Disturbance rejection by acceleration feedforward: Application to dynamic positioning ⋆

Øivind Kåre Kjerstad * **Roger Skjetne** * **Nils Albert Jenssen** **

* *Department of Marine Technology, Norwegian University of Science and Technology, Otto Nielsens vei 10, NO-7491 Trondheim, Norway.*
E-mails: oivind.k.kjerstad@ntnu.no and skjetne@ieee.org
** *Kongsberg Maritime AS, Kirkegaardsveien 45, NO-3601 Kongsberg, Norway.*
E-mail: nils.albert.jenssen@kongsberg.com

Abstract: This paper addresses environmental disturbance rejection in dynamic systems by means of acceleration feedforward. A feedforward structure calculates the environmental force and magnitude by comparing the filtered acceleration measurement with the acceleration due to actuation and known forces. Direct compensation of the environment by feedforward will alleviate the position and velocity feedback terms. The presented scheme compensates environmental disturbances affecting the system in addition to model uncertainties. First, the design is used for a mechanical system and illustrated for an inverted pendulum. Then, the main application, being a dynamic positioning system affected by ice forces in an Arctic operation, is controlled by acceleration feedforward to give enhanced performance.

Keywords: Disturbance rejection, Dynamic positioning, Feedforward compensation, Acceleration feedforward, Disturbance estimation, Arctic marine operations

1. INTRODUCTION

In motion control the main objective is to control the position and velocity of an object to a desired state by measurements of the position and velocity states only. Acceleration measurements are typically not used. Although these systems are proved stable and robust it is not given that the control laws provide adequate environmental compensation. Any inertial system controlled by state feedback influenced by a step in the disturbance force will have to perform unnecessary work as momentum has been gained before the control law begins compensation. By introducing an acceleration feedback, such effects can be caught at an earlier stage since the force disturbance is sensed directly and not through time integrals of the force.

Acceleration signals are seldom applied directly in marine motion control applications. Among the exceptions is the work by Lindegaard (2003), where an observer and a proportional acceleration feedback control law is derived. Here the acceleration term is used to virtually modify the vessel‘s mass as seen from the disturbance to obtain a favorable closed-loop dynamic response of the vessel. Also, Fossen et al. (2002) incorporated acceleration feedback in a design technique to virtually shape the kinetic energy of the vessel. For aerospace applications, on the other hand, acceleration measurements have been used to some extent; see for instance (Blakelock, 1991). Other relevant references are (de Jager, 1994), where the goal is to increase performance and robustness of mechanical systems by applying acceleration measurements, and (Kempf and Kobayashi, 1999) and (Chen et al., 2000), where disturbance observers are developed to attenuate the effect of the disturbances on the system.

As the acceleration is a proportional measure of the resulting force acting on an object, this can be utilized to derive the environmental contribution through a simple relationship, decomposing the absolute acceleration $\mathbf{a}$ into $\mathbf{a} = \mathbf{a}_a + \mathbf{a}_e$, where $\mathbf{a}_e$ represents the unknown environmental contribution and $\mathbf{a}_a$ represents the acceleration due to actuation and other known forces. As a result, a feedforward term can be applied in the control structure for direct compensation of the unknown environmental forces.

To obtain a feasible acceleration measurement, several challenges have to be overcome. First of all, the gravitational component must be removed as this will affect the linear accelerations during roll and pitch motion. For an inertial measurement unit (IMU) this can either be done by horizontal stabilization of the sensor platform or, by a dynamic compensation scheme. Second, the noise, bias, and nonlinearities of the accelerometers must be handled. Using an observer to produce a filtered acceleration signal based on the acceleration, velocity, and position measurements, significantly aids that problem, and this strategy is used in this paper in addition to assuming that the gravitational effect is removed (Lindegaard, 2003).

Notations: In GS, LAS, LES, UGAS, UGES, etc., stands G for Global, L for Local, S for Stable, U for Uniform, A for Asymptotic, and E for Exponential. Total time derivatives of $x(t)$ is denoted $\dot{x}$, $\ddot{x}$, $x^{(3)}$,.., $x^{(n)}$. The Euclidean vector norm is $|\mathbf{x}| := (\mathbf{x}^\top \mathbf{x})^{1/2}$, a general p-norm is $|.|_p$, and the distance to a set $\mathcal{M}$ is $|x|_{\mathcal{M}} := \inf\{|x-y| : y \in \mathcal{M}\}$. A diagonal matrix is de-

⋆ Supported by the Research Council of Norway under grant 199567/I40 : "Arctic DP".

noted $diag\{a_1, .., a_n\} \in \mathcal{R}^{n\times n}$. Stacking several vectors into one is denoted $\mathrm{col}(x, y, z) := [x^\top, y^\top, z^\top]^\top$, and whenever convenient, $|(x, y, z)| = |\mathrm{col}(x, y, z)|$.

