Chapter 5

Force and Motion (I)

Dynamics
vs
Kinematics

(1643.01.04 – 1727.03.31)
(Painted by Godfrey Kneller in 1702)

Key contents:

* Newton’s laws
* Some often-seen forces
* Inertial reference frames
* Applying Newton’s laws

Newtonian Mechanics:

Study of relation between force and acceleration of a body.

(caution)

Newtonian Mechanics does not hold good for all situations.

Examples:

1. Relativistic or near-relativistic motion (high-speed motion)
2. Motion of atomic-scale particles

‘Natural state’ of matter ?

Newton’s First Law:

**If no force acts on a body, the body’s velocity cannot change;
that is, the body cannot accelerate.**

⬇

If the body is at rest, it stays at rest. If it is moving, it continues to move with the same velocity (same magnitude *and same direction).*

Concept of force

Newton's Second Law

The net force on a body is equal to the product of the body's mass and its acceleration.

$$\vec{F}_{net} = m\vec{a}$$

In component form, $F_{net,x} = ma_x, \quad F_{net,y} = ma_y, \quad \text{and} \quad F_{net,z} = ma_z.$

The acceleration component along a given axis is caused *only by the sum of the force* components along that *same axis, and not by force components along any other axis.*

This is a statement to summarize what is observed and to describe the relationship among the three quantities: force, mass, and acceleration.

Concept of (inertia) mass

The SI unit of force is newton (N):

$$1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{kg m/s}^2.$$

TABLE 5-1

Units in Newton's Second Law (Eqs. 5-1 and 5-2)

System	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
CGS[a]	dyne	gram (g)	cm/s^2
British[b]	pound (lb)	slug	ft/s^2

[a] $1\ \text{dyne} = 1\ \text{g} \cdot \text{cm/s}^2$.
[b] $1\ \text{lb} = 1\ \text{slug} \cdot \text{ft/s}^2$.

Example: forces

Parts A, B, and C of Fig. 5-3 show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?

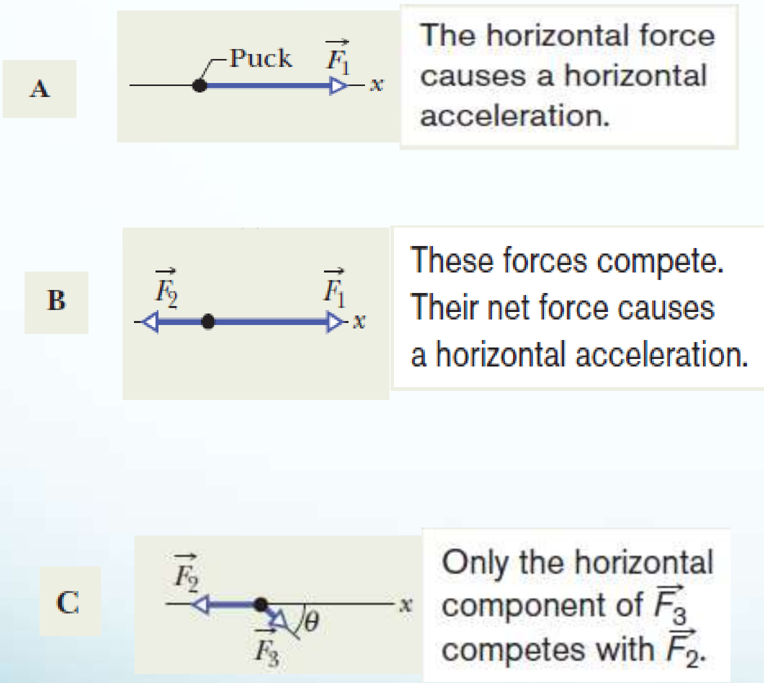


Fig. 5-3 In three situations, forces act on a puck that moves along an x axis.

Situation A: For Fig. 5-3*b*, where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0\text{ N}}{0.20\text{ kg}} = 20\text{ m/s}^2. \qquad \text{(Answer)}$$

The positive answer indicates that the acceleration is in the positive direction of the x axis.

Situation B: In Fig. 5-3*d*, two horizontal forces act on the puck, $\vec{F}_1$ in the positive direction of x and $\vec{F}_2$ in the negative direction. Now Eq. 5-4 gives us

$$F_1 - F_2 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{ N} - 2.0\text{ N}}{0.20\text{ kg}} = 10\text{ m/s}^2.$$

(Answer)

Thus, the net force accelerates the puck in the positive direction of the x axis.

