

2009 VLSI: Final Examination Solution

1.

(a.)

$$S = A \oplus B \oplus C$$

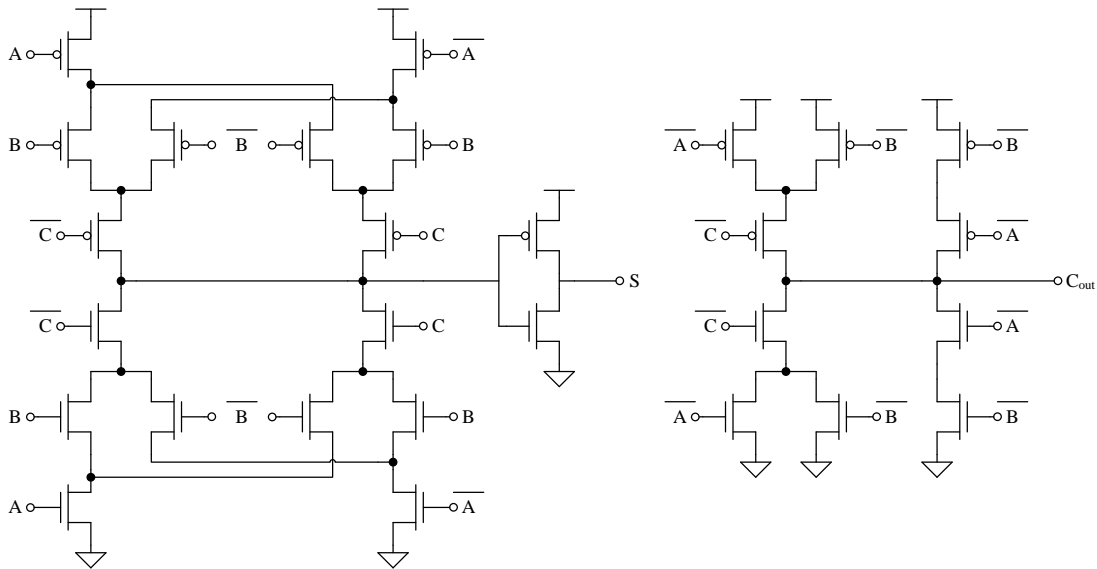
$$C_{out} = AB + BC + AC$$

(b.)

$$G = AB$$

$$P = A \oplus B$$

(c.)



2.

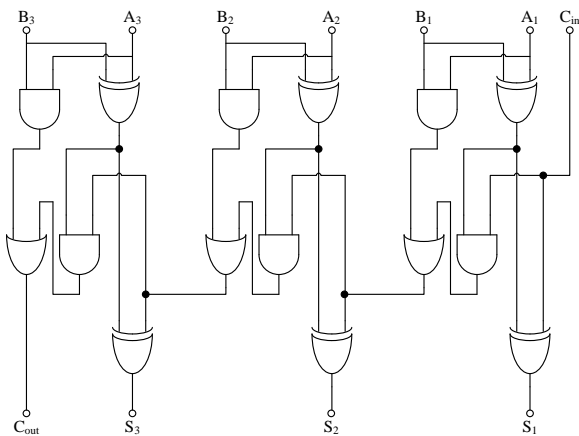
(a.)

$$G_{i:0} = G_i + P_i G_{i-1:0}$$

(b.)

$$C_i = G_i + P_i C_{i-1}$$

(c.)



(d.)

A_1 to C_{out}

3.

(a.)	F	(f.)	T
(b.)	T	(g.)	F
(c.)	T	(h.)	T
(d.)	T	(i.)	F
(e.)	T	(j.)	T

4.

(a.)

$$t_{pd} \leq T_C - (t_{setup} + t_{pcq})$$
$$\Rightarrow t_{pd} \leq 300p - (60p + 55p) = 185p$$

(b.)

$$t_{pd} \leq T_C - (2t_{pdq})$$
$$\Rightarrow t_{pd} \leq 300p - (2 \times 40p) = 220p$$

(c.)

$$t_{pd} \leq T_C - \max(t_{pdq}, t_{pcq} + t_{setup} - t_{pw})$$
$$\Rightarrow t_{pd} \leq 300p - \max(40p, 45p + 20p - 70p) = 300p - 40p = 260p$$

5.

