

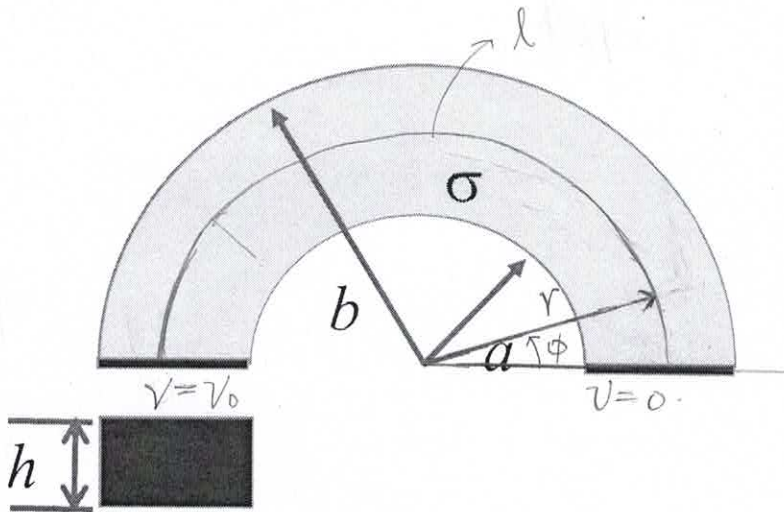
Your name: \_\_\_\_\_ ID: \_\_\_\_\_ Dec. 5<sup>th</sup>, 2020

EE214000 Electromagnetics, Fall, 2020

Quiz #12-2, Open books, notes (10 points), due in class, Monday, Dec. 7<sup>th</sup>, 2020

Late submission won't be accepted!

1. Calculate the resistance of the conductor (conductivity  $\sigma$ ) between the two black electrodes.



$$(1) R = \frac{l}{\sigma S} \Rightarrow G = \frac{\sigma S}{l}, \text{ where } l = \pi r$$

$$dG = \frac{\sigma ds}{l}, \text{ where } ds = h dr$$

$$\text{so } dG = \frac{\sigma h dr}{\pi r}$$

$$\text{then } G = \int_a^b \frac{\sigma h}{\pi r} dr = \frac{\sigma h}{\pi} \ln \frac{b}{a}$$

$$R = \frac{1}{G} = \frac{\pi}{\sigma h \ln(\frac{b}{a})} \quad *$$

(2) Set boundary condition  $\begin{cases} V=0 \text{ at } \phi=0 \\ V=V_0 \text{ at } \phi=\pi \end{cases}$

By Laplace equation:  $\nabla^2 V = 0$

Since no variation in  $z$  and  $r$ ,

$$\nabla^2 V = \frac{d^2 V}{d\phi^2} = 0 \Rightarrow V = C_1 \phi + C_2$$

Apply boundary conditions, we can get  $\begin{cases} C_1 = \frac{V_0}{\pi} \\ C_2 = 0 \end{cases}$

$$\text{then } V = \frac{V_0}{\pi} \phi$$

$$\vec{E} = -\nabla V = -\hat{a}_\phi \frac{\partial V}{\partial \phi} = -\hat{a}_\phi \frac{V_0}{\pi r}$$

$$\vec{J} = \sigma \vec{E} = -\sigma \frac{V_0}{\pi r} \hat{a}_\phi$$

$$I = \int_S \vec{J} \cdot d\vec{s}, \text{ where } d\vec{s} = -h dr \hat{a}_\phi$$

$$\Rightarrow I = \int_a^b \sigma \frac{V_0}{\pi r} h dr = \frac{\sigma h V_0}{\pi} \ln \frac{b}{a}$$

$$\text{And then } R = \frac{V_0}{I} = \frac{\pi}{\sigma h \ln \frac{b}{a}} \quad *$$