## Signals and Systems

Homework 10 — Due : May 17 2024

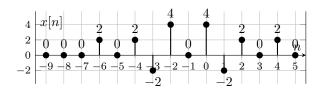
**Problem 1** (20 pts). Determine the Fourier transform of the following signals:

- (a)  $x_1[n] = \sin\left(\frac{\pi}{4}n\right) + \cos\left(\frac{2\pi}{3}n\right)$
- (b)  $x_3[n] = 2^n \sin(\frac{\pi}{4}n) u[-n]$

**Problem 2** (20 pts). Determine the inverse Fourier transform of the following signals:

- (a)  $X_1(e^{j\omega}) = \cos^2(3\omega) + \sin^2(\omega)$
- (b)  $X_2(e^{j\omega}) = A(\omega)e^{jB(\omega)}$ , where  $A(\omega) = \begin{cases} 0, & 0 \le |\omega| < \pi/3 \\ 1, & \pi/3 \le |\omega| < \pi \end{cases}$ , and  $B(\omega) = -\frac{2}{3}\omega$

**Problem 3** (30 pts). Let  $X(e^{j\omega})$  be the Fourier transform of the signal x[n].



Perform the following calculations without explicitly evaluating  $X(e^{j\omega})$ .

- (a) Find  $X(e^{j0})$ .
- (b) Find  $\not \subset X(e^{j\omega})$ .
- (c) Find  $X(e^{j\pi})$ .
- (d) Evaluate  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$ .
- (e) Evaluate  $\int_{-\pi}^{\pi} |\frac{d}{d\omega} X(e^{j\omega})|^2 d\omega$ .
- (f) Sketch y[n] such that  $\mathfrak{F}\{y[n]\} = \mathbf{Re}\{X(e^{j\omega})\}.$

**Problem 4** (20 pts). Let  $X(e^{j\omega})$  be the Fourier transform of a real signal x[n]. Show that x[n] can be written as

$$x[n] = \int_0^\pi \left[ B(\omega) \sin(\omega n) + C(\omega) \cos(\omega n) \right) ] \, d\omega$$

by finding expressions for  $B(\omega)$  and  $C(\omega)$  in terms of  $X(e^{j\omega})$ .

**Problem 5** (10 pts). Consider a signal y[n] with Fourier transform  $Y(e^{j\omega})$ . Suppose that  $y[n] = x_{(4)}[n]$ , where the signal x[n] has a Fourier transform  $X(e^{j\omega})$ . Determine a real number  $\alpha$  such that  $0 < \alpha < 2\pi$  and  $Y(e^{j\omega}) = Y(e^{j(\omega-\alpha)})$ .

**Problem 1** (20 pts). Determine the Fourier transform of the following signals:  $\frac{1}{2\pi} \int_{0}^{2\pi} 2\pi \, \delta(\omega - \omega_{\bullet}) e^{j\omega n} d\omega = e^{j\omega_{\bullet} n}$ (a)  $x_1[n] = \sin(\frac{\pi}{4}n) + \cos(\frac{2\pi}{3}n)$ 

(a) 
$$x_1[n] = \sin\left(\frac{\pi}{4}n\right) + \cos\left(\frac{2\pi}{3}n\right)$$

$$2\pi \int_0^{\pi} dx$$

$$3\ln\left(\frac{\pi}{4}n\right) = \frac{1}{2\pi}e^{j\frac{\pi}{4}n} - \frac{1}{2\pi}e^{-j\frac{\pi}{4}n}$$

$$F\left\{\sin\left(\frac{\pi}{4}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\frac{\pi}{j} \delta\left(w - \frac{\pi}{4} - 2\pi k\right) - \frac{\pi}{j} \delta\left(w + \frac{\pi}{4} - 2\pi k\right)\right]$$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2}e^{j\frac{2\pi}{3}n} + \frac{1}{2}e^{-j\frac{2\pi}{3}n}$$

