Quiz 4 solution

1. In Figure 1, a constant voltage source of 10 V is applied at t = 0 (which means no voltage applied to the circuit at t < 0). Given that $i_1(0^-) = 2$ A and $v_4(0^-) = 4$ V. Find all branch voltages (v_1 to v_4) and current (i_1 to i_4) at t = 0^+ and t = ∞. (20%, each 2.5%)

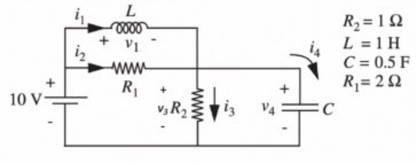

Figure 1.

There is current continuous at inductor

$i_1(0^+) = i_1(0^-) = 2$

There is voltage continuous at capacitor

$v_4(0^+) = v_4(0^-) = 4$

Use Ohm's law

$$i_2(0^+) = \frac{10 - v_4(0^+)}{R_1} = 3$$

$v_2(0^+) = 3 * 2 = 6 = v_1(0^+)$

And

$v_4(0^+) = v_3(0^+) = 4$

Thus,

$$i_3(0^+) = \frac{4}{1} = 4$$

Use KCL

$i_4(0^+) = i_1(0^+) + i_2(0^+) - i_3(0^+) = 1$

When t → ∞, it is steady state, and the inductor is treated as a short while the capacitor is treated as an open .

$v_1(\infty) = 0 = v_2\ (\infty)$

$i_4(\infty) = 0$

$v_4(\infty) = 10 = v_3\ (\infty)$

By Ohm's law

$i_2(\infty) = 0$

$$i_3(\infty) = \frac{10}{1} = 10$$

Use KCL

$i_1(\infty) = i_3(\infty) + i_4(\infty) - i_2(\infty) = 10$

Conclude

Time\value	i_1	i_2	i_3	i_4	units	V_1	V_2	V_3	V_4	units
0^+	2	3	4	1	A	6	6	4	4	V
∞	10	0	10	0	A	0	0	10	10	V

2. Consider the circuit in Figure 2. Assume the operational amplifier is ideal. Derive the second-order differential equation that shows how the voltage $v_2(t)$ is related to the input voltage $v_s(t)$. (20%)

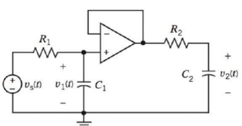

Figure 2.

$$\frac{v_s - v_1}{R_1} - C_1\frac{dv_1}{dt} = 0 \Rightarrow \frac{v_1}{R_1} + C_1\frac{dv_1}{dt} = \frac{v_s}{R_1} \quad — ①$$

$$\frac{v_1 - v_2}{R_2} - C_2\frac{dv_2}{dt} = 0 \Rightarrow \frac{v_2}{R_2} + C_2\frac{dv_2}{dt} = \frac{v_1}{R_2}$$

$$\Rightarrow v_1 = R_2C_2\frac{dv_2}{dt} + v_2 \quad — ②$$

Substitude ② into ① $\Rightarrow \frac{R_2C_2}{R_1}\frac{dv_2}{dt} + \frac{v_2}{R_1} + C_1R_2C_2\frac{d^2v_2}{dt^2} + C_1\frac{dv_2}{dt} = \frac{v_s}{R_1}$

$$\Rightarrow v_2'' + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right)v_2' + \frac{1}{R_1R_2C_1C_2}v_2 = \frac{1}{R_1R_2C_1C_2}v_s$$

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Quiz 4

3. KVL:

$$\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + \frac{V_c}{LC} = \frac{48}{LC}$$

$\Rightarrow$ $V_{c,p} = 48$

$V_{c,h} = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

characteristic equ: $s^2 + 2800s + 2.5\times10^7 = 0$

$\rightarrow s = -1400 \pm 4800j.$

$V_c = 48 + e^{-1400t}(K_1\cos(4800t) + K_2\sin(4800t))$

$K_1 = A_1 + A_2$ and $K_2 = jA_1 - jA_2$

$$\begin{cases} V_c(0) = 48 + K_1 = 0 \\ i_L(0) = C\cdot(-1400K_1 + 4800K_2) = 0 \end{cases} \Rightarrow \begin{cases} K_1 = -48 \\ K_2 = -14 \end{cases}$$

$$\Rightarrow V_c(t) = 48 + e^{-1400t}\left[-48\cos(4800t) - 14\sin(4800t)\right] \text{ (V)}$$

for $t \geq 0$

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4.

$R = 200\Omega$, $L = 50mH$, $C = 0.2\mu F$

$i_R + i_L + i_C = 0$

$\frac{V}{R} + \frac{1}{L}\int_0^t v\,d\tau + i_L(0) + C\frac{dv}{dt} = 0 \Rightarrow C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$

$\Rightarrow \frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0$, $s^2 + \frac{1}{200\times 0.2M}\frac{dv}{dt} + \frac{V}{50m\cdot 0.2\mu} = 0$

$s^2 + 25000s + 10^8 = 0$, $\Rightarrow s = -20000/-5000$

$\therefore v(t) = A_1e^{-20000t} + A_2e^{-5000t}$, $V(0) = A_1 + A_2 = 12$ ...①

$i_C(0) = 30mA$, $C\frac{dV}{dt}(t=0) = 30mA$

$-20000A_1 - 5000A_2 = 30\times10^{-3}\times\frac{1}{0.2\times10^{-6}} = 150000$

$A_1 + 0.25A_2 = 7.5$ ...②

solve ①,②, $A_1 = 6$, $A_2 = 6$

$V(t) = 6e^{-2000t} + 6e^{-5000t}$ #

$A_1 + A_2 = 12$

$-)\ A_1 + 0.25A_2 = 7.5$

$0.75A_2 = 4.5$

$A_2 = 6$

$A_1 = 6$