Proof SM-03.

① Claim: after step t, $\forall i.\ 1 \leq i \leq n.$ $S[i] = \sum_{j=i-2^t+1}^{i} X[i]$

② When $t=0$, $S[i] = X[i] = \sum_{j=\min(1,i-2^0+1)}^{i} X[i]$ # (let $X[k]=0$ $\forall k \leq 0$)

③ Assume that after step t, $\forall i.\ 1 \leq i \leq n.$

$$S[i] = \sum_{j=i-2^t+1}^{i} X[i]$$

After step $t+1$.

$$S[i] = S[i] + S[i-2^t] = \sum_{j=i-2^t+1}^{i} X[i] + \sum_{j=i-2^t-2^t+1}^{i-2^t} X[i]$$

$$= \sum_{j=i-2^{t+1}+1}^{i} X[i]$$ #

④ By M.I. after step $t = \lceil \lg n \rceil$.

$\forall i.\ 1 \leq i \leq n.$ $S[i] = \sum_{j=i-2^t+1}^{i} X[i] = \sum_{j=1}^{i} X[i]$ #

$(i-2^t+1 \leq i-n+1 \leq 1)$