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Next,

How to solve nonhomogeneous linear 2nd-order ODEs

Preliminary: About nonhomogeneous linear 2nd-order ODEs

① For a nonhomogeneous ODE , any function y_p satisfies is called a " ".

ex: $y'' + 9y = 27$,

② is called the " "

ex:

③ If a set y_1, y_2 is a fundamental set of & y_p is a of $L(y) = g(t)$, the general solution of the nonhomogeneous DE is expressed by their

④ If $L(y) = g_1(x)$ has a

then the general solution of $L(y)$

In the following, we will learn two techniques to solve $L(y) = g(t)$

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Method of "undetermined coefficients"

✡ Condition:

Q: What's your best guess of the particular y_p solution for

ex: $y'' - y' + y = 2\sin 4t$

$= 2t^2 + 1$

$= e^{3t}$

$= te^{3t}$

$= te^{3t} + 2\sin 4t + 2t^2 + 1$

Idea: For a nonhomogeneous ODE $L(y) = g(t)$,

General guess of particular solutions y_p for $L(y) = g(t)$

$g(t)$	guess of y_p

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Example1 : $y'' - 2y' - 3y = 4t - 5 + 6te^{2t}$

Example2 : $y'' - 5y' + 4y = 8e^{t}$

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Example 3 : $y''+2y'+y=e^t$

Remarks:

① General procedures:

1) Solve the

2) Categorize $g(t)$ by

then find

3) Obtain general solution by

② When y_p you guess belongs to , we just keep

③ This method can also be used to solve

ex: $y'''' + y'' = 1 - t^2 e^{-t}$

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<u>Method of "variation of parameters"</u>

* Condition :

Idea : Use the solution of

Q : How to find

Set $y_p =$ into $y'' + Py' + Qy = g(t)$

$\Rightarrow y_p'' + Py_p' + Qy_p$

$=$

$$\begin{cases} y_p = u_1 y_1 + u_2 y_2 \\ y_p' = u_1 y_1' + u_1' y_1 + u_2 y_2' + u_2' y_2 \\ y_p'' = u_1 y_1'' + 2u_1' y_1' + u_1'' y_1 \\ \quad + u_2 y_2'' + 2u_2' y_2' + u_2'' y_2 \end{cases}$$

Since we just need to find any u_1, u_2 that satisfy (*), we can choose the simplest case where

So now we have two unknowns, u_1, u_2, with two equations

$\{$

Find u_1', ① × $y_1 y_2' u_1' + y_2 y_2' u_2' = 0$

② ×

$(y_1 y_2' - y_1' y_2) u_1' = -y_2 g(t)$

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$$\Rightarrow \quad u_1' = \frac{-y_2 g(t)}{\qquad}$$

It's easier to express u_1', u_2' in a determinant form:

$$u_1' = \frac{-y_2 g(t)}{\qquad} = \underline{\qquad\qquad}$$

By a similar procedure, we can find

$$u_2' = \frac{y_1 g(t)}{\qquad} = \underline{\qquad\qquad}$$

Then, u_1, u_2 can be obtained by integration:

$u_1 =$ $\qquad\qquad\qquad$ $u_2 =$

$\Rightarrow$ general solution $y =$

<u>Example</u>: Solve $y'' - 4y' + 4y = (t+1)e^{2t}$ by variation of parameters

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Remarks:

① General procedures of method of "variation of parameters"

1) Express the DE in

2) Find

3) Set

② This method can also be used in higher-order nonhomogeneous ODE. ex: $y''' + Py'' + Qy' + Ry = g(t)$