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Properties of Laplace transform (LT)

Here we list some properties of LT to show why LT is useful to solve DEs :

<1> "derivation" property :

By LT, derivation (in t-domain) ⟶

ex: $\mathcal{L}\{y'\} =$

proof:

$$\mathcal{L}\{y'\} = \int_0^\infty y' e^{-st} dt = y e^{-st}\Big|_0^\infty - \int_0^\infty (-s) e^{-st} y \, dt$$

✡ LT of derivation :

$$\begin{cases} \mathcal{L}\{y'\} = \\ \mathcal{L}\{y''\} = \\ \quad \vdots \\ \mathcal{L}\{y^{(n)}\} = \end{cases}$$

It means ⟶

<2> "linearity" property

LT is a linear operation.

"linear" means :

<3> " " property

Many engineering / physics problems involve in time. ex:

Their LT are .

How to solve DEs by LT

General procedures:

Remarks about $\mathcal{L}^{-1}$ (inverse Laplace transform)

① We need to convert

② Properties of $\mathcal{L}^{-1}$

— "uniqueness" property:

If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}^{-1}\{F(s)\}$ can only be

— "linearity" property

Example 1: Use LT to solve $y' = y - 4e^{-t}$, $y(0) = 1$

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Example2: Use LT to solve $y''+4y'+20y=e^{-2t}$, $y(0)=y'(0)=0$

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By comparing with $\mathcal{L}\{y\}$, we find that the denominator of $\mathcal{L}\{y\}$ consists of product of

So from the roots of denominator of $\mathcal{L}\{y\}$, we can get qualitative analysis of the system response.

ex: $\mathcal{L}\{y\} = \frac{1}{(s^2+2s+2)(s+\frac{1}{10})}$, what can we say about the system response qualitatively?

Remark: Qualitative analysis of system response from LT

Use LT to solve DEs with { discontinuous / periodic } forcing

In many engineering/physics problems, the systems are described by constant-coefficient 2nd-order linear DEs.

Ex:

system
$y'' + py' + qy = f(t)$

The input may be "turned on" at a specific time, like a switch.

ex:

LT is particularly useful to solve problems with

In the following, we will discuss