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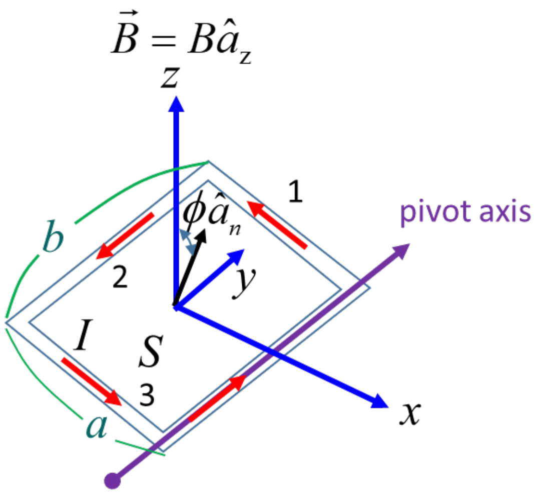
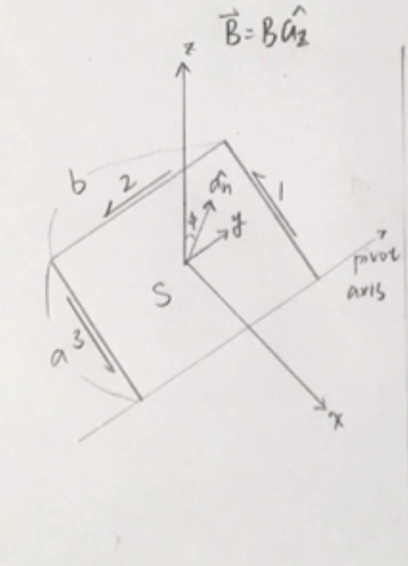
EE214000 Electromagnetics, Fall, 2020
Quiz #16-1, Open books, notes (22 points), due 11 pm, Wednesday, Dec. 30rd, 2020
(submission through iLMS)

Late submission won't be accepted!

1. Refer to the following coil with a current I in a magnetic field. Calculate the forces on the 1-3 wire segments and determine the torque on the wire loop. (8 points)





(1) $\vec{F_1} = I\vec{L_1} \times B\hat{a}_z = IaB\,\hat{a}_y$

$\vec{F_2} = I\vec{L_2} \times B\hat{a}_z = IbB(-\hat{a}_x)$

$\vec{F_3} = I\vec{L_3} \times B\hat{a}_z = IaB(-\hat{a}_y)$

$\vec{F_1} = IaB\,\hat{a}_y$

$\vec{F_2} = -IbB\,\hat{a}_x$

$\vec{F_3} = -IaB\,\hat{a}_y$

(2) $\vec{T} = \vec{m} \times \vec{B} = I\vec{S} \times \vec{B} = ISB\sin\phi\,(-\hat{a}_y) = IabB\sin\phi\,(-\hat{a}_y)$

$\vec{T} = -IabB\sin\phi\,\hat{a}_y$

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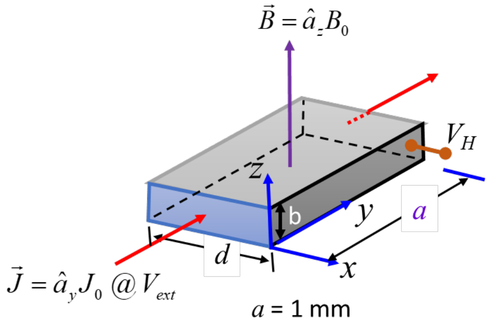
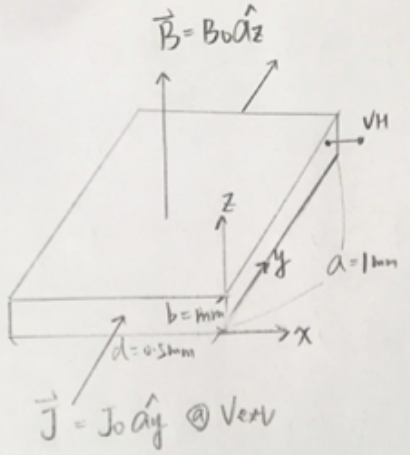
2. A piece of *n*-type semiconductor shown below is known to have a carrier density of 10^{19} electrons/cm^3. When under a magnetic field of 1 kG and applied with V_{ext} = 1 V, a uniform current of 1 A is generated along *y*. (9 points)

