Documentation for RVG maneuvering model in MCsim Python library

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May 4, 2022

1 Introduction

This note serves as documentation for the RVG maneuvering model in Python. Main parameters of RVG are provided in Table 1.

Table 1: Main parameters of RVG

Parameter	Value
Length between perpendiculars	$L_{pp} = 33.9$m
Breadth (moulded)	$B_m = 9.6$m
Draught (moulded)	$d_m = 2.7$ m

2 Overview

Two maneuvering models have been developed: A 3DOF horizontal plane model, and a pseudo-6DOF model; a 6DOF model where heave and pitch is "suppressed". Both models include a first order model of the actuator dynamics.

2.1 3DOF model

The 3DOF model simulates the 3DOF maneuvering equations of RVG. The model state, control inputs and disturbances are:

- State: 3DOF pose and body-fixed velocities, azimuth angle and propeller revolutions of the equivalent thruster, i.e., the two thrusters modelled as a single equivalent thruster located at centerline.
- Control input: Desired angle and propeller revolutions of the equivalent thruster.
- Disturbance: 3DOF body-fixed forces. Can be used to emulate wind and wave loads.

The function

dot_RVG_Man_3DOF_lq(x,u,w,parV,parA,parS):

returns the time derivative of the state vector. Vessel parameters, actuator parameters and simulation parameters are stored in parV, parA and parS, respectively. Here, **_lq** denotes that the model uses linear+quadratic damping formulation.

The function

int_RVG_Man_3DOF_lq(x,u,w,parV,parA,parS):

performs 1 time step numerical integration of the system response, using the 4th order Runge Kutta method.

2.2 6DOF model

The 6DOF model simulates the 3DOF maneuvering equations of RVG, together with the roll dynamics, and is thus more correctly labeled as a (3+1)DOF model. For practical reasons it has been implemented using a 6DOF equation of motion. This also enables expanding to a true 6DOF model in the future. Artificially large damping has been included to suppress heave and pitch motion. Additionally, the actuator-induced loads in heave and pitch have been omitted. The result is a model with negligible motion in heave and pitch.

The model state, control inputs and disturbances are:

- State: 6DOF position and orientation, and body-fixed velocities, azimuth angle and propeller revolutions of the equivalent thruster, i.e., the two thrusters modelled as a single equivalent thruster located at centerline.
- Control input: Desired angle and propeller revolutions of the equivalent thruster.
- Disturbance: 6DOF body-fixed forces. Can be used to emulate wind and wave loads.

The function

dot_RVG_Man_6DOF_lq(x,u,w,parV,parA,parS):

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The function

int_RVG_Man_6DOF_lq(x,u,w,parV,parA,parS):

performs 1 time step numerical integration of the system response, using the 4th order Runge Kutta method.

2.3 Vessel data

Several sources of vessel data are available, with unknown validity. For the numerical model reasonable data have been selected based on engineering judgement. The script **GenerateRVGManeuveringModelData** located in the *data* subfolder generates the model data. See comments therein.

2.4 Thruster data

The location of the thrusters, crudely estimated from drawings, are provided in Table 2. Thruster model data are qualitatively verified towards a black-box fmu model; see Section 6.

Table 2: Thruster locations of Gunnerus

Parameter	Value	Comment
Longitudinal	0m	Relative to aft perpendicular
Vertical	0.3m	Above baseline
Lateral	± 2.7 m	Relative to centerline

3 Mathematical preliminaries

The following definitions are used in the remainder of this document. For a vector $x = [x_1\ x_2\ x_3]^\top \in \mathbb{R}^3$ we define the skew-symmetric matrix cross-product operator $S : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ as

$$S(x) := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{1}$$

We define the selection vectors

$$\varepsilon_1 := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon_2 := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \varepsilon_3 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{2}$$

To reduce notational burden, functions are sometimes written without their arguments.

4 3DOF maneuvering model theory

We consider a ship with pose $\eta := [x\ y\ \psi]^\top \in \mathbb{R}^2 \times [-\pi, \pi]$, and body-fixed velocity vector $\nu := [u\ v\ r]^\top \in \mathbb{R}^3$.

4.1 Kinematics

Define the 3DOF rotation matrix about the vertical axis as

$$R_z(\psi) := \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

The kinematic equation relating ship pose with body-fixed velocities is given by

$$\dot{\eta} = R_z(\psi)\nu. \tag{4}$$

A steady (time-invariant) and uniform (in space) current is described by

- Current speed $U_c \in \mathbb{R}$.
- Flow direction $\beta_c \in [-\pi, \pi]$.