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$$\mathbb{E}\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\frac{\pi}{3}n}$$

$$Cos\left(\frac{3}{3}n\right) = \frac{1}{2}e^{3} + \frac{1}{2}e$$

$$F \left\{cos\left(\frac{2\pi}{3}n\right)\right\} = \frac{8}{2}\left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

$$\cos\left(\frac{3}{3}n\right) = \frac{1}{2}e^{3} + \frac{1}{2}e$$

$$F\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

(b)  $x_3[n] = 2^n \sin(\frac{\pi}{4}n) u[-n]$ 

$$Cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{\pi}{3}n} + \frac{1}{2} e^{-j\frac{\pi}{3}n}$$

$$\overline{F}\left\{cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\kappa k\right) + \pi\right]$$

$$\overline{F}\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(w - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{3} + \frac{1}{2} e^{3}$$

$$F\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(k\right) - \frac{2\pi}{3} - 2\pi k\right] + 7$$

$$Cos\left(\frac{3}{3}n\right) = \frac{1}{2}e^{3} + \frac{1}{2}e^{3}$$

$$\tilde{F}\left\{cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{j\frac{2\pi}{3}n}$$

$$\tilde{F}\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(k\right) - \frac{2\pi}{3} - 2\pi k\right) + \pi$$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\frac{2\pi}{3}n}$$

$$F\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi \cdot \delta\left(\omega + \frac{2\pi}{3} - 2\pi k\right)\right]$$

$$F\left\{\chi_{i}\left[n\right]\right\} = \sum_{k=-\infty}^{\infty} \left[\frac{\pi}{3} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) - \frac{\pi}{3} \delta\left(\omega + \frac{\pi}{4} - 2\pi k\right) + \pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right)\right\}$$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{j\frac{2\pi}{3}n}$$

$$\tilde{F}\left[\cos\left(\frac{2\pi}{3}n\right)\right]^{2} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

$$Cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2}e^{0.3} + \frac{1}{2}e^{0.3}$$

$$F\left\{cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

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 $X_{3}[n] = 2^{n} \cdot \frac{1}{2!} \cdot (e^{j\frac{\pi}{6}n} - e^{j\frac{\pi}{6}n}) u[-n]$ 

 $=\frac{1}{2 \cdot 1} \underbrace{\stackrel{\circ}{\mathcal{E}}}_{n=\infty} \left( \left[ 2 \cdot e^{\mathbf{j} (\omega - \frac{\pi}{6})} \right]^n - \left[ 2 \cdot e^{-\mathbf{j} (\omega + \frac{\pi}{6})} \right]^n \right)$ 

 $=\frac{1}{2i}\sum_{n=0}^{\infty}\left(\frac{e^{j(n-\frac{n}{2})}}{2}\right)^{n}-\frac{1}{2i}\sum_{n=0}^{\infty}\left(\frac{e^{j(n-\frac{n}{2})}}{2}\right)^{n}$ 

 $= \frac{1}{2j} \cdot \frac{2}{2 - e^{j(\omega - \frac{\pi}{6})}} - \frac{1}{2j} \cdot \frac{2}{2 - e^{j(\omega + \frac{\pi}{6})}}$ 

 $=\frac{1}{i}\left(\frac{1}{2-e^{i(e-\frac{\pi}{4})}}-\frac{1}{2-e^{i(e+\frac{\pi}{4})}}\right)$ 

 $=\frac{1}{2j}\lim_{n\to\infty}\frac{1-\left[2^{-i}e^{j(\omega-\frac{\pi}{4})}\right]^{n}}{1-2^{-i}e^{j(\omega-\frac{\pi}{4})}}-\frac{1}{2j}\sum_{n=0}^{\infty}\frac{1-\left[2^{-i}\cdot e^{j(\omega+\frac{\pi}{4})}\right]^{n}}{1-2^{-i}\cdot e^{j(\omega+\frac{\pi}{4})}}$ 

 $\frac{1}{2} \sum_{n=-\infty}^{\infty} 2^{n} \cdot \left(e^{j\frac{\pi}{q}n} - e^{-j\frac{\pi}{q}n}\right) e^{-j\omega n}$ 

