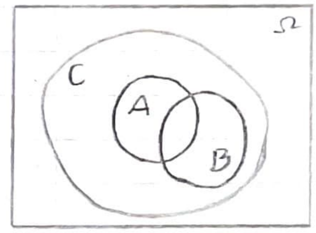
1. Given a Venn diagram shown on right. Prove that $\dfrac{P(A)}{P(B)} = \dfrac{P(A|C)}{P(B|C)}$

$$P(A|C) = \frac{P(A\cap C)}{P(C)}$$



$$P(B|C) = \frac{P(B\cap C)}{P(C)}$$

In the graph, we can know that $\begin{cases} P(A\cap C) = P(A) \\ P(B\cap C) = P(B) \end{cases}$

$$\frac{P(A|C)}{P(B|C)} = \frac{P(A)/P(C)}{P(B)/P(C)} = \frac{P(A)}{P(B)} \quad ※$$

2. Suppose that A, B, C are independent.
Use the definition of independence to show that A and $B\cup C$ are independent

$P(A\cap(B\cup C)) = P(A)P(B\cup C)$ if A, $B\cup C$ are independent

$$\begin{aligned} P(A\cap(B\cup C)) &= P((A\cap B)\cup(A\cap C)) \\ &= P(A\cap B) + P(A\cap C) - P((A\cap B)\cap(A\cap C)) \\ &= P(A\cap B) + P(A\cap C) - P(A\cap B\cap C) \end{aligned}$$

A, B, C independent

$$\begin{aligned} &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)\left[P(B) + P(C) - P(B)P(C)\right] = P(A)P(B\cup C) \quad ※ \end{aligned}$$

3. Given that $P(A|D) > P(B|D)$ and $P(A|D^c) > P(B|D^c)$
where A, B, D are 3 event and D^c is the complement of D

Show that $P(A) > P(B)$

According to Total Probability Theorem

$$P(A) = P(A|D) + P(A|D^c)$$

$$P(B) = P(B|D) + P(B|D^c)$$

$\because$ $P(A|D) > P(B|D)$
$P(A|D^c) > P(B|D^c)$ $\quad\therefore P(A) > P(B)$ ※

4. We roll fair 4-sided dice. Each of the 16 possible outcomes is assumed to be equally likely.
(a) Find the probability that at least one die is 4.

(4.1) (4.2) (4.3) (4.4) (3.4) (2.4) (1.4) $\Rightarrow \frac{7}{16}$ #

(b) Given that the two dies land on different numbers,
find the conditional probability that at least one die is a 3.

A : { two different numbers } $= 12/16 = 3/4$
B : { there's a 3 exist } $= 7/16$

$\Rightarrow P(A \cap B) = \frac{6}{16} = \frac{3}{8}$

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/8}{12/16} = \frac{1}{2}$$ ※

5. A circuit contains components that come from one of three manufactures : I II and III
When components are ordered, 40 % are ordered from I, 40% from II, 20 % from III
A component from I, II, III respectively has a probability of 0.005, 0.01, 0.05 of being defective.
If we found a component defective, what is the probability that it came from II ?

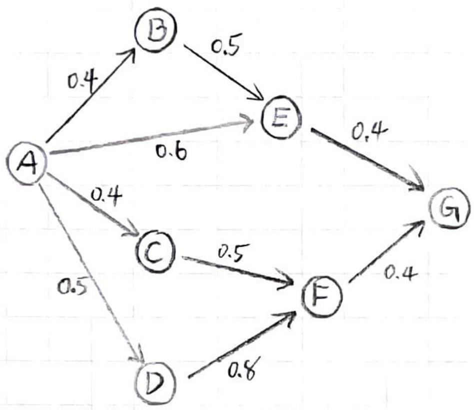
A : { having a defective component }
B : { having defective and its from II }

$$P(A) = 0.4 \times 0.005 + 0.4 \times 0.01 + 0.2 \times 0.05$$
$$= 0.002 + 0.004 + 0.01$$
$$= 0.016$$

$$P(B) = 0.4 \times 0.01 = 0.004$$

$$\Rightarrow \frac{P(B)}{P(A)} = \frac{0.004}{0.016} = \frac{1}{4}$$ ※

6 Given a computer network shown under.
where the number associated with each link is the probability of being "up",
and the probability that a link is "failed" is independent of each other,
Find the probability that there is a path connecting A and G.



分段討論

$$\begin{cases} P(A \to B \to E \text{ is up}) = 0.4 \times 0.5 = 0.2 \\ P(A \to E \text{ is up}) = 0.6 \end{cases}$$

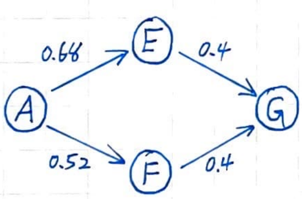
$\Rightarrow$ all route from A to E is up : $1 - [(1-0.2)(1-0.6)] = 1 - 0.32 = 0.68$

$$\begin{cases} P(A \to C \to F \text{ is up}) = 0.4 \times 0.5 = 0.2 \\ P(A \to D \to F \text{ is up}) = 0.5 \times 0.8 = 0.4 \end{cases}$$

$\Rightarrow$ all route from A to F is up : $1 - [(1-0.2)(1-0.4)] = 1 - 0.48 = 0.52$

$\Downarrow$

(simplify the graph)



$$\begin{cases} P(A \to E \to G) = 0.68 \times 0.4 = 0.272 \\ P(A \to F \to G) = 0.52 \times 0.4 = 0.208 \end{cases}$$

$\Rightarrow$ Probability of A to G is up :

$$[1 - (1-0.272)(1-0.208)] = 1 - (0.728)(0.792)$$

$$\fallingdotseq 1 - 0.576$$

$$\fallingdotseq 0.424 \quad \fallingdotseq 0.42 \;\#$$

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7. Prof. Berlin comes from a family of two children.
What is the probability that the other child is his sister?

(conditional probability)

$$\frac{P(\text{a boy and a girl})}{P(\text{a boy})} = \frac{2}{3} \quad \left(\frac{\{\text{Male Female}\}\ \{\text{Female Male}\}}{\{\text{Male Female}\}\ \{\text{Male Male}\}\ \{\text{Female Male}\}}\right)$$