(a.)

$$t_{pd} \leq T_C - (t_{setup} + t_{pcq} + t_{skew})$$
$$\Rightarrow t_{pd} \leq 300p - (60p + 55p + 40p) = 145p$$

(b.)

$$t_{pd} \leq T_C - (2t_{pdq})$$
$$\Rightarrow t_{pd} \leq 300p - (2 \times 40p) = 220p$$

(c.)

$$t_{pd} \leq T_C - \max(t_{pdq}, t_{pcq} + t_{setup} - t_{pw} + t_{skew})$$
$$\Rightarrow t_{pd} \leq 300p - \max(40p, 45p + 20p - 70p + 40p) = 300p - 40p = 260p$$

6.

(a.)

$$t_{cd} \geq t_{hold} - t_{ccq}$$

$$\Rightarrow t_{cd} \geq 30p - 40p = -20p$$

$$\Rightarrow t_{cd} \geq 0$$

(b.)

$$t_{cd1}, t_{cd2} \geq t_{hold} - t_{ccq} - t_{nonoverlap}$$

$$\Rightarrow t_{cd} \geq 25p - 40p - 60p = -75p$$

$$\Rightarrow t_{cd} \geq 0$$

(c.)

$$t_{cd} \geq t_{hold} - t_{ccq} + t_{pw}$$

$$\Rightarrow t_{cd} \geq 25p - 40p + 80p = 65p$$

7.

(a.)

$$t_{cd} \geq t_{hold} - t_{ccq} + t_{skew}$$

$$\Rightarrow t_{cd} \geq 30p - 40p + 40p = 20p$$

(b.)

$$t_{cd1}, t_{cd2} \geq t_{hold} - t_{ccq} - t_{nonoverlap} + t_{skew}$$

$$\Rightarrow t_{cd} \geq 25p - 40p - 60p + 40p = -35p$$

$$\Rightarrow t_{cd} \geq 0$$

(c.)

$$t_{cd} \geq t_{hold} - t_{ccq} + t_{pw} + t_{skew}$$

$$\Rightarrow t_{cd} \geq 25p - 40p + 80p + 40p = 105p$$

8.

(a.)

0.

There is no time borrowing technique in flop-based system.

(b.)

$$t_{borrow} \leq \frac{T_C}{2} - (t_{setup} + t_{nonoverlap})$$

$$\Rightarrow t_{borrow} \leq \frac{300p}{2} - (20p + 50p) = 80p$$

(c.)

$$t_{borrow} \leq t_{pw} - t_{setup}$$
$$\Rightarrow t_{borrow} \leq 70p - 20p = 50p$$

9.

$$t_{pd} \leq T_C - (2t_{setup} + 2t_{skew})$$
$$\Rightarrow t_{pd} \leq 400p - (2 \times 20p + 2 \times 40p) = 280p$$

10.

$$t_{pd} \leq T_C$$
$$\Rightarrow t_{pd} \leq 600p$$

11.

$$t_{borrow} \leq t_{overlap} - t_{hold} - t_{skew}$$
$$\Rightarrow t_{pd} \leq (0.6 - 0.25) \times 600p - 25p - 60p = 125p$$

12.

(a.)	F	(f)	T
(b.)	T	(g.)	F
(c.)	F	(h.)	T
(d.)	T	(i.)	F
(e.)	T	(j.)	T

13.

Data0:

$$5p \times \frac{3.3}{2} + 0.1p \times 0 = (5p + 0.1p) \times V_{data0}$$
$$\Rightarrow V_{data0} = 1.6176$$

Data1:

$$5p \times \frac{3.3}{2} + 0.1p \times 3.3 = (5p + 0.1p) \times V_{data1}$$
$$\Rightarrow V_{data1} = 1.6824$$

14.

(a.)	F	(f)	F
(b.)	F	(g.)	T
(c.)	T	(h.)	F
(d.)	T	(i.)	F
(e.)	F	(j.)	F

15.

$$F = \prod f_i = 25 \times 6 \times 10 = 1500$$

N	$N\sqrt[N]{F}$
5	21.5868
6	20.3002
7	19.8987
8	19.9573
9	20.2834

(a.)

$$N=7$$

(b.)

$$D_F = 7 \times \sqrt[7]{1500} = 19.8987$$

16.

$$G = (1)^N = 1$$

$$B = 1$$

$$H = \frac{20p}{10f} = 2000$$

$$F = GBH = 2000$$

N	$N\sqrt[N]{F}$
5	22.8653
6	21.2972
7	20.7336
8	20.6880
9	20.9423

(a.)