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\frac{2\pi}{3}n}$$

$$\tilde{F}\left\{\cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right]$$

$$Cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2} e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{j\frac{2\pi}{3}n}$$

$$\tilde{F}\left\{cos\left(\frac{2\pi}{3}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\pi \cdot \delta\left(\omega - \frac{2\pi}{3} - 2\pi k\right) + \pi\right\}$$

$$Cos(\frac{2\pi}{3}n) = \frac{1}{2}e^{j\frac{2\pi}{3}n} + \frac{1}{2}e^{-j\frac{2\pi}{3}n}$$

$$F(cos(\frac{2\pi}{3}n))^{\frac{7}{3}} = \sum_{k=-\infty}^{\infty} [\pi \cdot 3(\mu) - \frac{2\pi}{3}n] + \frac{\pi}{2}e^{-j\frac{2\pi}{3}n}$$

$$\mathcal{F}\left\{\sin\left(\frac{\pi}{4}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) - \frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right)\right] = \frac{1}{2} \left[e^{j\frac{2\pi}{3}n} + \frac{1}{2} e^{-j\frac{2\pi}{3}n}\right]$$

$$\operatorname{Sin}\left(\frac{\pi}{4}n\right) = \frac{1}{2j} e^{j\frac{\pi}{4}n} - \frac{1}{2j} e^{-j\frac{\pi}{6}n}$$

$$\operatorname{F}\left\{\sin\left(\frac{\pi}{4}n\right)\right\} = \sum_{k=-\infty}^{\infty} \left[\frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right) - \frac{\pi}{j} \delta\left(\omega - \frac{\pi}{4} - 2\pi k\right)\right\}$$

$$e^{-j\frac{x}{\phi}n}$$

$$e^{-2\pi k} - \frac{\pi}{2} \geq 0$$



**Problem 2** (20 pts). Determine the inverse Fourier transform of the following signals:

(a) 
$$X_1(e^{j\omega}) = \cos^2(3\omega) + \sin^2(\omega)$$

$$\sum_{n=-\infty}^{\infty} \delta[n-n]e^{-j\omega n} = e^{-j\omega n}$$

$$\cos^{2}(3\omega) = \frac{1}{4}e^{j6\omega} + \frac{1}{4}e^{-j6\omega} + \frac{1}{2}$$

$$F^{-1} \int \cos^{2}(3\omega) \int = \frac{1}{4} \delta[n+6] + \frac{1}{4}\delta[n-6] + \frac{1}{2}\delta[n]$$

$$\sin^{2}(w) = \left(-\frac{1}{4}\right)e^{32w} + \left(-\frac{1}{4}\right)e^{-32w} + \frac{1}{2}$$

$$F^{-1}\left\{\sin^2(\omega)\right\} = \left(-\frac{1}{4}\right)\delta[n+2] + \left(-\frac{1}{4}\right)\delta[n-2] + \frac{1}{2}\delta[n]$$

$$F^{-1}\{X, (e^{3\omega})\} = 3[n] - \frac{1}{4}3[n+2] - \frac{1}{4}3[n-2] + \frac{1}{4}3[n+6] + \frac{1}{4}3[n-6]$$

(b) 
$$X_2(e^{j\omega}) = A(\omega)e^{jB(\omega)}$$
, where  $A(\omega) = \begin{cases} 0, & 0 \le |\omega| < \pi/3 \\ 1, & \pi/3 \le |\omega| < \pi \end{cases}$ , and  $B(\omega) = -\frac{2}{3}\omega$ 

$$F^{-1}\left\{\chi_{2}(e^{j\omega})\right\} = \frac{1}{2\pi}\int_{-x}^{\pi}A(\omega)e^{-j\frac{2}{3}\omega}\cdot e^{j\omega n}d\omega$$

$$F^{-1}\left\{\chi_{2}(e^{j\omega})\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi}A(\omega)e^{-j\frac{2}{3}\omega}\cdot e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{\pi}e^{-j\frac{2}{3}\omega}\cdot e^{j\omega n}d\omega\right)$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\frac{\pi}{3}} e^{-j\frac{\lambda}{3}\omega} e^{j\omega n} d\omega + \int_{\frac{\pi}{3}}^{\pi} e^{-j\frac{\lambda}{3}\omega} e^{j\omega n} d\omega \right)$$

$$= \frac{1}{2\pi} \left( \int_{-\pi}^{-\frac{\pi}{3}} e^{-j\frac{\pi}{3}N} e^{j\omega n} d\omega + \int_{\frac{\pi}{3}}^{\pi} e^{-j\frac{\pi}{3}N} e^{j\omega} \right)$$

$$= \frac{1}{2\pi} \left( \frac{e^{j(n-\frac{3}{3})\omega}}{j(n-\frac{2}{3})} \Big|_{-\pi}^{-\frac{\pi}{3}} + \frac{e^{j(n-\frac{3}{3})\omega}}{j(n-\frac{2}{3})} \Big|_{\frac{\pi}{3}}^{\pi} \right)$$