Situation C: In Fig. 5-3*f*, force $\vec{F}_3$ is not directed along the direction of the puck's acceleration; only x component $F_{3,x}$ is. (Force $\vec{F}_3$ is two-dimensional but the motion is only one-dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. \qquad (5\text{-}5)$$

From the figure, we see that $F_{3,x} = F_3 \cos\theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

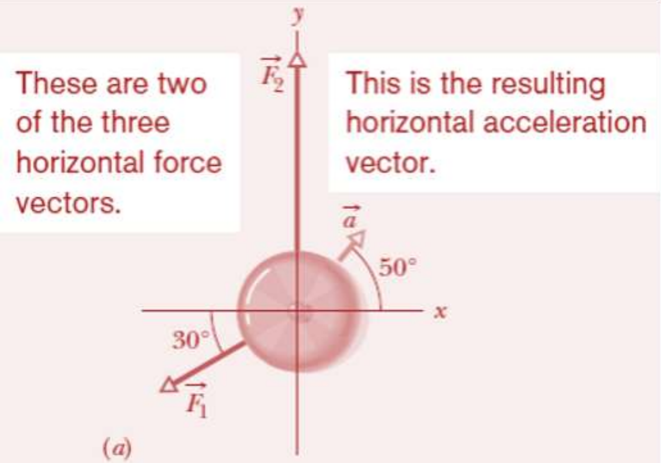
$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos\theta - F_2}{m}$$

$$= \frac{(1.0\text{ N})(\cos 30°) - 2.0\text{ N}}{0.20\text{ kg}} = -5.7\text{ m/s}^2.$$

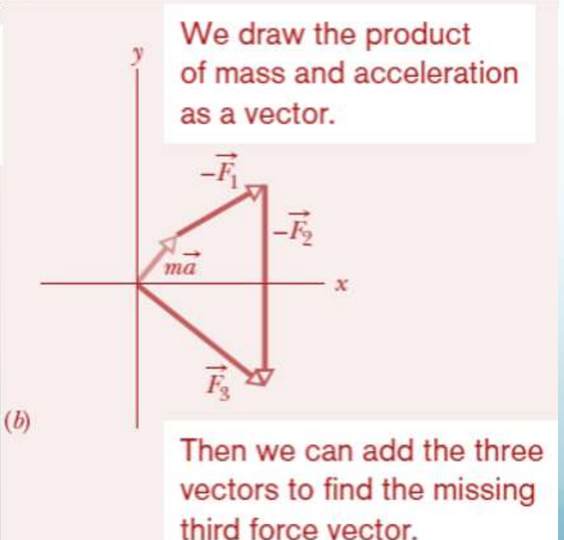
(Answer)

Example: 2-D forces

In the overhead view of Fig. 5-4a, a 2.0 kg cookie tin is accelerated at 3.0 m/s^2 in the direction shown by $\vec{a}$, over a frictionless horizontal surface. The acceleration is caused by three horizontal forces, only two of which are shown: $\vec{F}_1$ of magnitude 10 N and $\vec{F}_2$ of magnitude 20 N. What is the third force $\vec{F}_3$ in unit-vector notation and in magnitude-angle notation?



(*a*)



(*b*)

KEY IDEA

The net force $\vec{F}_{net}$ on the tin is the sum of the three forces and is related to the acceleration $\vec{a}$ via Newton's second law ($\vec{F}_{net} = m\vec{a}$). Thus,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = m\vec{a},$$

which gives us

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2.$$

Calculations:

***x* components:** Along the x axis we have

$$F_{3,x} = ma_x - F_{1,x} - F_{2,x}$$
$$= m(a\cos 50°) - F_1\cos(-150°) - F_2\cos 90°.$$

Then, substituting known data, we find

$$F_{3,x} = (2.0\text{ kg})(3.0\text{ m/s}^2)\cos 50° - (10\text{ N})\cos(-150°)$$
$$- (20\text{ N})\cos 90°$$
$$= 12.5\text{ N}.$$

***y* components:** Similarly, along the y axis we find

$$F_{3,y} = ma_y - F_{1,y} - F_{2,y}$$
$$= m(a\sin 50°) - F_1\sin(-150°) - F_2\sin 90°$$
$$= (2.0\text{ kg})(3.0\text{ m/s}^2)\sin 50° - (10\text{ N})\sin(-150°)$$
$$- (20\text{ N})\sin 90°$$
$$= -10.4\text{ N}.$$

Vector: In unit-vector notation, we can write

$$\vec{F}_3 = F_{3,x}\hat{i} + F_{3,y}\hat{j} = (12.5\text{ N})\hat{i} - (10.4\text{ N})\hat{j}$$
$$\approx (13\text{ N})\hat{i} - (10\text{ N})\hat{j}. \qquad \text{(Answer)}$$

We can now use a vector-capable calculator to get the magnitude and the angle of $\vec{F}_3$. We can also use Eq. 3-6 to obtain the magnitude and the angle (from the positive direction of the x axis) as

$$F_3 = \sqrt{F_{3,x}^2 + F_{3,y}^2} = 16\text{ N}$$

and

$$\theta = \tan^{-1}\frac{F_{3,y}}{F_{3,x}} = -40°. \qquad \text{(Answer)}$$

Quick question:

The graph shows the velocities of two objects as a function of time. During the intervals A, B, and C indicated, net forces F_A , F_B , and F_C act on the two objects, respectively. If the objects have equal mass, which one of the following choices is the correct relationship between the magnitudes of the three net forces?