2. ILLUSTRATIVE STUDY: A SCALAR MECHANICAL SYSTEM

Consider the mechanical system modeled according to Newton's 2nd law

$$m\ddot{\xi} = \sum f_i = \rho(\xi, \dot{\xi}) + u + d(t), \tag{1}$$

where $\xi \in \mathbb{R}$ is the positional state, $\rho(\cdot, \cdot)$ is a known force function, $u \in \mathbb{R}$ is the control force input, and $d(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is an exogenous force disturbance. We assume the disturbance $d(\cdot)$ is globally Lipschitz, that is, $\exists L > 0$ such that

$$|d(t) - d(\tau)| \leq L\,|t - \tau|\,, \forall t, \tau \in \mathbb{R}_{\geq 0}. \tag{2}$$

Let $\mathbf{x}_d(t) := \mathrm{col}(\xi_d(t), \dot{\xi}_d(t))$ be a desired position and velocity for the system in (1), and assume by construction that $\mathbf{x}_d(t)$ is bounded and absolutely continuous, and $\exists M > 0$ such that $|\dot{\mathbf{x}}_d(t)| \leq M$, a.a. $t \geq 0$.

The tracking control objective is to render the set

$$\mathcal{A} := \left\{(\mathbf{x}, t) \in \mathbb{R}^2 \times \mathbb{R}_{\geq 0} :\ \mathbf{x} - \mathbf{x}_d(t) = 0\right\} \tag{3}$$

UGAS, where $\mathbf{x} := \mathrm{col}\,(x_1, x_2) := \mathrm{col}(\xi, \dot{\xi})$.

2.1 Nominal disturbance-free case

In the disturbance-free case, $d(t) \equiv 0$, a PD-type control law that solves the control objective is

$$\begin{aligned} u &= \alpha(x, t) &(4)\\ &= -mk_1(x_1 - \xi_d) - mk_2(x_2 - \dot{\xi}_d) + m\ddot{\xi}_d - \rho\,(x_1, x_2)\,, \end{aligned}$$

resulting in the 'nominal' closed-loop system

$$\dot{\mathbf{x}} = \mathbf{A}\,(\mathbf{x} - \mathbf{x}_d(t)) + \dot{\mathbf{x}}_d(t) \tag{5}$$

where $\mathbf{A} \in \mathbb{R}^{2\times 2}$ is a Hurwitz matrix.

Proposition 1. The closed, forward invariant set $\mathcal{A}$ is UGES with respect to the closed-loop system (5).

Proof. Clearly, the linear system (5) is forward complete under the stated assumptions. Let $\mathbf{P} = \mathbf{P}^\top > 0$ be the solution to $\mathbf{PA} + \mathbf{A}^\top\mathbf{P} = -\mathbf{I}$. We will then show that

$$V(\mathbf{x}, t) = (\mathbf{x} - \mathbf{x}_d(t))^\top\mathbf{P}(\mathbf{x} - \mathbf{x}_d(t)) \tag{6}$$

is a smooth Lyapunov function for (5) with respect to the set $\mathcal{A}$. From the absolute continuity of $\mathbf{x}_d(t)$ and boundedness of $\dot{\mathbf{x}}_d(t)$, which implies that $\mathbf{x}_d(t)$ is globally Lipschitz, the following equivalence relation holds

$$c\,|\mathbf{x} - \mathbf{x}_d(t)| \leq |(\mathbf{x}, t)|_{\mathcal{A}} \leq |\mathbf{x} - \mathbf{x}_d(t)| \tag{7}$$

where $c = 1/(\sqrt{2}\max(1, M))$. This gives

$$p_m\,|(\mathbf{x}, t)|_{\mathcal{A}}^2 \leq V(\mathbf{x}, t) \leq \frac{p_M}{c^2}\,|(\mathbf{x}, t)|_{\mathcal{A}}^2 \tag{8}$$

where p_m and p_M are the minimum and maximum eigenvalues of $\mathbf{P}$, respectively. Differentiating (6) along the solutions of (5) gives

$$\dot{V} = \frac{\partial V}{\partial \mathbf{x}}\dot{\mathbf{x}} + \frac{\partial V}{\partial t} = -\,|\mathbf{x} - \mathbf{x}_d|^2 \leq -\,|(\mathbf{x}, t)|_{\mathcal{A}}^2 \tag{9}$$

and the conclusion of the proposition thereby follows from Lyapunov theorems for set stability; for instance, Theorem A.10 in (Skjetne, 2005).

2.2 Handling the disturbance by robust control

With the disturbance present, the goal is to recover the closed-loop behavior of (5) as close as possible. In this case, (5) is augmented with the additive term $\mathbf{g}d(t)$ where $\mathbf{g} := \mathrm{col}\left(0, \frac{1}{m}\right)$, to give the perturbed closed-loop system

$$\dot{\mathbf{x}} = \mathbf{A}\,(\mathbf{x} - \mathbf{x}_d(t)) + \dot{\mathbf{x}}_d(t) + \mathbf{g}d(t) \tag{10}$$

in accordance with (1). Assuming that the disturbance $d(t)$ is bounded, the time derivative of (6) becomes

$$\begin{aligned} \dot{V} &\leq -\,|(\mathbf{x}, t)|_{\mathcal{A}}^2 + \frac{\delta V}{\delta \mathbf{x}}\mathbf{g}d(t) \leq -\frac{1}{2}\,|(\mathbf{x}, t)|_{\mathcal{A}}^2 + k\,|d(t)|^2 \\ k &:= 2\left(\frac{p_M}{mc}\right)^2 &(11) \end{aligned}$$

which shows that the closed-loop system is ISS with respect to the 0-invariant set $\mathcal{A}$ (Lin, 1992).

