

Logic Circuits and Boolean Algebra

Chapter 2

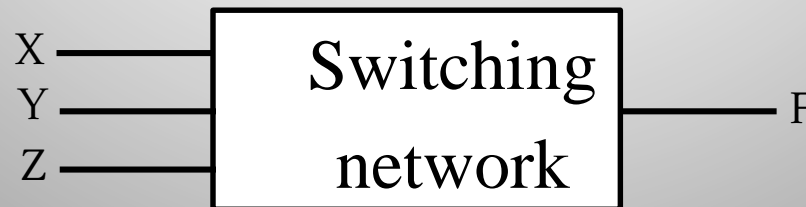
Outline

- Boolean Functions and Truth Tables
- AND, OR, NOT Gates
- Boolean Algebra and Logic Simplification
- Canonical Forms and Standard Forms
- Logic Diagrams
- Verilog
 - Synthesis
 - Simulation

Basic Operations (1/4)

- Binary logic consists of binary **variables** and a set of logical **operations**. The variables are designated by letters of the alphabet, such as A, B, C, x, y, z, etc, with each variable having two and only two distinct possible values: **1** and **0**, There are three basic logical operations: **AND**, **OR**, and **NOT**.

Boolean Variables: X, Y, Z ... takes “0”, “1”
“L”, “H”
“F”, “T”

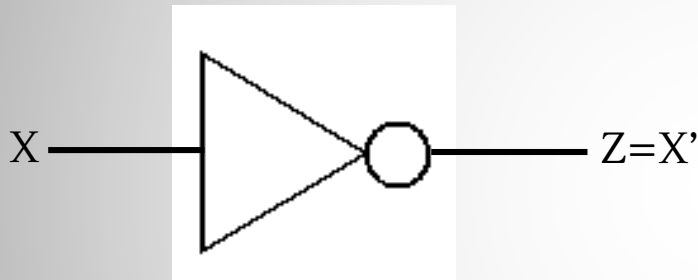


Basic Operations (2/4)

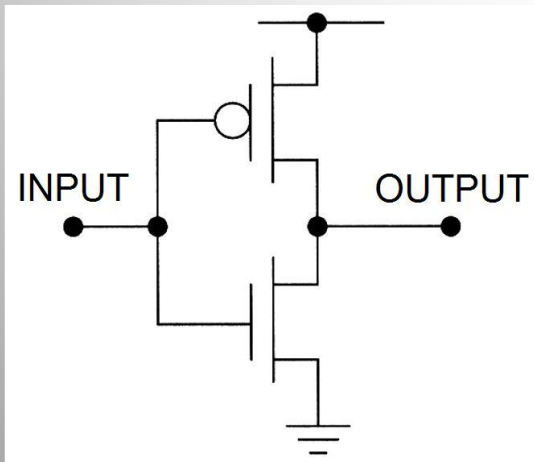
Complement (NOT or Inverter):

$$0 \leftrightarrow 1 \quad x \leftrightarrow x' \text{ (or } x \leftrightarrow \bar{x})$$

Truth Table

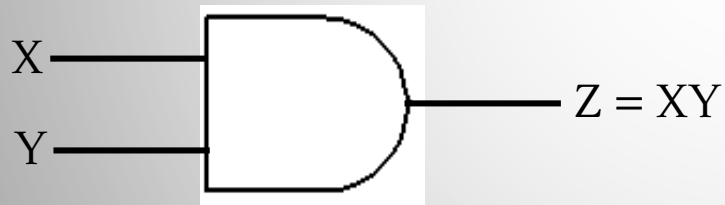
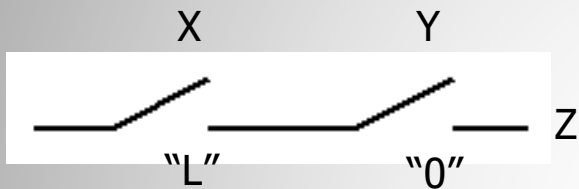


X	Z = X' or (\bar{X})
0	1
1	0



Basic Operations (3/4)

AND $0 \cdot 0 = 0$ $0 \cdot 1 = 0$ $1 \cdot 0 = 0$ $1 \cdot 1 = 1$



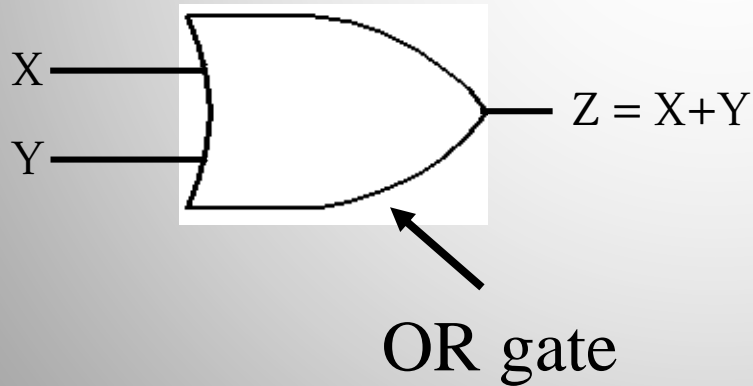
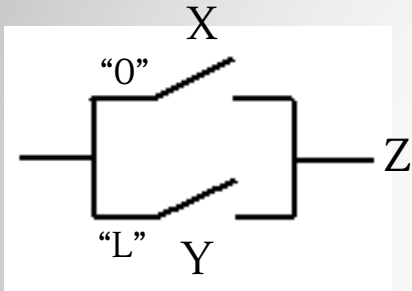
AND Gate

Truth Table

X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

Basic Operations (4/4)

$$OR \quad 0 + 0 = 0 \quad 0 + 1 = 1 \quad 1 + 0 = 1 \quad 1 + 1 = 1$$



Truth Table

X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

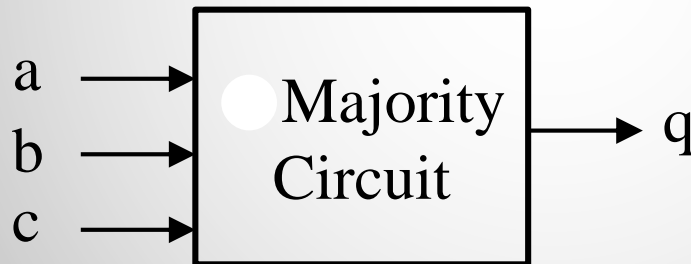
Ways to Describe A Logic Function

- Example – 3-Input Majority Circuit

- 1) English language description (specification)

- Outputs 1 if more inputs are 1 than are 0

- 1) Block Diagram



- 4) Logic equation
(Boolean equation)

- $q = (a \cdot b) + (a \cdot c) + (b \cdot c)$

- 3) Truth table

a	b	c	q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Notes on Notation

- For AND logic function:
 - Alternative symbol: \bullet or \wedge or $\&$
- For OR logic function:
 - Alternative symbol: $+$ or \vee or $|$
- For NOT logic function:
 - Alternative symbol: \neg or $'$ or \bar{a} or \sim or $!$

Truth Table

- A *truth table* of a Boolean function lists the output values for every possible input value combination in tabular form.