(1) What is the Hall voltage measured from this semiconductor? (2) What is the mobility of the electrons in this semiconductor? (3) what is the conductivity of this material?



a = 1 mm
b = 0.2 mm
d = 0.5 mm



$cm^3 = 10^6 m^3$

(1) $V_H = E_H d = \frac{J_0 B_0}{\rho} d = \frac{\frac{1}{(0.2\cdot10^{-3})(0.5\cdot10^{-3})}\cdot\left(\frac{1000}{10^4}\right)}{10^{19}\cdot10^{6}\cdot(1.6\times10^{-19})}\cdot 0.5\cdot10^{-3} = 3.125\cdot10^{-4}\ V$

(2) $J_0 = \rho u_e$, $u_e = \frac{J_0}{\rho} = \frac{\left(\frac{1}{0.2\cdot10^{-3}\times0.5\cdot10^{-3}}\right)}{10^{19}\cdot10^{6}\cdot1.6\cdot10^{-19}} = 6.25$ $\vec{u}_e = 6.25(-\hat{a}_y)$

$\vec{u}_e = -\mu_e \vec{E}_{ext} = -\mu_e \frac{V_{ext}}{a}\hat{a}_y = \mu_e \frac{1}{1\times10^{-3}}(-\hat{a}_y)$

$\mu_e \cdot \frac{1}{1\times10^{-3}} = 6.25$ $\mu_e = 6.25\cdot10^{-3}\ m^2/V\cdot s$

(3) $\sigma = \rho\mu_e = (10^{19}\cdot10^{6}\times1.6\cdot10^{-19})(6.25\cdot10^{-3}) = 10^4\ S/m$

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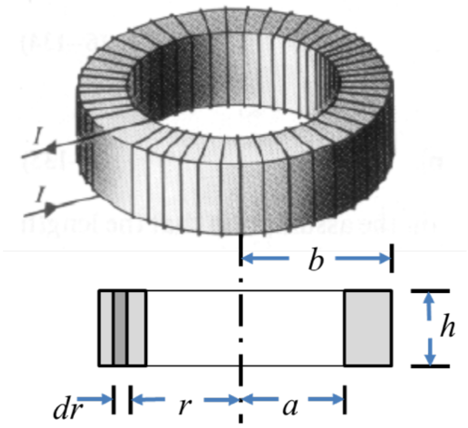
3. Calculate the magnetic energy (3 points) stored in the following N-turn toroid and deduce the inductance of it (2 points). Assume the ferromagnetic material in the toroid has a permeability of μ. (5 points)



(2) By Ampere's Law:

$$\oint_C \vec{H}\cdot d\vec{l} = H_\phi \cdot 2\pi r = NI \Rightarrow H_\phi = \frac{NI}{2\pi r} \Rightarrow B_\phi = \mu H_\phi = \frac{\mu NI}{2\pi r}$$

$$\Phi_{11} = \int_{S_1} \vec{B_1}\cdot d\vec{s} = \int_b^a B_\phi h\,dr = \frac{\mu NIh}{2\pi}\ln\frac{b}{a}$$

$$\Lambda_{11} = N_1\phi_{11} = \frac{\mu N^2 I h}{2\pi}\ln\frac{b}{a}$$

$$\Rightarrow L_{11} = \frac{\Lambda_{11}}{I_1} = \frac{\mu N^2 h}{2\pi}\ln\frac{b}{a}$$

(1) $$W_1 = \frac{1}{2}L_{11}I_1^2 = \frac{1}{2}\frac{\mu N^2 h I^2}{2\pi}\ln\frac{b}{a} = \frac{\mu N^2 I^2 h}{4\pi}\ln\frac{b}{a}$$

or $$W = \frac{1}{2\mu}\int_V B_\phi^2\,dv = \frac{1}{2\mu}\int_a^b \left(\frac{\mu NI}{2\pi r}\right)^2 h\,2\pi r\,dr$$

$$= \frac{\mu N^2 I^2 h}{4\pi}\int_a^b \frac{1}{r}\,dr = \frac{\mu N^2 I^2 h}{4\pi}\ln\frac{b}{a}$$