邏輯設計HW1_solution

備註1:第九題的(b)可以不用化簡，但之後作業的布林都要化簡。
備註2:如果有要算小數點，最多算到小數點後第四位就好。

1.24%

Decimal	Binary	Octal	Hexadecimal
519.67	10_0000_0111.1010	1007.5	207.A
$(77.375)_{10}$	**1001101.011**	$(115.3)_8$	$(4D.6)_{16}$
60.375	11_1100.011	**74.3**	3C.6
224.9375	1110_0000.1111	340.74	**E0.F**

(a) 519.67 轉成二進位：

- 整數部分：
  $519 = 512 + 7 = 512 + (4 + 3) = 512 + 4 + (2 + 1) = 2^9 + 2^2 + 2^1 + 2^0$
  小數點前第 1 , 2, 3, 10 位取1, 其他取0
  整數部分結果：10 0000 0111
- 小數部分：
  0.67 x 2 = 1.34 -> 小數點後第 1 位取1
  0.34 x 2 = 0.68 -> 小數點後第 2 位取0
  0.68 x 2 = 1.36 -> 小數點後第 3 位取1
  0.36 x 2 = 0.72 -> 小數點後第 4 位取0
  小數部分結果：0.1010

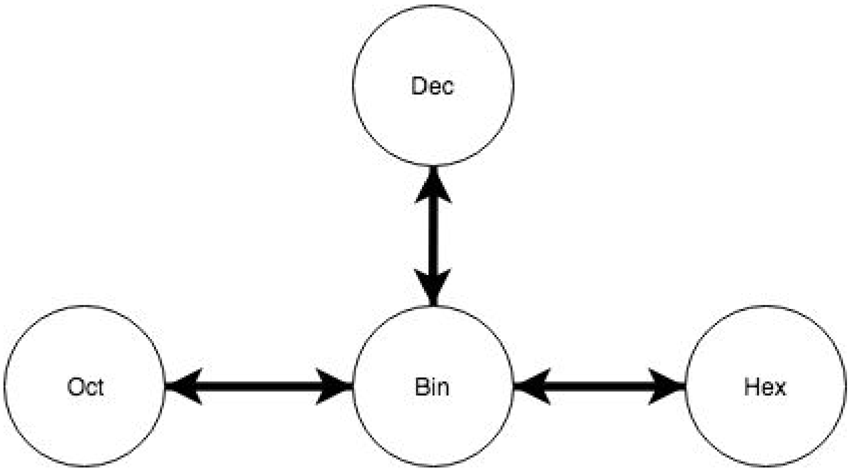
得最後結果$(519.67)_{10}$ = 10_0000_0111.1010

有了二進位數字之後即可用觀察的找出八進位、十六進位，轉換之前先將MSB補0成三個一組或者四個一組：

1000000111.1010 -> 001_000_000_111.101 = $(1007.5)_8$

1000000111.1010 -> 0010_0000_0111.1010 = $(207.A)_{16}$

若要八進位、十六進位之間轉換，須先轉換成二進位，再透過上述方法轉成二進位後才能用觀察的方式得出結果。結論如下：



任何轉換都可以透過二進位來達成，比方說Dec轉成Oct，可先將Dec轉成Bin，再用上述觀察的方法將Bin轉成Oct。

(b)

1001101.011 -> 001_001_101.011 = $(115.3)_8$

1001101.011 -> 0100_1101.0110 = $(4D.6)_{16}$

1001101.011 -> $2^6 + 2^3 + 2^2 + 2^0 + 2^{-2} + 2^{-3} = (77.375)_{10}$

也可利用 $(4D.6)_{16}$轉成十進位：

$(4D.6)_{16}$ -> $4 \times 16^1 + 13 \times 16^0 + 6 \times 16^{-1} = 64 + 13 + 0.375 = (77.375)_{10}$

(c)(d)方法同上

2.6%

$(E1BF)_{16} = (1110\_0001\_1011\_1111)_2 = (001\_110\_000\_110\_111\_111)_2 = (160677)_8$

3.6%

12-bits number is 1010_1011_0111

(a)BCD : 1010→無法表示，1011→無法表示，0111→7

BCD只定義0~9，所以1010、1011無法表示

(b)Excess-3 code : 784

Excess-3是將二進制轉成十進制再減去3

$(1010)_2 = (10)_{10}$ →10-3=7，$(1011)_2 = (11)_{10}$ →11-3=8，$(0111)_2 = (7)_{10}$ →7-3=4

4.6%

If you have 18 books and want to give each book a unique id with a binary number. If we want to use as least as possible the number of bits as the id, how many bits do you need?

