Q1. (a) min. $z = 2x_1 + 3x_2 + M\cdot\bar{x}_3 + 0\cdot x_4 + 0\cdot x_5 + M\cdot\bar{x}_6$

s.t.
$$3x_1 + 2x_2 + \bar{x}_3 = 14$$
$$4x_1 + 3x_2 + x_4 = 19$$
$$2x_1 - 4x_2 - x_5 + \bar{x}_6 = 12$$
$$x_1,\ x_2,\ \bar{x}_3,\ x_4,\ x_5,\ \bar{x}_6 \geq 0$$

$C = [2 \quad 3 \quad M \quad 0 \quad 0 \quad M]^T$, $B = \begin{bmatrix} 14 \\ 19 \\ 12 \end{bmatrix}$, $C^T - C_0^T A = [2 \quad 3 \quad M \quad 0 \quad 0 \quad M]$

$A = \begin{bmatrix} 3 & 2 & 1 & 0 & 0 & 0 \\ 4 & 3 & 0 & 1 & 0 & 0 \\ 2 & -4 & 0 & 0 & -1 & 1 \end{bmatrix}$. $\quad - [5M \quad -2M \quad M \quad 0 \quad -M \quad M]$

$C_0 = [M \quad 0 \quad M]^T$ $\quad = [2-5M \quad 3+2M \quad 0 \quad 0 \quad M \quad 0]$.

$-C_0^T B = -26M$,

	x_1	x_2	$\bar{x}_3$	x_4	x_5	$\bar{x}_6$		
$\bar{x}_3$	3^*	2	1	0	0	0	14	$\frac{14}{3}$
x_4	4	3	0	1	0	0	19	$\frac{19}{4}$
$\bar{x}_6$	2	-4	0	0	-1	1	12	6
	2	3	0	0	0	0	0	
	-5	2	0	0	1	0	-26	

$\frac{57}{3} - \frac{56}{3}$

$\frac{28}{3} \quad \frac{36}{3}$

$\frac{70}{3} - \frac{72}{3}$

x_1	1	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0	$\frac{14}{3}$
x_4	0	$\frac{1}{3}$	$-\frac{4}{3}$	1	0	0	$\frac{1}{3}$
$\bar{x}_6$	0	$-\frac{16}{3}$	$-\frac{2}{3}$	0	-1	1	$\frac{8}{3}$
	0	$\frac{5}{3}$	$-\frac{2}{3}$	0	0	0	$-\frac{28}{3}$
	0	$\frac{16}{3}$	$\frac{5}{3}$	0	1	0	$-\frac{2}{3}$

non-basic variable $\bar{x}_6$ is still in the system, there is no optimal solution for z.

(b) (i) length: $2.5x_1$, width: x_1, height: x_2

max. $2.5x_1^2 x_2$

s.t. $2.5x_1^2 + 2x_1x_2 + 5x_1x_2 = 15$

$x_1, x_2 > 0$

$\Rightarrow$ min. $-2.5x_1^2 x_2$

s.t. $2.5x_1^2 + 7x_1x_2 - 15 = 0$

$L = -2.5x_1^2 x_2 + \lambda(2.5x_1^2 + 7x_1x_2 - 15)$

$\frac{\partial L}{\partial x_1} = -5x_1x_2 + 5\lambda x_1 + 7x_2 = 0$, $\quad 5x_1x_2 - \frac{12.5}{7}x_1^2 - 7x_2 = 0$

$\frac{\partial L}{\partial x_2} = -2.5x_1^2 + 7\lambda x_1 = 0$, $\quad 2.5x_1^2 = 7\lambda x_1$, $\quad 2.5x_1 = 7\lambda$, $\quad \lambda = \frac{2.5}{7}x_1$

$\frac{\partial L}{\partial \lambda} = 2.5x_1^2 + 7x_1x_2 - 15 = 0$. $\quad \frac{12.5}{7}x_1^2 + 5x_1x_2 - \frac{75}{7} = 0$

$7\lambda x_1 + 7x_1x_2 = 15$

$7x_1(x_2 + \lambda) = 15$

$5x_1x_2 - 7x_2 = \frac{75}{7} - 5x_1x_2$

$10x_1x_2 - 7x_2 = \frac{75}{7}$

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Q2. (a) State space $S=\{0,1\}$.

$P_{00}=0.9$, $P_{01}=0.1$

$P_{10}=0.1$, $P_{11}=0.9$

TPM: $P=\begin{bmatrix}0.9 & 0.1\\ 0.1 & 0.9\end{bmatrix}$

(b) $\pi(n)=\pi(0)P^n$. $\pi(3)=\pi(0)P^3$

Q3. $S=\{0,1,2,3\}$, where:

0: no customer

1: one customer is being served

2: one is being surved, one is waiting

3: one is being surved, two are waiting

$$0 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 1 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 2 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 3$$

$q_{00}=-\lambda$, $q_{01}=\lambda$, $q_{02}=0$, $q_{03}=0$

$q_{10}=\mu$, $q_{11}=-(\lambda+\mu)$, $q_{12}=\lambda$, $q_{13}=0$

$q_{20}=0$, $q_{21}=\mu$, $q_{22}=-(\lambda+\mu)$, $q_{23}=\lambda$

$q_{30}=0$, $q_{31}=0$, $q_{32}=\mu$, $q_{33}=-\mu$

$$Q=\begin{bmatrix}-\lambda & \lambda & 0 & 0\\ \mu & -(\lambda+\mu) & \lambda & 0\\ 0 & \mu & -(\lambda+\mu) & \lambda\\ 0 & 0 & \mu & -\mu\end{bmatrix}$$

