$$\int_0^\infty e^{(j2\pi t-2)f}\,df \qquad H(f)=\begin{cases}-j & f>0\\ 0 & f=0\\ j & f<0\end{cases}$$

EE 3640 Communication Systems I
Spring 2023

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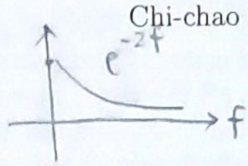



**Midterm Examination**
7:00pm to 10:00pm, April 21, 2023

**Problems for Solution:**

$u(f).e^{-2f}$

1. (15%) Consider the one-sided frequency function:

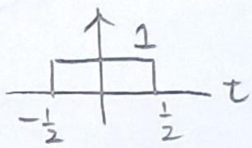
$$G(f)=\begin{cases}\exp(-2f), & f>0\\ \pi, & f=0\\ 0, & f<0.\end{cases}$$

2500

   (a) (7%) Evaluate its inverse Fourier transform $g(t)$. $Im = \mathcal{H}\{Re\}$

   (b) (8%) Show that $g(t)$ is complex and that its real and imaginary parts constitute a Hilbert-transform pair.

2. (15%) Consider the input




$$x(t)=\text{rect}(t/T)\cos(2\pi f_c t)$$

where $\text{rect}(t)=1$, for $|t|<1/2$, and $\text{rect}(t)=0$, elsewhere, to a linear filter with impulse response

$$h(t)=u(t)\cos(2\pi f_c t)$$

where $u(t)$ is the unit step function. Assume $f_c \gg 1/T$. Let $y(t)$ denote the output of this filter.

   (a) (4%) Find the complex envelope $\tilde{x}(t)$ of $x(t)$.

   (b) (4%) Find the complex envelope $\tilde{h}(t)$ of $h(t)$.

   (c) (7%) Use (a) and (b) to find $y(t)$ and sketch it.

3. (15%) Consider an AM modulator with the message signal

$$m(t)=3\cos(20\pi t)+7\cos(60\pi t).$$

$A_c\cos(2\pi f_c t)$

The unmodulated carrier ~~is given~~ by $100\cos(200\pi t)$, and the system operates with a percentage modulation of 50%.

$\mu=\max_t\{k_a m(t)\}=50\%$

   (a) (5%) Write an expression for the modulated signal $s(t)$.

   (b) (5%) Find the Fourier transform $S(f)$ of $s(t)$.

   (c) (5%) Determine the ratio of the power in the sidebands to the total power in $s(t)$.

V 4. (10%) Consider an SSB signal $s(t)$ given by

$$s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)$$ $\Delta f > 0.$

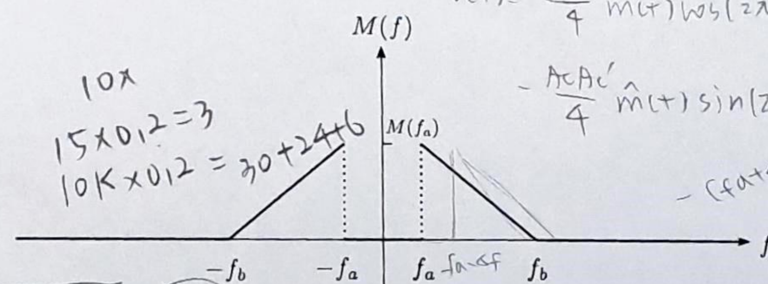
where $m(t)$ is the message signal and $\hat{m}(t)$ is its Hilbert transform. The local oscillator used for coherent demodulation of $s(t)$ has a (positive) frequency error $\Delta f$ measured with respect to the carrier frequency $f_c$ used to generate $s(t)$. Otherwise, there is perfect synchronism between the oscillator in the receiver and the oscillator supplying the carrier wave in the transmitter. That is, the carrier generated by the local oscillator in the receiver is

$$c'(t) = A_c' \cos\left[2\pi(f_c + \Delta f)t\right].$$

(a) (5%) Evaluate the demodulated signal $v(t)$.

(b) (5%) Suppose the Fourier transform $M(f)$ of $m(t)$ is given below. Plot the Fourier transform $V(f)$ of $v(t)$.

$v(t) = \frac{A_c A_c'}{4} m(t)\cos(2\pi\Delta f t) - \frac{A_c A_c'}{4}\hat{m}(t)\sin(2\pi\Delta f t)$




15×0.2=3

5×0.2=1

FDM

6+12+18+12+12 = 60

5. (10%) Five baseband message signals, $m_i(t)$, $i = 1, 2, 3, 4, 5$, with bandwidths of 5 kHz, 10 kHz, 15 kHz, 10 kHz, 10 kHz, respectively, are to be transmitted using frequency division multiplexing in the band from 800 kHz to 860 kHz over a single communication channel. To do this, we must select a modulation technique.

