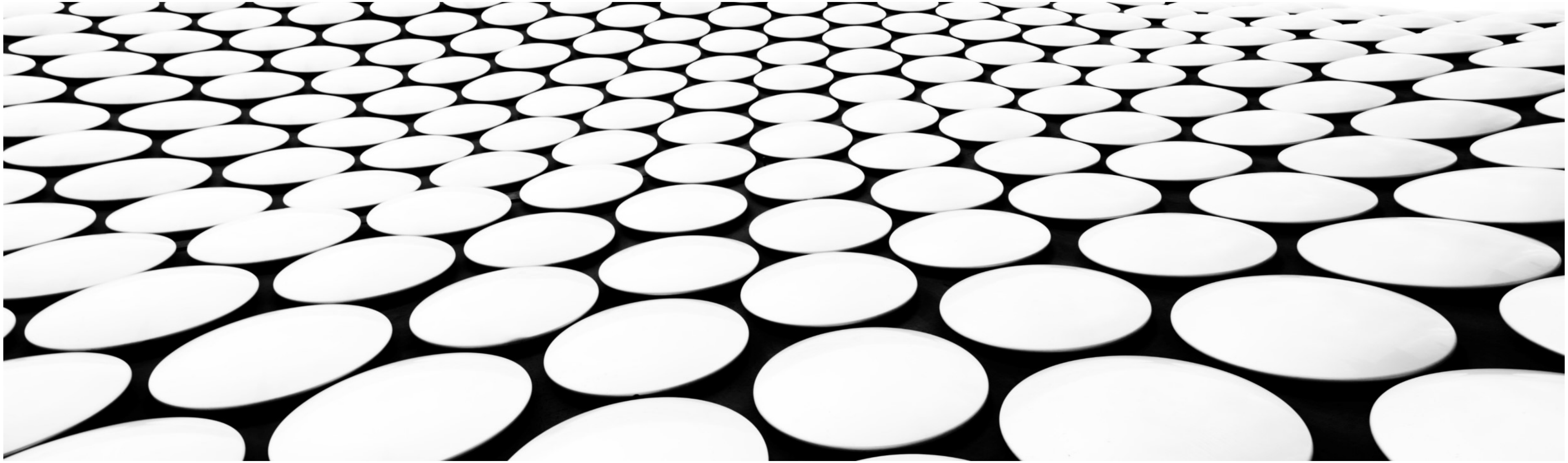
DISCRETE OPTIMIZATION



DEFINITION

- Discrete optimization is a branch of mathematical optimization
- deals with problems where the set of feasible solutions is discrete, meaning that the decision variables can only take on distinct, separate values.
- These values are typically non-continuous, and they can include integers, binary (0 or 1), or categorical choices.
- The goal of discrete optimization is to find the best combination or configuration of these discrete decisions to optimize an objective function, subject to certain constraints.

SUBCATEGORIES

- Integer Programming: Involves optimization problems where decision variables are required to take on integer values. This can include linear integer programming (LIP) and mixed-integer programming (MIP) where some variables can be continuous.
- Combinatorial Optimization: Focuses on problems where the goal is to find the best combination of elements from a finite set to optimize an objective function. Examples include the Traveling Salesman Problem (TSP), Knapsack Problem(KSP), and Graph Coloring Problem (GCP).
- Binary Optimization: A specialized form of integer programming where decision variables are restricted to binary values (0 or 1). Problems in this category include the 0-1 Knapsack Problem and Binary Linear Programming.
- ...

LINEAR INTEGER PROGRAMMING

- Minimize $c^T x$
- Subject to
  - $Ax \leq b$
  - $x \in Z^n$

BRANCH AND BOUND METHODS

- The main idea:
  - systematically search the solution space by dividing it into smaller subproblems,
  - solving each subproblem efficiently,
  - and using bounds to prune branches that cannot lead to an optimal solution.
- The method is particularly effective for problems where the solution space has a tree-like structure.

BRANCH AND BOUND WITH SIMPLEX FOR LIP AND MIP

Do iteratively:

1. Relaxation: integer constraints are relaxed, allowing the variables to take on continuous values using the Simplex algorithm.
2. Branching: create two subproblems to fix one variable having a real value.
3. Bounding: The lower and upper bounds for each subproblem are calculated. Bounds are estimated based on the relaxation.
4. Pruning: If the lower bound of a subproblem is worse than to the current best-known upper bound, the branch is pruned, as it cannot lead to an improved solution.

BRANCH AND BOUND WITH SIMPLEX FOR LIP AND MIP

Example:

Maximize $3x_1+5x_2$

Subject to:

$2x_1+4x_2 \leq 25$

$x_1 \leq 8$

$x_1, x_2 \in Z^+$