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Properties of Laplace transform (LT)

Here we list some properties of LT to show why LT is useful to solve DEs:

<1> "derivation" property:

By LT, derivation (in t -domain) \longrightarrow

$$\text{ex: } \mathcal{L}\{y'\} =$$

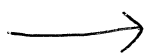
proof:

$$\mathcal{L}\{y'\} = \int_0^{\infty} y' e^{-st} dt = y e^{-st} \Big|_0^{\infty} - \int_0^{\infty} (-s) e^{-st} y dt$$

★ LT of derivation:

$$\left\{ \begin{array}{l} \mathcal{L}\{y'\} = \\ \mathcal{L}\{y''\} = \\ \vdots \\ \mathcal{L}\{y^{(n)}\} = \end{array} \right.$$

It means



<2> "linearity" property

LT is a linear operation

"linear" means:

<3> " " property

Many engineering / physics problems involve
in time. ex:

Their LT are

How to solve DEs by LT

General procedures:

Remarks about \mathcal{L}^{-1} (inverse Laplace transform)

① We need to convert

② Properties of \mathcal{L}^{-1}

— "uniqueness" property:

If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}^{-1}\{F(s)\}$ can only be

— "linearity" property

Example 1: Use LT to solve $y' = y - 4e^{-t}$, $y(0) = 1$

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Example 2: Use LT to solve $y'' + 4y' + 20y = e^{-2t}$, $y(0) = y'(0) = 0$

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By comparing with $\mathcal{L}\{y\}$, we find that the denominator of $\mathcal{L}\{y\}$ consists of product of

So from the roots of denominator of $\mathcal{L}\{y\}$, we can get qualitative analysis of the system response.

ex: $\mathcal{L}\{y\} = \frac{1}{(s^2 + 2s + 2)(s + \frac{1}{10})}$, what can we say about the system response qualitatively?

Remark: Qualitative analysis of system response from LT

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Use LT to solve DEs with { discontinuous forcing
periodic

In many engineering/physics problems, the systems are described by constant-coefficient 2nd-order linear DEs.

Ex:

$$\begin{array}{c} \text{system} \\ y'' + p y' + q y = f(t) \end{array}$$

The input may be "turned on" at a specific time, like a switch.

ex:

LT is particularly useful to solve problems with

In the following, we will discuss