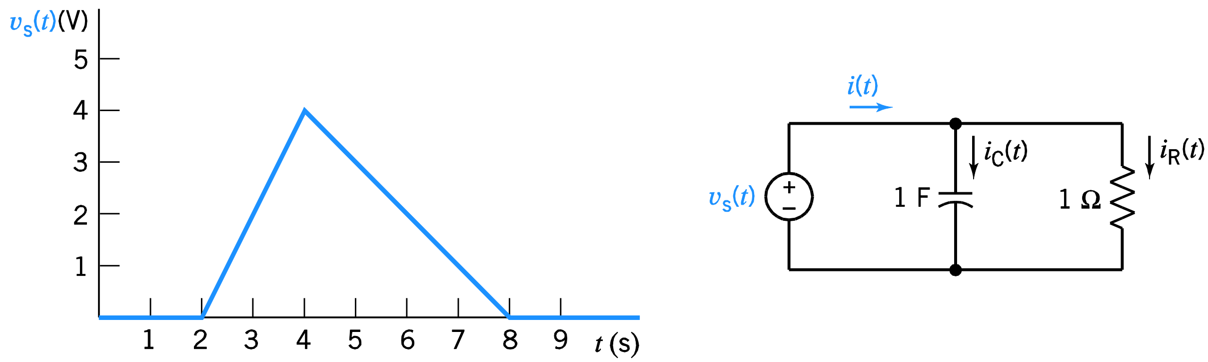
Chapter 7 Exercises

Exercise 7.2-1 Determine the current $i(t)$ for $t > 0$ for the circuit of Figure E 7.2-1*b* when $v_s(t)$ is the voltage shown in Figure E 7.2-1*a*.
Hint: Determine $i_C(t)$ and $i_R(t)$ separately, then use KCL.



(a) (b)

Figure E 7.2-1

Answer:

$$v(t)=\begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

$$i_C(t)=1\frac{d}{dt}v_s(t)=\begin{cases} 2 & 2<t<4 \\ -1 & 4<t<8 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad i_R(t)=1\,v_s(t)=\begin{cases} 2t-4 & 2<t<4 \\ 8-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

so

$$i(t)=i_C(t)+i_R(t)=\begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Exercise 7.3-1 A 200-μF capacitor has been charged to 100 V. Find the energy stored by the capacitor. Find the capacitor voltage at $t = 0^+$ if $v(0^-) = 100$ V.
Answer: $w(1) = 1$ J and $v(0^+) = 100$ V

Solution:

$$\mathcal{W} = \frac{Cv^2}{2} = \frac{1}{2}\left(2\times 10^{-4}\right)(100)^2 = \underline{1\text{ J}}$$

$$v_c\left(0^+\right) = v_c\left(0^-\right) = \underline{100\text{ V}}$$

Exercise 7.3-2 A constant current $i = 2$ A flows into a capacitor of 100μF after a switch is closed at $t = 0$. The voltage of the capacitor was equal to zero at $t = 0^-$. Find the energy stored at **(a)** $t = 1$ s and **(b)** $t = 100$ s.
Answer: $w(1) = 20$ kJ and $w(100) = 200$ MJ

Solution:

(a)

$$\mathcal{W}(t) = \mathcal{W}(0) + \int_0^t vi\,dt$$

First, $\mathcal{W}(0) = 0$ since $v(0) = 0$

Next, $v(t) = v(0) + \frac{1}{C}\int_0^t i\,dt = 10^4\int_0^t 2\,dt = \underline{2\times 10^4 t}$

$$\therefore\ \mathcal{W}(t) = \int_0^t \left(2\times 10^4\right)t\,(2)dt = 2\times 10^4 t^2$$

$$\mathcal{W}(1\text{s}) = 2\times 10^4\text{ J} = \underline{20\text{ kJ}}$$

(b)

$$\mathcal{W}(100\text{s}) = 2\times 10^4 (100)^2 = 2\times 10^8\text{ J} = \underline{200\text{ MJ}}$$

Exercise 7.4-1 Find the equivalent capacitance for the circuit of Figure E 7.4-1

Answer: $C_{eq} = 4$ mF

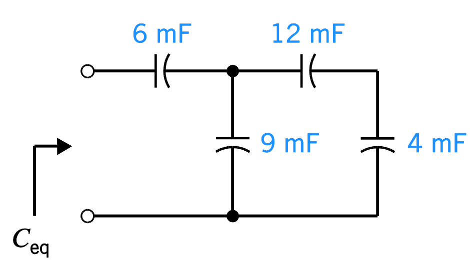
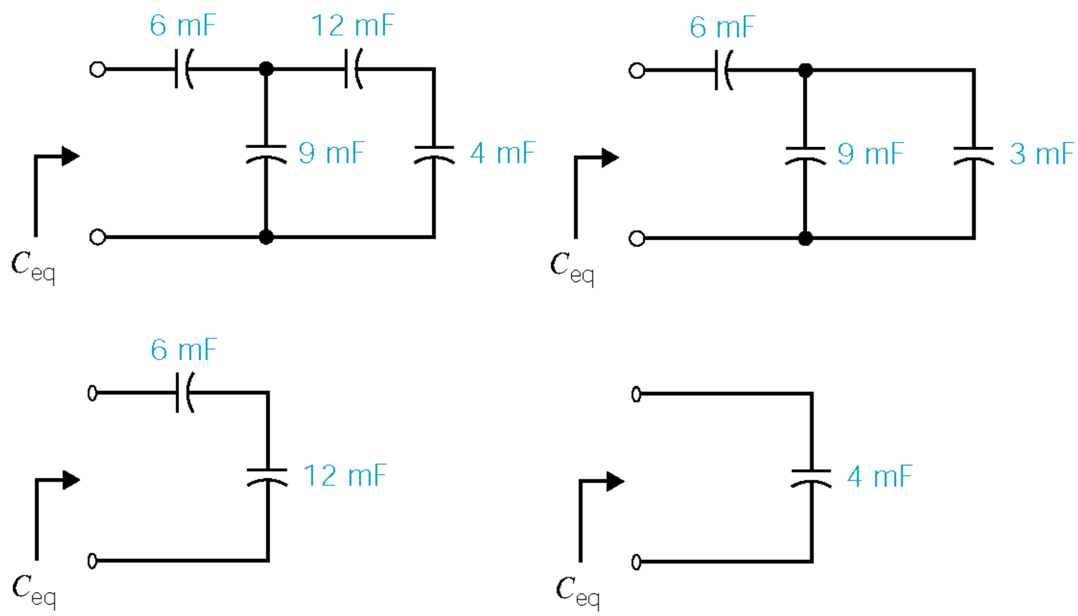


Figure E 7.4-1

Solution:



Exercise 7.4-2 Determine the equivalent capacitance C_{eq} for the circuit shown in Figure E 7.4-2.

Answer: 10/19 mF

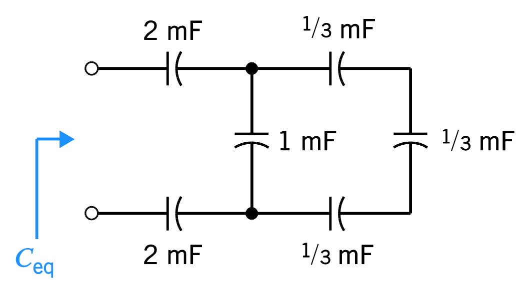


Figure E 7.4-2

Solution:

$$C_{eq1} = \frac{1}{\frac{1}{1/3}+\frac{1}{1/3}+\frac{1}{1/3}} = \frac{1}{9}, \quad C_{eq2} = 1 + C_{eq1} = \frac{10}{9}, \quad C_{eq} = \frac{1}{\frac{1}{2}+\frac{1}{2}+\frac{1}{C_{eq2}}} = \frac{1}{\frac{1}{2}+\frac{1}{2}+\frac{9}{10}} = \frac{1}{\frac{19}{10}} = \frac{10}{19}\ \text{mF}$$

Exercise 7.5-1 Determine the voltage $v(t)$ for $t > 0$ for the circuit of Figure E 7.5-1*b* when $i_s(t)$ is the current shown in Figure E 7.5-1*a*.

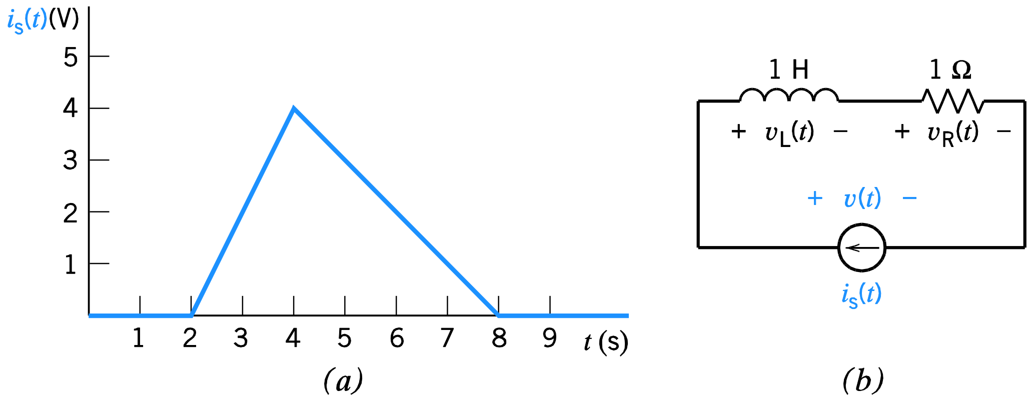


Figure E 7.5-1*b*

Hint: Determine $v_L(t)$ and $v_R(t)$ separately, then use KVL.

Answer:
$$v(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Solution:

$$v_L(t) = 1\frac{d}{dt}i_s(t) = \begin{cases} 2 & 2<t<4 \\ -1 & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

and

$$v_R(t) = 1\,i_s(t) = \begin{cases} 2t-4 & 2<t<4 \\ 8-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

so

$$v(t) = v_L(t) + v_R(t) = \begin{cases} 2t-2 & 2<t<4 \\ 7-t & 4<t<8 \\ 0 & \text{otherwise} \end{cases}$$

Exercise 7.7-1 Find the equivalent inductance of the circuit of Figure E 7.7-1.
Answer: L_{eq} = 14 mH

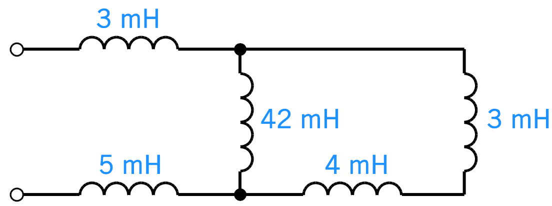
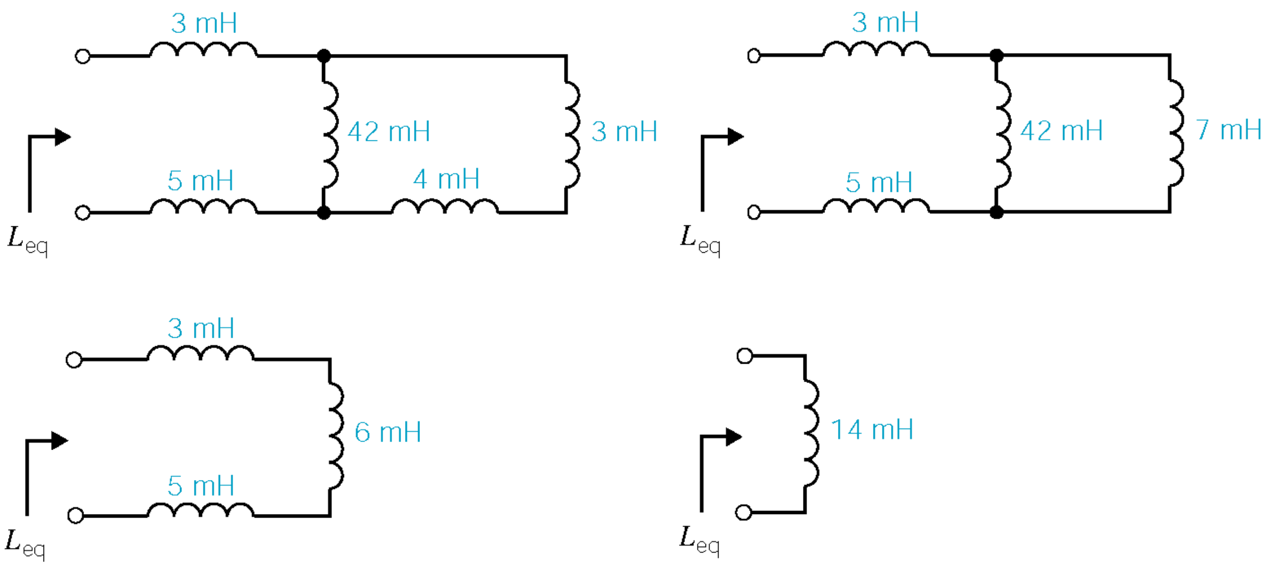


Figure E 7.7-1

Solution:



Exercise 7.7-2 Find the equivalent inductance of the circuit of Figure E 7.7-2.

