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系級

學號

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Examples of systems modeled by 2nd-order ODE with constant coefficients

Many physics and engineering problems may be modeled by 2nd-order ODEs with constant coefficients.

The following systems are completely different processes in different fields:

All yields the same 2nd-order ODE with constant coefficients.

⇒ Just by learning one ODE, we can interpret the results in terms of

Depending on the operation conditions, responses can be categorized as:

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Let's take the spring-mass system for example:

Case I: unforced, undamped

$\boxed{\text{DE}}$ $my''+ky=0$

$\boxed{\text{roots/solution}}$

$\boxed{\text{behavior/ physical meaning}}$

Case II: unforced, damped

$\boxed{\text{DE}}$ $my''+by'+ky=0$

$\boxed{\text{roots/solution}}$

$\boxed{\text{behavior/ physical meaning}}$

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科目

系級

學號

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Case III : forced, damped

Of particular interest is the response of the system when driven by a

DE

roots/solution

behavior/ physical meaning

— Initially, the response is

— As t increases,

So we call "y_h" is the

"y_p" is the

— The system eventually

Case IV : forced, undamped

DE

roots/solution

behavior/ physical meaning

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科目

系級

學號

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Example 1: Consider a harmonic oscillator modeled by $y'' + 2y' + 2y = \sin t$. Discuss the system behavior.

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科目

系級

學號

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Example 2: Consider a harmonic oscillator modeled by $y'' + 2y = \cos\omega t$. Discuss the system behavior.

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科目

系級

學號

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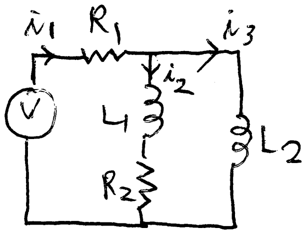
Remark: For such 2nd-order ODE, we can also rewrite the 2nd-order ODE as two

ex:

Q: How to solve a system of multiple ODEs?

Example: Parallel circuit



This parallel circuit is modeled by

$$\begin{cases} L_1\dfrac{di_2}{dt} + (R_1+R_2)\, i_2 + R_1 i_3 = V \\ L_2\dfrac{di_3}{dt} + \end{cases}$$

Given $L_1 = L_2 = 1$ H, $R_1 = 4\,\Omega$, $R_2 = 6\,\Omega$, $V = 10$ V, solve i_2, i_3.

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科目

系級

學號

姓名 P44

General procedures:

① Express ODEs by

② Eliminate the

③ Use   to find

                         to find

⇒ obtain

④