ME 7112: FINITE ELEMENT METHOD - Fall 2023

Assignment 4 (Due by 11:59 pm on 2023/11/14 Mon.)

Problem 1

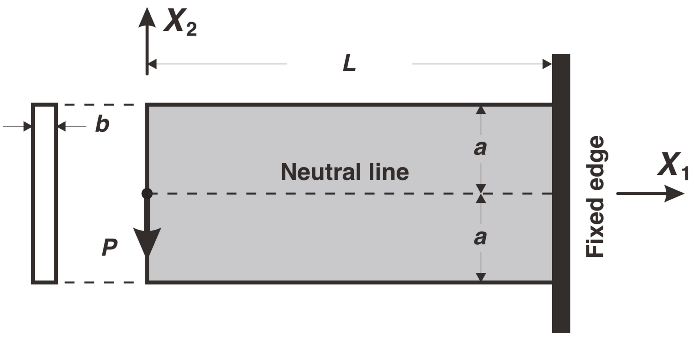
A rectangular beam in a state of plane stress show in the following figure is considered. The geometric properties are shown in the figure. The exact solution was found by Timoshenko and Goodier based on use of an Airy stress function solution. The exact solutions for displacements satisfying the displacement boundary conditions of $u_1 = u_2 = 0$ at $x_1 = L$ are:

$$u_1(x_1, x_2) = \frac{3P}{4Ea^3b}x_1^2x_2 - \frac{P}{4Ea^3b}(2+\nu)x_2^3 + \frac{3P}{2Ea^3b}(1+\nu)a^2x_2 - \frac{3PL^2x_2}{4Ea^3b}$$

and

$$u_2(x_1, x_2) = -\nu\frac{3P}{4Ea^3b}x_1x_2^2 - \frac{P}{4Ea^3b}x_1^3 + \frac{3PL^2}{4Ea^3b}x_1 - \frac{PL^3}{2Ea^3b}$$

where P is the applied end load at $(x_1 = 0, x_2 = 0)$, a is the half height of the beam cross section, b is the width of the beam cross section, L is the beam length, E is the Young's modulus, and ν is the Poisson's ratio. For the numerical computation, the parameters are chosen as: $P = 80\,\mathrm{kN}$, $a = 10\,\mathrm{mm}$, $b = 1\,\mathrm{mm}$, $E = 1000\,\mathrm{GPa}$, and $\nu = 0.3$. The beam is then modeled as an assemblage of *identical 4-node-quadrilateral elements*, each $5\,\mathrm{mm} \times 5\,\mathrm{mm}$.



1. Consider the two cases $L = 2a \times C$, and $C = 2$ and 3, what are the displacements at the application point of P based on the Airy stress function solution above?
2. By using your own FEM code, what the the displacements at the application point of P? Are those close to the exact solutions? Please provide your comments.

Problem 2

Following Problem 1, it is well known that as L is getting increased, the Euler-Bernoulli beam theory should be adopted for the beam deflection fields instead of the elasticity solution above. However, because the element interpolation functions are intrinsically unable to approximate accurately the material strain distributions in the beam with larger L (the beam is relatively long), we cannot get correct results through conventional finite element analysis discussed in Chapter 2. The phenomenon is commonly called *shear locking* of finite elements.

1. Consider the two cases $L = 2a \times C$, and $C = 8$ and 10, what are the displacements at the application point of P based on the Euler-Bernoulli beam therory?
2. By using your own FEM code, what are the the displacements at the application point of P? Are those close to the exact solutions? Please provide your comments.

Problem 3

The problem of a square steel plate totally fixed along all four edges and subjected to a concentrated load P of 100 lb at its center. Please determine the maximum vertical deflection of the plate. The following numerical informations should be considered:

- Plate width L : 20 in
- Plate thickness t : 0.1 in

- Young's modulus of steel E : 30 $\times$ 10^6 psi
- Poisson's ratio of steel ν : 0.3

1. By using your own FEM code, what is the maximum vertical deflection of the plate?
2. The exact solution of this problem is: $w_{\max} = 0.0056PL^2/D$, where $D = Et^3/(12(1-\nu^2))$. Is your solution solved by FEM close to the exact solutions? Please provide your comments.

Problem 4

Consider a cantilever beam with the following engineering data:

- Length of the beam: L : 30 in
- Young's modulus of steel E : 3 $\times$ 10^7 psi
- Moment of inertia I : 0.0833 in^3
- Cross-sectional area A : 1 in^2
- Mass density ρ : 0.00073 $\text{lb-s}^2/\text{in}^4$
- Poisson's ratio of steel ν : 0.3

1. By using your own FEM code, please determine the first three natural frequencies and mode shapes of vibrations for the cantilever beam.
2. The exact solution according to the beam theory for the natural frequencies of the cantilever are:
   $\omega_1 = \dfrac{3.516}{L^2}\left(\dfrac{EI}{\rho A}\right)^{1/2}$, and $\dfrac{\omega_2}{\omega_1} = 6.2669$, $\dfrac{\omega_3}{\omega_1} = 17.5475$
   Are your solutions solved by FEM close to the exact solutions? Please provide your comments.