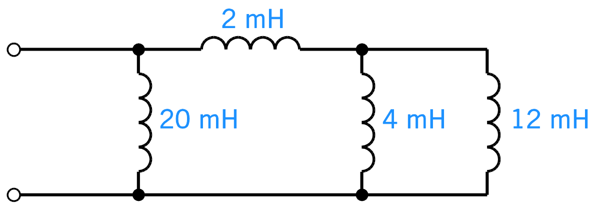
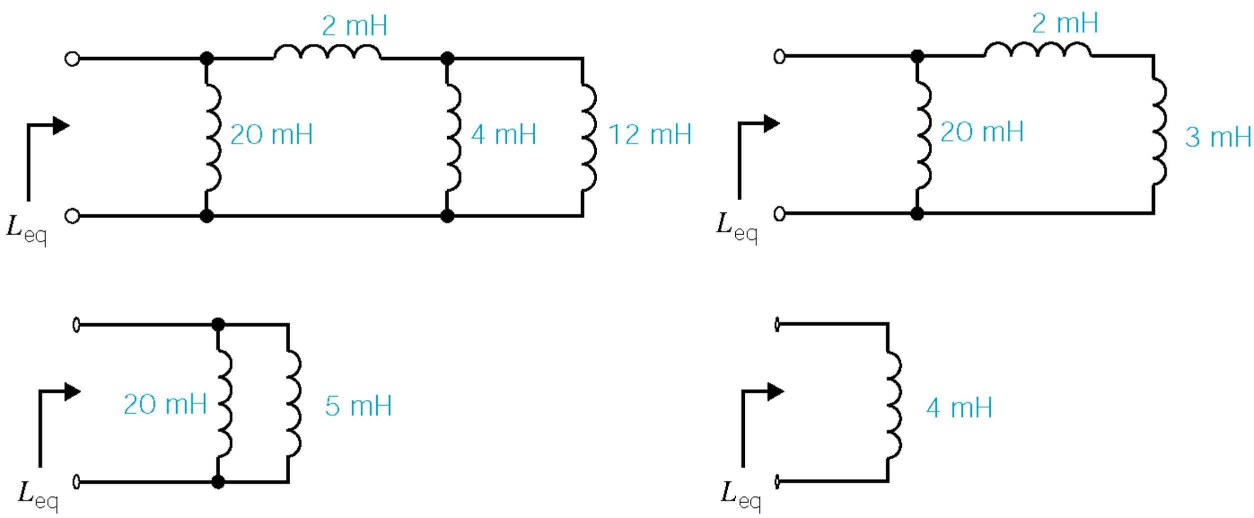


Figure E 7.7-2

Solution:



Section 7-2: Capacitors

P 7.2-1
Solution:

$$v(t) = v(0)+\frac{1}{C}\int_0^t i(\tau)\,d\tau \quad and \quad q = Cv$$

In our case, the current is constant so $\int_0^t i(\tau)\,d\tau$.

$$\therefore\; Cv(t) = Cv(0)+i\,t$$

$$\therefore\; t=\frac{q-Cv(0)}{i}=\frac{180\times10^{-6}-\left(20\times10^{-6}\right)(5)}{30\times10^{-3}}\;\underline{=2.7\text{ ms}}$$

P 7.2-2
Solution:

$$i(t)=C\frac{d}{dt}v(t)=\frac{1}{8}\frac{d}{dt}12\cos\left(2t+30^\circ\right)=\frac{1}{8}(12)(-2)\sin\left(2t+30^\circ\right)=3\cos\left(2t+120^\circ\right)\text{ A}$$

P 7.2-3
Solution:

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = \frac{1}{2\times10^{-12}}\int_0^t i(\tau)\,d\tau - 10^{-3}$$

$0 < t < 2\times10^{-9}$ $\quad i_s(t) = 0 \;\Rightarrow\; v(t) = \frac{1}{2\times10^{-12}}\int_0^t 0\,d\tau - 10^{-3} = -10^{-3}$

$2\times10^{-9} < t < 3\times10^{-9}$ $\quad i_s(t) = 4\times10^{-6}$ A

$$\Rightarrow\; v(t) = \frac{1}{2\times10^{-12}}\int_{2\text{ns}}^t \left(4\times10^{-6}\right)d\tau - 10^{-3} = -5\times10^{-3} + \left(2\times10^{6}\right)t$$

In particular, $v\left(3\times10^{-9}\right) = -5\times10^{-3} + \left(2\times10^{6}\right)\left(3\times10^{-9}\right) = 10^{-3}$

$3\times10^{-9} < t < 5\times10^{-9}$ $\quad i_s(t) = -2\times10^{-6}$ A

$$\Rightarrow\; v(t) = \frac{1}{2\times10^{-12}}\int_{3\text{ns}}^t \left(-2\times10^{-6}\right)d\tau + 10^{-3} = 4\times10^{-3} - \left(10^{6}\right)t$$

In particular, $v\left(5\times10^{-9}\right) = 4\times10^{-3} - \left(10^{6}\right)\left(5\times10^{-9}\right) = -10^{-3}$ V

$5\times10^{-9} < t$ $\quad i_s(t) = 0 \;\Rightarrow\; v(t) = \frac{1}{2\times10^{-12}}\int_{5\text{ns}}^t 0\,d\tau - 10^{-3} = -10^{-3}$ V

P 7.2-4
Solution:

(a)

$$i(t) = C\frac{d}{dt}v(t) = \begin{cases} 0 & 0 < t < 2 \\ \text{x(t)} & 2 < t < 6 \\ 0.8 & 6 < t \end{cases}$$

(b)

$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0) = 2\int_0^t i(\tau)\,d\tau$$

For $0 < t < 2$, $i(t) = 0$ A so $\quad v(t) = 2\int_0^t 0\,d\tau + 0 = 0$ V

For $2 < t < 6$, $i(t) = 0.2\,t - 0.4$ V so

$$v(t) = 2\int_1^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = 0.2t^2 - 0.8t + 0.8 \text{ V}$$

$v(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2$ V.

For $6 < t$, $i(t) = 0.8$ A so $\quad v(t) = 2\int_6^t 0.8\,d\tau + 3.2 = 1.6t - 6.4$ V

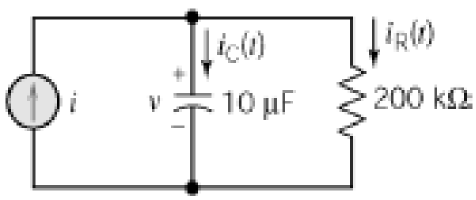
P 7.2-5
Solution:

$$v(t)=v(0)+\frac{1}{C}\int_0^t i(\tau)\,d\tau = 30+2.5\times10^4\int_0^t \left(6\times10^{-3}\right)e^{-6\tau}\,d\tau$$

$$=30+150\int_0^t e^{-6\tau}\,d\tau$$

$$=30+150\left[-\frac{1}{6}e^{-6\tau}\right]_0^t = \underline{55-25e^{-6t}\ \mathrm{V}}$$

P 7.2-6
Solution:



$$i_R=\frac{v}{200\times10^3}=\frac{1}{40}\left(1-2e^{-2t}\right)\times10^{-3}=25\left(1-2e^{-2t}\right)\ \mu\mathrm{A}$$

$$i_C = C\frac{dv}{dt}=\left(10\times10^{-6}\right)(-2)\left(-10\,e^{-2t}\right)=200\,e^{-2t}\ \mu\mathrm{A}$$

$$i = i_R+i_C = 200\,e^{-2t}+25-50\,e^{-2t}$$

$$=\underline{25+150e^{-2t}\ \ \mu\mathrm{A}}$$

P 7.2-7
Solution:

$$v(t)=1\int_0^t i(t)\,dt+2$$

$$=2 \qquad \text{for} \qquad 0\le t\le 2$$

$$=1\int_2^t 4\,dt+2=4(t-2)+2=4t-6 \qquad \text{for} \qquad 2\le t\le 3$$

$$=1\int_2^3 4\,dt+1\int_3^t -4\,dt+2=4-4(t-3)+2=-4t+18 \qquad \text{for} \qquad 3\le t\le 4$$

$$=1\int_2^3 4\,dt+1\int_3^4 -4\,dt+2=2 \qquad \text{for} \qquad t\ge 4$$

In summary

$$v(t)=\begin{cases} 2 & 0\le t\le 2 \\ 4t-6 & 2\le t\le 3 \\ -4t+18 & 3\le t\le 4 \\ 2 & 4\le t \end{cases}$$

P 7.2-8
Solution:

$$v(t)=v(0)+\frac{1}{C}\int_0^t i_s(t)\ dt=-6+5\int_0^t i_s(t)\ dt$$

For $0\le t\le 0.5\ \left(i_s(t)=8t\ \text{ for }\ 0\le t\le 0.5\right)$

$$v(t)=-6+5\int_0^t 8\tau\ d\tau=-6+40\left(\frac{\tau^2}{2}\right)\Bigg|_0^t=-6+20t^2$$

For example $v(0)=-6,\ v\left(\frac{1}{4}\right)=-4.75,\ v\left(\frac{1}{2}\right)=-1$

For $0.5\le t\le 1$

$$v(t)=-1+5\int_{0.5}^t 4\ d\tau=-1+20(t-05)=20t-11$$

For example $v(0.5)=-1,\ v(1)=9$

For $1\le t$ $$v(t)=9+5\int_1^t 0\ d\tau=9$$

In summary $$v(t)=\begin{cases}-6+20t^2 & 0\le t\le 0.5\text{ s}\\ 20t-11 & 0.5\le t\le 1\text{ s}\\ 9 & t\ge 1\text{ s}\end{cases}$$

P 7.2-9
Solution:
Representing $v_s(t)$ using equations of the straight line segments gives

$$v_s(t)=\begin{cases}0 & t\le 1\\ 32t-32 & 1\le t\le 2\\ -16t+64 & 2\le t\le 4\\ 0 & 4\le t\end{cases}$$

Use KCL to get

$$i(t)=\frac{1}{2}\frac{d}{dt}v_s(t)+\frac{v_s(t)}{5}=\begin{cases}0 & t\le 1\\ 16+\dfrac{32t-32}{8} & 1\le t\le 2\\ -8+\dfrac{64-16t}{8} & 2\le t\le 4\\ 0 & t\ge 4\end{cases}$$

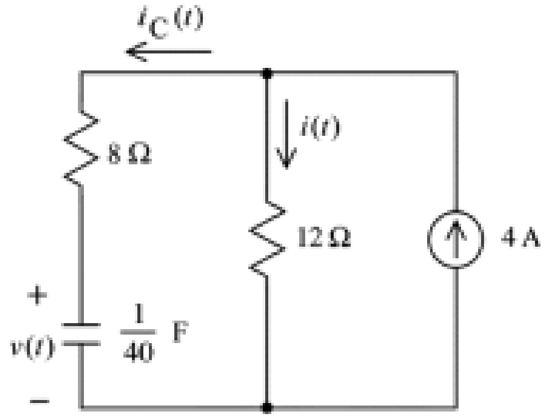
$$i(t)=\begin{cases}0 & t\le 1\\ 4t+12 & 1\le t\le 2\\ -2t & 2\le t\le 4\\ 0 & t\ge 4\end{cases}$$

P 7.2-10
Solution:

$$i_C(t) = \frac{1}{40}\frac{d}{dt}v(t)$$
$$= \frac{1}{40}\left(+20e^{-2t}\right)$$
$$= 0.5e^{-2t} \text{ A for } t > 0$$

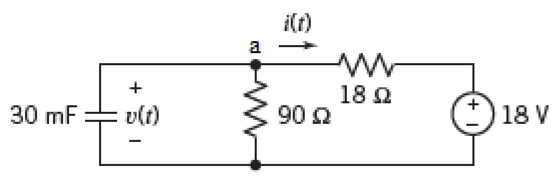
Apply KCL to get

$$i(t) = 4 - i_C(t) = 4 - 0.5e^{-2t} \text{ A for } t > 0$$



P 7.2-11
Solution:
Apply KCL to node a to get

$$i(t) + \frac{v(t)}{90} + 0.030\frac{d}{dt}v(t) = 0$$
$$i(t) = -\frac{10-8e^{-5t}}{90} - 0.030\frac{d}{dt}\left(10-8e^{-5t}\right) = -\frac{1}{9} - \frac{10}{9}e^{-5t} \text{ A for t} > 0$$



P 7.2-12
Solution:

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i_s(\tau)\,d\tau + v(t_0) = \frac{1}{\frac{1}{3}}\int_0^t i_s(\tau)\,d\tau - 12$$

$v(t) = 3\int_0^t 4\,d\tau - 12 = 12\,t - 12 \quad \text{for} \quad 0 < t < 4$ In particular, $v(4) = 36$ V.

$v(t) = 3\int_4^t (-2)\,d\tau + 36 = 60 - 6t \quad \text{for} \quad 4 < t < 10$ In particular, $v(10) = 0$ V.

$$v(t) = 3\int_{10}^t 0\,d\tau + 0 = 0 \quad \text{for} \quad 10 < t$$

P7.2-13
Solution:
The capacitor voltage is related to the capacitor current by

$$v(t)=\frac{1}{C}\int_0^t i(\tau)\,d\tau+v(0)$$

That is:

$$4-1.25e^{-1.2t}=\frac{1}{C}\int_0^t 3.75e^{-1.2\tau}\,d\tau+v(0)=\frac{3.75}{C(-1.2)}e^{-1.2\tau}\Big|_0^t+v(0)=\frac{-3.125}{C}\left(e^{-1.2t}-1\right)+v(0)$$

Equating the coefficients of $e^{-1.2t}$ gives

$$12.5=\frac{3.125}{C}\quad\Rightarrow\quad C=\frac{3.125}{12.5}=0.25=250\text{ mF}$$

P7.2-14
Solution:

Apply KVL to the mesh to get

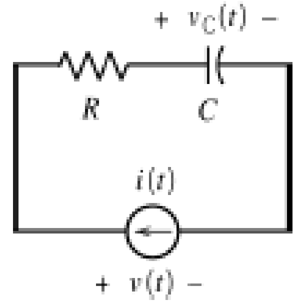
$$v(t)=8i(t)+v_C(t)=8i(t)+\left[\frac{1}{0.1}\int_0^t i(\tau)\,d\tau+v(0)\right]$$

That is,

$$\begin{aligned}v(t)&=8\left(3e^{-25t}\right)+\frac{1}{0.1}\int_0^t 3e^{-25\tau}\,d\tau-2\\&=24e^{-25t}+\frac{3}{0.1(-25)}\left(e^{-25t}-1\right)-2\\&=24e^{-25t}-1.2\left(e^{-25t}-1\right)-2\\&=22.8e^{-25t}-0.8\text{ V for }t>0\end{aligned}$$

P7.2-15
Solution:

Apply KVL to the mesh to get



$$v(t) = Ri(t) + v_C(t) = Ri(t) + \left[\frac{1}{C}\int_0^t i(\tau)\,d\tau + v(0)\right]$$

That is

$$9.8e^{-25t} + 0.6 = R\left(5e^{-25t}\right) + \left[\frac{1}{C}\int_0^t 5e^{-25\tau}d\tau - 2\right]$$

$$= 5Re^{-25t} + \frac{5}{C(-25)}\left(e^{-25t} - 1\right) - 2 = 5\left(R - \frac{1}{25C}\right)e^{-25t} + \frac{1}{5C} - 2$$

Equating coefficients gives

$$0.6 = \frac{1}{5C} - 2 \quad \Rightarrow \quad C = 0.08 = 80 \text{ mF}$$

and $$9.8 = 5\left(R - \frac{1}{25C}\right) = 5\left(R - \frac{1}{25(0.08)}\right) = 5(R - 0.5) \quad R = 2.46\ \Omega$$

P7.2-16
Solution:
Apply KCL at either node to get

$$0.3 - 1.6e^{-2t} = \frac{3 + 4e^{-2t}}{R} + C\frac{d}{dt}\left(3 + 4e^{-2t}\right)$$

$$= \frac{3 + 4e^{-2t}}{R} + (-2)4Ce^{-2t} = \frac{3}{R} + \left(\frac{4}{R} - 8C\right)e^{-2t}$$

Equating coefficients:

$$0.3 = \frac{3}{R} \quad \Rightarrow \quad R = 10\ \Omega \qquad \text{and} \qquad -1.6 = \frac{4}{10} - 8C \quad \Rightarrow \quad C = 0.25 \text{ F}$$

P7.2-17

Solution:

At $t = 0.5$ s

$$v(0.5) = 2(0.5) + 8.6 = 9.6 \text{ V}$$

For $0.5 \le t \le 1.5$

$$v(t) = \frac{1}{0.25}\int_{0.5}^{t} 2\,d\tau + 9.6 = 8\tau\Big|_{0.5}^{t} + 9.6 = 8(t-0.5) + 9.6 = 8t + 5.6 \text{ V}$$

At $t = 1.5$ s

$$v(1.5) = 8(1.5) + 5.6 = 17.6 \text{ V}$$

For $t \ge 1.5$

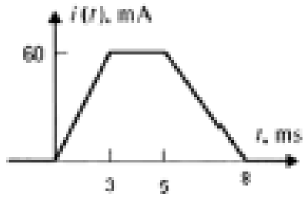
$$v(t) = \frac{1}{0.25}\int_{1.5}^{t} 0\,d\tau + 17.6 = 17.6$$

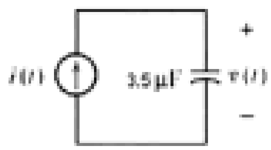
Checks:

At $t = 1.0$ s $\quad i(t) = \frac{1}{4}\frac{d}{dt}v(t) = \frac{1}{4}\frac{d}{dt}(8t + 5.6) = \frac{1}{4}(8) = 2 \text{ A}$

At $t = 0.5$ s $\quad v(0.5) = 8(0.5) + 5.6 = 9.6 \text{ V}$

P7.2-18
Solution:





$$v(t)=v(0)+\frac{1}{C}\int_0^t i(\tau)\,d\tau=v(0)+\frac{\text{"area under the curve"}}{C}=-30+\frac{\text{"area under the curve"}}{3.5\times10^{-6}}$$

$$v(0.002)=-30+\frac{\frac{1}{2}\left(40\times10^{-3}\right)\left(2\times10^{-3}\right)}{3.5\times10^{-6}}=-30+\frac{40}{3.5}=-18.6\text{ V}$$

(When calculating the value of $v(0.002)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 2 ms = 0.002 s.)

$$v(0.004)=-30+\frac{\frac{1}{2}\left(60\times10^{-3}\right)\left(3\times10^{-3}\right)+\left(60\times10^{-3}\right)\left(1\times10^{-3}\right)}{3.5\times10^{-6}}=-30+\frac{90+60}{3.5}=12.9\text{ V}$$

(When calculating the value of $v(0.004)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 4 ms = 0.004 s.)

$$v(0.008)=-30+\frac{\frac{1}{2}\left(60\times10^{-3}\right)\left(3\times10^{-3}\right)+\left(60\times10^{-3}\right)\left(2\times10^{-3}\right)+\frac{1}{2}\left(60\times10^{-3}\right)\left(3\times10^{-3}\right)}{3.5\times10^{-6}}=55.7\text{ V}$$

(When calculating the value of $v(0.008)$, "area under the curve" indicates the area under the graph of $i(t)$ versus t corresponding to the time interval 0 to 8 ms = 0.008 s.)

Section 7-3: Energy Storage in a Capacitor

P 7.3-1
Solution:

Given
$$i(t)=\begin{cases}0 & t<1\\ 0.6(t-1) & 1<t<3\\ 1.2 & t>3\end{cases}$$

The capacitor voltage is given by

$$v(t)=\frac{1}{0.8}\int_0^t i(\tau)\,d\tau+v(0)=1.25\int_0^t i(\tau)\,d\tau+v(0)$$

For $t<1$
$$v(t)=1.25\int_0^t 0\,d\tau+0=0$$

In particular, $v(1)=0$. For $1<t<3$

$$\begin{aligned}v(t)&=1.25\int_2^t 0.6(\tau-1)\,d\tau+0\\&=1.25\left(0.3\tau^2-0.6\tau\right)\Big|_1^t\\&=1.25\left(0.3t^2-0.6t+0.9\right)\text{ V}\\&=0.375\left(t^2-2t+3\right)\text{ V}\end{aligned}$$

In particular, $v(3)=2.25$ V. For $3<t$

$$\begin{aligned}v(t)&=1.25\int_3^t 1.2\,d\tau+2.25\\&=1.6\tau\Big|_3^t+2.25\\&=(1.5t-2.25)\text{ V}\\&=1.5(t-1.5)\text{ V}\end{aligned}$$

Now the power and energy are calculated as

$$p(t)=v(t)i(t)=\begin{cases}0 & t<1\\ 0.225(t-1)\left(t^2-2t+3\right) & 1<t<3\\ 1.8(t-1.5) & 3<t\end{cases}$$

and

$$\mathbf{W}(t)=\int_0^t p(\tau)\,d\tau=\begin{cases}0 & t<1\\ 0.06(t-2)\left(t^3-2t^2+6t\right)+5 & 1<t<3\\ 0.9\left(t^2-3t\right) & 3<t\end{cases}$$

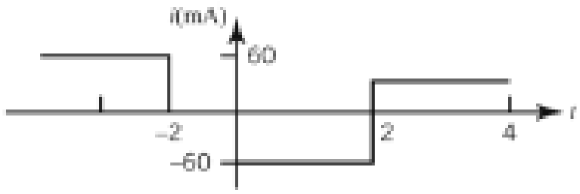
P 7.3-2
Solution:

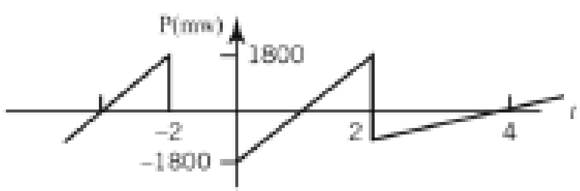
$$i_c = C\frac{dv}{dt} = \left(15\times10^{-6}\right)(-10)(-2000)e^{-2000t} = \underline{0.3e^{-2000t}\text{ A}} \Rightarrow \begin{cases} \underline{i_c(0) = 0.3\,A} \\ \underline{i_c(15ms) \;= 2.8\times10^{-14}\,A} \end{cases}$$

$$W(t) = \frac{1}{2}Cv^2(t) \quad \text{and} \quad v(0) = 10-10e^0 = 0 \Rightarrow \underline{W(0) \;= 0}$$

$$v\left(15\times10^{-3}\right) = 10-10e^{-30} = 10-9.36\times10^{-13} \cong 10 \Rightarrow \underline{W(10) = 7.5\times10^{-4}\text{ J}}$$

P 7.3-3
Solution:





$$i(t) \;=\; C\,\frac{dv_c}{dt} \text{ so read off slope of } v_c(t) \text{ to get } i(t)$$

$$p(t) \;=\; v_c(t)\,i(t) \text{ so multiply } v_c(t) \;\&\; i(t) \text{ curves to get } p(t)$$

P 7.3-4
Solution:

$$v_c(t) = v_c(0) + \frac{1}{4}\int_0^t i\,d\tau = v_c(0) + \frac{1}{4}\int_0^t 60\cos\left(10t+\frac{\pi}{6}\right)d\tau$$

$$= \left[v_c(0) - \frac{3}{2}\sin\frac{\pi}{6}\right] + \frac{3}{2}\sin\left(10t+\frac{\pi}{6}\right)$$

Now since $v_c(t)_{ave} = 0 \Rightarrow v_c(0) - \frac{3}{2}\sin\frac{\pi}{6} = 0$

$$\Rightarrow v_c(t) = \frac{3}{2}\sin\left(10t+\frac{\pi}{6}\right)\text{ V}$$

$$\therefore \text{W}_{\max} = \frac{1}{2}Cv^2_{c_{\max}} = \frac{\left(4\times10^{-6}\right)(1.5)^2}{2} = 4.5\ \mu J$$

First non-negative t for max energy occurs when :

$$10t + \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow \underline{t = \frac{\pi}{30} = 0.1047\,s}$$

P 7.3-5

Solution:

$$\text{Max. charge on capacitor} = Cv = \left(15\times10^{-6}\right)(9) = 135\ \mu C$$

$$\Delta t = \frac{\Delta q}{i} = \frac{135\times10^{-6}}{15\times10^{-6}} = \underline{9\ \text{sec}}\ \text{to charge}$$

$$\text{stored energy} = W = \frac{1}{2}Cv^2 = \frac{1}{2}\left(15\times10^{-6}\right)(9)^2 = \underline{607.5\ \mu J}$$

P 7.3-6

Solution:

We have $v(0^+) = v(0^-) = 3$ V

$$v_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + v_c(0) = 5\int_0^t 3\,e^{5t}\,dt + 3 = 3\left(e^{5t}-1\right)+3 = 3\,e^{5t}\ \text{V},\ 0<t<1$$

a) $$v(t) = v_R(t) + v_c(t) = 5\,i(t) + v_c(t) = 15\,e^{5t} + 3\,e^{5t} = \underline{18\,e^{5t}\ \text{V}},\ 0<t<1$$

b) $$W(t) = \frac{1}{2}Cv_c^2(t) = \frac{1}{2}\times0.2\left(3e^{5t}\right)^2 = 0.9e^{10t}\ \text{J} \Rightarrow \begin{cases} W(t)\big|_{t=0.2s} = \underline{6.65\ \text{J}} \\ W(t)\big|_{t=0.8s} = \underline{2.68\ \text{kJ}} \end{cases}$$

P7.3-7

Solution:

The capacitor acts like an open circuit when this circuit is at steady state.

(a) When the switch is closed and the circuit is at steady state, $v(t) = 6$ V . The energy stored by the capacitor is $W = \frac{1}{2}\left(2.2\times10^{-6}\right)\left(6^2\right) = 39.6\ \ \mu\text{J}$.

(b) When the switch is closed and the circuit is at steady state, $v(t) = 12$ V . The energy stored by the capacitor is $W = \frac{1}{2}\left(2.2\times10^{-6}\right)\left(12^2\right) = 158.4\ \ \mu\text{J}$.

Section 7-4: Series and Parallel Capacitors

P 7.4-1
Solution:

$$3\mu\text{F} \,|\, 6\mu\text{F} = 9\mu\text{F}$$

$$9\mu\text{F in series with } 6\mu\text{F} = \frac{9\mu\text{F}\cdot 6\mu\text{F}}{9\mu\text{F}+6\mu\text{F}} = 3.6\mu\text{F}$$

$$i(t) = 3.6\mu\text{F}\frac{d}{dt}(9\cos 100t) = (3.6\times 10^{-6})(9)(100)(-\sin 100t)\text{A} = \underline{-3.24\sin 100t \text{ mA}}$$

P 7.4-2
Solution:

$$4\ \mu\text{F in series with } 4\ \mu\text{F} = \frac{4\ \mu\text{F}\times 4\ \mu\text{F}}{4\ \mu\text{F}+4\ \mu\text{F}} = 2\ \mu\text{F}$$

$$2\ \mu\text{F} \,\|\, 2\ \mu\text{F} = 4\ \mu\text{F}$$

$$4\ \mu\text{F in series with } 4\ \mu\text{F} = 2\ \mu\text{F}$$

$$i(t) = (2\times 10^{-6})\frac{d}{dt}(5+3\,e^{-250t}) = (2\times 10^{-6})\,(0+3(-250)\,e^{-250t})\ \text{A} = \underline{-1.5\,e^{-250t}\ \text{mA}}$$

P 7.4-3
Solution:

$$C \text{ in series with } C = \frac{C\cdot C}{C+C} = \frac{C}{2}$$

$$C \,\|\, C \,\|\, \frac{C}{2} = \frac{5}{2}C$$

$$C \text{ in series with } \frac{5}{2}C = \frac{C\,\frac{5}{2}C}{C+\frac{5}{2}C} = \frac{5}{7}C$$

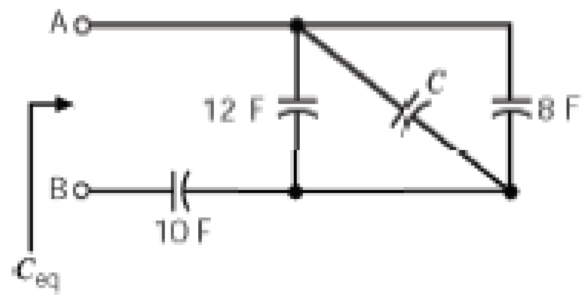
$$(25\times 10^{-3})\cos 250t = \left(\frac{5}{7}C\right)\frac{d}{dt}(14\sin 250t) = \left(\frac{5}{7}C\right)(14)(250)\cos 250t$$

$$\text{so } 25\times 10^{-3} = 2500\,C \Rightarrow C = 10\times 10^{-6} = 10\ \mu\text{F}$$

P7.4-4
Solution:
The 16 F capacitor is in series with a parallel combination of 4 F and 12 F capacitors. The capacitance of the equivalent capacitor is

$$\frac{16(4+12)}{16+(4+12)} = 8 \text{ F}$$

The 30 F capacitor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



Then

$$8 = C_{eq} = \frac{10(12+C+8)}{10+(12+C+8)} \quad \Rightarrow \quad C = 20 \text{ F}$$

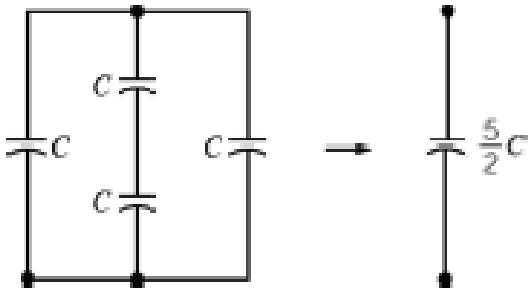
(Checked using LNAP 6/26/04)

P 7.4-5
Solution:

$$C_{eq} = \frac{1}{\frac{1}{70}+\frac{1}{25+20}+\frac{1}{40}+\frac{1}{50+70}} = 14.3 \text{ F}$$

P 7.4-6

Solution: First



Then

$$50 = C_{\text{eq}} = \frac{1}{\frac{1}{C}+\frac{2}{5\,C}+\frac{2}{5\,C}} \quad \Rightarrow \quad C = 90 \text{ mF}$$

(Checked using LNAP 6/26/04)

P 7.4-7

Solution:

(a) The energy stored in the 60 mF capacitor is $w_1 = \frac{1}{2}(0.060)3.6^2 = 0.3888 \text{ W}$ and the energy stored in the 20 mF capacitor is $w_2 = \frac{1}{2}(0.020)3.6^2 = 0.1296 \text{ J}$.

(b) One second after the switch opens, the voltage across the capacitors is $3.6e^{-2.5} = 0.2955 \text{ V}$. Then $w_1 = 2.620$ mJ and $w_2 = 0.873$ mJ.

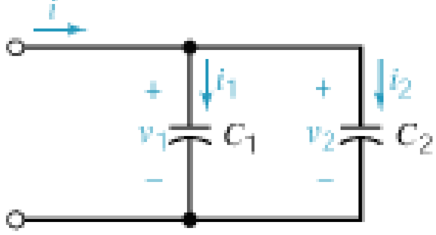
Next $C_{eq} = 0.06 + 0.02 = 80$ mF.

