1 (a) State and explain the three axioms

(a) Nonnegativety

(b) Addictivity

(c) Normalization

(b) Verify the following multiplication rule using the definitions of conditional probability

$$P(\bigcap_{i=1}^{n} A_i) = P(A_1) P(A_2|A_1) P(A_3|A_2 \cap A_1) \cdots P(A_n | \bigcap_{i=1}^{n-1} A_i)$$

$$= \cancel{P(A_1)} \cdot \frac{\cancel{P(A_1 \cap A_2)}}{\cancel{P(A_1)}} \cdot \frac{\cancel{P(A_3 \cap A_2 \cap A_1)}}{\cancel{P(A_1 \cap A_2)}} \cdots \frac{P(A_n \cap \cdots \cap A_2 \cap A_1)}{P(A_{n-1} \cap \cancel{A_{n-2}} \cap \cdots A_2 \cap A_1)}$$

$$= P(A_n \cap A_{n-1} \cap \cdots \cap A_2 \cap A_1) \ \#$$

(c) Given 2 events A, B, where $0 < P(B) < 1$
and the complement of B denoted by B^c
Show that if $P(A|B) > P(A|B^c)$ then $P(A \cap B) > P(A) P(B)$

$$\Rightarrow P(A|B) > P(A|B^c) \Rightarrow \frac{P(A \cap B)}{P(B)} > \frac{P(A \cap B^c)}{P(B^c)}$$

$$\Rightarrow P(A \cap B) P(B^c) > P(A \cap B^c) P(B)$$

$$\Rightarrow P(A \cap B)(1 - P(B)) > (P(A) - P(A \cap B)) P(B)$$

$$\Rightarrow P(A \cap B) - P(B) P(A \cap B) > P(A) P(B) - P(B) P(A \cap B) \Rightarrow P(A \cap B) > P(A) P(B) \ \#$$

2. Show that if A, B be independent events

(a) show that A, B^c are independent

$\Rightarrow$ We can be sure that $P(A \cap B) = P(A)P(B)$

$\Rightarrow P(A \cap B^c) = P(A) - P(A \cap B) = P(A) - P(A)P(B) = P(A)[1-P(B)] = P(A)P(B^c)$ #

(b) Based on the proof of 2(a), show that A^c, B^c are also independent

$\Rightarrow$ According to Total Probability Theorem

$P(B^c) = P(A \cap B^c) + P(A^c \cap B^c)$

$\Rightarrow P(A^c \cap B^c) = P(B^c) - P(A)P(B^c) = P(B^c)[1-P(A)] = P(B^c)P(A^c)$ #

3. We roll two fair 4-sided dices. Each of 16 outcomes is assumed equally likely.

(a) Given that the two dice land on different numbers,
find the conditional probability that at least one die is a 2.

A : { two different numbers } $= 12/16 = 3/4$
B : { there's a 2 exist } $= 7/16$

$\Rightarrow P(A \cap B) = \frac{6}{16} = \frac{3}{8}$

$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{3}{8}}{12/16} = \frac{1}{2}$ #

(b) Let E denote the event that the sum of two dies is 4
F denote the event that the first die equals 4
Determine whether E and F are dependent or independent of each other

$P(E) = 3/16$ $\quad P(F) = 4/16$ $\quad P(E \cap F) = 0$

$\Rightarrow P(E \cap F) \neq P(E)P(F)$ $\Rightarrow$ E, F and dependent #

4. A total of 80 percent of the female students and 70 percent male student who took the "probability Course" and passed midterm. Given that 40 percent of the class were female.

(a) What percentage of those students that passed the midterm are female?

$$\Rightarrow \frac{0.4 \times 0.8}{0.4 \times 0.8 + 0.6 \times 0.7} = \frac{32}{32+42} = \frac{32}{74} = \frac{16}{37} \ \#$$

(b) What percentage of those students in the class passed midterm?

$$\Rightarrow \frac{0.4 \times 0.8 + 0.6 \times 0.7}{1} = \frac{74}{100} = \frac{37}{50} \ \#$$

5. A team PASSION has 5 members, each of them can independently accomplish a mission X-Star with probability of $\frac{2}{3}$. Team PASSION is successful whenever at least one of its members accomplish the mission. Find the conditionally probability that exactly 2 member accomplished the mission, given that team PASSION is successful.

$$\Rightarrow \frac{C^5_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3}{1 - \left(\frac{1}{3}\right)^5} = \frac{10 \times 4}{3^5 - 1} = \frac{40}{242} = \frac{20}{121} \ \#$$