TABLE 2-2
Truth Table
for the Function $F = X + \overline{Y}Z$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Table 2-2 Truth Table for the Function $F = X + \overline{Y}Z$

Boolean Functions

- Examples

- $F_1 = x y z'$

- $F_2 = x + y'z$

- $F_3 = x' y' z + x' y z + x y'$

- $F_4 = x y' + x' z$

Boolean Functions

- Truth table of 2^n entries for n variables

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

- Examples

- $F_1 = x y z'$
- $F_2 = x + y'z$
- $F_3 = x' y' z + x' y z + x y'$
- $F_4 = x y' + x' z$

- Two Boolean expressions may specify the same function
 - e.g. $F_3 = F_4$

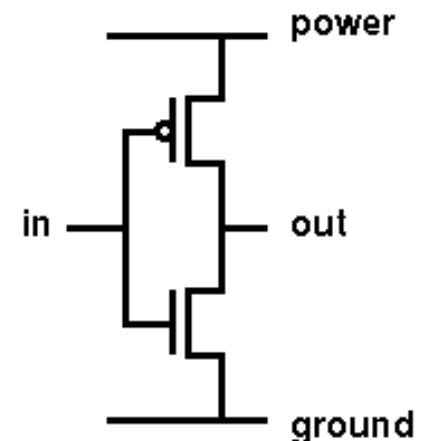
AND, OR, NOT Gates

- *Logic gates* are basic logic elements of a circuit
- They implement Boolean logic (use voltage to represent logic value)
- NOT gate, AND gate, OR gate implement the three basic logic operations

NOT gate:

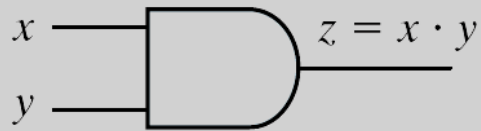


Gate representation

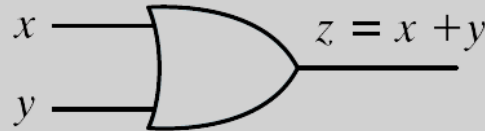


Switch-level model

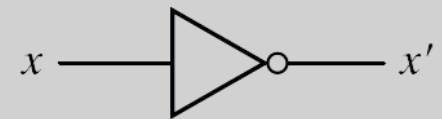
Graphic Symbols for Logic Gates



(a) Two-input AND gate



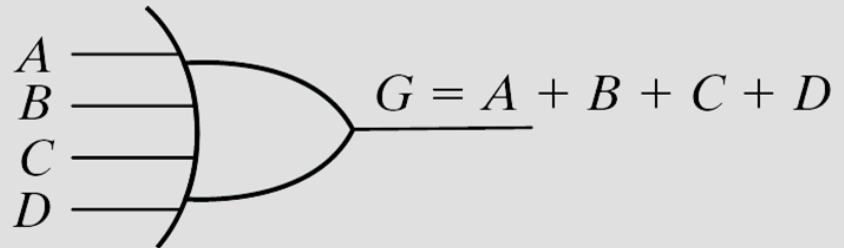
(b) Two-input OR gate



(c) NOT gate or inverter

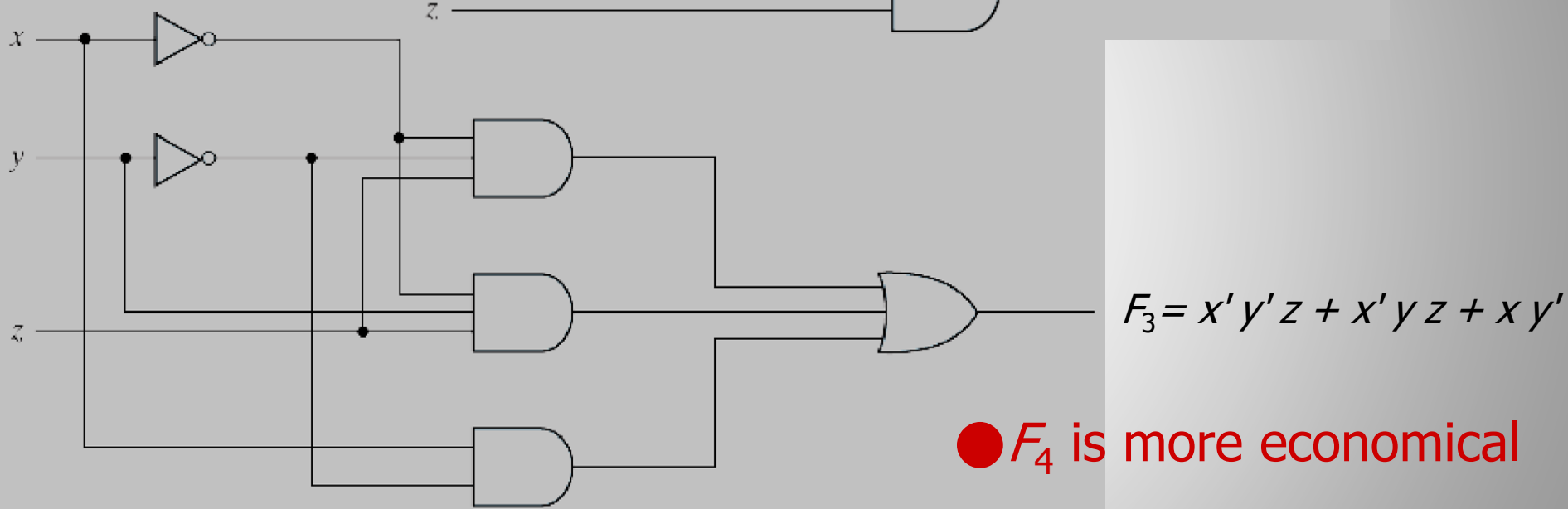
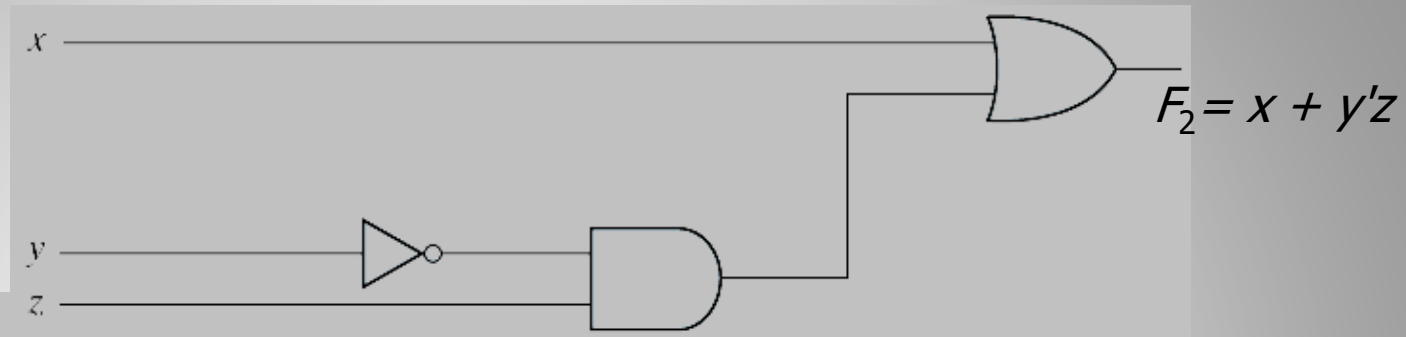


