Introduction to Probability

Quiz 1

October 4, 2022, 15:30 a.m. - 17:20 a.m.

Note: You have to answer the questions with supporting explanations if needed.

1. (a) State and explain the three probability axioms. (5%)

   (b) Given the three probability axioms and the definition of conditional probability are known in advance, and let $A_1, \cdots, A_n, \cdots, A_N$ be disjoint events that form a partition of the sample space and assume that $\mathbf{P}(A_n) > 0$, for all n. Then, for any event B, show that: $\mathbf{P}(B) = \sum_{i=1}^{N} \mathbf{P}(A_i)\mathbf{P}(B|A_i)$. (5%)

   (c) Given two events A and B, where $0 < \mathbf{P}(B) < 1$ and the complement of B is denoted by B^c.

   Show that if $\mathbf{P}(A|B) > \mathbf{P}(A|B^c)$ then $\mathbf{P}(A \cap B) > \mathbf{P}(A)\mathbf{P}(B)$. (10%)

2. Show that if A and B be independent events:

   (a) Show that A and B^c are independent. (10%)

   (a) Based on the proof of 2(a), show that A^c and B^c are also independent. (10%)

   Hint: $\mathbf{P}(A) = \mathbf{P}(A \cap B) + \mathbf{P}(A \cap B^c)$.

3. A team ASPIRATION has 5 members, each of them can independently accomplish a mission U-Win with probability 1/4. The team ~~PASSION~~ ASPIRATION is successful whenever at least one of its members accomplished the mission U-Win. Find the conditionally probability that exactly 3 members have accomplished the mission U-Win, given that the team ASPIRATION is successful. (10%)

4. Three friends, D, E, F, enter a round-robin tournament (i.e., only three persons in the tournament) in which each person plays every other person one time, no ties are allowed in a game. Assume $\mathbf{P}(D \text{ beats } E) = 0.3$, $\mathbf{P}(E \text{ beats } F) = 0.4$, and $\mathbf{P}(F \text{ beats } D) = 0.6$. Assume here that the outcomes of all the games are independent of one another.

   (a) For the probability that E wins the tournament. (10%)

   (b) For the probability that no one ~~wills~~ wins the tournament. (10%)

5. A discrete random variable Y has the range $R_y = \{1, 2, 3, 4, 5\}$.

   Given that $p_Y(y) = \frac{1}{y} p_Y(1)$ for $y = 2, 3, 4, 5$, Find $p_Y(y)$ and $\mathbf{E}[Y]$. (20%)

6. Two fair 6-sided dices (denote by $D1$ and $D2$, respectively) are rolled together one time. Let Z be the result of the number of $D1$ minus that of $D2$.

   (a) For the PMF of Z (10%)

   (b) For the PMF of X, $X = |Z|$. (10%)