

邏輯設計 HW5 Solution

1. (16%)

(a.) $0101 - 0110$

$$\begin{aligned} &0101 - 0110 \\ &= 0101 + 1010 \\ &= 1111(-1) \end{aligned}$$

unsigned representation:-0001

2's complement representation:1111

Decimal: $5 - 6 = -1$

(b.) $10110 - 1100$

$$\begin{aligned} &010110 - 001100 \\ &= 010110 + 110100 \\ &= 001010 \end{aligned}$$

Decimal: $22 - 12 = 10$

unsigned representation:001010

2's complement representation:001010

(c.) $1011110 - 1111110$

$$\begin{aligned} &1011110 - 1111110 \\ &= 1011110 + 10000010 \\ &= 1100000 \end{aligned}$$

Decimal: $94 - 126 = -32$

unsigned representation:-0100000

2's complement representation:1100000

(d.) $101010 - 101$

$$101010 - 000101$$

$$= 101010 + 111011$$

$$= 100101$$

Decimal: $42 - 5 = 37$

unsigned representation: 100101

2's complement representation: 0100101

2. (16%)

(a.) (b.) (c.) (d.) all no overflow

(a.) $0101 - 0110$

$$0101 - 0110$$

$$= 0101 + 1010$$

$$= \underline{1111}$$

Decimal: $5 - 6 = -1$

(b.) $10110 - 1100$

$$10110 - 1100$$

$$= 10110 + 00100$$

$$= \underline{11010}$$

Decimal: $-10 - (-4) = -6$

(c.) $1011110 - 1111110$

$$1011110 - 1111110$$

$$= 1011110 + 0000010$$

$$= \underline{1100000}$$

Decimal: $-34 - (-2) = -32$

(d.) $101010 - 101$

$$101010 - 111101$$

$$= 101010 + 000011$$

$$= \underline{101101}$$

Decimal: $-22 - (-3) = -19$

3.(16%)

(a.) (b.) (c.) (d.) all no overflow

(a.) 0101 - 0110

Decimal: $5 - 6 = -1$

轉 2's complement

0101 - 0110

= 0101 + 1010

= 1111

sign-magnitude representation:1001

2's complementation representation:1111

(b.) 10110 - 1100

Decimal: $-6 - (-4) = -2$

11001 - 11100

= 11010 + 00100

= 11110

sign-magnitude representation:10010

2's complementation representation:11110

(c.) 1011110 - 1111110

Decimal: $-30 - (-62) = 32$

1100010 - 1000010

= 1100010 + 0111110

= 0100000

sign-magnitude representation:0100000

2's complementation representation:0100000

(d.) 101010 - 101

Decimal: $-10 - (-1) = -9$

$110110 - 111111$
 $= 110110 + 000001$
 $= 110111$

sign-magnitude representation: 101001

2's complementation representation: 110111

4.(18%)

P.S. 此答案假設輸入 1000 為無效輸入；此題真值表若輸入 1000 輸出 1000 亦算對，即設計輸出為 unsigned number。

Sol 1

先畫出真值表

z	z	z	z	Z	Z	Z	Z
3	2	1	0	3	2	1	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	1
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	1
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	1
1	0	0	0	x	x	x	x
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

由真值表畫出 K-map，並求出 logic function。

$$Z_3 = 0$$

$$Z_2 = (z_3'z_2 + z_3z_2' + z_2z_1'z_0') \text{ or } (z_3'z_2 + z_3z_2' + z_3z_1'z_0')$$

