Part 1: Computing problems (55)

Please use your codes to solve the following mathematical problems. If the answer is not an integer, please round off to the **4th** decimal place.

(a) $x^3 + 4x^2 - 10 = 0 \quad$ for $1 \leq x \leq 2$ **(5)**

(b) $3.06 = \frac{(1-x)(3+x)}{x(1+x)}$ **(10)**

(c) Solve the following linear system. **(10)**

$a_{ii} = 3$

$a_{ij} = -1 \text{ if } |i - j| = 1$

$a_{ij} = 0 \text{ if } |i - j| > 1 \text{ where } i, j = 1, 2, \ldots 10$

$\underline{b}^T = [2\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 2]$

(d) Find the PA=LU factorization of the system and solve $\underline{x}$. **(20)**

$$
\begin{aligned}
1.19x_1 + \ 2.11x_2 - \ 100x_3 + x_4 &= 1.12, \\
14.2x_1 - 0.122x_2 + 12.2x_3 - x_4 &= 3.44, \\
100x_2 - 99.9x_3 + x_4 &= 2.15, \\
15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 &= 4.16.
\end{aligned}
$$

(e) Find a solution to the following nonlinear system:

$$
\begin{gathered}
\sin x + y^2 + \log_e z = 7 \\
3x + 2y - z^3 = -1 \\
x + y + z = 5
\end{gathered}
$$

Use the initial guess $[0, 2, 2]^T$. **(10)**

Part2: Written problems (45)

Problem #1 (5). Nested multiplication (or Horner's method) is an efficient method for evaluating polynomials. Please write down the nested form of the following polynomial:

$$f(x) = x^{14} - 4x^{11} + 5x^{8} + 3x^{5}$$

Problem #2 (10). Consider the equation $x^3 + x - 2 = 0$, with root $r = 1$. Add the term cx to both sides and divide by c to obtain $g(x)$.

(a) For what condition of c would FPI be linearly convergent to $r = 1$? **(5)**

(b) For what condition of c would FPI be faster than bisection? **(5)**

Problem #3 (10). A *permutation matrix* is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere. It can be used to reorder the rows of another matrix.

(a) How many permutation matrices of size 10×10 exist? **(5)**

(b) Please show how do we use a permutation matrix to make the following matrix strictly diagonally dominant. **(5)**

$$\boldsymbol{A} = \begin{bmatrix} 1 & -8 & -2 \\ 1 & 1 & 5 \\ 3 & -1 & 1 \end{bmatrix}$$

[Please turn over for Problem #4.]

Problem #4 (20). Given three circles with centers (x_1, y_1), (x_2, y_2), (x_3, y_3) and radii R_1, R_2 and R_3. We expand the radii of each circle ***by the same amount*** k to get an intersection point (the black dot in the figure). The goal is to compute the coordinate of the intersection point and k.

This problem can be addressed by formulating a nonlinear system and use multivariate Newton's method to solve it. Please write down the $F(.)$ and $D_F(.)$ needed for applying this method.

$$\boxed{\begin{array}{l} \underline{x}_0 = \text{initial vector} \\ \text{solve } D_F(\underline{x}_k)\underline{s} = -F(\underline{x}_k) \\ \underline{x}_{k+1} = \underline{x}_k + \underline{s} \quad \text{for } k = 0,1,2,\ldots \end{array}}$$