However, since the force disturbance in practice can change rapidly it may be difficult for the control law to dominate the disturbance by feedback fast enough. This motivates a feedforward term in the control law that instead can compensate the disturbance directly.

A first attempt towards this goal is to use adaptive control under the assumption that the disturbance is constant or at least slowly varying. An adaptive control law that accomplishes this is

$$\begin{aligned} \dot{\hat{d}} &= 2\gamma\mathbf{g}^\top\mathbf{P}\,(\mathbf{x} - \mathbf{x}_d)\,, \quad \gamma > 0 \\ u &= \alpha(x, t) - \hat{\delta}. &(12) \end{aligned}$$

UGS and convergence $|\mathbf{x}(t) - \mathbf{x}_d(t)| \to 0$ can be shown by differentiating the adaptive CLF $W(\mathbf{x}, \hat{d}, t) := V(\mathbf{x}, t) + \frac{1}{2\gamma}\tilde{d}^2$ where $\tilde{d} := d - \hat{d}$ (Krstic et al., 1995). In fact, an investigation of the closed-loop adaptive system will also reveal that the disturbance estimate $\hat{d}(t)$ must converge to the true value d if this is constant. This is convenient for accurate cancellation of the disturbance. However, as the disturbance becomes time-varying, exact cancellation is no longer possible, and the resulting estimation error must once again be attenuated by the feedback terms. As the disturbance gets more severe it will rapidly deteriorate the closed-loop performance. Another attempt could then be to use integral action, but that also has best effect when the disturbance is constant.

2.3 Acceleration feedforward

As an alternative to robust control measures, we propose to employ an acceleration measurement for direct feedforward compensation of $d(t)$. This is motivated by the fact that accelerometers today are available off-the-shelf as small, light, and inexpensive sensor devices. Furthermore, in many motion control applications, for instance dynamic positioning of ships, these measurements are already available in the installed equipment. However, accelerometers are known to be problematic as they produce noisy, biased, and scaled measurements. They are also affected by gravity, but this effect is as stated above assumed to be removed by a compensation scheme.

$$a_m = \epsilon(a_r + b) + w_1, \tag{13}$$

where a_r is the acceleration found using the system model. ϵ is the scale factor, and b is the bias. These are modeled as in (Vik, 2000),

$$\dot{\epsilon} = T_\epsilon^{-1}\epsilon + w_2 \tag{14}$$
$$\dot{b} = T_b^{-1} b + w_3 \tag{15}$$

where T_ϵ^{-1} and T_b^{-1} are the time constants of the scale factor and bias drift, and w_1, w_2, and w_3 are white noise.

The acceleration signal produced by (13) is not directly applicable in a closed-loop control system as it may inject noise into the system. This will increase the wear-and-tear on the actuator in addition to give deteriorated system performance. To employ disturbance rejection by acceleration feedforward, an observer must be developed to produce a filtered acceleration measurement. In the following, for the sake of the example, we assume that the actual acceleration in (1), can be estimated with a short time delay, to produce the measurement

$$a(t) = \ddot{\xi}(t-\delta) \tag{16}$$

where $\delta > 0$ is the time-delay from the instant of the actual acceleration to the filtered signal is available. An estimate of the disturbance force is then

$$\hat{d}(t) = ma(t) - u(t-\delta) - \rho(\xi(t-\delta), \dot{\xi}(t-\delta)) = d(t-\delta). \tag{17}$$

Based on this measurement we propose the control law

$$u = \alpha(x,t) - \hat{d}(t), \tag{18}$$

resulting in the closed-loop system

$$\dot{\mathbf{x}} = \mathbf{A}\left(\mathbf{x} - \mathbf{x}_d(t)\right) + \dot{\mathbf{x}}_d(t) + \mathbf{g}\tilde{d}(t), \tag{19}$$

where $\tilde{d}(t) := d(t) - \hat{d}(t) = d(t) - d(t-\delta)$. As seen, the faster the measurement and communication is done, the smaller the delay δ is, and consequently the more accurate the estimate of $d(t)$ becomes. Ideally, the limit as $\delta \to 0$ the estimation error $\tilde{d}(t)$ vanishes and the system becomes the unperturbed nominal system in (5). However, $\delta = 0$ cannot be possible as this would violate causality.

