## Introduction to Probability Midterm II

15:30-17:20, December 6, 2022

Note: You have to answer the questions with supporting explanations if needed.

1. A continuous random variable X has a probability density function (PDF) given below

$$f_X(x) = \begin{cases} a + bx, & \text{if } 0 \le x \le 2\\ 0, & \text{otherwise.} \end{cases}$$

If  $E[X] = \frac{10}{9}$ , find a and b. (20%)

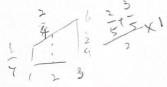
2. We are told that X is a normal distribution with mean 4 and variance 9. Given that a few CDF values of a standard normal are given as follows:

$$\Phi(0) = 0.5$$
,  $\Phi(0.5) = 0.6915$ ,  $\Phi(1.0) = 0.8413$ ,  $\Phi(1.5) = 0.9332$ ,  $\Phi(2.0) = 0.9772$  and  $\Phi(2.5) = 0.9938$ .

- (i) Find the probability  $P(1 \le X \le 10)$ . (10%)
- (ii) If Y = 2X + 1, Find the probability  $P(Y \le 18)$ . (10%)
- 3. You are allowed to play a game four times independently, each time your score (in a discrete value) will take values from the range 1 to 5 (that is, 1, 2, 3, 4, or 5), with probability 0.2, uniformly. Your final score will be the maximum (最大值) of the four scores and is modeled with a random variable X. Calculate the probability mass function (PMF) of X. (20%)
- 4. Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} x/4, & \text{if } 1 \le x \le 3\\ 0, & \text{otherwise.} \end{cases}$$

and Let A be the event  $\{X \ge 2\}$ . Find  $\mathbf{E}[X]$ ,  $\mathbf{P}(A)$ ,  $f_{X|A}(x)$ , and  $\mathbf{E}[X|A]$ . (20%)



5. Allen, a French chef in Taipei, has good days and bad days with equal frequency. On a good day, the time (in hours) it takes Allen to cook a duck breast (鴨胸肉) is described by the PDF of a continuous random variable G; but on a bad day, the time it takes is described by the PDF of a continuous random variable B:

$$f_G(g) = \begin{cases} 2, & \text{if } 1/2 \le g \le 1 \\ 0, & \text{otherwise,} \end{cases} \qquad f_B(b) = \begin{cases} 1, & \text{if } 1/2 \le b \le 3/2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional probability that today was a bad day, given that it took Allen less than 3/4 hour to cook a duck breast. (20%)

Hint: You can apply Bayes' rule for solving this problem.

- 6. Given two continuous random variables X and Y, each of which takes values between 0 and 1, with the joint PDF indicated below (the cutoff between probability density 0.8 and 1.6 occurs at X = 0.5 and Y = 0.5):
  - (i) Find the PDF of X. (5%)
  - (ii) Find the PDF of Y. (5%)
  - (iii) Find the conditional PDF  $f_{X|Y}(x|0.6)$ . (5%)
  - (iv) Are X and Y independent of each other? (5%)