The inertial frame current velocity vector $\nu_c^n \in \mathbb{R}^3$ is given by

$$\nu_c^n := U_c \begin{bmatrix} \cos(\beta_c) & \sin(\beta_c) & 0 \end{bmatrix}^\top. \tag{5}$$

The body-fixed current velocity $\nu_c \in \mathbb{R}^3$ becomes

$$\nu_c := [R_z(\psi)]^\top \nu_c^n. \tag{6}$$

The derivative of the body-fixed current velocity is given by

$$\dot{\nu}_c = \frac{d[R_z(\psi)]^\top}{dt}\nu_c^n = r[S(\varepsilon_3)R_z(\psi)]^\top \nu_c^n. \tag{7}$$

4.2 Kinetics

Define the fluid relative velocity $\nu_r \in \mathbb{R}^3$ as

$$\nu_r := \nu - \nu_c. \tag{8}$$

The nonlinear equation of motion of a ship is commonly stated as

$$M_{rb}\dot{\nu} + M_a\dot{\nu}_r + C_{rb}(\nu)\nu + C_a(\nu_r) + D(\nu_r)\nu_r = F, \tag{9}$$

where $M_{rb}, M_a \in \mathbb{R}^{3\times 3}$ are the rigid body and added mass matrices, respectively, $C_{rb}, C_a : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ are the rigid body and added mass Coriolis and centripetal force matrices, and $D : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is the hydrodynamic damping matrix. $F \in \mathbb{R}^3$ collects all external forces, e.g. actuator forces, wave and wind loads.

Eq. (9) can be solved using either ν or ν_r as the state. Choosing ν we obtain

$$\dot{\nu} = [M_{rb} + M_a]^{-1}(F - C_{rb}(\nu)\nu - C_a(\nu_r) - D(\nu_r)\nu_r + M_a\dot{\nu}_c), \tag{10}$$

where $\dot{\nu}_c$ is given by (7).

4.2.1 Coriolis and centripetal forces

We assume ships with port-starboard symmetry, and reference point along the centerline. This gives inertia matrices of the form

$$M = \begin{bmatrix} m_{11} & 0 & \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}. \tag{11}$$

The rigid body mass matrix is symmetric and constant. In general, the hydrodynamic added mass is non-symmetric and depends on both the mean speed, as well as the oscillation frequency of perturbations about the mean speed. For maneuvering control purposes, M_a is selected as the low-frequency asymptotic value about the service speed. The Coriolis and centripetal force matrices are parameterized as

$$C(\nu) := \begin{bmatrix} 0 & 0 & -m_{22}v - 0.5(m_{23} + m_{32})r \\ 0 & 0 & m_{11}u \\ m_{22}u + 0.5(m_{23} + m_{32})r & -m_{11}u & 0 \end{bmatrix}. \tag{12}$$

4.3 Hydrodynamic viscous loads

The model uses a "linear+quadratic" damping formulation given by

$$D(\nu_r) := D_l + D_u|u_r| + D_v|v_r| + D_r|r|, \tag{13}$$

where $D_l \in \mathbb{R}^{3\times 3}$ is the linear damping matrix, and $D_u, D_v, D_r \in \mathbb{R}^{3\times 3}$ are matrices of quadratic damping coefficients. The relation to second-order modulus functions using hydrodynamic derivatives is illustrated by an example: The viscous damping formulation proposed by Blanke (see [1, Section 7.1.2]),

$$D(\nu_r) = \begin{bmatrix} -X_{|u|u}|u_r| & 0 & 0 \\ 0 & -Y_{|v|v}|v_r| & -Y_{|v|r}|v_r| \\ 0 & -N_{|v|v}|v_r| & -N_{|v|r}|v_r| \end{bmatrix} \tag{14}$$

is represented by the matrices $D_l = D_r = 0$, and

$$D_u = \begin{bmatrix} -X_{|u|u} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -Y_{|v|v} & -Y_{|v|r} \\ 0 & -N_{|v|v} & -N_{|v|r} \end{bmatrix}. \tag{15}$$

5 6DOF maneuvering model theory

As stated, the 6DOF model is more accurately described as a (3+1)DOF model, that is, a 3DOF horizontal plane model, with roll added as an additional DOF, under the assumption of small roll angles. The main uncertainty of the 6DOF model is the vertical center of gravity and natural period in roll.

