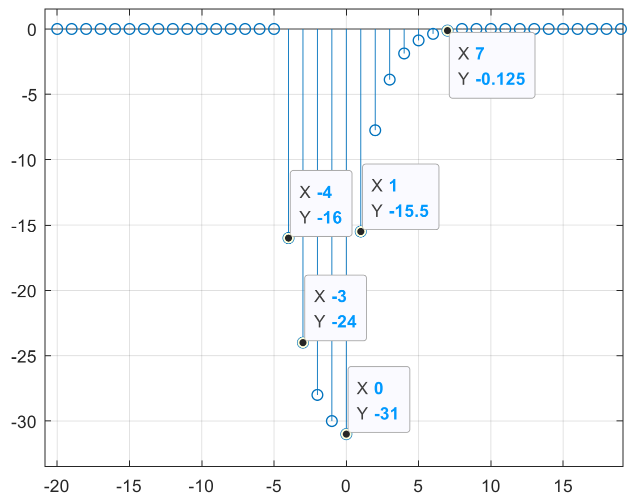
$$y[n] = x[n] * h[n] = \sum_{k=-3}^{4} \left(\frac{1}{2}\right)^{n-1} (u[n-k-4] - u[n-k+1])$$

$$-4 \le n \le -1, y[n] = \frac{-\left(\frac{1}{2}\right)^{-5} \times \left(1-\left(\frac{1}{2}\right)^{n+5}\right)}{1-\frac{1}{2}} = \left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{-5}$$

$$0 \le n \le 3,\ \ y[n] = \frac{-\left(\frac{1}{2}\right)^{n-1} \times \left(1-\left(\frac{1}{2}\right)^{5}\right)}{1-\frac{1}{2}} = \left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{n-5}$$

$$4 \le n \le 7,\ \ y[n] = \frac{-\left(\frac{1}{2}\right)^{3} \times \left(2^{8-n}-1\right)}{2-1} = \left(\frac{1}{2}\right)^{3} - \left(\frac{1}{2}\right)^{n-5}$$

$$y[n] = \begin{cases} \left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{-5} & , for -4 \le n \le -1 \\ \left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{n-5} & , for\ 0 \le n \le 3 \\ \left(\frac{1}{2}\right)^{n-5} - \left(\frac{1}{2}\right)^{3} & , for\ 4 \le n \le 7 \\ 0 & , otherwise \end{cases}$$



-16.0000 n=-4
-24.0000
-28.0000
-30.0000
-31.0000
-15.5000
-7.7500
-3.8750
-1.8750
-0.8750
-0.3750
-0.1250 n=7

8.

(a)

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$

$= \int_{0}^{\infty} e^{-2(\tau-3)}[u(t-\tau-3) - u(t-\tau-5)]d\tau$

is non zero only in the range $t-5 < \tau < t-3$,

For $t \le 3,\ y(t) = 0$

$3 < t \le 5,\ y(t) = \int_{0}^{t-3} e^{-2(\tau-3)}d\tau = \frac{1}{2}(1 - e^{-2(t-6)})$

$5 < t < \infty,\ y(t) = \int_{t-3}^{t-5} e^{-2(\tau-3)}d\tau = \frac{1}{2}(e^{-2(t-6)} - e^{-2(t-8)})$

$$y(t) = \begin{cases} \frac{1}{2}(e^{6} - e^{-2(t-6)}) & ,for\ 3 < t \le 5 \\ \frac{1}{2}(e^{-2(t-8)} - e^{-2(t-6)}) & ,for\ 5 < t < \infty \\ 0 & ,otherwise \end{cases}$$

(b)

$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$

$g(t) = \frac{dx(t)}{dt} * h(t) = e^{-2(t-6)}u(t-3) - e^{-2(t-8)}u(t-5)$

(c)

For any signal $x(t)$, *we have* $\frac{dx(t)}{dt} = x(t) * u_1(t)$ *(eq.(2.144))*

where $u_1(t)$ *is the unit doublet.*

$$g(t) = (dx(t)/dt) * h(t)$$

$$= (x(t) * u_1(t)) * h(t)$$

$$= (x(t) * h(t)) * u_1(t)$$

$$= y(t) * u_1(t) = dy(t)/dt$$

Thus, $g(t) = dy(t)/dt$.