$z_1z_0 \backslash z_3z_2$	00	01	11	10
00		1	1	x
01		1		1
11		1		1
10		1		1

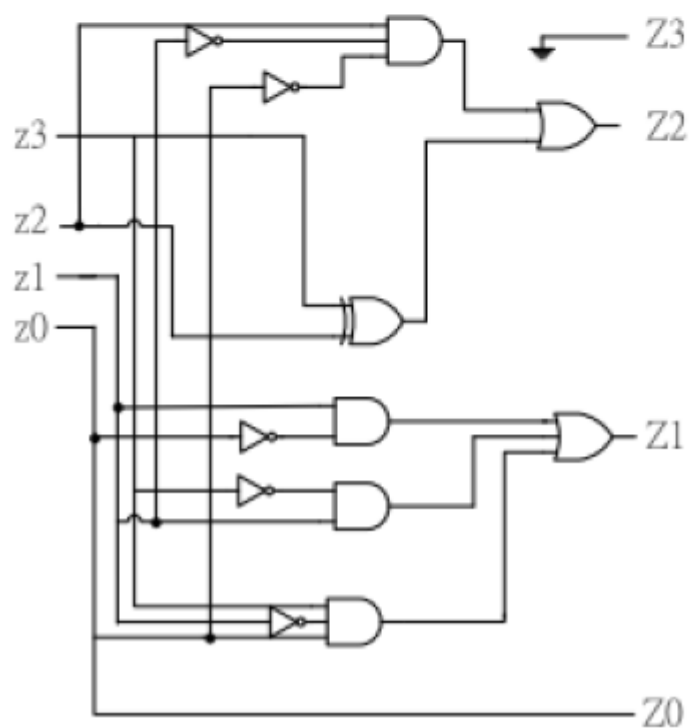
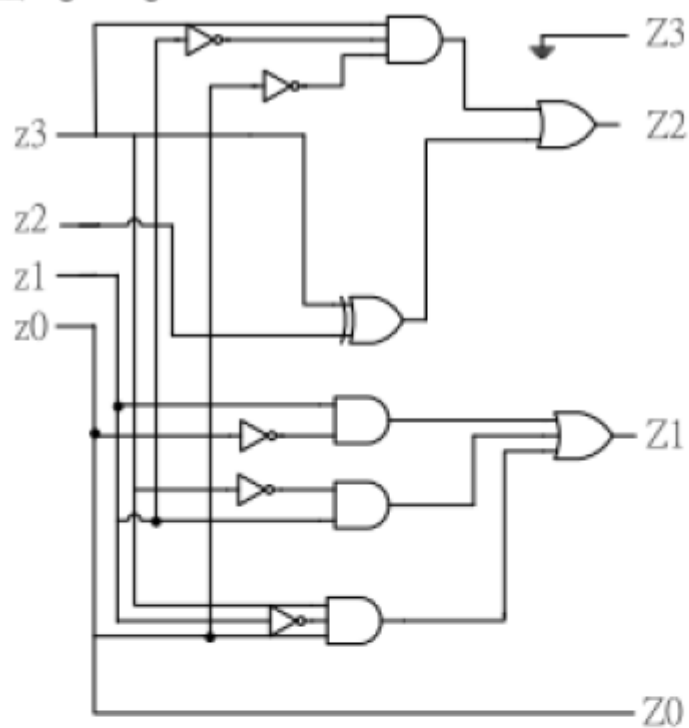
$$Z_1 = z_1z_0' + z_3'z_1 + z_3z_1'z_0$$