Ans:

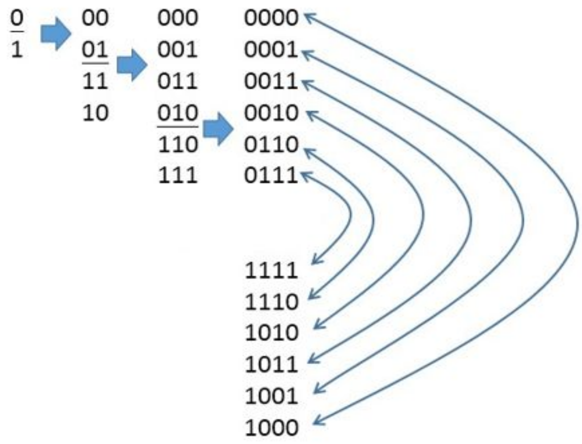
$18 < 2^5 \rightarrow$ 5-bit

要二進位做 18種分類，，2 的 4 次方為 16 不足表示，因此要 5 個 bit 才能分 18 種

5.6%

Find the Gray code with 12 code numbers

Ans:
依題意，要產生 12 組 Gray code，我們使用最簡單的鏡像法，過程如下



當然只要符合定義"任意相鄰兩數，只有一個Bit改變"，就可以是一種Gray code。所以答案是:

0000

0001

0011

0010

0110

0111

1111

1110

1010

1011

1001

1000

6.10%

Convert F to the other normal form and standard forms of sum of products and product of sums.

