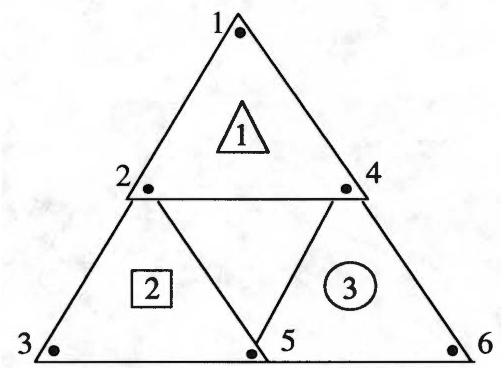
ME 7112: FINITE ELEMENT METHOD - Fall 2023

Assignment 3 (Due by 11:59 pm on 2023/10/30 Mon.)

Problem 1

Three identical constant-strain triangle elements are assembled in a two-dimensional plane strain problem as shown in the figure blew. Knowing that the side length of each triangle element is $L = 1$ mm, the materials properties including the Young's modulus $E = 2 \times 10^5$ MPa and the Poisson's raio $\nu = 0.33$:

1. Solve the assembled stiffness matrix of the system by your hand.
2. Solve 1. again using your own FEM code.
3. Make comparison between the answers of 1. and 2., and provide your comments.

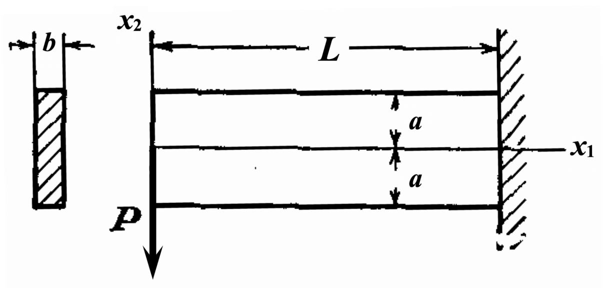


Problem 2

In this problem, the analysis focuses on a rectangular beam experiencing plane stress conditions, as illustrated in the accompanying figure. The precise characterization of the beam's geometry is delineated within the graphical representation. The fundamental solution, harking back to the seminal work of Timoshenko and Goodier, hinges on the strategic utilization of an Airy stress function approach. The specific context here entails displacement boundary conditions denoted by $u_1 = u_2 = 0$ at $x_1 = L$. The ensuing exact solutions for displacements, meticulously tailored to meet these stipulated conditions, are delineated as follows:

$$u_1(x_1, x_2) = \frac{3P}{4Ea^3b}x_1^2x_2 - \frac{P}{4Ea^3b}(2+\nu)x_2^3 + \frac{3P}{2Ea^3b}(1+\nu)a^2x_2 - \frac{3PL^2x_2}{4Ea^3b} \text{ and}$$
$$u_2(x_1, x_2) = -\nu\frac{3P}{4Ea^3b}x_1x_2^2 - \frac{P}{4Ea^3b}x_1^3 + \frac{3PL^2}{4Ea^3b}x_1 - \frac{PL^3}{2Ea^3b}$$

In the equations above, P represents the applied end load at the point $(x_1 = 0, x_2 = 0)$, a signifies half the height of the beam's cross section, b corresponds to the width of the beam's cross section, L denotes the length of the beam, E stands for Young's modulus, and ν embodies Poisson's ratio. For the purpose of numerical computations, specific values are attributed to these parameters: $P = 80\,\mathrm{kN}$, $a = 10\,\mathrm{mm}$, $b = 1\,\mathrm{mm}$, $L = 60\,\mathrm{mm}$, $E = 1000\,\mathrm{GPa}$, and $\nu = 0.3$. Notably, the beam is discretized into an assemblage of 48 identical quadrilateral elements, each measuring 5 mm × 5 mm for the purpose of modeling.



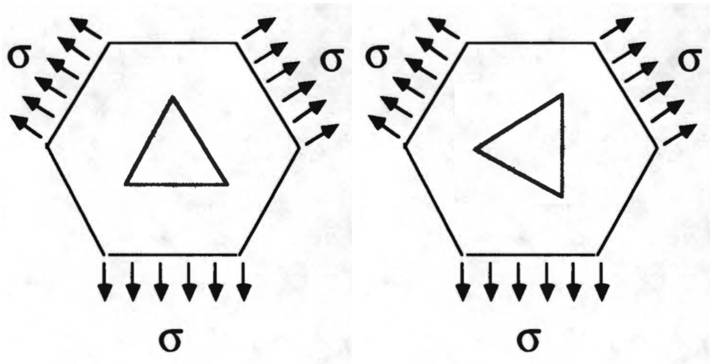
Please respond to the following questions:

1. Utilize your proprietary Finite Element Method (FEM) code, employing 4-node quadrilateral elements, to compute the numerical values of displacements $u_1(L/2, a/2)$ and $u_2(L/2, a/2)$.
2. Apply your personal FEM code utilizing 8-node quadrilateral elements to ascertain the numerical values of displacements $u_1(L/2, a/2)$ and $u_2(L/2, a/2)$.
3. Following the answers in 1. and 2., and in light of the exact solution provided earlier, kindly provide your comments on the implications of using linear and quadratic quadrilateral elements in the context of bending problems. Your comparative analysis should shed light on the advantages and limitations inherent in the application of these different element types in the analysis.

Problem 3

Two hexagonal bars have been pierced by triangular holes shown in the figures below and are loaded on three sides by uniform distributed stresses. The polygons are regular and are concentrically located such that their centroids coincide. Each represents a two-dimensional, plane strain problem, with the only difference being the orientation of the triangle. By using FEM, please determine which design has the greater localized stress.

(If necessary, set proper parameters of the calculation by yourself.)



Problem 4

Please numerially integrate the following functions using Gaussian integration:

1. $f(x) = 6 + 12\,x^3 + 4\,x^6$ over the interval $R: -1 \le x \le 1$.
2. $f(x, y) = 1 - 6\,x^2\,y$ over the region $R: 0 \le x \le 2$, $-1 \le y \le 1$.
3. $f(x, y) = 3 - x - y$ over the region bounded by the x-axis, and the lines $y = x$ and $x = 1$.
4. $f(x, y, z) = x\,y\,z$ over the region bounded by the coordinate planes and the plane $x = 2$, $y = 2$, and $z = 2$ in the first octant.

Is your answer close to the exact one? Please provide your comments.
