Part 1: Computing problems (50). Round off to the 4th decimal place if the answer is not an integer.

(a) Please find the least squares solution of *k* of the following equations. **(10 pts)**

$$\begin{bmatrix} 4 \\ 7 \\ 11 \end{bmatrix} k = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$$

4*k* = 3
7*k* = 5
11*k* = 8

$x^4+3x^3-33x^2+36x-23$
$4x^3+9x^2-66x+36$
$x^3-18x^2+36x-24$

(b) Given *f*(8.1) = 16.94410, *f*(8.3) = 17.56492, *f*(8.6) = 18.50515, *f*(8.7) = 18.82091, find the interpolating polynomial. Please report *f*(8.4). **(10 pts)**

(c) Please find x that minimizes $f(x) = x^4 + 3(x-2)^3 - 15x^2 + 1$ **(10 pts)**

2.66886 -5.5286

(d) Apply Gram-Schmidt orthogonalization to find the reduced QR factorization of the matrix:

$$\begin{bmatrix} 4 & 2 & 3 & 0 \\ -2 & 3 & -1 & 1 \\ 1 & 3 & -4 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & -2 \end{bmatrix}$$

Report *Q* and *R*. **(20 pts)**

$(x^2-4x+4)(x-2)$
$x^3-4x^2+4x-2x^2+8x-8$
$3x^3-6x^2+12x-8$
$3x^3-18x^2+36x-24$

$x^4+3x^3-18x^2+36x-24-15x^2+1$
$x^4+3x^3-33x^2+36x-23$
$4x^3+9x^2-66x+36$

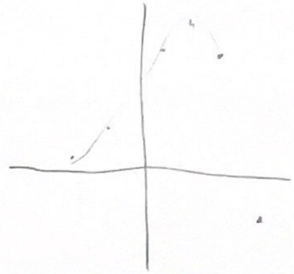
2.6688

Part 2: Written problems (50)

Problem #1 (10). Let *P*(x) be the interpolating polynomial for the data (0, 0), (0.5, *y*), (1, 3), and (2, 2). The coefficient of x^3 in *P*(x) is 6. Find *y*.

Problem #2 (10). What is the highest degree of the polynomial interpolating the following data?

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4



Problem #3 (10). Suppose we are given a set of n data points (t_1, y_1), ... (t_n, y_n). We choose the power law model $y = c_1 t^{c_2}$ to fit the data points. Below are the steps to compute c_1, c_2 and the least squares solution $\tilde{y}_i$ using linearization. Fill the '?' parts in steps 3 and 4.

1. Apply the natural logarithm to the model: $\ln y = \ln(c_1 t^{c_2}) = \ln c_1 + c_2 \ln t$
2. Let $k = \ln c_1$, and therefore, $\ln y = k + c_2 \ln t$.
3. Construct a matric $\mathrm{A} = [?]$ **(5 pts)**
4. Construct a vector $b = [?]$ **(5 pts)**
5. Let $x = \begin{bmatrix} k \\ c_2 \end{bmatrix}$. Solve the system of normal equations $A^T A x = A^T b$.

   Compute $c_1 = e^k$.
6. Compute the estimated $\tilde{y}_i = c_1 t_i{}^{c_2}$.

Problem #4 (20). The three-part problem is stated as follows: Divide 10 into three parts such that they shall be in continued proportion to each other and the product of the first and the last two shall be 6. Taking x, y, and z as three parts, this problem can be represented as a system as follows:

$$x + y + z = 10$$
$$x/y = y/z$$
$$xy = 6$$
$$yz = 6$$

$\frac{x}{y} = \frac{y}{z}$ $\frac{xz - y^2}{yz}$

$xz - y^2 = 0$

$y \times z$

It can be solved by the Gauss-Newton method discussed in the class. Please write down the vector $r()$ and the Jacobian matrix $D_r()$ for applying the Gauss-Newton method to find the least squares solution of the three-part problem.