(c) $w_{eq} = \frac{1}{2}(0.08)3.6^2 = 0.5184 \text{ J} = w_1 + w_2$

(d) $w_{eq} = \frac{1}{2}(0.08)(0.2955)^2 = 3.493 \text{ mJ} = w_1 + w_2$

P 7.4-8

Solution:



$$v_1 = v_2 \;\Rightarrow\; \frac{dv_1}{dt} = \frac{dv_2}{dt} \;\Rightarrow\; \frac{i_1}{C_1} = \frac{i_2}{C_2} \Rightarrow\; \underline{i_1 = \frac{C_1}{C_2}\, i_2}$$

$$\text{KCL: } i = i_1 + i_2 = \left(\frac{C_1}{C_2} + 1\right) i_2 \;\Rightarrow\; \underline{i_2 = \frac{C_2}{C_1 + C_2}\, i}$$

Section 7-5: Inductors

P 7.5-1
Solution:

Find max. voltage across coil: $v(t) = L\dfrac{di}{dt} = 250[150(400)\cos 400t]\text{ V}$

$\therefore\ v_{max} = 15\times10^6\text{ V}$ thus have a field of $\dfrac{15\times10^6}{2}\text{V / m}=7.5\times10^6\text{ V/m}$

which exceeds dielectric strength in air of 4×10^6 V/m

$\therefore$ We get a discharge as the air is ionized.

P 7.5-2
Solution:

$$v = L\frac{di}{dt} + R\,i = (0.2)\ (4e^{-t} - 4te^{-t}) + 20(4te^{-t}) = \underline{0.8e^{-t} + 79.2t\,e^{-t}\text{ V}}$$

P 7.5-3
Solution:

$$v(t) = (300\times10^{-3})\frac{d}{dt}(150\times10^{-3})\sin(500t - 30°) = (0.3)(0.15)(500)\cos(500t - 30°)$$
$$= 22.5\cos(500t - 30°)$$

P 7.5-4
Solution:

$$i_L(t) = \frac{1}{5\times10^{-3}}\int_0^t v_s(\tau)\,d\tau - 2\times10^{-6}$$

for $0 < t < 1\ \mu s$ $v_s(t) = 4$ mV

$$i_L(t) = \frac{1}{5\times10^{-3}}\int_0^t 4\times10^{-3}\ d\tau - 2\times10^{-6} = \left(\frac{4\times10^{-3}}{5\times10^{-3}}\right)t - 2\times10^{-6} = 0.8 - 2\times10^{-6}\text{ A}$$

$$i_L(1\mu s) = \left(\frac{4\times10^{-3}}{5\times10^{-3}}\,(1\times10^{-6})\right) - 2\times10^{-6} = -\frac{6}{5}\times10^{-6}\text{ A} = -1.2\text{ A}$$

for $1\mu s < t < 3\ \mu s$ $v_s(t) = -1$ mV

$$i_L(t) = \frac{1}{5\times10^{-3}}\int_{1\mu s}^t\left(-1\times10^{-3}\right)d\tau - \frac{6}{5}\times10^{-6} = -\frac{1\times10^{-3}}{5\times10^{-3}}(t - 1\times10^{-6}) - \frac{6}{5}\times10^{-6} = \left(-0.2\,t - 10^{-6}\right)\text{ A}$$

$$i_L\ (3\mu s) = \left(-\frac{1\times10^{-3}}{5\times10^{-3}} + 3\times10^{-6}\right) - 1\times10^{-6} = -1.6\ \mu\text{A}$$

for $3\mu s < t$ $v_s(t) = 0$ so $i_L(t)$ remains $-1.6\ \mu$A

P 7.5-5
Solution:
In general

$$v(t) = \left(4\times10^3\right)i_s\ (t) + \left(6\times10^{-3}\right)\frac{d}{dt}\ i_s(t)$$

For 0.5<t<1 μs $\quad i_s(t) = (1)\left(\frac{1.5\times10^{-3}}{1\times10^{-6}}\right)t = 1.5\times10^3\,t \Rightarrow \frac{d}{dt}i_s(t) = 1.5\times10^3$.

Consequently

$$v(t) = (4\times10^3)\left(1.5\times10^3\right)t + 6\times10^{-3}\left(1.5\times10^3\right) = \left(6\times10^6\,t + 9\right)\ \text{V}$$

For 1μs <t< 2μs $\quad i_s(t) = 1.5\ \text{mA} \Rightarrow \frac{d}{dt}\ i_s(t) = 0.$ Consequently

$$v(t) = (4\times10^3)\left(1.5\times10^{-3}\right) + \left(6\times10^{-3}\right)\times0 = 6\ \text{V}$$

For 2μs< t < 3μs $\quad i_s(t) = 6\times10^{-3} - \left(\frac{0.5\times10^{-3}}{0.5\times10^{-6}}\right)t \quad\Rightarrow\quad \frac{d}{dt}i_s(t) = -\frac{0.5\times10^{-3}}{0.5\times10^{-6}} = -10^3$.

Consequently

$$v(t) = \left(4\times10^3\right)\left(6\times10^{-3} - 10^3\,t\right) + 6\times10^{-3}\left(-10^3\right) = 18 - \left(4\times10^6\right)t$$

When 3μs <t< 4μs $\quad i_s(t) = -0.5\times10^{-3}$ and $\frac{d}{dt}i_s(t) = 0$. Consequently

$$v(t) = \left(4\times10^3\right)\left(0.5\times10^{-3}\right) = 2\ \text{V}$$

When 4μs< t < 5μs $\quad i_s(t) = \left(\frac{0.5\times10^{-3}}{0.5\times10^{-6}}\right)t - 4\times10^{-3} \Rightarrow \frac{d}{dt}i_s(t) = 1\times10^3$

$$v(t) = \left(4\times10^3\right)\left(10^3\,t - 4\times10^{-3}\right) + \left(6\times10^{-3}\right)\left(10^3\right) = -10 + \left(4\times10^6\right)t$$

When 5μs < t, then $i_s(t) = 1\text{mA} \Rightarrow \frac{d}{dt}i_s(t) = 0$. Consequently

$$v(t) = \left(4\times10^3\right)\left(1\times10^{-3}\right) = 4\text{V}.$$

P 7.5-6
Solution:

(a)

$$v(t) = L\frac{d}{dt}i(t) = \begin{cases} 0 & 0 < t < 2 \\ 0.1 & 2 < t < 6 \\ 0 & 6 < t \end{cases}$$

(b)

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0) = 2\int_0^t v(\tau)\,d\tau$$

For $0 < t < 2$, $v(t) = 0$ V so $i(t) = 2\int_0^t 0\,d\tau + 0 = 0$ A

For $2 < t < 6$, $v(t) = 0.2\,t - 0.4$ V so

$$i(t) = 2\int_2^t (0.2\tau - 0.4)\,d\tau + 0 = \left(0.2\tau^2 - 0.8\tau\right)\Big|_2^t = 0.2t^2 - 0.8t + 0.8 \text{ A}$$

$$i(6) = 0.2\left(6^2\right) - 0.8(6) + 0.8 = 3.2 \text{ A}.$$

For $6 < t$, $v(t) = 0.8$ V so

$$i(t) = 2\int_6^t 0.8\,d\tau + 3.2 = (1.6t - 6.4) \text{ A}$$

P 7.5-7
Solution:

$$i(t) = \frac{1}{150}\int_0^t 0\,dt + 0.03 = 0.03 \qquad \text{for} \qquad 0 < t < 1$$

so $i(1) = 0.03$

$$i(t) = \frac{1}{150}\int_1^t -4\,d\tau + 0.03 = \frac{-4(t-1)}{150} + 0.03 \qquad \text{for} \qquad 1 < t < 3$$

so $i(3) = -0.023$

$$i(t) = \frac{1}{150}\int_3^t 2\,d\tau - 0.023 = \frac{2(t-3)}{150} - 0.023 \qquad \text{for} \qquad 3 < t < 9$$

so $i(9) = 0.057$

$$i(t) = \frac{1}{150}\int_9^t 0\,d\tau + 0.057 = 0.057 \qquad \text{for} \qquad t > 9$$

P 7.5-8
Solution:

$$i(t)=\frac{1}{2}\int_0^t v(t)\,dt+1=\begin{cases} 1 & t\le 2 \\ \int_2^t d\tau+1=(t-2)+1=t-1 & 2\le t\le 4 \\ \frac{1}{2}\int_4^t d\tau+3=-\frac{1}{2}t+5 & 4\le t\le 6 \\ 2 & 6\le t \end{cases}$$

P 7.5-9
Solution:

$$i(t)=\frac{1}{200}\int_0^t -d\tau+0.025=\frac{-t}{200}+0.025 \qquad \text{for} \qquad 0<t<1$$

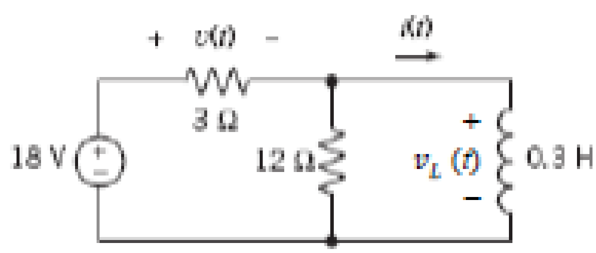
$$i(t)=\frac{1}{200}\int_1^t -2\,d\tau+0.02=\frac{-2(t-1)}{200}+0.02 \qquad \text{for} \qquad 1<t<4$$

$$i(t)=\frac{1}{200}\int_4^t d\tau-0.01=\frac{t-4}{200}-0.01 \qquad \text{for} \qquad 4<t<9$$

$$i(t)=0.015=15\text{ mA} \qquad t<9$$

P 7.5-10
Solution:

$$\begin{aligned} v_L(t)&=0.3\frac{d}{dt}i(t) \\ &=-9.6e^{-8t}\ \text{V} \quad \text{for } t>0 \end{aligned}$$



Use KVL to get

$$v(t)=18-\left(-9.6e^{-8t}\right)=18+9.6e^{-8t}\ \text{V} \quad \text{for } t>0$$

P 7.5-11

Solution:

Apply KVL to get

$$v(t)=9i(t)+7.5\frac{d}{dt}i(t)=9\left(3+2e^{-3t}\right)+7.5\frac{d}{dt}\left(3+2e^{-3t}\right)=27\left(1-e^{-3t}\right)\text{ V} \quad \text{for } t>0$$

P 7.5-12

Solution:

$$i(t)=\frac{1}{L}\int_{t_0}^{t}v_s(\tau)\,d\tau+i(t_0)=\frac{1}{\frac{1}{3}}\int_0^t v_s(\tau)\,d\tau-12$$

$$i(t)=3\int_0^t 4\,d\tau-12=12\,t-12 \quad \text{for} \quad 0<t<4$$ In particular, $i(4) = 36$ A.

$$i(t)=3\int_4^t(-2)\,d\tau+36=60-6t \quad \text{for} \quad 4<t<10$$ In particular, $i(10) = 0$ A.

$$i(t)=3\int_{10}^t 0\,d\tau+0=0 \quad \text{for} \quad 10<t$$

P7.5-14

Solution:

Apply KCL at either node to get

$$i(t)=\frac{v(t)}{R}+i_{\text{L}}(t)=\frac{v(t)}{R}+\left[\frac{1}{L}\int_0^t v(\tau)\,d\tau+i(0)\right]$$

That is

$$-1.2\,e^{-20t}-1.5=\frac{4e^{-20t}}{R}+\frac{1}{L}\int_0^t 4e^{-20\tau}\,d\tau-3.5=\frac{4e^{-20t}}{R}+\frac{4}{L(-20)}\left(e^{-20t}-1\right)-3.5$$

$$=\left(\frac{4}{R}-\frac{1}{5L}\right)e^{-20t}+\frac{1}{5L}-3.5$$

Equating coefficients gives

$$-1.5=\frac{1}{5L}-3.5 \quad \Rightarrow \quad L=0.1\text{ H}$$

and

$$-1.2=\frac{4}{R}-\frac{1}{5L}=\frac{4}{R}-\frac{1}{5(0.1)}=\frac{4}{R}-2 \quad \Rightarrow \quad R=5\,\Omega$$

P7.5-15
Solution:
At $t = 0.2$ s

$$i(0.2) = 5(0.2) - 4.6 = -3.6 \text{ A}$$

For $0.2 \le t \le 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.2}^{t} 25\,d\tau - 3.6 = 10\tau\,\Big|_{0.2}^{t} - 3.6 = 10(t-0.2) - 3.6 = 10t - 5.6 \text{ A}$$

At $t = 0.5$ s

$$i(0.5) = 10(0.5) - 5.6 = -0.6 \text{ A}$$

For $t \ge 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.5}^{t} 0\,d\tau - 0.6 = -0.6$$

Checks:

At $t = 0.2$ s $\quad i(0.2) = 10(0.2) - 5.6 = -3.6 \text{ A}$

For $0.2 \le t \le 0.5 \quad v(t) = 2.5\frac{d}{dt}i(t) = 2.5\frac{d}{dt}(10t - 5.6) = 2.5(10) = 25 \text{ V}$

$$-0.6 - (-3.6) = i(0.5) - i(0.2) = \frac{1}{2.5}\int_{0.2}^{0.5} 25\,d\tau = 10(0.5 - 0.2) = 3 \text{ A}$$

P7.5-16
Solution:

$$i(t) = i(0) + \frac{1}{L}\int_0^t v(\tau)\,d\tau = i(0) + \frac{\text{"area under the curve"}}{L} = 0.045 + \frac{\text{"area under the curve"}}{0.250}$$

$$i(0.001) = 0.045 + \frac{20(0.001)}{0.250} = 0.125 \text{ A} = 125 \text{ mA},$$

$$i(0.004) = 0.045 + \frac{20(0.002) + \frac{1}{2}20(0.002)}{0.250} = 0.285 \text{ A} = 285 \text{ mA}$$

$$i(0.006) = 0.045 + \frac{20(0.002) + \frac{1}{2}20(0.002) + 0}{0.250} = 0.285 \text{ A} = 285 \text{ mA}$$

P7.5-17
Solution:

First, $$\frac{d}{dt}i(t)=\frac{d}{dt}(0.3\cos(2t))=-(0.3)(2)\sin(2t)=-0.6\sin(2t)$$

The voltage of an inductor is proportional to the derivative of the current. The constant of proportionality is the inductance. We see that $v_a(t)$ is proportional to $\frac{d}{dt}i(t)$ and the constant of proportionality is positive. Consequently, element a is the inductor. Then

$$L=\frac{v_a(t)}{\frac{d}{dt}i(t)}=\frac{-10\sin(2t)}{-0.6\sin(2t)}=16.7\text{ H}$$

Next $$\int_{-\infty}^{t} i(\tau)\,d\tau=\int_{-\infty}^{t} 0.3\cos(2\tau)\,d\tau=\frac{0.3\sin(2\tau)}{2}=0.15\sin(2\tau)$$

The voltage of a capacitor is proportional to the integral of the current. The constant of proportionality is the reciprocal of the capacitance. We see that $v_b(t)$ is proportional to $\int_{-\infty}^{t} i(\tau)\,d\tau$ and the constant of proportionality is positive. Consequently, element b is the capacitor. Then

$$\frac{1}{C}=\frac{v_b(t)}{\int_{-\infty}^{t} i(\tau)\,d\tau}=\frac{10\sin(2t)}{0.15\sin(2t)}=66.7 \quad\Rightarrow\quad C=\frac{1}{66.7}=0.015\text{ F}$$

Finally, the voltage of element c is proportional to the current and the constant of proportionality is positive. Consequently, element c is the resistor and $R=\frac{v_c(t)}{i(t)}=33.3\;\Omega$.