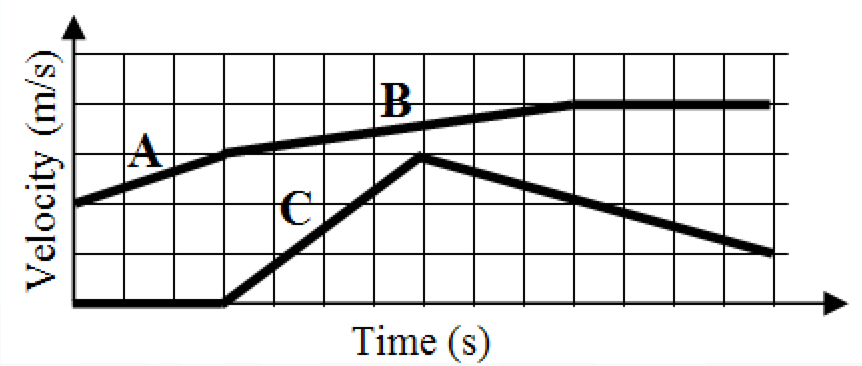
a) $F_A > F_B = F_C$

b) $F_C > F_A > F_B$

c) $F_A < F_B < F_C$

d) $F_A = F_B = F_C$

e) $F_A = F_C > F_B$



Newton's Third Law

When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction.

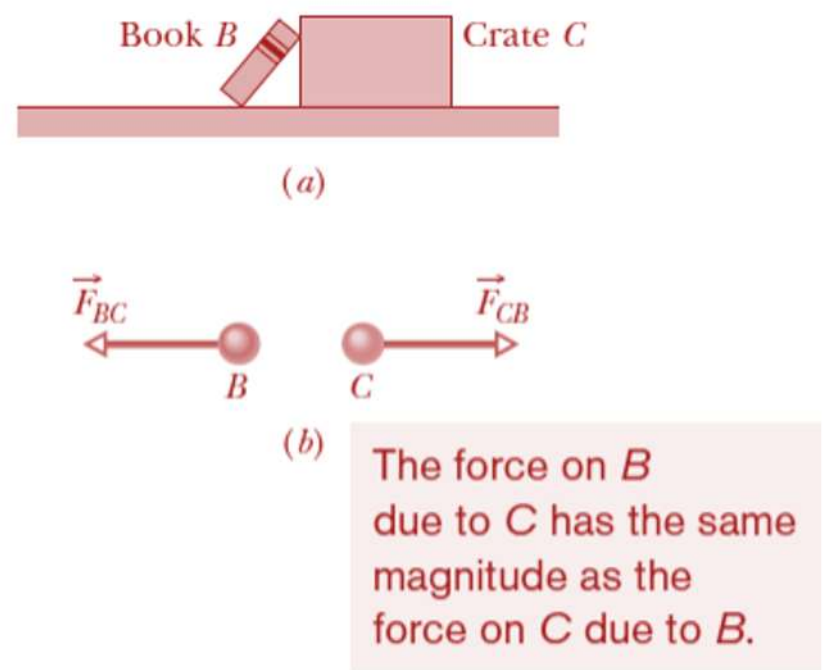


Fig. 5-10 (a) Book B leans against crate C. (b) Forces $\vec{F}_{BC}$ (the force on the book from the crate) and $\vec{F}_{CB}$ (the force on the crate from the book) have the same magnitude and are opposite in direction.

$$\vec{F}_{BC} = -\vec{F}_{CB}$$

The minus sign means that these two forces are in opposite directions.

It says actually the conservation of linear momentum.

Some often-seen forces

The gravitational force

A gravitational force on a body is a certain type of pull that is directed toward a second body.

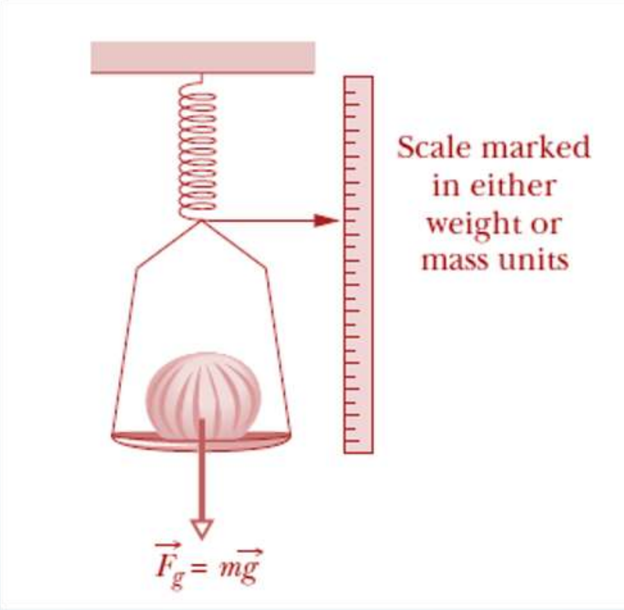
Here we consider the second body to be the earth.

