國立臺灣師範大學　　學年度第　　學期考試試卷　　年　月　日

得分：　科目：　系級：資工112　學號：40847014S　姓名：[illegible]

33

Part 1:

73

(a) 1.829384 ≈ 1.8294　Ans: 1.8294 ✓

15 (b) 2.690647 ≈ 2.6906　Ans: 2.6906 ✓

(c) 1.412395 ≈ 1.4124　Ans: 1.4124 ✓

0 (d)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2.0891 \\ -4.7224 \\ -3.6338 \\ -1.5185 \end{bmatrix}$$ ✗

(e)
$$x^{(1)} = [0.3333, 0.0, 0.5714]^T$$ ✓

10
$$x^{(2)} = [0.1429, -0.3571, 0.4286]^T$$ ✓

$$\text{ans} = [0.0351, -0.2368, 0.6579]^T$$ ✓

8 (f)
$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$PA = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \neq LU = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & 2 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

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~~Part 2~~ : 15

$\left(\frac{21}{x}\right)' = (21x^{-1})' = -21x^{-2}$　　$(x^{-1})' = -x^{-2}$

$r \approx 1.\sim$

Problem #1

1. $g(x)' = \left(\frac{20}{21}x + \left(\frac{1}{x}\right)^2\right)' = \frac{20}{21} + 2\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) = \frac{20}{21} - \frac{2}{x^3}$　　$|g(r)'| = \frac{18}{21}$ ✓

2. $g(x)' = \left(x - \frac{x^3-21}{3x^2}\right)' = 1 - \frac{(3x^2)(3x^2)-(x^3-21)(6x)}{6x^4} = 1 - \frac{21 \times 6x}{6x^4} = 1 - \frac{21}{x^3}$

$|g(r)'| = 0$ ✓

Ans: 3>4>1>2 ~~X~~

3. $g(x)' = \left(x - \frac{x^4-21x}{x^3-21}\right)' =$?

4. $g(x)' = \left(\left(\frac{21}{x}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(\frac{21}{x}\right)^{-\frac{1}{2}}\left(\frac{-21}{x^2}\right) = \frac{1}{2}\sqrt{\frac{x}{21}} \times \frac{-21}{x^2} = \frac{1}{2} \times \frac{-21}{\sqrt{21}} \times \frac{x^{\frac{1}{2}}}{x^2} = \frac{-21}{2\sqrt{21}} \times x^{-\frac{3}{2}} = \frac{-21}{2\sqrt{21}} \times \sqrt{\frac{1}{x^3}}$

$|g(r)'| = \frac{21}{2\sqrt{21}} \times \frac{1}{\sqrt{21}} = \frac{1}{2}$ ✓

Problem #2

5

$$\begin{bmatrix} 10 & 20 & 1 \\ 1 & 1.99 & 6 \\ 0 & 50 & 1 \end{bmatrix} \xrightarrow{\frac{1}{10}} \begin{bmatrix} 10 & 20 & 1 \\ 0 & -0.01 & 5.9 \\ 0 & 50 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & 20 & 1 \\ 0 & 50 & 1 \\ 0 & -0.01 & 5.9 \end{bmatrix}$$

?

$$\xrightarrow{\frac{0.01}{50}} \begin{bmatrix} 10 & 20 & 1 \\ 0 & 50 & 1 \\ 0 & 0 & 5.8998 \end{bmatrix}$$

$\frac{1}{10} > \frac{0.01}{50} = 0.0002$

Ans: $\frac{1}{10}$ ~~X~~

4

Problem #3

10

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 3 & 4 & 5 & 6 & 0 & 1 & 0 & 0 \\ 5 & 6 & 7 & 8 & 0 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|cccc} 5 & 6 & 7 & 8 & 0 & 0 & 1 & 0 \\ 3 & 4 & 5 & 6 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 7 & 8 & 9 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|cccc} 5 & 6 & 7 & 8 & 0 & 0 & 1 & 0 \\ 3 & 4 & 5 & 6 & 0 & 1 & 0 & 0 \\ 7 & 8 & 9 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 & 1 & 0 & 0 & 0 \end{array}\right]$$

Ans: $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ ✓

10

Problem #4

(a) $\begin{cases} u+4v=5 \\ v+2w=2 \\ 4u+3w=0 \end{cases}$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 2 \\ 4 & 0 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 0 & 3 \\ 0 & 1 & 2 \\ 1 & 4 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 0 & 3 \\ 1 & 4 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$ ✓

Ans: impossible, since the 4 & 3 cannot be in different rows. ✓

(b) $\begin{cases} u-8v-2w=1 \\ u+v+5w=4 \\ 3u-v+w=-2 \end{cases}$

$$\Rightarrow \begin{array}{c} \begin{matrix} u & v & w \end{matrix} \\ \begin{bmatrix} 1 & -8 & -2 \\ 1 & 1 & 5 \\ 3 & -1 & 1 \end{bmatrix} \end{array} \Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 1 & 1 & 5 \\ 1 & -8 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 & 1 \\ 1 & -8 & -2 \\ 1 & 1 & 5 \end{bmatrix}$$

Ans: $\begin{bmatrix} 3 & -1 & 1 \\ 1 & -8 & -2 \\ 1 & 1 & 5 \end{bmatrix}$ ✓