(a) Three-input AND gate

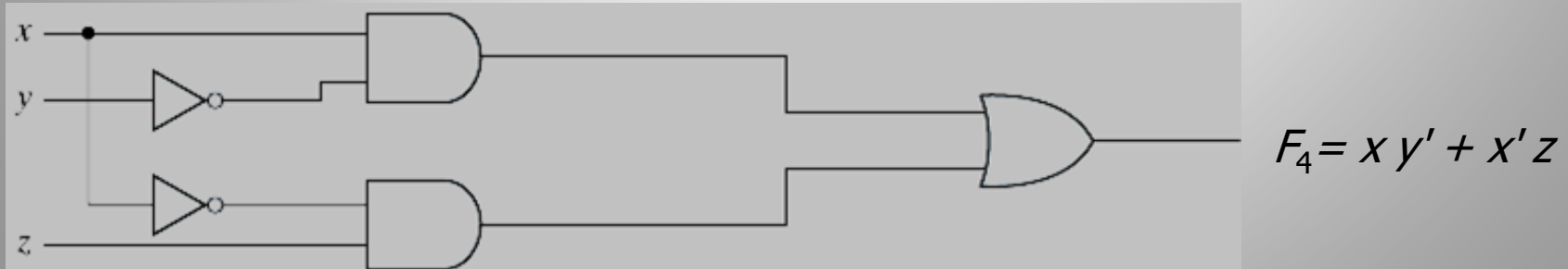


(b) Four-input OR gate

Implementation with Logic gates



● F_4 is more economical



*How to know if two circuits are
equivalent?*

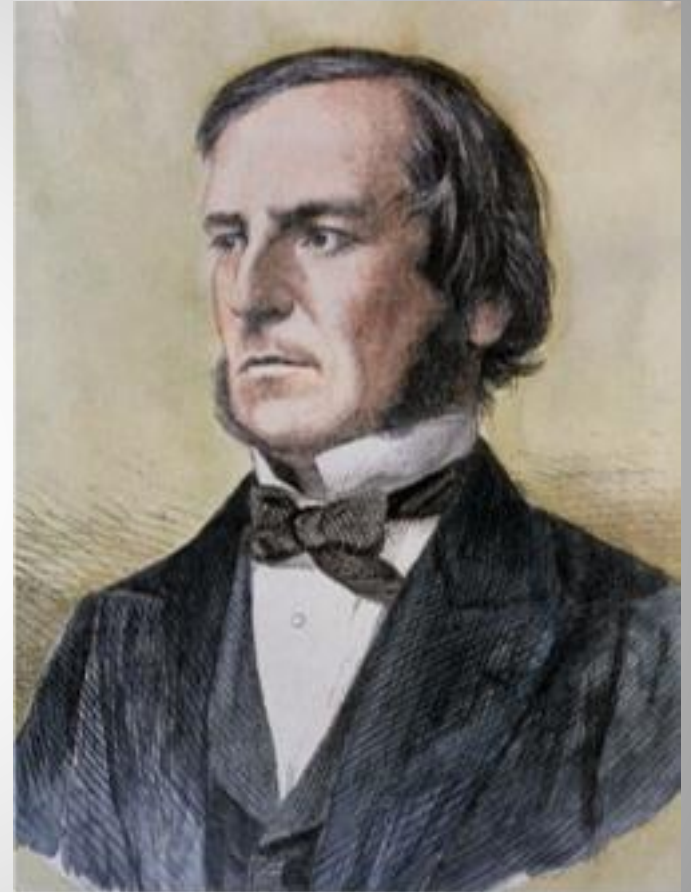
*How to get a more economical
implementation?*

Boolean Algebra

- Boolean algebra has applications in set theory, mathematical logic, digital system design.
- Algebra over
 - two elements: 0 and 1
 - two binary operations: + and \cdot
 - a complement operation: '
- The operations are defined as follows:
 - $0' = 1$ and $1' = 0$
 - $0 \cdot 0 = 0$, $0 \cdot 1 = 0$, $1 \cdot 0 = 0$, $1 \cdot 1 = 1$
 - $0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$, $1 + 1 = 1$

Boolean Logic

- George Boole (1815-1864)
 - An English mathematician, philosopher and logician
 - Mathematical Analysis of Logic (1847)
 - An Investigation of the Law of Thought (1854)
 - The inventor of Boolean logic, the basis of the modern digital computer



[Source: Wikipedia]

Operator Priority

- From high to low

- Parentheses

- NOT

- AND

- OR

- Examples

- $x \cdot y' + z$

- $(x \cdot y)' + z$

Axiomatic Approach of Boolean Algebra

- *Axiom*: a set of mathematical statement asserted to be true
- If we start with the following axioms (virtually stating the definitions of AND, OR, NOT with a set of math statements), then everything else in Boolean algebra can be derived from them
- Identity:
 - $1 \cdot x = x$
 - $0 + x = x$
- Annihilation:
 - $1 + x = 1$
 - $0 \cdot x = 0$
- Negation:
 - $0' = 1$
 - $1' = 0$

