# Introduction to Probability
## Quiz 3

November 22, 2022, 3:30 a.m. - 5:20 a.m.

**Note:** You have to answer the questions with supporting explanations if needed.

1. A continuous random variable $X$ has a probability density function (PDF) given below

$$f_X(x) = \begin{cases} a + bx^2, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

   If $\mathbf{E}[X]$=3/5, find $a$ and $b$. (15%)

2. The lifetimes of the computer chips produced by ChillX are normally distributed with mean $\mu = 1.5 \times 10^6$ hours and standard deviation $\sigma = 6 \times 10^5$ hours. What is the probability that the lifetime of a chip produced by ChillX is between $1.2 \times 10^6$ and $2.1 \times 10^6$ hours. (10%)

   **Hinit:** A few CDF values of a standard normal are given as follows:

$$\Phi(0) = 0.5, \Phi(0.5) = 0.6915, \Phi(1.0) = 0.8413, \Phi(1.5) = 0.9332 \text{ and } \Phi(2.0) = 0.9772.$$

3. Given that the time duration (in minutes) of your weekly meetup with Prof. Berlin can be modeled by an exponential random variable with parameter $\lambda = 1/5$.

   (i) Find the probability that a meetup will last less than 10 minutes. (10%)

   (ii) Suppose that a meetup has already lasted for 10 minutes. Find the probability that the meetup will lasted less than 5 more minutes. (10%)

   **Hint**: The PDF of an exponential random variable $X$ with parameter $\lambda$ is defined by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

$f(x) = \int \lambda e^{-\lambda x}$

$= -e^{\lambda x}$

4. We are told that the joint PDF of random variables $X$ and $Y$ is

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{9}xy, & 0 \le x \le 3, 0 \le y \le 2 \\ 0, & \text{otherwise} \end{cases}$$

   (i) Find the PDF of $X$ (That is $f_X(x)$). (10%)
   (ii) Find the PDF of $Y$ (That is $f_Y(y)$). (10%)
   (iii) Are $X$ and $Y$ independent of each other? (10%)
   (iv) Find $P(X + Y \le 1)$. (10%)

5. You are allowed to play a game four times independently, each time your score (in a discrete value) will take values from the range 1 to 5 (that is, 1, 2, 3, 4, or 5), with probability 0.2, uniformly. Your final score will be the minimum of the four scores and is modeled with a random variable $X$. Calculate the probability mass function (PMF) of $X$. (15%)

6. Consider a random variable $X$ with PDF

$$f_X(x) = \begin{cases} 2x/3, & \text{if } 1 < x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

   and let $A$ be the event $\{X \ge 1.8 \text{ or } X \le 1.2\}$. Calculate $E[X|A]$. (15%)