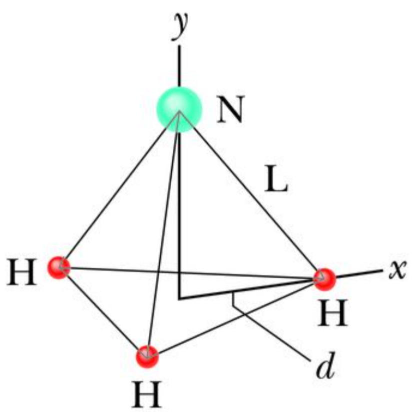
General Physics B1 - Homework Set 3

Due 11/18/2022, 5:00PM sharp. Please hand in your homework via eLearn.

1 points for each problem. Total:5 points

1.Center of Mass of an Ammonia Molecule

In the ammonia (NH3) molecule, three hydrogen (H) atoms form an equilateral triangle, with the center of the triangle at distance $d = 9.40 \times 10^{-11}m$ from each hydrogen atom. The nitrogen (N) atom is at the apex of a pyramid, with the three hydrogen atoms forming the base. The nitrogen-to-hydrogen atomic mass ratio is 13.9, and the nitrogen-to-hydrogen distance is $L = 10.14 \times 10^{-11}m$. What are the (a) x and (b) y coordinates of the molecule's center of mass if the coordinate system sets as shown in the following figure? (1point)

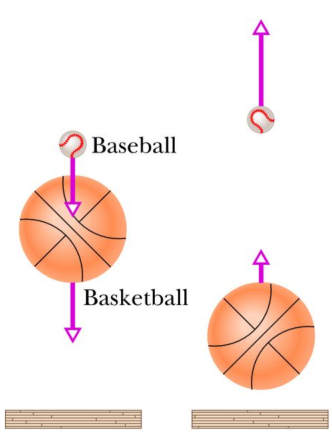


Solution:(a)The coordinate set as in the figure and the x component of center of mass's coordinate is $x_{COM} = \frac{1}{M}\sum m_i x_i$. Since the nitrogen atom's x coodrinate is at 0 and the hydrogen form an equilateral triangle and the origin is at the center of this triangle. We can find that the $x_{COM} = 0$.

(b)The y coordinate is $y_{COM} = \frac{1}{M}\sum m_i y_i$. Since all the hydrogen atom's y coordinate is 0, only nitrogen will contribute in the sum. The y coordinate of nitrogen atom is $y_N = \sqrt{L^2 - d^2} = 3.80 \times 10^{-11}m$. Assuming the mass of hydrogen is m, then mass of nitrogen is 13.9m. Thus, $y_{COM} = \frac{1}{M}\sum m_i y_i = \frac{1}{(13.9+3)m}(13.9m \times 3.80 \times 10^{-11}) = 3.13 \times 10^{-11}m$.

2.Elastic Collision

A small ball of mass m is aligned above a larger ball of mass M = 0.63 kg (with a slight separation, as with the baseball and basketball of Fig.(a), and the two are dropped simultaneously from a height of h = 1.8 m. (Assume the radius of each ball is negligible relative to h.) (a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what value of m results in the larger ball stopping when it collides with the small ball? (0.5points) (b) What height does the small ball then reach Fig.(b)? (0.5points)



(a) Before (b) After

Solution:

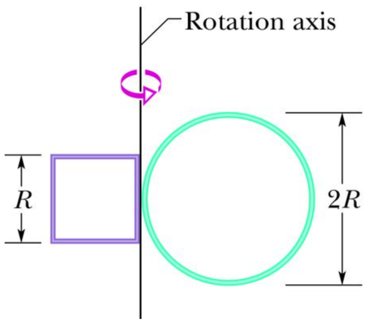
(a) Right before the basketball hit the floor, both basket ball and baseball have a velocity with magnitude $v_0 = \sqrt{2gh} = -5.94m/s$ and the negative sign means the direction of momention is pointing down. After the basketball hit the floor, the basketball will have a velocity with the same magnitude but the direction is upward because it rebounds elastically. Next, the collision process between basketball and baseball is elastic and the final velocity of basketball is zero. Therefore, we will have conservation of energy and conservation of momentum and the expression for final velocity of ellastic collision is valid.