5.1 Kinematics

The model uses the nonlinear 6DOF kinematic equation,

$$\dot{\eta} = J(\eta)\nu, \tag{16}$$

where $\eta \in \mathbb{R}^6$ is the position and orientation, $\nu \in \mathbb{R}^6$ is the body-fixed velocities, and $J : \mathbb{R}^6 \to \mathbb{R}^{6\times 6}$ is the transformation matrix; see [1].

5.2 Kinetics

The 6DOF numerical model solves the equation of motion

$$M_{rb}\dot{\nu} + M_a\dot{\nu}_r + C_{rb}(\nu)\nu + C_a(\nu_r) + D(\nu_r)\nu_r + K\eta = F, \tag{17}$$

i.e., the 6DOF version of (9), where the hydrostatic stiffness matrix $K \in \mathbb{R}^{6\times 6}$ is also included. Note the following:

- The hydrostatic restoring forces are modeled as linear, and thus valid for small angles.
- The current velocity is assumed acting in the body-fixed surge and sway direction, only.
- The rigid body kinetics are implemented as nonlinear in the horizontal DOFs (surge,sway,yaw) and linear in the vertical DOFs (heave,roll,pitch). That is, the Coriolis and centripetal forces are included for the horizontal DOFs only.

5.3 Nonlinear viscous damping in roll and sway

The roll-sway coupling terms are important for the roll dynamics, in particular the roll moment induced due to sideslip when turning. The nonlinear viscous forces in sway and roll are calculated as follows. Let z_b be the vertical position of the center of buoyancy in the body-fixed coordinate system. The fluid relative velocity at z_b is obtained as

$$v_b := v_r - z_b p, \tag{18}$$

where v_r is the fluid relative velocity in sway (at body-fixed origin), and p is the rotational velocity in roll. The sway force F_y and roll moment M_x is then calculated as

$$F_y := d_{vv}|v_b|v_b, \quad M_x := -F_y z_b, \tag{19}$$

where d_{vv} is the quadratic damping coefficient in sway. Additional nonlinear damping in sway and roll may be added as needed.

6 Azimuth thruster actuator model

RVG has two azimuth thrusters with a foil-shaped thruster body. At maneuvering speeds, the foils act as rudders. Kongsberg Maritime has provided a black box fmu [2], that takes relative fluid velocities, propeller revolutions and azimuth angle as input, and returns body-fixed actuator loads. While there exists a Python package for running fmus, this package does not work for all Python distributions. For this reason, a Python thruster model is developed, provided by the function **RVGazimuth_man** located in the **thrusters.py** module in the lib folder. Qualitative verification of the thruster model towards fmu data is provided by running the **verify_RVGazimuth_thruster.py** script located in the lib/testing folder. The Python thruster model has the additional benefit of being tractable, and can be used as basis for a control design model.

6.1 Actuator model theory

The two azimuth thrusters are modeled as an equivalent thruster located at centerline. The model is valid for maneuvering purposes; surge speed much larger than sway speed, and azimuth angle up to ±30 degrees (approximately). The *equivalent thruster model* implicitly assumes that both thrusters see the same fluid inflow angle and speed, an assumption which is not valid for large yaw velocities.

The actuator loads are separated into the propeller thrust load F_t, the foil drag load F_d and the foil lift load F_l. The thrust force is assumed quadratic in propeller revolutions ω,

$$F_t = C_t \omega^2, \tag{20}$$

where C_t is the thrust force coefficient. The thrust force is assumed acting in the propeller direction.

In the following, let $u, v \in \mathbb{R}$ be the thruster surge and sway velocity relative to the fluid, and $\alpha \in \mathbb{R}$ be the azimuth angle of the thruster. The relative fluid angle of attack ϕ and total fluid velocity V is then given by

$$\phi := \alpha - \text{atan2}(v, u), \quad V := \sqrt(u^2 + v^2). \tag{21}$$

The lift and drag loads are calculated as

$$F_d = 0.5\rho A_p C_d(\phi) V^2, \quad F_l = 0.5\rho C_l(\phi) V^2, \tag{22}$$

where A_p is the projected foil area. For small angles of attack the lift coefficient C_l can be assumed linear in ϕ,

$$C_l(\phi) = a_l \phi, \tag{23}$$

where $a_l >$ is a fixed parameter. Similarly, the variation in drag coefficient C_d is assumed to satisfy the relation

