proof for SM-01.

① To prevent confusion, let $I[1 \dots n]$ be the original input array.

② Claim: After step t ($k = m - t$). $\forall i$, $1 \le i \le 2^k$

$$A[i] = \sum_{j=(i-1)\cdot 2^t + 1}^{i \cdot 2^t} I[j].$$

③ When $t=0$, (before the algorithm)

$\forall i.\ 1 \le i \le 2^k = 2^{m-0} = n$

$$A[i] = I[i] = \sum_{j=(i-1)\cdot 2^0 + 1}^{i \cdot 2^0} I[j] \qquad \#$$

④ Assume that after step t,

$$\forall i\ \ 1 \le i \le 2^{m-t} \quad A[i] = \sum_{j=(i-1)\cdot 2^t + 1}^{i \cdot 2^t} I[j]$$

After step $(t+1)$

$$\forall i.\ 1 \le i \le 2^{m-t-1} \quad A[i] = A[2i-1] + A[2i]$$

$$= \sum_{j=(2i-2)\cdot 2^t + 1}^{(2i-1)\cdot 2^t} I[j] + \sum_{j=(2i-1)\cdot 2^t + 1}^{2i \cdot 2^t} I[j]$$

$$= \sum_{j=(i-1)\cdot 2^{t+1} + 1}^{i \cdot 2^{t+1}} I[j] \qquad \#$$

⑤ By M.I., $\forall t = 0, 1, \ldots, m-1$.

$$\forall i,\ 1 \le i \le 2^{m-t}.\quad A[i] = \sum_{j=(i-1)\cdot 2^t+1}^{i\cdot 2^t} I[j].$$

⑥ After step $t=m$, $\forall i,\ 1 \le i \le 2^{m-m}$ $\quad A[i] = \sum_{j=(i-1)\cdot 2^m+1}^{i\cdot 2^m} I[j]$

$$\Rightarrow A[1] = \sum_{j=1}^{n} I[j]$$ #