Section 7-6: Energy Storage in an Inductor

P 7.6-1
Solution:

$$v(t)=100\times10^{-3}\,\frac{d}{dt}\,i(t)=\begin{cases}0 & t<0\\ 0.4 & 0\le t\le 1\\ 0 & t>1\end{cases}$$

$$p(t)=v(t)\,i(t)=\begin{cases}0 & t<0\\ 1.6t & 0\le t\le 1\\ 0 & t>1\end{cases}$$

$$W(t)=\int_0^t p(\tau)\,d\tau=\begin{cases}0 & t<0\\ 0.8t^2 & 0<t<1\\ 0.8 & t>1\end{cases}$$

P 7.6-2

Solution:

$$p(t) = v(t)\, i(t) = \left[5\,\frac{d}{dt}(4\sin 2t)\right](4\sin 2t)$$
$$= 5\ (8\cos 2t)\ (4\sin 2t)$$
$$= 80\ [2\cos 2t\ \sin 2t]$$
$$= 80\ [\sin(2t+2t) + \sin(2t-2t)] = 80\ \sin 4t\ \text{W}$$

$$\mathcal{W}(t) = \int_0^t p(\tau)\, d\tau\ = 80\int_0^t \sin 4\tau\ d\tau = -\frac{80}{4}\ [\cos 4\tau\, |_0^t] = 20\ (1-\cos 4t)$$

$$i(t) = \frac{}{25\times 10^{-3}}\int_0^t 6\ \cos 100\tau\ d\tau + 0 = \frac{}{(25\times 10^{-3})(100)}\ [\sin 100\tau\ |_0^t] = 2.4\sin 100t$$

$$p(t) = v(t)\ i(t) = (6\cos 100\,t)(2.4\sin 100t) = 7.2\ [\ 2(\cos 100\,t)(\sin 100\,t)\ ]$$
$$= 7.2\ [\sin 200\,t + \sin 0] = 7.2\sin 200\,t$$

$$W(t) = \int_0^t p(\tau)\ d\tau = 7.2\int_0^t \sin\ 200\tau\ d\tau = -\frac{7.2}{200}\left[\cos\ 200\tau|_0^t\right]$$
$$= 0.036[1-\cos 200t]\ \text{J} =\ 36\ [1-\cos 200t\ ]\ \text{mJ}$$

P 7.6-4
Solution:

$$v(t) = L\frac{di}{dt} = \frac{1}{2}\frac{di}{dt}\quad \text{and}\quad i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ -2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases} \Rightarrow v(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$p(t)\ =\ v(t)i(t)\ = \begin{cases} 0 & t<0 \\ 2\,t & 0<t<1 \\ 2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases}$$

$$\mathcal{W}(t)\ =\ \mathcal{W}(t_0) + \int_{t_0}^t p(t)\, dt$$
$$i(t)\ =\ 0\ \text{ for }\ t<0\ \Rightarrow\ p(t)\ =\ 0\ \text{ for }\ t<0\ \Rightarrow\ \mathcal{W}(t_0) = 0$$

$$0<t<1:\ \mathcal{W}(t)\ =\ \int_0^t 2\,t\ dt\ =\ t^2$$
$$1<t<2:\ \mathcal{W}(t)\ =\ \mathcal{W}(1) + \int_1^t 2\,(t-2)\, dt\ =\ t^2\ -\ 4\,t\ +\ 4$$
$$t>2:\ \mathcal{W}(t)\ =\ \mathcal{W}(2)\ =\ 0$$

Section 7-7: Series and Parallel Inductors

P 7.7-1
Solution:

$$12\text{ H} \parallel 6\text{ H} = \frac{12\times 6}{12+6} = 4\text{ H} \quad \text{and} \quad 4\text{ H}+4\text{ H}=8\text{ H}$$

$$i(t)=\frac{1}{8}\int_0^t 6\cos 100\tau \, d\tau = \frac{6}{8\times 100}\left[\sin 100\tau|_0^t\right]=0.0075\sin 100t \text{ A}=7.5\sin 100t \text{ mA}$$

P 7.7-2
Soluton:

$$6\text{ mH}+6\text{ mH}=12\text{ mH} \quad , \quad 12\text{ mH}\parallel 12\text{ mH}=\frac{\left(12\times 10^{-3}\right)\times\left(12\times 10^{-3}\right)}{12\times 10^{-3}+12\times 10^{-3}}=6\text{ mH}$$

and $6\text{ mH}+6\text{mH}=12\text{ mH}$

$$v(t)=(12\times 10^{-3})\frac{d}{dt}(5+3e^{-250t})=(12\times 10^{-3})(0+3(-250)e^{-250t})=-9e^{-250t}\text{ V}$$

P 7.7-3
Solution:

$$L \parallel L = \frac{L\cdot L}{L+L} = \frac{L}{2} \quad \text{and} \quad L+L+\frac{L}{2}=\frac{5}{2}L$$

$$25\cos 250t = \left(\frac{5}{2}L\right)\frac{d}{dt}\left(\left(14\times 10^{-3}\right)\sin 250t\right)=\left(\frac{5}{2}L\right)(14\times 10^{-3})(250)\cos 250t$$

$$\text{so } L=\frac{25}{\frac{5}{2}(14\times 10^{-3})(250)}=2.86\text{ H}$$

P 7.7-4
Solution:

The equivalent inductance is: $$\frac{\left(\frac{L\times 2L}{L+2L}+L\right)\times L}{\left(\frac{L\times 2L}{L+2L}+L\right)+L}+2L=\frac{21}{8}L$$

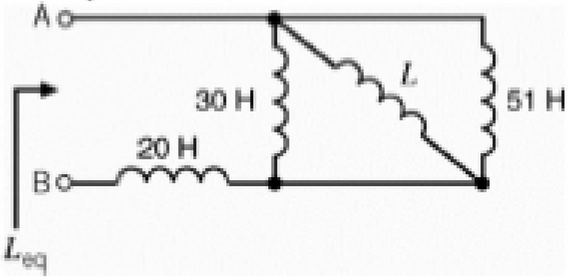
Then $$i(t)=\frac{1}{\frac{21}{8}L}\int_{-\infty}^t 4\cos(3\tau)\,d\tau=\frac{8}{21\times 4}\times\frac{4}{3}\sin(3t)=127\sin(3t) \text{ mA}$$

(Checked using LNAP 6/26/04)

P 7.7-5
Solution:
The 30 H inductor is in series with a parallel combination of 30 H and 70 H inductors. The inductance of the equivalent inductor is

$$30+\frac{70\times 30}{70+30}=51\text{ H}$$

The 40 H inductor is in parallel with a short circuit, which is equivalent to a short circuit. After making these simplifications, we have



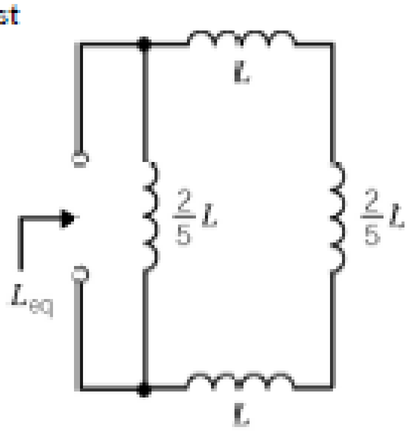
Then

$$28=L_{eq}=20+\frac{1}{\frac{1}{30}+\frac{1}{L}+\frac{1}{51}}\quad\Rightarrow\quad\frac{1}{30}+\frac{1}{L}+\frac{1}{51}=\frac{1}{8}\quad\Rightarrow\quad L=13.88\text{ H}$$

P 7.7-6
Solution:

$$L_{eq}=70+\frac{25\times 20}{25+20}+40+\frac{50\times 70}{50+70}=70+11.1+40+29.2=150.3\text{ H}$$

P 7.7-7
Solution:
First



Then

$$12=L_{eq}=\frac{\left(\frac{2}{5}L\right)\times\left(\frac{2}{5}L+2L\right)}{\left(\frac{2}{5}L\right)+\left(\frac{2}{5}L+2L\right)}=\frac{12}{35}L\quad\Rightarrow\quad L=35\text{ mH}$$

(Checked using LNAP 6/26/04)

P 7.7-8
Solution:

(a) The energy stored by the 0.5 H inductor is $w_1 = \frac{1}{2}(0.5)(0.8^2) = 0.16\ \text{J}$ and the energy stored by the 2 H inductor is $w_2 = \frac{1}{2}(2)(0.8^2) = 0.64\ \text{J}$.

(b) 200 ms after the switch opens the current in the inductors is $0.8e^{-0.4} = 0.536\ \text{A}$. Then $w_1 = \frac{1}{2}(0.5)(0.536^2) = 71.8\ \text{mJ}$ and $w_2 = \frac{1}{2}(2)(0.535^2) = 287.3\ \text{mJ}$.

Next, $L_{eq} = 2 + 0.5 = 2.5\ \text{H}$.

(c) $w_{eq} = \frac{1}{2}(2.5)(0.8^2) = 0.8\ \text{J} = w_1 + w_2$

(d) $w_{eq} = \frac{1}{2}(2.5)(0.536^2) = 359.12\ \text{mJ} = w_1 + w_2$

P 7.7-9
Solution:

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + i_1(t_0), \quad i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,dt + i_2(t_0) \qquad \text{but } i_1(t_0)=0 \text{ and } i_2(t_0)=0$$

$$i = i_1 + i_2 = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + \int_{t_0}^{t} v\,dt = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_{t_0}^{t} v\,dt = \frac{1}{L_P}\int_{t_0}^{t} v\,dt$$

$$\therefore \frac{i_1}{i} = \frac{\frac{1}{L_1}\int_{t_0}^{t} v\,dt}{\frac{1}{L_P}\int_{t_0}^{t} v\,dt} = \frac{\frac{1}{L_1}}{\frac{1}{L_1} + \frac{1}{L_2}} = \underline{\frac{L_2}{L_1 + L_2}}$$

P7.7-10
Solution:

(a) $C_{eq} = \frac{30(10)}{30+10} + 30 = 37.5\ \mu\text{F}$ (b) $L_{eq} = \frac{(60+40)(30)}{(60+40)+30} = 23.08\ \text{mH}$

(c) $R_{eq} = \frac{(10+8)(10)}{(10+8)+10} = 6.4\ \text{k}\Omega$

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P7.7-11

Solution:

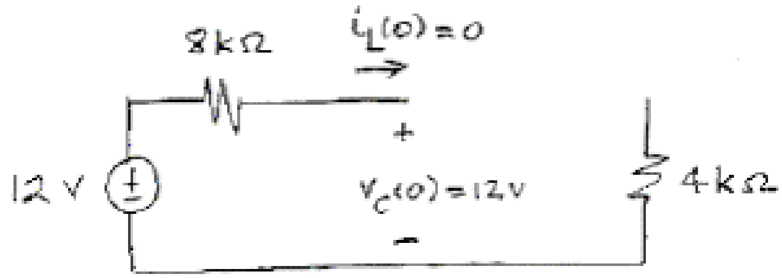
(a) $C_{eq} = \frac{(10+20)(15)}{(10+20)+15} = 10\ \mu\text{F}$ (b) $L_{eq} = \frac{(30)(6)}{30+6} + 10 = 15\ \text{mH}$

(c) $R_{eq} = \frac{(30)(40)}{30+40} + 16 = 33.14\ \text{k}\Omega$

Section 7-8: Initial Conditions of Switched Circuits

P 7.8-1

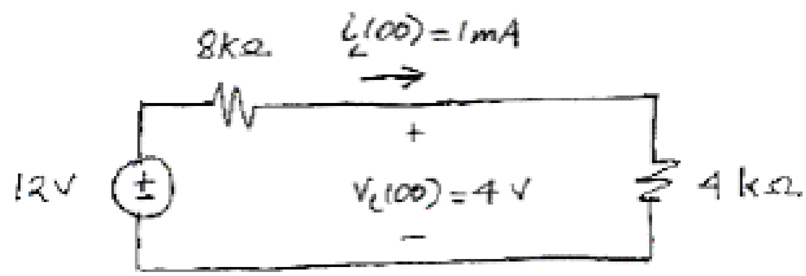
Solution:



Then

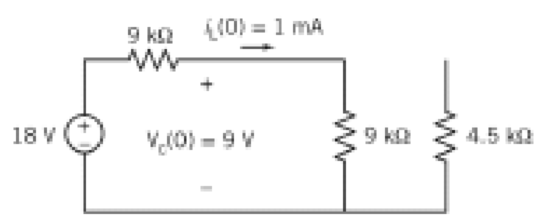
$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 12\ \text{V}$$

Next



P 7.8-2

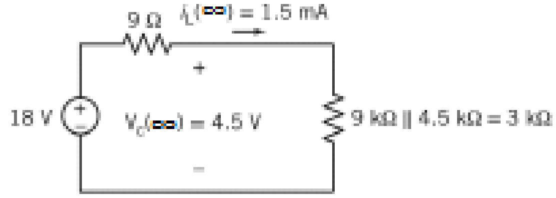
Solution:



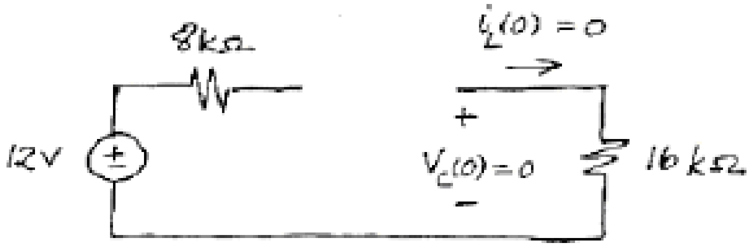
Then

$$i_L(0^+) = i_L(0^-) = 1\ \text{mA} \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 9\ \text{V}$$

Next



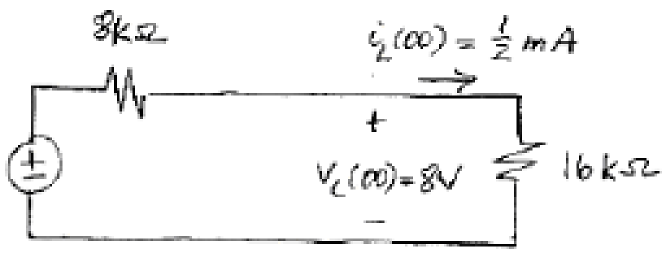
P 7.8-3
Solution:



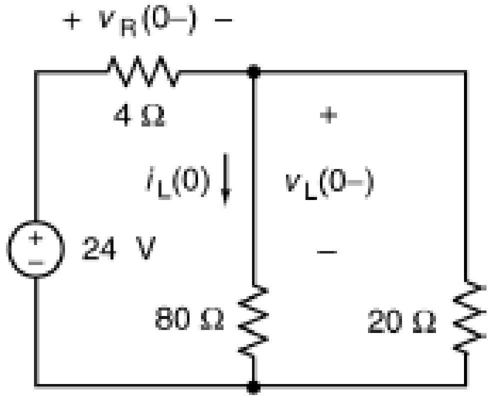
Then

$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 0 \text{ V}$$

Next



P7.8-4
Solution:
The circuit is at steady state immediately before the switch opens. We have



The inductor acts like a short circuit so $v_L(0-)=0$.

