Quantum Physics I Midterm Exam

2023/11/10 8:00~9:50

1. The wave function of a particle is given by

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$$\psi(x) = \begin{cases} Ae^{\alpha x}, & x < 0 \\ Be^{-\alpha x}, & x > 0 \end{cases},$$

where A, B, and α are positive real constants.

(a) Calculate A and B in terms of α so that $\psi(x)$ is normalized. (5 pts.)
(b) Where is the particle most likely to be found? (5 pts.)
(c) What is the probability of finding the particle between $x = 0$ and $1/\alpha$? (5 pts.)
(d) What is the expectation value of the particle's position x? (5 pts.)
(e) What is the expectation value of the particle's momentum p? (5 pts.)
(f) Calculate the particle's wave function in the momentum space $\phi(p)$. (5 pts.)

2. A particle in the harmonic potential $\frac{1}{2}m\omega^2x^2$ starts out in the state

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$$\Psi(x,0) = A[\psi_1(x) + 2\psi_2(x) + 2\psi_3(x)],$$

where $\psi_n(x)$ is the nth excited state of the quantum oscillator.

(a) Calculate A so that $\Psi(x,0)$ is normalized. (5 pts.)
(b) If you measure the particle's energy, what values might you get and with what probabilities? (5 pts.)
(c) Suppose the energy is measured (at time t_0) to be the highest value in (b). What is the expectation value of x after the energy measurement (for time $t > t_0$)? (5 pts.)
(d) What is the probability density of finding the particle at position x and time $t > t_0$ after the energy measurement in (c)? (5 pts.)

$$\hat{H} = \left(\frac{-\hbar^2}{2m}\left(\frac{\partial}{\partial x}\right)^2 + V\right)$$

3. Consider the potential

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$$ik\left(\frac{ik-l}{ik+l}\right) - l\left(\frac{2ik}{ik+l}\right)$$

$$V(x) = \begin{cases} 0, & x \le 0 \\ V_0, & x > 0 \end{cases},$$

$$= \frac{-k^2 - ikl - 2ik}{ik+l} \overset{?}{=} ik.$$

where V_0 and a are positive real numbers.

(a) Calculate the transmission coefficient of a particle with energy $E < V_0$. (10 pts.)
(b) Repeat (a) for $E > V_0$. (10 pts.)

$$ik(ik+l) = -k^2 + ikl.$$

4. The Hamiltonian for a certain three-level system is represented by the matrix

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$$\hat{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix},$$

where a, b, and c are real numbers.

(a) If you measure the energy, what are the possible values? (10 pts.)

(b) If the system starts out in the state

$$|S(0)\rangle = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix},$$

$$// \quad f_1\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} + f_2\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} + f_3\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(f_1+f_2) = d_1$$

$$\frac{1}{\sqrt{2}}(f_1-f_2) = d_2 \qquad \sqrt{2} f_1 = d_1 + d_2$$

what is $|S(t)\rangle$? What are the probabilities of measuring the energy at time t to be the values in (a)? (15 pts.)

5. Consider an experiment in which the position of an electron is measured by viewing the electron with a microscope while illuminating it. Use such a thought experiment to estimate the ultimate limit (minimal value) of $\Delta p_x \Delta x$ set by the uncertainty principle. (10 pts.)

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$$\frac{h}{\lambda}$$

$$\Delta p_x = |\vec{p}_h'| \sin\theta \sim |p_h'| \theta$$

$e^- \to mv$

$$\Delta x \propto |\vec{p}_h'| \cos\theta \sim |p_h'|$$

$$\Delta x \sim \frac{p_e'}{m_e} \cdot \frac{d}{p_h' \cos\theta}$$

photon