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1st-order ODEs (Ch 2)

In this class, we will introduce three approaches to solve 1st-order ODEs.

I. Qualitative approach
II. Numerical approach
III. Analytical approach

In this class, we are always solving the DE. It means, we are looking for "solution" of the DE. Before finding the solution, a fundamental question we should ask ourselves is

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Existence of a unique solution (Theorem 1.2.1)

For a 1st-order ODE $\frac{dy}{dt} = f(x, y)$ with initial value $y(t_0) = y_0$.

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I. Qualitative approach

Use graphical method to solve 1st-order ODEs by plotting the " · " (also called "direction field")

Example: Solve $\frac{dy}{dt} = 0.2ty$ by plotting the slope field.

observations:

① "$\frac{dy}{dt}$" means " ". Say, if a function $y(t)$ is a solution of this DE, its slope ($=\frac{dy}{dt}$) at (t_0, y_0) will be

② If we sketch $0.2ty$ in the ty plane, we can obtain a plot of " " →

Let's plot $0.2ty$ in the ty plane:

If "slopes" of solution look like this, what would the solution look like ?

For a 1st-order ODE $\frac{dy}{dt} = f(t,y)$, two important special cases : when the right-hand side (RHS) is only a function of t or only a function of y.

Case I: $\frac{dy}{dt} = f(t) \overset{ex:}{=\!=} 2t$

- for , slope is the same.
- geometrically, all the slopes on each line are parallel.

Feature : We can get infinite solutions from one solution curve by translating the curve

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Case II : $\frac{dy}{dt} = f(y) \overset{ex:}{=} 4y(1-y)$

(* Such DE is called " " DE, where the RHS does not depend on the independent variable t.

** Many of the DEs that arise in application are " "

— for , slope is the same.

— geometrically, all the slopes on each line are parallel.)

Feature:

① We can get infinite solutions from one solution curve by translating the curve

(ex:

② Depending on the values of the slopes, we can divide the slope field into the following regions:

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Remarks:

(1) Qualitative approach may give us infinite number (Graphical) of possible solutions. We need initial values to obtain a unique solution.

(2) Pros & cons of qualitative approach

Pros: visualize the solution in geometrical ways and visualize its long-term behavior

Cons: a rough sketch of the solution without precise values.

Example: An RC circuit