Contents

1 Basic

1.1 default code

```cpp
#include<bits/stdc++.h>
#define int long long
#define ld long double
#define endl '\n'
#define PB push_back
#define pii pair<int, int>
#define SZ(v) (int)v.size()
#define all(v) v.begin(), v.end()
#define ff first
#define ss second
#define PI acos(-1)
using namespace std;

void solve(){
}
signed main(){
    ios_base::sync_with_stdio(0),cin.tie(0);

    int T = 1;
    // cin >> T;
    while(T--){
        solve();
    }
}
```

1.2 .vimrc

```
sy on
set ai nu ru cul mouse=a bg=dark
set cin et ts=4 sw=4 sts=4
im jk <esc> | im kj <esc>
im ( ()<esc>i
im [ []<esc>i
im {<cr> {<cr>}<esc>ko
```

1.3 Increase Stack Size (linux)

```cpp
#include <sys/resource.h>
void increase_stack_size() {
  const rlim_t ks = 64*1024*1024;
  struct rlimit rl;
  int res=getrlimit(RLIMIT_STACK, &rl);
  if(res==0){
    if(rl.rlim_cur<ks){
      rl.rlim_cur=ks;
      res=setrlimit(RLIMIT_STACK, &rl);
} } }
```

1.4 Misc

```cpp
編譯參數：-std=c++14 -Wall -Wshadow (-fsanitize=
    undefined)

mt19937 gen(chrono::steady_clock::now().
    time_since_epoch().count());
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(gen); }

#define SECs ((double)clock() / CLOCKS_PER_SEC)

struct KeyHasher {
  size_t operator()(const Key& k) const {
    return k.first + k.second * 100000;
} };
typedef unordered_map<Key,int,KeyHasher> map_t;

__builtin_popcountll    // 二進位有幾個1
__builtin_clzll         // 左起第一個1之前0的個數
```

```
__builtin_parityll      // 1的個數的奇偶性
__builtin_mul_overflow(a,b,&h) // a*b是否溢位
```

1.5 check

```
#!/bin/bash
set -e
g++ ac.cpp -o ac.exe
g++ wa.cpp -o wa.exe

for((i=0;;i++))
do
    echo "$i"
    python3 gen.py > input
    ./ac.exe < input > ac.out
    ./wa.exe < input > wa.out
    diff ac.out wa.out || break
done
```

2 flow

2.1 ISAP

```
struct Maxflow {
  static const int MAXV = 20010;
  static const int INF  = 1000000;
  struct Edge {
    int v, c, r;
    Edge(int _v, int _c, int _r):
      v(_v), c(_c), r(_r) {}
  };
  int s, t;
  vector<Edge> G[MAXV*2];
  int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
  void init(int x) {
    tot = x+2;
    s = x+1, t = x+2;
    for(int i = 0; i <= tot; i++) {
      G[i].clear();
      iter[i] = d[i] = gap[i] = 0;
  } }
  void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v]) ));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  }
  int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
      Edge &e = G[p][i];
      if(e.c > 0 && d[p] == d[e.v]+1) {
        int f = dfs(e.v, min(flow, e.c));
        if(f) {
          e.c -= f;
          G[e.v][e.r].c += f;
          return f;
    } } }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
      d[p]++;
      iter[p] = 0;
      ++gap[d[p]];
    }
    return 0;
  }
  int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
  }
  void reset() {
    for(int i=0;i<=tot;i++) {
      iter[i]=d[i]=gap[i]=0;
} } }flow;
```

2.2 MinCostFlow

```
struct zkwflow{
  static const int maxN=10000;
  struct Edge{ int v,f,re; ll w;};
  int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
  vector<Edge> E[maxN];
  void init(int _n,int _s,int _t){
    n=_n,s=_s,t=_t;
    for(int i=0;i<n;i++) E[i].clear();
  }
  void addEdge(int u,int v,int f,ll w){
    E[u].push_back({v,f,(int)E[v].size(),w});
    E[v].push_back({u,0,(int)E[u].size()-1,-w});
  }
  bool SPFA(){
    fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
    queue<int> q; q.push(s); dis[s]=0;
    while (!q.empty()){
      int u=q.front(); q.pop(); vis[u]=false;
      for(auto &it:E[u]){
        if(it.f>0&&dis[it.v]>dis[u]+it.w){
          dis[it.v]=dis[u]+it.w;
          if(!vis[it.v]){
            vis[it.v]=true; q.push(it.v);
    } } } }
    return dis[t]!=LLONG_MAX;
  }
  int DFS(int u,int nf){
    if(u==t) return nf;
    int res=0; vis[u]=true;
    for(int &i=ptr[u];i<(int)E[u].size();i++){
      auto &it=E[u][i];
      if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
        int tf=DFS(it.v,min(nf,it.f));
        res+=tf,nf-=tf,it.f-=tf;
        E[it.v][it.re].f+=tf;
        if(nf==0){ vis[u]=false; break; }
      }
    }
    return res;
  }
  pair<int,ll> flow(){
    int flow=0; ll cost=0;
    while (SPFA()){
      fill_n(ptr,n,0);
      int f=DFS(s,INT_MAX);
      flow+=f; cost+=dis[t]*f;
    }
    return{ flow,cost };
  } // reset: do nothing
} flow;
```

2.3 Dinic

```
struct Dinic{
  struct Edge{ int v,f,re; };
  int n,s,t,level[MXN];
  vector<Edge> E[MXN];
  void init(int _n, int _s, int _t){
    n = _n; s = _s; t = _t;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void add_edge(int u, int v, int f){
    E[u].PB({v,f,SZ(E[v])});
    E[v].PB({u,0,SZ(E[u])-1});
  }
  bool BFS(){
    for (int i=0; i<n; i++) level[i] = -1;
    queue<int> que;
    que.push(s);
    level[s] = 0;
    while (!que.empty()){
      int u = que.front(); que.pop();
      for (auto it : E[u]){
        if (it.f > 0 && level[it.v] == -1){
          level[it.v] = level[u]+1;
          que.push(it.v);
    } } }
    return level[t] != -1;
  }
  int DFS(int u, int nf){
    if (u == t) return nf;
    int res = 0;
    for (auto &it : E[u]){
      if (it.f > 0 && level[it.v] == level[u]+1){
        int tf = DFS(it.v, min(nf,it.f));
        res += tf; nf -= tf; it.f -= tf;
        E[it.v][it.re].f += tf;
```

```
        if (nf == 0) return res;
    } }
    if (!res) level[u] = -1;
    return res;
  }
  int flow(int res=0){
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
} }flow;
```

2.4 Kuhn Munkres 最大完美二分匹配

```
struct KM{ // max weight, for min negate the weights O(
    n^3)
  int n, mx[MXN], my[MXN], pa[MXN];
  ll g[MXN][MXN], lx[MXN], ly[MXN], sy[MXN];
  bool vx[MXN], vy[MXN];
  void init(int _n) { // 1-based
    n = _n;
    for(int i=1; i<=n; i++) fill(g[i], g[i]+n+1, 0);
  }
  void addEdge(int x, int y, ll w) {g[x][y] = w;}
  void augment(int y) {
    for(int x, z; y; y = z)
      x=pa[y], z=mx[x], my[y]=x, mx[x]=y;
  }
  void bfs(int st) {
    for(int i=1; i<=n; ++i) sy[i]=INF, vx[i]=vy[i]=0;
    queue<int> q; q.push(st);
    for(;;) {
      while(q.size()) {
        int x=q.front(); q.pop(); vx[x]=1;
        for(int y=1; y<=n; ++y) if(!vy[y]){
          ll t = lx[x]+ly[y]-g[x][y];
          if(t==0){
            pa[y]=x;
            if(!my[y]){augment(y);return;}
            vy[y]=1, q.push(my[y]);
          }else if(sy[y]>t) pa[y]=x,sy[y]=t;
      } }
      ll cut = INF;
      for(int y=1; y<=n; ++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
      for(int j=1; j<=n; ++j){
        if(vx[j]) lx[j] -= cut;
        if(vy[j]) ly[j] += cut;
        else sy[j] -= cut;
      }
      for(int y=1; y<=n; ++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){augment(y);return;}
        vy[y]=1, q.push(my[y]);
  } } }
  ll solve(){
    fill(mx, mx+n+1, 0); fill(my, my+n+1, 0);
    fill(ly, ly+n+1, 0); fill(lx, lx+n+1, -INF);
    for(int x=1; x<=n; ++x) for(int y=1; y<=n; ++y)
      lx[x] = max(lx[x], g[x][y]);
    for(int x=1; x<=n; ++x) bfs(x);
    ll ans = 0;
    for(int y=1; y<=n; ++y) ans += g[my[y]][y];
    return ans;
} }graph;
```

2.5 Directed MST

```
/* Edmond's algoirthm for Directed MST
 * runs in O(VE) */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF  = 2147483647;
struct Edge{
  int u, v, c;
  Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
  fill(con, con+V+1, 0);
  int r1 = 0, r2 = 0;
  while(1){
    fill(mnInW, mnInW+V+1, INF);
    fill(prv, prv+V+1, -1);
    REP(i, 1, E){
      int u=edges[i].u, v=edges[i].v, c=edges[i].c;
      if(u != v && v != root && c < mnInW[v])
        mnInW[v] = c, prv[v] = u;
    }
    fill(vis, vis+V+1, -1);
    fill(cyc, cyc+V+1, -1);
    r1 = 0;
    bool jf = 0;
    REP(i, 1, V){
      if(con[i]) continue ;
      if(prv[i] == -1 && i != root) return -1;
      if(prv[i] > 0) r1 += mnInW[i];
      int s;
      for(s = i; s != -1 && vis[s] == -1; s = prv[s])
        vis[s] = i;
      if(s > 0 && vis[s] == i){
        // get a cycle
        jf = 1; int v = s;
        do{
          cyc[v] = s, con[v] = 1;
          r2 += mnInW[v]; v = prv[v];
        }while(v != s);
        con[s] = 0;
    } }
    if(!jf) break ;
    REP(i, 1, E){
      int &u = edges[i].u;
      int &v = edges[i].v;
      if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
      if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
      if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
      if(u == v) edges[i--] = edges[E--];
  } }
  return r1+r2;
}
```

3 Math

3.1 Martix fast pow

```
struct mat{
  int a[500][500], r, c;
  mat(int _r, int _c){r=_r, c=_c; memset(a, 0, sizeof(a
      ));}
  void build(){ for(int i=0; i<r; i++) a[i][i] = 1; }
};
mat operator *(mat x, mat y){
  mat z = mat(x.r, y.c);
  for(int i=0; i<x.r; i++)
    for(int j=0; j<x.c; j++)
      for(int k=0; k<y.c; k++)
        z.a[i][j] += x.a[i][k]*y.a[k][j]%MOD %= MOD;
  return z;
}
mat matpow(mat x, int y){
  mat ret = mat(x.r, x.c);
  ret.build();
  while(y){
    if(y & 1) ret = ret * x;
    x = x * x;
    y >>= 1;
  }
  return ret;
}
```

3.2 FFT

```
const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
typedef complex<ld> cplx; //real() ,imag()
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
```

```
  for(int i=0; i<=MAXN; i++)
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
  int basic = MAXN / n;
  int theta = basic;
  for(int m=n; m>=2; m>>=1){
    int mh = m>>1;
    for(int i=0; i<mh; i++){
      cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                         : i*theta%MAXN];
      for(int j=i; j<n; j+=m){
        int k = j + mh;
        cplx x = a[j] - a[k];
        a[j] += a[k];
        a[k] = w * x;
      }
    }
    theta = (theta*2) % MAXN;
  }
  int i = 0;
  for(int j=1; j<n-1; j++){
    for(int k=n>>1; k>(i ^= k); k>>=1);
    if(j < i) swap(a[i], a[j]);
  }
  if(inv) for(i=0; i<n; i++) a[i] /= n;
}
cplx arr[MAXN+1];
inline void mul(vector<int> &a, vector<int> &b, vector<
    int> &ans){
  int _n=a.size(), _m=b.size();
  int n=1, sum=_n+_m-1;
  while(n<sum) n<<=1;
  for(int i=0; i<n; i++){
    double x = (i<_n?a[i]:0), y = (i<_m?b[i]:0);
    arr[i] = complex<double>(x+y, x-y);
  }
  fft(n, arr);
  for(int i=0; i<n; i++)
    arr[i] = arr[i]*arr[i];
  fft(n, arr, true);
  for(int i=0; i<sum; i++)
    ans[i] = (int)(arr[i].real()/4+0.5);
}
```

