Hw 2021-5

1. Find the eigenvalues of linear operator T, determine a basis for each eigenspace,

where $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -4x_1 + 6x_2 \\ 2x_2 \\ -5x_1 + 5x_2 + x_3 \end{bmatrix}$

$$\det(A - \lambda I_3) = \det \begin{bmatrix} -4-\lambda & 6 & 0 \\ 0 & 2-\lambda & 0 \\ -5 & 5 & 1-\lambda \end{bmatrix} = -(\lambda+4)(\lambda-2)(\lambda-1)$$

eigenvalue 1, 2, -4

$$\text{(i)}\ A - I_3 = \begin{bmatrix} -5 & 6 & 0 \\ 0 & 1 & 0 \\ -5 & 5 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{(ii)}\ A - 2I_3 = \begin{bmatrix} -6 & 6 & 0 \\ 0 & 0 & 0 \\ -5 & 5 & -1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{(iii)}\ A + 4I_3 = \begin{bmatrix} 0 & 6 & 0 \\ 0 & 6 & 0 \\ -5 & 5 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Ans: $1, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $2, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$; $-4, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ ※