Signals and Systems

Homework 2 — Due : Mar. 08 2024

Problem 1 (16 pts)**.** Determine whether or not each of the following discrete-time signal is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$

(b) $x[n] = \cos(\frac{n}{8} - \pi)$

(c) $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

Problem 2 (27 pts)**.** Express each of the following complex numbers in Cartesian form ($x + jy$):

(a) $\frac{1}{2}e^{j\pi}$

(b) $\frac{1}{2}e^{-j\pi}$

(c) $e^{j\pi/2}$

(d) $e^{-j\pi/2}$

(e) $e^{j5\pi/2}$

(f) $\sqrt{2}e^{j\pi/4}$

(g) $\sqrt{2}e^{j9\pi/4}$

(h) $\sqrt{2}e^{-j9\pi/4}$

(i) $\sqrt{2}e^{-j\pi/4}$

Problem 3 (27 pts)**.** Express each of the following complex numbers in polar form ($re^{j\theta}$, with $-\pi < \theta \leq \pi$):

(a) 5

(b) -2

(c) $-3j$

(d) $\frac{1}{2} - j\frac{\sqrt{3}}{2}$

(e) $1 + j$

(f) $(1 - j)^2$

(g) $j(1 - j)$

(h) $\frac{1+j}{1-j}$

(i) $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}}$

Problem 4 (30 pts)**.** Determine the fundamental period of the signal $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

$W_0 N = 2\pi m \qquad N = \frac{2\pi}{W_0} m$

$N = m\left(\frac{2\pi}{W_0}\right)$

$W_0 N = 2\pi m$

Problem 1 (16 pts). Determine whether or not each of the following discrete-time signal is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$

(b) $x[n] = \cos(\frac{n}{8} - \pi)$

(c) $x[n] = \cos(\frac{\pi}{8}n^2)$

(d) $x[n] = \cos(\frac{\pi}{2}n)\cos(\frac{\pi}{4}n)$

(a) $\frac{\omega_0}{2\pi} = \frac{3}{7} \in \mathbb{Q} \Rightarrow x[n]$ is periodic.

$N = m \cdot \frac{2\pi}{\omega_0} = m \cdot \frac{7}{3} \Rightarrow$ fundamental period is 7

(b) $\left(2\pi \div \frac{1}{8}\right) \notin \mathbb{Q} \Rightarrow x[n]$ is not periodic.

(c) $2\pi \cdot \frac{8}{\pi} = 16$

m is int.

for $n = 16m$ $n^2 \% 16 = 0$ for $n = 16m+8$ $n \% 16 = 0$

for $n = 16m+1$ $n^2 \% 16 = 1$ for $n = 16m+9$ $n \% 16 = 1$

for $n = 16m+2$ $n^2 \% 16 = 4$ for $n = 16m+10$ $n \% 16 = 4$

for $n = 16m+3$ $n^2 \% 16 = 9$ for $n = 16m+11$ $n \% 16 = 9$

for $n = 16m+4$ $n^2 \% 16 = 0$ for $n = 16m+12$ $n \% 16 = 0$

for $n = 16m+5$ $n^2 \% 16 = 9$ for $n = 16m+13$ $n \% 16 = 9$

for $n = 16m+6$ $n^2 \% 16 = 4$ for $n = 16m+14$ $n \% 16 = 4$

for $n = 16m+7$ $n^2 \% 16 = 1$ for $n = 16m+15$ $n \% 16 = 1$

$\Rightarrow x[n]$ is periodic, and the fundamental period is 8.

(d) $\cos\left(\frac{\pi}{4}n+\frac{\pi}{2}n\right)=\cos\left(\frac{\pi}{4}n\right)\cos\left(\frac{\pi}{2}n\right)-\sin\left(\frac{\pi}{4}n\right)\sin\left(\frac{\pi}{2}n\right)$

$\cos\left(\frac{\pi}{2}n-\frac{\pi}{4}n\right)=\cos\left(\frac{\pi}{4}n\right)\cos\left(\frac{\pi}{2}n\right)+\sin\left(\frac{\pi}{4}n\right)\sin\left(\frac{\pi}{2}n\right)$

$\cos\left(\frac{\pi}{4}n\right)\cos\left(\frac{\pi}{2}n\right)=\frac{1}{2}\left[\cos\left(\frac{3\pi}{4}n\right)+\cos\left(\frac{\pi}{4}n\right)\right]$

$\cos\left(\frac{3\pi}{4}n\right)$ is periodic, the fundamental period is 8.

$\cos\left(\frac{\pi}{4}n\right)$ is periodic, the fundamental period is 8.

$\Rightarrow x[n]$ is periodic, and the fundamental period is 8.

