# ME 7112: FINITE ELEMENT METHOD - Fall 2023

## Assignment 2 (Due by 11:59 pm on 2023/10/09 Mon.)

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### Problem 1

Please complete the Problem 1 and 2 of the Assignment 1.

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### Problem 2

Please complete the Problem 3 and 4 of the Assignment 1.

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### Problem 3

In this problem, our goal is to analyze the voltage distribution and electric current flow through a conducting planar solid material subjected to prescribed voltage and prescribed external current density on its boundary. For a 2D material point, its electric field, current density, and electric potential density can be analytically described as:

$E_i = -\dfrac{\partial V}{\partial x_i}$

$I_i = \sigma E_i$ (the Ohm's law), and

$\pi = \dfrac{1}{2}\sum_{i=1}^{2} E_i\, I_i$ respectively,

where $V$ is the voltage, $E_i$ is the electric field along the $i$-th direction ( $i = 1$ and $2$ ), $I_i$ is the current density along the $i$-th direction ( $i = 1$ and $2$ ), $\sigma$ is the material conductiviry, and $\pi$ is the electric potential density. Consider a generic triangle element with the corner $a$, $b$, $c$, the voltage distribution on the element can be estimated by interpolating the voltage values at the nodes $V^a$, $V^b$, and $V^c$ as:

$V(x_1, x_2) = N_a(x_1, x_2)\, V^a + N_b(x_1, x_2)\, V^b + N_c(x_1, x_2)\, V^c$

where the interpolation functions $N_a$, $N_b$, and $N_c$ (also called as shape functions) are exactly the same as those used for 2D triangular elements and discussed in class.

Please derive that the electric potential owned by a triangular element of area $A^e$ is:

$\pi^e = \dfrac{1}{2}\boldsymbol{V^e}^{\intercal}\, \boldsymbol{k}^e\, \boldsymbol{V^e}$,

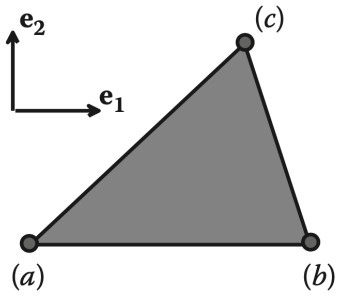
where

$\boldsymbol{k}^e = A^e\, \sigma\, \boldsymbol{B}^{\intercal}\boldsymbol{B}$,

$\boldsymbol{B} = \begin{bmatrix} \dfrac{\partial N_a}{\partial x_1} & \dfrac{\partial N_b}{\partial x_1} & \dfrac{\partial N_c}{\partial x_1} \\ \dfrac{\partial N_a}{\partial x_2} & \dfrac{\partial N_b}{\partial x_2} & \dfrac{\partial N_c}{\partial x_2} \end{bmatrix}$,

and

$\boldsymbol{V^e} = [V^a\ V^b\ V^c]^{\intercal}$ .

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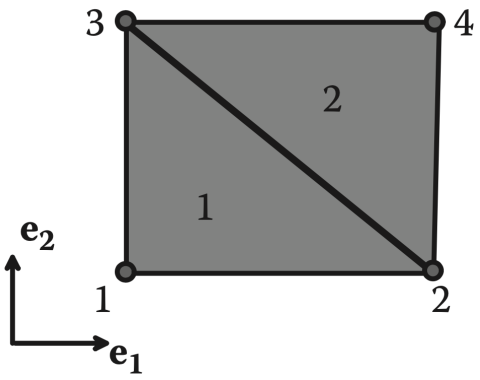
Problem 4

1. Following Problem 3, make your own FEM code for the problems of voltage distribution in solid conducting materials.

2. Test your code by solving the voltage and the electric current flow distribution in such materials.
   (1) Apply a uniform current to the top surface through the simple two-element mesh shown below.
   (2) Following (1), check your solution against the analytical result.




   (Please design the calculation parameters properly used in this problem.)

3. Consider a plate made of conducting solid materials with a circular hole at its center. Afterward, prescribe voltages on both left and right hand sides of the plate.
   (1) Please solve the voltage and the electric current flow distribution in the plate by your FEM code.
   (2) Please observe the the voltage and the electric current flow distribution around the hole, and make some interesting comments.
   (Please design the calculation parameters properly used in this problem.)