Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

Your name: ________________ ID: __________________ Nov. 29th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #12-1, Open books, notes (22 points), due 11 pm, Wednesday, Dec. 2nd, 2020

Late submission won't be accepted!

1. On Slide 11 of Lec. 12, the condition to have no charge accumulation at two lossy dielectric is

$$\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}.$$

Give a **thorough explanation** on the consequence of such a condition. (5 points)

Ans: From the charging and discharging of an RC circuit, the charging time constant is the same as the discharging time constant. So, imagine that the charges flow from Dielectric 1 to Dielectric 2, accumulating at a rate of

$$\tau_{r1} = \frac{\varepsilon_1}{\sigma_1}.$$

However, the excess charge accumulated on the interface is relaxed into the 2nd dielectric at a rate of

$$\tau_{r2} = \frac{\varepsilon_2}{\sigma_2}.$$

If $\tau_{r1} = \tau_{r2}$, there won't be accumulation of the charges at the interface. Therefore, the condition to have not charges at the interface is $\frac{\sigma_1}{\sigma_2} = \frac{\varepsilon_1}{\varepsilon_2}$.

2. Previously, we asserted that there's no electric field in a good conductor. Explain why it takes time for a good conductor to reach the zero-field condition upon some excitation. (3 points) Assume the dielectric constant of silver is equal to the vacuum value. Estimate the time constant, within which the electric in silver is not zero, when an external electric field excite a piece of silver. (3 points)

Ans: Upon excitation, it takes time to move charges in a conductor to offset the external electric field. For a good conductor such as copper, the time constant is about 10^{-19} sec., as given in the lecture note. For Silver, the conductivity is 6.17×10^7 S/m.

The time constant $\tau_r = \frac{\varepsilon_0}{\sigma} = 1.43 \times 10^{-19}$ sec.

Prof. Yen-Chieh Huang
Dept of Electrical Engineering
National Tsing-Hua University
office: Delta 856
ext: 62340
email:ychuang@ee.nthu.edu.tw

---

3. An electromagnetic wave has an oscillating electric field. In most electromagnetics textbooks, you are taught that an electromagnetic wave cannot penetrate a metal, because the electric field in a conductor is zero. Would you be able to explain why a hard *x*-ray or gamma ray (wavelength ~ 1 Angstrom or frequency ~ 3×10^{18} Hz) can penetrate metal? (5 points)

Ans: The oscillation period of a one Angstrom-wavelength electromagnetic wave is ~$1/3\times10^{18}$ ~ 3×10^{-19} sec, which is comparable to the charge relaxation time in a good conductor. This means that, with such a high-frequency excitation, the electric field in a metal cannot be zero, because, in each oscillation cycle, the time is not enough for the charges to move and compensate the incident electric field. Therefore, the field of a hard x-ray or gamma-ray wave can "enter" a metal.

4. A leaky capacitor consists of two parallel plate electrodes as shown below. The two electrodes are separated by a distance d and biased with a voltage of V0, each having an area of S. The material sandwiched between the two electrodes has a permittivity ε and conductivity σ. Ignore the fringe field near the electrode edges. (1) What is the capacitance of this capacitor? (2 points) (2) What is the resistance between the two electrodes? (2 points) (3) What is the electric power dissipated in this capacitor? (2 points)

Ans: (1) The capacitance is $C = \frac{Q}{V} = \frac{\vec{D}\cdot\vec{S}}{dE} = \frac{\varepsilon ES}{dE} = \frac{\varepsilon S}{d}$

(2) The resistance can be calculated from $RC = \frac{\varepsilon}{\sigma}$, given by $R = \frac{d}{\sigma S}$

(3) The electric power dissipated through the leaky capacitor is

$$P = IV = \frac{V_0^2}{R} = \frac{V_0^2}{d}\sigma S\ .$$