$$= \frac{1}{\pi \left(n - \frac{2}{3}\right)} \cdot \left[ Sin\left(\left(n - \frac{2}{3}\right)\pi\right) - Sin\left(\left(n - \frac{2}{3}\right)\frac{\pi}{3}\right) \right]$$
$$= SinC\left(n\pi - \frac{2\pi}{3}\right) - 3 SinC\left(\frac{\pi}{3}n - \frac{2\pi}{4}\right)$$

$$\sin C \left( \frac{\pi}{3} n - \frac{2\pi}{q} \right)$$

**Problem 3** (30 pts). Let 
$$X(e^{j\omega})$$
 be the Fourier transform of the signal  $x[n]$ .

Perform the following calculations without explicitly evaluating 
$$X(e^{j\omega})$$
.

$$\chi(o^{jo}) = 5, \quad \chi[n] e^{-jwn} = 5, \quad \chi[n] = 12$$

(a) Find  $X(e^{j0})$ .

$$\chi(e^{j\circ}) = \underset{n=-\infty}{\overset{\infty}{\leq}} \chi[n] e^{-jwn} = \underset{n=-\infty}{\overset{\infty}{\leq}} \chi[n] = 12$$

(b) Find 
$$\not \preceq X(e^{j\omega})$$
.

(b) Find 
$$\sqrt{A}(e^{a})$$
.

$$\chi[n-1]$$
 is real and even.  $F\{\chi[n-1]\}=e^{j\omega}\chi(e^{j\omega})$  is also real and even. 
$$\neq e^{j\omega}\chi(e^{j\omega})= \chi(\omega_{\chi}-\omega)= \chi(\omega_{\chi}-\omega)= \chi(e^{j\omega})= \omega$$

(c) Find 
$$X(e^{j\pi})$$
.

$$\chi(e^{j\pi}) = \sum_{n=-\infty}^{\infty} \chi[n] e^{-j\pi n} = \sum_{n=-\infty}^{\infty} \chi[n] \left\{ \cos(\pi n) - j \sin(\pi n) \right\}$$

$$= (2 + 2 + 4 + 2) \times 2 = 20$$

(d) Evaluate 
$$\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$$
.
$$\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 2\pi \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cdot e^{j\omega} d\omega$$

$$2\pi \int_{-\pi}^{\pi} \chi(0) = 2\pi \cdot \chi(0) = 8\pi$$

(e) Evaluate 
$$\int_{-\pi}^{\pi} \left| \frac{d}{d\omega} X(e^{j\omega}) \right|^2 d\omega$$
.

$$\frac{d}{du} \chi(e^{jw}) = \frac{d}{du} \underset{n=-\infty}{\overset{\circ}{\rightleftharpoons}} \chi[n] e^{-jten} = \underset{n=-\infty}{\overset{\circ}{\rightleftharpoons}} (-jn) \chi[n] e^{-jten}$$

$$\mathcal{F}^{-1}\left\{\frac{d}{d\omega}\,\chi(e^{j\omega})\right\} = -j\,n\,\chi[n]$$

$$\int_{-\bar{\kappa}}^{\bar{\kappa}}\,\left|\frac{d}{d\omega}\,\chi(e^{j\omega})\right|^2d\omega = 2\pi\,\sum_{n=-\infty}^{\infty}\,\left|-j\,n\,\chi[n]\right|^2 = 2\pi\cdot\sum_{n=-\infty}^{\infty}\,n^2\,\chi[n] = 2\pi\cdot392 = 784\,\pi$$

(f) Sketch 
$$y[n]$$
 such that  $\mathfrak{F}\{y[n]\} = \operatorname{Re}\{X(e^{j\omega})\}.$ 

$$y[n] = \frac{1}{2} \left(\chi[n] + \chi[-n]\right)$$

 $x[n] = \int_{0}^{n} \left[ B(\omega) \sin(\omega n) + C(\omega) \cos(\omega n) \right] d\omega$ 

by finding expressions for 
$$B(\omega)$$
 and  $C(\omega)$  in terms of  $X(e^{j\omega})$ .

