Problem 1

(一) $\quad -\frac{1}{8}z^{-1}Y(z)+\frac{1}{4}Y(z)+z^{+1}Y(z)=-2z^{-1}X(z)+X(z)$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{-2z^{-1}+1}{-\frac{1}{8}z^{-1}+\frac{1}{4}+z^{+1}}=\frac{0+z^{-1}-2z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$$
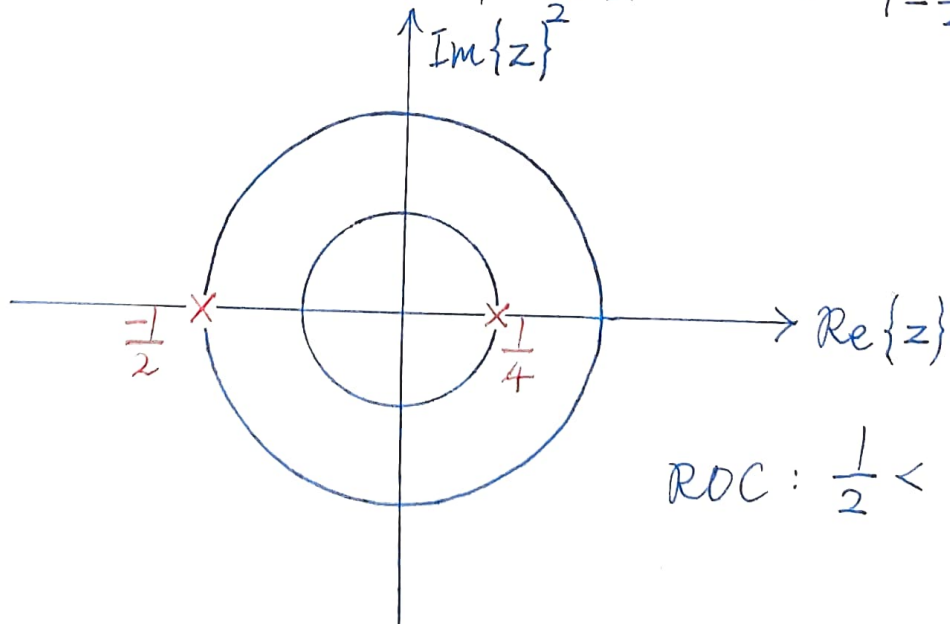
$$=\frac{z^{-1}(1-2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}=A+\frac{B}{1+\frac{1}{2}z^{-1}}+\frac{C}{1-\frac{1}{4}z^{-1}}$$

$$C=\frac{z^{-1}(1-2z^{-1})}{1+\frac{1}{2}z^{-1}}\bigg|_{z^{-1}=4}=\frac{4(1-8)}{1+2}=\frac{-28}{3}$$

$$B=\frac{z^{-1}(1-2z^{-1})}{1-\frac{1}{4}z^{-1}}\bigg|_{z^{-1}=-2}=\frac{-2(1+4)}{1+\frac{1}{2}}=\frac{-10}{\frac{3}{2}}=\frac{-20}{3}$$

