ME 7112: FINITE ELEMENT METHOD - Fall 2023

Assignment 6 (Due by 11:59 pm on 2023/12/11 Mon.)

Problem 1

Please write your own program for solving the following system of nonlinear equations for x_1 and x_2 by the Newton-Raphson method:

$$\begin{cases} x_1(x_1^2 + x_2^2 + 1)^{1/3} = \dfrac{9}{2} \\ x_2(x_1^2 + x_2^2 + 1)^{1/4} = -\dfrac{5}{2} \end{cases}$$

Problem 2

Please consider a long bar with its cross section h by h and length L. The bar is fully fixed at one of its ends and subjected to the displacement bondary condition at the other end for tensile testing with a maximum applied strain of +5%. Assume that the material with Poisson's ratio of 0.33 has a hypoelastic constitutive equation, so that the stress-strain relation has the form as described in our class.

1. While h = 0.5 cm and L = 10 cm, calculate the tensile stress curve versus strains of the bar with material properties: σ_0 = 5 MPa, n = 2, and ϵ_0 = 0.1.
2. Following 1., calculate the stress curve versus strains of the bar with material properties: σ_0 = 5 MPa, n = 10, and ϵ_0 = 0.1.
3. Following 1., calculate the stress curve versus strains of the bar with material properties: σ_0 = 12.5 MPa, n = 2, and ϵ_0 = 0.1.
4. Compare the stress curves versus strains obtained in 1, 2, and 3. Provide your comments on the material parameters of σ_0 and n.
5. Pick up one representive element in your mesh model of the bar. Compare the numerical stress values of the bar with the exact stress evaluation of the hypoelastic materials under various levels of applied strains.

Problem 3

Similar to Problem 2, please consider a long bar with its cross section h by h and length L. The bar is fully fixed at one of its ends and subjected to the displacement bondary condition at the other end for tensile testing with a maximum applied strain of +80%. Assume that the material has a hyperelastic constitutive equation, so that the stress-strain relation has the form as described in our class.

1. While h = 0.5 cm and L = 10 cm, calculate the tensile stress curve versus strains of the bar with material properties: μ_1 = 4.285714 MPa and K_1 = 20 MPa.
2. Following 1., calculate the tensile stress curve versus strains of the bar with material properties: μ_1 = 6.428571 MPa and K_1 = 30 MPa.
3. Following 1., calculate the stress curve versus strains of the bar with material properties: μ_1 = 4.137931 MPa and K_1 = 40 MPa.
4. Compare the stress curves versus strains obtained in 1, 2, and 3. Provide your comments on the material parameters of μ_1 and K_1.
5. Pick up one representive element in your mesh model of the bar. Compare the numerical stress values of the bar with the exact stress evaluation of the hyperelastic materials under various levels of applied strains.