NATIONAL TAIWAN NORMAL UNIVERSITY
Department of Computer Science and Information Engineering

Numerical Methods

Midterm Examination
Friday 11/06/2020

Instructions:

- This exam contains two parts: computing problems and written problems. You have 100 minutes.
- This exam is closed book. No hand-held devices are permitted.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it.
- **Good luck!**

Part 1: Computing problems (50)

Please use your codes to solve the following mathematical problems. If the answer is not an integer, please round off to the **4th** decimal place.

(a) $e^x + 2^{-x} + 2\cos x - 6 = 0$ for $1 \le x \le 2$ **(5)**

(b) $x^3 - 2x^2 - 5 = 0$ for $1 \le x \le 4$ **(5)**

(c) $x^2 - 4x + 4 - \ln x = 0$ for $1 \le x \le 2$ **(5)**

(d) Solve the following linear system. **(10)**

$$\pi x_1 + \sqrt{2}x_2 - x_3 + x_4 = 0$$

$$ex_1 - x_2 + x_3 + 2x_4 = 1$$

$$x_1 + x_2 - \sqrt{3}x_3 + x_4 = 2$$

$$-x_1 - x_2 + x_3 - \sqrt{5}x_4 = 3$$

(e) Find the first two iterations, as well as the solution, of the Jacobi method for the following linear system, using $x^{(0)} = [0\ 0\ 0]^T$. **(10)**

$$3x_1 - x_2 + x_3 = 1$$

$$3x_1 + 6x_2 + 2x_3 = 0$$

$$3x_1 + 3x_2 + 7x_3 = 4$$

(f) Return the *P*, *L* and *U* of the *PA* = *LU* factorization of the matric: **(15)**

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

Part2: Written problems (50)

Problem #1 (20). The following four fixed-point iteration methods are proposed to compute $21^{1/3}$. Write down their speed of convergence and rank them in order based on the convergence speed, assuming $x_0 = 1$.

1. $g(x) = \frac{20x+(21/x^2)}{21}$ 2. $g(x) = x - \frac{x^3-21}{3x^2}$

3. $g(x) = x - \frac{x^4-21x}{x^2-21}$ 4. $g(x) = \left(\frac{21}{x}\right)^{1/2}$

Problem #2 (10).

In the process of finding LU factorization of $A = \begin{bmatrix} 10 & 20 & 1 \\ 1 & 1.99 & 6 \\ 0 & 50 & 1 \end{bmatrix}$, what is the largest magnitude multiplier l_{ij} needed?

Problem #3 (10). Please change four entries of the left most matrix to make the matrix equation correct:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Problem #4 (10). For each system, rearrange the equations to form a strictly diagonally dominant system and write the rearranged system in matrix form.

(a)
$$\begin{aligned} u + 4v &= 5 \\ v + 2w &= 2 \\ 4u + 3w &= 0 \end{aligned}$$

(b)
$$\begin{aligned} u - 8v - 2w &= 1 \\ u + v + 5w &= 4 \\ 3u - v + w &= -2 \end{aligned}$$