Part 1: Computing problems (50). Round off to the 4th decimal place if the answer is not an integer.

(a) Let $f(x)$ be a degree 3 polynomial through the points:
$f(1.00) = 0.1924$
$f(1.05) = 0.2414$
$f(1.10) = 0.2933$
$f(1.15) = 0.3492$
Calculate $f(1.09)$. **(8 pts)** 0.282635

(b) In Newton's divided difference method a polynomial for interpolating n points $\{x_1, x_2, \ldots, x_n\}$ can be written in this form:

$$P(x) = c_0 + c_1(x - x_1) + c_2(x - x_1)(x - x_2) + \cdots + c_{n-1}(x - x_1)\cdots(x - x_{n-1})$$

Find the coefficients c_i (i = 0 to 5) of the interpolating polynomial of the following 6 points. **(12 pts)**

x	-2	-1	0	1	2	3
$P(x)$	1	4	11	16	13	-4

c_i 1, 3, 2, -1, 0, 0
i 0, 1, 2, 3, 4, 5

(c) Apply Gram-Schmidt orthogonalization to find the reduced QR factorization of the matrix:

$$\begin{matrix} 2 & -1 & 0 & -1 \\ 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{matrix}$$

Report Q and R. **(20 pts)**

(d) Find the point (x, y) that minimizes the sum of squares distance to the following circles. **(10 pts)**
C1: center (1.8, 2.2), radius 1.2
C2: center (0.8, -2), radius 2.2
C3: center (-4.2, -2.1), radius 1.2
C4: center (-4, 2.5), radius 2.6

(1.312472, 0.126904)

Part 2: Written problems (50)

$$f(x)-P_9(x)=\frac{(x-x_1)(x-x_2)\cdots(x-x_9)}{9!}f^{(9)}(x)$$

Problem #1 (10). Assume that the polynomial $P_9(x)$ interpolates the function $f(x) = e^{-2x}$ at the 10 evenly spaced points $x = 0, 1/9, 2/9, 3/9, \ldots, 8/9, 1$. Find an upper bound for the error $|f(1/2) - P_9(1/2)|$.

≈ 0

$2.5964556 \times 10^{-10}$

8.83112×10^{-3}

$f'(x)=-2e^{-2x}$ $f^{(2)}(x)=4e^{-2x}$ $f^{(n)}(x)=(-2)^n e^{-2x}$ $f^{(9)}(x)=(-2)^9 e^{-2x}=-512e^{-2x}$

Problem #2 (20). We are given three data points as follows:

x	-1	0	1
y	1	-1	-1

Determine the interpolating polynomial of lowest degree possible using

(a) Lagrange's interpolation formula

(b) Newton's interpolation formula

x^2-x-1.

Problem #3 (20). The Gauss-Newton method for solving the nonlinear least squares problem is described as follows.

$$\boxed{\begin{aligned}&\underline{x}_0 = \text{initial vector}\\ &\underline{x}_{k+1} = \underline{x}_k - \left(Dr(\underline{x}_k)^T Dr(\underline{x}_k)\right)^{-1} Dr(\underline{x}_k)^T r(\underline{x}_k) \text{ for } k = 0,1,2,\ldots\end{aligned}}$$

Please find the matrix Dr needed for applying Gauss-Newton iteration to the model-fitting problem with three data points $(t_1, y_1), (t_2, y_2), (t_3, y_3)$.

(a) translated exponential $y = c_3 + c_1 e^{c_2 t}$

(b) power law $y = c_1 t^{c_2}$

$y=c_3+c_1e^{c_2x}$ $r(c)=c_3+c_1e^{c_2x}-y$

$(1+e^{x})'$ $\cancel{y}=1+e^{x}-y$

$$\begin{cases} y_1=c_3+c_1e^{c_2t_1}\\ y_2=c_3+c_1e^{c_2t_2}\\ y_3=c_3+c_1e^{c_3t_3}\end{cases}$$

$$\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}$$