Part2: Written problems (50)

Problem #1 (20). The following four fixed-point iteration methods are proposed to compute $21^{1/3}$. Write down their speed of convergence and rank them in order based on the convergence speed, assuming $x_0 = 1$.

1. $g(x) = \frac{20x+(21/x^2)}{21}$
2. $g(x) = x - \frac{x^3-21}{3x^2}$
3. $g(x) = x - \frac{x^4-21x}{x^2-21}$
4. $g(x) = \left(\frac{21}{x}\right)^{1/2}$

Problem #2 (10). In the process of finding LU factorization of $A = \begin{bmatrix} 10 & 20 & 1 \\ 1 & 1.99 & 6 \\ 0 & 50 & 1 \end{bmatrix}$, what is the largest magnitude multiplier l_{ij} needed?

Problem #3 (10). Please change four entries of the left most matrix to make the matrix equation correct:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 5 & 6 & 7 & 8 \\ 7 & 8 & 9 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Problem #4 (10). For each system, rearrange the equations to form a strictly diagonally dominant system and write the rearranged system in matrix form.

(a)
$$\begin{aligned} u + 4v &= 5 \\ v + 2w &= 2 \\ 4u + 3w &= 0 \end{aligned}$$

(b)
$$\begin{aligned} u - 8v - 2w &= 1 \\ u + v + 5w &= 4 \\ 3u - v + w &= -2 \end{aligned}$$