Suppose a body of mass m is in free fall with the free-fall acceleration of magnitude g. The force that the body feels as a result is:

$$F_g = mg.$$

The weight, W, of a body is equal to the magnitude F_g of the gravitational force on the body.

$$W = mg \text{ (weight)},$$



Some often-seen forces

The normal force:

When a body presses against a surface, the surface (even a seemingly rigid one) deforms and pushes on the body with a normal force, $\boldsymbol{F}_N$, that is perpendicular to the surface.

In the figure, forces F_g and F_N are the only two forces on the block and they are both vertical. Thus, for the block we can write Newton's second law for a positive-upward *y axis*,

($F_{net,\,y} = ma_y$), as:

$$F_N - F_g = ma_y.$$

$$F_N - mg = ma_y.$$

$$F_N = mg + ma_y = m(g + a_y)$$

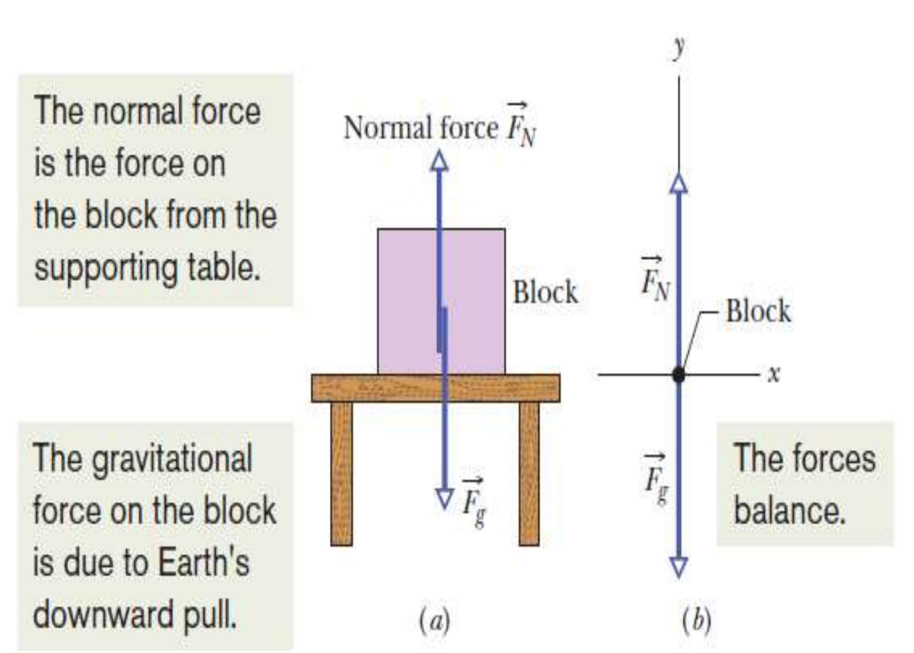


Fig. 5-7 (a) A block resting on a table experiences a normal force perpendicular to the tabletop. (b) The free-body diagram for the block.

Some often-seen forces

Friction

If we either slide or attempt to slide a body over a surface, the motion is resisted by a bonding between the body and the surface.

The resistance is considered to be a single force called the frictional force. This force is directed along the surface, opposite the direction of the intended motion.

More will be discussed in the next chapter.

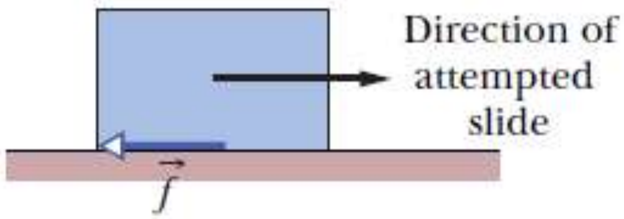


Fig. 5-8 A frictional force $\vec{f}$ opposes the attempted slide of a body over a surface.

Some often-seen forces

Tension

When a cord is attached to a body and pulled taut, the cord pulls on the body with a force *T directed away from the* body and along the cord.