Theorem 2. The closed-loop system (19) with $\tilde{d}(t)$ as input, is globally input-to-state stable (ISS) with respect to the closed 0-invariant set $\mathcal{A}$ in (3), and the solution $\mathbf{x}(t)$ of (19) converges to the set

$$\mathcal{B} := \left\{ (\mathbf{x}, t) : |(\mathbf{x}, t)|_{\mathcal{A}} \leq \sqrt{\frac{3kp_M}{c^2 p_m}} L\delta \right\}. \tag{20}$$

Proof. We check that (6) is an ISS-Lyapunov function for (19). Differentiating it along the solutions of (19) gives in accordance with (11) the bound

$$\dot{V} \leq -\frac{1}{2}\left|(\mathbf{x},t)\right|_{\mathcal{A}}^2 + k|\tilde{d}(t)|^2 \tag{21}$$

From the boundedness of $\mathbf{x}_d(t)$, the system is finite escape-time detectable through $|\cdot|_{\mathcal{A}}$ (Teel, 2002; Skjetne, 2005). Forward completeness follows then from (7), (8), and (21), and this proves the ISS part. To show convergence to $\mathcal{B}$ we use (8) and the global Lipschitz property of $d(\cdot)$ to get

$$\begin{aligned} \dot{V} &\leq -\frac{c^2}{2p_M} V + k\left(L\delta\right)^2 \\ &\leq -\frac{c^2}{6p_M} V, \qquad \forall V \geq \frac{3kp_M (L\delta)^2}{c^2} \end{aligned} \tag{22}$$

This last inequality shows that the trajectory $(\mathbf{x}(t), t)$ must converge to the set

$$\mathcal{B}' = \left\{ (\mathbf{x}, t) : V(\mathbf{x}, t) \leq \frac{3kp_M (L\delta)^2}{c^2} \right\} \tag{23}$$

which is contained in $\mathcal{B}$.

2.4 Example: Inverted pendulum

To illustrate the use of acceleration feedforward, we consider as an example a fixed suspension point inverted pendulum, modeled by the scalar differential equation

$$mL^2\ddot{\theta} = mLg\sin\theta + u + d(t) \tag{24}$$

where θ, m, L, g, u, and $d(t)$ is the angle deviation from the upright position, the mass located at the end of the pendulum, the length of the pendulum, the gravity constant, the control input, and a disturbance force, respectively. For convenience the mass and pendulum length is set to $m = 1$ and $L = 1$ such that

$$\ddot{\theta} = g\sin\theta + u + d(t). \tag{25}$$

This system clearly match (1) by making the appropriate substitutions.

By assuming we have an acceleration signal produced by (13) available, a simple observer is

$$\dot{\hat{\omega}} = g\sin\hat{\theta} + u + k_1 a_f + k_2\tilde{\omega} + k_3\tilde{\theta} \tag{26}$$
$$\dot{\hat{\theta}} = \hat{\omega} + k_4\tilde{\theta}, \tag{27}$$

where $\tilde{\omega} = \omega - \hat{\omega}$ and $\tilde{\theta} = \theta - \hat{\theta}$. The filtered acceleration term a_f is modeled by

$$\dot{a}_f = k_a(-a_f + a_m - g\sin\hat{\theta} - u), \tag{28}$$

where a_m is the output of the accelerometer. This is the same strategy for incorporating acceleration measurements into an observer as in (Lindegaard, 2003).

To extract the disturbance estimate from the acceleration signal, the contributions of the state-dependent terms and the control law are subtracted as

$$\hat{d}(t) = \dot{\hat{\omega}}(t) - (g\sin\hat{\theta}(t) + u(t)). \tag{29}$$

The behavior of the control law incorporating the acceleration feedforward (18) is in the following simulation compared to the nominal control law (4), which was shown to ensure an ISS property with respect to the disturbance, and the adaptive control law (12), which ensures disturbance compensation by an adaptive feedforward term.

According to (4), the nominal control law is chosen as

$$u = \alpha(\theta, \dot{\theta}) = -k_1\theta - k_2\omega - g\sin\theta \tag{30}$$

where the feedback gains are set to $k_1 = 25$ and $k_2 = 15$. In the adaptive control law, the adaptation gain is set to $\gamma = 8$, while the gains of the observer was set to $k_a = 0.1$, $k_1 = 1$, $k_2 = 10$, $k_3 = 40$, and $k_3 = 75$. The accelerometer was set up with

$$\dot{\epsilon} = -0.1\epsilon + w_2 \tag{31}$$
$$\dot{b} = -b + w_3, \tag{32}$$

where w_1 was set to have noise power 0.005, w_2 set to 10, and w_3 set to 0.01.