$$A=H(z)\bigg|_{z^{-1}=0}-B-C=0-\frac{-28}{3}-\frac{-20}{3}=16$$

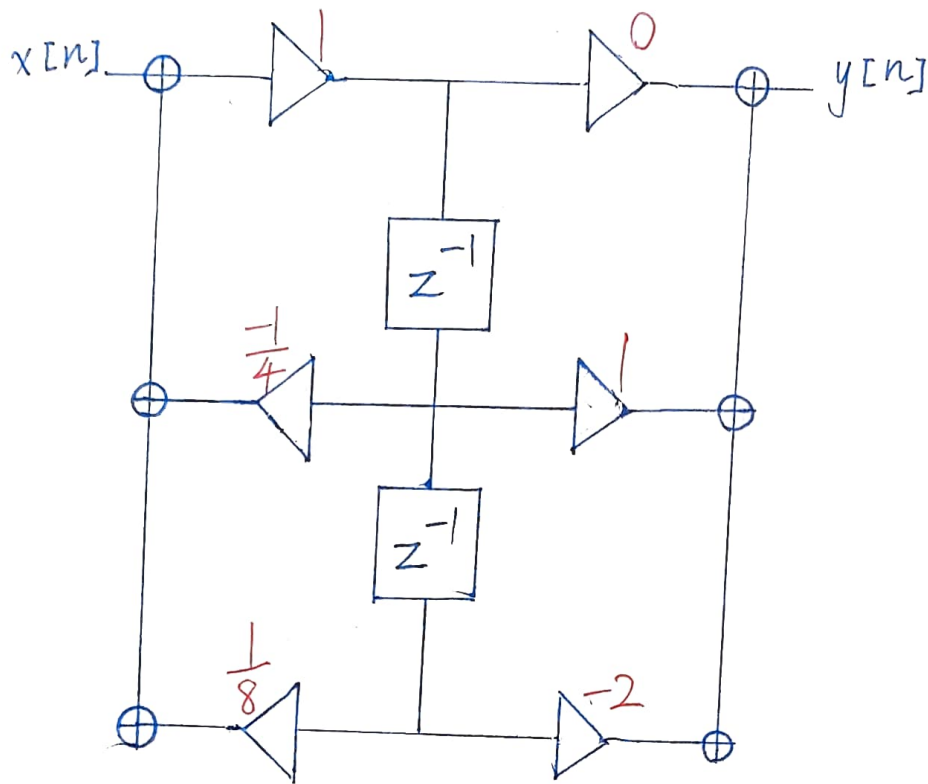$$H(z)=16+\frac{-20}{3}\frac{1}{1-\frac{-1}{2}z^{-1}}+\frac{-28}{3}\frac{1}{1-\frac{1}{4}z^{-1}}$$

ROC: $\frac{1}{2}<|z|$

$$h[n]=16\,\delta[n]+\frac{-20}{3}\left(\frac{-1}{2}\right)^{n}u[n]+\frac{-28}{3}\left(\frac{1}{4}\right)^{n}u[n]$$

Problem 1 (continued)
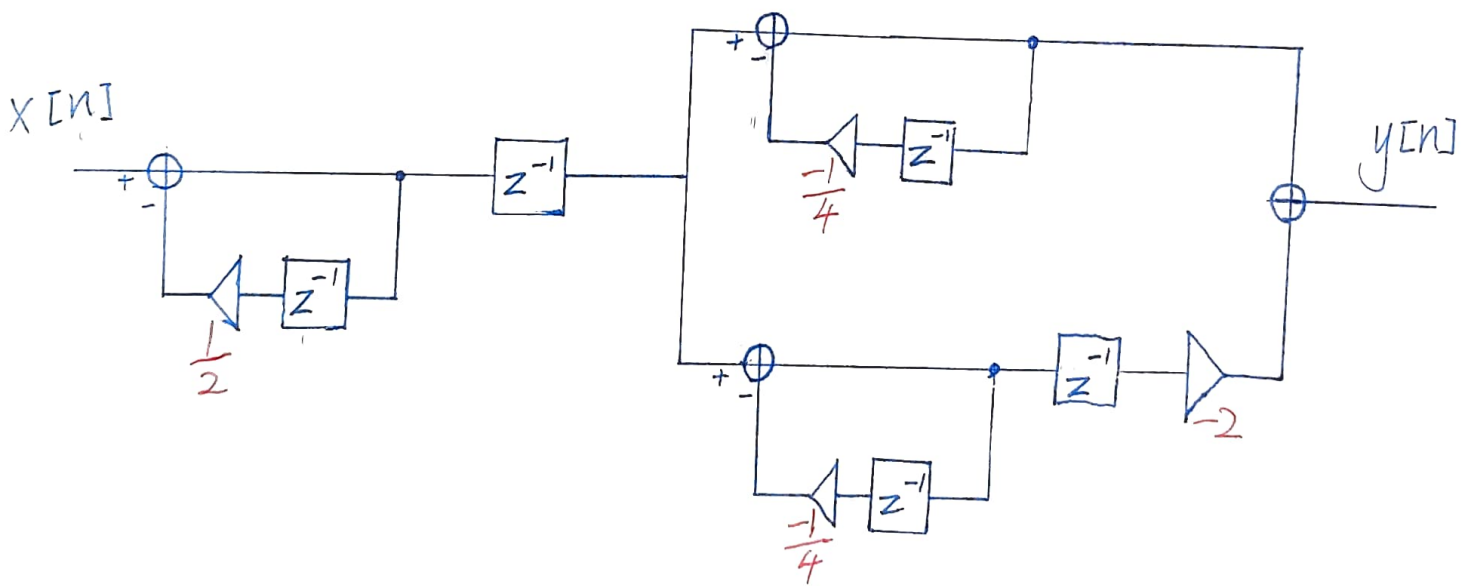
(二)

