REVISION OF MATRICES

$$A = \left[\begin{array}{c|c|c} 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \end{array}\right]$$

then,

$$A^T = \left[\begin{array}{c|c|c} 1 & 4 & 7 \\ \hline 2 & 5 & 8 \\ \hline 3 & 6 & 9 \end{array}\right]$$

$$\left[\begin{array}{c|c|c} 1 & 2 & 3 \\ \hline 4 & 5 & 6 \\ \hline 7 & 8 & 9 \end{array}\right] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\text{x}1 + 2\text{x}2 + 3\text{x}3 \\ * \\ * \end{bmatrix} = \begin{bmatrix} 14 \\ * \\ * \end{bmatrix}$$

DETERMINANTS

If

$$A = \left[\begin{array}{c|c} a_{11} & a_{12} \\ \hline a_{21} & a_{22} \end{array}\right]$$

then

$$det\ A = a_{11}a_{22} - a_{12}a_{21}$$

If

$$A = \left[\begin{array}{c|c|c} a_{11} & a_{12} & a_{13} \\ \hline a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right]$$

then

$$det\ A = a_{11} \left|\begin{array}{c|c} a_{22} & a_{23} \\ \hline a_{32} & a_{33} \end{array}\right| - a_{21} \left|\begin{array}{c|c} a_{12} & a_{13} \\ \hline a_{32} & a_{33} \end{array}\right| + a_{31} \left|\begin{array}{c|c} a_{12} & a_{13} \\ \hline a_{22} & a_{23} \end{array}\right|$$

Example :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

then

$$det\ A = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}$$

$$= 1(45-48) - 4(18-24) + 7(12-15)$$

$$= 0$$

4.3 Inverse of a matrix

If $det(A) \neq 0$, then the inverse of A is defined by

$$A^{-1} = \frac{adj(A)}{det(A)};\ adj(A) = (\gamma_{ij})^T$$

where

$$\gamma_{ij} = (-1)^{i+j} det(M_{ij})$$

is called the co-factor and $det(M_{ij})$ is called a minor. M_{ij} is the same as the matrix A except that its ith row and jth column have been removed. Note that M_{ij} is always an $(n-1)$x$(n-1)$ matrix.

$$\begin{bmatrix} s+2 & 3 & 0 \\ 2 & s+1 & 0 \\ -4 & 0 & s-2 \end{bmatrix}^{-1} = A(s)^{-1}$$

$$= \frac{\begin{bmatrix} (s+1)(s-2) & -3(s-2) & 0 \\ -2(s-2) & (s+2)(s-2) & 0 \\ 4(s+1) & -12 & (s+1)(s+2)-6 \end{bmatrix}}{p(s)}$$

adj $[A(s)]$

where,

$$p(s)$$
$$= det(A(s))$$
$$= (s-2)((s+1)(s+2)-6)$$

$$\text{adj}\,(A(s)) = \begin{bmatrix} (s+1)(s-2) & -2(s-2) & 4(s+1) \\ -3(s-2) & (s+2)(s-2) & -12 \\ 0 & 0 & (s+1)(s+2)-6 \end{bmatrix}^T$$

Some useful formulae :

(i) If

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

then

$$A^{-1} = \frac{1}{det\ A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$det\ A = a_{11}a_{22} - a_{12}a_{21}$$

(ii) If

$$A = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}$$

then

$$A^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 \\ 0 & \frac{1}{a_{22}} \end{bmatrix}$$