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Higher-order ODEs (Ch 4)

Preliminary:

① Definitions of ODEs:

— linear v.s. nonlinear (review)

— homogeneous v.s. nonhomogeneous

② Notations:

$$\frac{d^n y}{dt^n} =$$

ex: $y'' + 3y' - 4y = 0$

③ About "existence and uniqueness of a solution"

④ In ch 4, we discuss analytical methods to solve

homogeneous 2nd-order ODEs $\Bigg\{$

nonhomogeneous 2nd-order ODEs $\Bigg\{$

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How to solve $L(y)=0$

Preliminary: about $L(y)=0$

① For a homogeneous 2nd-order ODE $L(y)=0$

– There exists

– The general solution of $L(y)$

② How to check if functions $y_1, y_2, \cdots y_n$ are

→ By

③ From experiences, functions

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Method of "reduction of order"

Condition :

Idea :

Example 1 : $y'' - y = 0$, given one solution $y_1 = e^t$

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General procedures of method of "reduction of order":

Express the ODE by its standard form:

$$y'' + P y' + Q y = 0$$

$$\downarrow$$

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Method of "characteristic equation"

✻ Condition: used for ODEs with

ex:

Idea: By observation, the 1st & 2nd derivative are related by

Given $ay'' + by' + cy = 0$

Remarks:

① By this method, solve ODE

② This method also works for

3 cases of roots and the solutions

Case I: $2y'' - 5y' - 3y = 0$

Case II: $y'' + 4y' + 7y = 0$

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Case Ⅳ : $y'' - 10y' + 25y = 0$

If one solution is given → use

☆ For repeated roots, we just need to multiply
The same rule can also be applied to
ex : $y''' = 0$

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How to solve $L(y)=g(t)$

Preliminary: about $L(y)=g(t)$

① For $L(y)=g(t)$, any function

ex: $y''+9y=27$

② $L(y)=0$ is called the "

ex: $y''+9y=27$

③ If the general solution of $L(y)=0$ is $y=c_1y_1+c_2y_2$

& y_p is the particular solution of

the general solution of $L(y)=g(t)$ is

④ If $L(y)=g_1(t)$ has a particular solution

$L(y)=g_2(t)$ "

$L(y)=g_3(t)$ "

the general solution of $L(y)=g_1(t)+g_2(t)+g_3(t)$ is

In the following, we will learn two methods to solve $L(y)=g(t)$

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Method of "undetermined coefficients"

✡ Condition:

Q: What's your best guess of the particular y_p solution for

ex: $y'' - y' + y = 2\sin 4t$

$= 2t^2 + 1$

$= e^{3t}$

$= te^{3t}$

$= te^{3t} + 2\sin 4t + 2t^2 + 1$

Idea: For a nonhomogeneous ODE $L(y) = g(t)$,

General guess of particular solutions y_p for $L(y) = g(t)$

$g(t)$	guess of y_p

Example 1: $y'' - 2y' - 3y = 4t - 5 + 6te^{2t}$

Example 2: $y'' - 5y' + 4y = 8e^{t}$

Example 3: $y'' - 2y' + y = e^{t}$

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General procedures of method of "undetermined coefficients":

① Solve

② Categorize

③ Obtain general solution by

Remark: This method can also be applied to the 1st & higher order.

ex_1 : $y' - 3y = 6$