$z_1z_0 \backslash z_3z_2$	00	01	11	10
00				x
01			1	1
11	1	1		
10	1	1	1	1

$$Z_0 = z_0$$

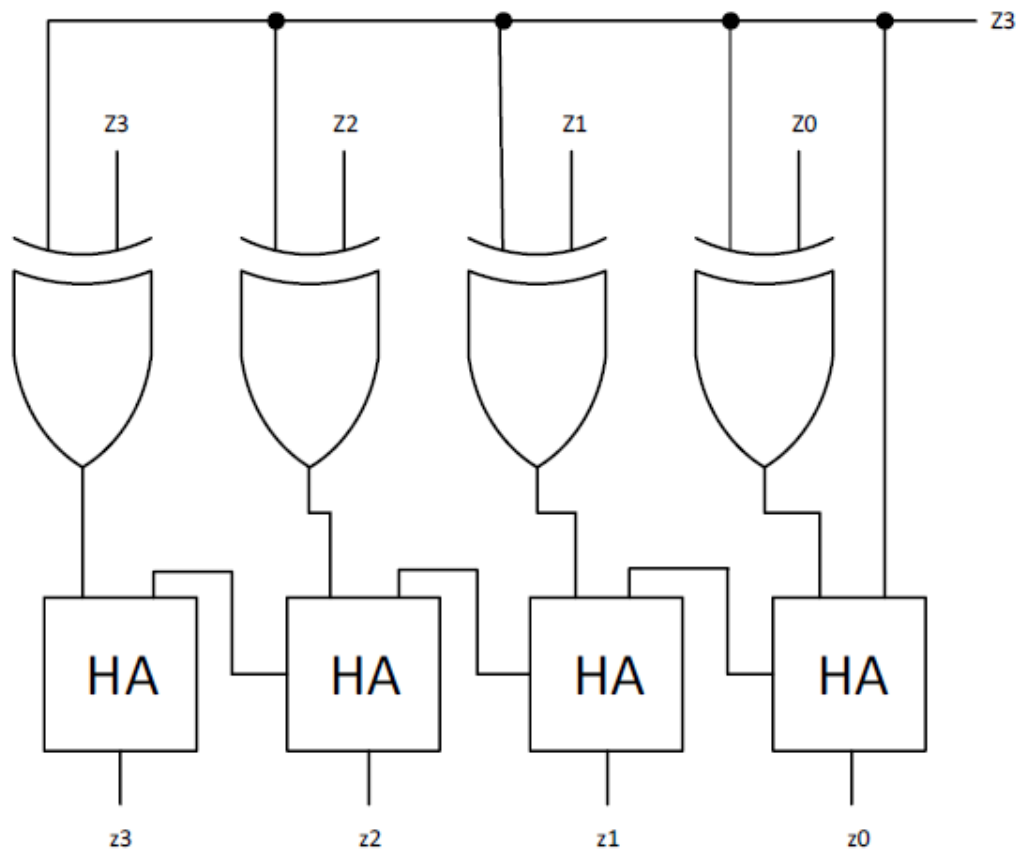
$z_1z_0 \backslash z_3z_2$	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

畫出 logic diagram

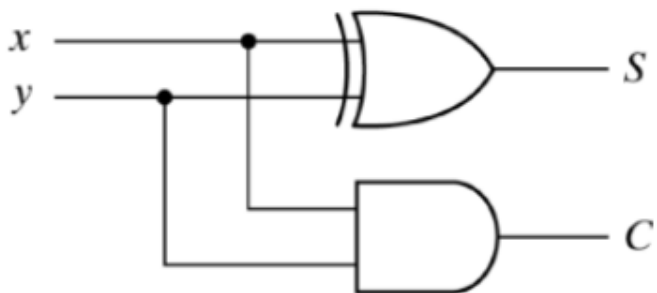


sol2.

