H.W.4　張芯瑜　109060013

(1a) 1° $n=1$, $F_1 = F_0 + 2 = 3 + 2 = 5$ ∴ True.

2° Assume $n = k-1$ is True.

s.t. $F_{k-1} = F_0 \times F_1 \times \cdots \times F_{k-2} + 2 = 2^{2^{k-1}} + 1$

then $F_k = 2^{2^k} + 1$

$= (2^{2^{k-1}} + 1)(2^{2^{k-1}} - 1) + 2$

$= F_{k-1} \times (F_{k-1} - 2) + 2$

$= F_{k-1} (F_0 \times F_1 \times \cdots \times F_{k-2}) + 2$

$= F_0 \times F_1 \times \cdots \times F_{k-2} \times F_{k-1} + 2$ #

3° By 1°.2°, We proved.

(1b) Choose 2 num from F

$\begin{cases} F_n = F_0 \times \cdots \times F_{n-1} + 2 \\ F_{n+k} = F_0 \times \cdots \times F_{n+k-1} + 2 \end{cases}$

Assume p is F_n's 因數, then ∵ $F_{n+k} = F_0 \times \cdots \times \underbrace{F_n}_{p\text{的倍數}} \times \cdots \times F_{n+k-1} + 2$

↳ ∵ $F_n = \underbrace{F_0 \times \cdots F_{n-1}}_{\text{非2的倍數}} + 2$ ∴ $p \neq 2$ 且 $p \neq 1$

$\Rightarrow F_{n+k} = p \times (\text{———}) + 2$ ↷ 不是 p 的倍數

∴ Fermat Num 必定互質.

(1c) By (1b), ∵ 兩兩互質

∴ 質因數必定不同!!

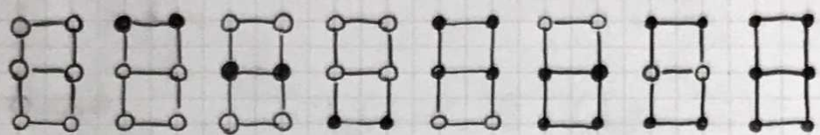
(1d) ∵ Fermat Num 有無限個 且 (1c)

∴ prime 有無限多個 !!

(2) 1°  $\longrightarrow 2^3$ kinds of ways for each column

∵ have 9 columns

∴ two of them must be same.

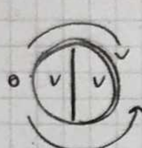
2° 

⇒任一種有2個時，必會有相同顏色的點形成四边形

∴在 3×9 個桌中，必有由4相同color的桌

所形成的四边形。

(3) 1° Base : $n=2^1$

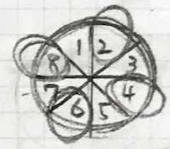


(不管正負，1就可以到对面)

2° For $n>2$,

每次選偶數和 $\frac{n}{2}$ 跳!!

跳到新的一對　　跳到對面

必定可把偶數位的cake吃完 

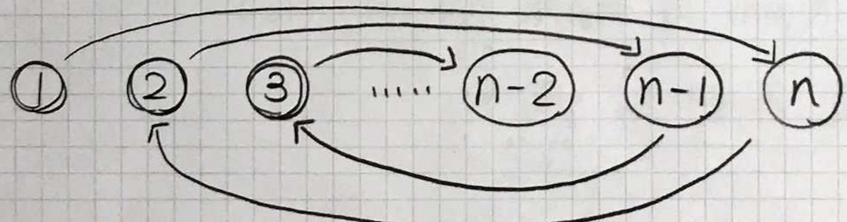
接著跳1個位置，不論正負都可換到奇數位。

然後再一樣選偶枚和 $\frac{n}{2}$ 跳，就可以把奇數位

走完，也就可以吃到所有cake!!

(4) $\forall\, n$, we can set $f(i)$ as:

$f(1)=1$, $f(2)=n$, $f(3)=2$, $f(4)=n-1$, ...



$|f(i)-f(i+1)|$ would all be distinct.

(5) As $x \in X$，x 只有 k 位數

$$
\left.\begin{aligned}
x_1 &= 0.\cancel{0}\text{———} \\
x_2 &= 0.\ \cancel{0}\text{———} \\
&\ \vdots \\
x_j &= 0.\ \text{———}\cancel{0}
\end{aligned}\right\downarrow
$$

最多只能不同到 k

$\therefore j = k$

$$x = 0.\underbrace{\text{———}}_{k}$$

但 X_{k+1} 之後的項有可能與 x 一樣，所以證明不成立。