Figure 1 shows a comparison of performance in four cases, the first case without disturbances as the nominal response, the second with a constant disturbance, the third with a sine wave disturbance, and in the final case a random walk disturbance. For the constant disturbance, the acceleration and the adaptive schemes have similar performance by exact cancellation of the disturbance when the adaptive variable has converged. In the case with the sine wave disturbance, on the other hand, it is seen in

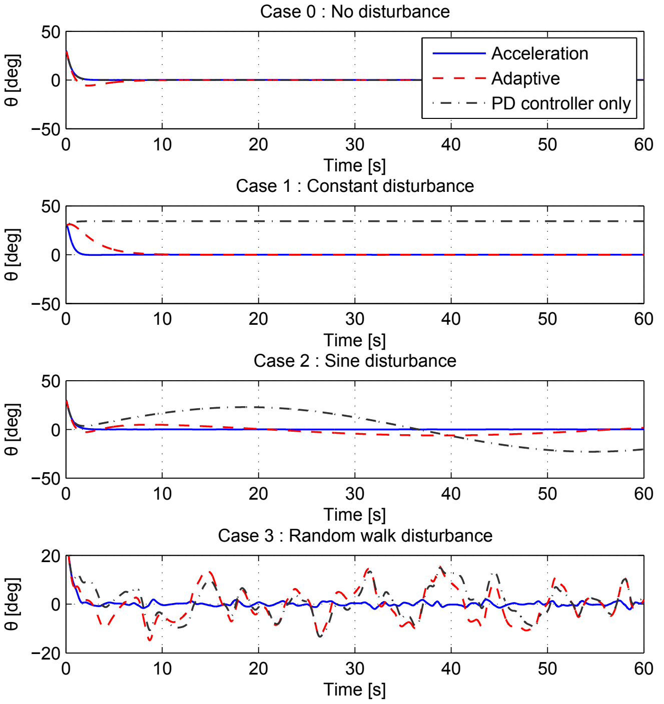

Figure 1. Angular responses to different disturbances affecting the inverted pendulum.

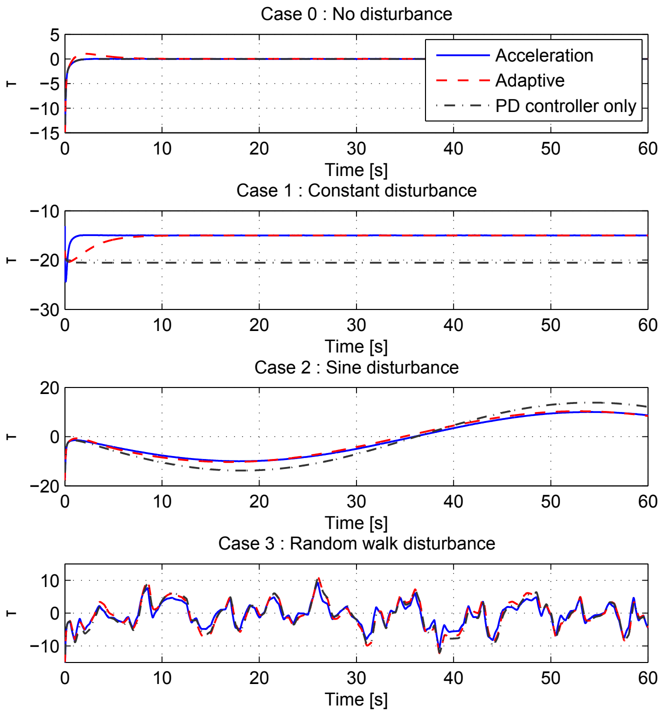

Figure 2. Input vector of the inverted pendulum subject to different disturbances.

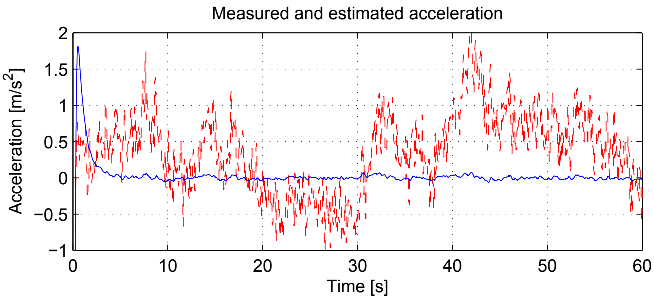

Figure 3. Measured and estimated acceleration subject to a constant disturbance.

Figure 1 that the performance of the adaptive system has deteriorated significantly. Finally, in the last case with a random walk disturbance the performance of the adaptive scheme is as bad as the PD control law without disturbance rejection. The acceleration feedforward control law, on the other hand, shows good performance in all cases, even for unknown disturbances with random characteristics. This statement is also justified by the plots of the control effort in Figure 2. Here it is seen that the acceleration feedforward control law is not more aggressive than needed, nor does it apply an increased average control effort.

It should also be emphasized that even though the feedforward control law is subject to a noisy acceleration measurement, it still outperforms the other two. Figure 3 features the measured and estimated acceleration subject to a constant disturbance.

3. CASE STUDY: DYNAMIC POSITIONING SYSTEM

Dynamic positioning of marine surface vessels is subject to severe, time-varying disturbances due to wind, waves, and currents. In addition, DP operations have recently turned towards Arctic offshore fields where ice is the governing environmental force. Ice floes acting on a vessel performing stationkeeping, is a major disturbance characterized by a rapid force build-up and release process governed by the geometry and composition of the ice floes. In such an environment, the use of integral action can be inadequate. The problem becomes even more difficult due to insufficient models of the ice-hull interaction forces. In this context, acceleration feedforward is proposed to complement integral action for environmental disturbance rejection.