3.3 NTT

```
// Remember coefficient are mod P
/* p=a*2^n+1
   n    2^n         p          a    root
   16   65536       65537      1    3
   20   1048576     7340033    7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
  static LL bigmod(LL a, LL b) {
    LL res = 1;
    for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P)
      if(b&1) res=(res*bs)%P;
    return res;
  }
  static LL inv(LL a, LL b) {
    if(a==1)return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  }
  LL omega[MAXN+1];
  NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
      omega[i] = (omega[i-1]*r)%P;
  }
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
    int basic = MAXN / n , theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
      int mh = m >> 1;
      for (int i = 0; i < mh; i++) {
        LL w = omega[i*theta%MAXN];
        for (int j = i; j < n; j += m) {
          int k = j + mh;
          LL x = a[j] - a[k];
          if (x < 0) x += P;
          a[j] += a[k];
          if (a[j] > P) a[j] -= P;
          a[k] = (w * x) % P;
        }
      }
      theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
      for (int k = n >> 1; k > (i ^= k); k >>= 1);
      if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
      LL ni = inv(n,P);
      reverse( a+1 , a+n );
      for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
} } };
const LL P=2013265921,root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

3.4 BigInt

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s;
  int vl, v[LEN];
  //  vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
    s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
  } }
  Bigint(string str) {
    s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1;
    }
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
      num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
    } }
    if (num) push_back(num);
    n();
  }
  int len() const {
    return vl;  // return SZ(v);
  }
  bool empty() const { return len() == 0; }
  void push_back(int x) {
    v[vl++] = x; // v.PB(x);
  }
  void pop_back() {
    vl--; // v.pop_back();
  }
  int back() const {
    return v[vl-1]; // return v.back();
  }
  void n() {
    while (!empty() && !back()) pop_back();
  }
  void resize(int nl) {
    vl = nl;
    fill(v, v+vl, 0);
    //    v.resize(nl);
    //    fill(ALL(v), 0);
  }
  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
```

```
  for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
}
friend std::ostream& operator << (std::ostream& out,
    const Bigint &a) {
  if (a.empty()) { out << "0"; return out; }
  if (a.s == -1) out << "-";
  out << a.back();
  for (int i=a.len()-2; i>=0; i--) {
    char str[10];
    snprintf(str, 5, "%.4d", a.v[i]);
    out << str;
  }
  return out;
}
int cp3(const Bigint &b)const {
  if (s != b.s) return s - b.s;
  if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()-b.len();//int
  for (int i=len()-1; i>=0; i--)
    if (v[i]!=b.v[i]) return v[i]-b.v[i];
  return 0;
}
bool operator<(const Bigint &b)const
  { return cp3(b)<0; }
bool operator<=(const Bigint &b)const
  { return cp3(b)<=0; }
bool operator==(const Bigint &b)const
  { return cp3(b)==0; }
bool operator!=(const Bigint &b)const
  { return cp3(b)!=0; }
bool operator>(const Bigint &b)const
  { return cp3(b)>0; }
bool operator>=(const Bigint &b)const
  { return cp3(b)>=0; }
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
  return r;
}
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  for (int i=0; i<nl; i++) {
    if (i < len()) r.v[i] += v[i];
    if (i < b.len()) r.v[i] += b.v[i];
    if(r.v[i] >= BIGMOD) {
      r.v[i+1] += r.v[i] / BIGMOD;
      r.v[i] %= BIGMOD;
  } }
  r.n();
  return r;
}
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];
    if (r.v[i] < 0) {
      r.v[i] += BIGMOD;
      r.v[i+1]--;
  } }
  r.n();
  return r;
}
Bigint operator * (const Bigint &b) {
  Bigint r;
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
  for (int i=0; i<len(); i++) {
    for (int j=0; j<b.len(); j++) {
      r.v[i+j] += v[i] * b.v[j];
      if(r.v[i+j] >= BIGMOD) {
        r.v[i+j+1] += r.v[i+j] / BIGMOD;
        r.v[i+j] %= BIGMOD;
  } } }
    r.n();
    return r;
  }
  Bigint operator / (const Bigint &b) {
    Bigint r;
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      }
      r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
  }
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
} };
```

3.5 Miller Rabin

```
// n < 4,759,123,141        3 :  2, 7, 61
// n < 1,122,004,669,633    4 :  2, 13, 23, 1662803
// n < 3,474,749,660,383          6 :  pirmes <= 13
// n < 2^64                       7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
LL magic[]={}
bool witness(LL a,LL n,LL u,int t){
  if(!a) return 0;
  LL x=mypow(a,u,n);
  for(int i=0;i<t;i++) {
    LL nx=mul(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  }
  return x!=1;
}
bool miller_rabin(LL n) {
  int s=(magic number size)
  // iterate s times of witness on n
  if(n<2) return 0;
  if(!(n&1)) return n == 2;
  ll u=n-1; int t=0;
  // n-1 = u*2^t
  while(!(u&1)) u>>=1, t++;
  while(s--){
    LL a=magic[s]%n;
    if(witness(a,n,u,t)) return 0;
  }
  return 1;
}
```

3.6 Faulhaber ($\sum_{i=1}^{n} i^p$)

```
/* faulhaber's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {
  int a=x,b=mod,a0=1,a1=0,b0=0,b1=1;
  while(b) {
    int q,t;
    q=a/b; t=b; b=a-b*q; a=t;
    t=b0; b0=a0-b0*q; a0=t;
    t=b1; b1=a1-b1*q; a1=t;
  }
```

```
  return a0<0?a0+mod:a0;
}
inline void pre() {
  /* combinational */
  for(int i=0;i<=MAXK;i++) {
    cm[i][0]=cm[i][i]=1;
    for(int j=1;j<i;j++)
      cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);
  }
  /* inverse */
  for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);
  /* bernoulli */
  b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
  for(int i=2;i<MAXK;i++) {
    if(i&1) { b[i]=0; continue; }
    b[i]=1;
    for(int j=0;j<i;j++)
      b[i]=sub(b[i],
               mul(cm[i][j],mul(b[j], inv[i-j+1])));
  }
  /* faulhaber */
  // sigma_x=1~n {x^p} =
  //   1/(p+1) * sigma_j=0~p {C(p+1,j)*Bj*n^(p-j+1)}
  for(int i=1;i<MAXK;i++) {
    co[i][0]=0;
    for(int j=0;j<=i;j++)
      co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]))
          ;
  }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n,int p) {
  int sol=0,m=n;
  for(int i=1;i<=p+1;i++) {
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
  }
  return sol;
}
```

3.7 Chinese Remainder

```
LL x[N],m[N];
LL CRT(LL x1, LL m1, LL x2, LL m2) {
  LL g = __gcd(m1, m2);
  if((x2 - x1) % g) return -1;// no sol
  m1 /= g; m2 /= g;
  pair<LL,LL> p = gcd(m1, m2);
  LL lcm = m1 * m2 * g;
  LL res = p.first * (x2 - x1) * m1 + x1;
  return (res % lcm + lcm) % lcm;
}
LL solve(int n){ // n>=2,be careful with no solution
  LL res=CRT(x[0],m[0],x[1],m[1]),p=m[0]/__gcd(m[0],m
      [1])*m[1];
  for(int i=2;i<n;i++){
    res=CRT(res,p,x[i],m[i]);
    p=p/__gcd(p,m[i])*m[i];
  }
  return res;
}
```

3.8 Pollard Rho

```
// does not work when n is prime  O(n^(1/4))
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
  if(!(n&1)) return 2;
  while(true){
    LL y=2, x=rand()%(n-1)+1, res=1;
    for(int sz=2; res==1; sz*=2) {
      for(int i=0; i<sz && res<=1; i++) {
        x = f(x, n);
        res = __gcd(abs(x-y), n);
      }
      y = x;
    }
    if (res!=0 && res!=n) return res;
} }
```

3.9 Josephus Problem

```
int josephus(int n, int m){ //n人 每m次
    int ans = 0;
    for (int i=1; i<=n; ++i)
        ans = (ans + m) % i;
    return ans;
}
```

3.10 ax+by=gcd

```
PII gcd(int a, int b){
  if(b == 0) return {1, 0};
  PII q = gcd(b, a % b);
  return {q.second, q.first - q.second * (a / b)};
}
```

3.11 Romberg 定積分

```
// Estimates the definite integral of
// \int_a^b f(x) dx
template<class T>
double romberg( T& f, double a, double b, double eps=1e
    -8){
  vector<double>t; double h=b-a,last,curr; int k=1,i=1;
  t.push_back(h*(f(a)+f(b))/2);
  do{ last=t.back(); curr=0; double x=a+h/2;
    for(int j=0;j<k;j++) curr+=f(x), x+=h;
    curr=(t[0] + h*curr)/2; double k1=4.0/3.0,k2
        =1.0/3.0;
    for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];
      t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
    } t.push_back(curr); k*=2; h/=2; i++;
  }while( fabs(last-curr) > eps);
  return t.back();
}
```

3.12 Prefix Inverse

```
void solve( int m ){
  inv[ 1 ] = 1;
  for( int i = 2 ; i < m ; i ++ )
    inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}
```

3.13 Roots of Polynomial 找多項式的根

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ]; // a[0..n](coef) must be
    filled
int n; // degree of polynomial must be filled
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
  double tmp=1,sum=0;
  for(int i=0;i<=n;i++)
  { sum=sum+a[i]*tmp; tmp=tmp*x; }
  return sum;
}
double binary(double l,double r,double a[],int n){
  int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
  if(sl==0) return l; if(sr==0) return r;
  if(sl*sr>0) return inf;
  while(r-l>eps){
    double mid=(l+r)/2;
    int ss=sign(f(a,n,mid));
    if(ss==0) return mid;
    if(ss*sl>0) l=mid; else r=mid;
  }
  return l;
}
void solve(int n,double a[],double x[],int &nx){
  if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
  double da[10], dx[10]; int ndx;
  for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
  solve(n-1,da,dx,ndx);
  nx=0;
  if(ndx==0){
    double tmp=binary(-inf,inf,a,n);
    if (tmp<inf) x[++nx]=tmp;
    return;
  }
  double tmp;
  tmp=binary(-inf,dx[1],a,n);
```

```
  if(tmp<inf) x[++nx]=tmp;
  for(int i=1;i<=ndx-1;i++){
    tmp=binary(dx[i],dx[i+1],a,n);
    if(tmp<inf) x[++nx]=tmp;
  }
  tmp=binary(dx[ndx],inf,a,n);
  if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]
```

3.14 Primes

```
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 1000000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[ N ] , p_tbl[ N ];
vector<int> primes;
void sieve() {
  mu[ 1 ] = p_tbl[ 1 ] = 1;
  for( int i = 2 ; i < N ; i ++ ){
    if( !p_tbl[ i ] ){
      p_tbl[ i ] = i;
      primes.push_back( i );
      mu[ i ] = -1;
    }
    for( int p : primes ){
      int x = i * p;
      if( x >= M ) break;
      p_tbl[ x ] = p;
      mu[ x ] = -mu[ i ];
      if( i % p == 0 ){
        mu[ x ] = 0;
        break;
} } } }
vector<int> factor( int x ){
  vector<int> fac{ 1 };
  while( x > 1 ){
    int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
    while( x % p == 0 ){
      x /= p;
      for( int i = 0 ; i < fn ; i ++ )
        fac.PB( fac[ pos ++ ] * p );
  } }
  return fac;
}
```

3.15 Phi

```
ll phi(ll n){   // 計算小於n的數中與n互質的有幾個
    ll res = n, a=n;    // O(sqrtN)
    for(ll i=2;i*i<=a;i++){
        if(a%i==0){
            res = res/i*(i-1);
            while(a%i==0) a/=i;
    }   }
    if(a>1) res = res/a*(a-1);
    return res;
}
```

3.16 Result

- Lucas' Theorem :
  For $n, m \in \mathbb{Z}^*$ and prime P, $C(m,n)$ mod $P = \Pi(C(m_i, n_i))$ where m_i is the i-th digit of m in base P.
- Stirling approximation :
  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n e^{\frac{1}{12n}}$
- Stirling Numbers(permutation $|P| = n$ with k cycles):
  $S(n,k) =$ coefficient of x^k in $\Pi_{i=0}^{n-1}(x+i)$
- Stirling Numbers(Partition n elements into k non-empty set):
  $S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$
- Pick' s Theorem : $A = i + b/2 - 1$
  A: Area , i: grid number in the inner , b: grid number on the side
- Catalan number : $C_n = \binom{2n}{n}/(n+1)$
  $C_n^{n+m} - C_{n+1}^{n+m} = (m+n)!\frac{n-m+1}{n+1}$ for $n \geq m$
  $C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
  $C_0 = 1$ and $C_{n+1} = 2(\frac{2n+1}{n+2})C_n$
  $C_0 = 1$ and $C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$ for $n \geq 0$
- Euler Characteristic:
  planar graph: $V - E + F - C = 1$
  convex polyhedron: $V - E + F = 2$
  V, E, F, C: number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :
  $A_{ii} = deg(i), A_{ij} = (i,j) \in E \ ? -1 : 0$, Deleting any one row, one column, and cal the det(A)
- Polya' theorem (c is number of color , m is the number of cycle size):
  $(\sum_{i=1}^{m} c^{gcd(i,m)})/m$
- Burnside lemma:
  $|X/G| = \frac{1}{|G|}\sum_{g \in G}|X^g|$
- 錯排公式: (n 個人中，每個人皆不再原來位置的組合數):
  $dp[0] = 1; dp[1] = 0;$
  $dp[i] = (i-1) * (dp[i-1] + dp[i-2]);$
- Bell 數 (有 n 個人, 把他們拆組的方法總數) :
  $B_0 = 1$
  $B_n = \sum_{k=0}^{n} s(n,k)$ $(second - stirling)$
  $B_{n+1} = \sum_{k=0}^{n}\binom{n}{k}B_k$
- Wilson's theorem :
  $(p-1)! \equiv -1 (mod\ p)$
- Fermat's little theorem :
  $a^p \equiv a (mod\ p)$
- Euler's totient function:
  $A^{B^C}\ mod\ p = pow(A, pow(B, C, p-1)) mod\ p$
- 歐拉函數降冪公式:
  A^B mod $C = A^{B \bmod \phi(c)+\phi(c)}$ mod C
- 6 的倍數:
  $(a-1)^3 + (a+1)^3 + (-a)^3 + (-a)^3 = 6a$

4 Geometry

4.1 definition

