Quiz 2 solution

1.

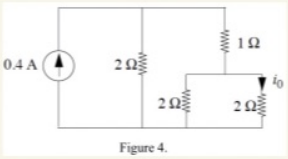
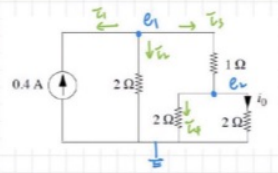


Figure 4.

4.



$i_0 = \frac{e_2}{2}$

KCL @ e_1 : $-0.4 + \frac{e_1}{2} + \frac{e_1 - e_2}{1} = 0$

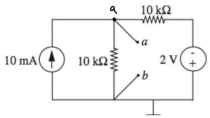
KCL @ e_2 : $\frac{e_2 - e_1}{1} + \frac{e_2}{2} + \frac{e_2}{2} = 0$

$\Rightarrow e_1 = 0.4\,V$, $e_2 = 0.2\,V$

$\Rightarrow i_0 = \frac{0.2}{2} = 0.1\,A$ #

2.

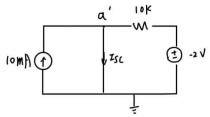
2. Find the Thevenin equivalent circuit of the circuit at terminals a, b, in the following figure. (18%)



KCL at a

$10mA + \frac{-2 - V_a}{10k} + \frac{-V_a}{10k} = 0$

$\frac{2V_a + 2}{10^4} = 10^{-2} \Rightarrow 2V_a + 2 = 100$, $V_a = 49\,V = V_{oc}$ #



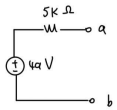
KCL at a'

$10mA - I_{sc} + \frac{-2}{10k} = 0$

$I_{sc} = 10mA - 0.2mA = 9.8\,mA$ #

$R_{TH} = \frac{V_{oc}}{I_{sc}} = 5\,K\Omega$

ANS:



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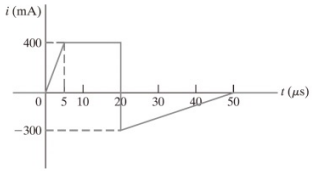
3.

3. The current is applied to a capacitor as shown in the following figure. The capacitance of this capacitor is 1 μF. The initial voltage of the capacitor is zero.

(a) Derive and plot the voltage across the capacitor. (16%)

(b) Determine the energy stored in the capacitor at t = 30 μs. (8%)



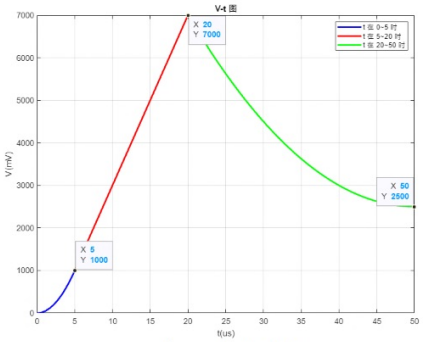
a. Function of current at each section : $\begin{cases} 0\sim5: i = 80t \\ 5\sim20: i = 400 \\ 20\sim50: i = -500 + 10t \end{cases}$

For capacitor, we have formula $i = C\frac{dV}{dt} \rightarrow V = \frac{1}{C}\int i\,dt$

Therefore, function of voltage at each section : $\begin{cases} 0\sim5: V\ is\ quadratic \\ 5\sim20: V\ is\ linear \\ 20\sim50: V\ is\ quadratic \end{cases}$

Moreover, we can calculate the area (or integral) to obtain $\begin{cases} t = 0\ us, V = 0\ V \\ t = 5\ us, V = 1\ V \\ t = 20\ us, V = 7\ V \\ t = 50\ us, V = 2.5\ V \end{cases}$

So, we can plot V-t diagram as below :



Or slove for the exact function $\begin{cases} 0\sim5: V = 40t^2 \\ 5\sim20: V = 400t - 1000 \\ 20\sim50: V = -500t + 5t^2 + 15000 \end{cases}$, *where unit* $= mV$

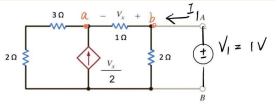
b.

i @ 30 us $= -200$ mA

V @ 30 us $= 7 + (-300 - 200) * \frac{10}{2} = 4.5$ V

energy stored in capacitor : $U = \frac{1}{2}CV^2 = \frac{1}{2} * 1\,\mu * 4.5^2 = 10.125\ \mu J$

4.



a: $\frac{V_x}{1} + \frac{V_x}{2} = \frac{1 - V_x}{5} \Rightarrow V_x = \frac{2}{17}\ V$

b: $I_1 = \frac{V_x}{1} + \frac{1}{2} = \frac{21}{34}\ A$

$R_{TH} = \frac{V_1}{I_1} = \frac{1}{\frac{21}{34}} = \frac{34}{21}\ \Omega = R_N$

equivalent circuit $\Rightarrow$ A — $R_N = \frac{34}{21}\ \Omega$ — B

$i_N = \frac{V_{TH}}{R_{TH}} = \frac{0\ V}{R_{TH}} = 0\ A$

Quiz 2

5. $i_C = C\frac{dV_o}{dt} = \frac{V_s - V_o}{R}$

$\Rightarrow RC\frac{dV_o}{dt} + V_o = V_s$

① for V_{op}

guess $V_{op} = V_s$

$\rightarrow RC\frac{dV_s}{dt} + V_s = V_s = V_s \Rightarrow V_{op} = V_s$.

② for V_{oh}:

$RC\frac{dV_{oh}}{dt} + V_{oh} = 0$

guess $V_{oh} = Ae^{st}$

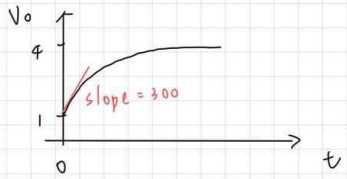
$\Rightarrow RC\frac{d(Ae^{st})}{dt} + Ae^{st} = 0 \rightarrow sRCAe^{st} + Ae^{st} = 0$

$\Rightarrow A(sRC+1)e^{st} = 0 \Rightarrow s = -\frac{1}{RC} \rightarrow V_{oh} = Ae^{-\frac{t}{RC}}$

$\Rightarrow V_o = V_s + Ae^{-\frac{t}{RC}}, \quad t \geq 0$

$V_o(t=0) = V_s + A = 1 \rightarrow A = 1 - V_s$

$\Rightarrow V_o = 4 - 3e^{-\frac{t}{0.01}} \quad (V)$



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