

Part 1: Computing problems (55)

Please use your codes to solve the following mathematical problems. If the answer is not an integer, please round off to the **4th** decimal place.

(a) $x^3 + 4x^2 - 10 = 0$ for $1 \leq x \leq 2$ (5)

(b) $3.06 = \frac{(1-x)(3+x)}{x(1+x)}$ (10)

(c) Solve the following linear system. (10)

$$a_{ii} = 3$$

$$a_{ij} = -1 \text{ if } |i - j| = 1$$

$$a_{ij} = 0 \text{ if } |i - j| > 1 \text{ where } i, j = 1, 2, \dots, 10$$

$$\underline{b}^T = [2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 2]$$

(d) Find the PA=LU factorization of the system and solve \underline{x} . (20)

$$1.19x_1 + 2.11x_2 - 100x_3 + x_4 = 1.12,$$

$$14.2x_1 - 0.122x_2 + 12.2x_3 - x_4 = 3.44,$$

$$100x_2 - 99.9x_3 + x_4 = 2.15,$$

$$15.3x_1 + 0.110x_2 - 13.1x_3 - x_4 = 4.16.$$

(e) Find a solution to the following nonlinear system:

$$\sin x + y^2 + \log_e z = 7$$

$$3x + 2y - z^3 = -1$$

$$x + y + z = 5$$

Use the initial guess $[0, 2, 2]^T$. (10)

Part2: Written problems (45)

Problem #1 (5). Nested multiplication (or Horner's method) is an efficient method for evaluating polynomials. Please write down the nested form of the following polynomial:

$$f(x) = x^{14} - 4x^{11} + 5x^8 + 3x^5$$

Problem #2 (10). Consider the equation $x^3 + x - 2 = 0$, with root $r = 1$. Add the term cx to both sides and divide by c to obtain $g(x)$.

- (a) For what condition of c would FPI be linearly convergent to $r = 1$? (5)
- (b) For what condition of c would FPI be faster than bisection? (5)

Problem #3 (10). A *permutation matrix* is a square binary matrix that has exactly one entry of 1 in each row and each column and 0s elsewhere. It can be used to reorder the rows of another matrix.

- (a) How many permutation matrices of size 10×10 exist? (5)
- (b) Please show how do we use a permutation matrix to make the following matrix strictly diagonally dominant. (5)

$$A = \begin{bmatrix} 1 & -8 & -2 \\ 1 & 1 & 5 \\ 3 & -1 & 1 \end{bmatrix}$$

[Please turn over for Problem #4.]

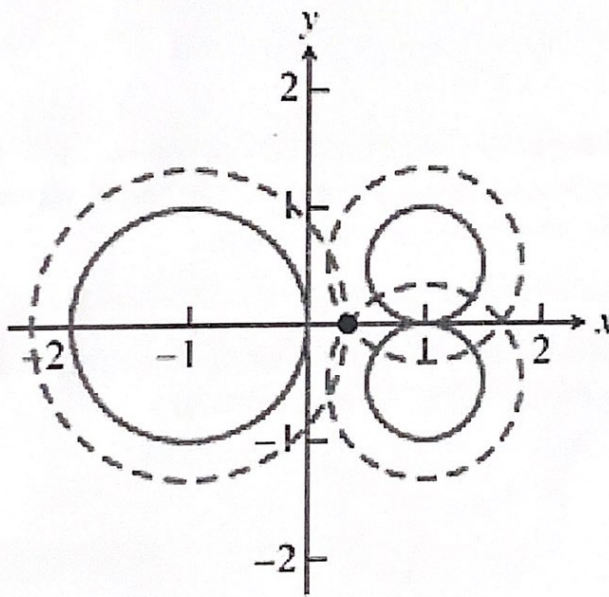
Problem #4 (20). Given three circles with centers (x_1, y_1) , (x_2, y_2) , (x_3, y_3) and radii R_1 , R_2 and R_3 . We expand the radii of each circle **by the same amount** k to get an intersection point (the black dot in the figure). The goal is to compute the coordinate of the intersection point and k .

This problem can be addressed by formulating a nonlinear system and use multivariate Newton's method to solve it. Please write down the $F(\cdot)$ and $D_F(\cdot)$ needed for applying this method.

\underline{x}_0 = initial vector

solve $D_F(\underline{x}_k)\underline{s} = -F(\underline{x}_k)$

$\underline{x}_{k+1} = \underline{x}_k + \underline{s}$ for $k = 0, 1, 2, \dots$



$$(x - x_1)^2 + (y - y_1)^2 = R_1^2$$

$$(x - x_2)^2 + (y - y_2)^2 = R_2^2$$

$$(x - x_3)^2 + (y - y_3)^2 = R_3^2$$