$$F(A,B,C,D) = \Sigma(0,1,3,5,7,9,13,14,15)$$

Normal form：

$F = A'B'C'D' + A'B'C'D + A'B'CD + A'BC'D + A'BCD + AB'C'D + ABC'D + ABCD' + ABCD$

Sum of product：

$F = A'B'C'(D'+D) + A'(B'+B)CD + A'BC'D + A(B'+B)C'D + ABC(D'+D)$

$\Rightarrow A'B'C' + A'CD + A'BC'D + AC'D + ABC$

$\Rightarrow (A'B'C' + A'B'C'D) + A'BC'D + A'CD + AC'D + ABC$

$\Rightarrow A'B'C' + A'(B'+B)C'D + A'CD + AC'D + ABC$

$\Rightarrow A'B'C' + (A'C'D + A'CD) + (A'C'D + AC'D) + ABC$

$\Rightarrow A'B'C' + A'D + C'D + ABC$

Normal form：

$F' = A'B'CD' + A'BC'D' + A'BCD' + AB'C'D' + AB'CD' + AB'CD + ABC'D'$

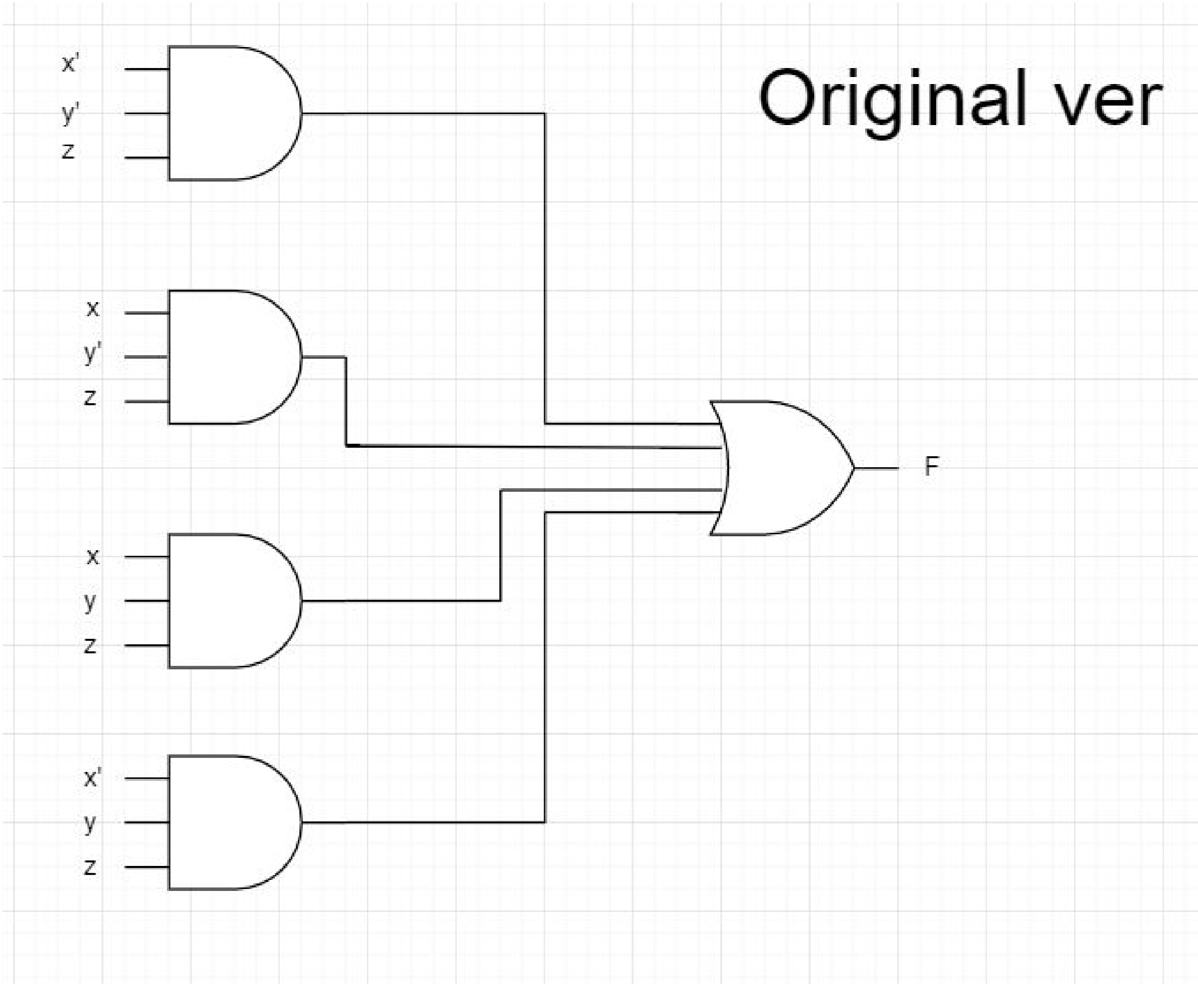
$F = (F')' = (A + B + C' + D)(A + B' + C + D)(A + B' + C' + D)(A' + B + C + D)(A' + B + C' + D)$
$(A' + B + C' + D')(A' + B' + C + D)$

