EE214000 Electromagnetics, Fall, 2020

Homework #4, due in class at 12 pm, noon, Monday, Nov. 30, 2020

Late submission won't be accepted!

Problems P.4-6, P.4-10, P. 4-17, P.4-19, P. 4-22, P.4-24 in DK Cheng's textbook

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P.4–6 Assume that the space between the inner and outer conductors of a long coaxial cylindrical structure is filled with an electron cloud having a volume density of charge $\rho = A/r$ for $a < r < b$, where a and b are, the radii of the inner and outer conductors, respectively. The inner conductor is maintained at a potential V_0, and the outer conductor is grounded. Determine the potential distribution in the region $a < r < b$ by solving Poisson's equation.

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解：由泊松方程，$\nabla^2 V = -\dfrac{A}{\varepsilon r} \Rightarrow \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial v}{\partial r}\right) = -\dfrac{A}{\varepsilon r}$

解得 $V = -\dfrac{A}{\varepsilon}r + c_1 lnr + c_2$

$$\begin{cases} \text{在 } r=a \text{ 時}, V_0 = -\dfrac{A}{\varepsilon}a + C_1 lna + C_2, \\ \text{在 } r=b \text{ 時}, 0 = -\dfrac{A}{\varepsilon}b + C_1 lnb + C_2. \end{cases}$$

聯立解得，$C_1 = \dfrac{\dfrac{A}{\varepsilon}(b-a) - V_0}{ln(b/a)}, \quad C_2 = \dfrac{V_0 lnb + \dfrac{A}{\varepsilon}(alnb - blna)}{ln(b/a)}$

P.4–10 A straight conducting wire of radius a is parallel to and at height h from the surface of the earth. Assuming that the earth is perfectly conducting, determine the capacitance and the force per unit length between the wire and the earth.

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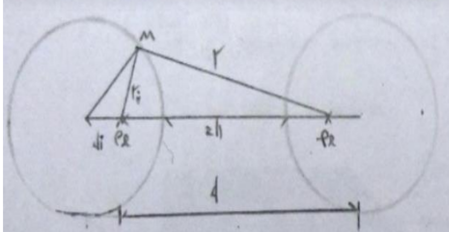
P. 4—10 解：參考例 4—4.

$$C' = \frac{2\pi\varepsilon_0}{ln[(h/a) + \sqrt{(h/a)^2 - 1}]} = \frac{2\pi\varepsilon_0}{\cosh^{-1}(h/a)} \text{(F/m)}$$



cf $V=\frac{\rho_\ell}{2\pi\epsilon_0}\ln\frac{r_-}{r_+}$

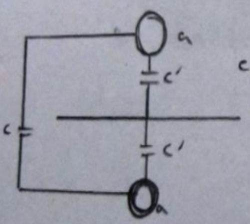
$$\frac{r_-}{r_+}=\frac{r}{r_i}=\frac{d}{a}=\frac{a}{d_i}\to d_i=\frac{a^2}{d}$$

$$d=2h+2a-\frac{a^2}{d}\to d^2=(2h+2a)d-a^2$$

$$d=(a+h)+\sqrt{h^2+2ah}$$

r_0：空間中任意位置

$$\to V_M=\frac{\rho_\ell}{2\pi\epsilon_0}\left(\ln\frac{r_0}{r_i}-\ln\frac{r_0}{r}\right)=\frac{\rho_\ell}{2\pi\epsilon_0}\ln\left(\frac{d}{a}\right)$$



$c=c'//c'$

$$C=\frac{2\pi\epsilon_0}{\ln\left[\frac{h}{a}+\sqrt{\left(\frac{h}{a}\right)^2-1}\right]}=\frac{2\pi\epsilon_0}{\cosh^{-1}\left(\frac{h}{a}\right)}$$

$$W=\frac{1}{2}CV^2=\frac{\rho_\ell^2}{4\pi\epsilon_0}\cosh^{-1}\left(\frac{h}{a}\right)$$

$$\therefore \vec{F}=-\nabla W=-\frac{\partial}{\partial h}W=\frac{+\rho_\ell^2}{4\pi\epsilon_0}\frac{1}{\sqrt{h^2-a^2}\,\mathrm{arccosh}^2\left(\frac{h}{a}\right)}\quad \#$$

P.4–17 Two dielectric media with dielectric constants ϵ_1 and ϵ_2 are separated by a plane boundary at $x = 0$, as shown in Fig. 4–23. A point charge Q exists in medium 1 at distance d from the boundary.

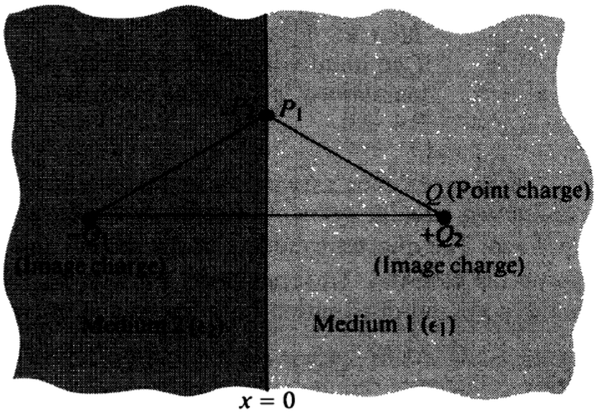


FIGURE 4–23
Image charges in dielectric media (Problem P.4–17).

a) Verify that the field in medium 1 can be obtained from Q and an image charge $-Q_1$, both acting in medium 1.
b) Verify that the field in medium 2 can be obtained from Q and an image charge $+Q_2$ coinciding with Q, both acting in medium 2.
c) Determine Q_1 and Q_2. (*Hint:* Consider neighboring points P_1 and P_2 in media 1 and 2, respectively, and require the continuity of the tangential component of the E-field and of the normal component of the **D**-field.)

