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EE214000 Electromagnetics, Fall, 2020
Quiz #2, Open books, notes (39 points)

1. What is the phase angle of the imaginary unit $\sqrt{-1}$. (2 points)

Ans: The imaginary unit $\sqrt{-1} = j$ is on the vertical axis on the complex plane.

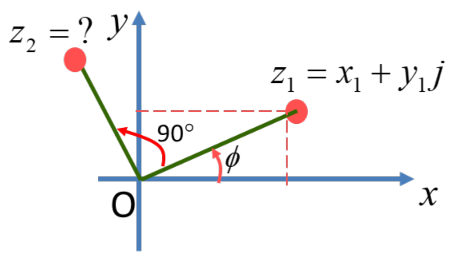
Therefore, the phase angle of $\sqrt{-1} = j$ is simply 90^0 or $\pi/2$.

2. A and B are real numbers. What is the complex conjugate of $z = \dfrac{1 + Ae^{j\phi}}{A - jB}$. (2 points)

Ans: To write the complex conjugate of z, simple replace j with – j in the expression, yielding

$$z^* = \frac{1 + Ae^{-j\phi}}{A + jB}$$

3. If you rotate the complex number $z_1 = x_1 + jy_1$ on the polar-coordinate plane by +90°, what is the resulting complex number z_2? (5 points)



Ans: $z_1 = x_1 + y_1 j = r_1 e^{j\phi} \Rightarrow z_2 = r_1 e^{j(\phi + 90^o)} = r_1 \cos(\phi + 90^o) + jr_1 \sin(\phi + 90^o)$

$\Rightarrow$ $z_2 = -r_1 \sin\phi + jr_1 \cos\phi = -y + jx$

4. Express z = 4+3j in the polar form (3 points) and mark it (2 points) and its complex conjugate (2 points) on the polar coordinate system.

Ans: $z = 4 + 3j = \sqrt{4^2 + 3^2}\, e^{j\tan^{-1}(3/4)} = 5e^{j37^\circ}$. Its complex conjugate of is

$z^* = 4 - 3j = 5e^{-j37^\circ}$. The following diagram shows the locations of z and z*.

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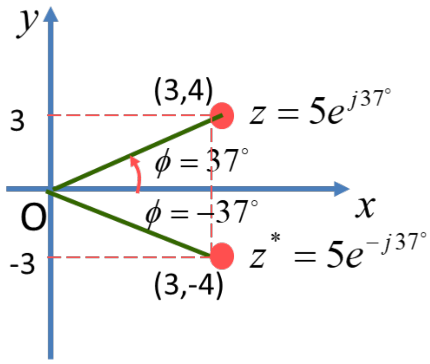

5. Calculate the division $z_3 = z_1 / z_2$ and express the result in polar form, z1 = 1 + j and z2 = 4+ 3j. (5 points)

Ans: $z_3 = z_1 / z_2 = \dfrac{1+j}{4+3j} = \dfrac{\sqrt{2}e^{j45^\circ}}{5e^{j37^0}} = 0.28e^{j8^\circ}$.

6. For a harmonic wave expressed as $A(z,t) = A_0\cos(\omega t - kz + \phi)$, what is the phasor of this wave? (3 points)

Ans: $A(z,t) = A_0\cos(\omega t - kz + \phi) = \mathrm{Re}(A_0 e^{-jkz+j\phi} \times e^{j\omega t}) = \mathrm{Re}(\hat{A}(z) \times e^{j\omega t})$. Therefore, the phasor of that expression is

$$\hat{A}(z)\times = A_0 e^{-jkz+j\phi}.$$

7. For a time-harmonic wave function expressed by A, what is the phasor expression of the wave equation $\nabla^2 A - \dfrac{1}{c^2}.\dfrac{\partial^2 A}{\partial t^2} = 0$, where c is a constant? (5 points)

Ans: Replace the time derivative with jω and A with $\hat{A}$ in the wave equation. The result is $\nabla^2 A - \dfrac{1}{c^2}.\dfrac{\partial^2 A}{\partial t^2} = 0 \Rightarrow \nabla^2 \hat{A} - (j\omega)^2 \dfrac{1}{c^2}.\hat{A} = 0 \Rightarrow \nabla^2 \hat{A} + \dfrac{\omega^2}{c^2}.\hat{A} = 0$

8. For the RC circuit shown below, if the driving voltage is a sinusoidal input with a frequency of 60 Hz, given by

$$\tilde{v}_s(t) = 100\cos(2\pi \times 60t + \pi / 6) \text{ volts}$$

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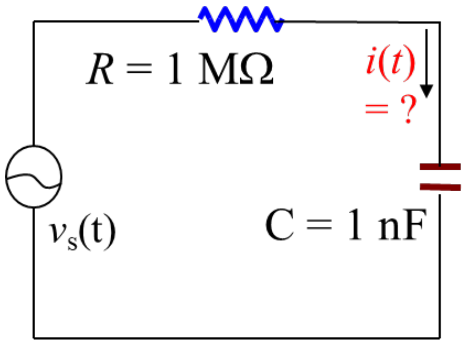
what is the current in the circuit? (10 points)



Ans: Use phasor notions to write

$$\hat{I} = \frac{\hat{V}_s}{R + \dfrac{1}{j\omega C}} = \frac{100 e^{j\pi/6}}{10^6 + \dfrac{1}{j2\pi \times 60 \times 10^{-9}}} = 3.5 \times 10^{-5} e^{j99.3^\circ}\ \text{A}$$

Restore the current back to a real value by using

$$\tilde{i}(t) = \text{Re}[\hat{I} e^{j\omega t}] = 3.5 \times 10^{-5} \cos(2\pi \times 60 t + 99.3^\circ)\ \text{A}$$