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Higher-order ODEs (Ch 4)

· Preliminary :

① Definitions of DEs

— linear v.s. nonlinear (review)

ex: $y''' + 3y - 4y = e^{t}$

$(1-t)\,y'' + t\,y' - 2y^2 = 0$

— homogeneous v.s. nonhomogeneous

ex: $y'' + 3y - 4y = 0$

$y'' + 3y - 4y = e^t$

② Notations:

$D^{(n)}(y) = \dfrac{d^n y}{dt^n}$ ex: $\dfrac{d^2y}{dt^2} \Rightarrow$

$y'' + 3y' - 4y = 0 \Rightarrow$

$\Rightarrow$

So for homogeneous ODE $\rightarrow$

nonhomogeneous ODE $\rightarrow$

③ About the "existence" and "uniqueness" of a solution:

<u>Existence of a unique solution</u> (Theorem 4.1.1)

For a 2nd-order ODE $y'' + p(t)y' + q(t)y = g(t)$ with $y(t_0) = y_0$, $y'(t_0) = y_0'$.

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學號

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④ In ch4, we discuss analytical techniques to solve
homogeneous linear 2nd-order ODEs

nonhomogeneous linear 2nd-order ODEs

How to solve homogeneous linear 2nd-order ODEs

Preliminary: About homogeneous linear 2nd-order ODEs

① For a homogeneous 2nd-order ODE $L(y)=0$

- There exists

- The general solution of the homogeneous 2nd-order ODE is formed by the superposition of the set

② How to check if functions $y_1, y_2, \cdots$ are "linearly independent"?

→ By

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③ From experiences, functions with different   or with different   are linearly independent.

ex:

Method of "reduction of order" (ch4.2)

✡ condition:

Idea: For a 2nd-order ODE, if one solution is given, the 2nd-order ODE can be reduced to 1st-order ODE.

Example 1: Solve $y''-y=0$, given one solution $y_1=e^t$.

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General procedures of method of "reduction of order"

Given a homogeneous linear 2nd-order ODE and a given solution y_1.

① Write the DE in its "   " : $y'' + Py' + Qy = 0$

② Set   , and find y_2' y_2''

③ Plug in y_2, y_2', y_2'' into DE :

$$y_2'' + Py_2' + Qy_2 = 0$$

$\Rightarrow$

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Method of "characteristic equation" (ch 4.3)

✡ Condition:

Idea: By observation, the 1st and 2nd derivative are related by a constant multiple of itself. The most reasonable guess of such function is

Given a homogeneous 2nd-order ODE with constant coefficients

$$ay'' + by' + cy = 0$$

$\Rightarrow$

Remarks:

① The use of "characteristic equation" is the most efficient method to solve homogeneous ODE with constant coefficients. Because

② This method can also be applied to

Examples of 3 types of roots & their solutions

Example 1: $2y'' - 5y' - 3y = 0$

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Example 2: $y'' + 4y' + 7y = 0$

* A more general form is to use and express the solutions in terms of

Example 3: $y'' - 10y' + 25y = 0$

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Summary : roots of characteristic equation and the corresponding solutions

Constant coeff DE : $ay'' + by' + cy = 0$

↓

Characteristic eq.

↓

roots

Case I : roots $m_1 \neq m_2$:

general solution $y =$

Case II : roots $m_1 \neq m_2$:

general solution $y =$

Case III : roots $m_1 = m_2$

general solution $y =$