direct form :

$$-\frac{1}{8}y[n-2] + \frac{1}{4}y[n-1] + y[n] = -2x[n-2] + x[n-1]$$

Problem 1 (continued)

cascade form

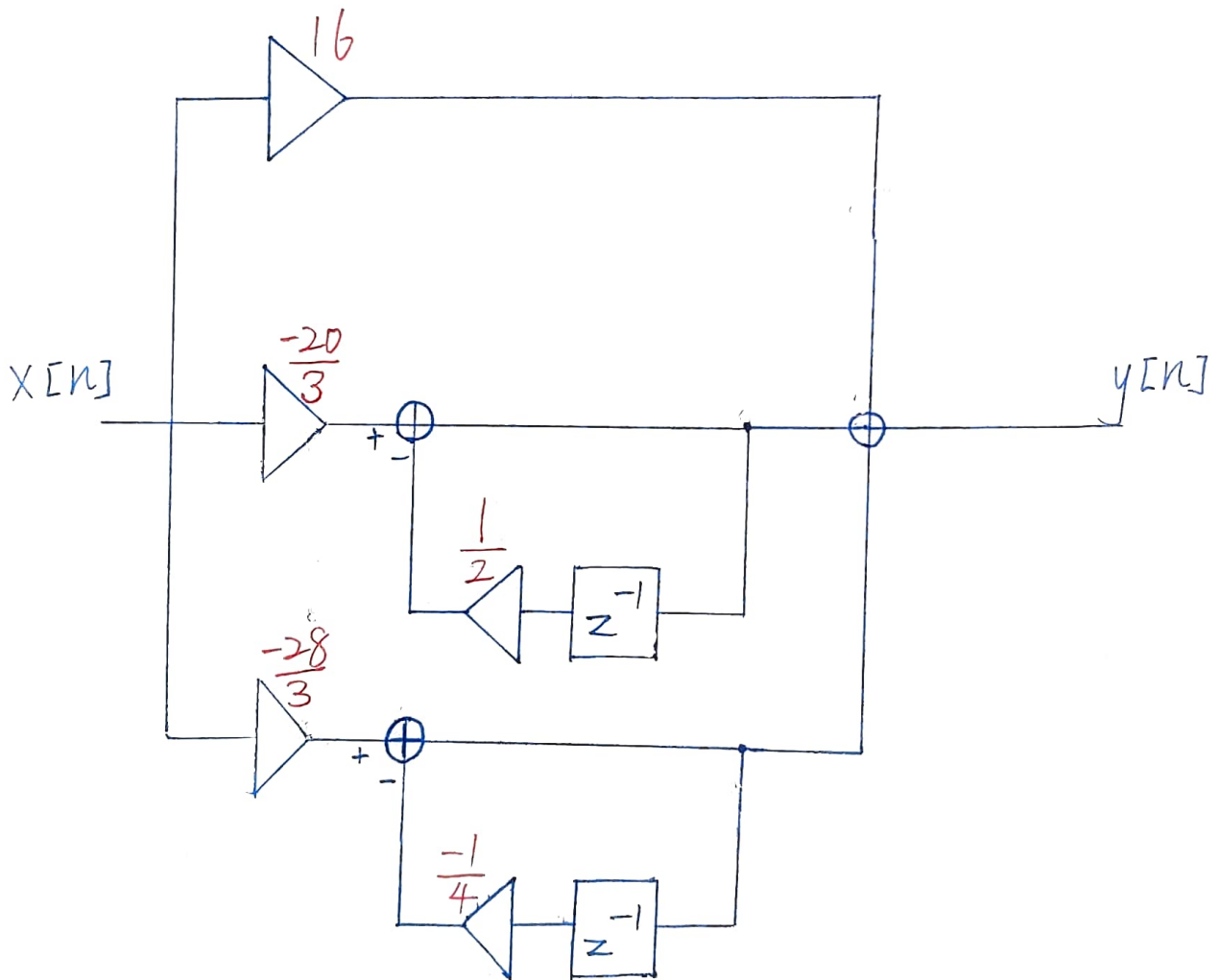$$H(z) = \frac{z^{-1}(1-2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$$H(z) = \frac{1}{1+\frac{1}{2}z^{-1}}(z^{-1})\left[\frac{1}{1+\frac{-1}{4}z^{-1}} + (-2)\frac{1}{1+\frac{-1}{4}z^{-1}}(z^{-1})\right]$$

