EE306002: Probability **Spring 2022**
Department of Electrical Engineering
National Tsing Hua University

Homework #4
Coverage: chapter 6–7
Due date: 5 May, 2022

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Problem 6.1.12. (12.5 points) Let X denote the lifetime of a radio, in years, manufactured by a certain company. The probability density function of X is given by

$$f(x) = \begin{cases} \frac{1}{15}e^{-x/15} & 0 \le x < \infty \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that, of eight such radios, at least four last more than 15 years?

Problem 6.2.8. (12.5 points) Let X be a Cauchy random variable with the probability density function

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty.$$

Find the probability density function of $Z = \arctan X$.

Problem 6.3.12. (12.5 points) Let X be a random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x|} \quad -\infty < x < \infty.$$

Calculate $Var(X)$.

Problem 6.3.16. (12.5 points) Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{1}{\pi}x\sin x & 0 \le x < \pi \\ 0 & \text{otherwise.} \end{cases}$$

Prove that

$$E(X^{n+1}) + (n+1)(n+2)E(X^{n-1}) = \pi^{n+1}.$$

Problem 7.1.6. (12.5 points) A point is selected at random on a line segment of length ℓ. What is the probability that the longer segment is at least twice as long as the shorter segment?

Problem 7.2.22. (12.5 points) Let $\alpha \in (-\infty, \infty)$ and $Z \sim N(0,1)$. Find $E(e^{\alpha Z})$.

Problem 7.3.8. (12.5 points) Suppose that, the time, in minutes, between two customers ordering pizza at an Italian restaurant is exponential random variable with parameter λ. What is the probability that (a) no customer orders pizza during the next t minutes; (b) the next pizza order is placed in at least t minutes but no later than s minutes ($t < s$)?

Problem 7.4.4. (12.5 points) Let X be the time to failure of a mobile robot that is used as an automated guide vehicle. Suppose that $E(X) = 7$ years, and X has a gamma distribution with $r = 4$. Find $Var(X)$ and $P(5 \leq X \leq 7)$.

References

[1] Saeed Ghahramani, *Fundamentals of Probability: With Stochastic Processes,* Chapman and Hall/CRC; 4th edition (September 4, 2018)