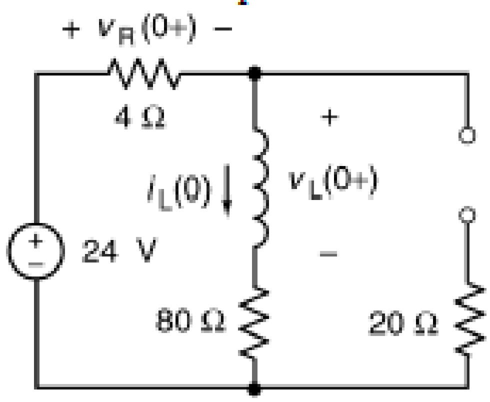
Noticing that the 80 Ω and 20 Ω are connected in parallel and using voltage division:

$$v_R(0-)=\frac{4}{4+(80\,\|\,20)}(24)=\frac{4}{4+16}(24)=4.8\text{ V}$$

Using current division:

$$i_L(0)=\left(\frac{20}{80+20}\right)\frac{24}{4+(80\,\|\,20)}=\frac{1}{5}\left(\frac{24}{4+16}\right)=0.24\text{ A}$$

The inductor current does not change instantaneously so $i_L(0+)=i_L(0-)\equiv i_L(0)$. Immediately after the switch opens we have:



$$v_R(0+)=4i_L(0)=4(0.24)=0.96\text{ V}$$

Using KVL:

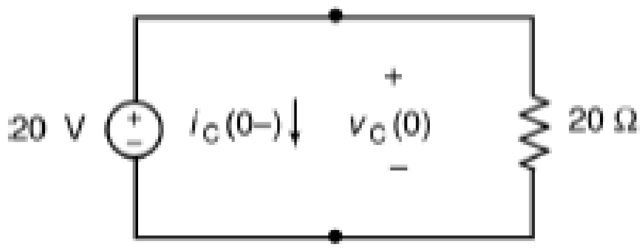
$$v_R(0+)+v_L(0+)+80i_L(0)-24=0$$
$$0.96+v_L(0+)+80(0.24)-24=0$$
$$v_L(0+)=3.84\text{ V}$$

$$v(t)=75-82e^{-7t}=R\left(5+2e^{-7t}\right)+L\frac{d}{dt}\left(5+2e^{-7t}\right)$$
$$=R\left(5+2e^{-7t}\right)+L\left((-7)2e^{-7t}\right)=5R+(2R-14L)e^{-7t}$$

Equation coefficients gives $75=5R \;\Rightarrow\; R=15\,\Omega$ and

and $-82=2R-14L=30-14L \;\Rightarrow\; L=\dfrac{82+30}{14}=8\text{ H}$

P7.8-5
Solution:
The circuit is at steady state immediately before the switch opens. We have

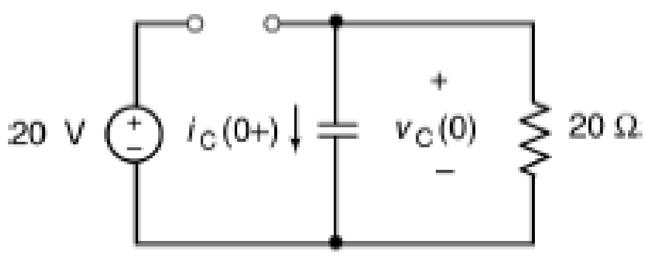


The capacitor acts like an open circuit so $i_C(0-)=0$.

The capacitor voltage is equal to the voltage source voltage:

$$v_C(0)=20\text{ V}$$

The capacitor does not change instantaneously so $v_C(0+)=v_C(0-) = v_C(0)$. Immediately after the switch opens we have:



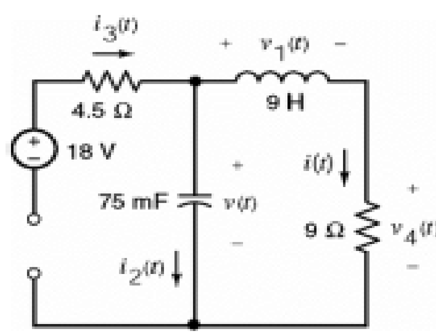
Applying KCL at the top node of the capacitor, we see that:

$$i_C(0+)+\frac{v_C(0)}{20}=0$$

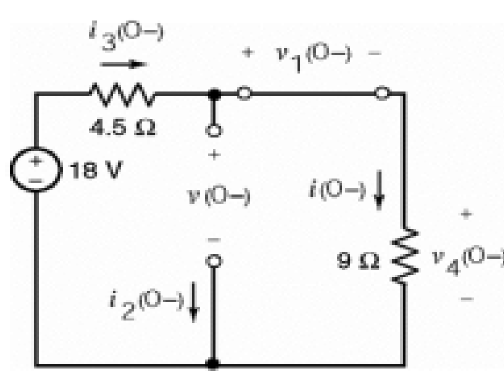
$$i_C(0+)=-\frac{v_C(0)}{20}=-1\text{ A}$$

P7-8.6
Solution:
The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t=0$, with the switch closed and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



$$i_3(0-)=i(0-)=\frac{18}{13.5}=1.33\text{ A}$$

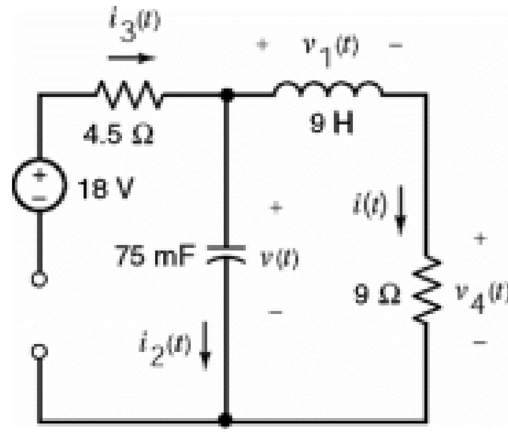
$$v_4(0-)=v(0-)=9i(0-)=12\text{ V}$$

$$v_1(0-)=0\text{ V and } i_2(0-)=0\text{ A}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+)=v(0-)=8\text{ V and } i(0+)=i(0-)=1.33\text{ A}$$

After the switch opens the circuit looks like this:



From KCL:

$$i_3(t)=0 \text{ A and } i_2(t)=-i(t)$$

From KVL:

$$v_1(t)+9i(t)=v(t)$$

From Ohm's Law:

$$v_4(t)=9i(t)$$

At $t=0+$

$$i_3(0+)=0 \text{ A and } i_2(0+)=-i(0+)=-1.33 \text{ A}$$

$$v_1(0+)=v(0+)-9i(0+)=12-9(1.333)=0 \text{ V}$$

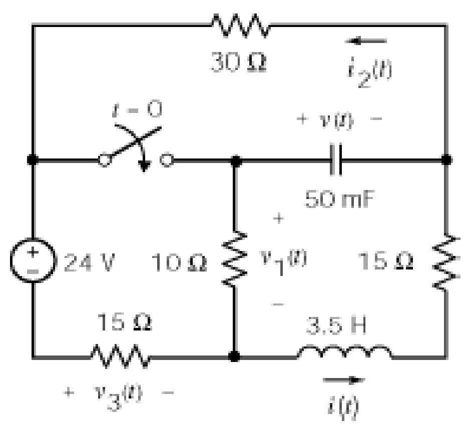
$$v_4(0+)=9i(0+)=12 \text{ V}$$

P 7.8-7

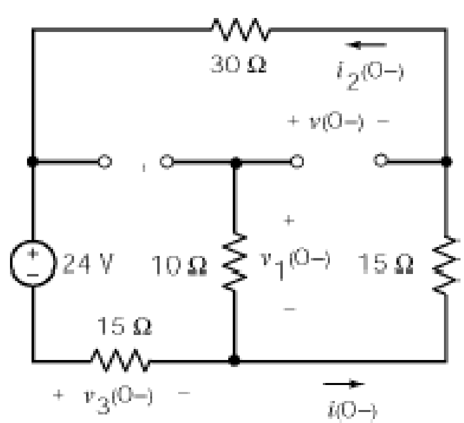
Solution:

The capacitor voltage and inductor current don't change instantaneously and so are the keys to solving this problem.

Label the capacitor voltage and inductor current as shown.



Before $t = 0$, with the switch open and the circuit at steady state, the inductor acts like a short circuit and the capacitor acts like an open circuit.



$$i_2(0-)=i(0-)=\frac{24}{60}=-0.4 \text{ A}$$

$$v_1(0-)=0 \text{ V}$$

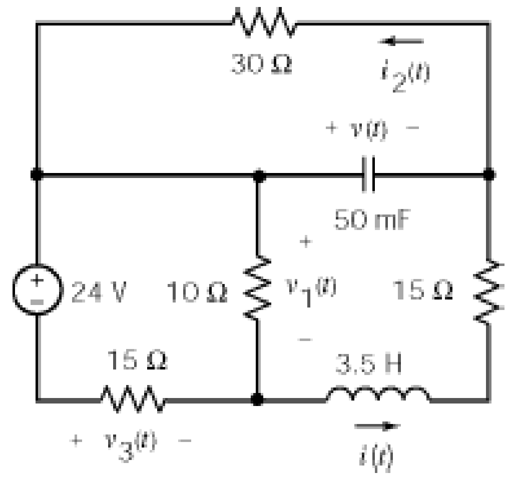
$$v(0-)-15i(0-)=v_1(0-) \quad\Rightarrow\quad v(0-)=-6 \text{ V}$$

$$v_3(0-)=15i(0-)=-6 \text{ V}$$

The capacitor voltage and inductor current don't change instantaneously so

$$v(0+)=v(0-)=-6 \text{ V} \text{ and } i(0+)=i(0-)=-0.4 \text{ A}$$

After the switch closes the circuit looks like this:



From Ohm's Law:

$$i_2(t) = -\frac{v(t)}{30}$$

From KVL:

$$v_1(t) = v_3(t) + 24$$

From KCL:

$$\frac{v_1(t)}{10} + \frac{v_3(t)}{15} = i(t)$$

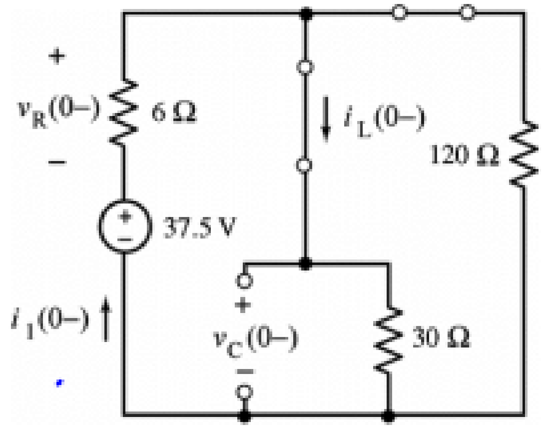
At $t = 0+$

$$i_2(0+) = -\frac{v(0+)}{30} = 0.2 \text{ A}$$

$$\left.\begin{array}{c} v_1(0+) = v_3(0+) + 24 \\ \dfrac{v_1(0+)}{10} + \dfrac{v_3(0+)}{15} = i(0+) \end{array}\right\} \Rightarrow \; v_1(0+) = 7.2 \text{ V and } v_3(0+) = -16.8 \text{ V}$$

P7.8-8
Solution:
Because

- This circuit has reached steady state before the switch opens at time $t = 0$.
- The only source is a constant voltage source.



At t=0−, the capacitor acts like an open circuit and the inductor acts like a short circuit. From the circuit

$$i_1(0-) = \frac{37.5}{6+(30\|120)} = \frac{37.5}{6+24} = 1.25 \text{ A},$$

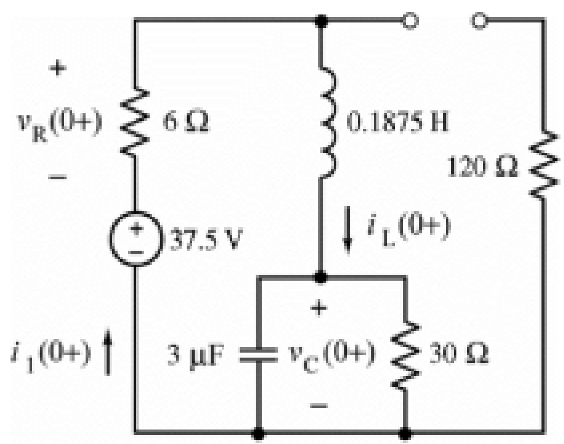
$$i_L(0-) = \left(\frac{120}{30+120}\right) i_1(0-) = 1 \text{ A},$$

$$v_C(0-) = 30 i_L(0-) = 30 \text{ V}$$

and

$$v_R(0-) = 6 i_1(0-) = 7.5 \text{ V}$$

The capacitor voltage and inductor current don't change instantaneously so



$$v_C(0+) = v_C(0-) = 30 \text{ V} \text{ and}$$

$$i_L(0+) = i_L(0-) = 1 \text{ A}$$

Apply KCL at the top node to see that

$$i_1(0+) = i_L(0+) = 1 \text{ A}$$

From Ohm's law

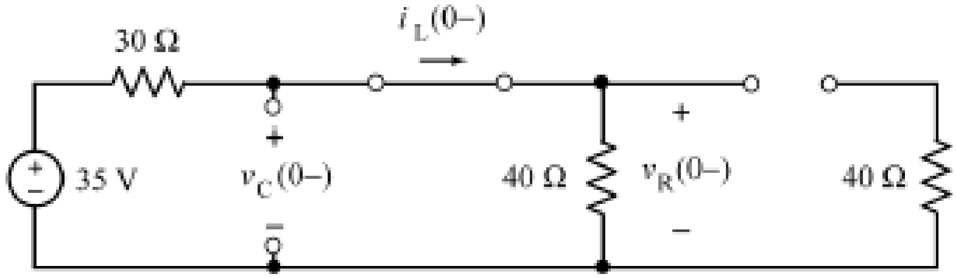
$$v_R(0+) = 6 i_1(0+) = 6 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

P7.8-9
Solution:
Because

- This circuit has reached steady state before the switch closes at time $t = 0$.
- The only source is a constant voltage source.