Therefore final velocity of basketball $v_{basketball-f} = \frac{M-m}{M+m}v_0 - \frac{2m}{M+m}v_0 = \frac{M-3m}{M+m}v_0$. Thus, if the final velocity of basketball is zero, the requirement is $m = \frac{M}{3} = 0.21kg$.

(b)The final velocity of baseball $v_{baseball-f} = \frac{2M}{M+m}v_0 + \frac{M-m}{M+m}v_0 = \frac{3M-m}{M+m}v_0 = \frac{3M-\frac{M}{3}}{M+\frac{M}{3}}v_0 = 2v_0 = 11.88m/s$. Thus, the final height is $h = \frac{v^2_{baseball-f}}{2g} = 7.2m$

3. Rotational Inertia and Angular Momentum

As shown in the following figure, a rigid structure consisting of a circular hoop of radius R and mass m, and a square made of four thin bars, each of length R and mass m. The rigid structure rotates at a constant speed about a vertical axis, with a period of rotation of 2.5 s. Assuming R = 0.50 m and m = 2.0 kg, calculate (a) the structure's rotational inertia about the axis of rotation and (0.5points) (b) its angular momentum about that axis. (0.5points)



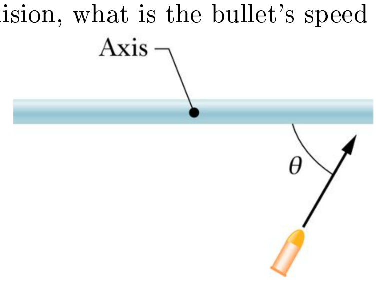
Solution:

(a) For the square, the rotational inertia $I_{square} = MR^2 + 2 \times [\frac{1}{12}MR^2 + M(\frac{R}{2})^2] + 0 = \frac{5}{3}MR^2$. For the circle loop, we can start with rotaional inertia $= \frac{1}{2}MR^2$ when the rotation is panetrating the circle center. (You can find this in the course slides.) Then use parallel axis theorem, we have $I_{circle} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$. Therefore, the total rotaional intertia $I_{total} = I_{square} + I_{circle} = \frac{5}{3}MR^2 + \frac{3}{2}MR^2 = \frac{19}{6}MR^2 = 1.58kgm^2$

(b) The angular moment $L = I_{total}\omega = 3.97kgm/s^2$

4.Conservation of Angular Momentum

In the following fig. (an overhead view), a uniform thin rod of length 0.500 m and mass 4.00 kg can rotate in a horizontal plane about a vertical axis through its center. The rod is at rest when a 3.00 g bullet traveling in the rotation plane is fired into one end of the rod. In the view from above, the bullet's path makes angle j = 60.0° with the rod. If the bullet lodges in the rod and the angular velocity of the rod is 10 rad/s immediately after the collision, what is the bullet's speed just before impact?



Solution:

During the process, there is no external torque exerting on the system. Therefore, the angular momentum is conserved:

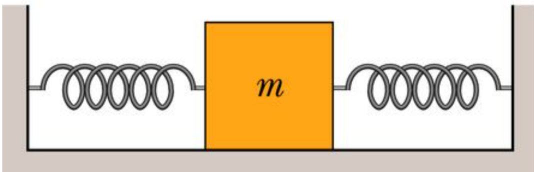
$L_{initial} = rpsin\theta = (\frac{L}{2})mvsin\theta$

$L_{final} = I\omega = [\frac{1}{12}ML^2 + m(\frac{L}{2})^2]\omega$

Thus the velocity of bullet: $v = \frac{[\frac{1}{12}ML^2+m(\frac{L}{2})^2]\omega}{\frac{L}{2}msin\theta} = 1285m/s$

5.Oscillation with two springs

In the following figure, two springs are attached to a block that can oscillate over a frictionless floor. If the left spring is removed, the block oscillates at a frequency of 30 Hz. If, instead, the spring on the right is removed, the block oscillates at a frequency of 45 Hz. At what frequency does the block oscillate with both springs attached? (1point)



olution:

The oscillation frequency of a mass m attached to one spring with spring constant k is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

For the setup in the figure, the two springs exert forces that have the same direction and sum up at all time. Therefore, we can conclude that $f_{tot} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$.

Thus, $f_{tot}^2 = (\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}})^2 = (\frac{1}{2\pi})^2\frac{k_1+k_2}{m} = f_1^2 + f_2^2$.

And $f_{tot} = \sqrt{f_1^2 + f_2^2} = 54Hz$.