TD 3

Exercise 1:

Calculate the gradient and the Hessian of the following functions:

1. $f(x, y) = x^2 + 2y^2 - 3xy$.
2. $g(x_1, x_2, x_3) = e^{x_1} + x_1^2 x_3 - x_1 x_2 x_3$
3. $h(x, y) = 3x^3 - 4x^2y + 2y^2$.

Exercise 2:

For the function $f(x, y) = x^4 + 4x^2y + 2y^4$, compute the determinant of the Hessian matrix and determine whether the function is convex or concave.

Exercise 3:

Consider the function $f(x, y, z) = x^2 + 2y^2 + 3z^2 - 2xy + 4xz - 5yz$. Calculate the gradient and Hessian matrix of this function, and determine its convexity or concavity.