At t=0−, the capacitor acts like an open circuit and the inductor acts like a short circuit.

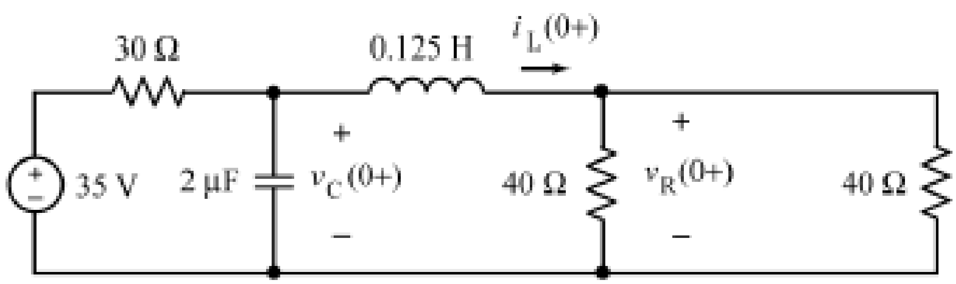


From the circuit $\quad i_L(0-)=\dfrac{35}{30+40}=0.5\text{ A},\quad v_R(0-)=40\,i_L(0-)=20\text{ V},$

And $\quad v_C(0-)=v_R(0-)=20\text{ V}$

The capacitor voltage and inductor current don't change instantaneously so

$$v_C(0+)=v_C(0-)=20\text{ V and } i_L(0+)=i_L(0-)=0.5\text{ A}$$



$$v_R(0+)=40\left(\frac{40}{40+40}\right)i_L(0+)=10\text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

P 7.8-10

Solution: Apply KVL to the left mesh to obtain

$$v_s(t)=100\big(i_1(t)+i_2(t)\big)+0.005\frac{d}{dt}i_1(t)$$

Apply KVL to the outside loop to obtain

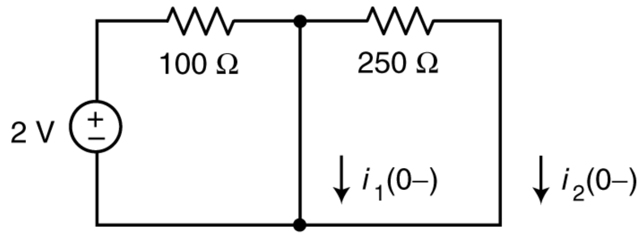
$$v_s(t) = 100(i_1(t) + i_2(t)) + 250 i_2(t) + 0.002 \frac{d}{dt} i_2(t)$$

Immediately after $t = 0$, at $t = 0+$, we have

$$\frac{d}{dt} i_1(0+) = \frac{8 - 100(i_1(0+) + i_2(0+))}{0.005}$$

and
$$\frac{d}{dt} i_2(0+) = \frac{8 - (100 i_1(0+) + 250 i_2(0+))}{0.002}$$

where 8 = $v_s(0+)$. The initial inductor currents can be calculated from:



Inductor currents must be continuous so

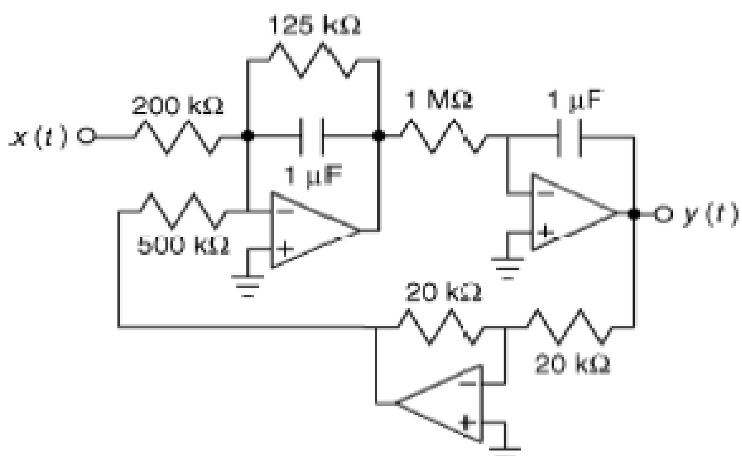
$$i_1(0+) = i_1(0-) = \frac{2}{100} = 0.02 \text{ A} \quad \text{and} \quad i_2(0+) = i_2(0-) = 0$$

Substituting these values into the equations for $\frac{d}{dt} i_1(0+)$ and $\frac{d}{dt} i_2(0+)$ gives

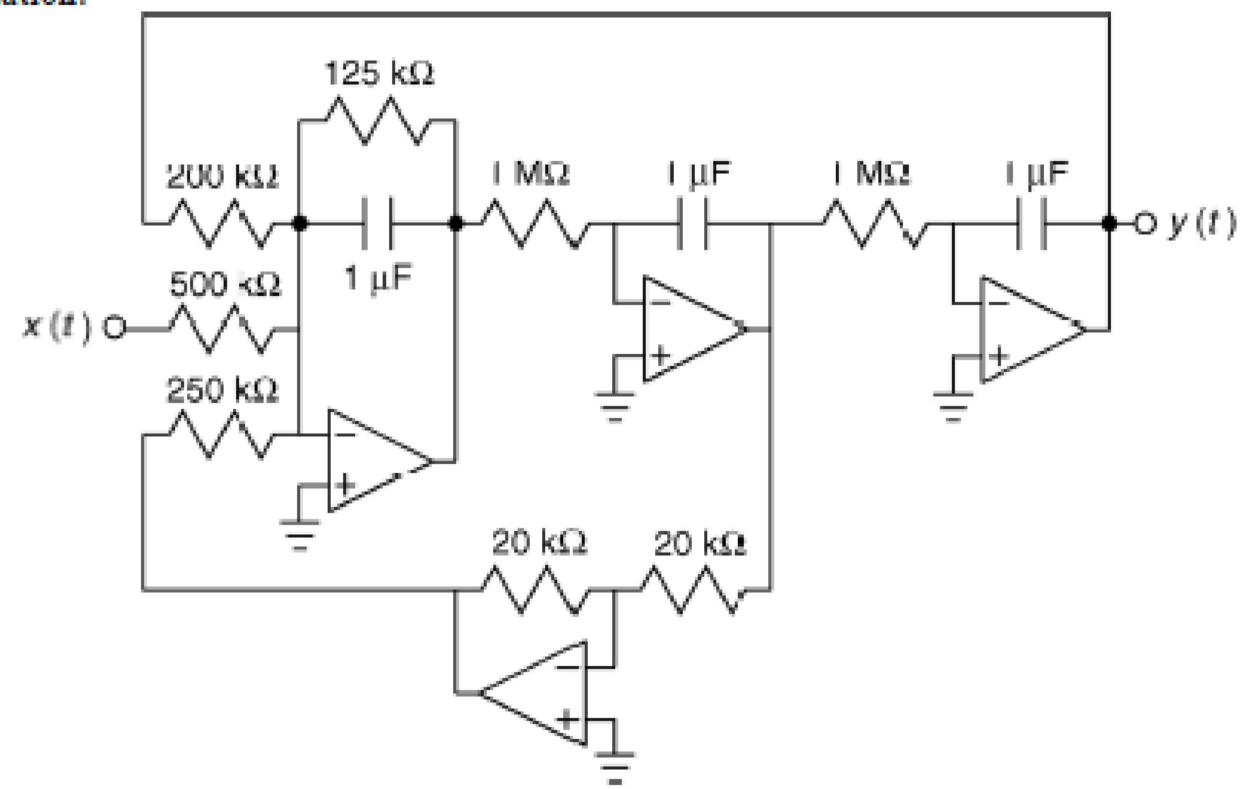
$$\frac{d}{dt} i_1(0+) = \frac{8 - 100(0.02 + 0)}{0.005} = 1200 \ \frac{\text{A}}{\text{s}} \quad \text{and} \quad \frac{d}{dt} i_2(0+) = \frac{8 - (100(0.02) + 0)}{0.002} = 3000 \ \frac{\text{A}}{\text{s}}$$

Section 7-9: Operational amplifier Circuits and Linear Differential Equations

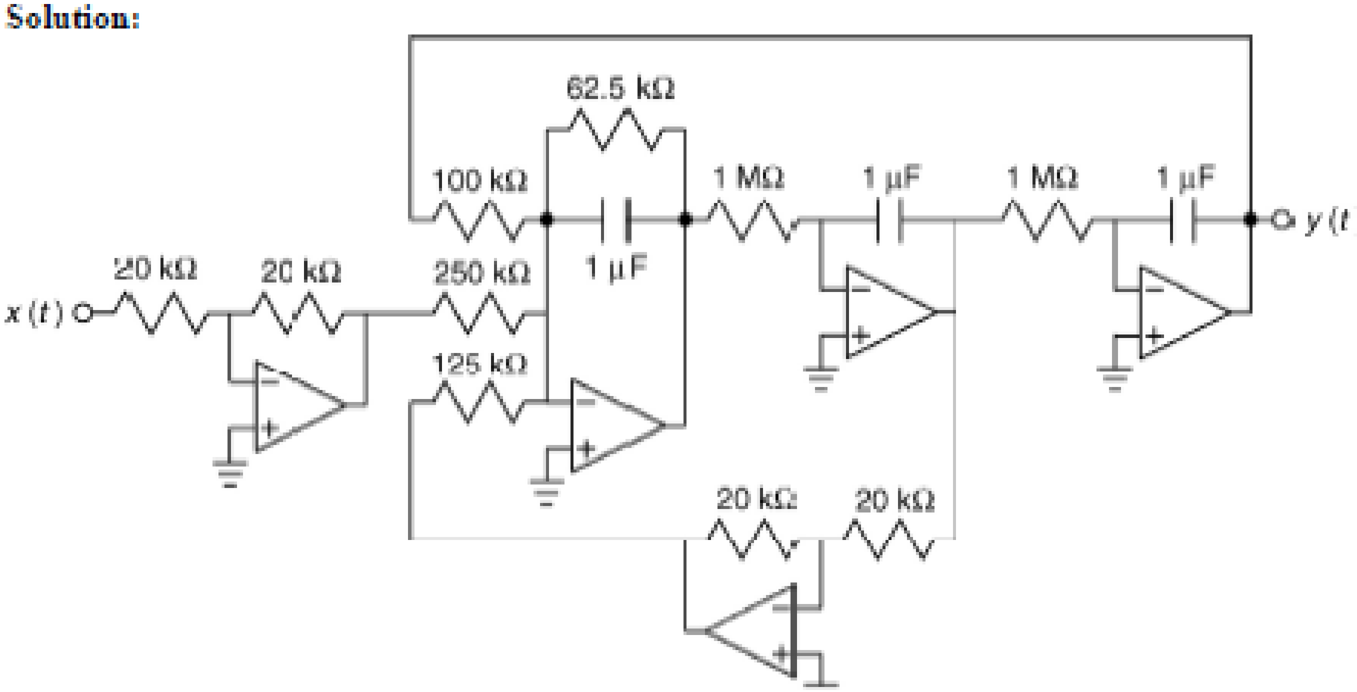
P 7.9-1
Solution:



P 7.9-2
Solution:



P 7.9-3
Solution:



Section 7.11 How Can We Check…?

P 7.11-1

Solution: We need to check the values of the inductor current at the ends of the intervals.

$$\text{at } t=1 \quad 0.025 \stackrel{?}{=} -\frac{1}{25} + 0.065 = 0.025 \quad (\text{Yes!})$$

$$\text{at } t=3 \quad -\frac{3}{25} + 0.065 \stackrel{?}{=} \frac{3}{50} - 0.115$$

$$-0.055 = -0.055 \quad (\text{Yes!})$$

$$\text{at } t=9 \quad \frac{9}{50} - 0.115 \stackrel{?}{=} 0.065$$

$$0.065 = 0.065 \quad (\text{Yes!})$$

The given equations for the inductor current describe a current that is continuous; as must be the case since the given inductor voltage is bounded.

P 7.11-2

Solution:

We need to check the values of the inductor current at the ends of the intervals.

$$-\frac{1}{300} + 0.0375 \stackrel{?}{=} -\frac{1}{150} + 0.045 \qquad (\text{No!})$$

$$-\frac{4}{150} + 0.045 \stackrel{?}{=} \frac{4}{150} - 0.045 \qquad (\text{No!})$$

The equation for the inductor current indicates that this current changes instantaneously at $t = 4$s. This equation cannot be correct.

Design Problems

DP 7-1
Solution:

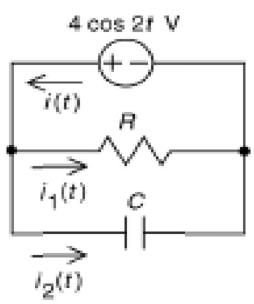
a) $\frac{d}{dt}v(t) = -13.5\, e^{-4.5t}$ is proportional to $i(t)$ so the element is a capacitor. $C = \dfrac{i(t)}{\frac{d}{dt}v(t)} = 0.3$ F.

b) $\frac{d}{dt}i(t) = -13.5\, e^{-1.5t}$ is proportional to $v(t)$ so the element is an inductor. $L = \dfrac{v(t)}{\frac{d}{dt}i(t)} = 0.3$ H.

c) $v(t)$ is proportional to $i(t)$ so the element is a resistor. $R = \dfrac{v(t)}{i(t)} = 2\ \Omega$.

