Ex. Find the PMF p_X, that $X = \max\{X_1\ X_2\ X_3\}$, Range $X = 1 \sim 10$, discrete

① Find CDF first:

$$F_X(k) = P(X \le k)$$
$$= P(X_1 \le k \cap X_2 \le k \cap X_3 \le k)$$
$$= P(X_1 \le k)\,P(X_2 \le k)\,P(X_3 \le k) = \left(\frac{k}{10}\right)^3$$

② Find the PMF:

$$p_X(k) = F_X(k) - F_X(k-1) = \left(\frac{k}{10}\right)^3 - \left(\frac{k-1}{10}\right)^3$$

Ex. Find the PMF p_X, that $X = \min\{X_1\ X_2\ X_3\}$, Range $X = 81 \sim 85$, discrete

① Find CDF:

$$P(X > k) = P(\min\{X_1\ X_2\ X_3\} > k)$$

$$\Rightarrow 1 - F_X(k) = P(X_1 > k)\,P(X_2 > k)\,P(X_3 > k)$$

$$= [1 - P(X_1 > k)][1 - P(X_2 > k)][1 - P(X_3 > k)]$$

$$\Rightarrow F_X(k) = 1 - [1 - P(X_1 > k)]^3$$

$$F_X(k) = \begin{cases} 0, & \text{for } k \le 80 \\ 1 - \left[1 - \frac{k-80}{5}\right]^3, & \text{for } k = 81, 82, 83, 84, 85 \\ 0, & \text{for } k > 85 \end{cases}$$

② Find PMF:

$$p_X(k) = \begin{cases} 0, & \text{for } k \le 80 \\ 1 - \left[1 - \frac{k-80}{5}\right]^3, & \text{for } k = 81 \\ \left[1 - \frac{k-81}{5}\right]^3 - \left[1 - \frac{k-80}{5}\right]^3, & \text{for } k = 82, 83, 84, 85 \\ 0, & \text{for } k > 85 \end{cases}$$

Ex. Find the PMF p_X, that $X = \max\{X_1\ \ 2X_2\ \ 3X_3\}$, Range $X = 1, 2, 3$

x_1	1	2	3			
x_2		2		4	6	
x_3			3		6	9

$$F_X(x) = P(X \le x) = P(X_1 \le x)\,P(2X_2 \le x)\,P(3X_3 \le x)$$

$$
\begin{aligned}
F_X(9) &= 1 \times 1 \times 1 = 1 \\
F_X(6) &= 1 \times 1 \times \tfrac{2}{3} = \tfrac{2}{3} \\
F_X(4) &= 1 \times \tfrac{2}{3} \times \tfrac{1}{3} = \tfrac{2}{9} \\
F_X(3) &= 1 \times \tfrac{1}{3} \times \tfrac{1}{3} = \tfrac{1}{9} \\
F_X(2) &= \tfrac{1}{3} \times \tfrac{1}{3} \times 0 = 0 \\
F_X(1) &= 0
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
p_X(3) &= F_X(3) - F_X(2) = \tfrac{1}{9} \\
p_X(4) &= F_X(4) - F_X(3) = \tfrac{1}{9} \\
p_X(6) &= F_X(6) - F_X(4) = \tfrac{4}{9} \\
p_X(9) &= F_X(9) - F_X(6) = \tfrac{1}{3}
\end{aligned}
$$

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Ex. Find the PMF p_X, that $X = \min\{X_1\ \ 2X_2\ \ 3X_3\}$, Range $X = 1, 2, 3$

x_1	1	2	3			
x_2		2		4	6	
x_3			3		6	9

$$
\begin{aligned}
F_X(x) = P(X \le x) &= 1 - P(X > x) \\
&= 1 - P(X_1 > x)\,P(2X_2 > x)\,P(3X_3 > x)
\end{aligned}
$$

$$
\begin{aligned}
F_X(9) &= 0 \\
F_X(6) &= 0 \\
F_X(4) &= 0 \\
F_X(3) &= 1 - 0 \times \tfrac{2}{3} \times \tfrac{2}{3} = 1 \\
F_X(2) &= 1 - \tfrac{1}{3} \times \tfrac{2}{3} \times 1 = \tfrac{7}{9} \\
F_X(1) &= 1 - \tfrac{2}{3} \times 1 \times 1 = \tfrac{1}{3} \\
F_X(0) &= 0
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
p_X(3) &= F_X(3) - F_X(2) = \tfrac{2}{9} \\
p_X(2) &= F_X(2) - F_X(1) = \tfrac{4}{9} \\
p_X(1) &= F_X(1) - F_X(0) = \tfrac{1}{3}
\end{aligned}
$$

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