Problem 1   (continued)

parallel form

$$H(z) = 16 + \left(\frac{-20}{3}\right)\left(\frac{1}{1 + \frac{1}{2}z^{-1}}\right) + \left(\frac{-28}{3}\right)\left(\frac{1}{1 + \frac{-1}{4}z^{-1}}\right)$$
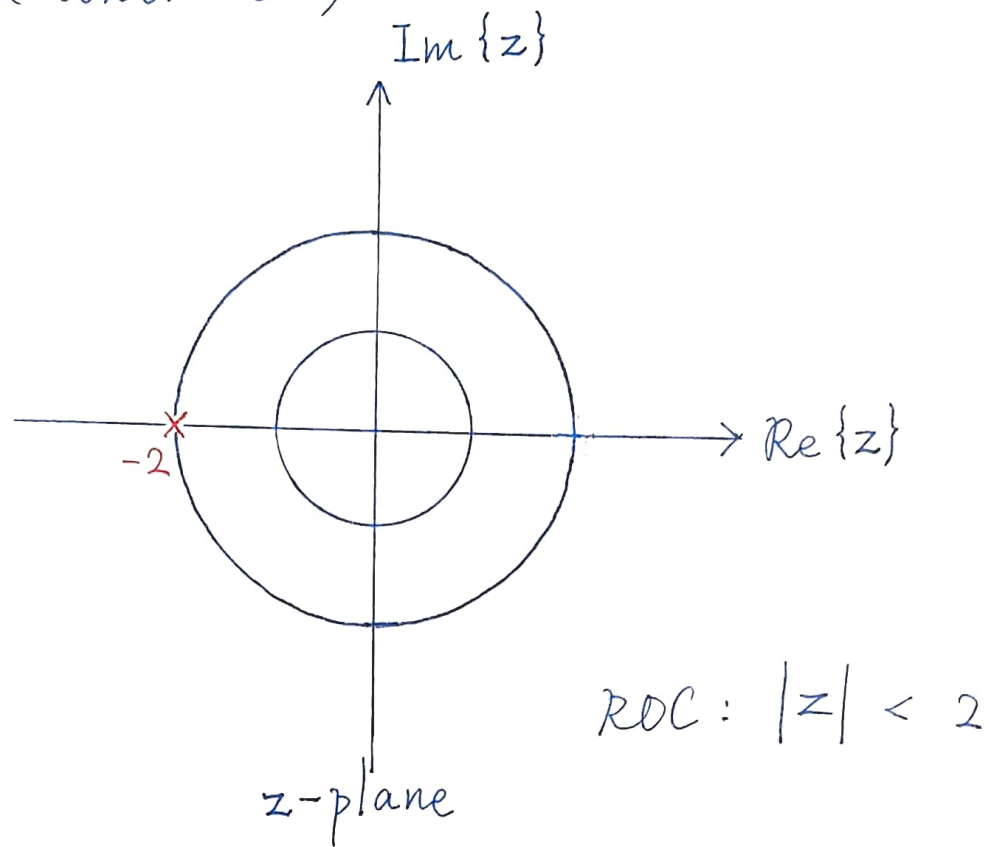
Problem 2