$$C_d(\phi) = C_{d0} + a_d |\phi|, \tag{24}$$

where C_{d0} is base drag for $\phi = 0$ and $a_d > 0$ is a fixed parameter. By definition, F_d and F_l act inline and perpendicular to the relative fluid velocity, respectively. The foil loads acting in the thruster-fixed x and y directions are then given by

$$F_{fx} = -F_d \cos(\phi) + F_l \sin(\phi), \tag{25}$$

$$F_{fy} = F_l \cos(\phi) + F_d \sin(phi). \tag{26}$$

The total surge force F_x and sway force F_y acting in the vessel coordinate system are obtained as

$$F_x = F_t \cos(\alpha) + F_{fx} \cos(\alpha) - F_{fy} \sin(\alpha) \tag{27}$$

$$F_y = F_t \sin(\alpha) + F_{fx} \sin(\alpha) F_{fy} \cos(\alpha). \tag{28}$$

The roll moment M_x and yaw moment M_z are obtained as

$$M_x = -z_t F_y, \quad M_z = x_t F_y, \tag{29}$$

where z_t and x_t is the vertical and longitudinal equivalent thruster position in the chosen reference frame.

6.2 Actuator model parameters

The model parameters, provided in Table 3 below, are obtained by a rough comparison towards an fmu dataset, see figures 1, 2 and 3. The results verify that the model structure is qualitatively correct. Note that A_p does not reflect the actual foil projected area, but is scaled to provide reasonable force magnitudes.

Table 3: Thruster model data

Parameter	Value
Thrust force coefficient	$C_t = 1.8$
Foil projected area	$A_p = 9$
Lift parameter	$a_l = 1$
Drag parameter	$a_d = 0.4$
Base drag	$C_{d0} = 0.2$

6.3 First order actuator dynamics

The actuator dynamics are given by a rate-limited first order model,

$$\dot{x} = -\text{sat}\left(\frac{x - x_d}{T}, a\right), \tag{30}$$

where $x \in \mathbb{R}$ is the actuator state (rudder angle or propeller revolutions), $x_d \in \mathbb{R}$ is the desired actuator state, $T > 0$ is a time constant, and $a > 0$ is the maximum rate for $\dot{x}$. The saturating function $\text{sat} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is defined as

$$\text{sat}(s, a) := \text{sign}(s) \min(|s|, a). \tag{31}$$

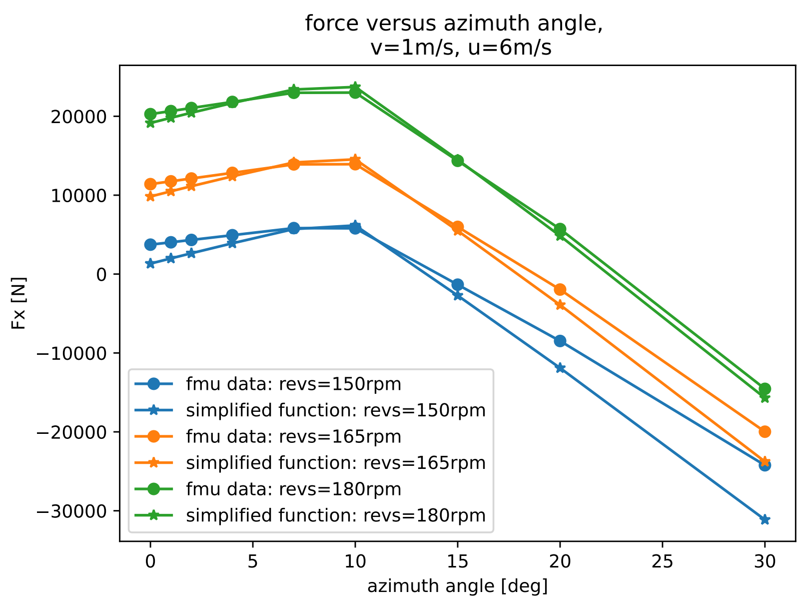

Figure 1: Actuator surge force versus azimuth angle. *Simplified function* is the model presented herein, while *fmu data* is dataset obtained from the black box fmu. Fixed $u = 6, v = 1$, and three values of propeller revolutions ω.

Note that the default time constants and maximum rates are a pure guess, due to lack of relevant data.