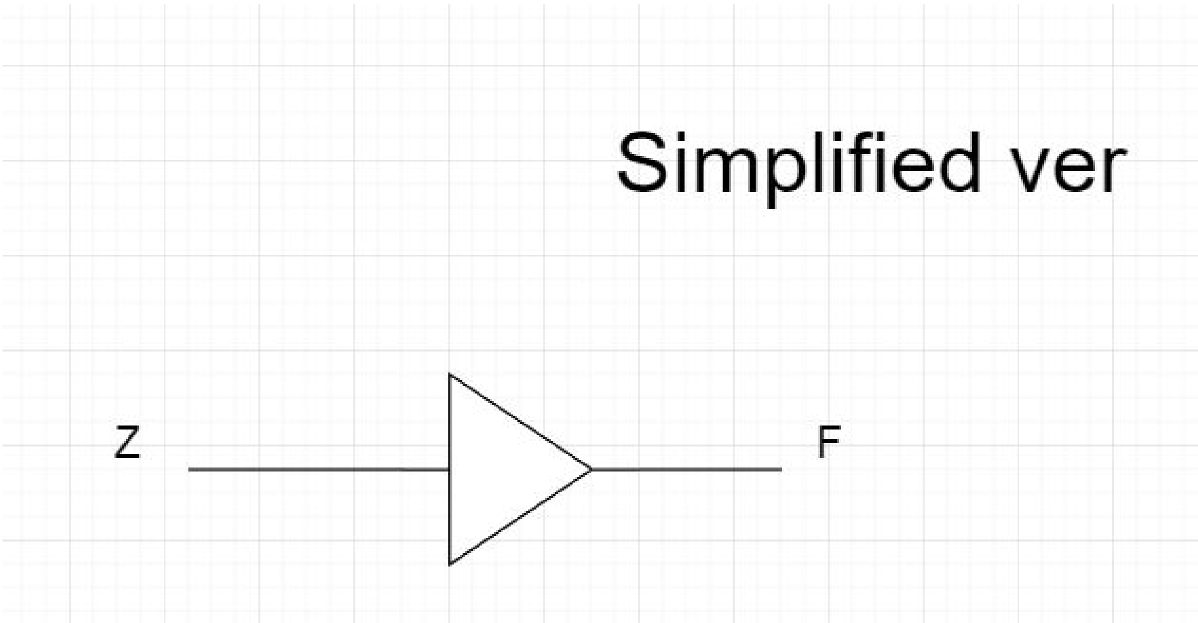
Product of sum：

1. $F = (A'B'CD' + A'BC'D' + A'BCD' + AB'C'D' + AB'CD' + AB'CD + ABC'D')'$
   $\Rightarrow ((m_{10} + m_{11}) + (m_8 + m_{12}) + (m_4 + m_{12}) + (m_2 + m_6))'$
   $\Rightarrow (AB'C + AC'D' + BC'D' + A'CD')'$
   $\Rightarrow (A' + B + C')(A' + C + D)(B' + C + D)(A + C' + D)$
2. $F = (A'B'CD' + A'BC'D' + A'BCD' + AB'C'D' + AB'CD' + AB'CD + ABC'D')'$
   $\Rightarrow ((m_{10} + m_{11}) + (m_8 + m_{12}) + (m_4 + m_6) + (m_2 + m_6))'$
   $\Rightarrow (AB'C + AC'D' + A'BD' + A'CD')'$
   $\Rightarrow (A' + B + C')(A' + C + D)(A + B' + D)(A + C' + D)$
3. $F = (A'B'CD' + A'BC'D' + A'BCD' + AB'C'D' + AB'CD' + AB'CD + ABC'D')'$
   $\Rightarrow ((m_{10} + m_{11}) + (m_8 + m_{12}) + (m_4 + m_6) + (m_2 + m_{10}))'$
   $\Rightarrow (AB'C + AC'D' + A'BD' + B'CD')'$
   $\Rightarrow (A' + B + C')(A' + C + D)(A + B' + D)(B + C' + D)$
4. $F = (A'B'CD' + A'BC'D' + A'BCD' + AB'C'D' + AB'CD' + AB'CD + ABC'D')'$
   $\Rightarrow ((m_{10} + m_{11}) + (m_8 + m_{10}) + (m_4 + m_{12}) + (m_2 + m_6))'$
   $\Rightarrow (AB'C + AB'D' + BC'D' + A'CD')'$
   $\Rightarrow (A' + B + C')(A' + B + D)(B' + C + D)(A + C' + D)$

7.10%(a)

$$x'y'z + xy'z + xyz + x'yz = x'z(y + y') + xz(y + y') = x'z + xz = z(x + x') = z$$

