

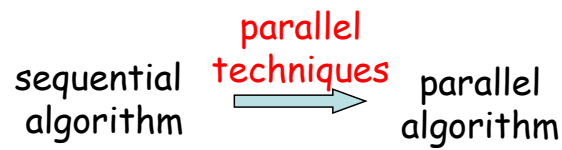
- 平行計算 (parallel computation) 是一門非常精緻迷人的藝術

1-3a

- 開課目的不是要讓大家成為專家
- 是為了讓大家欣賞和感動

- prerequisite: algorithms

(email your proofs in 1 week)



- 這是一門選修課

- 希望上課聽懂就可以，沒有作業
- 只有邏輯，~~沒有數學~~
- 需要一些簡單的數學分析 (痛恨數學分析的，要趕快退選)
- 有考試，需要花時間準備 (不喜歡考試的，要趕快退選)
- 會花很多時間討論 (痛恨上課講話的，要趕快退選)
- 缺課不得超過3次 (含請假) (不喜歡上課的，要趕快退選)
- 不得使用手機或筆電 (每次扣5分，手機中毒的趕快退選)
- 沒有預設進度，請踴躍發問

\*可能有加分作業 (自願)

補充講義：eLearn 上有電子檔

algo 學分證明：

email (subject: 平行計算學分證明王小明)

attachment (not a link; with a highlight)

no response (inform ones not sending)

加簽：

沒有加簽

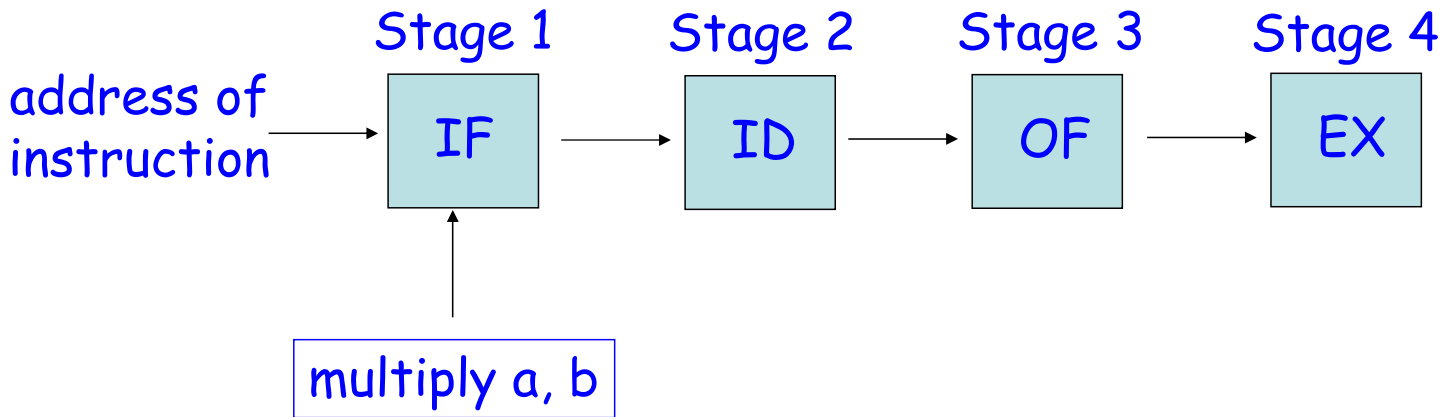
(請透過校務資訊系統選課等待退選)

