(i) Let 
$$u^{(\infty)}(x) = Ax + B$$

$$\begin{cases} u^{(\infty)}(0) = B = 0 \\ u^{(\infty)}(1) = A1 = 0 \end{cases}$$

$$u^{(\infty)}(1) = A1 = 0$$

$$u^{(\infty)}(1) = A1 = 0$$
(ii) Assume  $u(x,t) = X(x)T(t)$ 

(11) Assume 
$$U(x,t) = X(x)T(t)$$

$$\frac{X''}{X} = \frac{1}{\sqrt{x}} \frac{T'}{x} = -K^2$$

$$= \begin{cases} X(x) = C \cos k_0 x + D \sin k x \\ T(t) = E e^{-\alpha^2 k^2 t} \end{cases} = +5$$

Using boundary condition

$$=) U(x,t) = \int_{n=1}^{\infty} A_n \sin k_n x e^{-x^2 k_n^2 t} k_n = \frac{m}{2}$$

$$A_n = \frac{2}{2} \int_0^{\infty} f(x) \sin \frac{m}{2} x dx \qquad (= +10)$$
I.L.

(河道)速度。 双复发的上"三流》 剩慢 或写一身超敏, 多較慢

對一個 +2 全對 +5

(iv) 爾安超越 
$$\Rightarrow$$
 steady-stric 為厚麵能均勻分佈 
$$U^{(\infty)}(x) = \frac{20}{2} \times \frac{1}{2} = 10$$

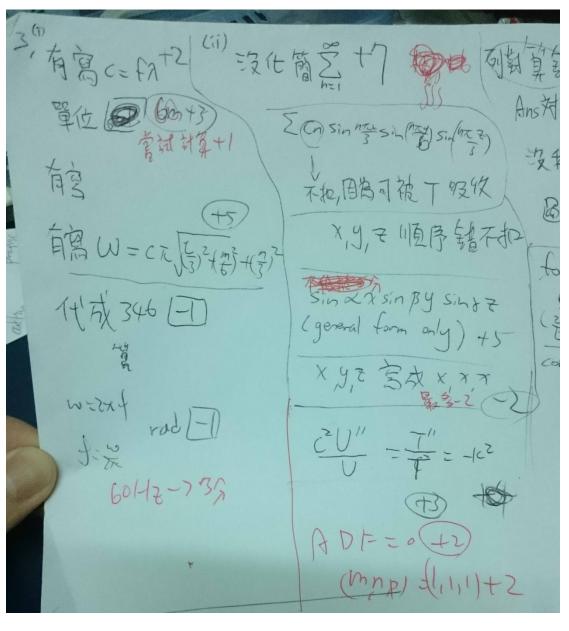
回 富岛福雯的 constant 回宫在10,包

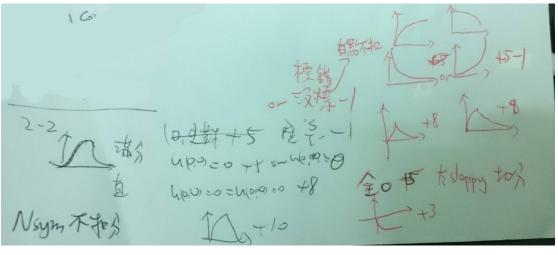
Let 
$$u^{(P)}(x,0) = S(x)$$
  
 $\Rightarrow u^{(P)}(x,t) = 1$   
 $\Rightarrow u^{(P)}(x,t) = 1$ 

then, 
$$u(x,t) = f(x) \leftrightarrow u^{(p)}(x,t)$$
 (through x)
$$= \int_{-\infty}^{\infty} f(\tau) u^{(p)}(x-\tau, +) d\tau$$

$$u(v,t) = \int_{-\infty}^{\infty} f(v) u^{(p)}(-\tau, t) d\tau$$

$$convolution (A7)$$





4. (i) We can expand a well-behaved function to an orthogonal and complete basis. 
$$sin(n_{L}^{T}x)$$
 is a complete basis, we can write  $g(x,t) = \sum_{n=1}^{\infty} G_{n}(t) sin(n_{L}^{T}x)$ 

(ii) 
$$\begin{cases} y(x,t) = \sum_{n=1}^{\infty} b_n(t) \sin(n\frac{\pi}{L}x) \\ c\frac{\partial y}{\partial x^2} = \frac{d^{\frac{1}{2}}}{\partial t^2} + a\frac{dy}{dt} - g(x,t) \end{cases}$$

$$\Rightarrow -C^{2} \sum_{h=1}^{\infty} \left(\frac{h^{\pi}}{L}\right)^{2} b_{n}(t) \sin \left(n \frac{\pi}{L} \right)$$

$$= \sum_{h=1}^{\infty} b_{n}''(t) \sin \left(n \frac{\pi}{L} \right) + \sum_{h=1}^{\infty} a b_{n}''(t) \sin \left(n \frac{\pi}{L} \right)$$

=> Compare the coefficient of 
$$sin(n = x)$$

$$c^{2} \left(\frac{n\pi}{T}\right)^{2} b_{n}(t) + b_{n}''(t) + a b_{n}'(t) = G_{n}(t)$$

(iii) 
$$\begin{cases} y(x,0) = 0 \implies y(x,0) = \sum_{h=1}^{\infty} b_h(0) \sin(h \frac{\pi}{L} x) = 0 \\ y_{\epsilon}(x,0) = 0 \implies y_{\epsilon}(x,0) = \sum_{h=1}^{\infty} b_h'(0) \sin(h \frac{\pi}{L} x) = 0 \end{cases}$$

From (ii)'s solution

$$b_n''(t) + a b_n'(t) + a b_n$$

where  $W_n = \frac{Cn\pi}{V}$ 

有智生形式

+2~+5

in general

Therefore, we just need to solve steady-state solution. Try bolt) = A e int and Gott = 90 cosut = 90 Re[eint] -Aw2 eint + a Aiweint + wn Aeint = 9, eint  $A\left(-\omega^{2}+i\alpha\omega+\omega_{n}^{2}\right)=g.$ 9. (wn'-w')+iaw We can express A in exponential notation IA/eip c = 20 [aplace equation N(Wn2-W2)2 + AW2 - 1 - 12/24 + 25 (Aplace 5 %) by ph tan = = \_ +5 兄番出 セファイ Then we have 7 /2 bn(+) = Re[lAleide int] 给为外群 如如常 cos (wt-4) V (m2-m2) 2 + a2m2 8/2 where  $tan \phi = \frac{aw}{w_n^2 - w}$ 

Part of the same

YEAR

(IV) Because it has "aye" torm, waves will decay. The shape will decrease exponentially and it con't expressed by two waves reflection in the walls that is 1 F(x+cu) + G(x-ct) W1/4 (" 1) - " W ) We can express A confer to Non the Yes 20 見まな センフェ Then we have 7 Fuit of 1 1 | A | = (+) , d 海局外 - (05 (wt-4) J. W. + 1 (" W-1, W.) Where tong = Quy 0=(0) d = (+) d sive - 5 => (05(-4) = Sin (in (wt-4)) (05 \$ = 4 sin \$ p.