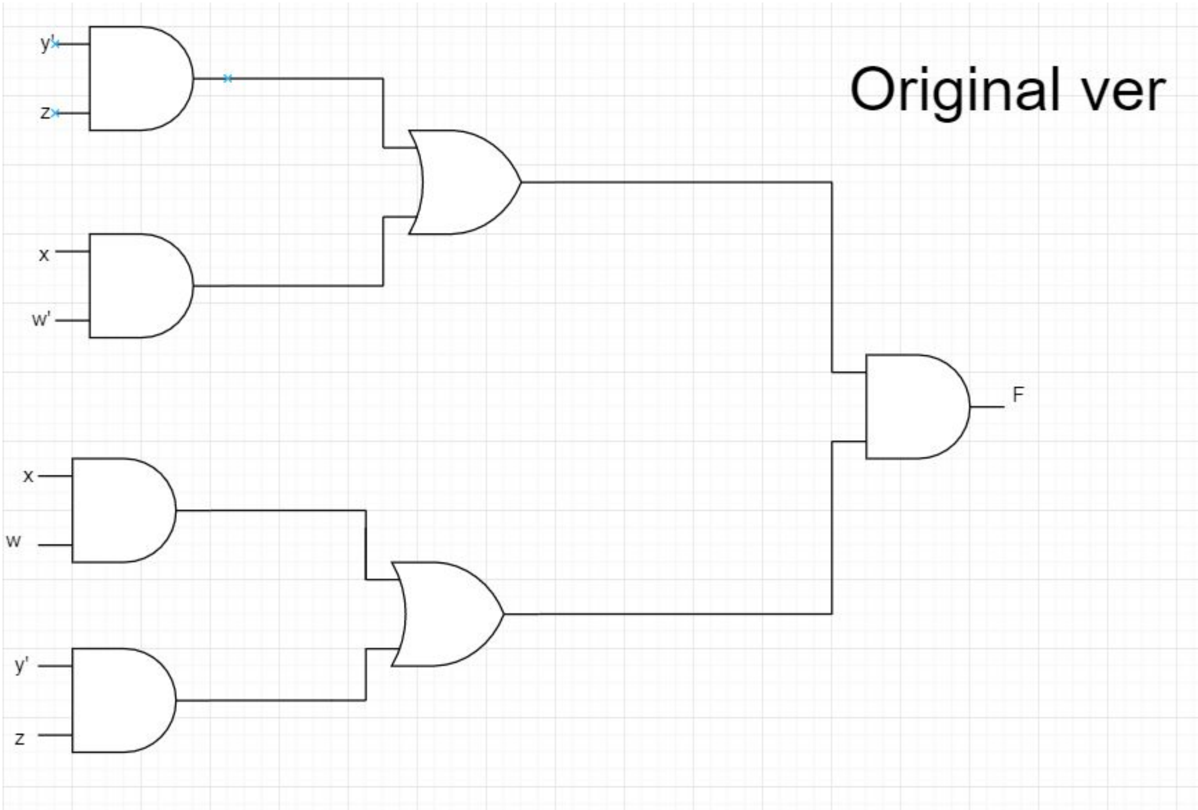


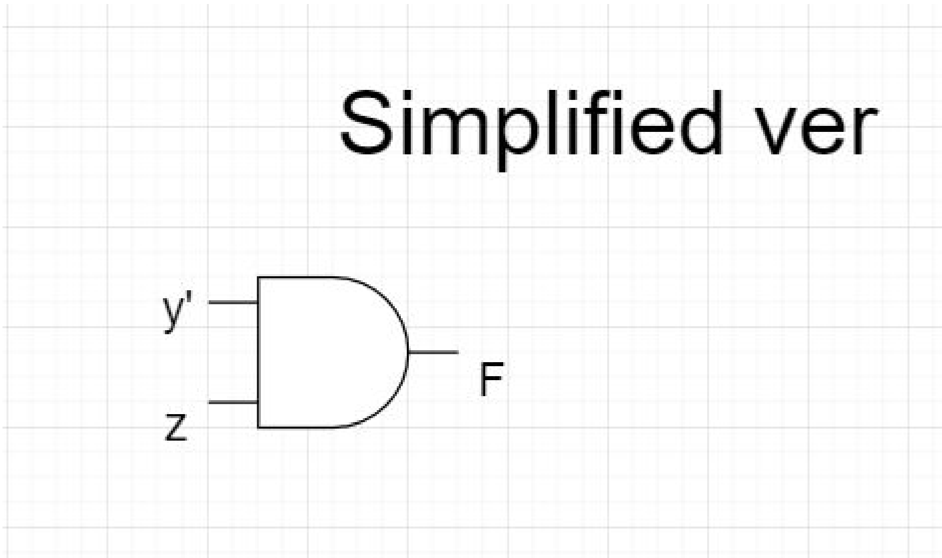


(b)

