EE6203 Exercises

Exercise 1

Q1.1. Find the $\mathcal{Z}$ transform of k^3. (Ans: $\dfrac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$)

Q1.2. Find the $\mathcal{Z}$ transform of t^2e^{-at}. (Ans: $\dfrac{T^2z^{-1}e^{-aT}(1+z^{-1}e^{-aT}}{(1-z^{-1}e^{-aT})^3}$)

Q1.3. Find the $\mathcal{Z}$ transform of the following $x(k)$:

$$x(k) = 9\ k\ (2^{k-1}) - 2^k + 3 \qquad k = 0, 1, 2, 3, \cdots$$

(Ans: $\dfrac{2+z^{-2}}{(1-2z^{-1})^2(1-z^{-1})}$)

Q1.4. Find the $\mathcal{Z}$ transform of

$$y(k) = \sum_{h=0}^{k} a^h$$

where a is a constant. (Ans: $\dfrac{1}{(1-z^{-1})(1-az^{-1})}$)

Q1.5. Find the $\mathcal{Z}$ transform of the curve $x(t)$ as shown below. Assume that the sampling period $T = 1$s. (Ans: $\dfrac{z^{-3}(1+z^{-1}+z^{-2})}{3(1-z^{-1})}$)

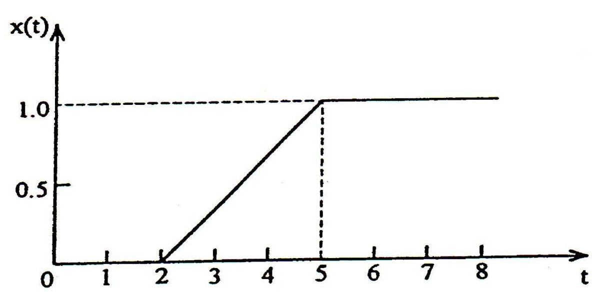

Figure 1.1

Exercise 2

Q2.1 (B-2-8) Find the inverse $\mathcal{Z}$ transform of

$$X(z) = \frac{1 + 2z + 3z^2 + 4z^3 + 5z^4}{z^4}$$

(Ans: $x(0) = 5$, $x(1) = 4$, $x(2) = 3$, $x(3) = 2$, $x(4) = 1$, $x(k) = 0$ for $k \geq 5$)

Q2.2 (B-2-9) Using the partial fraction expansion method, find the inverse $\mathcal{Z}$ transform of

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2}$$

(Ans: $x(k) = -\frac{25}{3}(0.5)^k + \frac{25}{3}(0.8)^k - 2k(0.8)^{k-1}, \quad k = 0,\ 1,\ 2,\ 3,\ \cdots$)

Q2.3 (B-2-10) Given the $\mathcal{Z}$ transform

$$X(z) = \frac{z^{-1}}{(1 - z^{-1})(1 + 1.3z^{-1} + 0.4z^{-2})}$$

determine the initial and final values of $x(k)$. Also find $x(k)$, the inverse $\mathcal{Z}$ transform of $X(z)$, in a closed form. (Ans: $x(0) = 0$, $x(\infty) = 1/2.7$)

Q2.4 (B-2-11) Find the inverse $\mathcal{Z}$ transform (using the inversion integral method) of

$$X(z) = \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}}$$

(Ans: $x(0) = 1$, $x(1) = 2$, and $x(k) = 1$ for $k \geq 2$)

Q2.5 (B-2-11) Find the inverse $\mathcal{Z}$ transform of

$$X(z) = \frac{z^{-3}}{(1 - z^{-1})(1 - 0.2z^{-1})}$$

(Ans: $x(k) = 0$ for $k = 0, 1, 2$; $x(k) = 1.25(1 - 0.2^{k-2})$ for $k = 3, 4, 5, \cdots$)

Exercise 3

Q3.1 Solve the following difference equation:

$$x(k+2)-2x(k+1)+x(k)=\delta(k),\ x(k)=0 \text{ for } k<0,$$

where $\delta(k)$ is a unit impulse function. [Ans: $x(k)=\delta(k)-1(k)+k,\ k=0,1,2,.....$]

Q3.2 (Problem B-3-4) Consider a transfer function system

$$X(s)=\frac{s+3}{(s+1)(s+2)}$$

Obtain the pulse transfer function by two different methods. [Ans: $\frac{1+e^{-T}(1-2e^{-T})z^{-1})}{(1-e^{-T}z^{-1})(1-2e^{-2T}z^{-1})}$]

