Documentation for RVG maneuvering model in MCsim Python library

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1 Introduction

This note serves as documentation for the RVG maneuvering model. Note that the model is currently in development stage. Main parameters of RVG are provided in Table 1.

Table 1: Main parameters of RVG

Parameter	Value
Length between perpendiculars	$L_{pp} = 33.9$m
Breadth (moulded)	$B_m = 9.6$m
Draught (moulded)	$d_m = 2.7$ m

2 Overview

The model simulates the 3DOF maneuvering equations of RVG, combined with a first order model of the two azimuth thrusters. Desired thruster forces and azimuth angles are considered control inputs. The model states, control inputs and disturbances are:

- States: 3DOF pose and body-fixed velocities, thrust force and azimuth angle of the two azimuth thrusters.
- Control input: Desired force and angle of the two azimuth thrusters.
- Disturbance: 3DOF body-fixed forces. Can be used to emulate wind and wave loads.

The function

dot_RVG_Man_3DOF_lq(x,u,w,parV,parA,parS):

returns the time derivative of the state vector. Vessel parameters, actuator parameters and simulation parameters are stored in parV, parA and parS, respectively. Here, **_lq** denotes that the model uses linear+quadratic damping formulation.

The function

int$_R VG_M an_3 DOF_l q(x, u, w, parV, parA, parS)$: performs 1 step numerical integration of the system respons, using the 4th order Runge Kutta method.

2.1 Vessel data

HOLD

2.2 Thruster data

The location of the thrusters, crudely estimated from drawings, are provided in Table 2.

Table 2: Thruster locations of Gunnerus

Parameter	Value	Comment
Longitudinal	0m	Relative to aft perpendicular
Vertical	0.3m	Above baseline
Lateral	$\pm$2.7 m	Relative to centerline

3 Mathematical preliminaries

The following definitions are used in the remainder of this document. For a vector $x = [x_1\ x_2\ x_3]^\top \in \mathbb{R}^3$ we define the skew-symmetric matrix cross-product operator $S : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ as

$$S(x) := \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{1}$$

We define the selection vectors

$$\varepsilon_1 := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon_2 := \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \varepsilon_3 := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{2}$$

4 Maneuvering model

We consider a ship with pose $\eta := [x\ y\ \psi]^\top \in \mathbb{R}^2 \times [-\pi, \pi]$, and body-fixed velocity vector $\nu := [u\ v\ r]^\top \in \mathbb{R}^3$.

4.1 Kinematics

Define the 3DOF rotation matrix about the vertical axis as

$$R_z(\psi) := \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

The kinematic equation relating ship pose with body-fixed velocities is given by

$$\dot{\eta} = R_z(\psi)\nu. \tag{4}$$

A steady (time-invariant) and uniform (in space) current is described by

- Current speed: $U_c \in \mathbb{R}$.
- Flow direction: $\beta_c \in [-\pi, \pi]$.

The inertial frame current velocity vector $\nu_c^n \in \mathbb{R}^3$ is given by

$$\nu_c^n := U_c \begin{bmatrix} \cos(\beta_c) & \sin(\beta_c) & 0 \end{bmatrix}^\top. \tag{5}$$

The body-fixed current velocity $\nu_c \in \mathbb{R}^3$ becomes

$$\nu_c := [R_z(\psi)]^\top \nu_c^n. \tag{6}$$

The derivative of the body-fixed current velocity is given by

$$\dot{\nu}_c = \frac{d[R_z(\psi)]^\top}{dt}\nu_c^n = r[S(\varepsilon_3)R_z(\psi)]^\top \nu_c^n. \tag{7}$$

4.2 Kinetics

Define the fluid relative velocity $\nu_r \in \mathbb{R}^3$ as

$$\nu_r := \nu - \nu_c. \tag{8}$$

The nonlinear equation of motion of a ship is commonly stated as

$$M_{rb}\dot{\nu} + M_{rb}\dot{\nu}_r + C_{rb}(\nu)\nu + C_a(\nu_r) + D(\nu_r)\nu_r = F, \tag{9}$$

where $M_{rb}, M_a \in \mathbb{R}^{3\times3}$ are the rigid body and added mass matrices, respectively, $C_{rb}, C_a : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ are the rigid body and added mass Coriolis and centripetal force matrices, and $D : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ is the hydrodynamic damping matrix. $F \in \mathbb{R}^3$ collects all external forces, e.g. actuator forces, wave and wind loads.