$$(y'z+xw')(xw+y'z)=(y'z+x)(y'z+w')(y'z+x)(y'z+w)=(y'z+x)(y'z+ww')$$

$$=(y'z+x)(y'z)=y'z+xy'z=y'z$$





a,b兩題各為五分;畫簡過程一分圖皆為兩分

8.12% 表格minterms,maxterms源自馬席彬教授上課講義

(a)

xyz	F
000	0
001	1
010	0
011	1
100	0
101	1
110	0
111	1

minterms: standard product, m_j

−an AND term of the n variables, w/ each variable being primed if the corresponding bit is 0 and unprimed if a 1.

n variables → 2^n minterms

−E.g.: minterms for 3 variables

maxterm: standard sum, $M_j (= m_j')$

−an OR term of the n variables, w /each variable being unprimed if the corresponding bit is 0 and primed if a 1.

n variables → 2^n maxterms

−E.g.: maxterms for 3 variables

	x y z	Minterms	Notation	Maxterms	Notation
0	0 0 0	x′y′z′	m_0	x+y+z	M_0
1	0 0 1	x′y′z	m_1	x+y+z′	M_1
2	0 1 0	x′yz′	m_2	x+y′+z	M_2
3	0 1 1	x′yz	m_3	x+y′+z′	M_3
4	1 0 0	xy′z′	m_4	x′+y+z	M_4
5	1 0 1	xy′z	m_5	x′+y+z′	M_5
6	1 1 0	xyz′	m_6	x′+y′+z	M_6
7	1 1 1	xyz	m_7	x′+y′+z′	M_7

(b)

Sum-of-minterms

$F = \sum(1,3,5,7) = m_1 + m_3 + m_5 + m_7$

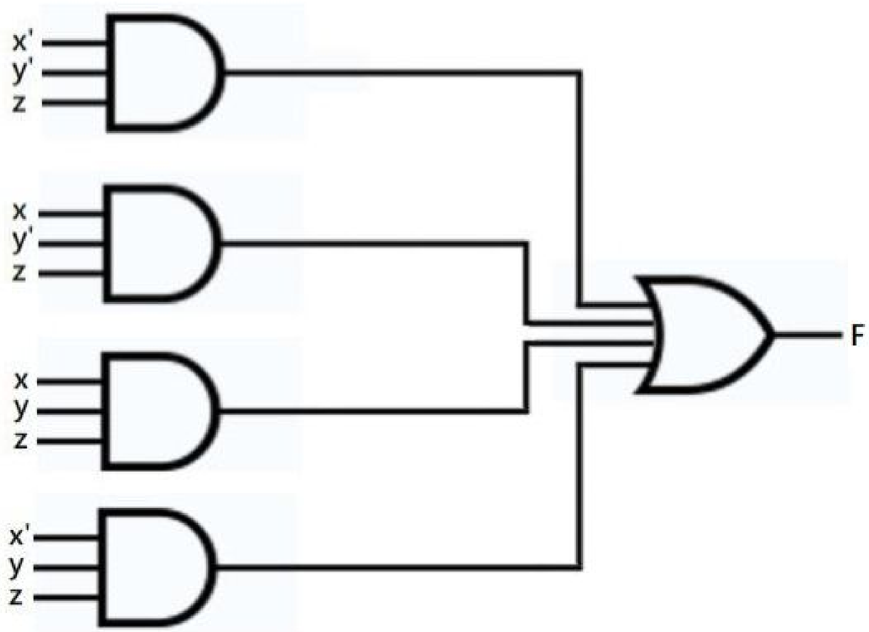
$F = x'y'z + xy'z + xyz + x'yz$

Product-of-maxterms

$F = \prod(0,2,4,6) = M_0 M_2 M_4 M_6$

$F = (x + y + z)(x + y' + z)(x' + y + z)(x' + y' + z)$

(c)



