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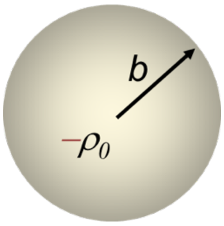
EE214000 Electromagnetics, Fall, 2020
Quiz #9-1, Open books, notes (30 points), due 11 pm, Wednesday, Nov. 4th, 2020
(請上傳至 iLMS 作業區)

Late submission won't be accepted!

1. Continue the charge ball calculation in the lecture to determine the electric potential and electric field outsider the ball (R > b). Verify the results by using the Gauss-law calculations. (9 points)



Outside the ball, $R > b$ and $\rho = 0$

Laplace's equation : $\nabla^2 V_0 = 0 \Rightarrow \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 \frac{dV_0}{dR}\right) = 0$ —— (1)

integrate (1) $\Rightarrow \frac{dV_0}{dR} = \frac{C_2}{R^2} \Rightarrow \vec{E_0} = -\nabla V_0 = -\frac{dV_0}{dR}\hat{a}_R = -\frac{C_2}{R^2}\hat{a}_R$ —— (2)

The integration consistant C_2 can be found by equating $\vec{E_0}$ and $\vec{E_i}$ at $R=b$, so there is no discontinuity in medium characteristics.

$\frac{C_2}{b^2} = \frac{\rho_0}{3\epsilon_0}b \Rightarrow C_2 = \frac{\rho_0 b^3}{3\epsilon_0}$ — (3) $\Rightarrow \vec{E_0} = -\frac{\rho_0 b^3}{3\epsilon_0 R^2}\hat{a}_R$ — (4), $R > b$

put (3) into (2), and integrate it. $\Rightarrow V_0 = -\frac{\rho_0 b^3}{3\epsilon_0 R} + C_2'$

C_2' will vanish since V_0 is zero at infinite $R \to \infty$. $\Rightarrow V_0 = -\frac{\rho_0 b^3}{3\epsilon_0 R}$ — (5)

Gauss's Law : $\epsilon_0 \oint_S \vec{E}\cdot d\vec{s} = \int_V \nabla\cdot\vec{D}\,dv = \int_V \rho\,dv = Q$

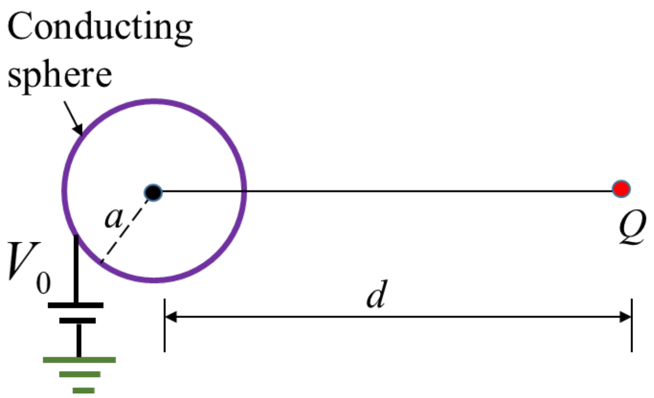
$$\Rightarrow \begin{cases} \epsilon_0 \oint_S E_R\,dS_R = \epsilon_0 E_R \oint_S dS_R = \epsilon_0 E_R \cdot 4\pi R^2 = Q \\ \int_V \rho\,dv = \rho\cdot\left(\frac{4}{3}\pi b^3\right) = (-\rho_0)\cdot\left(\frac{4}{3}\pi b^3\right) = Q \end{cases}$$

$\Rightarrow \epsilon_0 E_R \cdot 4\pi R^2 = -\frac{4\pi}{3}\rho_0 b^3 \Rightarrow E_R = -\frac{\rho_0 b^3}{3\epsilon_0 R^2} \Rightarrow \vec{E_0} = E_R\hat{a}_R = -\frac{\rho_0 b^3}{3\epsilon_0 R^2}\hat{a}_R$ — (6)

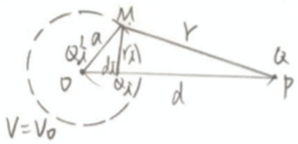
$\vec{E_0} = -\nabla V_0 \Rightarrow V_0 = -\frac{\rho_0 b^3}{3\epsilon_0 R}$ — (7)

We can get (6) = (4) and (7) = (5), so the result is verified by using Gauss's Law calculation.