# A pipelined computer

1-5a

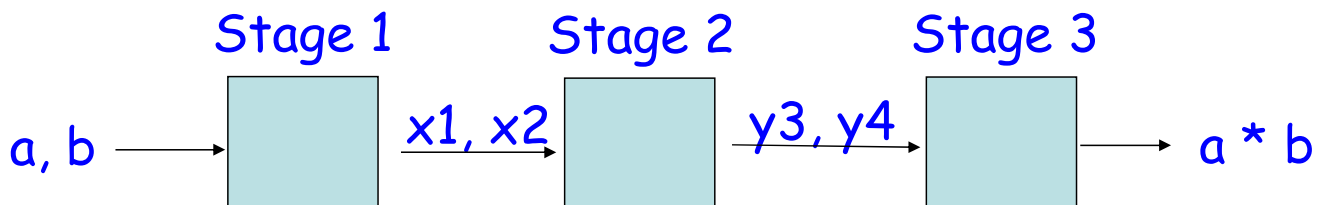
An instruction cycle

- instru. fetch
- instru. decoding
- operand fetch
- execution



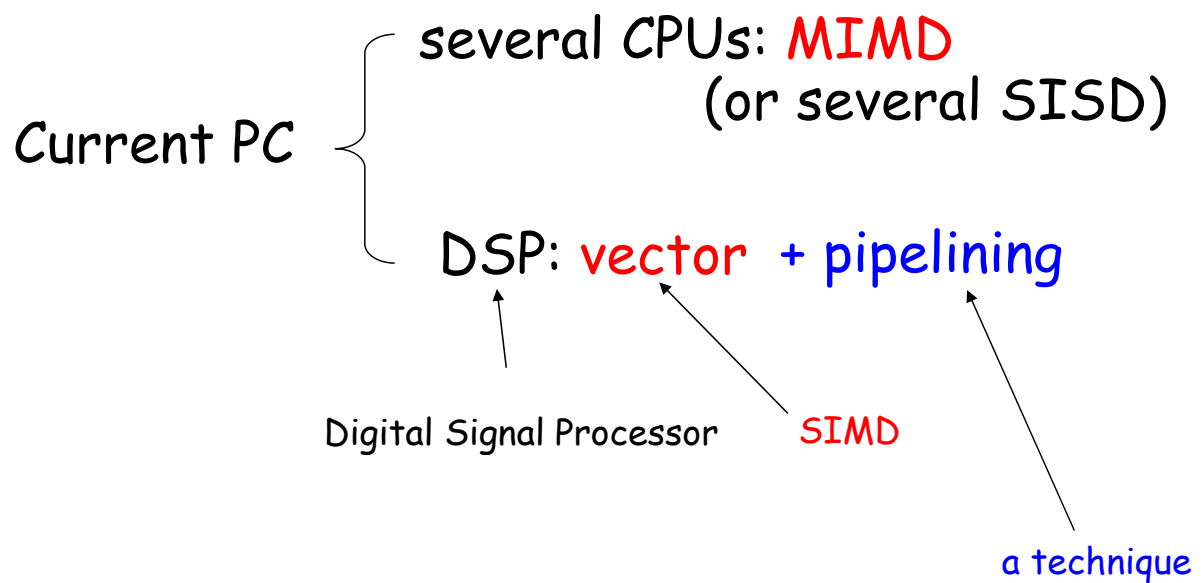
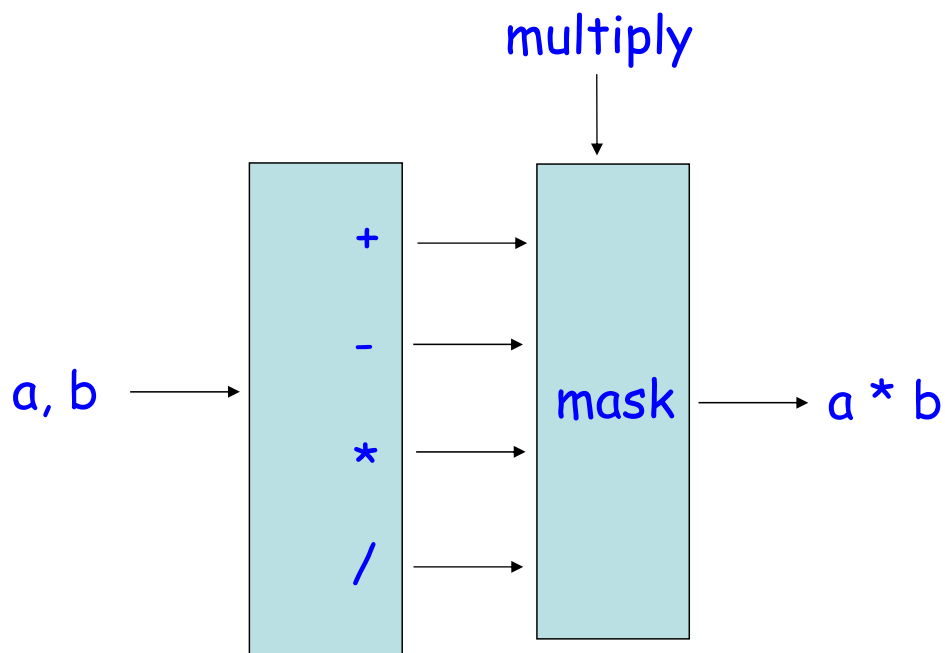
1-5b