```
typedef long double ld;
const ld eps = 1e-8;
int dcmp(ld x) {
  if(abs(x) < eps) return 0;
  else return x < 0 ? -1 : 1;
}
struct Pt {
  ld x, y;
  Pt(ld _x=0, ld _y=0):x(_x), y(_y) {}
  Pt operator+(const Pt &a) const {
    return Pt(x+a.x, y+a.y);  }
  Pt operator-(const Pt &a) const {
    return Pt(x-a.x, y-a.y);  }
  Pt operator*(const ld &a) const {
    return Pt(x*a, y*a);  }
  Pt operator/(const ld &a) const {
    return Pt(x/a, y/a);  }
  ld operator*(const Pt &a) const {
    return x*a.x + y*a.y;  }
  ld operator^(const Pt &a) const {
    return x*a.y - y*a.x;  }
  bool operator<(const Pt &a) const {
    return x < a.x || (x == a.x && y < a.y); }
    //return dcmp(x-a.x) < 0 || (dcmp(x-a.x) == 0 &&
        dcmp(y-a.y) < 0); }
  bool operator==(const Pt &a) const {
    return dcmp(x-a.x) == 0 && dcmp(y-a.y) == 0;  }
};
ld norm2(const Pt &a) {
  return a*a; }
ld norm(const Pt &a) {
  return sqrt(norm2(a)); }
Pt perp(const Pt &a) {
  return Pt(-a.y, a.x); }
Pt rotate(const Pt &a, ld ang) {
  return Pt(a.x*cos(ang)-a.y*sin(ang), a.x*sin(ang)+a.y
      *cos(ang)); }
struct Line {
  Pt s, e, v; // start, end, end-start
```

```
  ld ang;
  Line(Pt _s=Pt(0, 0), Pt _e=Pt(0, 0)):s(_s), e(_e) { v
       = e-s; ang = atan2(v.y, v.x); }
  bool operator<(const Line &L) const {
    return ang < L.ang;
} };
struct Circle {
  Pt o; ld r;
  Circle(Pt _o=Pt(0, 0), ld _r=0):o(_o), r(_r) {}
};
```

4.2 極角排序

```
bool cmp(const Pt& lhs, const Pt rhs){
    if((lhs < Pt(0, 0)) ^ (rhs < Pt(0, 0)))
        return (lhs < Pt(0, 0)) < (rhs < Pt(0, 0));
    return (lhs ^ rhs) > 0;
} // 從 270 度開始逆時針排序

sort(P.begin(), P.end(), cmp);
```

4.3 Intersection of 2 lines

```
Pt LLIntersect(Line a, Line b) {
  Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
  ld f1 = (p2-p1)^(q1-p1),f2 = (p2-p1)^(p1-q2),f;
  if(dcmp(f=f1+f2) == 0)
    return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
  return q1*(f2/f) + q2*(f1/f);
}
```

4.4 halfPlaneIntersection

```
// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
  return dcmp(L.v^(p-L.s)) > 0;
} // segment should add Counterclockwise
// assume that Lines intersect
vector<Pt> HPI(vector<Line>& L) {
  sort(L.begin(), L.end()); // sort by angle
  int n = L.size(), fir, las;
  Pt *p = new Pt[n];
  Line *q = new Line[n];
  q[fir=las=0] = L[0];
  for(int i = 1 ; i < n ; i++) {
    while(fir < las && !onleft(L[i], p[las-1])) las--;
    while(fir < las && !onleft(L[i], p[fir])) fir++;
    q[++las] = L[i];
    if(dcmp(q[las].v^q[las-1].v) == 0) {
      las--;
      if(onleft(q[las], L[i].s)) q[las] = L[i];
    }
    if(fir < las) p[las-1] = LLIntersect(q[las-1], q[
        las]);
  }
  while(fir < las && !onleft(q[fir], p[las-1])) las--;
  if(las-fir <= 1) return {};
  p[las] = LLIntersect(q[las], q[fir]);
  int m = 0;
  vector<Pt> ans(las-fir+1);
  for(int i = fir ; i <= las ; i++) ans[m++] = p[i];
  return ans;
}
```

4.5 Convex Hull

```
double cross(Pt o, Pt a, Pt b){
  return (a-o) ^ (b-o);
}
vector<Pt> convex_hull(vector<Pt> pt){
  sort(pt.begin(),pt.end());
  int top=0;
  vector<Pt> stk(2*pt.size());
  for (int i=0; i<(int)pt.size(); i++){
    while (top >= 2 && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-2],stk[top-1],pt[i
        ]) <= 0)
      top--;
    stk[top++] = pt[i];
  }
  stk.resize(top-1);
  return stk;
}
```

4.6 Convex Hull 3D

```
struct Pt{
  Pt cross(const Pt &p) const
  { return Pt(y * p.z - z * p.y, z * p.x - x * p.z, x *
       p.y - y * p.x); }
} info[N];
int mark[N][N],n, cnt;;
double mix(const Pt &a, const Pt &b, const Pt &c)
{ return a * (b ^ c); }
double area(int a, int b, int c)
{ return norm((info[b] - info[a]) ^ (info[c] - info[a])
    ); }
double volume(int a, int b, int c, int d)
{ return mix(info[b] - info[a], info[c] - info[a], info
    [d] - info[a]); }
struct Face{
  int a, b, c; Face(){}
  Face(int a, int b, int c): a(a), b(b), c(c) {}
  int &operator [](int k)
  { if (k == 0) return a; if (k == 1) return b; return
      c; }
};
vector<Face> face;
void insert(int a, int b, int c)
{ face.push_back(Face(a, b, c)); }
void add(int v) {
  vector <Face> tmp; int a, b, c; cnt++;
  for (int i = 0; i < SIZE(face); i++) {
    a = face[i][0]; b = face[i][1]; c = face[i][2];
    if(Sign(volume(v, a, b, c)) < 0)
    mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] =
        mark[c][a] = mark[a][c] = cnt;
    else tmp.push_back(face[i]);
  } face = tmp;
  for (int i = 0; i < SIZE(tmp); i++) {
    a = face[i][0]; b = face[i][1]; c = face[i][2];
    if (mark[a][b] == cnt) insert(b, a, v);
    if (mark[b][c] == cnt) insert(c, b, v);
    if (mark[c][a] == cnt) insert(a, c, v);
}}
int Find(){
  for (int i = 2; i < n; i++) {
    Pt ndir = (info[0] - info[i]) ^ (info[1] - info[i])
        ;
    if (ndir == Pt()) continue; swap(info[i], info[2]);
    for (int j = i + 1; j < n; j++) if (Sign(volume(0,
        1, 2, j)) != 0) {
      swap(info[j], info[3]); insert(0, 1, 2); insert
          (0, 2, 1); return 1;
} } return 0; }
int main() {
  for (; scanf("%d", &n) == 1; ) {
    for (int i = 0; i < n; i++) info[i].Input();
    sort(info, info + n); n = unique(info, info + n) -
        info;
    face.clear(); random_shuffle(info, info + n);
    if (Find()) { memset(mark, 0, sizeof(mark)); cnt =
        0;
      for (int i = 3; i < n; i++) add(i); vector<Pt>
          Ndir;
      for (int i = 0; i < SIZE(face); ++i) {
        Pt p = (info[face[i][0]] - info[face[i][1]]) ^
               (info[face[i][2]] - info[face[i][1]]);
        p = p / norm( p ); Ndir.push_back(p);
      } sort(Ndir.begin(), Ndir.end());
      int ans = unique(Ndir.begin(), Ndir.end()) - Ndir
          .begin();
      printf("%d\n", ans);
    } else printf("1\n");
} }
double calcDist(const Pt &p, int a, int b, int c)
{ return fabs(mix(info[a] - p, info[b] - p, info[c] - p
    ) / area(a, b, c)); }
//compute the minimal distance of center of any faces
```

```
double findDist() { //compute center of mass
  double totalWeight = 0; Pt center(.0, .0, .0);
  Pt first = info[face[0][0]];
  for (int i = 0; i < SIZE(face); ++i) {
    Pt p = (info[face[i][0]]+info[face[i][1]]+info[face
        [i][2]]+first)*.25;
    double weight = mix(info[face[i][0]] - first, info[
        face[i][1]]
        - first, info[face[i][2]] - first);
    totalWeight += weight; center = center + p * weight
        ;
  } center = center / totalWeight;
  double res = 1e100; //compute distance
  for (int i = 0; i < SIZE(face); ++i)
    res = min(res, calcDist(center, face[i][0], face[i
        ][1], face[i][2]));
    return res; }
```

4.7 Farthest pair

```
double FarthestPair(vector<Pt> arr){
    //Need to make convex hull first
    double ret=0;
    for(int i = 0, j = i+1; i<arr.size(); i++){
        while(distance(arr[i], arr[j]) <= distance(arr[
            i], arr[(j+1)%arr.size()]) ){
            j = (j+1) % arr.size();
        }
        ret = max(ret, distance(arr[i],arr[j]));
    }
    return ret;
}
```

4.8 Intersection of 2 segments

```
int ori( const Pt& o , const Pt& a , const Pt& b ){
  LL ret = ( a - o ) ^ ( b - o );
  return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
             const Pt& q1 , const Pt& q2 ){
  if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
    if( ori( p1 , p2 , q1 ) ) return false;
    return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
           ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
           ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
           ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
  }
  return (ori( p1, p2, q1 ) * ori( p1, p2, q2 )<=0) &&
         (ori( q1, q2, p1 ) * ori( q1, q2, p2 )<=0);
}
```

4.9 Intersection of circle and segment

```
bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
  Pt dp = p2 - p1;
  double a = dp * dp;
  double b = 2 * ( dp * ( p1 - cc.O ) );
  double c = cc.O * cc.O + p1 * p1 - 2 * ( cc.O * p1 )
      - cc.R * cc.R;
  double bb4ac = b * b - 4 * a * c;
  return !( fabs( a ) < eps or bb4ac < 0 );
}
```

4.10 Intersection of polygon and circle

```
ld PCIntersect(vector<Pt> v, Circle cir) {
  for(int i = 0 ; i < (int)v.size() ; ++i) v[i] = v[i]
      - cir.o;
  ld ans = 0, r = cir.r;
  int n = v.size();
  for(int i = 0 ; i < n ; ++i) {
    Pt pa = v[i], pb = v[(i+1)%n];
    if(norm(pa) < norm(pb)) swap(pa, pb);
    if(dcmp(norm(pb)) == 0) continue;
    ld s, h, theta;
    ld a = norm(pb), b = norm(pa), c = norm(pb-pa);
    ld cosB = (pb*(pb-pa))/a/c, B = acos(cosB);
    if(cosB > 1) B = 0;
    else if(cosB < -1) B = PI;
    ld cosC = (pa*pb)/a/b, C = acos(cosC);
    if(cosC > 1) C = 0;
    else if(cosC < -1) C = PI;
    if(a > r) {
      s = (C/2)*r*r;
      h = a*b*sin(C)/c;
      if(h < r && B < PI/2) s -= (acos(h/r)*r*r - h*
          sqrt(r*r-h*h));
    }
    else if(b > r) {
      theta = PI - B - asin(sin(B)/r*a);
      s = 0.5*a*r*sin(theta) + (C-theta)/2*r*r;
    }
    else s = 0.5*sin(C)*a*b;
    ans += abs(s)*dcmp(v[i]^v[(i+1)%n]);
  }
  return abs(ans);
}
```

4.11 Point In Polygon

