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## Properties of Laplace transform (LT)

Here we list some properties of LT to show why LT is useful to solve DEs:

<1> "derivation" property:

By LT, derivation (in  $t$ -domain)  $\longrightarrow$

$$\text{ex: } \mathcal{L}\{y'\} =$$

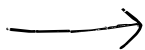
proof:

$$\mathcal{L}\{y'\} = \int_0^{\infty} y' e^{-st} dt = y e^{-st} \Big|_0^{\infty} - \int_0^{\infty} (-s) e^{-st} y dt$$

\* LT of derivation:

$$\left\{ \begin{array}{l} \mathcal{L}\{y'\} = \\ \mathcal{L}\{y''\} = \\ \vdots \\ \mathcal{L}\{y^{(n)}\} = \end{array} \right.$$

It means



<2> "linearity" property

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<3> " " property

Many engineering / physics problems involve in time. ex:

Their LT are

## How to solve DEs by LT

General procedures:

Remarks about  $\mathcal{L}^{-1}$  (inverse Laplace transform)

① We need to convert

② Properties of  $\mathcal{L}^{-1}$

— "uniqueness" property:

If  $\mathcal{L}\{f(t)\} = F(s)$ , then  $\mathcal{L}^{-1}\{F(s)\}$  can only be

— "linearity" property

Example 1: Use LT to solve  $y' = y - 4e^{-t}$ ,  $y(0) = 1$

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Example 2: Use LT to solve  $y'' + 4y' + 20y = e^{-2t}$ ,  $y(0) = y'(0) = 0$

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By comparing with  $\mathcal{L}\{y\}$ , we find that the denominator of  $\mathcal{L}\{y\}$  consists of product of

So from the roots of denominator of  $\mathcal{L}\{y\}$ , we can get qualitative analysis of the system response.

ex:  $\mathcal{L}\{y\} = \frac{1}{(s^2 + 2s + 2)(s + \frac{1}{10})}$ , what can we say about the system response qualitatively?

Remark: Qualitative analysis of system response from LT