EE306002: Probability **Spring 2022**
College of Electrical Engineering
National Tsing Hua University

Homework #2
Coverage: chapter 3–4
Due date: 29 March, 2022

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Problem 3.1.14. (10 points) Prove that if $P(A) = a$ and $P(B) = b$, then $P(A|B) \geq (a + b - 1)/b$.

Problem 3.2.14. (10 points) In a series of games, the winning number of the nth game, $n = 1, 2, 3, \ldots$, is a number selected at random from the set of integers $\{1, 2, ..., n + 2\}$. Don bets on 1 in each game and says that he will quit as soon as he wins. What is the probability that he has to play indefinitely?
Hint: Let A_n be the event that Don loses the first n games. To calculate the desired probability, $P(\lim_{n\to\infty} A_n)$, use Theorem 1.8: For any increasing or decreasing sequence of events, $\{E_n\}_{n\geq 1}$,

$$\lim_{n\to\infty} P(E_n) = P(\lim_{n\to\infty} E_n).$$

Problem 3.3.18. (10 points) A number is selected at random from the set $\{1, 2, 3, \ldots, 20\}$. Then a second number is selected randomly between 1 and the first number selected. What is the probability that the second number is 5?

Problem 3.4.14. (10 points) A certain cancer is found in one person in 5000. If a person does have the disease, in 92% of the cases the diagnostic procedure will show that he or she actually has it. If a person does not have the disease, the diagnostic procedure in one out of 500 cases gives a false positive result. Determine the probability that a person with a positive test result has cancer.

Problem 3.5.40. (10 points) A fair coin is tossed n times. Show that the events "at least two heads" and "one or two tails" are independent if $n = 3$ but dependent if $n = 4$.

Problem 4.2.16. (10 points) In a small town there are 40 taxis, numbered 1 to 40. Three taxis arrive at random at a station to pick up passengers. What is the probability that the number of at least one of the taxis is less than 5 ?

Problem 4.3.8. (10 points) From 18 potential women jurors and 28 potential men jurors, a jury of 12 is chosen at random. Let X be the number of women selected. Find the probability mass function of X.

Problem 4.4.14. (10 points) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 0 & x < -3 \\ 3/8 & -3 \leq x < 0 \\ 1/2 & 0 \leq x < 3 \\ 3/4 & 3 \leq x < 4 \\ 1 & x \geq 4. \end{cases}$$

Calculate $E(X)$, $E(X^2 - 2|X|)$, and $E(X|X|)$.

Problem 4.5.6. (10 points) Let X be a discrete random variable with the set of possible values $\{x_1, x_2, \ldots, x_n\}$; X called a **discrete uniform random variable** if

$$P(X = x_i) = \frac{1}{n},\ 1 \leq i \leq n.$$

Find $E(X)$ and $Var(X)$ for the special case, where $x_i = i,\ 1 \leq i \leq n$. Note that $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ and $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 4.6.1 (10 points) Mr. Norton owns two appliance stores. In store 1 the number of TV sets sold by a salesperson is, on average, 13 per week with a standard deviation of 5. In store 2 the number of TV sets sold by a salesperson is, on average, 7 with a standard deviation of 4. Mr. Norton has a position open for a person to sell TV sets. There are two applicants. Mr. Norton asked one of them to work in store 1 and the other in store 2, each for one week. The salesperson in store 1 sold 10 sets, and the salesperson in store 2 sold 6 sets. Based on this information, which person should Mr. Norton hire?

References

[1] Saeed Ghahramani, *Fundamentals of Probability: With Stochastic Processes,* Chapman and Hall/CRC; 4th edition (September 4, 2018)