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P.4—17 解：在 $x=0$ 處要求的邊界條件爲：$V_1=V_2$ 且 $\varepsilon_1 \frac{\partial v_1}{\partial x}=\varepsilon_2 \frac{\partial v_2}{\partial x}$.

依圖 4—23 及相關假定，知：

$$V_1=\frac{Q}{4\pi\varepsilon_1\sqrt{(x-d)^2+y^2+z^2}}-\frac{Q_1}{4\pi\varepsilon_1\sqrt{(x+d)^2+y^2+z^2}}$$

$$V_2=\frac{Q+Q_2}{4\pi\varepsilon_2\sqrt{(d+x)^2+y^2+z^2}}$$

爲滿足條件，要求 $\frac{Q-Q_1}{\varepsilon_1}=\frac{Q+Q_2}{\varepsilon_2}$ 且 $Q+Q_1=Q+Q_2$

則 $Q_1=Q_2=\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}Q$

P.4–19 In what way should we modify the solution in Eq. (4–114) for Example 4–7 if the boundary conditions on the top, bottom, and right planes in Fig. 4–17 are $\partial V/\partial n = 0$?

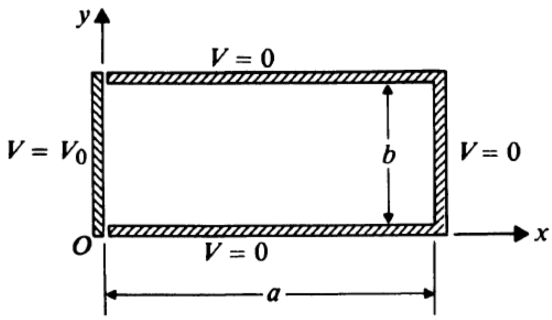

FIGURE 4–17
Cross-sectional figure for Example 4–7.

The desired potential distribution within the enclosed region in Fig. 4–17 is a summation of $V_n(x, y)$ in Eq. (4–111):

$$\begin{aligned} V(x, y) &= \sum_{n=1}^{\infty} C_n' \sinh \frac{n\pi}{b}(x-a) \sin \frac{n\pi}{b} y \\ &= \frac{4V_0}{\pi} \sum_{n=\text{odd}}^{\infty} \frac{\sinh [n\pi(a-x)/b]}{n \sinh (n\pi a/b)} \sin \frac{n\pi}{b} y, \end{aligned} \qquad (4\text{–}114)$$

$n = 1, 3, 5, \ldots,$

$0 < x < a \quad$ and $\quad 0 < y < b.$

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P. 4—19 解：$V_n(x,y)=C_n \cosh \frac{n\pi}{b}(x-a)\cos \frac{n\pi}{b} y.$

P.4–22 Consider a metallic rectangular box with sides a and b and height c. The side walls and the bottom surface are grounded. The top surface is isolated and kept at a constant potential V_0. Determine the potential distribution inside the box.

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P.4—22 解：$V(x,y,z)=\sum_{m}\sum_{n}C_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sinh k_{mn}z$

其中 $k_{mn}=\sqrt{(\frac{m\pi}{a})^2+(\frac{n\pi}{b})^2}$

在 $z=c, V(x,y,c)=V_0=\sum_{m}\sum_{n}C_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\sinh k_{mn}c$

$$\Rightarrow C_{mn}=\begin{cases}\dfrac{16V_0}{mn\pi^2\sinh k_{mn}c}, & m、n \text{ 爲奇數.}\\ 0, & m、n \text{ 爲偶數.}\end{cases}$$

P.4–24 An infinitely long, thin conducting circular cylinder of radius b is split in four quarter-cylinders, as shown in Fig. 4–25. The quarter-cylinders in the second and fourth quadrants are grounded, and those in the first and third quadrants are kept at potentials V_0 and $-V_0$, respectively. Determine the potential distribution both inside and outside the cylinder.

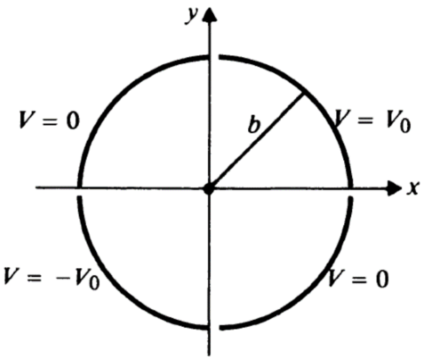

FIGURE 4–25
Cross section of long circular cylinder split in four quarters (Problem P.4–24).

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P.4—24 解：内部：$V(r,\phi)=\frac{2V_0}{\pi}\sum_{n=odd}\frac{1}{n}(\frac{r}{b})^n[\sin n\phi+\sin n(\phi+\frac{\pi}{2})], r<b$

外部：$V(r,\phi)=\frac{2V_0}{\pi}\sum_{n=odd}\frac{1}{n}(\frac{b}{r})^n[\sin n\phi+\sin n(\phi+\frac{\pi}{2})], r>b.$