```
int ptInPoly(vector<Pt> ps,Pt p){
  int c=0;
  for(int i=0;i<ps.size();++i){
    int a=i,b=(i+1)%ps.size(); Line l(ps[a],ps[b]);
    Pt q=l.s+l.v*((l.v*(p-l.s))/norm2(l.v)); // project
    if(norm(p-q)<eps&&onseg(q,l)) return 1; // boundary
    if(dcmp(ps[a].y-ps[b].y)==0&&dcmp(ps[a].y-p.y)==0)
      continue;
    if(ps[a].y>ps[b].y) swap(a,b);
    if(ps[a].y<=p.y&&p.y<ps[b].y&&p.x<=ps[a].x+(ps[b].x
        -ps[a].x)/(ps[b].y-ps[a].y)*(p.y-ps[a].y)) ++c;
  }
  return (c&1)*2; // 0: outside, 1: boundary, 2: inside
} // check whether a point is in a polygon
```

4.12 Intersection of 2 circles

```
vector<Pt> interCircle( Pt o1 , D r1 , Pt o2 , D r2 ){
  if( norm( o1 - o2 ) > r1 + r2 ) return {};
  if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
      return {};
  D d2 = ( o1 - o2 ) * ( o1 - o2 );
  D d = sqrt(d2);
  if( d > r1 + r2 ) return {};
  Pt u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
  D A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
  Pt v = Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
  return {u+v, u-v};
}
```

4.13 Circle cover

```
#define N 1021
#define D long double
struct CircleCover{
  int C; Circ c[ N ]; //填入C(圓數量),c(圓陣列)
  bool g[ N ][ N ], overlap[ N ][ N ];
  // Area[i] : area covered by at least i circles
  D Area[ N ];
  void init( int _C ){ C = _C; }
  bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
    Pt o1 = a.O , o2 = b.O;
    D r1 = a.R , r2 = b.R;
    if( norm( o1 - o2 ) > r1 + r2 ) return {};
    if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
        return {};
    D d2 = ( o1 - o2 ) * ( o1 - o2 );
    D d = sqrt(d2);
    if( d > r1 + r2 ) return false;
    Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
    D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
  }
  struct Teve {
    Pt p; D ang; int add;
    Teve() {}
    Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
    bool operator<(const Teve &a)const
    {return ang < a.ang;}
  }eve[ N * 2 ];
```

```
  // strict: x = 0, otherwise x = -1
  bool disjuct( Circ& a, Circ &b, int x )
  {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
  bool contain( Circ& a, Circ &b, int x )
  {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
  bool contain(int i, int j){
    /* c[j] is non-strictly in c[i]. */
    return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
                contain(c[i], c[j], -1);
  }
  void solve(){
    for( int i = 0 ; i <= C + 1 ; i ++ )
      Area[ i ] = 0;
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        overlap[i][j] = contain(i, j);
    for( int i = 0 ; i < C ; i ++ )
      for( int j = 0 ; j < C ; j ++ )
        g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjuct(c[i], c[j], -1));
    for( int i = 0 ; i < C ; i ++ ){
      int E = 0, cnt = 1;
      for( int j = 0 ; j < C ; j ++ )
        if( j != i && overlap[j][i] )
          cnt ++;
      for( int j = 0 ; j < C ; j ++ )
        if( i != j && g[i][j] ){
          Pt aa, bb;
          CCinter(c[i], c[j], aa, bb);
          D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
          D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
          eve[E ++] = Teve(bb, B, 1);
          eve[E ++] = Teve(aa, A, -1);
          if(B > A) cnt ++;
        }
      if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
      else{
        sort( eve , eve + E );
        eve[E] = eve[0];
        for( int j = 0 ; j < E ; j ++ ){
          cnt += eve[j].add;
          Area[cnt] += (eve[j].p ^ eve[j + 1].p) * 0.5;
          D theta = eve[j + 1].ang - eve[j].ang;
          if (theta < 0) theta += 2.0 * pi;
          Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * 0.5;
}}}}};
```

4.14 Convex Hull trick

```
/* Given a convexhull, answer querys in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
  int n;
  vector<Pt> a;
  vector<Pt> upper, lower;
  Conv(vector<Pt> _a) : a(_a){
    n = a.size();
    int ptr = 0;
    for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
    for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
    for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
    upper.push_back(a[0]);
  }
  int sign( LL x ){ // fixed when changed to double
    return x < 0 ? -1 : x > 0; }
  pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
    int l = 0, r = (int)conv.size() - 2;
    for( ; l + 1 < r; ){
      int mid = (l + r) / 2;
      if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
      else l = mid;
    }
    return max(make_pair(det(vec, conv[r]), r),
               make_pair(det(vec, conv[0]), 0));
  }
  void upd_tang(const Pt &p, int id, int &i0, int &i1){
    if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
    if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
  }
  void bi_search(int l, int r, Pt p, int &i0, int &i1){
    if(l == r) return;
    upd_tang(p, l % n, i0, i1);
    int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
      if (smid == sl) l = mid;
      else r = mid;
    }
    upd_tang(p, r % n, i0, i1);
  }
  int bi_search(Pt u, Pt v, int l, int r) {
    int sl = sign(det(v - u, a[l % n] - u));
    for( ; l + 1 < r; ) {
      int mid = (l + r) / 2;
      int smid = sign(det(v - u, a[mid % n] - u));
      if (smid == sl) l = mid;
      else r = mid;
    }
    return l % n;
  }
  // 1. whether a given point is inside the CH
  bool contain(Pt p) {
    if (p.X < lower[0].X || p.X > lower.back().X)
        return 0;
    int id = lower_bound(lower.begin(), lower.end(), Pt
        (p.X, -INF)) - lower.begin();
    if (lower[id].X == p.X) {
      if (lower[id].Y > p.Y) return 0;
    }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
    id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        , INF), greater<Pt>()) - upper.begin();
    if (upper[id].X == p.X) {
      if (upper[id].Y < p.Y) return 0;
    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
  }
  // 2. Find 2 tang pts on CH of a given outside point
  // return true with i0, i1 as index of tangent points
  // return false if inside CH
  bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
  }
  // 3. Find tangent points of a given vector
  // ret the idx of vertex has max cross value with vec
  int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
  }
  // 4. Find intersection point of a given line
  // return 1 and intersection is on edge (i, next(i))
  // return 0 if no strictly intersection
  bool get_intersection(Pt u, Pt v, int &i0, int &i1){
   int p0 = get_tang(u - v), p1 = get_tang(v - u);
   if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
     if (p0 > p1) swap(p0, p1);
     i0 = bi_search(u, v, p0, p1);
     i1 = bi_search(u, v, p1, p0 + n);
     return 1;
   }
   return 0;
}  };
```

4.15 Tangent line of two circles

```
vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
  // sign1 = 1 for outer tang, -1 for inter tang
  vector<Line> ret;
  double d_sq = norm2( c1.O - c2.O );
  if( d_sq < eps ) return ret;
  double d = sqrt( d_sq );
  Pt v = ( c2.O - c1.O ) / d;
  double c = ( c1.R - sign1 * c2.R ) / d;
  if( c * c > 1 ) return ret;
  double h = sqrt( max( 0.0 , 1.0 - c * c ) );
  for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
    Pt n = { v.X * c - sign2 * h * v.Y ,
             v.Y * c + sign2 * h * v.X };
    Pt p1 = c1.O + n * c1.R;
    Pt p2 = c2.O + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
      p2 = p1 + perp( c2.O - c1.O );
    ret.push_back( { p1 , p2 } );
  }
  return ret;
}
```

4.16 Minimum distance of two convex

```
double TwoConvexHullMinDis(Pt P[],Pt Q[],int n,int m){
  int mn=0,mx=0; double tmp,ans=1e9;
  for(int i=0;i<n;++i) if(P[i].y<P[mn].y) mn=i;
  for(int i=0;i<m;++i) if(Q[i].y>Q[mx].y) mx=i;
  P[n]=P[0]; Q[m]=Q[0];
  for (int i=0;i<n;++i) {
    while(tmp=((Q[mx+1]-P[mn+1])^(P[mn]-P[mn+1]))>((Q[
        mx]-P[mn+1])^(P[mn]-P[mn+1]))) mx=(mx+1)%m;
    if(tmp<0) // pt to segment distance
      ans=min(ans,dis(Line(P[mn],P[mn+1]),Q[mx]));
    else // segment to segment distance
      ans=min(ans,dis(Line(P[mn],P[mn+1]),Line(Q[mx],Q[
          mx+1])));
    mn=(mn+1)%n;
  }
  return ans;
}
```

4.17 Poly Union

```
struct PY{
  int n; Pt pt[5]; double area;
  Pt& operator[](const int x){ return pt[x]; }
  void init(){ //n,pt[0~n-1] must be filled
    area=pt[n-1]^pt[0];
    for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
    if((area/=2)<0)reverse(pt,pt+n),area=-area;
} };
PY py[500]; pair<double,int> c[5000];
inline double segP(Pt &p,Pt &p1,Pt &p2){
  if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
  return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){ //py[0~n-1] must be filled
  int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
  for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
  for(i=0;i<n;i++){
    for(ii=0;ii<py[i].n;ii++){
      r=0;
      c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
      for(j=0;j<n;j++){
        if(i==j) continue;
        for(jj=0;jj<py[j].n;jj++){
          ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]))
              ;
          tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj
              +1]));
          if(ta==0 && tb==0){
            if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[
                i][ii])>0&&j<i){
              c[r++]=make_pair(segP(py[j][jj],py[i][ii
                  ],py[i][ii+1]),1);
              c[r++]=make_pair(segP(py[j][jj+1],py[i][
                  ii],py[i][ii+1]),-1);
            }
          }else if(ta>=0 && tb<0){
            tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
            td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
            c[r++]=make_pair(tc/(tc-td),1);
          }else if(ta<0 && tb>=0){
            tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
            td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
            c[r++]=make_pair(tc/(tc-td),-1);
      } } }
      sort(c,c+r);
      z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
          =0;
      for(j=1;j<r;j++){
        w=min(max(c[j].first,0.0),1.0);
        if(!d) s+=w-z;
        d+=c[j].second; z=w;
      }
      sum+=(py[i][ii]^py[i][ii+1])*s;
  } }
  return sum/2;
}
```

4.18 Lower Concave Hull

```
struct Line {
  mutable ll m, b, p;
  bool operator<(const Line& o) const { return m < o.m;
       }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->m == y->m) x->p = x->b > y->b ? inf : -inf;
    else x->p = div(y->b - x->b, x->m - y->m);
    return x->p >= y->p;
  }
  void insert_line(ll m, ll b) {
    auto z = insert({m, b, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
        erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll eval(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.m * x + l.b;
  }
};
```

4.19 Min Enclosing Circle

```
struct Mec{  // return pair of center and r
  int n;
  Pt p[ MXN ], cen;
  double r2;
  void init( int _n , Pt _p[] ){
    n = _n;
    memcpy( p , _p , sizeof(Pt) * n );
  }
  double sqr(double a){ return a*a; }
  Pt center(Pt p0, Pt p1, Pt p2) {
    Pt a = p1-p0;
    Pt b = p2-p0;
    double c1=norm2( a ) * 0.5;
    double c2=norm2( b ) * 0.5;
    double d = a ^ b;
    double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
    double y = p0.Y + (a.X * c2 - b.X * c1) / d;
    return Pt(x,y);
  }
  pair<Pt,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){
      if (norm2(cen-p[i]) <= r2) continue;
```

```
      cen = p[i];
      r2 = 0;
      for (int j=0; j<i; j++){
        if (norm2(cen-p[j]) <= r2) continue;
        cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
        r2 = norm2(cen-p[j]);
        for (int k=0; k<j; k++){
          if (norm2(cen-p[k]) <= r2) continue;
          cen = center(p[i],p[j],p[k]);
          r2 = norm2(cen-p[k]);
    } } }
    return {cen,sqrt(r2)};
} }mec;
```

4.20 Min/Max Enclosing Rectangle