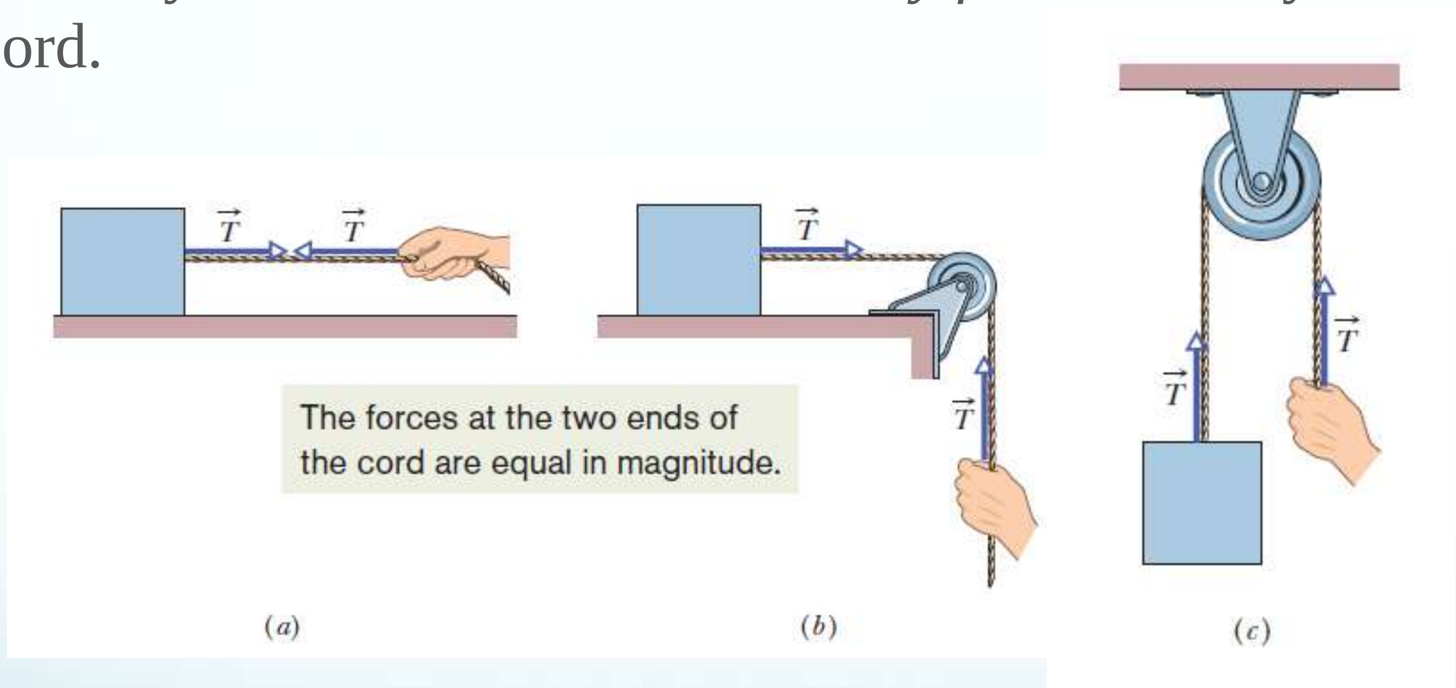
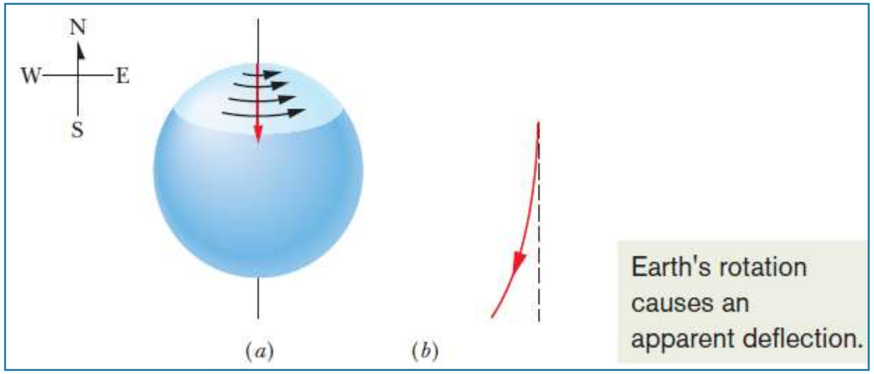


Fig. 5-9 (*a)* The cord, pulled taut, is under tension. *If its mass is negligible*, the cord pulls on the body and the hand with force ***T***, even if the cord runs around a massless, frictionless pulley as in (*b) and (c)*.

Inertial Reference Frames

An inertial reference frame is one in which Newton's laws hold.



(*a*) *The path of a puck sliding from the north pole as seen from a stationary* point in space. Earth rotates to the east. (*b*) *The path of the puck as seen from the ground*.

The apparent deflection is not caused by a force, but by the fact that we see the puck from a rotating frame. In this situation, the ground is a **non-inertial frame.**

Other examples for non-inertial frames?

All the frames at a constant velocity w.r.t. an inertial frame are inertial frames.

Is there a 'true' inertial frame ?

