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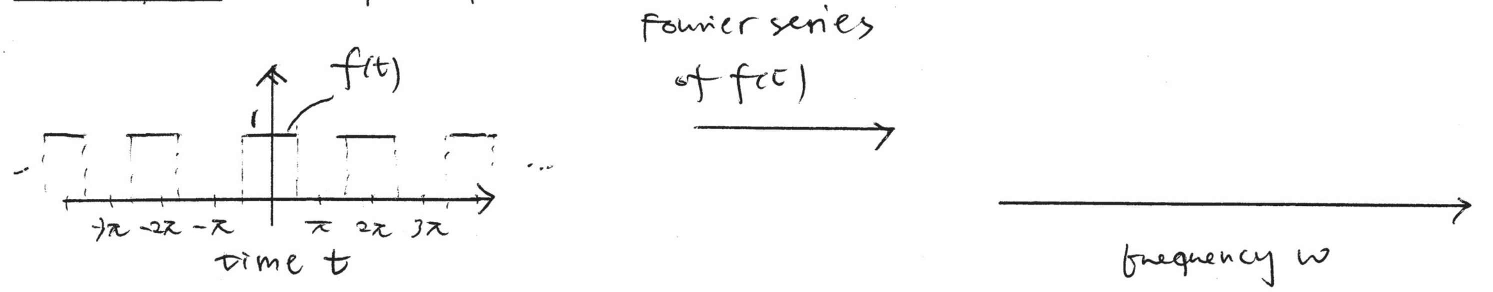
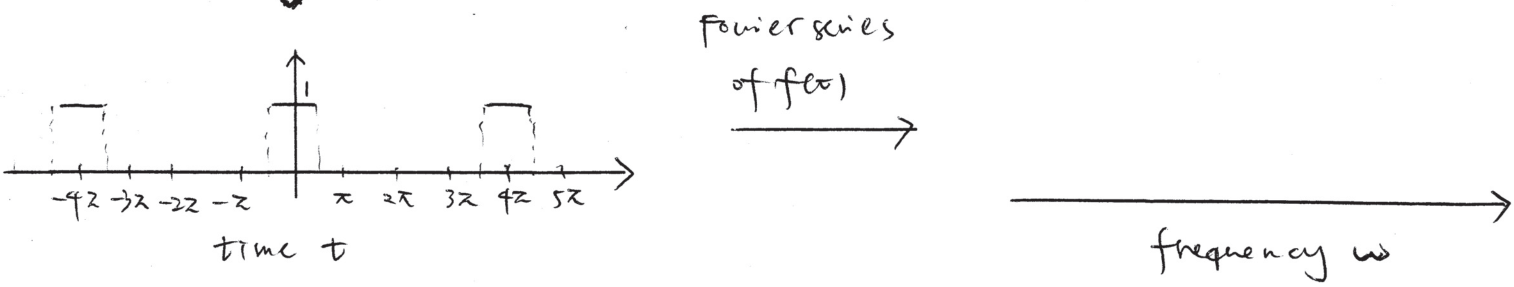
# From Fourier series to Fourier integral to Fourier transform

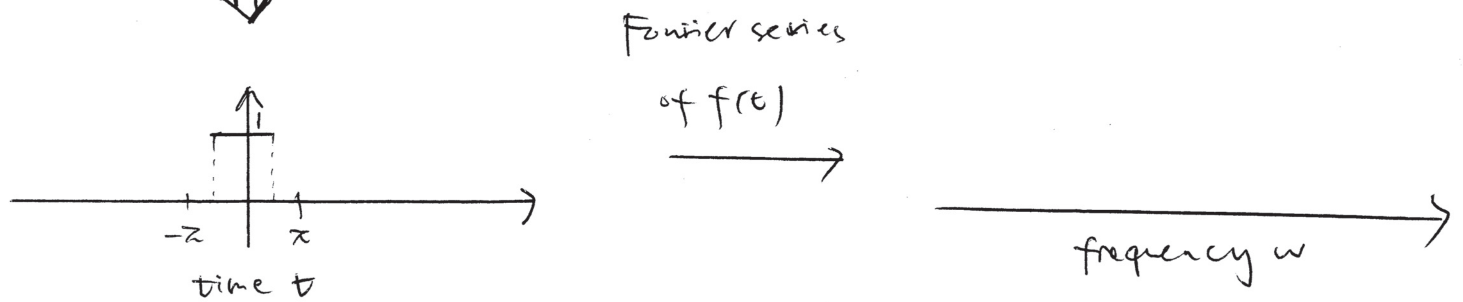
## From Fourier series to Fourier integral

Fourier series is a superposition of periodic sine & cosine. So a function represented by a Fourier series is also periodic. However, from a practical perspective, what we deal with is mostly "nonperiodic." Let's see how a nonperiodic function can be evolved from a periodic one by the following example.

Example : A square pulse train



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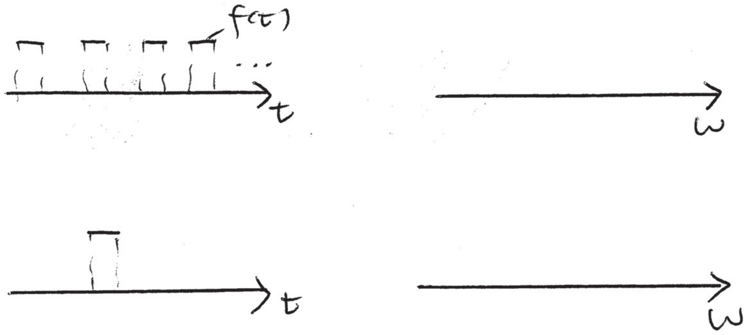
## General expression

time domain | frequency domain | Fourier series of $f(t)$




Depending on an even or odd $f(t)$, the Fourier integral can be expressed as

1) If $f(t)$ is even

$$f(t) = \frac{2}{\pi}\int_0^\infty A(\omega)\cos\omega t\, d\omega \text{ : Fourier cosine integral}$$

where $A(\omega) =$

2) If $f(t)$ is odd

$$f(t) = \frac{2}{\pi}\int_0^\infty B(\omega)\sin\omega t\, d\omega \text{ :}$$

where $B(\omega) =$

$\boxed{\text{From Fourier integral to Fourier transform}}$

In Fourier integral, we use two coefficients

We can actually "combine" sine and cosine by

Therefore, by Euler's formula, we come up with

Fourier integral : $f(t) = \frac{1}{\pi}\int_0^\infty A(w)\cos wt +$

Fourier integral in "complex form" : $f(t) =$