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Part 1.

87

(a) $0.282635 \approx 0.2826$ ✓

(b) $C_0=1 \quad C_1=3 \quad C_2=2 \quad C_3=1 \quad C_4=0 \quad C_5=0$ ✓

(c)

$$Q=\begin{bmatrix} 0.5547 & -0.438529 & -0.597614 & -0.377964 \\ 0.5547 & 0.701646 & 0.239046 & -0.377964 \\ 0 & 0 & 0 & 0 \\ 0.5547 & -0.438529 & 0.597614 & 0.377964 \\ 0.27735 & 0.350823 & -0.478091 & 0.755929 \end{bmatrix}$$

$$R=\begin{bmatrix} 3.605551 & 0.277350 & 1.1094 & 0.27735 \\ 0 & 2.631174 & 0.263117 & 0.350823 \\ 0 & 0 & 0.83666 & 0.717137 \\ 0 & 0 & 0 & 1.511858 \end{bmatrix}$$

✓

(d) $(x,y)=(-1.312472, 0.126904)$ ✓

Part 2.

5

#1.

$f(x)=e^{-2x},\ f'(x)=-2e^{-2x},\ f^{(2)}(x)=4e^{-2x} \Rightarrow f^{(n)}(x)=(-2)^n e^{-2x}$

$f^{(9)}(x)=(-2)^9 e^{-2x}=-512e^{-2x}.$

$$f(x)-P_9(x)=\frac{(x-x_1)(x-x_2)\cdots\cdots(x-x_9)}{9!\ \ 10!}f^{(9)}(x)=\frac{\left(\frac{1}{2}\times\frac{7}{18}\times\frac{5}{18}\times\frac{3}{18}\times\frac{1}{18}\times\frac{-1}{18}\times\frac{-3}{18}\times\frac{-5}{18}\times\frac{-7}{18}\right)}{362880}\times(-512\cdot e^{-2x})$$

$$\approx -7.0578998\times10^{-10\ 1}\times e^{-2x}$$

$$\left|f\left(\tfrac{1}{2}\right)-P_9\left(\tfrac{1}{2}\right)\right|=7.0578998\times10^{-10}\times e^{-1}\approx 7.057899\times10^{-10}\times0.36\approx 0$$

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#2

(a)

$$P_3(x)=1\times\frac{(x-0)(x-1)}{(-1-0)(-1-1)}-1\times\frac{(x+1)(x-1)}{(0+1)(0-1)}-1\times\frac{(x+1)(x-0)}{(1+1)(1-0)}$$

$$=\frac{1}{2}x(x-1)+(x+1)(x-1)-\frac{1}{2}x(x+1)$$

$$=-x+x^2-1=x^2-x-1$$ ✓

(b)

$$P_3(x)=f[x_1]+f[x_1\ x_2](x-x_1)+f[x_1\ x_2\ x_3](x-x_1)(x-x_2)$$

$$=1+(x+1)\times\frac{-1-1}{0+1}+(x+1)(x-0)\times\frac{\frac{-1+1}{1-0}-\frac{-1-1}{0+1}}{1+1}$$

$$=1-2(x+1)+x(x+1)$$

$$=x^2-x-1$$ ✓

$\frac{0+2}{2}$

$1-2x-2+x^2+x$

#3 (12)

(a) (7)

$$W_r = \begin{bmatrix} e^{c_2 t_1} & c_1 t_1 e^{c_2 t_1} & 1 \\ e^{c_2 t_2} & c_1 t_2 e^{c_2 t_2} & 1 \\ e^{c_2 t_3} & c_1 t_3 e^{c_2 t_3} & 1 \end{bmatrix} \#\#$$

(b) (5)

$$W_r = \begin{bmatrix} t_1^{c_2} & c_1 t_1^{c_2} \ln t_1 \\ t_2^{c_2} & c_1 t_2^{c_2} \ln t_2 \\ t_3^{c_2} & c_1 t_3^{c_2} \ln t_3 \end{bmatrix} \#\#$$