Q3.3 (Problem B-3-15) Obtain the closed-loop pulse transfer function of the system shown in Figure 3.2 below. [Ans: $\frac{C(z)}{R(z)}=\frac{G(z)}{1+H_2(z)GH_1(z)}$]

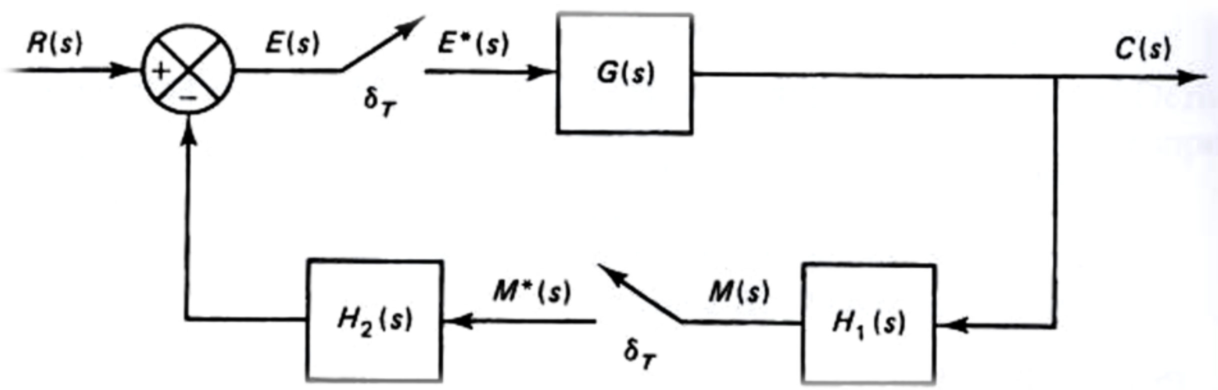

Figure 3.2 Discrete-time control system

Exercise 4

Q4.1 (Problem B-3-17) Consider the discrete-time control system shown in the Figure below. Obtain the discrete-time output *C(z)* and the continuous-time output *C(s)* in terms of the input and the transfer functions of the blocks.

(Ans: $C(z) = \frac{G_1(z)G_2(z)R(z)}{1+G_1(z)HG_2(z)+G_2(z)}$ and $C(s) = \frac{G_2(s)G_1^*(s)R^*(s)}{1+G_1^*(s)(HG_2)^*(s)+G_2^*(s)}$)

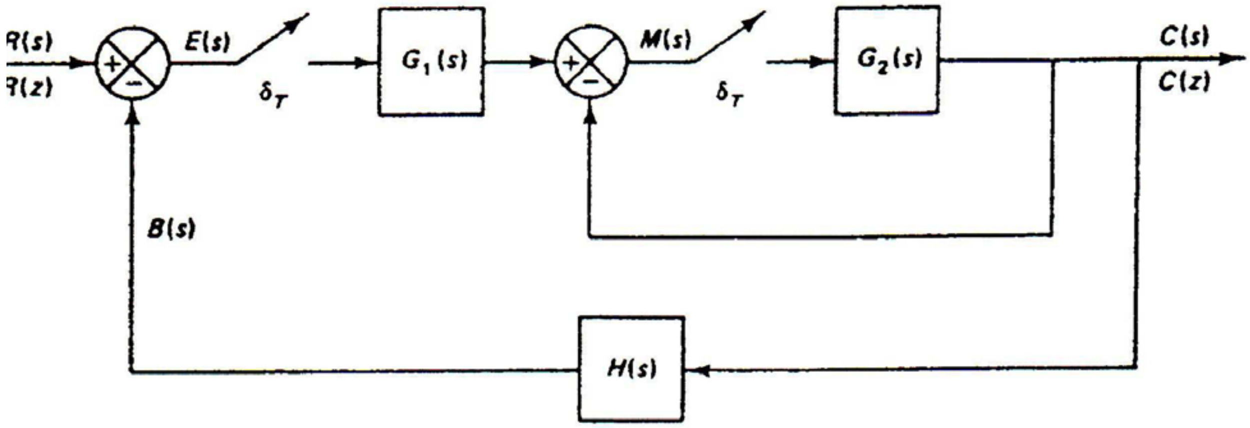

Figure 4.1 Discrete-time control system

Q4.2 (Problem B-3-18) Consider the discrete-time control system shown in the figure below. Obtain the output sequence *c(kT)* of the system when it is subjected to a unit-step input. Assume that the sampling period T is 1 sec.