```
/******* NEED REVISION *******/
/* uva819 - gifts large and small */
#define MAXN 100005
const double eps=1e-8;
const double inf=1e15;
class Coor {
 public:
  double x,y;
  Coor() {}
  Coor(double xi,double yi) { x=xi; y=yi; }
  Coor& operator+=(const Coor &b) { x+=b.x; y+=b.y;
      return *this; }
  const Coor operator+(const Coor &b) const { return (
      Coor)*this+=b; }
  Coor& operator-=(const Coor &b) { x-=b.x; y-=b.y;
      return *this; }
  const Coor operator-(const Coor &b) const { return (
      Coor)*this-=b; }
  Coor& operator*=(const double b) { x*=b; y*=b; return
       *this; }
  const Coor operator*(const double b) const { return (
      Coor)*this*=b; }
  Coor& operator/=(const double b) { x/=b; y/=b; return
       *this; }
  const Coor operator/(const double b) const { return (
      Coor)*this/=b; }
  const bool operator<(const Coor& b) const { return y<
      b.y-eps||fabs(y-b.y)<eps&&x<b.x; }
  const double len2() const { return x*x+y*y; }
  const double len() const { return sqrt(len2()); }
  const Coor perp() const { return Coor(y,-x); }
  Coor& standardize() {
    if(y<0||y==0&&x<0) {
      x=-x;
      y=-y;
    }
    return *this;
  }
  const Coor standardize() const { return ((Coor)*this)
      .standardize(); }
};
double dot(const Coor &a,const Coor &b) { return a.x*b.
    x+a.y*b.y; }
double dot(const Coor &o,const Coor &a,const Coor &b) {
     return dot(a-o,b-o); }
double cross(const Coor &a,const Coor &b) { return a.x*
    b.y-a.y*b.x; }
double cross(const Coor &o,const Coor &a,const Coor &b)
     { return cross(a-o,b-o); }
Coor cmpo;
const bool cmpf(const Coor &a,const Coor &b) {
  return cross(cmpo,a,b)>eps||fabs(cross(cmpo,a,b))<eps
      &&
    dot(a,cmpo,b)<-eps;
}
class Polygon {
 public:
  int pn;
  Coor p[MAXN];
  void convex_hull() {
    int i,tn=pn;
    for(i=1;i<pn;++i) if(p[i]<p[0]) swap(p[0],p[i]);
    cmpo=p[0];
    std::sort(p+1,p+pn,cmpf);
    for(i=pn=1;i<tn;++i) {
      while(pn>2&&cross(p[pn-2],p[pn-1],p[i])<=eps) --
          pn;
      p[pn++]=p[i];
    }
    p[pn]=p[0];
  }
};
Polygon pol;
double minarea,maxarea;
int slpn;
Coor slope[MAXN*2];
Coor lrec[MAXN*2],rrec[MAXN*2],trec[MAXN*2],brec[MAXN
    *2];
inline double xproject(Coor p,Coor slp) { return dot(p,
    slp)/slp.len(); }
inline double yproject(Coor p,Coor slp) { return cross(
    p,slp)/slp.len(); }
inline double calcarea(Coor lp,Coor rp,Coor bp,Coor tp,
    Coor slp) {
  return (xproject(rp,slp)-xproject(lp,slp))*(yproject(
      tp,slp)-yproject(bp,slp)); }
  inline void solve(){
    int i,lind,rind,tind,bind,tn;
    double pro,area1,area2,l,r,m1,m2;
    Coor s1,s2;
    pol.convex_hull();
    slpn=0; /* generate all critical slope */
    slope[slpn++]=Coor(1.0,0.0);
    slope[slpn++]=Coor(0.0,1.0);
    for(i=0;i<pol.pn;i++) {
      slope[slpn]=(pol.p[i+1]-pol.p[i]).standardize();
      if(slope[slpn].x>0) slpn++;
      slope[slpn]=(pol.p[i+1]-pol.p[i]).perp().
        standardize();
      if(slope[slpn].x>0) slpn++;
    }
    cmpo=Coor(0,0);
    std::sort(slope,slope+slpn,cmpf);
    tn=slpn;
    for(i=slpn=1;i<tn;i++)
      if(cross(cmpo,slope[i-1],slope[i])>0) slope[slpn
        ++]=slope[i];
    lind=rind=0; /* find critical touchpoints */
    for(i=0;i<pol.pn;i++) {
      pro=xproject(pol.p[i],slope[0]);
      if(pro<xproject(pol.p[lind],slope[0])) lind=i;
      if(pro>xproject(pol.p[rind],slope[0])) rind=i;
    }
    tind=bind=0;
    for(i=0;i<pol.pn;i++) {
      pro=yproject(pol.p[i],slope[0]);
      if(pro<yproject(pol.p[bind],slope[0])) bind=i;
      if(pro>yproject(pol.p[tind],slope[0])) tind=i;
    }
    for(i=0;i<slpn;i++) {
      while(xproject(pol.p[lind+1],slope[i])<=xproject(
            pol.p[lind],slope[i])+eps)
        lind=(lind==pol.pn-1?0:lind+1);
      while(xproject(pol.p[rind+1],slope[i])>=xproject(
            pol.p[rind],slope[i])-eps)
        rind=(rind==pol.pn-1?0:rind+1);
      while(yproject(pol.p[bind+1],slope[i])<=yproject(
            pol.p[bind],slope[i])+eps)
        bind=(bind==pol.pn-1?0:bind+1);
      while(yproject(pol.p[tind+1],slope[i])>=yproject(
            pol.p[tind],slope[i])-eps)
        tind=(tind==pol.pn-1?0:tind+1);
      lrec[i]=pol.p[lind];
      rrec[i]=pol.p[rind];
      brec[i]=pol.p[bind];
      trec[i]=pol.p[tind];
    }
    minarea=inf; /* find minimum area */
    for(i=0;i<slpn;i++) {
      area1=calcarea(lrec[i],rrec[i],brec[i],trec[i],
          slope[i]);
      if(area1<minarea) minarea=area1;
    }
    maxarea=minarea; /* find maximum area */
    for(i=0;i<slpn-1;i++) {
      l=0.0; r=1.0;
      while(l<r-eps) {
        m1=l+(r-l)/3;
```

```
        m2=l+(r-l)*2/3;
        s1=slope[i]*(1.0-m1)+slope[i+1]*m1;
        area1=calcarea(lrec[i],rrec[i],brec[i],trec[i],
            s1);
        s2=slope[i]*(1.0-m2)+slope[i+1]*m2;
        area2=calcarea(lrec[i],rrec[i],brec[i],trec[i],
            s2);
        if(area1<area2) l=m1;
        else r=m2;
      }
      s1=slope[i]*(1.0-l)+slope[i+1]*l;
      area1=calcarea(lrec[i],rrec[i],brec[i],trec[i],s1
          );
      if(area1>maxarea) maxarea=area1;
    }
  }
int main(){
  int i,casenum=1;
  while(scanf("%d",&pol.pn)==1&&pol.pn) {
    for(i=0;i<pol.pn;i++)
      scanf("%lf %lf",&pol.p[i].x,&pol.p[i].y);
    solve();
    //minarea, maxarea
  }
}
```

4.21 Area of Rectangles

```
struct AreaofRectangles{
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
    ll n, id, sid;
    pair<ll,ll> tree[MXN<<3];   // count, area
    vector<ll> ind;
    tuple<ll,ll,ll,ll> scan[MXN<<1];
    void pull(int i, int l, int r){
        if(tree[i].first)  tree[i].second = ind[r+1] -
            ind[l];
        else if(l != r){
            int mid = (l+r)>>1;
            tree[i].second = tree[cl(i)].second + tree[
                cr(i)].second;
        }
        else    tree[i].second = 0;
    }
    void upd(int i, int l, int r, int ql, int qr, int v
        ){
        if(ql <= l && r <= qr){
            tree[i].first += v;
            pull(i, l, r); return;
        }
        int mid = (l+r) >> 1;
        if(ql <= mid) upd(cl(i), l, mid, ql, qr, v);
        if(qr > mid) upd(cr(i), mid+1, r, ql, qr, v);
        pull(i, l, r);
    }
    void init(int _n){
        n = _n; id = sid = 0;
        ind.clear(); ind.resize(n<<1);
        fill(tree, tree+(n<<2), make_pair(0, 0));
    }
    void addRectangle(int lx, int ly, int rx, int ry){
        ind[id++] = lx; ind[id++] = rx;
        scan[sid++] = make_tuple(ly, 1, lx, rx);
        scan[sid++] = make_tuple(ry, -1, lx, rx);
    }
    ll solve(){
        sort(ind.begin(), ind.end());
        ind.resize(unique(ind.begin(), ind.end()) - ind
            .begin());
        sort(scan, scan + sid);
        ll area = 0, pre = get<0>(scan[0]);
        for(int i = 0; i < sid; i++){
            auto [x, v, l, r] = scan[i];
            area += tree[1].second * (x-pre);
            upd(1, 0, ind.size()-1, lower_bound(ind.
                begin(), ind.end(), l)-ind.begin(),
                lower_bound(ind.begin(),ind.end(),r)-
                ind.begin()-1, v);
            pre = x;
        }
        return area;
    }   }rect;
```

4.22 Min dist on Cuboid

```
typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
  if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
  if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                        x0+L, y0, H, W, L);
  if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                        x0, y0+W, L, H, W);
  if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                        x0-H, y0, H, W, L);
  if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                        x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
  if( z1!=0 && z1!=H ){
    if( y1==0 || y1==W )
      swap(y1,z1), swap(y2,z2), swap(W,H);
  else swap(x1,z1), swap(x2,z2), swap(L,H);
  }
  if (z1==H) z1=0, z2=H-z2;
  r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
  return r;
}
```

4.23 Heart of Triangle

```
Pt inCenter( Pt &A,  Pt &B,  Pt &C) { // 內心
  double a = norm(B-C), b = norm(C-A), c = norm(A-B);
  return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a,  Pt &b,  Pt &c) { // 外心
  Pt bb = b - a, cc = c - a;
  double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
  return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a,  Pt &b,  Pt &c) { // 垂心
  Pt ba = b - a, ca = c - a, bc = b - c;
  double Y = ba.Y * ca.Y * bc.Y,
    A = ca.X * ba.Y - ba.X * ca.Y,
    x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
    y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
  return Pt(x0, y0);
}
```

5 Graph

5.1 MaximumClique 最大團

```
#define N 111
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int linkto[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      linkto[i].reset(); v[i].reset();
  } }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int popcount(const Int& val)
  { return val.count(); }
  int lowbit(const Int& val)
  { return val._Find_first(); }
  int ans , stk[N];
  int id[N] , di[N] , deg[N];
  Int cans;
  void maxclique(int elem_num, Int candi){
    if(elem_num > ans){
      ans = elem_num; cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
    }
    int potential = elem_num + popcount(candi);
    if(potential <= ans) return;
    int pivot = lowbit(candi);
```

```
    Int smaller_candi = candi & (~linkto[pivot]);
    while(smaller_candi.count() && potential > ans){
      int next = lowbit(smaller_candi);
      candi[next] = !candi[next];
      smaller_candi[next] = !smaller_candi[next];
      potential --;
      if(next == pivot || (smaller_candi & linkto[next
          ]).count()){
        stk[elem_num] = next;
        maxclique(elem_num + 1, candi & linkto[next]);
  } } }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) linkto[di[i]][di[j]] = 1;
    Int cand; cand.reset();
    for(int i = 0 ; i < n ; i ++) cand[i] = 1;
    ans = 1;
    cans.reset(); cans[0] = 1;
    maxclique(0, cand);
    return ans;
} }solver;
```

5.2 MaximalClique 極大團

```
#define N 80
struct MaxClique{ // 0-base
  typedef bitset<N> Int;
  Int lnk[N] , v[N];
  int n;
  void init(int _n){
    n = _n;
    for(int i = 0 ; i < n ; i ++){
      lnk[i].reset(); v[i].reset();
  } }
  void addEdge(int a , int b)
  { v[a][b] = v[b][a] = 1; }
  int ans , stk[N], id[N] , di[N] , deg[N];
  Int cans;
  void dfs(int elem_num, Int candi, Int ex){
    if(candi.none()&&ex.none()){
      cans.reset();
      for(int i = 0 ; i < elem_num ; i ++)
        cans[id[stk[i]]] = 1;
      ans = elem_num; // cans is a maximal clique
      return;
    }
    int pivot = (candi|ex)._Find_first();
    Int smaller_candi = candi & (~lnk[pivot]);
    while(smaller_candi.count()){
      int nxt = smaller_candi._Find_first();
      candi[nxt] = smaller_candi[nxt] = 0;
      ex[nxt] = 1;
      stk[elem_num] = nxt;
      dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
  } }
  int solve(){
    for(int i = 0 ; i < n ; i ++){
      id[i] = i; deg[i] = v[i].count();
    }
    sort(id , id + n , [&](int id1, int id2){
          return deg[id1] > deg[id2]; });
    for(int i = 0 ; i < n ; i ++) di[id[i]] = i;
    for(int i = 0 ; i < n ; i ++)
      for(int j = 0 ; j < n ; j ++)
        if(v[i][j]) lnk[di[i]][di[j]] = 1;
    ans = 1; cans.reset(); cans[0] = 1;
    dfs(0, Int(string(n,'1')), 0);
    return ans;
} }solver;
```

5.3 Strongly Connected Component