Theorems

- *Theorems* are logical consequences of the axioms
- Some useful theorems for logic simplification:

Commutative

$$x \cdot y = y \cdot x$$

$$x + y = y + x$$

Associative

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$x + (y + z) = (x + y) + z$$

Distributive

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

$$x + (y \cdot z) = (x + y) \wedge (x + z)$$

Idempotence

$$x \cdot x = x$$

$$x + x = x$$

Complementation

$$x \cdot x' = 0$$

$$x + x' = 1$$

Absorption

$$x \cdot (x + y) = x$$

$$x + (x \cdot y) = x$$

Combining

$$(x \cdot y) + (x \cdot y') = x$$

$$(x + y) \cdot (x + y') = x$$

Theorems

DeMorgan's

$$(x \cdot y)' = x' + y'$$

$$(x + y)' = x' \cdot y'$$

Involution

$$(x')' = x$$

Consensus

$$\begin{aligned} & (x \cdot y) + (x' \cdot z) + (y \cdot z) \\ &= (x \cdot y) + (x' \cdot z) \end{aligned}$$

$$\begin{aligned} & (x + y) \cdot (x' + z) \cdot (y + z) \\ &= (x + y) \cdot (x' + z) \end{aligned}$$

Example

- Prove $x \cdot y = y \cdot x$. (commutative)

Example

- Prove $x \cdot (y + z) = x \cdot y + x \cdot z$. (distributive)

Proof. We can prove it by perfect induction.

x	y	z	$y + z$	$x \cdot (y + z)$	$x \cdot y$	$x \cdot z$	$(x \cdot y) + (x \cdot z)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

It can be seen that the statement is true for all possible values of x , y , and z , hence the statement is proved.

Duality

$$1. X + 0 = X$$

$$2. X + 1 = 1$$

$$3. X + X = X$$

$$5. X + X' = 1$$

$$6. X + Y = Y + X$$

$$7. (X + Y) + Z = X + (Y + Z)$$

$$8. X(Y + Z) = XY + XZ$$

$$12. (X + Y)' = X'Y'$$

$$1D. X \cdot 1 = X$$

$$2D. X \cdot 0 = 0$$

$$3D. X \cdot X = X$$

$$5D. X \cdot X' = 0$$

$$6D. XY = YX$$

$$7D. (XY)Z = X(YZ)$$

$$8D. X + YZ = (X + Y)(X + Z)$$

$$12D. (XY)' = X' + Y'$$

- *Duality principle*: the dual of any true statement is also a true statement
- The dual is obtained by interchanging OR and AND operations, interchanging 0 and 1
- Duality holds in the axioms, and all of the Boolean algebra derived from them

Remarks

- In general, Boolean properties can be proved
 - 1) By using axioms or known theorems/Boolean properties
 - 2) By checking their validity for all possible combinations of variables
 - Simply with truth table
 - Mathematicians call this proof technique perfect induction

Example

- Prove the *DeMorgan's Theorems*

- $(x + y)' = x' y'$
- $(x y)' = x' + y'$

Proof. We can prove them by perfect induction.

x	y	$x+y$	$(x+y)'$	x'	y'	$x'y'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

It can be seen that $(x+y)' = x' y'$ is true for all possible values of x and y , hence the statement is proved.

DeMorgan Graphically

- NAND: $z = (x y)' = x' + y'$



- NOR: $z = (x + y)' = x' y'$

Example: Simplify Logic Expression using Boolean Properties

$$f(a,b,c) = (a \wedge c) \vee (a \wedge b \wedge c) \vee (\neg a \wedge b \wedge c) \vee (a \wedge b \wedge \neg c)$$

$$= (a \wedge c) \vee \underline{(a \wedge b \wedge c)} \vee (\neg a \wedge b \wedge c) \vee (a \wedge b \wedge \neg c) \vee (a \wedge b \wedge \neg c) \vee \underline{(a \wedge b \wedge c)}$$

By Idempotence ($x = x \vee x$)

$$= (a \wedge c) \vee (\neg a \wedge b \wedge c) \vee \underline{(a \wedge b \wedge c) \vee (a \wedge b \wedge \neg c) \vee (a \wedge b \wedge c)}$$

By Absorption Property
 $x \vee (x \wedge y) = x$

$$= (a \wedge c) \vee (b \wedge c) \vee (a \wedge b)$$

By Combining Property

 *Majority function*

Example: Simplify Logic Expression using Boolean Properties

$$f(a,b,c) = ac + abc + a'bc + abc'$$

$$= ac + (abc + a'bc) + (abc + abc')$$

By Idempotence ($x = x \vee x$)

$$= ac + bc + ab$$

By Combining Property



Majority function

Example

- Simplify the following Boolean expression using known Boolean properties
- $$\begin{aligned} f(x, y) &= (x \wedge (y \vee \neg x)) \vee \neg(\neg x \wedge \neg y) \\ &= x(y+x')+(x'y')' \\ &= xy+ x+y \\ &= x+y \end{aligned}$$

*How to implement arbitrary
functions using AND, OR, NOT
gate?*

Minterms and Maxterms

- *Minterms*

- minterm: an AND term consisting of all literals in their normal form or in their complement form
- For two binary vars x and y , the minterms are
 - $xy, xy', x'y, x'y'$
- n variables $\rightarrow 2^n$ minterms
- Also called *standard products*

- *Maxterms*

- maxterm: an OR term consisting of all literals in their normal form or in their complement form
- Also called *standard sums*

Minterms and Maxterms

Maxterm M_j is the complement of minterm m_j , and vice versa

Table 2.3
Minterms and Maxterms for Three Binary Variables

x	y	z	Minterms		Maxterms	
			Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Sum of Minterms

- A Boolean function can be expressed by
 - a truth table, or
 - sum of minterms that produces output 1
- e.g. $f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$
 $f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$

Table 2.4
Functions of Three Variables

x	y	z	Function f_1	Function f_2
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Product of Maxterms

- The complement of a function can be expressed by sum of minterms that produce output 0 for the original function
- e.g. If $f = m_1 + m_4 + m_7$, then

$$f' = m_0 + m_2 + m_3 + m_5 + m_6$$

- Notice that

$$f = (f')'$$

$$= m'_0 m'_2 m'_3 m'_5 m'_6$$

$$= M_0 M_2 M_3 M_5 M_6$$

- Any Boolean function can be expressed as
 - a sum of minterms, or a product of maxterms
 - both forms are *Canonical forms*

Conversion between Canonical Forms

- Example
 - $F(A,B,C) = \sum m(1,4,5,6,7)$
 $\Rightarrow F'(A,B,C) = \sum m(0,2,3)$
 \Rightarrow By DeMorgan's theorem
 $F(A,B,C) = \prod M(0,2,3)$
- Interchange the symbols \sum and \prod and list those numbers missing from the original form

Example

- $F = xy + x'z$
- $F(x, y, z) = \sum m(1, 3, 6, 7)$
- $F(x, y, z) = \prod M(0, 2, 4, 5)$

Table 2.6

Truth Table for $F = xy + x'z$

<i>x</i>	<i>y</i>	<i>z</i>	<i>F</i>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Standard Forms

- Two kinds of *Standard forms*
 - Sum of products (SOP) form
 - e.g. $f = xy + x'y + y'z$
 - Product of sums (POS) form
 - e.g. $g = x(y'+z)(x'+y+z'+w)$
 - SOP/POS form of a function is not unique e.g. f can also be expressed as $y + y'z$, or $y + z$, etc.

