Q1. (a)

	1	2	3	S	ui	
1	$\boxed{20}$ 40	$\boxed{25}$ 10	$\boxed{26}$ 1	50	5	5
2	$\boxed{20}$ 1	$\boxed{24}$ 55	$\boxed{24}$ 5	60	4	4
3	$\boxed{18}$ 3	$\boxed{23}$ 3	$\boxed{20}$ 70*	70	0	2
D	40	65	75	180		
Vi	15	20	20			
	2	1	4			

	2	3	S	
1	$\boxed{25}$	$\boxed{26}$	10	1
2	$\boxed{24}$	$\boxed{24}$	60	0
3	$\boxed{23}$	$\boxed{20}$	70	3
D	65	75	140	
	1	4		

	2	3		
1	$\boxed{25}$	$\boxed{26}$	10	1
2	$\boxed{24}$	$\boxed{24}$	60	0
D	65	5	70	
	1	2		

	2	
1	$\boxed{25}$	10
2	$\boxed{24}$	55
D	65	

(i) $$\min.\ 20x_{11} + 25x_{12} + 26x_{13} + 20x_{21} + 24x_{22} + 24x_{23} + 18x_{31} + 23x_{32} + 20x_{33}$$

s.t. $$x_{11} + x_{12} + x_{13} = 50$$
$$x_{21} + x_{22} + x_{23} = 60$$
$$x_{31} + x_{32} + x_{33} = 70$$
$$x_{11} + x_{21} + x_{31} = 40$$
$$x_{12} + x_{22} + x_{32} = 65$$
$$x_{13} + x_{23} + x_{33} = 75$$

(ii) $$f_{min} = 40\times20 + 10\times25 + 55\times24 + 5\times24 + 70\times20 = 3890.$$

(iii) $$26 - \Delta - 5 - 20 = 1 - \Delta \geq 0$$
$$\Delta \leq 1.$$

(b) (i) height: x_1, radius: x_2

min. $f = 2\pi(x_1 + x_2)x_2 = 2\pi x_2^2 + 2\pi x_1 x_2$

s.t. $\pi x_1 x_2^2 \geq 10 \Rightarrow -\pi x_1 x_2^2 + 10 \leq 0$

$x_1, x_2 > 0$

$L = 2\pi x_2^2 + 2\pi x_1 x_2 + \mu(-\pi x_1 x_2^2 + 10)$

$\frac{\partial L}{\partial x_1} = 2\pi x_2 - \pi\mu x_2^2 = 0$, $\mu x_2 = 2$

$\frac{\partial L}{\partial x_2} = 4\pi x_2 + 2\pi x_1 - 2\pi\mu x_1 x_2 = 0$, $4\pi x_2 + 2\pi x_1 - 4\pi x_1 = 0$

$x_1 = 2x_2$

$\mu(-\pi x_1 x_2^2 + 10) = 0$

$\pi x_1 x_2^2 = 10$, $x_2^3 = \frac{5}{\pi}$.

$\Rightarrow x_1 = 2.336$, $x_2 = 1.168$

$\mu = 1.712$

$\nabla g^T Y = [-\pi x_2^2 \quad -2\pi x_1 x_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= -\pi x_2^2 y_1 - 2\pi x_1 x_2 y_2 = 0 = -4.286 y_1 - 17.143 y_2 = 0$

$y_1 = -4y_2$.

$Y^T \nabla^2 L_{xx} Y = [y_1 \quad y_2] \begin{bmatrix} 0 & 2\pi - 2\pi\mu x_2 \\ 2\pi - 2\pi\mu x_2 & 4\pi - 2\pi\mu x_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [y_1 \quad y_2] \begin{bmatrix} 0 & -6.28 \\ -6.28 & -12.56 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [-6.28 y_2 \quad -6.28 y_1 - 12.56 y_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= -6.28 y_1 y_2 - 6.28 y_1 y_2 - 12.56 y_2^2$

$= -12.56(y_1 y_2 - y_2^2)$

$= 12.56 \times 5 y_2^2 > 0$.