```
// u or v: addEdge(u, ~v); addEdge(v, ~u);
// u => v: addEdge(~u, ~v); addEdge(v, u);
// bln[i] == bln[n+i] => no solution
struct Scc{// 0-based
  int n, nScc, vst[MXN], bln[MXN];
  vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
    for (int i=0; i<MXN; i++)
      E[i].clear(), rE[i].clear();
  }
  void addEdge(int u, int v){
    E[u].PB(v); rE[v].PB(u);
  }
  void DFS(int u){
    vst[u]=1;
    for(auto v : E[u]) if(!vst[v]) DFS(v);
    vec.PB(u);
  }
  void rDFS(int u){
    vst[u] = 1; bln[u] = nScc;
    for(auto v : rE[u]) if(!vst[v]) rDFS(v);
  }
  void solve(){
    nScc = 0;
    vec.clear();
    fill(vst, vst+n+1, 0);
    for(int i=0; i<n; i++)
      if(!vst[i]) DFS(i);
    reverse(vec.begin(),vec.end());
    fill(vst, vst+n+1, 0);
    for(auto v : vec)
      if(!vst[v]){
          rDFS(v); nScc++;
      }
  }
}scc;
```

5.4 Dynamic MST

```
/* Dynamic MST O( Q lg^2 Q )
 (qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
 delete an edge: (i, \infty)
 add an edge: change from \infty to specific value   */
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
  int root=xx; while(a[root]) root=a[root];
  int next; while((next=a[xx])){a[xx]=root; xx=next; }
  return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
  if(Q==1){
    for(int i=1;i<=n;i++) a[i]=0;
    z[ qx[0] ]=qy[0]; tz = z;
    for(int i=0;i<m1;i++) id[i]=i;
    sort(id,id+m1,cmp); int ri,rj;
    for(int i=0;i<m1;i++){
      ri=find(x[id[i]]); rj=find(y[id[i]]);
      if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
    }
    printf("%lld\n",ans);
    return;
  }
  int ri,rj;
  //contract
  kt=0;
  for(int i=1;i<=n;i++) a[i]=0;
  for(int i=0;i<Q;i++){
    ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[
        ri]=rj;
  }
  int tm=0;
  for(int i=0;i<m1;i++) extra[i]=true;
  for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
  for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
  tz=z; sort(id,id+tm,cmp);
  for(int i=0;i<tm;i++){
    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
      a[ri]=rj; ans += z[id[i]];
```

```
      kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
  } }
  for(int i=1;i<=n;i++) a[i]=0;
  for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
  int n2=0;
  for(int i=1;i<=n;i++) if(a[i]==0)
  vd[i]=++n2;
  for(int i=1;i<=n;i++) if(a[i])
  vd[i]=vd[find(i)];
  int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
  for(int i=0;i<m1;i++) app[i]=-1;
  for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
    app[qx[i]]=m2; m2++;
  }
  for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[
      i]]; }
  for(int i=1;i<=n2;i++) a[i]=0;
  for(int i=0;i<tm;i++){
    ri=find(vd[ x[id[i]] ]);  rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
      a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
      Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
  } }
  int mid=Q/2;
  solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
  solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
  scanf("%d%d",&n,&m);
  for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
  scanf("%d",&Q);
  for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i
      ]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
```

5.5 Maximum General graph Matching

```
// should shuffle vertices and edges
const int N=100005,E=(2e5)*2+40;
struct Graph{ // 1-based; match: i <-> lnk[i]
  int to[E],bro[E],head[N],e,lnk[N],vis[N],stp,n;
  void init(int _n){
    stp=0; e=1; n=_n;
    for(int i=1;i<=n;i++) head[i]=lnk[i]=vis[i]=0;
  }
  void add_edge(int u,int v){
    to[e]=v,bro[e]=head[u],head[u]=e++;
    to[e]=u,bro[e]=head[v],head[v]=e++;
  }
  bool dfs(int x){
    vis[x]=stp;
    for(int i=head[x];i;i=bro[i]){
      int v=to[i];
      if(!lnk[v]){ lnk[x]=v,lnk[v]=x; return true; }
    }
    for(int i=head[x];i;i=bro[i]){
      int v=to[i];
      if(vis[lnk[v]]<stp){
        int w=lnk[v]; lnk[x]=v,lnk[v]=x,lnk[w]=0;
        if(dfs(w)) return true;
        lnk[w]=v,lnk[v]=w,lnk[x]=0;
      }
    }
    return false;
  }
  int solve(){
    int ans=0;
    for(int i=1;i<=n;i++) if(!lnk[i]) stp++,ans+=dfs(i)
        ;
    return ans;
  }
}graph;
```

5.6 Minimum General Weighted Matching

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match)
  static const int MXN = 105;
  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;
  void init(int _n) {
    n = _n;
    for( int i = 0 ; i < n ; i ++ )
      for( int j = 0 ; j < n ; j ++ )
        edge[ i ][ j ] = 0;
  }
  void add_edge(int u, int v, int w)
  { edge[u][v] = edge[v][u] = w; }
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){
      if (u != v && match[u] != v && !onstk[v]){
        int m = match[v];
        if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
          dis[m] = dis[u] - edge[v][m] + edge[u][v];
          onstk[v] = 1;
          stk.PB(v);
          if (SPFA(m)) return true;
          stk.pop_back();
          onstk[v] = 0;
    } } }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  }
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
      match[i] = i+1;
      match[i+1] = i;
    }
    while (true){
      int found = 0;
      for( int i = 0 ; i < n ; i ++ )
        onstk[ i ] = dis[ i ] = 0;
      for (int i=0; i<n; i++){
        stk.clear();
        if (!onstk[i] && SPFA(i)){
          found = 1;
          while (SZ(stk)>=2){
            int u = stk.back(); stk.pop_back();
            int v = stk.back(); stk.pop_back();
            match[u] = v;
            match[v] = u;
      } } }
      if (!found) break;
    }
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
  }
}graph;
```

5.7 BCC based on vertex

```
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n; nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();
  }
  void addEdge(int u, int v)
  { E[u].PB(v); E[v].PB(u); }
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
          int z;
```

```
        sccv[nScc].clear();
        do {
          z = stk[--top];
          sccv[nScc].PB(z);
        } while (z != v);
        sccv[nScc++].PB(u);
      }
    }else
      low[u] = min(low[u],dfn[v]);
  } }
  vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++)
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
      }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
  }
}graph;
```

5.8 Min Mean Cycle 最小平均數環

```
/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
  struct Edge { int v,u; double c; };
  int n, m, prv[V][V], prve[V][V], vst[V];
  Edge e[E];
  vector<int> edgeID, cycle, rho;
  double d[V][V];
  void init( int _n )
  { n = _n; m = 0; }
  // WARNING: TYPE matters
  void addEdge( int vi , int ui , double ci )
  { e[ m ++ ] = { vi , ui , ci }; }
  void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
      fill(d[i+1], d[i+1]+n, inf);
      for(int j=0; j<m; j++) {
        int v = e[j].v, u = e[j].u;
        if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
          d[i+1][u] = d[i][v]+e[j].c;
          prv[i+1][u] = v;
          prve[i+1][u] = j;
  } } } }
  double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
      double avg=-inf;
      for(int k=0; k<n; k++) {
        if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i
            ])/(n-k));
        else avg=max(avg,inf);
      }
      if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    fill(vst,0); edgeID.clear(); cycle.clear(); rho.
        clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
      vst[st]++;
      edgeID.PB(prve[i][st]);
      rho.PB(st);
    }
    while (vst[st] != 2) {
      if(rho.empty()) return inf;
      int v = rho.back(); rho.pop_back();
      cycle.PB(v);
      vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
} }mmc;
```

5.9 Directed Graph Min Cost Cycle

```
// works in O(N M)
#define INF 100000000000000LL
#define N 5010
#define M 200010
struct edge{
  int to; LL w;
  edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
  LL d; int u, next;
  node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
  vector<edge> g[N], grev[N];
  LL dp[N][N], p[N], d[N], mu;
  bool inq[N];
  int n, bn, bsz, hd[N];
  void b_insert(LL d, int u){
    int i = d/mu;
    if(i >= bn) return;
    b[++bsz] = node(d, u, hd[i]);
    hd[i] = bsz;
  }
  void init( int _n ){
    n = _n;
    for( int i = 1 ; i <= n ; i ++ )
      g[ i ].clear();
  }
  void addEdge( int ai , int bi , LL ci )
  { g[ai].push_back(edge(bi,ci)); }
  LL solve(){
    fill(dp[0], dp[0]+n+1, 0);
    for(int i=1; i<=n; i++){
      fill(dp[i]+1, dp[i]+n+1, INF);
      for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
        for(int k=0; k<(int)g[j].size(); k++)
          dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                                 dp[i-1][j]+g[j][k].w);
    } }
    mu=INF; LL bunbo=1;
    for(int i=1; i<=n; i++) if(dp[n][i] < INF){
      LL a=-INF, b=1;
      for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
        if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
          a = dp[n][i]-dp[j][i];
          b = n-j;
        } }
        if(mu*b > bunbo*a)
          mu = a, bunbo = b;
    }
    if(mu < 0) return -1; // negative cycle
    if(mu == INF) return INF; // no cycle
    if(mu == 0) return 0;
    for(int i=1; i<=n; i++)
      for(int j=0; j<(int)g[i].size(); j++)
      g[i][j].w *= bunbo;
    memset(p, 0, sizeof(p));
    queue<int> q;
    for(int i=1; i<=n; i++){
      q.push(i);
      inq[i] = true;
    }
    while(!q.empty()){
      int i=q.front(); q.pop(); inq[i]=false;
      for(int j=0; j<(int)g[i].size(); j++){
        if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
          p[g[i][j].to] = p[i]+g[i][j].w-mu;
          if(!inq[g[i][j].to]){
            q.push(g[i][j].to);
            inq[g[i][j].to] = true;
    } } } }
    for(int i=1; i<=n; i++) grev[i].clear();
    for(int i=1; i<=n; i++)
      for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w += p[i]-p[g[i][j].to];
        grev[g[i][j].to].push_back(edge(i, g[i][j].w));
      }
```

```
    LL mldc = n*mu;
    for(int i=1; i<=n; i++){
      bn=mldc/mu, bsz=0;
      memset(hd, 0, sizeof(hd));
      fill(d+i+1, d+n+1, INF);
      b_insert(d[i]=0, i);
      for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
          b[k].next){
        int u = b[k].u;
        LL du = b[k].d;
        if(du > d[u]) continue;
        for(int l=0; l<(int)g[u].size(); l++) if(g[u][l
            ].to > i){
          if(d[g[u][l].to] > du + g[u][l].w){
            d[g[u][l].to] = du + g[u][l].w;
            b_insert(d[g[u][l].to], g[u][l].to);
      } } }
      for(int j=0; j<(int)grev[i].size(); j++) if(grev[
          i][j].to > i)
        mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
    }
    return mldc / bunbo;
} }graph;
```

