國立清華大學

科目

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Definitions & terminology (ch 1.1 & 1.2)

① If y is a function of t $\xrightarrow[\text{as}]{\text{expressed}}$

Derivative of y with respect to t $\longrightarrow$

where t : independent variable

y : dependent variable

② Notations for "derivative"

③ Classification of DEs

— by "type" : ODE → one independent variable

ex :

PDE → two or more independent variables

ex :

— by "order" : check out the order of the highest derivative

ex : $y''' - y' + 6 = 0$

$y'' + 5(y')^3 - 4y = e^x$

— by "linearity" : check out

if { or power of $y, y', y'', y''', \ldots$ is more than 1 → nonlinear

otherwise → linear

ex : $(1-y)y' + 2y = e^x$, $y'' + \sin y = 0$, $y^{(4)} + y^2 = 0$

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④ Express a DE in "normal form" or "differential form"

ex:

$$4x\frac{dy}{dx}+y=x$$

"normal form": $\frac{dy}{dx}=\frac{x-y}{4x}$

"differential form":

⑤ Solution forms of DEs

explicit solution: expressed as $y=\phi(x)$

(dependent variable is expressed only in terms of independent variable)

$$=\sqrt{c-\frac{x^2}{2}}$$

$$=-\sqrt{c-\frac{x^2}{2}}$$

implicit solution: expressed as $\frac{1}{2}x^2+y^2=c$

⑥ Problems can be categorized into Initial-value problem (IVP) or boundary-value problem (BVP) depending on the conditions given.

ex: For a DE: $y''-y=0$

if given $y(1)=0$, $y'(1)=0$ $\Rightarrow$

if given $y'(0)=0$, $y'(\pi)=0$ $\Rightarrow$

⑦ A DE always has more than one solution.

ex: $y''=1 \Rightarrow y=\frac{x^2}{2}+bx+c$, where b, c are any constant.