

Introduction to Probability

Quiz 2

October 21, 2022, 9:10 a.m. - 10:00 a.m.

Note: You have to answer the questions with supporting explanations if needed.

- Two random variables X and Y whose joint probability mass function (PMF) is given in the right figure, and a new random variable Z is defined by $Z = X + 2Y$.

- (1) Calculate $E[X]$, $E[Y]$ and $E[Z]$ (30%)
- (2) Determine whether X and Y are independent of each other or not. (15%)
- (3) Given an event $A = \{X \geq 2, Y \leq 2\}$ occurs, determine whether X and Y are independent of each other or not. (15%)

Joint PMF $p_{X,Y}(x,y)$

	1	2	3	
4	2/12	0	0	2
3	1/12	2/12	0	3
2	1/12	1/12	2/12	4
1	0	1/12	2/12	5
	1	2	3	

- Given that X is a random variable with PMF $p_X(x)$ and $g(X)$ is a function of X .

- (1) Show that the expectation of the random variable $g(X)$ can be expressed by

$$E[g(X)] = \sum_x g(x)p_X(x). \quad (15\%)$$

- (2) Given the variance of a random variable X is defined by $\text{var}(X) = E[(X - E[X])^2]$ and the expression shown in Problem 2 (1), show that

$$\text{var}(X) = E[X^2] - (E[X])^2. \quad (15\%)$$

- Given that X is a random variable, which has $E[X] = 5$ and $\text{var}(X) = 4$. Compute $E[(2X + 1)(X + 3)]$. (10%)

- An explorer is trapped in an abandoned mine (廢棄礦坑) containing 3 doors. The first door leads to a tunnel that will take him to safety after 2 hours of travel. The second door leads him to a tunnel that will return him to the mine after 3 hours of travel. The third door leads him to a tunnel that will return him to the mine after 4 hours of travel. If we assume that each door is equally likely to be chosen by the explorer, what is the expected length of time (in hours) until the explorer reaches safety? (20%)

8
50
55
3

$$3X + 2Y + 2$$

$$E[X] = \sum_x x p_X(x)$$

$$\binom{7+1}{x} \left(\frac{1}{3}\right)^x \left(\frac{1}{3}\right)^{7-x} \frac{1}{3}$$

$$\sum_x 2x E[X] p_X(x)$$

$$= 2 E[X] \sum_x x p_X(x)$$

$$= 2 E[X] E[X]$$

$$8 + 50 + 35 + 3$$

$$96$$