5.10 K-th Shortest Path

```
// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
  struct nd{
    int u, v; ll d;
    nd(int ui = 0, int vi = 0, ll di = INF)
    { u = ui; v = vi; d = di; }
  };
  struct heap{
    nd* edge; int dep; heap* chd[4];
  };
  static int cmp(heap* a,heap* b)
  { return a->edge->d > b->edge->d; }
  struct node{
    int v; ll d; heap* H; nd* E;
    node(){}
    node(ll _d, int _v, nd* _E)
    { d =_d; v = _v; E = _E; }
    node(heap* _H, ll _d)
    { H = _H; d = _d; }
    friend bool operator<(node a, node b)
    { return a.d > b.d; }
  };
  int n, k, s, t;
  ll dst[ N ];
  nd *nxt[ N ];
  vector<nd*> g[ N ], rg[ N ];
  heap *nullNd, *head[ N ];
  void init( int _n , int _k , int _s , int _t ){
    n = _n; k = _k; s = _s; t = _t;
    for( int i = 1 ; i <= n ; i ++ ){
      g[ i ].clear(); rg[ i ].clear();
      nxt[ i ] = NULL; head[ i ] = NULL;
      dst[ i ] = -1;
  } }
  void addEdge( int ui , int vi , ll di ){
    nd* e = new nd(ui, vi, di);
    g[ ui ].push_back( e );
    rg[ vi ].push_back( e );
  }
  queue<int> dfsQ;
  void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q;
    Q.push(node(0, t, NULL));
    while (!Q.empty()){
      node p = Q.top(); Q.pop();
      if(dst[p.v] != -1) continue;
      dst[ p.v ] = p.d;
      nxt[ p.v ] = p.E;
      dfsQ.push( p.v );
      for(auto e: rg[ p.v ])
        Q.push(node(p.d + e->d, e->u, e));
  } }
  heap* merge(heap* curNd, heap* newNd){
    if(curNd == nullNd) return newNd;
    heap* root = new heap;
    memcpy(root, curNd, sizeof(heap));
    if(newNd->edge->d < curNd->edge->d){
      root->edge = newNd->edge;
      root->chd[2] = newNd->chd[2];
      root->chd[3] = newNd->chd[3];
      newNd->edge = curNd->edge;
      newNd->chd[2] = curNd->chd[2];
      newNd->chd[3] = curNd->chd[3];
    }
    if(root->chd[0]->dep < root->chd[1]->dep)
      root->chd[0] = merge(root->chd[0],newNd);
    else
      root->chd[1] = merge(root->chd[1],newNd);
    root->dep = max(root->chd[0]->dep, root->chd[1]->
        dep) + 1;
    return root;
  }
  vector<heap*> V;
  void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
      int u = dfsQ.front(); dfsQ.pop();
      if(!nxt[ u ]) head[ u ] = nullNd;
      else head[ u ] = head[nxt[ u ]->v];
      V.clear();
      for( auto&& e : g[ u ] ){
        int v = e->v;
        if( dst[ v ] == -1 ) continue;
        e->d += dst[ v ] - dst[ u ];
        if( nxt[ u ] != e ){
          heap* p = new heap;
          fill(p->chd, p->chd+4, nullNd);
          p->dep = 1;
          p->edge = e;
          V.push_back(p);
      } }
      if(V.empty()) continue;
      make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
      for( size_t i = 0 ; i < V.size() ; i ++ ){
        if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
        else V[i]->chd[2]=nullNd;
        if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
        else V[i]->chd[3]=nullNd;
      }
      head[u] = merge(head[u], V.front());
  } }
  vector<ll> ans;
  void first_K(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
      Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
      node p = Q.top(), q; Q.pop();
      ans.push_back( p.d );
      if(head[ p.H->edge->v ] != nullNd){
        q.H = head[ p.H->edge->v ];
        q.d = p.d + q.H->edge->d;
        Q.push(q);
      }
      for( int i = 0 ; i < 4 ; i ++ )
        if( p.H->chd[ i ] != nullNd ){
          q.H = p.H->chd[ i ];
          q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
              edge->d;
          Q.push( q );
  } }   }
  void solve(){ // ans[i] stores the i-th shortest path
    dijkstra();
    build();
    first_K(); // ans.size() might less than k
} }solver;
```

5.11 SPFA

```
#define MXN 200005
struct SPFA{
  int n;
  LL inq[MXN], len[MXN];
  vector<LL> dis;
  vector<pair<int, LL>> edge[MXN];
  void init(int _n){
    n = _n;
    dis.clear(); dis.resize(n, 1e18);
    for(int i = 0; i < n; i++){
      edge[i].clear();
      inq[i] = len[i] = 0;
  } }
  void addEdge(int u, int v, LL w){
    edge[u].push_back({v, w});
  }
  vector<LL> solve(int st = 0){
    deque<int> dq; //return {-1} if has negative cycle
    dq.push_back(st); //otherwise return dis from st
    inq[st] = 1; dis[st] = 0;
    while(!dq.empty()){
      int u = dq.front();  dq.pop_front();
      inq[u] = 0;
      for(auto [to, d] : edge[u]){
        if(dis[to] > d+dis[u]){
          dis[to] = d+dis[u];
          len[to] = len[u]+1;
          if(len[to] > n) return {-1};
          if(inq[to]) continue;
          (!dq.empty()&&dis[dq.front()] > dis[to]?
              dq.push_front(to) : dq.push_back(to));
          inq[to] = 1;
    } } }
    return dis;
} }spfa;
```

5.12 Count Cycles

```
// ord = sort by deg decreasing, rk[ord[i]] = i
// D[i] = edge point from rk small to rk big
for (int x : ord) { // c3
  for (int y : D[x]) vis[y] = 1;
  for (int y : D[x]) for (int z : D[y]) c3 += vis[z];
  for (int y : D[x]) vis[y] = 0;
}
for (int x : ord) { // c4
  for (int y : D[x]) for (int z : adj[y])
    if (rk[z] > rk[x]) c4 += vis[z]++;
  for (int y : D[x]) for (int z : adj[y])
    if (rk[z] > rk[x]) --vis[z];
} // both are O(M*sqrt(M))
```

5.13 差分約束

約束條件 $V_j - V_i \leq W$ addEdge(V_i, V_j, W) and run bellman-ford or spfa

5.14 eulerPath

```
#define FOR(i,a,b) for(int i=a;i<=b;i++)
int dfs_st[10000500],dfn=0;
int ans[10000500],cnt=0,num=0;
vector<int>G[1000050];
int cur[1000050];
int ind[1000050],out[1000050];
void dfs(int x){
    FOR(i,1,n)sort(G[i].begin(),G[i].end());
    dfs_st[++dfn]=x;
    memset(cur,-1,sizeof(cur));
    while(dfn>0){
        int u=dfs_st[dfn];
        int complete=1;
        for(int i=cur[u]+1;i<G[u].size();i++){
            int v=G[u][i];
            num++;
            dfs_st[++dfn]=v;
            cur[u]=i;
            complete=0;
            break;
        }
        if(complete)ans[++cnt]=u,dfn--;
    }
}
bool check(int &start){
    int l=0,r=0,mid=0;
    FOR(i,1,n){
        if(ind[i]==out[i]+1)l++;
        if(out[i]==ind[i]+1)r++,start=i;
        if(ind[i]==out[i])mid++;
    }
    if(l==1&&r==1&&mid==n-2)return true;
    l=1;
    FOR(i,1,n)if(ind[i]!=out[i])l=0;
    if(l){
        FOR(i,1,n)if(out[i]>0){
            start=i;
            break;
        }
        return true;
    }
    return false;
}
int main(){
    cin>>n>>m;
    FOR(i,1,m){
        int x,y;scanf("%d%d",&x,&y);
        G[x].push_back(y);
        ind[y]++,out[x]++;
    }
    int start=-1,ok=true;
    if(check(start)){
        dfs(start);
        if(num!=m){
            puts("What a shame!");
            return 0;
        }
        for(int i=cnt;i>=1;i--)
            printf("%d ",ans[i]);
        puts("");
    }
    else puts("What a shame!");
}
```

6 Tree

6.1 LCA

```
const int MXN = 2e5 + 5;
const int lgN = __lg(MXN);
int tin[MXN], tout[MXN], anc[MXN][lgN+5], ti = 0;
vector<int> E[MXN];
void dfs(int x, int f){
  anc[x][0] = f;
  tin[x] = ti++;
  for(auto i : E[x]){
    if(i == f) continue;
    dfs(i, x);
  }
  tout[x] = ti++;
}
void init(int n){
  for(int i=1; i<=lgN; i++)
    for(int u=0; u<n; u++)
      anc[u][i] = anc[anc[u][i-1]][i-1];
}
bool inanc(int x, int y){
  return tin[x]<=tin[y]&&tout[x]>=tout[y];
}
int lca(int x, int y){
  if(isanc(x, y)) return x;
  if(isanc(y, x)) return y;
  for(int i=lgN; i>=0; i--)
    if(!isanc(anc[x][i], y))
      x = anc[x][i];
  return anc[x][0];
}
```

6.2 TreeHash

```
// 比較兩棵樹的hash值，判斷是否為同構(Isomorphism)
#define ull unsigned long long
const ull mask = std::mt19937_64(time(nullptr))();
ull hash[N];
vector<ull> trees;
```

```
ull shift(ull x) {
  x ^= mask;
  x ^= x << 13;
  x ^= x >> 7;
  x ^= x << 17;
  x ^= mask;
  return x;
}
// rooted tree
// 對於根節點做DFS，找到整棵樹的hash值
void getHash(int x, int fa){
  hash[x] = 1;
  for(auto i : edge[x]){
    if(i == fa) continue;
    getHash(i, x);
    hash[x] += shift(hash[i]);
  }
  trees.insert(hash[x]);
}
// unrooted tree
// 找到樹的重心(最多兩個)，分別對重心做DFS，找到整棵樹
    的hash值(pair)
int sz[N], w[N];
vector<int> centroid;
void getCentroid(int x, int fa){
  sz[x] = 1;
  w[x] = 0;
  for(auto i : edge[x]){
    if(i == fa) continue;
    getCentroid(i, x);
    sz[x] += sz[i];
    w[x] = max(w[x], sz[i]);
  }
  w[x] = max(w[x], n-sz[x]);
  if(w[x] <= n/2)
    centroid.PB(x);
}
ull getHash(int x, int fa){
  ull hash = 1;
  for(auto i : edge[x]){
    if(i == fa) continue;
    hash += shift(getHash(i, x));
  }
  return hash;
}
```

7 String

7.1 PalTree

```
// len[s]是對應的回文長度
// num[s]是有幾個回文後綴
// cnt[s]是這個回文子字串在整個字串中的出現次數
// fail[s]是他長度次長的回文後綴，aba的fail是a
const int MXN = 1000010;
struct PalT{
  int nxt[MXN][26],fail[MXN],len[MXN];
  int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
  int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
  char s[MXN]={-1};
  int newNode(int l,int f){
    len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
    memset(nxt[tot],0,sizeof(nxt[tot]));
    diff[tot]=(l>0?l-len[f]:0);
    sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
    return tot++;
  }
  int getfail(int x){
    while(s[n-len[x]-1]!=s[n]) x=fail[x];
    return x;
  }
  int getmin(int v){
    dp[v]=fac[n-len[sfail[v]]-diff[v]];
    if(diff[v]==diff[fail[v]])
        dp[v]=min(dp[v],dp[fail[v]]);
    return dp[v]+1;
  }
  int push(){
    int c=s[n]-'a',np=getfail(lst);
    if(!(lst=nxt[np][c])){
      lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
      nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
    }
    fac[n]=n;
    for(int v=lst;len[v]>0;v=sfail[v])
        fac[n]=min(fac[n],getmin(v));
    return ++cnt[lst],lst;
  }
  void init(const char *_s){
    tot=lst=n=0;
    newNode(0,1),newNode(-1,1);
    for(;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
  }
}palt;
```

7.2 LIS

```
vector<int> getLIS(vector<int> v){
    //run in O(nlogn)
    vector<int> lis;
    for(auto i : v){
        if(lis.empty() || lis.back() < i)
            lis.push_back(i);
        else
            *lower_bound(lis.begin(), lis.end(), i) = i
                ;
    }
    return lis;
}
```

7.3 LCS to LIS

```
(1) LCS problem:

index: 0 1 2 3 4 5 6
--------------------
s1:    a b a c d
s2:    d b a a b c a

(2)matched positions:

  a     a     a     b     b
(0,2) (0,3) (0,6) (1,1) (1,4)
  a     a     a     c     d
(2,2) (2,3) (2,6) (3,5) (4,0)

(3)sort all pairs:

increasing in 1st components.
decreasing in 2nd components if ties.

(4) 1D LIS:

use 2nd components to LIS
```

7.4 KMP

```
/* len-failure[k]:
在k結尾的情況下，這個子字串可以由開頭
長度為(len-failure[k])的部分重複出現來表達

failure[k]為次長相同前綴後綴
如果我們不只想求最多，而且以0-base做為考量
，那可能的長度由大到小會是
failuer[k]、failure[failuer[k]-1]
、failure[failure[failuer[k]-1]-1]..
直到有值為0為止 */
int failure[MXN];
vector<int> KMP(string& t, string& p){
    vector<int> ret;
    if (p.size() > t.size()) return;
    for (int i=1, j=failure[0]=-1; i<p.size(); ++i){
        while (j >= 0 && p[j+1] != p[i])
            j = failure[j];
        if (p[j+1] == p[i]) j++;
        failure[i] = j;
    }
    for (int i=0, j=-1; i<t.size(); ++i){
        while (j >= 0 && p[j+1] != t[i])
            j = failure[j];
        if (p[j+1] == t[i]) j++;
```

```
        if (j == p.size()-1){
            ret.push_bck( i - p.size() + 1 );
            j = failure[j];
}   }   }
```

7.5 SAIS

```
const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
  bool _t[N*2];
  int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
      hei[N], r[N];
  int operator [] (int i){ return _sa[i]; }
  void build(int *s, int n, int m){
    memcpy(_s, s, sizeof(int) * n);
    sais(_s, _sa, _p, _q, _t, _c, n, m);
    mkhei(n);
  }
  void mkhei(int n){
    REP(i,n) r[_sa[i]] = i;
    hei[0] = 0;
    REP(i,n) if(r[i]) {
      int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
      while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
      hei[r[i]] = ans;
    }
  }
  void sais(int *s, int *sa, int *p, int *q, bool *t,
      int *c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
        ]-1]]++] = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i
        ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
        ]]]=p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
      neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
          [i])*sizeof(int));
      ns[q[lst=sa[i]]]=nmxz+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
        nsa[i]]]]] = p[nsa[i]]);
  }
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
  // should padding a zero in the back
  // ip is int array, len is array length
  // ip[0..n-1] != 0, and ip[len] = 0
  ip[len++] = 0;
  sa.build(ip, len, 128);
  for (int i=0; i<len; i++) {
    H[i] = sa.hei[i + 1];
    SA[i] = sa._sa[i + 1];
  }
  // resulting height, sa array \in [0,len)
}
```