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Q4. (a) 0: low of sales for current month
1: high of sales for current month

Strategy I: $P_{00} = \frac{3}{4}$, $P_{01} = \frac{1}{4}$ $\quad P_1 = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$P_{10} = \frac{1}{2}$, $P_{11} = \frac{1}{2}$

Strategy II: $P_{00} = \frac{1}{2}$, $P_{01} = \frac{1}{2}$ $\quad P_2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

$P_{10} = \frac{1}{4}$, $P_{11} = \frac{3}{4}$

Strategy III: $P_{00} = \frac{1}{2}$, $P_{01} = \frac{1}{2}$ $\quad P_3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$P_{10} = \frac{1}{2}$, $P_{11} = \frac{1}{2}$

(b) I: $E(T_0) = \frac{1}{1-P_{00}} = 4$, $E(T_1) = 2$.

II: $E(T_0) = 2$, $E(T_1) = 4$.

III: $E(T_0) = 2$. $E(T_1) = 2$.

Strategy II is the best.

(c) I: $Y = YP$, $\sum_i y_i = 1 \Rightarrow [y_0 \;\; y_1]\begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = [y_0 \;\; y_1]$

$\begin{cases} \frac{3}{4}y_0 + \frac{1}{2}y_1 = y_0 \\ \frac{1}{4}y_0 + \frac{1}{2}y_1 = y_1 \end{cases}$ $\quad \frac{1}{4}y_0 = \frac{1}{2}y_1$

$y_1 = \frac{1}{2}y_0$

$y_0 + \frac{1}{2}y_0 = 1$. $y_0 = \frac{2}{3}$

$y_1 = \frac{1}{3}$

II:

$[y_0 \;\; y_1]\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} = [y_0 \;\; y_1]$

$\begin{cases} \frac{1}{2}y_0 + \frac{1}{4}y_1 = y_0 \\ \frac{1}{2}y_0 + \frac{3}{4}y_1 = y_1 \end{cases}$ $\quad \frac{1}{4}y_1 = \frac{1}{2}y_0$.

$y_0 = \frac{1}{3}$, $y_1 = \frac{2}{3}$

III:

$[y_0 \;\; y_1]\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = [y_0 \;\; y_1]$

$\frac{1}{2}y_0 + \frac{1}{2}y_1 = y_0 = y_1 = \frac{1}{2}$

(d)

I: $\frac{2}{3} \times 40000 + \frac{1}{3} \times 80000 = \frac{16}{3} \times 10^4$

II: $\frac{1}{3} \times 40000 + \frac{2}{3} \times 80000 - 20000 = \frac{14}{3} \times 10^4$

III: $\frac{1}{2} \times 40000 + \frac{1}{2} \times 80000 - \frac{1}{2} \times 20000$

$= 5 \times 10^4$

Strategy I is the best.

Q5. (a) Option 1:

(i) $L = \frac{\lambda}{\mu - \lambda} = \frac{100}{1-100}$

(ii) $Q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

(iii) $W = \frac{1}{\mu-\lambda}$

(iv) $D = \frac{\lambda}{\mu(\mu-\lambda)}$

(b)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2013-2014

EE6204 – Systems Analysis

April/May 2014 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises SEVEN (7) pages.
2. Answer ALL questions.
3. Appendix A is given on pages 6 and 7.

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1. There are two parts in question 1 and they are part (a) and part (b).

(a) A local company produces a product for three exporters. The company has three warehouses to supply the product required by the exporters. The quantity of the product required for each exporter and the unit cost ($/ton) in shipping the product from the various warehouses to the exporters are given in the following table.

Shipping cost ($) per ton

		Exporters 1	2	3	Supply (ton)
Warehouses	1	20	25	26	50
	2	20	24	24	60
	3	18	23	20	70
Demand (ton)		40	65	75	

Table 1

The company would like to minimize the total shipping cost.

(i) Create a mathematical formulation for the above transportation problem.

(3 marks)

(ii) Use Vogel's method to construct an initial basic feasible solution and apply the transportation algorithm to obtain an optimal solution. Show all your calculations.

(10 marks)

Note: Question No. 1 continues on Page 2

(iii) If the unit shipping cost of shipping the product from warehouse 1 to exporter 3 is reduced by $\$\Delta$ /ton, determine the range of values of Δ for which the optimal solution in part 1(a)(ii) remains optimal.

(2 marks)

(b) A closed cylindrical container is designed to have a volume of no less than 10 m^3. The aim is to design the container with the smallest surface area possible. (Note: Cylinder Volume $=\pi h r^2$, Cylinder Surface $=2\pi(h+r)r$, where h is the height and r is the radius of the base surface.)

x_1 x_2

(i) Formulate a nonlinear program for the above problem. You may denote the height of the container be x_1 (m) and the radius of the base surface be x_2 (m).