DP 7-2
Solution:
(a)

$$1.131\cos(2t+45°) = 1.131\left[\cos(45°)\cos(2t) - \sin(45°)\sin(2t)\right]$$
$$= 0.8\cos 2t - 0.8\sin 2t$$



The first term is proportional to the voltage. Associate it with the resistor. The noticing that

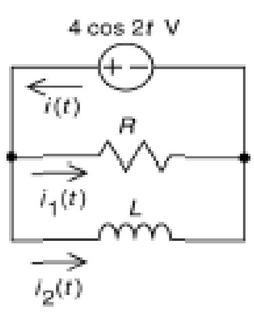
$$\int_{-\infty}^{t} v(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2\tau\, d\tau = 2\sin 2t$$
$$\frac{d}{dt}v(t) = \frac{d}{dt}4\cos 2t = -8\sin 2t$$

associate the second term with a capacitor to get the minus sign. Then

$$R = \frac{4\cos 2t}{i_1(t)} = \frac{4\cos 2t}{0.8\cos 2t} = 5\ \Omega \text{ and}$$
$$C = \frac{i_2(t)}{\frac{d}{dt}4\cos 2t} = \frac{-0.8\sin 2t}{-8\sin 2t} = 0.1\text{ F}$$

(b)

$$1.131\cos(2t-45°) = 1.131\left[\cos(-45°)\cos(2t) - \sin(-45°)\sin(2t)\right]$$
$$= 0.8\cos 2t + 0.8\sin 2t$$



The first term is proportional to the voltage. Associate it with the resistor. Then noticing that

$$\int_{-\infty}^{t} v(\tau)\,d\tau = \int_{-\infty}^{t} 4\cos 2\tau\, d\tau = 2\sin 2t$$
$$\frac{d}{dt}v(t) = \frac{d}{dt}4\cos 2t = -8\sin 2t$$

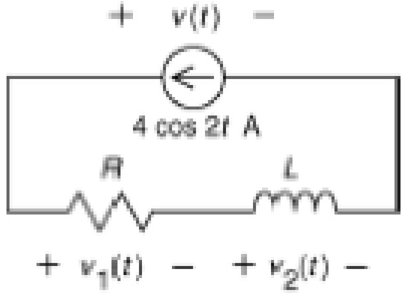
associate the second term with an inductor to get the plus sign. Then

$$R = \frac{4\cos 2t}{i_1(t)} = \frac{4\cos 2t}{0.8\cos 2t} = 5\ \Omega \text{ and}$$
$$L = \frac{\int_{-\infty}^{t} 4\cos 2\tau\, d\tau}{i_2(t)} = \frac{2\sin 2t}{0.8\sin 2t} = 2.5\text{ H}$$

DP 7-3

Solution:

a)



$$11.31\cos(2t+45°)=11.31[\cos(45°)\cos(2t)-\sin(45°)\sin(2t)]$$
$$=8\cos 2t-8\sin 2t$$

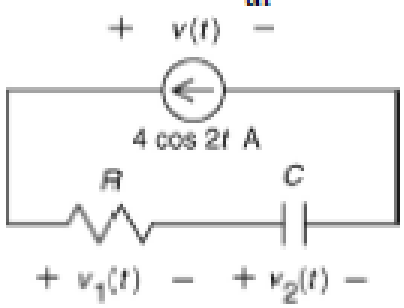
The first term is proportional to the voltage. Associate it with the resistor. The noticing that

$$\int_{-\infty}^{t} i(\tau)\,d\tau=\int_{-\infty}^{t} 4\cos 2t\,d\tau=2\sin 2t$$
$$\frac{d}{dt}i(t)=\frac{d}{dt}4\cos 2t=-8\sin 2t$$

associate the second term with an inductor to get the minus sign. Then

$$R=\frac{v_1(t)}{4\cos 2t}=\frac{8\cos 2t}{4\cos 2t}=2\,\Omega \text{ and } L=\frac{v_2(t)}{\frac{d}{dt}4\cos 2t}=\frac{-8\sin 2t}{-8\sin 2t}=1\text{ H}$$

b)



$$11.31\cos(2t+45°)=11.31[\cos(-45°)\cos(2t)-\sin(-45°)\sin(2t)]$$
$$=8\cos 2t+8\sin 2t$$

The first term is proportional to the voltage. Associate it with the resistor. The noticing that

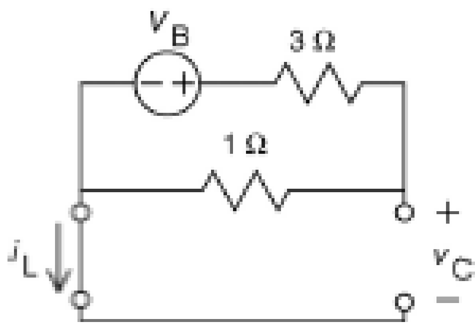
$$\int_{-\infty}^{t} i(\tau)\,d\tau=\int_{-\infty}^{t} 4\cos 2t\,d\tau=2\sin 2t$$
$$\frac{d}{dt}i(t)=\frac{d}{dt}4\cos 2t=-8\sin 2t$$

associate the second term with a capacitor to get the minus sign. Then

$$R=\frac{v_1(t)}{4\cos 2t}=\frac{8\cos 2t}{4\cos 2t}=2\,\Omega \text{ and } C=\frac{\int_{-\infty}^{t} 4\cos 2t\,d\tau}{v_2(t)}=\frac{2\sin 2t}{8\sin 2t}=0.25\text{ F}$$

DP 7-4
Solution:

at t=0$^-$

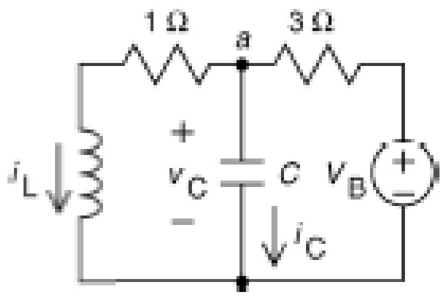


$$i_L\left(0^-\right)=0$$

By voltage division: $v_C\left(0^-\right)=\dfrac{V_B}{4}$

We require $v_C\left(0^-\right)=3$ V so V_B = 12 V

at t=0$^+$



Now we will check $\left.\dfrac{dv_C}{dt}\right|_{t=0^+}$

First: $i_L\left(0^+\right)=i_L\left(0^-\right)=0$

and $v_C\left(0^+\right)=v_C\left(0^-\right)=3$ V

Apply KCL at node a: $$i_L\left(0^+\right)+i_C\left(0^+\right)=\frac{V_B-v_C\left(0^+\right)}{3}$$

$$0+i_C\left(0^+\right)=\frac{12-3}{3}\Rightarrow i_C\left(0^+\right)=3\text{ A}$$

Finally $$\left.\frac{dv_C}{dt}\right|_{t=0^+}=\frac{i_C\left(0^+\right)}{C}=\frac{3}{0.125}=24\ \frac{\text{V}}{\text{s}}$$

as required.

DP 7-5
Solution:

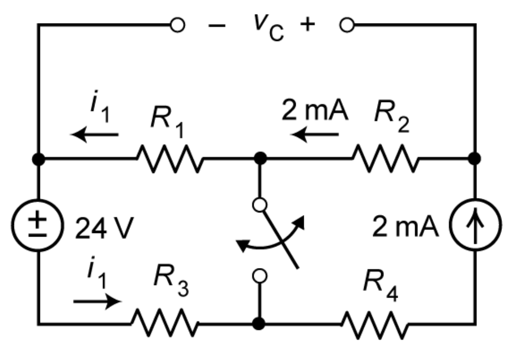
We require $\frac{1}{2}L\,i_L^2=\frac{1}{2}C\,v_C^2$ where i_L and v_c are the steady-state inductor current and capacitor voltage. At steady state, $i_L=\dfrac{v_C}{R}$. Then

$$L\left(\frac{v_C}{R}\right)^2=C\,v_C^2\quad\Rightarrow\quad C=\frac{L}{R^2}\quad\Rightarrow\quad R=\sqrt{\frac{L}{C}}=\sqrt{\frac{10^{-2}}{10^{-6}}}=\sqrt{10^4}=10^2\ \Omega$$

DP 7-6

Solution:

A capacitor in a **steady state** circuit with constant inputs (that is, a dc circuit) acts like an open circuit:



The **steady state** current in resistor R_2 is 2 mA both when the switch is open and when it is closed. Also the **steady state** current in resistor R_3 is equal to the **steady state** current in resistor R_1 as shown in the above figure.

The current in resistor R_1 is given by

$$i_1 = \begin{cases} -2 \text{ mA} & \text{the switch is open} \\ \dfrac{24}{R_1 + R_3} & \text{the switch is closed} \end{cases}$$

Use Ohm's law and KVL to express the **steady state** voltage across the capacitor as

$$v_C = \begin{cases} \left(R_1 + R_2\right)0.002 & \text{the switch is open} \\ -\dfrac{R_1}{R_1 + R_3}24 + R_2\left(0.002\right) & \text{the switch is closed} \end{cases}$$

The specification requires

$$10 = v_C = \left(R_1 + R_2\right)0.002 \quad \Rightarrow \quad R_1 + R_2 = \frac{10}{0.002} = 5000$$

when the switch is open, and

$$-10 = -\frac{R_1}{R_1 + R_3}24 + R_2\left(0.002\right)$$

when the switch is closed.

Let's try R_1 = 4 kΩ and R_2 = 1 kΩ. The specification is satisfied when the switch is open. When the switch is closed we have

$$-10=-\frac{4000}{4000+R_3}24+1000(0.002) \quad\Rightarrow\quad -12=-\frac{4000}{4000+R_3}24 \quad\Rightarrow\quad R_3=4\text{ k}\Omega$$

The specifications are satisfied when R_1 = 4 kΩ, R_2 = 1 kΩ and R_3 = 4 kΩ . Any value of R_4 will do. It's hard to argue against choosing R_4 = 0, in which case resistor R_4 is just a wire.

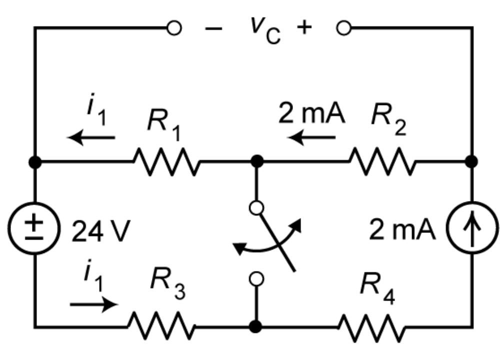
In summary, the specifications are satisfied when

R_1 = 4 kΩ, R_2 = 1 kΩ, R_3 = 4 kΩ and R_4 = 0

DP 7-7

Solution:

A capacitor in a **steady state** circuit with constant inputs (that is, a dc circuit) acts like an open circuit:



The **steady state** current in resistor R_2 is 2 mA both when the switch is open and when it is closed. Also the **steady state** current in resistor R_3 is equal to the **steady state** current in resistor R_1 as shown in the above figure.

The current in resistor R_1 is given by

$$i_1=\begin{cases} -2\text{ mA} & \text{the switch is open} \\ \dfrac{24}{R_1+R_3} & \text{the switch is closed} \end{cases}$$

Use Ohm's law and KVL to express the **steady state** voltage across the capacitor as

$$v_C = \begin{cases} (R_1 + R_2)0.002 & \text{the switch is open} \\ -\dfrac{R_1}{R_1 + R_3} 24 + R_2(0.002) & \text{the switch is closed} \end{cases}$$

The specification requires

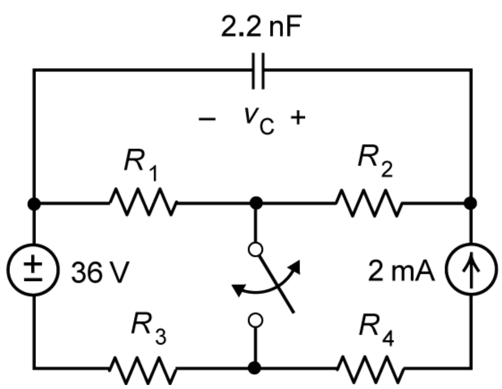
$$4 = v_C = (R_1 + R_2)0.002 \quad \Rightarrow \quad R_1 + R_2 = \frac{4}{0.002} = 2000$$

when the switch is open, and

$$-30 = -\frac{R_1}{R_1 + R_3} 24 + R_2(0.002)$$

when the switch is closed. There is no choice of positive resistances R_2, R_2 and R_2 that will satisfy this equation. The specifications cannot be satisfied.

We might be able to satisfy the specifications if we increased the voltage source-voltage to something a bit larger than 24 V. Let's try 36 V:



Now

$$v_C = \begin{cases} (R_1 + R_2)0.002 & \text{the switch is open} \\ -\dfrac{R_1}{R_1 + R_3} 30 + R_2(0.002) & \text{the switch is closed} \end{cases}$$

Let's try R_1 = 1 kΩ and R_2 = 1 kΩ. The specification is satisfied when the switch is open. When the switch is closed we have

$$-30 = -\frac{1000}{1000 + R_3} 36 + 1000(0.002) \quad \Rightarrow \quad -32 = -\frac{1000}{1000 + R_3} 36 \quad \Rightarrow \quad R_3 = 125\ \Omega$$

The specifications are satisfied when R_1 = 4 kΩ, R_2 = 1 kΩ and R_3 = 125 Ω . Any value of R_4 will do. It's hard to argue against choosing R_4 = 0, in which case resistor R_4 is just a wire.

In summary, if the voltage source-voltage the specifications are satisfied when

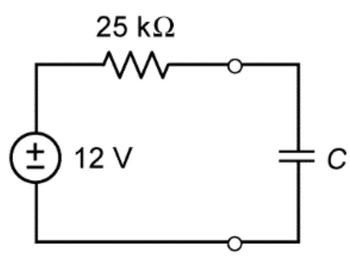
$$R_1 = 4\ \text{k}\Omega,\ \ R_2 = 1\ \text{k}\Omega,\ R_3 = 4\ \text{k}\Omega \text{ and } R_4 = 0$$

If the circuit cannot be changed, the specifications cannot be satisfied.

DP7-8

Solution:

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit (see the solution to problem DP 5-11).



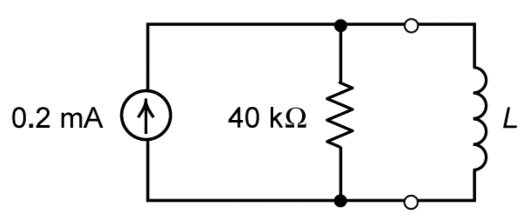
We require: $$2\times10^{-3} = 25\times10^{3} C \quad \Rightarrow \quad C = \frac{2}{25}\times10^{-6} = 80\ \text{nF}$$

Consequently 80 nF is the value of the capacitance *C* in the circuit shown in Figure DP 7-8*b* that will cause the time constant to be $\tau = 2\ \text{ms}$.

DP7-9

Solution:

Replace the part of the circuit connected to the resistor having resistance *R* by its Norton equivalent circuit (see the solution to problem DP5-5).



We require: $$2\times10^{-6} = \frac{L}{40000} \quad \Rightarrow \quad L = 80\times10^{-3} = 80\ \text{mH}$$

Consequently 80 mH is the value of the inductance *L* in the circuit shown in Figure DP 7-9(b) that will cause the time constant to be $\tau = 2\ \mu\text{s}$.