## A pipelined multiplier (floating-point numbers)



Some ALU

Instruction: multiply a, b



# An Example of SIMD

## Odd-Even Transposition Sort on a Linear Array

Odd iteration:

**for** each  $P_i$ ,  $1 \leq i \leq n$ , **pardo**  
**if**  $i$  **is odd** **then**  
 $P_i$  Compare&Exchange with  $P_{i+1}$

Even iteration:

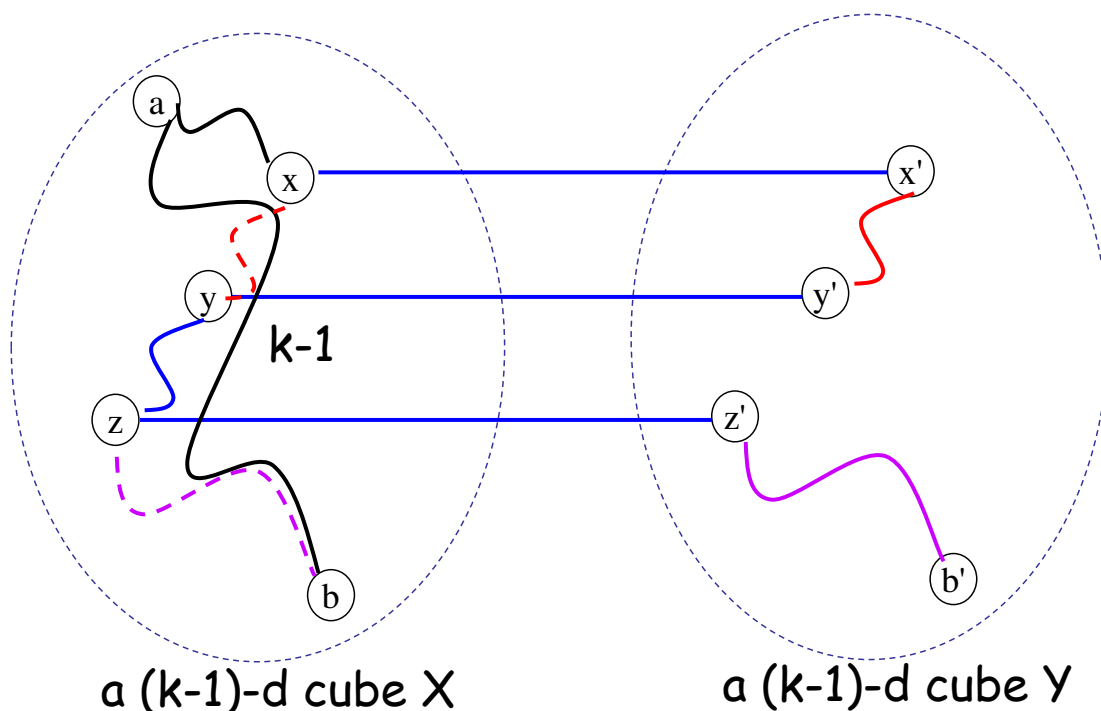
**for** each  $P_i$ ,  $1 \leq i \leq n$ , **pardo**  
**if**  $i$  **is even** **then**  
 $P_i$  Compare&Exchange with  $P_{i+1}$

Given  $d(a, b)$  in  $X$  is  $k-1$ , prove that  $d(a, b') = k$   
 (assume that  $d(a, b') \leq k$  is proved)