Sum-of-Product & Product-of-Sum Forms

- Sum-of-product (SOP) form
 - $AB + A'BC' + B'D + C$
 - $AB'C + A'C$
- Product-of-sum (POS) form
 - $(A + B)(A' + D + E)$
 - $(A + C)(B + D)(B' + C' + D')CE'$

Transforming to Canonical Form

- Any logic expression can be transformed to Canonical form by factoring about each input variable using the identity:



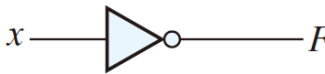
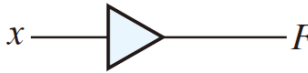
$$f(x_1, \dots, x_i, \dots, x_n) \\ = (x_i \wedge f(x_1, \dots, 1, \dots, x_n)) \vee (\overline{x_i} \wedge f(x_1, \dots, 0, \dots, x_n))$$

- E.g., $f(a,b,c) = (a \wedge c) \vee (b \wedge c) \vee (a \wedge b)$

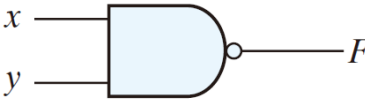
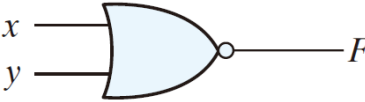

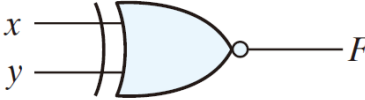
$$\begin{aligned} f(a, b, c) &= (a \wedge f(1, b, c)) \vee (a' \wedge f(0, b, c)) \\ &= (a \wedge (c \vee (b \wedge c) \vee b)) \vee (a' \wedge (b \wedge c)) \\ &= (a \wedge c) \vee (a \wedge b \wedge c) \vee (a \wedge b) \vee (a' \wedge b \wedge c) \end{aligned}$$

- Repeat with b and c (exercise)

Digital Logic Gates (1/2)

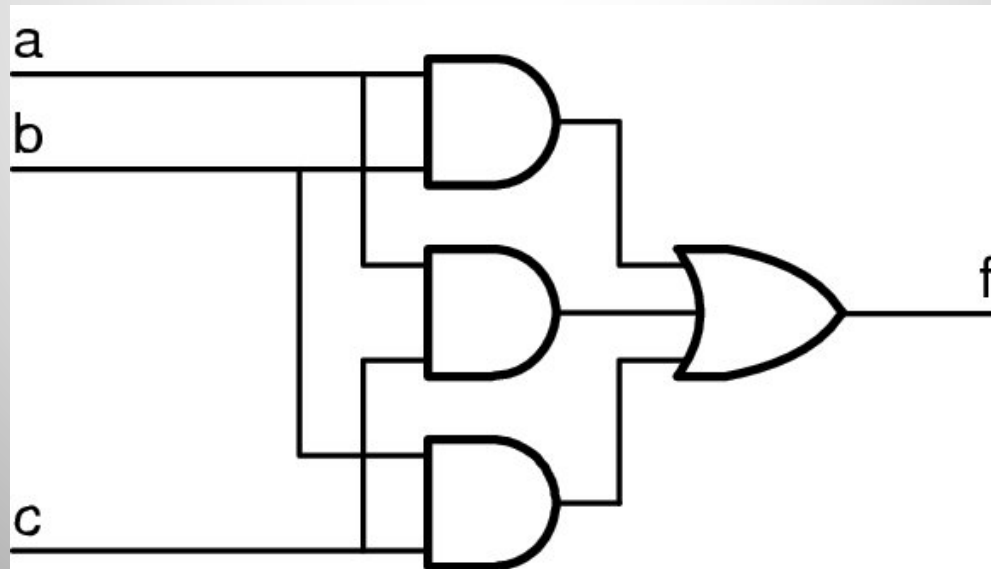
Name	Graphic symbol	Algebraic function	Truth table															
AND		$F = x \cdot y$	<table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1
x	y	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = x + y$	<table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	1
x	y	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
Inverter		$F = x'$	<table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	x	F	0	1	1	0									
x	F																	
0	1																	
1	0																	
Buffer		$F = x$	<table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table>	x	F	0	0	1	1									
x	F																	
0	0																	
1	1																	

Digital Logic Gates (2/2)

NAND	 $F = (xy)'$	<table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	1	0	1	1	1	0	1	1	1	0
x	y	F															
0	0	1															
0	1	1															
1	0	1															
1	1	0															
NOR	 $F = (x + y)'$	<table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	0
x	y	F															
0	0	1															
0	1	0															
1	0	0															
1	1	0															
Exclusive-OR (XOR)	 $F = xy' + x'y$ $= x \oplus y$	<table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	0
x	y	F															
0	0	0															
0	1	1															
1	0	1															
1	1	0															
Exclusive-NOR or equivalence	 $F = xy + x'y'$ $= (x \oplus y)'$	<table> <tr> <th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	1
x	y	F															
0	0	1															
0	1	0															
1	0	0															
1	1	1															