$$y[n] + 2y[n-1] = x[n] + 2x[n-2]$$

$$Y(z) + 2z^{-1}Y(z) = X(z) + 2z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-2}}{1 + 2z^{-1}}$$

$$= \frac{1 + 2z^{-1} - 2z^{-1} + 2z^{-2}}{1 + 2z^{-1}}$$

$$= 1 + \frac{-2z^{-1} + 2z^{-2}}{1 + 2z^{-1}}$$

$$= 1 - 2z^{-1} \cdot \frac{1 - z^{-1}}{1 + 2z^{-1}}$$

$$= 1 - 2z^{-1} \cdot \frac{1 + 2z^{-1} - 3z^{-1}}{1 + 2z^{-1}}$$

$$= 1 - 2z^{-1}\left(1 - 3\frac{1}{1 + 2z^{-1}}z^{-1}\right)$$

$$= 1 - 2z^{-1} - 2z^{-1}\left(-3\frac{1}{1 + 2z^{-1}}z^{-1}\right)$$

$$= 1 - 2z^{-1} + 6z^{-2}\frac{1}{1 - (-2)z^{-1}}$$

Problem 2 (continued)

Im {z}

→ Re {z}

−2

z-plane

ROC : $|z| < 2$

$$H(z) = 1 - 2(z^{-1}) + 6(z^{-2})\left(\frac{1}{1-(-2)z^{-1}}\right)$$

$$h[n] = \delta[n] - 2\delta[n-1] + 6\delta[n-2] * \left[-(-2)^n u[-n-1]\right]$$

$$x[n] = \delta[n-2]$$

$$y[n] = x[n] * h[n]$$

$$= \delta[n-2] - 2\delta[n-3] - 6(-2)^{n-4} u[-n+3]$$

Problem 3

$$X(z) = \frac{1}{1 - \frac{1}{6}z^{-1}}$$

$$Y(z) = H(z)\, X(z) = \frac{1 - \frac{2}{9}z^{-1}}{1 - \frac{1}{3}z^{-1}} \cdot \frac{1}{1 - \frac{1}{6}z^{-1}}$$