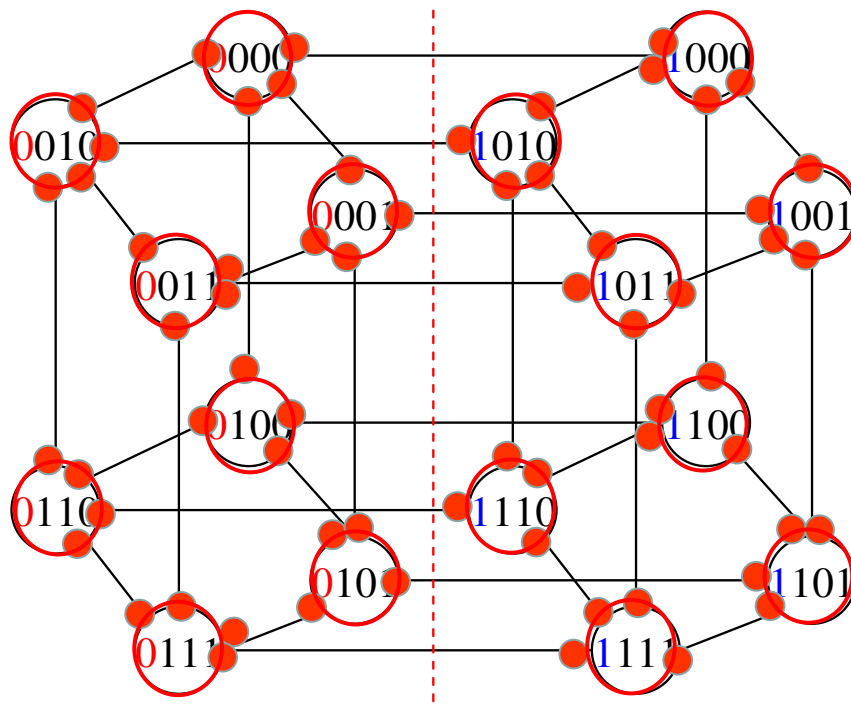
1-8a

By **contradiction**, assume that  $d(a, b') < k$

Then, in  $X$ ,  $d(a, b) < k - 1$ , a **contradiction**.



1-8b



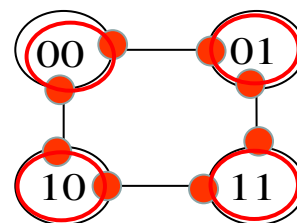
4-d CCC

4x2<sup>4</sup> processors

4-cube 2<sup>4</sup> processors

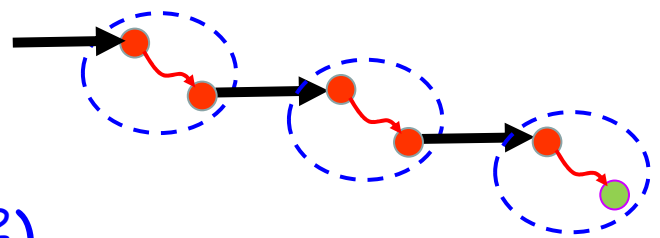
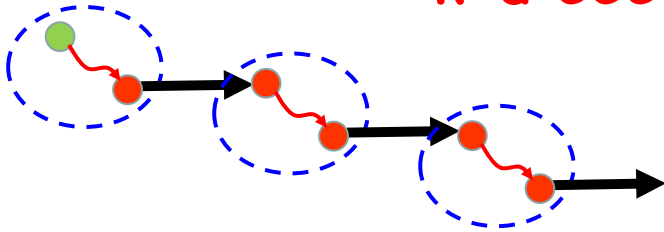
1-8c

2-d CCC



2-cube

## k-d CCC

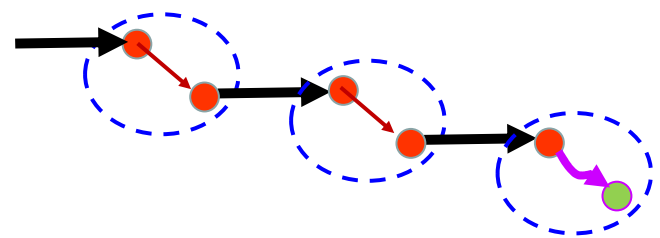
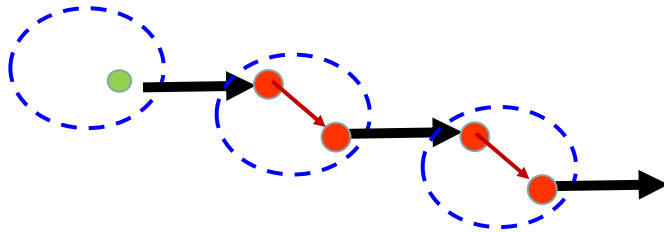


diameter ???

$$k + (k+1) \times (k-1) = O(k^2)$$

(or  $(k+1) \times (k/2)$ )