$A_c k_a m(t)\cos(2\pi f_c t)$ 60K

(a) (5%) Can DSB-SC be used? Why or why not?

carrier?

(b) (5%) If VSB (with a full upper sideband and a 20% lower sideband) transmission is used, specify the carrier frequency and the band of frequencies occupied for each message signal assuming that no bandwidth is allocated to guardbands.

FM 6. (25%) A carrier wave of frequency 25 MHz is frequency-modulated by a sinusoidal wave of amplitude 5 volts and frequency 10 kHz. The frequency sensitivity of the modulator $k_f$ is 4 kHz per volt. Let the resulting FM signal be denoted as $s_1(t)$.

(a) (5%) Determine the approximate transmission bandwidth $B_T$ of $s_1(t)$, using Carson's rule.

(b) (5%) Determine the approximate transmission bandwidth by considering only those side frequencies whose amplitudes exceed 1 percent of the unmodulated carrier amplitude.

20+18

800+6+12+18+2

800+6+12

+3

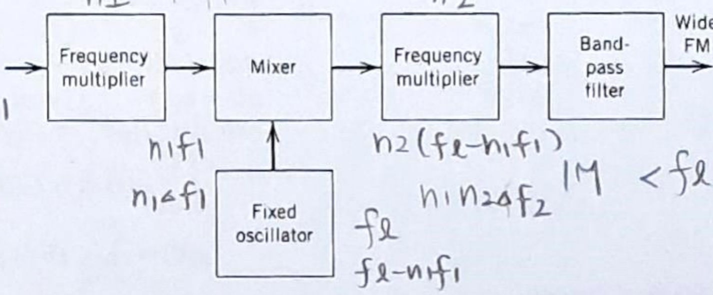
V

(c) (5%) Suppose this FM signal $s_1(t)$ is further applied as the input to the system shown below. Let the output signal be $s_2(t)$. The multiplication ratios of the first and second frequency multipliers are $n_1$ and $n_2$, respectively. If we would like to have the carrier frequency of $s_2(t)$ at 92 MHz and the frequency deviation as 80 kHz, determine the multiplication ratios $n_1, n_2$ and the frequency $f_l$ of the local oscillator. Note that due to the limitation of the available local oscillators and frequency multipliers, $f_l$ can only be in the range of 1 to 10 MHz and the multiplication ratios can only be 2 or 3. $n_1, n_2$

$n_1$ $\qquad$ $n_2$

$f_1$ $\qquad$ $f_2 = 92M$

$\Delta f_1$ $\qquad$ $\Delta f_2 = 80K$




$n_1 f_1$ $\qquad$ $n_2(f_l - n_1 f_1)$

$n_1 \Delta f_1$ $\qquad$ $n_1 n_2 \Delta f_2$ $\qquad$ $1M < f_l < 10M$ $\qquad$ $n_1 = n_2 = 2$

$f_l$

$f_l - n_1 f_1$ $\qquad$ 2, 2, 20

$n_1 \Delta f_1$ $\qquad$ $20 \times 2 \times 2 = 80$

(d) (5%) Now repeat (a) and (b) for $s_2(t)$. BT ? → discrete

(e) (5%) For the frequency components in $s_2(t)$, what is the frequency separation of the adjacent side frequencies?

7. (10%) Consider a first-order phase-locked loop (PLL) described by the following differential equation:

$$\frac{d\phi_e(t)}{dt} + 2\pi K_0 \sin[\phi_e(t)] = \frac{d\phi_1(t)}{dt}$$

where $\phi_1(t)$ is the phase to be tracked, $\phi_e(t)$ is the phase error, and $K_0$ is a constant called the loop-gain parameter. This PLL can be used as an FM demodulator. Assume

$$\phi_1(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau$$

output

where $m(t)$ is the modulating message signal and $k_f$ is a constant. The output is given by $v(t) = (K_0/k_v)\phi_e(t)$ where $k_v$ is a constant. Suppose the following approximation is employed to ~~linearize the~~ differential equation: $\sin[\phi_e(t)] \approx \phi_e(t)$ when $\phi_e(t)$ is small. linearize Please use Fourier transform to find the steady-state response of $v(t)$. Also assume for the frequency range concerned, the loop gain $K_0$ is much larger than $|f|$, i.e., $K_0 \gg |f|$. Explain how demodulation can be achieved.