7.6 Trie

```
// cnt[i] 有多少字串以此節點結尾(字串出現次數)
// sz[i] 有多少字串前綴包括此節點
struct trie{
  trie *nxt[26];
  int cnt, sz;
  trie(){cnt=0; sz=0; memset(nxt, 0, sizeof(nxt));}
};

trie root = new trie();
void insert(string s){
  trie *now = root;
  for(auto c : s){
    now->sz++;
    if(now->nxt[c-'a'] == nullptr)
      now->nxt[c-'a'] = new trie();
    now = now->nxt[c-'a'];
  }
  now->cnt++; now->sz++;
}
int query(string s){
  trie *now = root;
  for(auto c : s){
    if(now->nxt[c-'a'] == nullptr)
      return 0;
    now = now->nxt[c-'a'];
  }
  return now->cnt;
}
```

7.7 Z Value

```
int z[MAXN];
void Z_value(const string& s) { //z[i] = lcp(s[1...],s[
    i...])
  int i, j, left, right, len = s.size();
  left=right=0; z[0]=len;
  for(i=1;i<len;i++) {
    j=max(min(z[i-left],right-i),0);
    for(;i+j<len&&s[i+j]==s[j];j++);
    z[i]=j;
    if(i+z[i]>right) {
      right=i+z[i];
      left=i;
}   }   }
```

7.8 ZValue Palindrome

```
void z_value_pal(char *s,int len,int *z){
  len=(len<<1)+1;
  for(int i=len-1;i>=0;i--)
    s[i]=i&1?s[i>>1]:'@';
  z[0]=1;
  for(int i=1,l=0,r=0;i<len;i++){
    z[i]=i<r?min(z[l+l-i],r-i):1;
    while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
        ++z[i];
    if(i+z[i]>r) l=i,r=i+z[i];
} }
```

7.9 Smallest Rotation

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
  int a = 0, N = s.size(); s += s;
  rep(b,0,N) rep(k,0,N) {
    if(a+k == b || s[a+k] < s[b+k])
      {b += max(0, k-1); break;}
    if(s[a+k] > s[b+k]) {a = b; break;}
  } return a;
}
```

7.10 Cyclic LCS

```
#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
  int i=r+al,j=bl,l=0;
  while(i>r) {
    char dir=pred[i][j];
```

```
    if(dir==LU) l++;
    i+=mov[dir][0];
    j+=mov[dir][1];
  }
  return l;
}
inline void reroot(int r) { // r = new base row
  int i=r,j=1;
  while(j<=bl&&pred[i][j]!=LU) j++;
  if(j>bl) return;
  pred[i][j]=L;
  while(i<2*al&&j<=bl) {
    if(pred[i+1][j]==U) {
      i++;
      pred[i][j]=L;
    } else if(j<bl&&pred[i+1][j+1]==LU) {
      i++;
      j++;
      pred[i][j]=L;
    } else {
      j++;
} } }
int cyclic_lcs() {
  // a, b, al, bl should be properly filled
  // note: a WILL be altered in process
  //       -- concatenated after itself
  char tmp[MAXL];
  if(al>bl) {
    swap(al,bl);
    strcpy(tmp,a);
    strcpy(a,b);
    strcpy(b,tmp);
  }
  strcpy(tmp,a);
  strcat(a,tmp);
  // basic lcs
  for(int i=0;i<=2*al;i++) {
    dp[i][0]=0;
    pred[i][0]=U;
  }
  for(int j=0;j<=bl;j++) {
    dp[0][j]=0;
    pred[0][j]=L;
  }
  for(int i=1;i<=2*al;i++) {
    for(int j=1;j<=bl;j++) {
      if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
      else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
      if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
      else if(a[i-1]==b[j-1]) pred[i][j]=LU;
      else pred[i][j]=U;
  } }
  // do cyclic lcs
  int clcs=0;
  for(int i=0;i<al;i++) {
    clcs=max(clcs,lcs_length(i));
    reroot(i+1);
  }
  // recover a
  a[al]='\0';
  return clcs;
}
```

8 Data Structure

8.1 Segment tree

```
#define cl(x) (x<<1)
#define cr(x) (x<<1|1)
#define M ((l+r)>>1)
const int N = 5e5 + 5;
struct SegTree{
  int seg[N<<2], lz[N<<2];
  void push(int i, int l, int r){
    if(lz[i]){
        if(l != r){ lz[cl(i)] += lz[i]; lz[cr(i)] += lz
            [i]; }
        seg[i] += lz[i];
        lz[i] = 0;
    }
  }
  void pull(int i, int l, int r){
    if(l == r) return;
    push(cl(i), l, M);
    push(cr(i), M+1, r);
    seg[i] = seg[cl(i)] + seg[cr(i)];
  }
  void build(int i, int l, int r){
    if(l == r){ seg[i] = 0; return; }
    build(cl(i), l, M);
    build(cr(i), M+1, r);
    pull(i, l, r);
  }
  void upd(int i, int l, int r, int nl, int nr, int v){
    push(i, l, r);
    if(nl <= l && r <= nr){
        lz[i] += v;
        return;
    }

    if(nl <= M) upd(cl(i), l, M, nl, nr, v);
    if(M < nr) upd(cr(i), M+1, r, nl, nr, v);
    pull(i, l, r);
  }
  int qry(int i, int l, int r, int nl, int nr){
    push(i, l, r);
    if(nl <= l && r <= nr) return seg[i];

    int ret = 0;
    if(nl <= M) ret += qry(cl(i), l, M, nl, nr);
    if(M < nr) ret += qry(cr(i), M+1, r, nl, nr);
    return ret;
  }
}Seg;
```

8.2 Treap

```
struct Treap{
  int sz , val , pri , tag;
  Treap *l , *r;
  Treap( int _val ){
    val = _val; sz = 1;
    pri = rand(); l = r = NULL; tag = 0;
  }
};
void push( Treap * a ){
  if( a->tag ){
    Treap *swp = a->l; a->l = a->r; a->r = swp;
    int swp2;
    if( a->l ) a->l->tag ^= 1;
    if( a->r ) a->r->tag ^= 1;
    a->tag = 0;
} }
inline int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
  a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
  if( !a || !b ) return a ? a : b;
  if( a->pri > b->pri ){
    push( a );
    a->r = merge( a->r , b );
    pull( a );
    return a;
  }else{
    push( b );
    b->l = merge( a , b->l );
    pull( b );
    return b;
} }
void split_kth( Treap *t , int k, Treap*&a, Treap*&b ){
  if( !t ){ a = b = NULL; return; }
  push( t );
  if( Size( t->l ) + 1 <= k ){
    a = t;
    split_kth( t->r , k - Size( t->l ) - 1 , a->r , b )
        ;
    pull( a );
  }else{
    b = t;
    split_kth( t->l , k , a , b->l );
    pull( b );
} }
void split_key(Treap *t, int k, Treap*&a, Treap*&b){
```

```
  if(!t){ a = b = NULL; return; }
  push(t);
  if(k<=t->val){
    b = t;
    split_key(t->l,k,a,b->l);
    pull(b);
  }
  else{
    a = t;
    split_key(t->r,k,a->r,b);
    pull(a);
} }
```

8.3 Disjoint Set

```
struct DisjointSet {
  int fa[MXN], h[MXN], top;
  struct Node {
    int x, y, fa, h;
    Node(int _x = 0, int _y = 0, int _fa = 0, int _h =
        0)
        : x(_x), y(_y), fa(_fa), h(_h) {}
  } stk[MXN];
  void init(int n) {
    top = 0;
    for (int i = 1; i <= n; i++) fa[i] = i, h[i] = 0;
  }
  int find(int x) { return x == fa[x] ? x : find(fa[x])
      ; }
  void merge(int u, int v) {
    int x = find(u), y = find(v);
    if (h[x] > h[y]) swap(x, y);
    stk[top++] = Node(x, y, fa[x], h[y]);
    if (h[x] == h[y]) h[y]++;
    fa[x] = y;
  }
  void undo(int k=1) {  //undo k times
    for (int i = 0; i < k; i++) {
      Node &it = stk[--top];
      fa[it.x] = it.fa;
      h[it.y] = it.h;
} } }djs;
```

8.4 Sparse Table

```
struct ST{
  int n, st[N][20];
  ST(int _n){ n = _n; memset(st, 0, sizeof(st));}
  void build(int a[]){
    for(int i=1; i<=n; i++)
      st[i][0] = a[i];
    for(int j=1; j<20; j++)
      for(int i=1; i+(1<<j)-1<=n; i++)
        st[i][j] = min(st[i][j-1], st[i + (1 << (j-1))
            ][j-1]);
  }
  int query(int l, int r){
    int q = log2(r-l+1);
    return min(st[l][q], st[r - (1<<q) + 1][q]);
  }
}st;
```

8.5 Black Magic

```
#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
  // Insert some entries into s.
  set_t s; s.insert(12); s.insert(505);
  // The order of the keys should be: 12, 505.
  assert(*s.find_by_order(0) == 12);
  assert(*s.find_by_order(3) == 505);
  // The order of the keys should be: 12, 505.
  assert(s.order_of_key(12) == 0);
  assert(s.order_of_key(505) == 1);
  // Erase an entry.
  s.erase(12);
  // The order of the keys should be: 505.
  assert(*s.find_by_order(0) == 505);
  // The order of the keys should be: 505.
  assert(s.order_of_key(505) == 0);

  heap h1 , h2; h1.join( h2 );

  rope<char> r[ 2 ];
  r[ 1 ] = r[ 0 ]; // persistenet
  string t = "abc";
  r[ 1 ].insert( 0 , t.c_str() );
  r[ 1 ].erase( 1 , 1 );
  cout << r[ 1 ].substr( 0 , 2 );
}
```

9 Others

9.1 SOS dp

```
for(int i = 0; i<(1<<N); ++i)
  F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<
    N); ++mask){
  if(mask & (1<<i))
    F[mask] += F[mask^(1<<i)];
}
```

9.2 Number of Occurrences of Digit

```
int dp[MAXN][MAXN], a[MAXN];
int dfs(int pos, bool leadZero, bool bound, int sum,
    int digit) {
    if (!pos) return sum;
    if (!leadZero && !bound && dp[pos][sum] != -1)
        return dp[pos][sum];
    int top = bound ? a[pos] : 9, ans = 0;
    for (int i = 0; i <= top; ++i)
        ans += dfs(pos - 1, !(i || !leadZero), bound &&
             i == a[pos], sum + ((i == digit) && (i ||
            !leadZero)), digit);
    if (!leadZero && !bound) dp[pos][sum] = ans;
    return ans;
}
int pre(int r, int digit) { //return num of digit in
    [1, r]
    int cnt = 0;
    memset(dp, -1, sizeof dp);
    while (r != 0)
        a[++cnt] = r % 10, r /= 10;
    return dfs(cnt, 1, 1, 0, digit);
}
```

9.3 Find max tangent(x,y is increasing)

```
const int MAXN = 100010;
Pt sum[MAXN], pnt[MAXN], ans, calc;
inline bool cross(Pt a, Pt b, Pt c){
  return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
}//pt[0]=(0,0);pt[i]=(i,pt[i-1].y+dy[i-1]),i=1~n;dx>=l
double find_max_tan(int n,int l,LL dy[]){
  int np, st, ed, now;
  sum[0].x = sum[0].y = np = st = ed = 0;
  for (int i = 1, v; i <= n; i++)
    sum[i].x=i,sum[i].y=sum[i-1].y+dy[i-1];
  ans.x = now = 1,ans.y = -1;
  for (int i = 0; i <= n - l; i++){
    while(np>1&&cross(pnt[np-2],pnt[np-1],sum[i]))
      np--;
    if (np < now && np != 0) now = np;
    pnt[np++] = sum[i];
    while(now<np&&!cross(pnt[now-1],pnt[now],sum[i+l]))
      now++;
    calc = sum[i + l] - pnt[now - 1];
    if (ans.y * calc.x < ans.x * calc.y)
      ans = calc,st = pnt[now - 1].x,ed = i + l;
  }
  return (double)(sum[ed].y-sum[st].y)/(sum[ed].x-sum[
      st].x);
}
```