Applying Newton's Laws

Example:

Figure 5-12 shows a block *S (the sliding block) with mass M =3.3 kg. The block is free to move along a horizontal* frictionless surface and connected, by a cord that wraps over a frictionless pulley, to a second block *H (the hanging block), with mass m 2.1 kg. The cord and pulley have negligible* masses compared to the blocks (they are "massless"). The hanging block *H falls as the sliding block S accelerates* to the right. Find (a) the acceleration of block *S, (b) the acceleration* of block *H, and (c) the tension in the cord.*

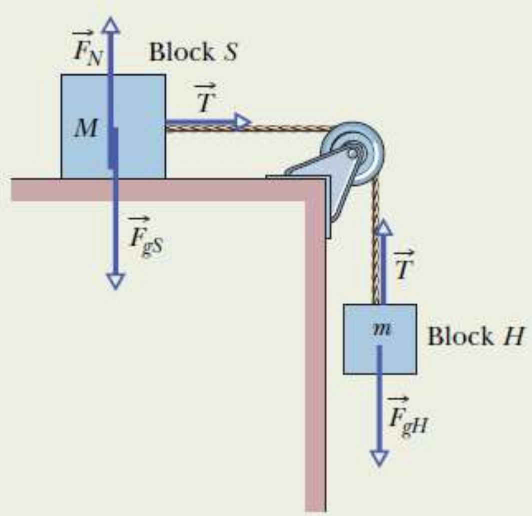


Fig. 5-13 The forces acting on the two blocks of Fig. 5-12.

Key Ideas:

1. Forces, masses, and accelerations are involved, and they should suggest Newton's second law of motion: $\vec{F} = m\vec{a}$

2. The expression $\vec{F} = m\vec{a}$ is a vector equation, so we can write it as three component equations.

3. Identify the forces acting on each of the bodies and draw free body diagrams.

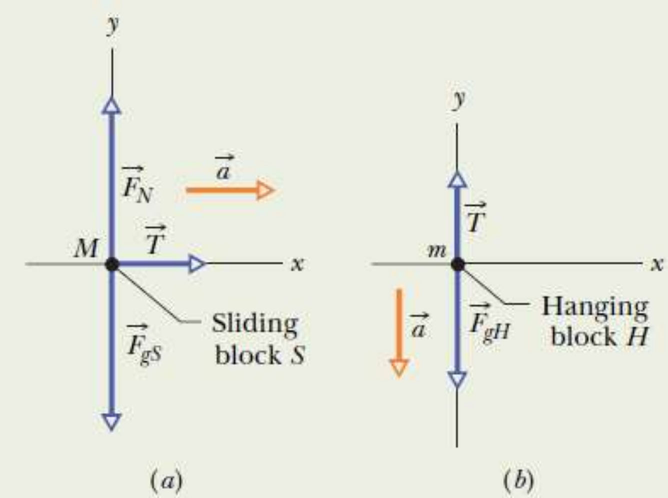


Fig. 5-14 (*a*) A free-body diagram for block *S* of Fig. 5-12. (*b*) A free-body diagram for block *H* of Fig. 5-12.

Applying Newton's Laws

Example: cont.

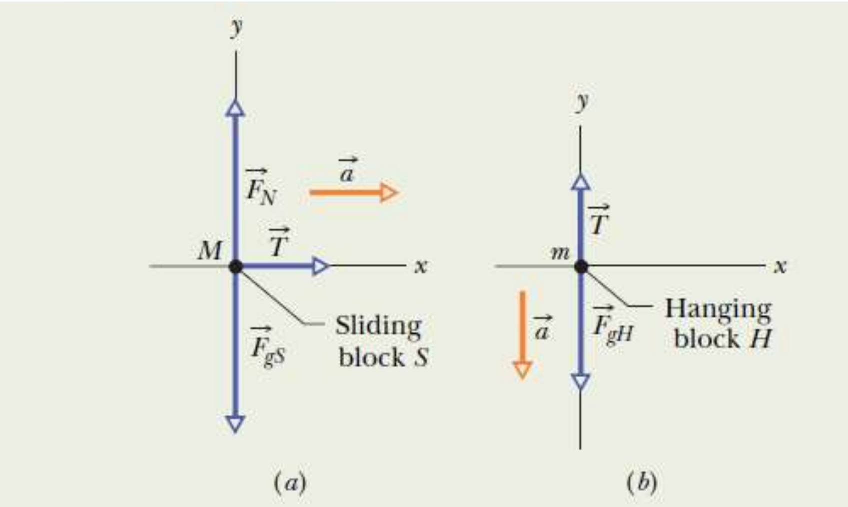


Fig. 5-14 (*a*) A free-body diagram for block *S* of Fig. 5-12. (*b*) A free-body diagram for block *H* of Fig. 5-12.

From the free body diagrams, write Newton's Second Law $\vec{F} = m\vec{a}$ in the vector form, assuming a direction of acceleration for the whole system.

