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Method of "variation of parameters"

※ Condition :

Idea : Use the solution of

Q : How to find

Set $y_p =$ into $y'' + Py' + Qy = g(t)$

$\Rightarrow y_p'' + P y_p' + Q y_p$

$=$

$$\begin{cases} y_p = u_1 y_1 + u_2 y_2 \\ y_p' = u_1 y_1' + u_1' y_1 + u_2 y_2' + u_2' y_2 \\ y_p'' = u_1 y_1'' + 2u_1' y_1' + u_1'' y_1 \\ \quad + u_2 y_2'' + 2u_2' y_2' + u_2'' y_2 \end{cases}$$

Since we just need to find any u_1, u_2 that satisfy (*), we can choose the simplest case where

So now we have two unknowns, u_1, u_2, with two equations

$\begin{cases}$

Find u_1', ① × $y_1 y_2' u_1' + y_2 y_2' u_2' = 0$

② × $\dfrac{}{(y_1 y_2' - y_1' y_2) u_1' = -y_2 g(t)}$

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$$\Rightarrow \quad u_1' = \frac{-y_2 g(t)}{\qquad}$$

It's easier to express u_1', u_2' in a determinant form:

$$u_1' = \frac{-y_2 g(t)}{\qquad} = \frac{\qquad}{\qquad}$$

By a similar procedure, we can find

$$u_2' = \frac{y_1 g(t)}{\qquad} = \frac{\qquad}{\qquad}$$

Then, u_1, u_2 can be obtained by integration:

$u_1 =$ $\qquad$ $u_2 =$

$\Rightarrow$ general solution $y =$

Example: Solve $y'' - 4y' + 4y = (t+1)e^{2t}$ by variation of parameters

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Remarks :

① General procedures of method of "variation of parameters"

1) Express the DE in

2) Find

3) Set

② This method can also be used in higher-order nonhomogeneous ODE. ex: $y''' + Py'' + Qy' + Ry = g(t)$