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EE214000 Electromagnetics, Fall 2020

Your name: _________________ ID: __________________ Oct. 12th, 2020

EE214000 Electromagnetics, Fall, 2020
Quiz #6-1, Open books, notes (31 points), due 11 pm, Wednesday, Oct. 14th, 2020
(email solutions to 劉峰麒 alex851225@gmail.com)

Late submission won't be accepted!

1. (review) Use the Gauss law to calculate the electric field at R from a point charge q located at the origin in a spherical coordinate system. (3 points) (2) Given the point charge q at the origin, calculate the work you have to do to move another point charge Q from (R_1, θ_1, ϕ_1) to a distance (R_2, θ_2, ϕ_2). (3 points)

Ans: As given in the lecture, the electric field is

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

In such a spherical symmetric system, a surface at a constant R is an equal potential surface, θ and ϕ are irrelevant to the calculations. The work to bring Q from R_1 to R_2 is simply

$$V(R_2) - V(R_1) = \frac{q}{4\pi\varepsilon_0 R_2} - \frac{q}{4\pi\varepsilon_0 R_1}$$

2. Explain why the static electric field in an ideal conductor must be zero. (3 points)

Ans:
By definition, an ideal or perfect conductor has an infinite amount of free electrons in it. Under the excitation of an external electric field, if the electric field inside the perfect conductor is not zero, the free electrons will be moved by the field to build up a field in the opposite direction until the two fields cancel each other exactly to stop the moving of the free electrons. In equilibrium, the net electric field inside a perfect conductor will settle to zero.

3. Explain why the electric field lines entering a perfect conductor must be along the surface normal of the conductor. (3 points)

Ans: For electrostatics, the tangential electric fields across a material boundary must be continuous. However, the electric field inside a perfect conductor is zero and thus the tangential component of the electric field on the outer surface of the conductor can only be zero. Therefore, there only exists a normal component of the electric field on the outer surface of the conductor.

4. In Sec. 6.4, we first derived the induced surface charge of the conducting sphere at $R = a$, given by

$$\rho_s = \hat{a}_{n2} \cdot \vec{D} = -\frac{q}{4\pi a^2}$$

Argue from charge conservation that the surface charge at $R = b$ must be

$$\rho_s = \frac{q}{4\pi b^2}.$$

(3 points)

Ans: The conductor between $a \le R \le b$ is neutral. The total amount of surface charge at $R = a$ surface must be the same as that at $R = b$, except that the signs of the charges are opposite. Apparently, the total charge at $R = a$ is

$$-\frac{q}{4\pi a^2} \times 4\pi a^2 = -q$$

From charge conservation, there will be $+q$ distributed uniformly on the surface of $R = b$. Therefore, the surface charge at $R = b$ must be

$$\rho_s = \frac{q}{4\pi b^2}.$$

5. What is the physical meaning of the polarization density vector $\vec{P}$? How is it related to the electric field? (3 points)

Ans: The polarization density vector of the total vector sum of the dipole moments is a differential volume of a material average over the volume. It is proportional to the electric field at the same point, given by

$$\vec{P} = \varepsilon_0 \chi_e \vec{E},$$

where χe is the electric susceptibility of the dielectric.

6. Compare the electric field intensity at a distance R from a point charge q in vacuum and in a space filled with a dielectric having relative permittivity $\varepsilon_r > 1$.
(3 points)

Ans: In vacuum, from Gauss law with $\varepsilon_r = 1$, the electric field intensity at R is given by

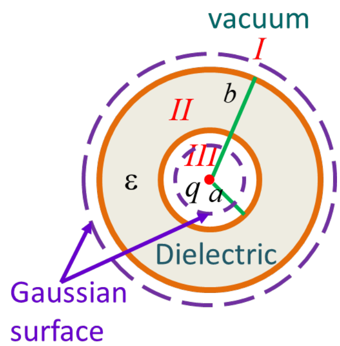
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

In dielectric, from Gauss law with $\varepsilon r > 1$, the electric field intensity at R is given by

$$\vec{E} = \frac{q}{4\pi\varepsilon_0\varepsilon_r R^2}\hat{a}_R.$$

For a typical $\varepsilon_r > 1$ in a dielectric, the electric field in a dielectric is reduced by a factor of ε_r.

7. In the example of Sec. 6.2, what are the total polarization charges induced at spherical surfaces of $R = a$ and b. Do the answers agree with or violate the charge conservation? Take the relative permittivity of the dielectric to be ε_r (5 points).



Ans: At radius $R = a$, the polarization density vector and the surface polarization charge density are given by

$$\vec{P}(R=a) = (1-\frac{1}{\varepsilon_r})\vec{D} = (1-\frac{1}{\varepsilon_r})\frac{q}{4\pi a^2}\hat{a}_R$$

and

$$\rho_s = \vec{P}(R=a)\cdot(-\hat{a}_R) = -(1-\frac{1}{\varepsilon_r})\frac{q}{4\pi a^2},$$

respectively.

Integrate the total spherical surface at $R = a$ to get the total polarization charge of

$$\rho_s \times 4\pi a^2 = -(1-\frac{1}{\varepsilon_r})q. \qquad (1)$$

On the $R = b$ surface, the polarization density vector and the polarization surface charge density are

$$\vec{P}(R=b) = (1-\frac{1}{\varepsilon_r})\vec{D} = (1-\frac{1}{\varepsilon_r})\frac{q}{4\pi b^2}\hat{a}_R$$

and

$$\rho_s = \vec{P}(R=b)\cdot(\hat{a}_R) = +(1-\frac{1}{\varepsilon_r})\frac{q}{4\pi b^2},$$

respectively.

Integrate the total spherical surface at $R = b$ to get the total polarization charge of

$$\rho_s \times 4\pi b^2 = +(1 - \frac{1}{\varepsilon_r})q \,. \tag{2}$$

(1) and (2) have an equal magnitude and opposite signs, as they should be for charge conservation. * Note the induced charge is less than q. Compare it with that in Prob. 3.

8. What are the boundary conditions for the tangential and normal components of E at a dielectric-dielectric interface, where the relative permittivities of the 1st and 2nd dielectrics are ε_{r1} and ε_{r2}, respectively (5 points).

Ans:

The tangential components of E are continuous or $E_{1t} = E_{2t}$.

Since there's no surface charge on the dielectric surface, the normal components of D are continuous on the boundary or $D_{1n} = D_{2n} \Rightarrow \varepsilon_{r1} E_{1n} = \varepsilon_{r2} E_{2n}$.