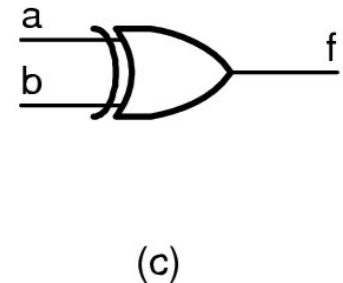
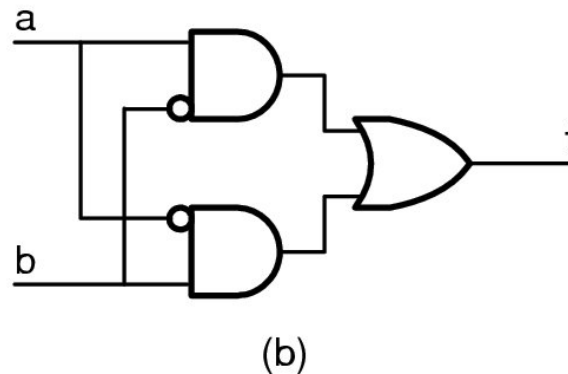
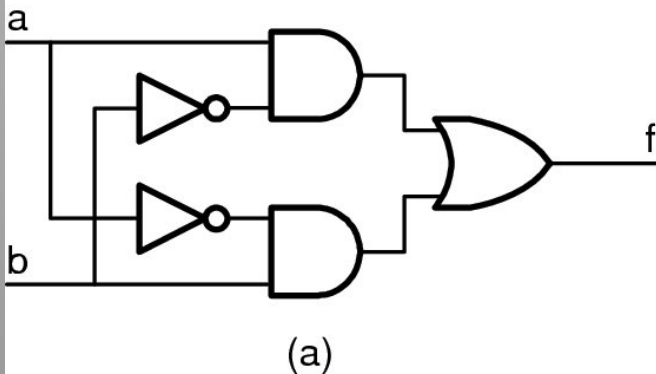
From Equations to Gates

- Logical functions can be represented by a (gate-level) **logic diagram**
 - A (gate-level) **schematic** drawing of gate symbols connected by lines
- E.g., $f = a \cdot c + b \cdot c + a \cdot b$



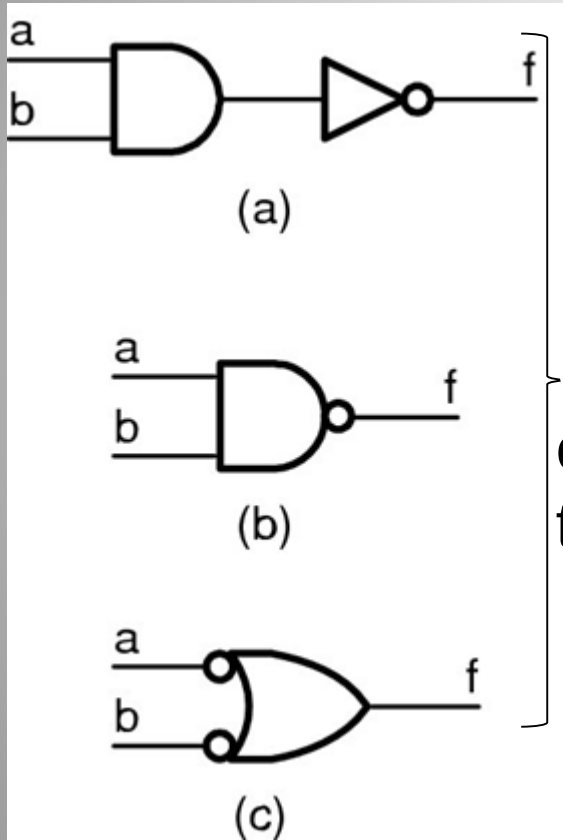
From Equations to Gates (cont)

- Using AND/OR/INV gates, we can draw logic diagram (schematic) for any Boolean expression
- E.g., exclusive-or (XOR) function
$$f = a \cdot b' + a' \cdot b$$

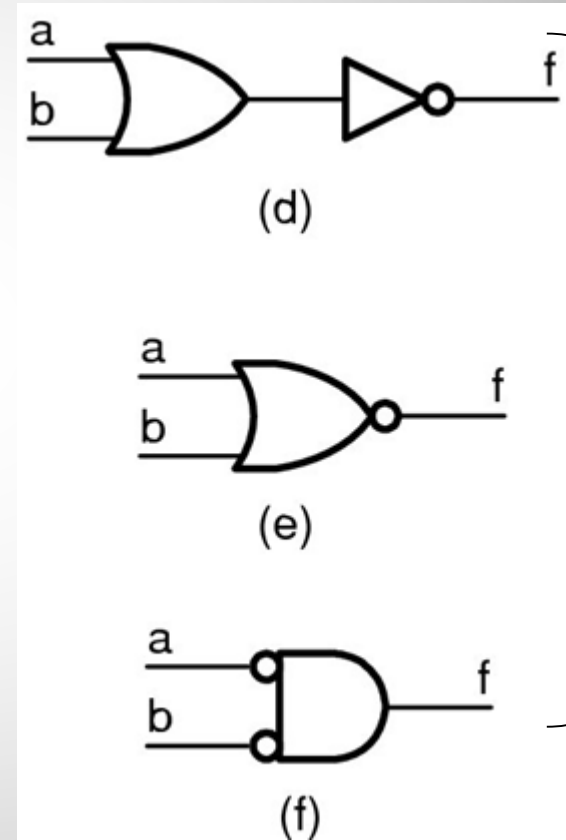


Moving Inversion Bubbles

(Review of DeMorgan's Law)