$$= \frac{A}{1 - \frac{1}{3}z^{-1}} + \frac{B}{1 - \frac{1}{6}z^{-1}}$$

$$A = \frac{1 - \frac{2}{9}z^{-1}}{1 - \frac{1}{6}z^{-1}}\Bigg|_{z^{-1} = 3} = \frac{1 - \frac{2}{3}}{1 - \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$
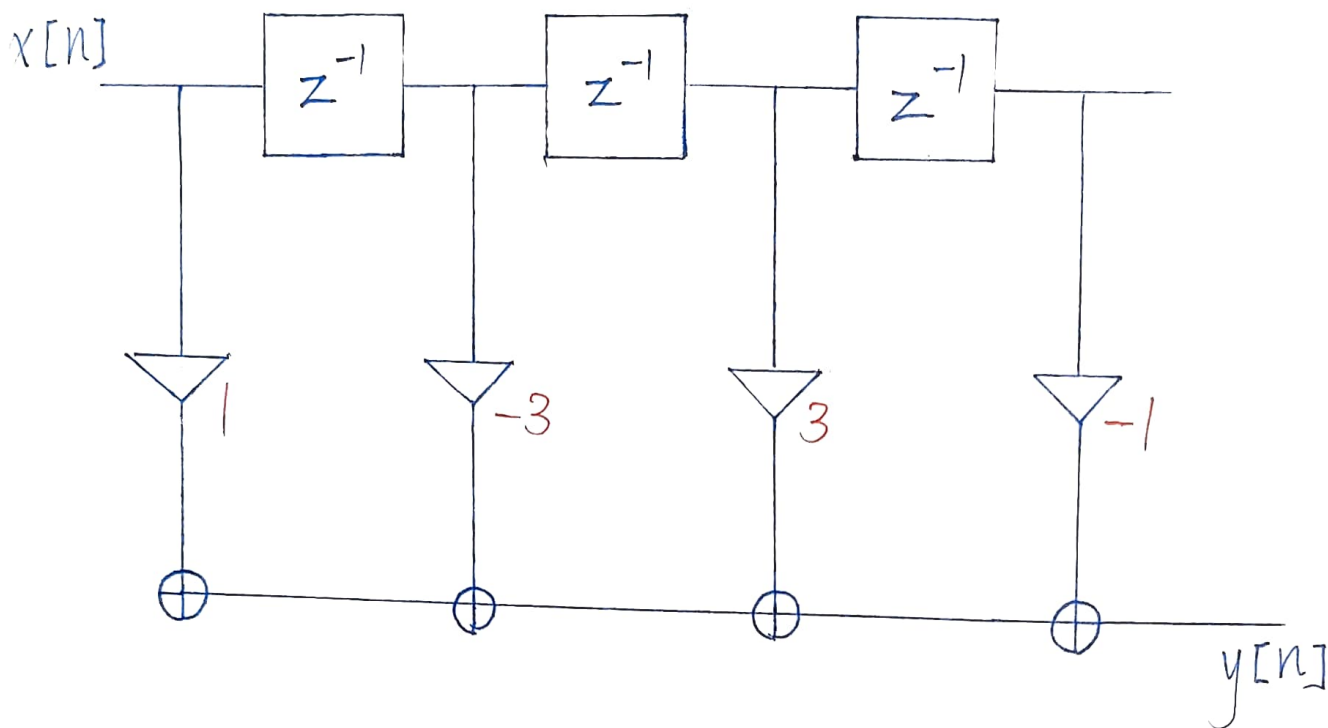
$$B = \frac{1 - \frac{2}{9}z^{-1}}{1 - \frac{1}{3}z^{-1}}\Bigg|_{z^{-1} = 6} = \frac{1 - \frac{4}{3}}{1 - 2} = \frac{-\frac{1}{3}}{-1} = \frac{1}{3}$$

$$Y(z) = \frac{2}{3} \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{1}{3} \frac{1}{1 - \frac{1}{6}z^{-1}}$$

$$ROC : \frac{1}{6} < |z| < \frac{2}{3}$$

$$y[n] = \frac{2}{3}\left[-\left(\frac{1}{3}\right)^n u[-n-1]\right] + \frac{1}{3}\left[\left(\frac{1}{6}\right)^n u[n]\right]$$

Problem 4

x[n]

$$x[n] = 10 + 4\cos\left[0.5\pi(n) + \frac{\pi}{4}\right] + 5\,\delta[n-3]$$

$$y[n] = 1\left\{10 + 4\cos\left[0.5\pi(n) + \frac{\pi}{4}\right] + 5\,\delta[n-3]\right\}$$

$$-3\left\{10 + 4\cos\left[0.5\pi(n-1) + \frac{\pi}{4}\right] + 5\,\delta[n-4]\right\}$$

$$+3\left\{10 + 4\cos\left[0.5\pi(n-2) + \frac{\pi}{4}\right] + 5\,\delta[n-5]\right\}$$

$$-1\left\{10 + 4\cos\left[0.5\pi(n-3) + \frac{\pi}{4}\right] + 5\,\delta[n-6]\right\}$$

