Signals and Systems

Homework 11 — Due : May 24 2024

Problem 1 (30 pts)**.** Consider a causal and stable LTI system S whose input $x[n]$ and output $y[n]$ are related through the second-order difference equation

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n].$$

Determine the frequency response and the impulse response for the system.

Problem 2 (30 pts)**.** Determine the Laplace transform, its poles and zeros, and the associated ROC, for the following functions of time:

(a) $x(t) = e^{-3t}u(t) + e^{-4t}u(t)$

(b) $x(t) = e^{3t}\sin(4t)u(-t)$

Problem 3 (40 pts)**.** Consider the expression

$$\frac{(s-2)(s-3)}{(s+2)(s-\sqrt{5})(s^2-s+1)}.$$

(a) Find all zeros and poles.

(b) How many signals have a Laplace transform that may be expressed as this expression in its region of convergence?

Problem 1 (30 pts). Consider a causal and stable LTI system S whose input $x[n]$ and output $y[n]$ are related through the second-order difference equation

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Determine the frequency response and the impulse response for the system.

Let $\mathcal{F}\{y[n]\} = Y(e^{j\omega})$, $\mathcal{F}\{\delta[n]\} = 1$, $\mathcal{F}\{h[n]\} = H(e^{j\omega})$

$$H(e^{j\omega}) - \frac{1}{6}e^{-j\omega}H(e^{j\omega}) + \frac{1}{6}e^{-j\omega 2}H(e^{j\omega}) = 1$$

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}(e^{-j\omega})^2} = \frac{6}{(-e^{-j\omega}+2)(e^{-j\omega}+3)} = \frac{6}{5}\left(\frac{1}{-e^{-j\omega}+2}+\frac{1}{e^{-j\omega}+3}\right)$$

$$= \frac{3}{5}\left(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right) + \frac{2}{5}\left(\frac{1}{1-\frac{-1}{3}\cdot e^{-j\omega}}\right)$$

$$\mathcal{F}^{-1}\{H(e^{j\omega})\} = \frac{3}{5}\cdot\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\cdot\left(\frac{-1}{3}\right)^n u[n]$$

Problem 2 (30 pts). Determine the Laplace transform, its poles and zeros, and the associated ROC, for the following functions of time:

(a) $x(t) = e^{-3t}u(t) + e^{-4t}u(t)$

$$\mathcal{L}\{e^{-3t}u(t)\} = \int_0^{\infty} e^{-3t}\cdot e^{-st}\,dt = \frac{1}{s+3} \qquad \mathcal{L}\{e^{-4t}u(t)\} = \int_0^{\infty} e^{-4t}\cdot e^{-st}\,dt = \frac{1}{s+4}$$

$$\mathcal{L}\{x(t)\} = X(s) = \frac{1}{s+3} + \frac{1}{s+4} = \frac{2s+7}{(s+3)(s+4)}$$

zero: $s = \frac{-7}{2}$, pole: $s=-3$, $s=-4$, ROC: $\mathrm{Re}\{s\} > -3$

(b) $x(t) = e^{3t}\sin(4t)u(-t)$

$$\mathcal{L}\{x(t)\} = \mathcal{L}\left\{\frac{1}{2j}\left(e^{(3+4j)t} - e^{(3-4j)t}\right)u(-t)\right\} = \frac{1}{2j}\int_0^{\infty}\left(e^{-(3+4j)t} - e^{-(3-4j)t}\right)e^{st}\,dt$$

$$= \frac{1}{2j}\left(\frac{1}{-s+3+4j} - \frac{1}{-s+3-4j}\right) = \frac{-4}{(s-3-4j)(s-3+4j)}$$

No zeros. pole: $s = 3+4j$, $s = 3-4j$, ROC: $\mathrm{Re}\{s\} < 3$

Problem 3 (40 pts). Consider the expression

$$\frac{(s-2)(s-3)}{(s+2)(s-\sqrt{5})(s^2-s+1)}.$$

(a) Find all zeros and poles.

$s^2 - s + 1 = 0$, $s = \frac{1 \pm \sqrt{3}j}{2}$

zero: $s=2$, $s=3$. pole: $s=-2$, $s=\sqrt{5}$, $s=\frac{1+\sqrt{3}j}{2}$, $s=\frac{1-\sqrt{3}j}{2}$

(b) How many signals have a Laplace transform that may be expressed as this expression in its region of convergence?

4 種 : $Re\{s\} < -2$, $-2 < Re\{s\} < \frac{1}{2}$, $\frac{1}{2} < Re\{s\} < \sqrt{5}$, $Re\{s\} > \sqrt{5}$