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EE214000 Electromagnetics, Fall 2020

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EE214000 Electromagnetics, Fall, 2020
Quiz #14-1, Open books, notes (25 points), due 11 pm, Wednesday, Dec. 16th, 2020
(submission through iLMS)

Late submission won't be accepted!

1. Explain why you can't use the Ampere's law $\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$ to calculate the magnetic flux density at $P(r,\phi,0)$? Of course, you could try it to get a different answer but the answer is wrong (why?). (5 points)

In vacuum, $\nabla\times\vec{B} = \mu_0 \vec{J} \Rightarrow \oint_C \vec{B}\cdot d\vec{l} = \mu_0 I$

Now, we observe from $P(r, \phi, 0)$. Since z component is zero, we can not see the whole ring, we can just integrate some part of magnetic flux density by Ampere's Law. Besides, the result of calculation may be quite different, since the object we observe may not be a uniform ring. The observed object may be nonuniform and asymmetry, and that's why the answer will be wrong if we use Ampere's Law.

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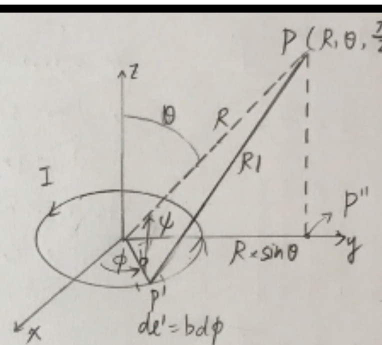
2. Step by step, write down the derivation of the far-zone magnetic flux density for a magnetic dipole. (6 points)



$\hat{a}_\phi = -\hat{a}_x$

Due to symmetry, $\vec{A} = A_\phi \hat{a}_\phi$

We know $\vec{A} = \frac{\mu_0 I}{4\pi} \oint_{C'} \frac{d\vec{l}'}{R_1}$

$d\vec{l}' = (-\hat{a}_x \sin\phi' + \hat{a}_y \cos\phi') b \, d\phi'$

$\Rightarrow \vec{A} = -\hat{a}_x \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{b \sin\phi'}{R_1} d\phi'$

$\vec{A} = \hat{a}_\phi \frac{\mu_0 I b}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin\phi'}{R_1} d\phi'$,

$R_1^2 = R^2 + b^2 - 2bR\cos\psi = R^2 + b^2 - 2bR\sin\theta\sin\phi'$

$\frac{1}{R_1} = \frac{1}{R}\left(1 + \frac{b^2}{R^2} - \frac{2b}{R}\sin\theta\sin\phi'\right)^{1/2}$

If $R^2 \gg b^2$; we can get:

$\frac{1}{R_1} \cong \frac{1}{R}\left(1 - \frac{2b}{R}\sin\theta\sin\phi'\right)^{1/2} \cong \frac{1}{R}\left(1 + \frac{b}{R}\sin\theta\sin\phi'\right)$

$\vec{A} = \hat{a}_\phi \frac{\mu_0 I b}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{R}\left(1 + \frac{b}{R}\sin\theta\sin\phi'\right)\sin\phi' \, d\phi'$

$\vec{A} = \hat{a}_\phi \frac{\mu_0 I b}{2\pi R} \int_{-\pi/2}^{\pi/2} \left(1 + \frac{b}{R}\sin\theta\sin\phi'\right)\sin\phi' \, d\phi' = \hat{a}_\phi \frac{\mu_0 I b^2}{4R^2}\sin\theta$

$\Rightarrow \vec{B} = \nabla \times \vec{A} = \frac{\mu_0 I b^2}{4R^3}\left(\hat{a}_R \, 2\cos\theta + \hat{a}_\theta \sin\theta\right)$

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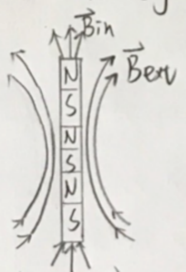
3. Explain why the magnetic flux density *B* can be greatly increased nearby or inside a ferromagnetic material subject to an external current (6 points); whereas the electric field intensity *E* is usually reduced inside a dielectric material given an external charge. (3 points) Graphic illustrations along with text explanations are encouraged

(1) Consider the figure :



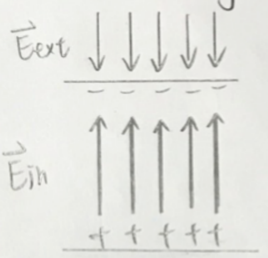
$\vec{B}_{net} = \vec{B}_{ext} + \vec{B}_{in}$

An external current generates an external magnetic field that aligns the magnetic dipoles in a ferromagnetic material. Since the magnetic field enters S pole and exits N pole of the aligned dipoles, the external field and the dipole field are aligned along the same direction.

$\Rightarrow \vec{B}_{net} = \vec{B}_{ext} + \vec{B}_{in}$

The net effect is an increase of the B field.

(2) Consider the figure :

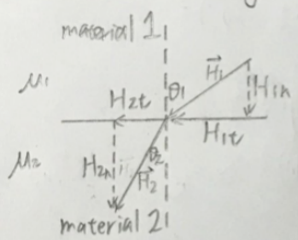


$\vec{E}_{net} = \vec{E}_{ext} - \vec{E}_{in}$

An external $\vec{E}_{ext}$ will also align the electric dipoles in a dielectric material in a way that the induced electric field in the material is along the opposite direction of the external field. As a result, the net field in the material is reduced according to $\vec{E}_{net} = \vec{E}_{ext} - \vec{E}_{in}$

4. In electrostatics, you learned that the electric field lines entering a perfect conductor along the surface normal of the conductor. How does the magnetic field lines in vacuum enter a non-conducting magnetic material with $\mu_r \to \infty$? (5 points)

4. Consider the figure:



material 1: vacuum

material 2: non-conducting magnetic material

$\Rightarrow J_s = 0$ and $H_{1t} = H_{2t}$

Without a surface current, B.C.: $H_{1t} = H_{2t}$ and $B_{1n} = B_{2n}$

tangential component: $\begin{cases} H_{1t} = H_1 \sin\theta_1 \\ H_{2t} = H_1 \sin\theta_1 \end{cases}$

normal component: $\begin{cases} H_{1n} = H_1 \cos\theta_1 \\ \Rightarrow B_{2n} = B_{1n} = \mu_1 H_1 \cos\theta \end{cases}$

$\Rightarrow H_{2n} = \dfrac{B_{2n}}{\mu_2} = \dfrac{\mu_1}{\mu_2} H_1 \cos\theta_1$

$\Rightarrow \tan\theta_2 = \dfrac{H_{2t}}{H_{2n}} = \dfrac{\mu_2}{\mu_1} \tan\theta_1$

Here, $\mu_r = \mu_2$.

$\mu_r \to \infty \Rightarrow \mu_2 \to \infty$ & $\tan\theta_2 \to \infty$

$\tan\theta_2 \to \infty \Rightarrow \theta_2 \to 90^\circ$

$\Rightarrow$ The magnetic field lines will propagate along the surface of the intersection.