Eq. (9) can be solved using either ν or ν_r as the state. Choosing ν we obtain

$$\dot{\nu} = [M_{rb} + M_a]^{-1}(F - C_{rb}(\nu)\nu - C_a(\nu_r) - D(\nu_r)\nu_r + M_a\dot{\nu}_c), \tag{10}$$

where $\dot{\nu}_c$ is given by (7).

4.2.1 Coriolis and centripetal forces

We assume ships with port-starboard symmetry, and reference point along the center line. This gives inertia matrices of the form

$$M = \begin{bmatrix} m_{11} & 0 & \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}. \tag{11}$$

The rigid body mass matrix is symmetric, while the added mass matrix is in general non-symmetric. The Coriolis and centripetal force matrices are parametrized as

$$C(\nu) := \begin{bmatrix} 0 & 0 & -m_{22}v - 0.5(m_{23}+m_{32})r \\ 0 & 0 & m_{11}u \\ m_{22}u + 0.5(m_{23}+m_{32})r & -m_{11}u & 0 \end{bmatrix}. \tag{12}$$

4.3 Hydrodynamic viscous loads

The model uses a "linear+quadratic" damping formulation given by

$$D(\nu_r) := D_l + D_u|u_r| + D_v|v_r| + D_r|r|, \tag{13}$$

where $D_l \in \mathbb{R}^{3\times3}$ is the linear damping matrix, and $D_u, D_v, D_r \in \mathbb{R}^{3\times3}$ are matrices of quadratic damping coefficients. The relation to second-order modulus functions using hydrodynamic derivatives is illustrated by an example. The viscous damping formulation proposed by Blanke (see [1, Section 7.1.2]),

$$D(\nu_r) = \begin{bmatrix} -X_{|u|u}|u_r| & 0 & 0 \\ 0 & -Y_{|v|v}|v_r| & -Y_{|v|r}|v_r| \\ 0 & -N_{|v|v}|v_r| & -N_{|v|r}|v_r| \end{bmatrix} \tag{14}$$

is represented by the matrices $D_l = D_r = 0$, and

$$D_u = \begin{bmatrix} -X_{|u|u} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -Y_{|v|v} & -Y_{|v|r} \\ 0 & -N_{|v|v} & -N_{|v|r} \end{bmatrix}. \tag{15}$$

5 Actuator models

5.1 First order actuator models

The actuator dynamics are modelled as

$$\dot{x} = -T(x - x_d), \tag{16}$$

where x is the actuator state (e.g. rudder angle), x_d is the desired actuator state, and T is a diagonal matrix of time constants.

5.2 Azimuth thruster loads

Consider an azimuth thruster located at a position $r_t := [x_t \ y_t \ z_t]^\top \in \mathbb{R}^3$ in the body-fixed coordinate system, with thrust force $\tau \in \mathbb{R}$ and azimuth angle $\alpha \in [-\pi, \pi]$. The body-fixed actuator force vector $F : \mathbb{R}^2 \to \mathbb{R}^6$ is given by

$$F(\tau, \alpha) = \tau \begin{bmatrix} R_z(\alpha)\varepsilon_1 \\ S(r_t)R_z(\alpha)\varepsilon_1 \end{bmatrix}, \quad \varepsilon_1 := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top, \tag{17}$$

which implicitly defines the convention that $\tau > 0$ and $\alpha = 0$ represents a pure surge force in the forward direction. Expanding we obtain

$$F(\tau, \alpha) = \tau \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \\ 0 \\ -z_t \sin(\alpha) \\ z_t \cos(\alpha) \\ -y_t \cos(\alpha) + x_t \sin(\alpha) \end{bmatrix}. \tag{18}$$

Note that (17) does not contain "rudder" loads, that are important for foil-shaped azimuth thrusters at forward speeds.

References

[1] Thor I Fossen. *Handbook of marine craft hydrodynamics and motion control.* John Wiley & Sons, 2011.