Identify the net forces for the sliding and the hanging blocks:

$$F_{\text{net},x} = Ma_x \quad F_{\text{net},y} = Ma_y \quad F_{\text{net},z} = Ma_z$$

For the sliding block, *S*, which does not accelerate vertically.

$$F_{\text{net},y} = Ma_y \Rightarrow F_N - F_{gS} = 0 \quad \text{or} \quad F_N = F_{gS}.$$

Also, for S, in the x direction, there is only one force component, which is $F_{\text{net},x} = Ma_x \Rightarrow T = Ma.$

For the hanging block, because the acceleration is along the y axis,

$$T - F_{gH} = ma_y. \Rightarrow T - mg = -ma.$$

With some algebra,

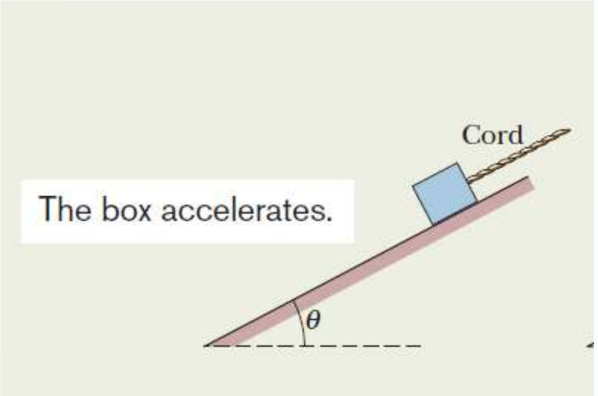
$$a = \frac{m}{M+m}g = \frac{2.1 \text{ kg}}{3.3 \text{ kg} + 2.1 \text{ kg}}(9.8 \text{ m/s}^2) = 3.8 \text{ m/s}^2$$

$$T = \frac{Mm}{M+m}g = \frac{(3.3 \text{ kg})(2.1 \text{ kg})}{3.3 \text{ kg} + 2.1 \text{ kg}}(9.8 \text{ m/s}^2) = 13 \text{ N}.$$

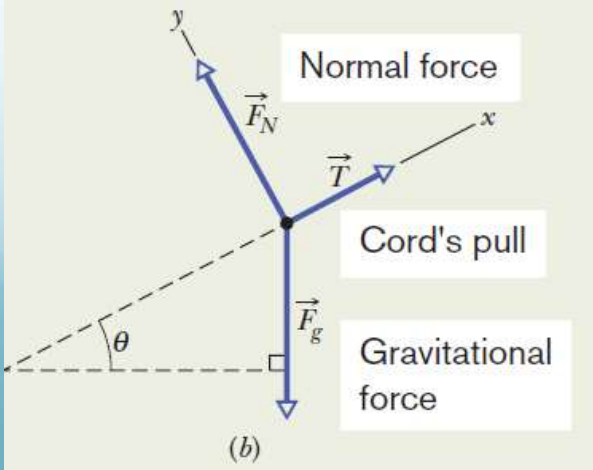
Applying Newton's Laws

Example:

In Fig. a, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at θ= 30°.The box has mass m =5.00 kg, and the force from the cord has magnitude T =25.0 N. What is the box's acceleration component a along the inclined plane?

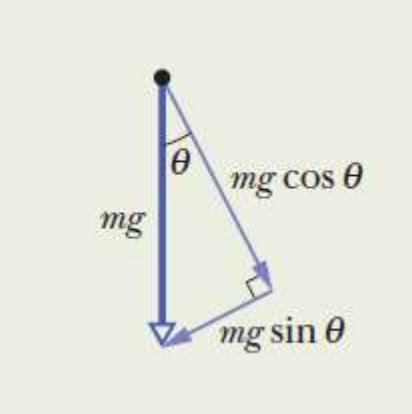


(a)



(b)

For convenience, we draw a coordinate system and a free-body diagram as shown in Fig. b. The positive direction of the x axis is up the plane. Force from the cord is up the plane and has magnitude T=25.0 N. The gravitational force is downward and has magnitude mg =(5.00 kg)(9.8 m/s2) =49.0 N. Also, the component along the plane is down the plane and has magnitude mg sin θ as indicated in the following figure. To indicate the direction, we can write the down-the-plane component as *-mg sin θ*.



Using Newton's Second Law, we have :

$$T - mg \sin\theta = ma.$$

which gives: $a = 0.100 \text{ m/s}^2,$

The positive result indicates that the box accelerates up the plane.

Applying Newton's Laws

Example:

In Fig. 5-17*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

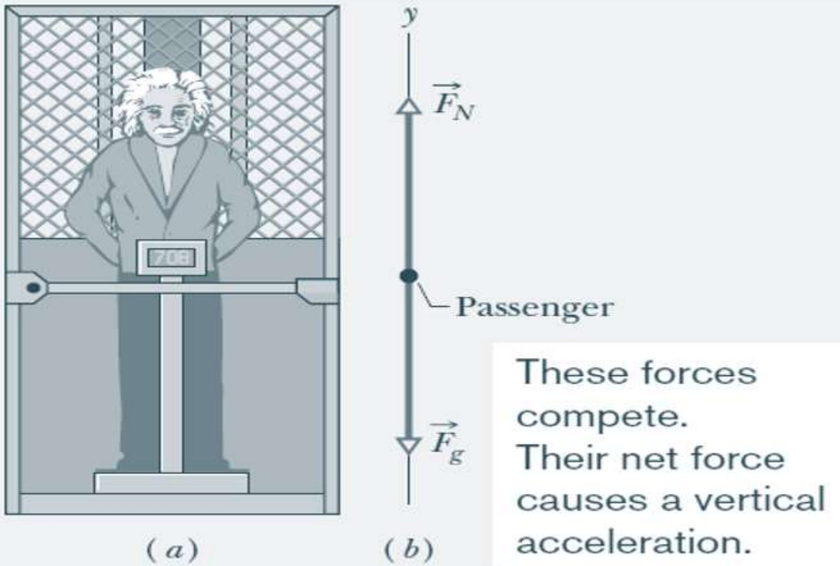


Fig. 5-17 (*a*) A passenger stands on a platform scale that indicates either his weight or his apparent weight. (*b*) The free-body diagram for the passenger, showing the normal force $\vec{F}_N$ on him from the scale and the gravitational force $\vec{F}_g$.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

- The reading is equal to the magnitude of the normal force on the passenger from the scale.

- We can use Newton's Second Law only in an inertial frame. If the cab accelerates, then it is *not an inertial frame*. So we choose the ground to be our inertial frame and make any measure of the passenger's acceleration relative to it.

Calculations: Because the two forces on the passenger and his acceleration are all directed vertically, along the y axis in Fig. 5-17*b*, we can use Newton's second law written for y components ($F_{\text{net},y} = ma_y$) to get

$$F_N - F_g = ma$$

$$F_N = F_g + ma.$$

This tells us that the scale reading, which is equal to F_N, depends on the vertical acceleration. Substituting mg for F_g gives us

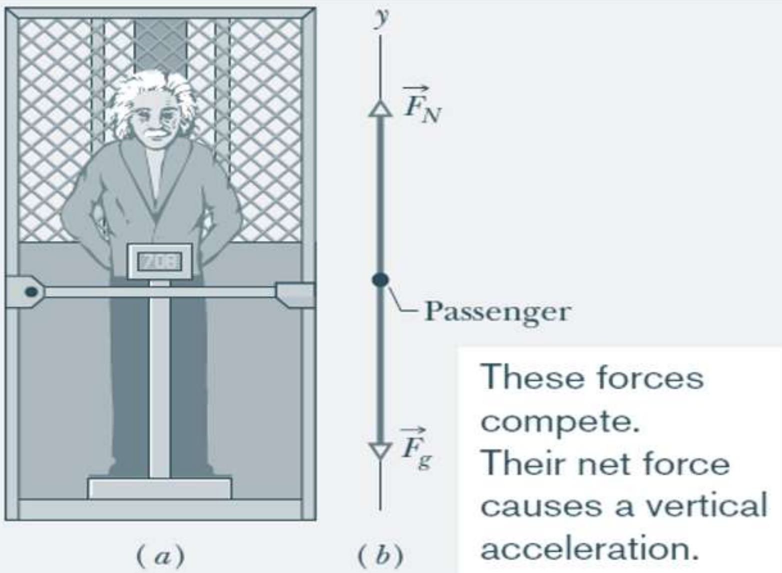
$$F_N = m(g + a) \quad \text{(Answer)} \qquad (5\text{-}28)$$

for any choice of acceleration a.

Applying Newton's Laws

Example: cont.

In Fig. 5-17*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.



(*a*) (*b*)

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

For any constant velocity (zero or otherwise), the acceleration a of the passenger is zero.

Calculation: Substituting this and other known values into Eq. 5-28, we find

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 0) = 708 \text{ N.} \qquad \text{(Answer)}$$

This is the weight of the passenger and is equal to the magnitude F_g of the gravitational force on him.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

Calculations: For $a = 3.20 \text{ m/s}^2$, Eq. 5-28 gives

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 3.20 \text{ m/s}^2) = 939 \text{ N,} \qquad \text{(Answer)}$$

and for $a = -3.20 \text{ m/s}^2$, it gives

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 - 3.20 \text{ m/s}^2) = 477 \text{ N.} \qquad \text{(Answer)}$$

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger, and what is the magnitude $a_{p,cab}$ of his acceleration as measured in the frame of the cab? Does $\vec{F}_{net} = m\vec{a}_{p,cab}$?

Calculation: The magnitude F_g of the gravitational force on the passenger does not depend on the motion of the passenger or the cab; so, from part (b), F_g is 708 N. From part (c), the magnitude F_N of the normal force on the passenger during the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

$$F_{net} = F_N - F_g = 939 \text{ N} - 708 \text{ N} = 231 \text{ N,} \qquad \text{(Answer)}$$

during the upward acceleration. However, his acceleration $a_{p,cab}$ relative to the frame of the cab is zero. Thus, in the noninertial frame of the accelerating cab, F_{net} is not equal to $ma_{p,cab}$, and Newton's second law does not hold.

Key contents:

* Newton’s laws
* Some often-seen forces
* Inertial reference frames
* Applying Newton’s laws