$V(\infty)$

$V(t) = \left(\frac{K_0}{K_V}\right)\phi_e(t)$ $\qquad$ $\frac{K_0}{K_V}\,\frac{K_f}{S + K_0}$



$V(f) = \frac{K_0}{K_V} \cdot \frac{K_f}{jf + K_0} M(f)$

## A.1 Properties of the Fourier transform

| Property | Mathematical Description |
|---|---|
| 1. Linearity | $ag_1(t) + bg_2(t) \rightleftharpoons aG_1(f) + bG_2(f)$ where $a$ and $b$ are constants |
| 2. Time scaling | $g(at) \rightleftharpoons \frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$ where $a$ is a constant |
| 3. Duality | If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$ |
| 4. Time shifting | $g(t - t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$ |
| 5. Frequency shifting | $\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f - f_c)$ |
| 6. Area under $g(t)$ | $\int_{-\infty}^{\infty} g(t)\,dt = G(0)$ |
| 7. Area under $G(f)$ | $g(0) = \int_{-\infty}^{\infty} G(f)\,df$ |
| 8. Differentiation in the time domain | $\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$ |
| 9. Integration in the time domain | $\int_{-\infty}^{t} g(\tau)d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$ |
| 10. Conjugate functions | If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$ |
| 11. Multiplication in the time domain | $g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G_2(f-\lambda)\,d\lambda$ |
| 12. Convolution in the time domain | $\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)d\tau \rightleftharpoons G_1(f)G_2(f)$ |
| 13. Rayleigh's energy theorem | $\int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2 df$ |

## A.2 Fourier-transform pairs

| Time Function | Fourier Transform |
|---|---|
| $\text{rect}\left(\frac{t}{T}\right)$ | $T\,\text{sinc}(fT)$ |
| $\text{sinc}(2Wt)$ | $\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$ |
| $\exp(-at)u(t), \quad a > 0$ | $\frac{1}{a + j2\pi f}$ |
| $\exp(-a\lvert t\rvert), \quad a > 0$ | $\frac{2a}{a^2 + (2\pi f)^2}$ |
| $\exp(-\pi t^2)$ | $\exp(-\pi f^2)$ |
| $\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$ | $T\,\text{sinc}^2(fT)$ |
| $\delta(t)$ | $1$ |
| $1$ | $\delta(f)$ |
| $\delta(t - t_0)$ | $\exp(-j2\pi f t_0)$ |
| $\exp(j2\pi f_c t)$ | $\delta(f - f_c)$ |
| $\cos(2\pi f_c t)$ | $\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$ |
| $\sin(2\pi f_c t)$ | $\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$ |
| $\text{sgn}(t)$ | $\frac{1}{j\pi f}$ |
| $\frac{1}{\pi t}$ | $-j\,\text{sgn}(f)$ |
| $u(t)$ | $\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$ |
| $\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$ | $\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$ |

*Notes:* $u(t)$ = unit step function
$\delta(t)$ = Dirac delta function
$\text{rect}(t)$ = rectangular function
$\text{sgn}(t)$ = signum function
$\text{sinc}(t)$ = sinc function

## A.3 Trigonometric identities

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$

$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

1%= 0,01

## A.4 Table of Bessel functions

Jn(2)

| | $J_n(x)$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n$ \ $x$ | 0.5 | 1 | 2 | 3 | 4 | 6 | 8 | 10 | 12 |
| 0 | 0.9385 | 0.7652 | 0.2239 | −0.2601 | −0.3971 | 0.1506 | 0.1717 | −0.2459 | 0.0477 |
| 1 | 0.2423 | 0.4401 | 0.5767 | 0.3391 | −0.0660 | −0.2767 | 0.2346 | 0.0435 | −0.2234 |
| 2 | 0.0306 | 0.1149 | 0.3528 | 0.4861 | 0.3641 | −0.2429 | −0.1130 | 0.2546 | −0.0849 |
| 3 | 0.0026 | 0.0196 | 0.1289 | 0.3091 | 0.4302 | 0.1148 | −0.2911 | 0.0584 | 0.1951 |
| 4 | 0.0002 | 0.0025 | 0.0340 | 0.1320 | 0.2811 | 0.3576 | −0.1054 | −0.2196 | 0.1825 |
| 5 | — | 0.0002 | 0.0070 | 0.0430 | 0.1321 | 0.3621 | 0.1858 | −0.2341 | −0.0735 |
| 6 | | — | 0.0012 | 0.0114 | 0.0491 | 0.2458 | 0.3376 | −0.0145 | −0.2437 |
| 7 | | | 0.0002 | 0.0025 | 0.0152 | 0.1296 | 0.3206 | 0.2167 | −0.1703 |
| 8 | | | — | 0.0005 | 0.0040 | 0.0565 | 0.2235 | 0.3179 | 0.0451 |
| 9 | | | | 0.0001 | 0.0009 | 0.0212 | 0.1263 | 0.2919 | 0.2304 |
| 10 | | | | — | 0.0002 | 0.0070 | 0.0608 | 0.2075 | 0.3005 |
| 11 | | | | | — | 0.0020 | 0.0256 | 0.1231 | 0.2704 |
| 12 | | | | | | 0.0005 | 0.0096 | 0.0634 | 0.1953 |
| 13 | | | | | | 0.0001 | 0.0033 | 0.0290 | 0.1201 |
| 14 | | | | | | — | 0.0010 | 0.0120 | 0.0650 |

1%=0,01