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Properties of Laplace Transform (LT)

Here we list some properties of LT to show why LT is useful to solve DEs :

<1> "derivation" property :

By LT, derivation (in t-domain) ⟶

ex: $\mathcal{L}\{y'\} =$

proof:

$$\mathcal{L}\{y'\} = \int_0^\infty y' e^{-st} dt = y e^{-st} \Big|_0^\infty - \int_0^\infty (-s) e^{-st} y \, dt$$

☆ LT of derivation :

$$\begin{cases} \mathcal{L}\{y'\} = \\ \mathcal{L}\{y''\} = \\ \vdots \\ \mathcal{L}\{y^{(n)}\} = \end{cases}$$

It means
⟶

<2> "linearity" property

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<3> " " property

Many engineering / physics problems involve in time. ex:

Their LT are .

How to solve DEs by LT

General procedures :

Remarks about $\mathcal{L}^{-1}$ (inverse Laplace transform)

① We need to convert

② Properties of $\mathcal{L}^{-1}$

— "uniqueness" property :

If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}^{-1}\{F(s)\}$ can only be

— "linearity" property

Example 1 : Use LT to solve $y' = y - 4e^{-t}$, $y(0) = 1$

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Example2 : Use LT to solve $y'' + 4y' + 20y = e^{-2t}$, $y(0) = y'(0) = 0$

By comparing with $\mathcal{L}\{y\}$, we find that the denominator of $\mathcal{L}\{y\}$ consists of product of

So from the roots of denominator of $\mathcal{L}\{y\}$, we can get qualitative analysis of the system response.

ex: $\mathcal{L}\{y\} = \frac{1}{(s^2+2s+2)(s+\frac{1}{10})}$, what can we say about the system response qualitatively?

Remark: Qualitative analysis of system response from LT