(d)

$$
\begin{aligned}
F &= x'y'z + xy'z + xyz + x'yz \\
&= y'z(x' + x) + yz(x + x') \\
&= y'z + yz \\
&= z
\end{aligned}
$$

$G = z$

minimum number of literals: 1

- A literal is a variable or its complement in a Boolean expression

(e)

G Truth table

z	G
0	0
1	1

F Truth table

xyz	F
000	0
001	1
010	0
011	1
100	0
101	1
110	0
111	1

(f)

F literals: 12

G literals: 1

9.10%

For the Gray code with 4 bit ($g_3g_2g_1g_0$), use a 4-bit binary code ($b_3b_2b_1b_0$) as inputs to generate the code

(a) Derive the related truth table,

(b) Find the logic functions for each g_i.

(a)

$b_3b_2b_1b_0$	$g_3g_2g_1g_0$
0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1000	1100
1001	1101
1010	1111
1011	1110
1100	1010
1101	1011
1110	1001
1111	1000

(b)

$g_0 = b_3'b_2'b_1'b_0 + b_3'b_2'b_1b_0' + b_3'b_2b_1'b_0 + b_3'b_2b_1b_0' + b_3b_2'b_1'b_0 + b_3b_2'b_1b_0' + b_3b_2b_1'b_0 + b_3b_2b_1b_0'$

$= b_1'b_0 + b_1b_0'$

$g_1 = b_3'b_2'b_1b_0' + b_3'b_2'b_1b_0 + b_3'b_2b_1'b_0' + b_3'b_2b_1'b_0 + b_3b_2'b_1b_0' + b_3b_2'b_1b_0 + b_3b_2b_1'b_0' + b_3b_2b_1'b_0$

$= b_2b_1' + b_2'b_1$

$g_2 = b_3'b_2b_1'b_0' + b_3'b_2b_1'b_0 + b_3'b_2b_1b_0' + b_3'b_2b_1b_0 + b_3b_2'b_1'b_0' + b_3b_2'b_1'b_0 + b_3b_2'b_1b_0' + b_3b_2'b_1b_0$

$= b_3'b_2 + b_3b_2'$

$g_3 = b_3b_2'b_1'b_0' + b_3b_2'b_1'b_0 + b_3b_2'b_1b_0' + b_3b_2'b_1b_0 + b_3b_2b_1'b_0' + b_3b_2b_1'b_0 + b_3b_2b_1b_0' + b_3b_2b_1b_0$

$= b_3b_2' + b_3b_2$

$= b_3$

10.10%

(a)3%

((x + w')' + w'y'z + (x + z)'(x + y))'

= (x'w + w'y'z + x'yz')'

= (x'w)'(w'y'z)'(x'yz')'

= (x + w')(w + y + z')(x + y' + z)

= (x + w')(wx + wy' + wz + xy + yy' + yz + xz' + y'z' + z'z)

= (x + w')(wx + wy' + wz + xy + yz + xz' + y'z')

= (wx + wxy' + wxz + xy + xyz + xz' + xy'z') + (w'wx + w'wy' + w'wz + w'xy + w'yz + w'xz' + w'y'z')

= wx + wxy' + wxz + xy + xyz + xz' + xy'z' + w'xy + w'yz + w'xz' + w'y'z'

= wx + xy + xz' + w'yz + w'y'z'

(b)3%

(x(yz' + y'z)' + wy(y' + x'z))'

= (x(yz' + y'z)')'(wy(y' + x'z))'

= (x' + yz' + y'z)((w' + y') + y(x + z'))

= (x' + yz' + y'z)(w' + y' + xy + yz')

= w'x' + x'y' + (x'yz' + w'yz' + xyz' + yz') + (w'y'z + y'z)

= w'x' + x'y' + yz' + y'z

(c)2%

(x+y)'+z'(x'+z)'

= x'y' + z'xz'

= x'y' + xz'

(d)2%

$(xy'+z)'(w+y'z)$

$= ((x' + y)z')(w + y'z)$

$= wx'z' + wyz'$