Problem 5

(1)

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \frac{-1}{4} \delta[n-1] * \left[\left(\frac{1}{3}\right)^n u[n]\right]$$

$$X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{-1}{4} z^{-1} \frac{1}{1 - \frac{1}{3}z^{-1}}$$

$$y[n] = \frac{3}{2}\left[\left(\frac{1}{2}\right)^n u[n]\right]$$

$$Y(z) = \frac{3}{2} \frac{1}{1 - \frac{1}{2}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{2} \frac{1}{1 - \frac{1}{2}z^{-1}}}{\frac{1}{1 - \frac{1}{3}z^{-1}} + \frac{-1}{4} z^{-1} \frac{1}{1 - \frac{1}{3}z^{-1}}}$$

$$= \frac{\frac{3}{2} \frac{1}{1 - \frac{1}{2}z^{-1}}}{\frac{1 - \frac{1}{4}z^{-1}}{1 - \frac{1}{3}z^{-1}}}$$

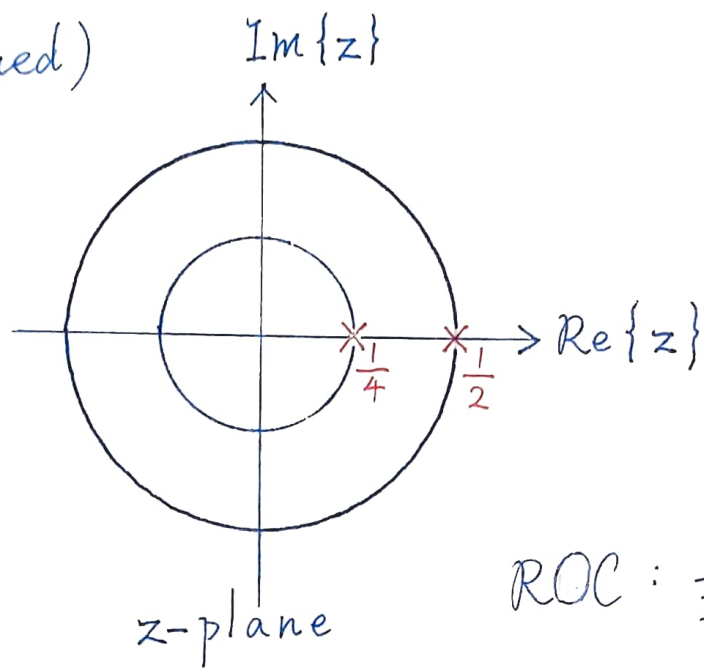$$= \frac{3}{2} \frac{1 - \frac{1}{3}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{3}{2}\left[\frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - \frac{1}{4}z^{-1}}\right]$$

$$A = \frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}}\bigg|_{z^{-1}=2} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

$$B = \frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}\bigg|_{z^{-1}=4} = \frac{\frac{-1}{3}}{-1} = \frac{1}{3}$$

$$H(z) = \frac{3}{2}\left(\frac{2}{3} \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{3} \frac{1}{1 - \frac{1}{4}z^{-1}}\right)$$

Problem 5 (continued)

Im{z}

z-plane

ROC : $\frac{1}{2} < |z|$

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{2} \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{4}\right)^n u[n]$$

Problem 5 (continued)

(2)

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\left(1 - \frac{1}{4}z^{-1}\right) + \frac{1}{2}\left(1 - \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

$$= \frac{\frac{3}{2} - \frac{1}{2}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = \frac{3}{2}X(z) - \frac{1}{2}z^{-1}X(z)$$

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = \frac{3}{2}x[n] - \frac{1}{2}x[n-1]$$