7 Concluding remarks

Both models are believed to qualitatively capture the maneuvering dynamics of RVG, and are suitable for control system development and testing. However, they have not been verified towards sea trials due to lack of data. Below is a list of recommendations for further work:

- Further develop the 6DOF model to also capture the heave and pitch dynamics.
- Include fluid memory effects and wave loads. Note that this is nontrivial, since it involves combining maneuvering theory with seakeeping theory.
- Obtain data for actuator dynamics, and update model parameters accordingly.
- Obtain an estimate of relevant parameters for the roll dynamics: roll natural period, vertical center of gravity, damping level. Tune the model

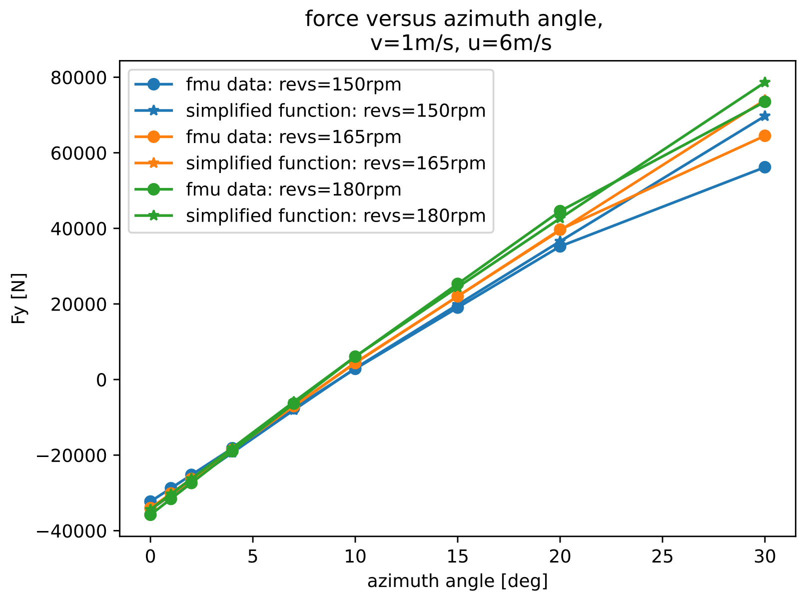

Figure 2: Actuator sway force versus azimuth angle. *Simplified function* is the model presented herein, while *fmu data* is dataset obtained from the black box fmu. Fixed $u = 6, v = 1$ and three values of propeller revolutions ω.

accordingly.

- Obtain an estimate of turning circle, and tune the model accordingly.

References

[1] Thor I Fossen. *Handbook of marine craft hydrodynamics and motion control*. John Wiley & Sons, 2011.

[2] *PEP 8 Python style guide*. `https://open-simulation-platform.github.io/cosim-demo-app/gunnerus-path-following`.

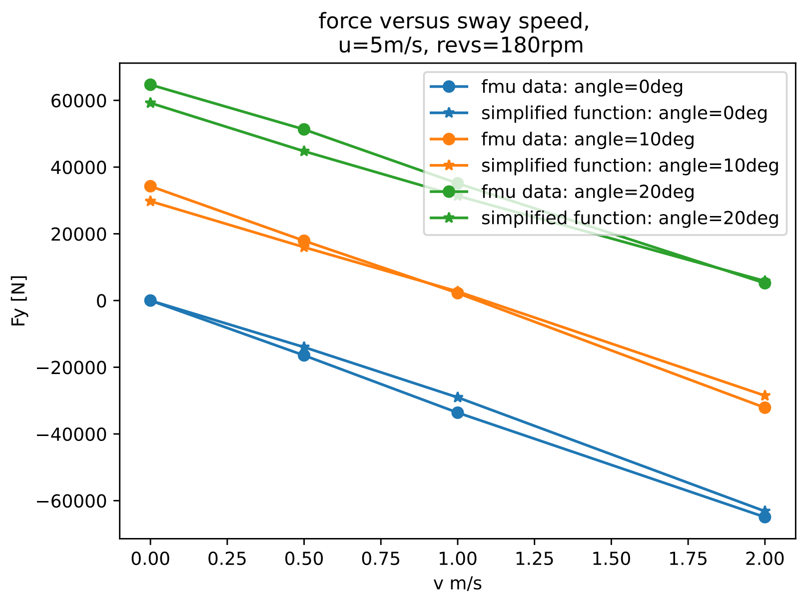

Figure 3: Actuator sway force versus sway speed. *Simplified function* is the model presented herein, while *fmu data* is dataset obtained from the black box fmu. Fixed $u = 5$, propeller revolutions $\omega = 180$, and three values of azimuth angle α.