ME 7112: FINITE ELEMENT METHOD - Fall 2023

Assignment 5 (Due by 11:59 pm on 2023/11/27 Mon.)

Problem 1

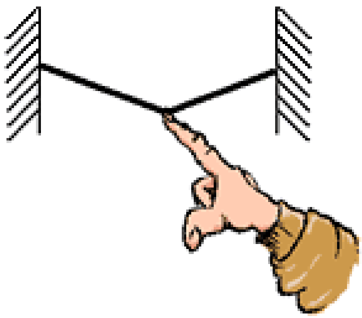
A nylon string of length L = 5 cm and density ρ = 1.14 g/cm^3, fixed at both ends with tension P = 10 N, is plucked at its midpoint. The initial displacement distribution of the string is denoted as $u(x, t=0)$ function as shown below. Then, the string is released without initial velocities:

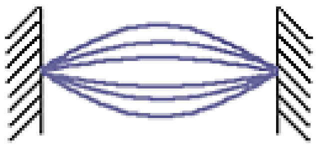
(1) Please determine the subsequent motion history of the string $u(x,t)$ by your FEM code.

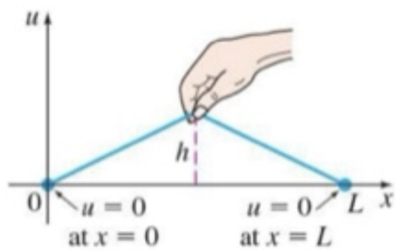
(2) The exact solution of $u(x,t)$ is:

$$u(x,t) = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi\, x}{L} \cos\frac{nc\pi\, t}{L}$$

where $C_n = \frac{8h}{\pi^2 n^2} \sin\frac{n\pi}{2}$ when n is an odd number ($C_n = 0$ when n: even number), c is $\sqrt{P/\rho}$. Is your solution solved by FEM close to the exact solutions? Please provide your comments.







Problem 2

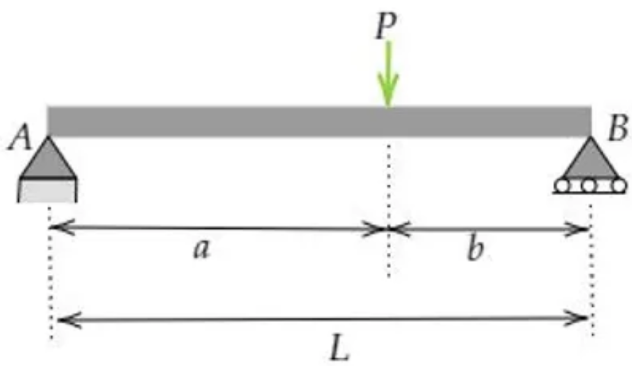
Consider a copper pinned-pinned beam with no initial motion subjected to a harmonic force: $P(x,t) = P_0 \sin\omega t$ at $x = a$, as shown below.

(1) Determine the steady motion $u(x,t)$ the beam by your FEM code.

(2) The exact solution of $u(x,t)$ is:

$$u(x,t) = \frac{2P_0}{\rho AL} \sum_{n=1}^{\infty} \frac{1}{\omega_n^2 - \omega^2} \sin\frac{n\pi a}{L} \sin\frac{n\pi x}{L} \sin\omega t$$

where ω_n is the natural frequency of the n-th mode of such beam, \rho is the density of the copper material, and A is the cross-sectional area of the beam. Please also apply the following data for your analysis: P_0 = 10 N, ω = 0.1 ω_1, Young's modulus of copper E = 130 GPa, the mass density of copper ρ = 8.96 g/cm^3, the width and height of the beam cross-section are both 3 cm, and L is 15 cm. Is your solution solved by FEM close to the exact solutions? Please provide your comments.



Problem 3

Suppose we could convert the beam in Problem 2 to a fixed-free axial bar with initial axial displacement condition $u_0(x) = u(x, t = 0) = \dfrac{F_0 x}{EA}$ caused by an suddenly axial tensile force F_0 (applied and removed immediately). Consider F_0 = 1 N, and please determine the vibration solution of the mechanical system:

(1) Apply your own FEM code.

(2) Compare your solution in (1) with the exact solution:

$$u(x,t) = \sum_{n=0}^{\infty} C_n \sin\frac{(2n+1)\pi x}{2L} \cos\frac{(2n+1)\pi ct}{2L}$$

where $C_n = \dfrac{2}{L}\displaystyle\int_0^L u_0(x) \sin\frac{(2n+1)\pi x}{2L} dx$, and $c = \sqrt{E/\rho}$. Is your solution solved by FEM close to the exact solutions? Please provide your comments.

$u = 0$