(3 marks)

(ii) Apply the method of Lagrange multipliers to the nonlinear program in part 1(b)(i) to determine the dimensions of the container (in m) as well as the final surface area. Show all your steps.

(12 marks)

2. A fresh graduate has received three job offers (J1, J2, J3) from three different companies. He needs to decide which offer to accept. The most relevant criteria for selection are Salary (S), Welfare (W) and Skill Match (M). The pairwise comparison matrices are shown below.

Criterion

	S	W	M
S	1	4/2	4/8
W	2/4	1	3/8
M	8/4	8/3	1

Salary (S)

	J1	J2	J3
J1	1	1	1/2
J2	1	1	1/2
J3	2	2	1

Welfare (W)

	J1	J2	J3
J1	1	4/3	4/5
J2	3/4	1	3/5
J3	5/4	5/3	1

Note: Question No. 2 continues on Page 3

Skill Match (M)

	J1	J2	J3
J1	1	1/2	1/3
J2	2	1	2/3
J3	3	3/2	1

(a) Is the matrix Criterion consistent? Please give your detailed explanation.

(2 marks)

(b) Compute the priority of each criterion (S, W, and M) based on its contribution to the overall goal of selecting the best offer.

(4 marks)

(c) Compute the consistency ratio and comment on its acceptability. For 3 criteria, the consistency index of a randomly generated pairwise comparison matrix (RI) is 0.58.

(4 marks)

(d) Determine the overall priority for each offer, and recommend the best offer to accept.

(5 marks)

3. A company is expanding its sales network by building a new branch. There are three possible locations A, B, C, which are suitable for this branch. To build the branch at Location A, the annual operational cost is \$500K. The annual sales profit may be either \$800K with 85% probability or \$400K with 15% probability. At location B the annual operational cost is \$350K. The sales profit is either \$700K with 60% probability or \$500K with 40% probability. At location C the annual operational cost is \$200K. But, the company is allowed to have an advertising campaign at location C, which costs \$200K per year. Without advertising, the sales profit is either \$300K with 80% probability or \$250K with 20% probability. With advertising the sales profit can be increased to either \$700K with 90% probability or \$450K with 10% probability.

(a) Please draw a decision tree of this problem.

(2 marks)

(b) Which location should the company choose and why?

(5 marks)

(c) Assume that the utility function for the aforementioned cases is defined as

$$U(x) = (x + 100) / 450,$$

where x denotes the net profit with the unit of \$1K (e.g., we write 700 to denote \$700K), which is defined as the subtracting the sales profit subtracting the operational cost. Determine the location, which has the maximal expected utility of the net profit.

(5 marks)

Note: Question No. 3 continues on Page 4

(d) Following the setup in 3(c), calculate the risk premium of the decision on the location which has the maximal expected utility of the net profit. Is the decision maker risk neutral, risk averse or risk taking?

(3 marks)

4. A car dealer specializes in selling *Toyota Wish*, a popular model of family car. The sales of this car fluctuate between two levels - "Low" and "High" - depending on two factors: (1) whether the car dealer advertises, and (2) the price of COE (Certificate of Entitlement). The second factor is out of the car dealer's control, but the first factor is within the car dealer's control and he is trying to determine his advertising strategy. There are three advertising strategies:

(I) never advertise,
(II) always advertise,
(III) advertise when sales are low but not to advertise when sales are high.

Running advertisements in any month of a year has its primary positive impact on sales in the following month. Therefore, at the beginning of each month, the needed information is available to forecast accurately whether sales will be low or high that month and to decide whether to advertise during that month. When advertising is done during a month, the probability of having high sales the next month is 1/2 or 3/4, depending on whether the current month's sales are low or high. These probabilities are reduced to 1/4 or 1/2, depending on whether the current month's sales are low or high, when advertising is not done during the current month. The cost of advertising is \$20,000 for an entire month in which it is done. The car dealer's monthly profits (excluding advertising costs) are \$80,000 when sales are high but only \$40,000 when sales are low.

	$i-1$	i
	low	$\frac{1}{2}$
	high	$\frac{3}{4}$

Let state 0 indicate the "Low" level of sales and state 1 indicate the "High" level of sales during the current month, where each transition of the process goes from one month to the next.