by finding expressions for 
$$D(\omega)$$
 and  $C(\omega)$  in terms of  $A(\varepsilon)$ 

$$\gamma = \frac{1}{\sqrt{\pi}} \left( \sqrt{2} \alpha^{2} \right) e^{2\pi i n} dn$$

$$V = \frac{1}{\sqrt{n}} \left( \sqrt{n} \right) \left( \sqrt{n} \right) = \frac{1}{\sqrt{n}} \left( \sqrt{n} \right) \left($$

$$\chi[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \chi(e^{j\omega}) e^{j\omega n} d\omega$$

$$\chi(h) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \chi(e^{x}) e^{x} dx$$

$$=\frac{1}{1000}\int_{0}^{\pi} \chi(e^{3\omega}) \log(\omega n) +$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}\chi\left(\ell^{3\omega}\right)\left[\cos\left(\omega n\right)\right]+$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \chi(e^{j\omega}) [\cos(\omega n) + j \sin(\omega n)] d\omega$$

$$= \int_{-\pi}^{\pi} \int \frac{\chi(e^{j\omega})}{\cos(\omega h)} + \frac{j}{-j} \chi(e^{j\omega})$$

$$= \int_{-\pi}^{\pi} \left[ \frac{\chi(e^{i\omega})}{2\pi} \cos(\omega n) + \frac{j}{2\pi} \chi(e^{j\omega}) \sin(\omega n) \right] d\omega$$

$$= \int_{-\pi}^{\pi} \left[ \frac{\chi(e^{j\omega})}{2\pi} \operatorname{as}(\omega n) + \frac{j \chi(e^{j\omega})}{2\pi} \operatorname{sin}(\omega n) \right] d\omega$$

$$=\int_{0}^{\pi}\left[\frac{\chi(e^{j\omega})}{2\pi}\cos(\omega n)+\frac{j\chi(e^{j\omega})}{2\pi}\sin(\omega n)\right]d\omega+\int_{0}^{\pi}\left[\frac{\chi(e^{-j\omega})}{2\pi}\cos(-\omega n)+\frac{j\chi(e^{j\omega})}{2\pi}\sin(-\omega n)\right]d\omega$$

$$= \int_0^{\pi} \frac{1}{2\pi} [\chi(e^{j\omega}) + \chi(e^{-j\omega})] \cos(\omega n) d\omega + \int_0^{\pi} \frac{j}{2\pi} [\chi(e^{j\omega}) - \chi(e^{-j\omega})] \sin(\omega n) d\omega$$

$$\beta(\omega) = \frac{-1}{\pi} \operatorname{Im} \{ \chi(e^{i\omega}) \}$$

$$C(\omega) = \frac{1}{\pi} \operatorname{Re} \left\{ K(e^{j\omega}) \right\}$$

**Problem 4** (20 pts). Let 
$$X(e^{j\omega})$$
 be the Fourier transform of a real signal  $x[n]$ . Show that  $x[n]$  can be written as

$$(e^{i\omega}) - X(e^{-i\omega})$$
  $\sin(\omega n) d\omega$ 

$$= \int_{0}^{\pi} \frac{1}{\pi} \operatorname{Ref} \left\{ \left( e^{j\omega} \right) \right\} \cos(\omega n) d\omega + \int_{0}^{\pi} \frac{-1}{\pi} \operatorname{Im} \left\{ \left( e^{j\omega} \right) \right\} \sin(\omega n) d\omega$$

**Problem 5** (10 pts). Consider a signal y[n] with Fourier transform  $Y(e^{j\omega})$ . Suppose that  $y[n] = x_{(4)}[n]$ , where the signal x[n] has a Fourier transform  $X(e^{j\omega})$ . Determine a real number  $\alpha$  such that  $0 < \alpha < 2\pi$  and  $Y(e^{j\omega}) = Y(e^{j(\omega-\alpha)})$ . a is a period of Y(ein)  $Y(e^{j\omega}) = \underset{n=-\infty}{\overset{\infty}{\succeq}} \chi_{(n)}[n]e^{-j\omega n} = \underset{n=-\infty}{\overset{\infty}{\succeq}} \chi[n]e^{-j\omega 4n} = \chi(e^{j\psi})$  $\frac{2\pi}{4}=\frac{\pi}{2}\quad ,\ \alpha=\frac{\pi}{2}$