Problem 2 (27 pts). Express each of the following complex numbers in Cartesian form $(x + jy)$:

(a) $\frac{1}{2}e^{j\pi}$ (b) $\frac{1}{2}e^{-j\pi}$ (c) $e^{j\pi/2}$

(d) $e^{-j\pi/2}$ (e) $e^{j5\pi/2}$ (f) $\sqrt{2}e^{j\pi/4}$

(g) $\sqrt{2}e^{j9\pi/4}$ (h) $\sqrt{2}e^{-j9\pi/4}$ (i) $\sqrt{2}e^{-j\pi/4}$

(a) $\frac{1}{2}e^{j\pi} = \frac{1}{2}\cos\pi + \frac{1}{2}j\sin\pi$

$= -\frac{1}{2}$

(b) $\frac{1}{2}e^{-j\pi} = \frac{1}{2}\cos\pi - \frac{1}{2}j\sin\pi$

$= -\frac{1}{2}$

(c) $e^{\frac{\pi}{2}j} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$

$= j$

(d) $e^{-\frac{\pi}{2}j} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$

$= -j$

(e) $e^{\frac{5}{2}\pi j} = \cos\frac{5}{2}\pi + j\sin\frac{5}{2}\pi$

$= j$

(f) $\sqrt{2}\,e^{\frac{\pi}{4}j} = \sqrt{2}\cos\frac{\pi}{4} + \sqrt{2}\,j\sin\frac{\pi}{4}$

$= 1 + j$

(g) $\sqrt{2}\,e^{\frac{9}{4}\pi j} = \sqrt{2}\cos\frac{9}{4}\pi + \sqrt{2}\,j\sin\frac{9}{4}\pi$

$= 1 + j$

(h) $\sqrt{2}\,e^{-\frac{9}{4}\pi j} = \sqrt{2}\cos\frac{9}{4}\pi - \sqrt{2}\,j\sin\frac{9}{4}\pi$

$= 1 - j$

(i) $\sqrt{2}\,e^{-\frac{\pi}{4}j} = \sqrt{2}\cos\frac{\pi}{4} - \sqrt{2}\,j\sin\frac{\pi}{4}$

$= 1 - j$

Problem 3 (27 pts). Express each of the following complex numbers in polar form ($re^{j\theta}$, with $-\pi < \theta \leq \pi$):

(a) 5 (b) -2 (c) $-3j$

(d) $\frac{1}{2} - j\frac{\sqrt{3}}{2}$ (e) $1 + j$ (f) $(1-j)^2$

(g) $j(1-j)$ (h) $\frac{1+j}{1-j}$ (i) $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}}$

(a) $5 = 5\cos 0 + 5j\sin 0$

$= 5e^{j0}$

(b) $-2 = 2\cos\pi + 2j\sin\pi$

$= 2e^{j\pi}$

(c) $-3j = 3\cos\frac{-1}{2}\pi + 3j\sin\frac{-1}{2}\pi$

$= 3e^{j\cdot(-\frac{\pi}{2})}$

(d) $\frac{1}{2} - j\frac{\sqrt{3}}{2} = \cos\frac{\pi}{3} - j\sin\frac{\pi}{3}$

$= e^{-j\frac{\pi}{3}}$

(e) $1+j = \sqrt{2}\cos\frac{\pi}{4} + \sqrt{2}j\sin\frac{\pi}{4}$

$= \sqrt{2}e^{j\frac{\pi}{4}}$

(f) $(1-j)^2 = \left(\sqrt{2}\cos\frac{\pi}{4} - \sqrt{2}j\sin\frac{\pi}{4}\right)^2$

$= 2\cdot e^{-j\frac{\pi}{2}}$

(g) $j(1-j) = 1+j = \sqrt{2}\cos\frac{\pi}{4} + \sqrt{2}j\sin\frac{\pi}{4}$

$= \sqrt{2}e^{j\frac{\pi}{4}}$

(h) $\frac{1+j}{1-j} = \frac{\sqrt{2}e^{j\frac{\pi}{4}}}{\sqrt{2}e^{-j\frac{\pi}{4}}} = e^{j\frac{\pi}{2}}$

(i) $\frac{\sqrt{2}+j\sqrt{2}}{1+j\sqrt{3}} = \frac{2e^{j\frac{\pi}{4}}}{2e^{j\frac{\pi}{3}}} = e^{-j\frac{\pi}{12}}$

Problem 4 (30 pts). Determine the fundamental period of the signal $x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$.

the fundamental period of $e^{j\frac{4\pi}{7}n}$ is 7 $\quad 2\pi \times \frac{7}{4\pi} = \frac{7}{2}$

the fundamental period of $e^{j\frac{2\pi}{5}n}$ is 5 $\quad 2\pi \times \frac{5}{2\pi} = 5$

$\Rightarrow$ the fundamental period of $x[n]$ is 35