

$$20x = \frac{21}{x^2} \quad x^3 = \frac{21}{20} \quad x = \sqrt[3]{\frac{21}{20}}$$

## Part2: Written problems (50)

$$x^3 = 21$$

$$\frac{20}{21}x + \frac{1}{x^2}$$

$$\sqrt[3]{21}$$

**Problem #1 (20).** The following four fixed-point iteration methods are proposed to compute  $21^{1/3}$ . Write down their speed of convergence and rank them in order based on the convergence speed, assuming  $x_0 = 1$ .

$$\frac{20}{21}x + \left(\frac{21}{x^2}\right)^2$$

$$1. \quad g(x) = \frac{20x + (21/x^2)}{21}$$

$$2. \quad g(x) = x - \frac{x^3 - 21}{3x^2}$$

$$20 - 20 = 18$$

$$x^3 - (3\sqrt[3]{21})^3$$

$$3. \quad g(x) = x - \frac{x^4 - 21x}{x^2 - 21}$$

$$4. \quad g(x) = \left(\frac{21}{x}\right)^{1/2}$$

$$6x^4 - 6x^4 + 21x$$

**Problem #2 (10).**

$$\frac{x(x^3 - 21)}{x^2 - 21}$$

$$-4x^5 + 84x^3 + 21x^2 - 44$$

$$-4x^5 + 84x^3 + 21x^2 - 44$$

$$-(-2x^5 + 42x^2)$$

In the process of finding LU factorization of  $A = \begin{bmatrix} 10 & 20 & 1 \\ 1 & 1.99 & 6 \\ 0 & 50 & 1 \end{bmatrix}$ , what is

the largest magnitude multiplier  $l_{ij}$  needed?

$$\frac{0.01}{50} = \frac{0.02}{100}$$

$$50x = 0.01 \quad x = \frac{0.01}{50} = \frac{0.01}{50}$$

$$\frac{1}{10}$$

$$\begin{bmatrix} 10 & 20 & 1 \\ 0 & 50 & 1 \\ 0 & 50 & 1 \end{bmatrix}$$

**Problem #3 (10).** Please change four entries of the left most matrix to make the matrix equation correct:

$$1 \ 0 \ 0 \ 0$$

$$2x^3 - 42x + 21$$

$$1 + 0 - 21 \quad \frac{2+0-42+21}{2+0-84+21+0+44}$$

$$4x^5 - 21x^4 + 21x^2 - 2x^5 + 21x^2$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

**Problem #4 (10).** For each system, rearrange the equations to form a strictly diagonally dominant system and write the rearranged system in matrix form.

$$(a) \quad \begin{aligned} u + 4v &= 5 \\ v + 2w &= 2 \\ 4u + 3w &= 0 \end{aligned}$$

$$\begin{aligned} u + 4v + 0w &= 5 \\ 0u + v + 2w &= 2 \\ 4u + 0v + 3w &= 0 \end{aligned}$$

$$(b) \quad \begin{aligned} u - 8v - 2w &= 1 \\ u + v + 5w &= 4 \\ 3u - v + w &= -2 \end{aligned}$$

$$\begin{bmatrix} 4 & 0 & 3 \\ 0 & 4 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} &2+0-42 \\ &1+0-21 \quad \frac{2+0-84+0+0+21}{2+0-42} \\ &+0-42+0+0 \\ &-42+0+44 \end{aligned}$$