(a) Construct the (one-step) transition probability matrix for each of the three advertising strategies.

(6 marks)

(b) For each of the three advertising strategies, find the mean sojourn time for each state. Which of these strategies is the best if we want to have high sales for as long a time as possible on the average?

(5 marks)

(c) Determine the steady-state probabilities for each of the three advertising strategies.

(9 marks)

(d) Find the long run expected average profit (including a deduction for advertising costs) per month for each of the three advertising strategies. Which of these strategies is the best according to this measure of performance?

(5 marks)

5. Peter is the manager of a new fast-food restaurant which is opening soon. Peter estimates that customers will arrive randomly (a Poisson process) at a mean rate of $\lambda=100$ 100 per hour during the busiest times of the day. Peter plans to have three employees directly serving the customers. He now needs to decide how to organize these employees.

Option 1 is to have three cash registers with one employee at each cash register to take the orders and get the food and drinks. In this case, it is estimated that the average time to serve each customer would be 1 minute, and the distribution of service times is assumed to be exponential. $m=3$ $\mu=1$

Option 2 is to have one cash register with the three employees working as a team to serve each customer. One would take the order, a second would get the food, $m=1$ and the third would get the drinks. Peter estimates that this would reduce the average time to serve each customer to 20 seconds, with the same assumption of exponential service times. $\mu=3$

Peter wants to choose the option that would provide the best service to his customers.

(a) For each of the two options, calculate the following measures of performance:

(i) the mean number of customers in the system, L
(ii) the mean number of customers in the queue, Q
(iii) the mean waiting time in the system, W
(iv) the mean waiting time in the queue. P

(8 marks)

(b) Compare the measures of performance calculated in part 5(a) and explain why these comparisons make sense intuitively.

(4 marks)

(c) Which measure do you think would be the most important to the customers? Why? Which option is better with respect to this measure?

(3 marks)

Appendix A

M/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \rho^k(1-\rho), \qquad k \geq 1 \\
L &= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \\
Q &= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \\
W &= \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \\
D &= D = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}
\end{aligned}
$$

M/M/1/N Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= \left(\sum_{k=0}^{N} \rho^k\right)^{-1} = \frac{1-\rho}{1-\rho^{N+1}} \\
\pi_k &= \rho^k \pi_0 = \frac{\rho^k(1-\rho)}{1-\rho^{N+1}}, \qquad 0 \leq k \leq N \\
L &= \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
Q &= \frac{\rho^2[1-\rho^N - N\rho^{N-1}(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
W &= \frac{1-\rho^N - N\rho^N(1-\rho)}{\mu(1-\rho)(1-\rho^{N+1})} \\
D &= \frac{\rho[1-\rho^N - N\rho^{N-1}(1-\rho)]}{\mu(1-\rho)(1-\rho^{N+1})}
\end{aligned}
$$

M/M/m Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{\lambda}{m\mu} \\
\pi_0 &= \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!}\right]^{-1} \\
\pi_k &= \pi_0 \begin{cases} \dfrac{(m\rho)^k}{k!}, & 0 \le k \le m-1 \\[2ex] \dfrac{m^m \rho^k}{m!}, & k \ge m \end{cases} \\
L &= \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} \\
Q &= \sum_{k=m}^{\infty} (k-m)\pi_k = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} \\
W &= \frac{L}{\lambda} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2} + \frac{1}{\mu} \\
D &= W - \frac{1}{\mu} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2}
\end{aligned}$$

M^b/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{b\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \begin{cases} \left(\dfrac{\lambda+\mu}{\mu}\right)^{k-1} \dfrac{\lambda}{\mu}\pi_0 & 1 \le k \le b \\[2ex] \dfrac{\lambda+\mu}{\mu}\pi_{k-1} - \dfrac{\lambda}{\mu}\pi_{k-b-1} & k \ge b+1 \end{cases} \\
L &= \frac{\rho(1+b)}{2(1-\rho)} \\
Q &= L - \rho = \frac{\rho(b-1+2\rho)}{2(1-\rho)} \\
W &= \frac{L}{\lambda b} = \frac{1+b}{2\mu(1-\rho)} \\
D &= W - \frac{1}{\mu} = \frac{b+2\rho-1}{2\mu(1-\rho)}
\end{aligned}$$

End of Paper