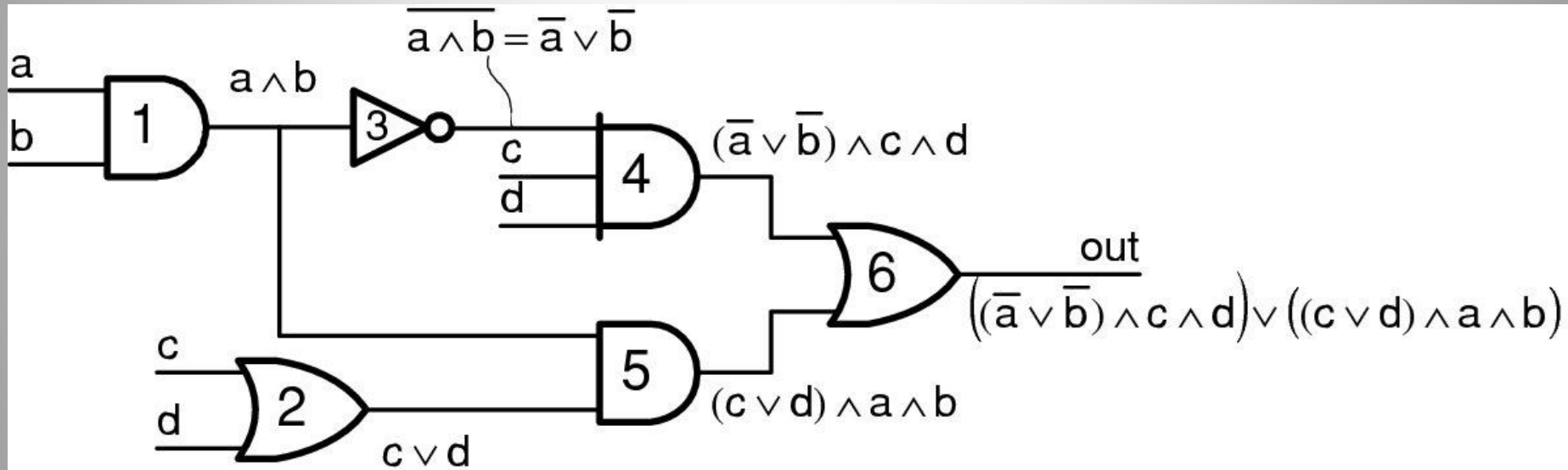


all are
equivalent
to NAND



all are
equivalent
to NOR

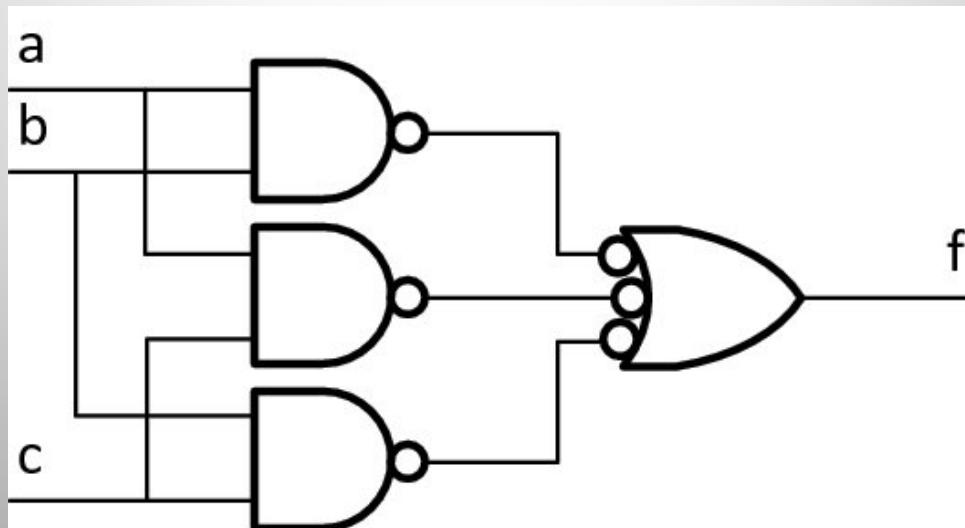
Converting from Logic Diagram to Equation



Example: From Equation to Schematic

- Draw the schematic for a three-input majority function with NAND gates only

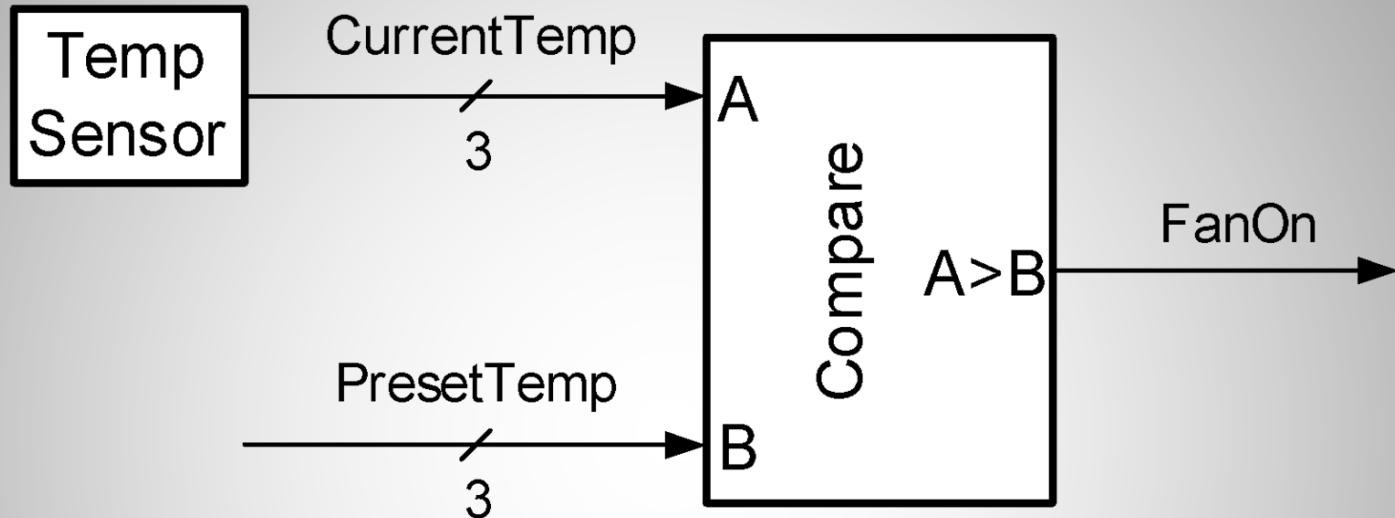
$$\begin{aligned} f(a, b, c) &= (a \wedge b) \vee (a \wedge c) \vee (b \wedge c) \\ &= \overline{\overline{(a \wedge b) \vee (a \wedge c) \vee (b \wedge c)}} \\ &= \overline{(a \wedge b)} \wedge \overline{(a \wedge c)} \wedge \overline{(b \wedge c)} \end{aligned}$$



Basic Concepts of Verilog

- A *Hardware Description Language (HDL)*
- Used as *design description* for
 - *Synthesis* – implement hardware with a network of logic gates/cells in ASIC or FPGA
 - *Simulation* – see what your hardware will do before you build it
- Basic unit is a *module*
- Modules have
 - Module declaration
 - Input and output declarations
 - Internal signal declarations
 - Logic definition
 - *assign* statements
 - *case* statements
 - Submodule instantiations

Example: Verilog for Thermostat



```
module Thermostat(presetTemp, currentTemp, fanOn) ;  
    input [2:0] presetTemp, currentTemp ; // temp inputs, 3 bits  
        each  
    output fanOn ;                        // true when current > preset  
  
    wire fanOn ;  
    assign fanOn = (currentTemp > presetTemp) ;  
endmodule
```

Example: Verilog for Thermostat

```
module Thermostat(presetTemp, currentTemp, fanOn) ;  
    input [2:0] presetTemp, currentTemp ;    // temperature inputs, 3 bits each  
    output fanOn ; // true when current > preset  
  
    wire fanOn ;  
    assign fanOn = (currentTemp > presetTemp) ;  
endmodule
```

