proof of SM-02.

① To prevent confusion. let $\omega(i)$ be the original weight assign to node #$i$. (denoted as $n_i$)

② Since input is a tree, for each node, it has unique walk to reach the root:

$$n_i = n_{p_i(0)} \longrightarrow n_{p_i(1)} \longrightarrow n_{p_i(2)} \rightarrow \cdots \qquad \longrightarrow n_{p_i(\ell)} \rightarrow \cdots$$

and $\forall \ell \geq n$. $p_i(\ell) = \text{root}$. (Since there are only $n$ nodes)

Furthermore. let $p_d(\ell)$ be the first node that $p_d(\ell) = \text{root}$

$$\forall L \geq d \,.\ \sum_{\ell=0}^{L} \omega(p_i(\ell)) = \sum_{\ell=0}^{d} \omega(p_i(\ell))$$

③ Claim: After step $t$, $W[i] = \sum_{\ell=0}^{2^t-1} \omega(p_i(\ell))$

and $S[i] = p_i(2^t)$

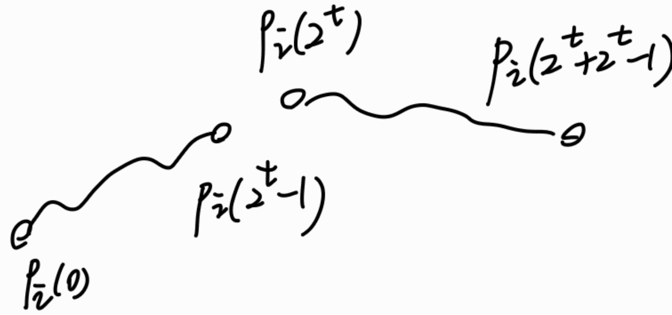
④ When $t=0$, $W[i] = \omega(p_i(0)) = \sum_{\ell=0}^{2^0-1} \omega(p_i(\ell))$

and $S[i] = p_i(1) = p_i(2^0)$

⑤ Assume that after step $t$. $W[i] = \sum_{\ell=0}^{\#\, 2^t-1} \omega(p_i(\ell))$

and $S[i] = p_i(2^t)$

After step $t+1$,




$$W[i] = W[i] + W[S[i]]$$

$$= W[i] + W[p_i(2^t)]$$

$$= \sum_{l=0}^{2^t-1} p_i(l) + \sum_{l=0}^{2^t-1} p_{p_i(2^t)}(l)$$

$$= \sum_{l=0}^{2^t-1} p_i(l) + \sum_{l=2^t}^{2^t+2^t-1} p_i(l) = \sum_{l=0}^{2^{t+1}-1} p_i(l)$$

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$$S[i] = S[S[i]] = S[p_i(2^t)] = p_{p_i(2^t)}(2^t) = p_i(2^t+2^t)$$

$$= p_i(2^{t+1})$$

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(b) By M.I. $\forall t = 1, 2, \ldots \lceil \lg n \rceil$, $W[i] = \sum_{l=0}^{2^t-1} \omega(p_i(l))$

and $S[i] = p_i(2^t)$

After $t = \lceil \lg n \rceil$, $W[i] = \sum_{l=0}^{2^t-1} \omega(p_i(l)) = \sum_{l=0}^{d} \omega(p_i(l))$

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since $\forall i.\ d \leq n-1 \leq 2^t - 1 = 2^{\lceil \lg n \rceil} - 1$