(Ans: $c(k) = 1-(1-K)^k$,

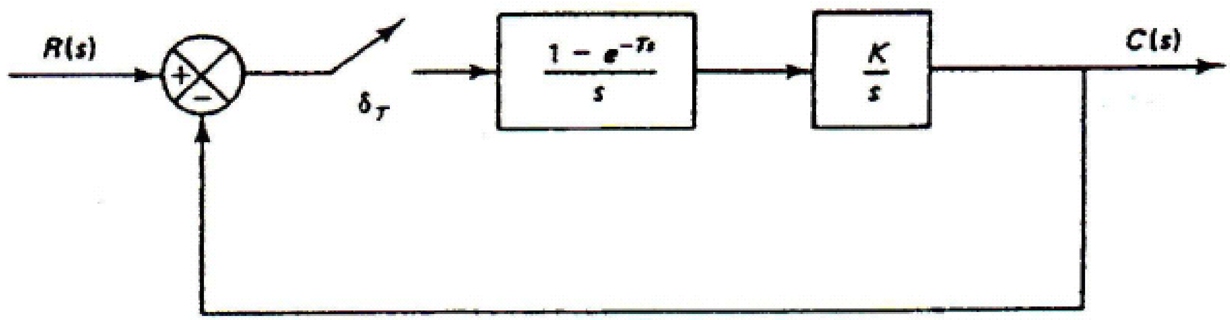

Figure 4.2 Discrete-time control system

Q4.3 (Problem B-3-21) Obtain the closed loop transfer function *C(z)/R(z)* of the digital control system shown in Figure 4.3. Assume the pulse transfer function of the plant is *G(z).*

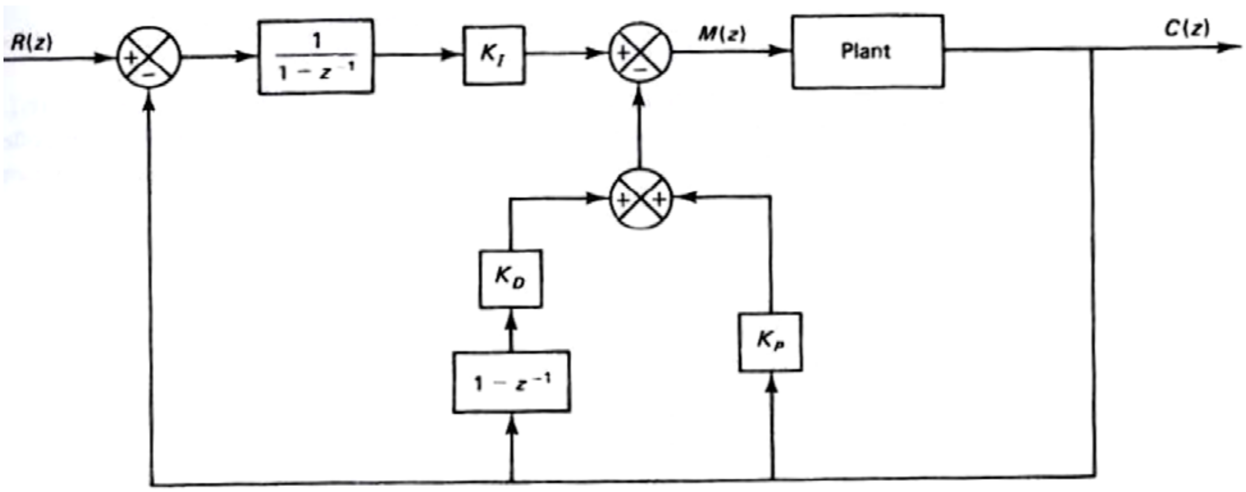

Figure 4.3 Discrete-time control system

[Ans: $\frac{C(z)}{R(z)} = \frac{G(z)\frac{K_I}{1-z^{-1}}}{1+G(z)[\frac{K_I}{1-z^{-1}}+K_p+K_D(1-z^{-1})]}$]