$$Z = |z|$$



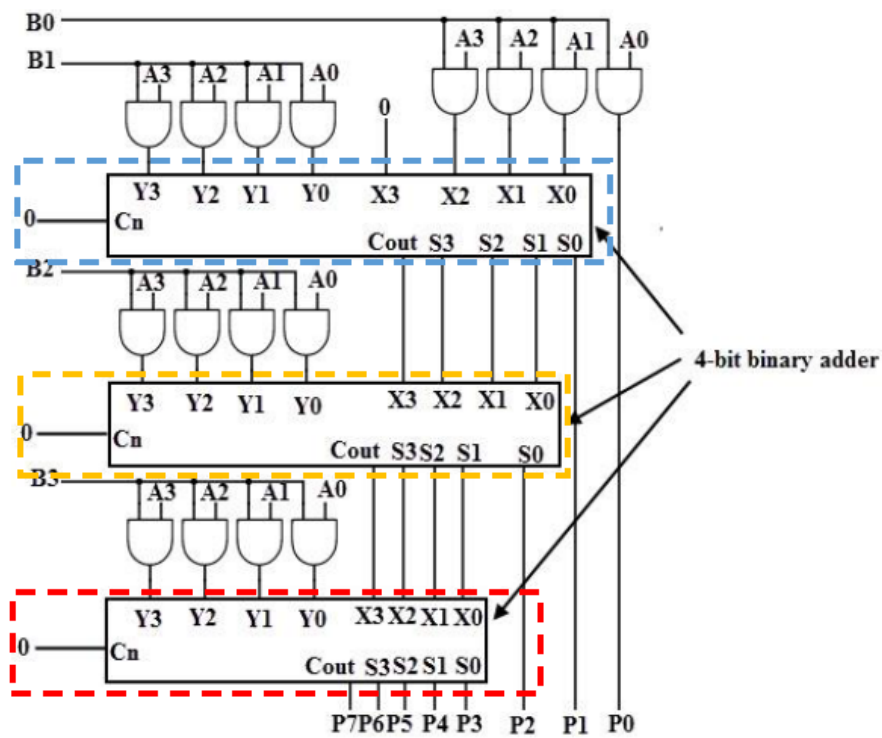
Half adder



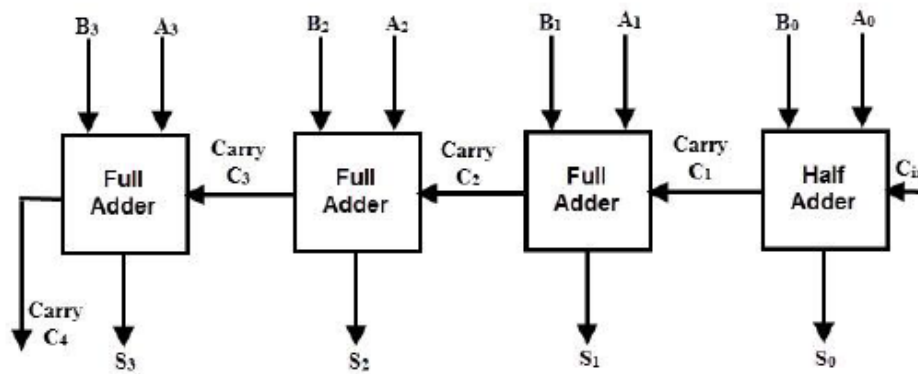
5.(18%)

4-bit x 4-bit multiplier (unsigned)

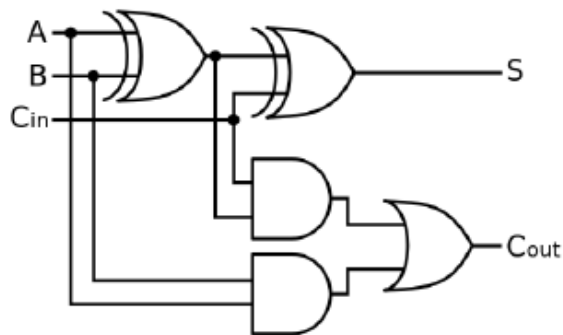
$$\begin{array}{r}
 \begin{array}{cccc}
 & A3 & A2 & A1 & A0 \\
 \times & B3 & B2 & B1 & B0 \\
 \hline
 & A3B0 & A2B0 & A1B0 & A0B0 \\
 & A3B1 & A2B1 & A1B1 & A0B1 \\
 & A3B2 & A2B2 & A1B2 & A0B2 \\
 + & A3B3 & A2B3 & A1B3 & A0B3 \\
 \hline
 P7 & P6 & P5 & P4 & P3 & P2 & P1 & P0
 \end{array}
 \end{array}$$



4-bit binary adder



Full adder



6.(16%)

