CS2336 Discrete Mathematics

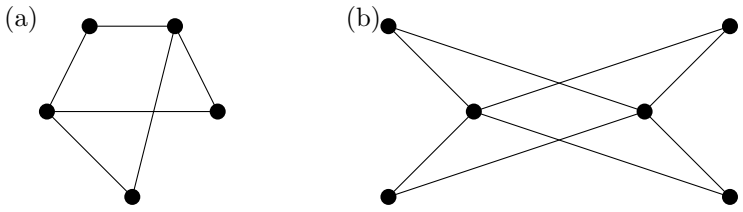
Homework 6
Tutorial: December 30, 2019
Exam 3: Januray 06, 2020 (2.5 hours)

Problems marked with * will be explained in the tutorial.

1. Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that
   (a) $2e/v \geq m$
   (b) $2e/v \leq M$

2. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

3. (*) For each of the following graphs, determine whether it is bipartite.



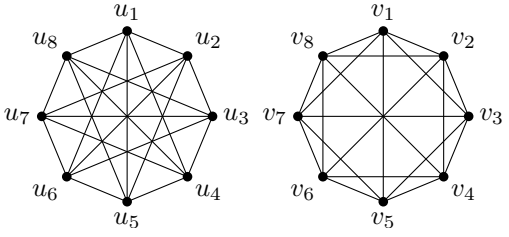
4. (*) If G is a graph, the *complement* of G, denoted by $\overline{G}$, is a graph with the same vertex set, such that an edge e exists in $\overline{G}$ if and only if e does not exist in G.

   Suppose it is known that a simple graph G has 15 edges and its complement graph $\overline{G}$ has 13 edges. How many vertices does G have?

5. Show that if G is a simple graph with n vertices, then the union of G and $\overline{G}$ is K_n.

6. (*) Two graphs G and H are *isomorphic* if there is a one-to-one correspondence f between the vertices of G and the vertices of H, such that u, v are adjacent in G if and only if $f(u), f(v)$ are adjacent in H. (See Lecture Note 13, pages 26 and 27 for some examples.)

   Determine whether the following two graphs are isomorphic.



7. Suppose that G and H ae isomorphic simple graphs. Show that their complementary graphs $\overline{G}$ and $\overline{H}$ are also isomorphic.