Example: Verilog for Thermostat

● Module declaration

● I/O list

```
module Thermostat(presetTemp, currentTemp, fanOn) ;  
    input [2:0] presetTemp, currentTemp ;    // temperature inputs, 3 bits each  
    output fanOn ; // true when current > preset
```

● Declare I/Os

● 3-bit wide signals

```
    wire fanOn ;  
    assign fanOn = (currentTemp > presetTemp) ;  
endmodule
```

● An assign statement defines a signal with an equation

● A wire is a signal set with an assign statement or connected to a module

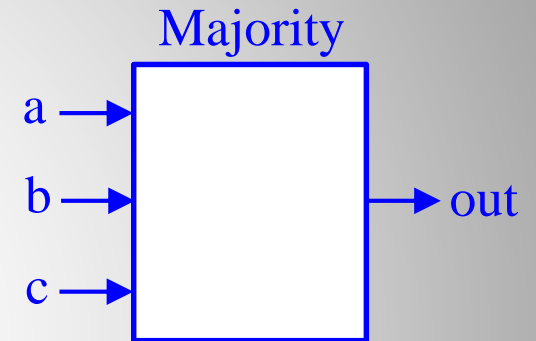
Verilog Description of Majority Function: Majority.v

● Module name

```
module Majority(a, b, c, out);  
  input a, b, c;  
  output out;
```

```
  wire out;  
  assign out = (a & b) | (a & c) | (b & c);  
endmodule
```

Block Diagram



● Can be replaced with

```
● wire out = (a & b) | (a & c) | (b & c);
```

Logic Function in Verilog

- Combinational logic in Verilog

Wire type (signal type)

```
wire out;
```

```
assign out = (a & b) | (a & c) | (b & c);
```

Signal IS NOT variable

- Alternatively,

$$f(a, b, c) = \overline{\overline{(a \wedge b)} \wedge \overline{(a \wedge c)} \wedge \overline{(b \wedge c)}}$$

```
assign out = ~(~(a & b) & ~(a & c) & ~(b & c));
```

- Less readable
- Not necessary since synthesis tool performs optimization for you

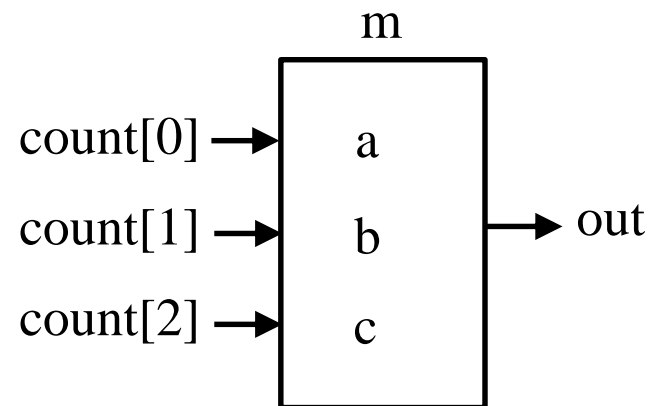
Test Stimulus (Testbench): test.v

Register type (signal type), 3-bit vector

```
module test;
    reg [2:0] count;    // input - three bit counter
    wire out;           // output of majority
    // instantiate the block
    Majority m(count[0], count[1], count[2], out);
    // generate all eight input patterns
    initial begin
        count = 3'b000;
        repeat (8) begin
            #100
            $display("in = %b, out = %b", count, out);
            count = count + 3'b001;
        end
    end
endmodule
```

Delay

test



Verilog Simulation Output

- There is 100-unit delay time between two outputs
- Simulation result

in = 000, out = 0

in = 001, out = 0

in = 010, out = 0

in = 011, out = 1

in = 100, out = 0

in = 101, out = 1

in = 110, out = 1

in = 111, out = 1

Example: Days in Month Function

```
module DaysInMonth(month, days) ;  
    input [3:0] month ; // month of the year 1 = Jan, 12 = Dec  
    output [4:0] days ; // number of days in month  
  
    reg [4:0] days ;  
  
    always @(month) begin // evaluate whenever month changes  
        case(month)  
            2: days = 5'd28 ;           // February has 28  
            4,6,9,11: days = 5'd30 ; // 30 days have September ...  
            default: days = 5'd31 ; // all the rest have 31...  
        endcase  
    end  
endmodule
```


Example: Days in Month Function

```
module DaysInMonth(month, days)
    input [3:0] month
    output [4:0] days
    reg [4:0] days ;
    always @ (month) begin
        case (month)
            1: days = 5'd28 ;
            4,6,9,11: days = 5'd30 ;
            default: days = 5'd31 ; // all the rest have 31...
        endcase
    end
endmodule
```

● Reg defines a signal set in an *always* block. It does **NOT** define a register.

● Always block evaluates each time activity list changes. In this case each time *month changes*

● Case statement selects statement depending on value of argument. Like a truth table.

● Can have multiple values per statement

● Default covers values not listed.

Verilog design style – for Synthesizable Modules

1. Combinational modules use only
 1. Assign statements
 2. Case or Casex statements (with default)
 3. If statements – only if all signals have a default assignment
 4. Instantiations of other combinational modules
2. Sequential modules use only
 1. Combinational logic
 2. Explicitly declared registers (flip-flops)
3. Do not use
 1. Loops
 2. Always blocks except for case, casex, or if
4. Do use
 1. Signal concatenation, e.g., $\{a, b\} = \{c, d\}$
 2. Signal subranges, e.g., $a[7:1] = b[6:0]$;
5. Logic is organized into small modules
 1. Leaf modules not more than 40 lines
 2. If it could be made two modules, it should be

Verilog design style – for Synthesizable Modules (cont)

6. Use lots of comments

1. Comments themselves
2. Meaningful signal names – tempHigh, not th
3. Meaningful module names – DaysInMonth, not Mod3

7. Activation lists for case statements include **ALL** inputs (or use *)

8. Constants

1. All constants are `defined if used more than once
2. Width of all constants is specified 5'd31, not 31

9. Signals

1. Buses (multi-bit signals) are numbered high to low
 - e.g., wire [31:0] bus
2. All signals should be high-true (except primary inputs and outputs)

10. Visualize the logic your Verilog will generate.

- If you can't visualize it, the result will not be pretty

Summary

- Boolean algebra
 - Rules to manipulate logic equations
- Logic diagrams
 - Representation of logic equations
- Verilog
 - Describe hardware for simulation and synthesis