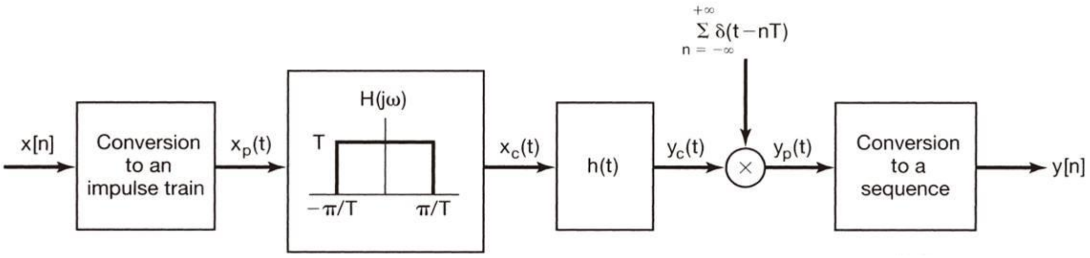
Examination #3

6/16 (Th) R8R9

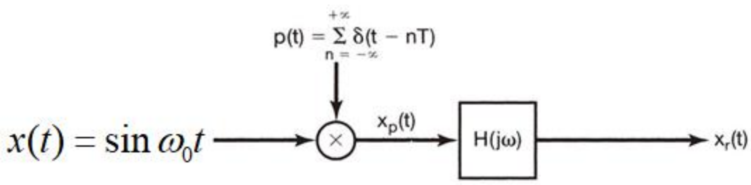
[Online; open-book]

1. (10%) Consider a band-limited CT signal $x_c(t)$. It is properly sampled, being spaced T seconds apart, and then converted to a DT signal $x[n]$. Determine the relationship between the two signal energies.

2. (20%) For the DT sampling system shown below, you know the CT-LTI system is characterized by the differential equation $\frac{d^2y_c(t)}{dt^2}+7\frac{dy_c(t)}{dt}+10y_c(t)=3x_c(t)$. The entire system ($x[n]$ as input and $y[n]$ as output) is equivalent to a causal DT-LTI system. Determine the equivalent system frequency response $H(e^{j\omega})$ and the impulse response $h[n]$.



3. (20%) For the figure below, derive the recovered signal $x_r(t)$ with various sampling frequencies: (a) $\omega_s=3\omega_0$; (b) $\omega_s=2\omega_0$; (c) $\omega_s=1.5\omega_0$; and (d) $\omega_s=\omega_0$.



4. (10%) Derive the Final-Value Theorem for Laplace transform. Which states if $x(t)$ is a right-sided signal and has a finite limit as t approaches infinity, then $\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$.

5. (20%) Consider a CT-LTI system described by the differential equation

$$\frac{d^3y(t)}{dt^2}-2\frac{d^2y(t)}{dt^2}-5\frac{dy(t)}{dt}+6y(t)=x(t)\,.$$

(a) (5%) Determine $H(s)$ as a ratio of two polynomials in *s*. Also provide the pole-zero plot.

(b) (5%) Determine the system impulse response *h(t)* if the system is stable.

(c) (5%) Determine *h(t)* if the system is causal.

(d) (5%) Determine *h(t)* if the system is *neither* stable *nor* causal.

6. (20%) What we covered in Chapter 10 are called *bilateral* z-transform $X(z)$. There is also a thing called *unilateral* z-transform, defined as $X_{uni}(z)=\sum_{n=0}^{\infty}x[n]z^{-n}$.

(a) (5%) Write down the mathematical relationship between *bilateral and unilateral* z-transform.

(b) (15%) For $x[n]=a^{n+2}u[n+2]$, derive both $X(z)$ and $X_{uni}(z)$.