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Your name: ________________ ID: __________________ Oct. 19th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #1-1, Open books, notes (21 points), due 11 pm, Wednesday, Oct. 21st, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. When you design a capacitor, what are the key parameters to increase its capacitance? (6 points)

Ans: From the formula of a parallel-plate capacitor,

$$C = \frac{\varepsilon S}{d}$$

one could in general increase the capacitance of a capacitor by increasing the electrode areas, the permittivity of the dielectric between the electrodes, and decreasing the separation of the electrodes.

2. In Sec. 7.3, for a cylindrical capacitor of length L, we derive a capacitance given by

$$C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln(b/a)}.$$

Show that, when the radii a and b are big, the expression converges to

$$C = \frac{\varepsilon S}{d}$$

with $d = b - a$. (5 points)

Ans: Write $\frac{b}{a} = 1 + \frac{b-a}{a}$, with $\frac{b-a}{a} << 1$, when a and b are big. Use Taylor's expansion to get

$$\ln\frac{b}{a} = \ln(1 + \frac{b-a}{a}) \sim \frac{b-a}{a}.$$

Substitute it into $C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln(b/a)}$ to obtain

$$C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln(b/a)} \sim \frac{2\pi\varepsilon L}{(b-a)/a} = \frac{2\pi a L\varepsilon}{b-a} = \frac{S\varepsilon}{d},$$

where $b - a = d$ and $S = 2\pi aL$.

3. In Sec. 7.4, we derive the expression

$$C \equiv \frac{Q}{V} = \frac{4\pi\varepsilon}{(1/a - 1/b)}$$

for the capacitance of a spherical capacitor. Argue that when radii a and b are big, the expression also converge to

$$C = \frac{\varepsilon S}{d}.$$ (4 points)

Ans: Continue to write $C \equiv \frac{Q}{V} = \frac{4\pi\varepsilon}{(1/a - 1/b)} \rightarrow C \equiv \frac{Q}{V} = \frac{4\pi ab\varepsilon}{b-a}$. When a and b are big, $b - a = d$ and the two spherical areas $4\pi a^2, 4\pi b^2$ converge to a geometric mean $S = 4\pi ab$. Therefore,

$$C \equiv \frac{Q}{V} = \frac{4\pi ab\varepsilon}{b-a} \rightarrow \frac{S\varepsilon}{d}.$$

4. If you have a few capacitors in your hands and you want to connect them together to have a high capacitance for your circuit, would you choose serial or parallel connections for your capacitors? (3 points)

Ans: To solve this problem, one could of course prove Cp > Cs from the following two formulas for serial and parallel capacitors

$$\frac{1}{C_{sr}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots + \frac{1}{C_n} \quad \text{and} \quad C_p = C_1 + C_2 + C_3 \ldots + C_n.$$

However, from the circuit diagrams shown in Sec. 7.4, one can already see that the parallel capacitors store more charges from an increased area. Therefore, to increase the capacitance, parallel connection is the choice.

5. For a high-speed circuit containing R and C, if you would like to have a signal bandwidth > 1 GHz, what is the requirement on the RC time constant of the circuit? (3 points)

Ans: Consider a sinusoidal signal of 1 GHz in the circuit. Since the charging and discharging time in the circuit has to be less than 1/(1 GHz) ~ 1 ns to support the 1 GHz signal, the RC time constant of the circuit has to be less than ~1 ns.