2. A conducting sphere of radius a is maintained at a constant voltage of V_0. A point charge of Q is placed at a distance d from the center of the conducting sphere, where $d > a$. Find out the locations and values of the image charges that can replace the spherical boundaries. (8 points) What is the total charge induced on the surface of the sphere? (4 points)



(1)



Keep Q_i at d_i and installed another Q_i' at O.

$$V_M = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{Q_i}{r_i}\right) + \frac{1}{4\pi\varepsilon_0}\frac{Q_i'}{a} = 0 + V_0 \Rightarrow V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q_i'}{a}$$

$$\frac{r_i}{r} = -\frac{Q_i}{Q} = \text{constant.} \quad d_i = \frac{a^2}{d}, \quad Q_i = -Q\frac{a}{d} \Rightarrow Q_i' = V_0 \cdot 4\pi\varepsilon_0 a$$

(2) let the total charge induced on the surface of the sphere be $Q_{induced}$

$$\frac{Q_{induced}}{4\pi\varepsilon_0 a} = V_0 \Rightarrow Q_{induced} = V_0 4\pi\varepsilon_0 a$$

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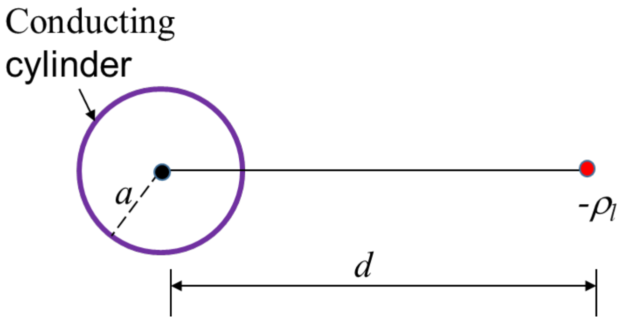
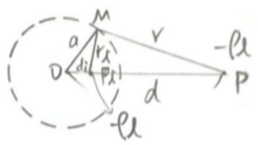
3. An infinitely long conducting cylinder of radius of a is installed in parallel with an infinitely long wire with a line charge density of $-\rho_l$, as shown below. The separation of the two objects is d.



(a) What is the electric potential on the conduction cylinder? (3 points)
(b) What is the capacitance per unit length of this signal line? (3 points)
(c) What is the total surface charge per unit length along the longitudinal direction induced on the conducting cylinder? (3 points)

(a)



put a image charge $-\rho_l$ at d_i

$$V_M = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{r_0}{r} - \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{r_0}{r_i} = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\frac{r_i}{r} = \text{constant}$$

$$\Rightarrow \frac{r_i}{r} = \text{constant}.$$

choose d_i such that $\triangle OPM \sim \triangle OMP_i \Rightarrow \frac{r_i}{r} = \frac{d_i}{a} = \frac{a}{d} = \text{constant} \Rightarrow d_i = \frac{a^2}{d}$

$$V_M = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_i}{r}\right) = \frac{-\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{a}{d}\right) = \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{d}{a}\right)$$

(b)

$$\rho_l = C_l V_M \Rightarrow C_l = \frac{\rho_l}{V_M} = \frac{\rho_l}{\frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{d}{a}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{d}{a}\right)}$$

(C_l: capacitance per unit length of this signal line)

(c)

$$Q = C_l V_M = \frac{2\pi\varepsilon_0}{\ln\left(\frac{d}{a}\right)} \cdot \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{d}{a}\right) = \rho_l$$

(Q: total surface charge per unit length along the longitudinal direction)