Another aspect justifying disturbance rejection by feedforward is the need to instantly counteract the large momentum of the vessel if set in motion by the environment. If only compensated by feedback, this will imply excessive work and higher fuel consumption as the motion must be stopped and reversed to correct the deviation.

Consider the standard DP vessel model (Fossen, 2002)

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu} \tag{33}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} = \boldsymbol{\tau} + \boldsymbol{\tau}_e, \tag{34}$$

where $\boldsymbol{\tau}_e$ is a disturbance force acting on the vessel. As explained above, an observer has to be constructed (Lindegaard, 2003) with the structure proposed as:

$$\mathbf{M}\dot{\hat{\boldsymbol{\nu}}} = \boldsymbol{\tau} - \mathbf{D}\hat{\boldsymbol{\nu}} + \mathbf{k}_1\mathbf{a}_f + \mathbf{k}_2\tilde{\boldsymbol{\nu}} + \mathbf{k}_3\mathbf{R}(\psi)^\top\tilde{\boldsymbol{\eta}} \tag{35}$$

$$\dot{\hat{\boldsymbol{\eta}}} = \mathbf{R}(\psi)\hat{\boldsymbol{\nu}} + \mathbf{k}_4\tilde{\boldsymbol{\eta}} \tag{36}$$

$$\dot{\mathbf{a}}_f = \mathbf{k}_a\left(-\mathbf{a}_f + \dot{\boldsymbol{\nu}} - \left(\mathbf{M}^{-1}\left(\boldsymbol{\tau} - \mathbf{D}\hat{\boldsymbol{\nu}}\right)\right)\right). \tag{37}$$

This outputs the filtered position, velocity, and acceleration as used in the control law, proposed as

$$\boldsymbol{\tau} = \boldsymbol{\alpha}(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, t) - \boldsymbol{\tau}_{ff}(t) \tag{38}$$

where $\boldsymbol{\alpha}(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, t)$ is a conventional PID control law defined as

$$\boldsymbol{\alpha}(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, t) = \mathbf{K}_p\left(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d(t)\right) + \mathbf{K}_d\left(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}_d(t)\right) + \mathbf{K}_i \int_0^t \left(\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d(\tau)\right) d\tau \tag{39}$$

which is proven to be LAS and GAS if $\mathbf{K}_i = 0$ by Fossen (2002). The acceleration feedforward term is assigned as

$$\boldsymbol{\tau}_{ff}(t) = \mathbf{M}\dot{\hat{\boldsymbol{\nu}}}(t-\delta) + \mathbf{D}\hat{\boldsymbol{\nu}}(t-\delta) - \boldsymbol{\tau}(t-\delta) \tag{40}$$

such that $\boldsymbol{\tau}_{ff}$ becomes an estimate of the environmental forces and moments acting on the vessel. A causality problem is avoided since historic values of $\dot{\hat{\boldsymbol{\nu}}}$ and $\boldsymbol{\tau}$ are used. Note that this estimate will also encapsulate contributions from model uncertainties.

Inserting (38) and (40) into (34) gives

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} = \boldsymbol{\alpha}(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, t) + \boldsymbol{\tau}_e(t) - \boldsymbol{\tau}_e(t-\delta), \tag{41}$$

where the damping $\mathbf{D}\boldsymbol{\nu}$ helps stabilize the system. As the disturbance is canceled by the feedforward, the need for integral effect is reduced compared to a conventional PID approach. However, as the disturbance cancellation is imperfect some integral effect is appropriate.

3.1 Example: DP of a supply vessel

A simulator of a DP supply vessel, with (38) as the control law, is created by the Marine Systems Simulator (2010) toolbox for Matlab/Simulink. The respective mass and damping matrices are

$$\mathbf{M} = \begin{bmatrix} 5.3122 \cdot 10^6 & 0 & 0 \\ 0 & 8.2831 \cdot 10^6 & 0 \\ 0 & 0 & 3.7454 \cdot 10^9 \end{bmatrix} \tag{42}$$

$$\mathbf{D} = \begin{bmatrix} 5.0242 \cdot 10^4 & 0 & 0 \\ 0 & 2.7229 \cdot 10^5 & -4.3933 \cdot 10^6 \\ 0 & -4.3933 \cdot 10^6 & 4.1894 \cdot 10^8 \end{bmatrix}. \tag{43}$$

The ice force disturbances used in the simulation, which is seen in Figure 4, is a pre-recorded time series obtained by model scale experiments; see (Jenssen et al., 2009). These experimental data are recorded for a model of a larger vessel than the one used in this example, and have been scaled accordingly. Using these ice forces open-loop in the simulation does not correspond exactly to the expected ice-hull interaction forces. However, it verifies the performance of the acceleration feedforward scheme and its ability to handle real acceleration measurements.

The control law is set up to obtain and maintain the position $[5 \quad 5]$ with $45°$ heading starting at $[0 \quad 0]$ with $0°$. The PID control law was set up using the following gains

$$\mathbf{K}_p = 10^4 \cdot diag\{20, 17, 5, 2500\} \tag{44}$$
$$\mathbf{K}_d = 10^5 \cdot diag\{35, 25, 5000\} \tag{45}$$
$$\mathbf{K}_i = 10^3 \cdot diag\{15, 15, 300\}. \tag{46}$$

Since this amount of integral effect is not necessary when using the feedforward, it was set to $\mathbf{K}_i = 0$ when the feedforward was enabled.