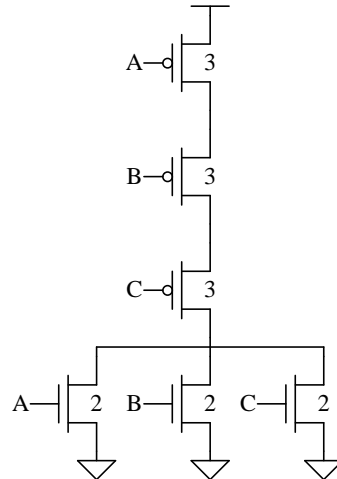
$$N=8$$

(b.)

$$D_F = 8 \times \sqrt[8]{2000} = 20.6880 = 4.1376 \times FO4 \text{ inverter delay}$$

17.

(a.)



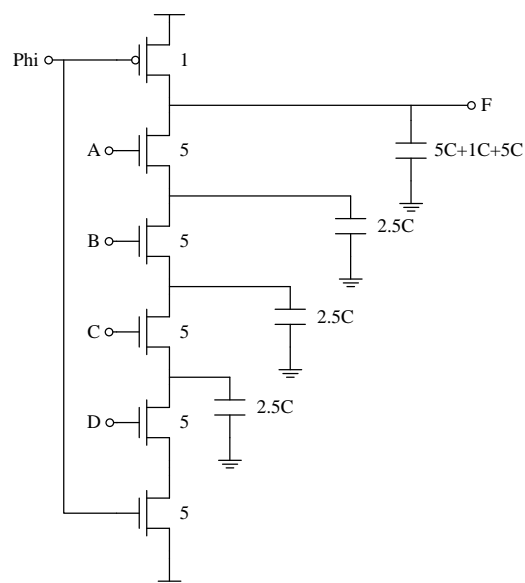
(b.)

$$g_u = \frac{5}{1.5} = 3.3333$$

$$g_d = \frac{5}{6} = 0.8333$$

$$g_{avg} = \frac{3.3333 + 0.8333}{2} = 2.0833$$

18.



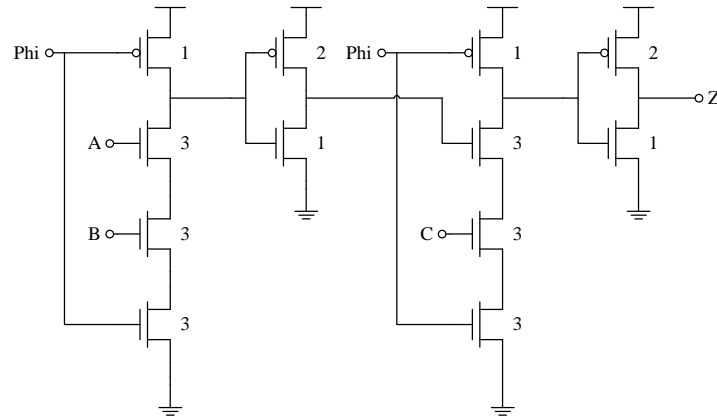
$$Q = 11C \times V_{DD} = (11C + 3 \times 2.5C) \times V_x$$

$$V_x = \frac{11}{18.5} V_{DD} = 0.5946 V_{DD}$$

$$\text{Charge Sharing Noise: } V_{DD} - 0.5946 V_{DD} = 0.4054 V_{DD}$$

19.

(a.)



(b.)

$$G = \frac{3}{3} \times \frac{3}{3} \times \frac{3}{3} \times \frac{3}{3} = 1$$

$$B = 1$$

$$H = \frac{300}{20} = 15$$

$$F = GBH = 15$$

$$D_F = 4 \times \sqrt[4]{15} = 7.8720$$

20.

$$G = \frac{4}{3} \times \frac{5}{3} \times \frac{5}{3} = \frac{100}{27}$$

$$B = 3 \times 2 = 6$$

$$H = \frac{80}{5} = 16$$

$$F = GBH = \frac{9600}{27} = \frac{3200}{9}$$

$$\hat{f} = \sqrt[3]{F} = 7.0844$$

$$P = 2 + 3 + 2 = 7$$

$$D = 3\hat{f} + P = 28.2532$$