## k-d CCC

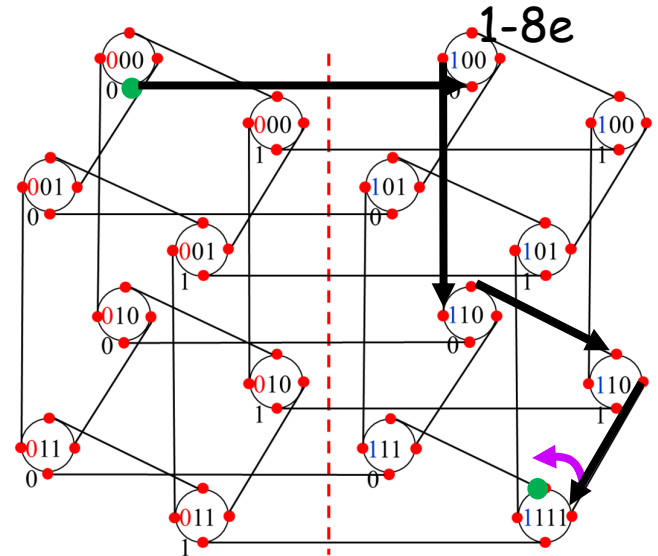


diameter ???

better estimation ???

$$k + (k-1) + \lfloor k/2 \rfloor$$

(correct:  $2k - 2 + \lfloor k/2 \rfloor$  for  $k \geq 4$ )

~~1-8e~~

Current PC: several CPUs on a common bus  
 MIMD (or several SISD)  
 shared-memory

Super-Computer: up to several  $10^3$  CPUs  
 MIMD/SIMD  
 intersection network

蓋房子（每位工人每天工錢 1 萬元）

sequential:

$$p = 1, T_s = 100$$

parallel:

$$p = 5, T_p = 25$$

$$\text{speedup} = T_s / T_p = 100 / 25 = 4$$

$$\text{cost} = p * T_p = 5 * 25 = 125 \text{ (萬元)}$$

$$\begin{aligned} \text{efficiency} &= T_s / \text{cost} = 100 / 125 = 0.8 \\ &= \text{speedup} / p = 4 / 5 = 0.8 \end{aligned}$$

蓋房子（每位工人每天工錢 1 萬元）

sequential:

$$p = 1, T_s = 100$$

parallel:

$$p = 10, T_p = 20$$

$$\text{speedup} = T_s / T_p = 100/20 = 5$$

$$\text{cost} = p * T_p = 10 * 20 = 200 \text{ (萬元)}$$

$$\begin{aligned} \text{efficiency} &= T_s / \text{cost} = 100 / 200 = 0.5 \\ &= \text{speedup} / p = 5/10 = 0.5 \end{aligned}$$

## Usage

1-11c

	speedup $T_s / T_p$	cost $p * T_p$	efficiency $T_s / \text{cost}$ speedup / p
experiment	✓	✓	✓
algorithm	✗ ( $\theta$ or $\Omega$ )	✓ (O or $\theta$ )	✗ ( $\theta$ or $\Omega$ )

\*experiment: in CPU time

\*algorithms:

- usually,  $T_p$  first; then cost (or # of PEs)
- if speedup/efficiency are needed,  
in  $\theta$  notation (suggested)



## Odd-even transposition sort

sequential:

$$p = 1, T_s = O(n \lg n)$$

parallel:

$$p = n, T_p = O(n)$$

$$\text{speedup} = T_s / T_p = \theta(\lg n)$$

$$\text{cost} = p * T_p = \theta(n * n) = \theta(n^2)$$

$$\begin{aligned} \text{efficiency} &= T_s / \text{cost} = \theta(n \lg n / n^2) = \theta(\lg n / n) \\ &= \text{speedup} / p = \theta(\lg n / n) \end{aligned}$$

\* Note: treat all  $O$  as  $\theta$

## Another parallel sorting

sequential:

$$p = 1, T_s = O(n \lg n)$$

parallel:

$$p = n^{0.5}, T_p = O(n^{0.5} \lg n)$$

$$\text{speedup} = T_s / T_p = \theta(n^{0.5}) \text{ or } n^{0.5}$$

$$\text{cost} = p * T_p = \theta(n \lg n) \text{ or } n \lg n$$

$$\begin{aligned} \text{efficiency} &= T_s / \text{cost} = \theta(n \lg n / n \lg n) = \theta(1) \text{ or } 1 \\ &= \text{speedup} / p = \theta(n^{0.5} / n^{0.5}) = \theta(1) \text{ or } 1 \end{aligned}$$

\* Note: treat all  $O$  as  $\theta$  (or simply remove the notation)