The acceleration feedforward term τ_{ff} is given by (40) where $\dot{\hat{\nu}}(t-\delta)$ is obtained from the observer, and the measurement delay is $\delta = 0.1$.

The observer gains in (35)-(37) were set to:

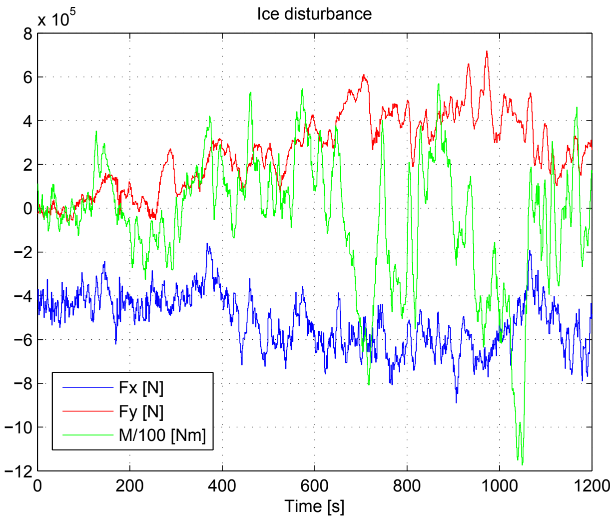

Figure 4. Ice interaction forces and moment affecting the vessel during DP. Courtesy to Jenssen et al. (2009).

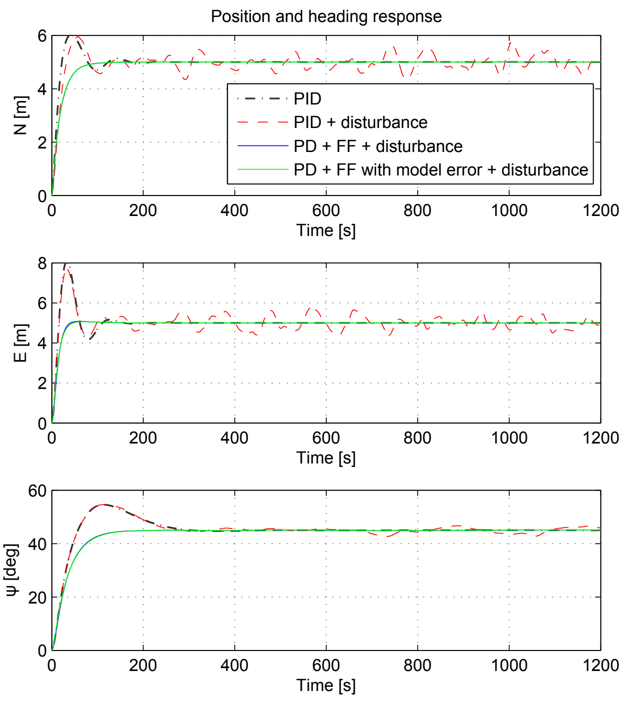

Figure 5. Position and heading response of the supply vessel subject to a gusty disturbance.

$$\mathbf{k}_a = diag\{0.5, 0.5, 0.5\} \tag{47}$$
$$\mathbf{k}_1 = diag\{1, 1, 1\} \tag{48}$$
$$\mathbf{k}_2 = diag\{25, 25, 25\} \tag{49}$$
$$\mathbf{k}_3 = diag\{25, 25, 25\} \tag{50}$$
$$\mathbf{k}_4 = diag\{20, 10, 10\}. \tag{51}$$

The accelerometers producing the acceleration measurement used in the observer were modeled as in (13). The bias and scale factors were set up for all DOFs as

$$\dot{\epsilon} = -0.001\epsilon + w_2 \tag{52}$$
$$\dot{b} = -0.001b + w_3, \tag{53}$$

where w_1, w_2, and w_3 are white noise with power 10^{-10}, 1.0, and 0.02, respectively.

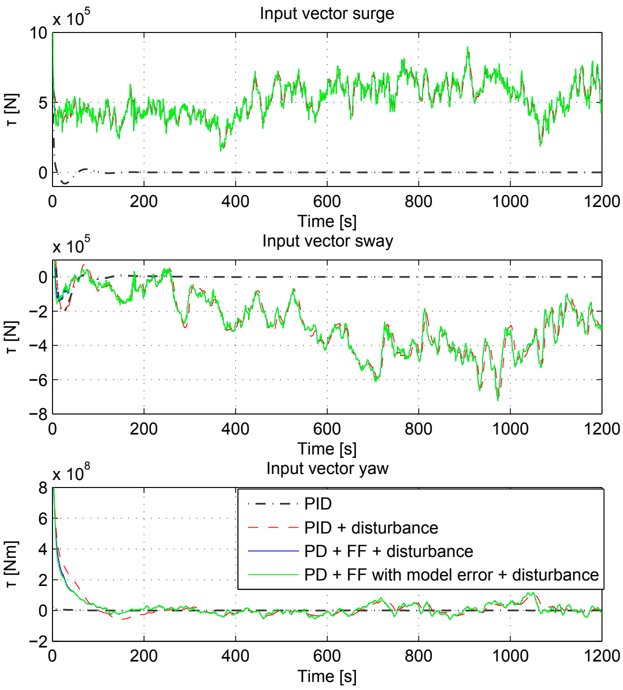

Figure 6. Input vector of the supply vessel subject to a gusty disturbance.

