TD 4

Exercise 1:

The steepest descent method, also known as gradient descent, is an iterative optimization technique that relies on the gradient of a function to minimize it. The update formula for the steepest descent method is:

$$x_{k+1} \leftarrow x_k - \tau_k . \nabla f(x_k)$$

Consider the function $f(x)=x_1^2+3.5x_2^2-4x_1x_2-2x_2$.

1. Use the steepest descent method to find the minimum of this function starting from an initial guess $x_0=[1,1]$. Perform two iterations of the method with a step size of $\tau_k=0.1$, then with Barzilai and Borwein step size

$$\tau_k = \frac{\|x_k - x_{k-1}\|^2}{(x_k - x_{k-1})'(\nabla f(x_k) - \nabla f(x_{k-1}))}$$

Exercise 2:

The Newton-Raphson method is an iterative optimization technique that uses the gradient (first derivative) and the Hessian (second derivative) of a function to find its minimum. The general update formula for the Newton-Raphson method is:

$$x_{k+1} = x_k - \tau_k [\nabla^2 f(x_k)]^{-1} . \nabla f(x_k)$$

Consider the function $f(x)=x_1^2+3.5x_2^2-4x_1x_2-2x_2$. Use the Newton-Raphson method to find the minimum of this function starting from an initial guess $x_0=[1,1]$. Perform two iterations of the method.