Four different scenarios were simulated. First, the acceleration feedforward was disabled for the disturbance-free case. Second, the disturbances were enabled to illustrate the PID performance alone. The third case illustrates the performance improvement with the acceleration feedforward enabled. In the last case, the system matrices are made uncertain, by setting them to $\hat{\mathbf{M}} = 0.75\mathbf{M}$ and $\hat{\mathbf{D}} = 0.9\mathbf{D}$, showing that the acceleration feedforward is effective also for unmodeled dynamics. This is a realistic scenario as the vessel weight will vary with its loading condition.

Figure 5 features the position and heading response of the vessel where it is seen that the use of the feedforward term improves performance by its rapid cancellation of disturbances and unmodeled dynamics. Due to the ideal actuation system, the acceleration feedforward control law produces an almost perfect response. This is not realistic when the thruster system is included in the loop, with its rate limitations and maximum bollard pull. In Figure 6 it is seen though that the control input is not more aggressive by the use of acceleration feedforward, but rather responds more quickly to the emerging disturbances.

It should be noted that an improved response with the PID could have been achieved with better tuning. However, as the disturbances and unmodeled dynamics are canceled by the acceleration feedforward, this caters for an easier tuning of the PID terms in the control law as it virtually experiences only a calm sea state. The performance, though, is limited by the actuation device's ability to change thrust magnitude and direction fast enough.

Note, also that in conventional DP systems, the wind influence is regarded an issue due to problems of obtaining a correct measurement of the wind force, and then to calculate the correct feedforward compensation. By using the proposed approach, this is a lesser problem.

4. CONCLUSIONS

In this paper we have presented a design scheme augmenting conventional control design by a disturbance rejection based on acceleration feedforward. The approach is applicable for a wide range of applications where disturbances and unwanted effects can be suppressed to give a resulting linear system characteristics. As an acceleration measurement captures the resulting force acting on the system, the disturbance rejection will offer compensation for environmental forces and model uncertainties. In turn this will enable for easier tuning of the feedback control terms.

REFERENCES

Blakelock, J.H. (1991). *Automatic Control of Aircraft and Missiles*. Wiley-Interscience.

Chen, W., Ballance, D., Gawthrop, P., and O'Reilly, J. (2000). A nonlinear disturbance observer for robotic manipulators. *Industrial Electronics, IEEE Transactions on*, 47(4), 932–938.

de Jager, B. (1994). Acceleration assisted tracking control. *IEEE Control Systems Magazine*, 14, 20–27.

Fossen, T.I. (2002). *Marine Control Systems: Guidance, Navigation and Control of Ships, Rigs, and Underwater Vehicles*. Marine Cybernetics.

Fossen, T.I., Lindegaard, K.P., and Skjetne, R. (2002). Inertia shaping techniques for marine vessels using acceleration feedback. In *Proc IFAC World Congress*.

Jenssen, N.A., Muddesitti, S., Phillips, D., and Backstrom, K. (2009). DP in ice conditions. In *Dynamic Positioing Conference*.

Kempf, C. and Kobayashi, S. (1999). Disturbance observer and feedforward design for a high-speed direct-drive positioning table. *IEEE Transactions on Control Systems Technology*, 7(5), 513–526.

Krstic, M., Kanellakopoulos, I., and Kokotovic, P.V. (1995). *Nonlinear and Adaptive Control Design (Adaptive and Learning Systems for Signal Processing, Communications and Control Series)*. Wiley-Interscience.

Lin, Y. (1992). *Lyapunov Function Techniques for Stabilization*. PhD thesis, New Brunswick Rutgers, State Univ. New Jersey, New Brunswick, NJ, USA.

Lindegaard, K.P.W. (2003). *Acceleration Feedback in Dynamic Positioning*. Ph.D. thesis, Norwegian University of Science and Technology.

Marine Systems Simulator (2010). URL `http://www.marinecontrol.org/download.html`.

Skjetne, R. (2005). *The Maneuvering Problem*. PhD thesis, NTNU, Dept. Eng. Cybernetics, Trondheim, Norway.

Teel, A.R. (2002). Notes on nonlinear control and analysis. Lecture Notes: Courses ECE 236, 237, and 594D.

Vik, B. (2000). *Nonlinear Design and Analysis of Integrated GPS and Inertial Navigation Systems*. PhD thesis, NTNU, Dept. Eng. Cybernetics, Trondheim, Norway.