$\pi Q=0$, $\sum_j \pi_j=1$

$$\begin{cases}-\lambda\pi_0+\mu\pi_1=0\\ \lambda\pi_0-(\lambda+\mu)\pi_1+\mu\pi_2=0\\ \lambda\pi_1-(\lambda+\mu)\pi_2+\mu\pi_3=0\\ \lambda\pi_2-\mu\pi_3=0\\ \pi_0+\pi_1+\pi_2+\pi_3=1\end{cases}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2012-2013

EE6204 – Systems Analysis

April/May 2013 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises SEVEN (7) pages.
2. Answer ALL questions.
3. Appendix A is given on pages 6 and 7.

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1. (a) Use the Simplex Method to solve the following Linear Programming problem and determine the minimum value z, if applicable. (Show your working and do not use the graphic method.)

$$\text{Minimize} \quad z = 2x_1 + 3x_2$$

$$\text{Subject to} \quad \begin{aligned} 3x_1 + 2x_2 &= 14 \\ 4x_1 + 3x_2 &\le 19 \\ 2x_1 - 4x_2 &\ge 12 \\ x_1, x_2 &\ge 0 \end{aligned}$$

$-4x_2 = -28 + 6x_1$

$8x_1 \geq 40$

$x_1 \geq 5$

(15 marks)

(b) A customer requires a rectangular container (without a cover) for material storage. The width and length of the container are to be x_1 m and 2.5 x_1 m, respectively. The total surface area of the container is fixed at 15 m^2. Determine the dimensions of the container to maximize the volume of the container.

(i) Formulate the above a Nonlinear Programming problem assuming the height of the container as x_2 m. Use Lagrangian multipliers to solve this Nonlinear Programming problem. Show all your calculations.

(12 marks)

(ii) Suppose that the total surface area of the container is 16 m^2, rather than 15 m^2. Use sensitivity analysis to estimate the volume of the container if all 16 m^2 material is completely used up.

(3 marks)

2. There is a digital communication system transmitting either "0" or "1". The transmission needs to go through a sequence of processes before the digits are received by receivers. Each process receives "0" or "1" and it shall output "0" or "1" after internal signal processing. However, due to noises and other factors, the output digit of each process may not be the same as the received digit. This is considered as an error bit. Assume the probability of an error bit is 0.1 for every process.

   (a) Model this communication problem as a Markov chain in the state space and with the state transition probability matrix.

   (10 marks)

   (b) Consider a case of transmitting a digit "1". Calculate the probability of digit "1" being output after 3 processes?

   (5 marks)

   (c) If the output of one process is "1", what is the probability of having "1" as the output of each process in two successive processes?

   (5 marks)

3. On one day in a bank, only one service counter was open. There was no customer when the bank was open. The arrival time of the first 20 customers was recorded as follows: 15, 17, 23, 24, 25, 31, 39, 55, 58, 62, 63, 65, 68, 80, 82, 85, 89, 97, 99 and 103 minutes, respectively. The number of arriving customers forms a Poison process. The time taken to serve one customer at the counter is a random variable with an exponential distribution. The average serving time for one customer was 4 minutes.

   In the bank, a customer is immediately served as long as the service counter is available. If a new customer arrives in the bank while the service counter is engaged by another customer, the customer shall queue up for service. The past statistical data show that the customers of this bank have a common behaviour described as follows. If one customer arrives in the bank and sees two customers in the queue, the customer will leave and will not return on the same day.

   (a) Write down the state space and the diagram of this queuing model.

   (8 marks)

   (b) Write down the transition rate matrix Q.

   (4 marks)

   (c) Write down the rate balance equations.

   (4 marks)

   (d) If the bank's business keeps running like this (that is, one counter is open and the customer behaviors are the same), what is the daily average number of customers in the bank?

   (4 marks)

4. A software company needs to select a brand of computer for a new project. The two possible candidates are A and B, and the most relevant criteria for selection are CPU speed (S), functional versatility (V), and multimedia capability (M). The pairwise comparison matrices are shown as follows.

0.38855833333
1.6158
1.12765

3.246436643

CI 0.1232183217

CR = 0.2124453822 > 0.1

3.0375875403
3.156475874
3.544343653

Criterion

	S	V	M	
S	1	1/3	1/4	0.1279
V	3	1	2	0.5119
M	4	1/2	1	0.3601

Speed

	A	B	
A	1	1/4	0.2
B	4	1	0.8

Versatility

	A	B	
A	1	3	0.75
B	1/3	1	0.25

Multimedia

	A	B	
A	1	1/2	0.3333
B	2	1	0.6667

(a) Compute the priority of each criterion (S, V, and M) based on its contribution to the overall goal of selecting the best computer.

(4 marks)

(b) Compute the consistency ratio and comment on its acceptability. For the 3 criteria, the consistency index of a randomly generated pairwise comparison matrix (RI) is 0.58.

(6 marks)

(c) Determine the overall priority for each brand, and recommend which brand is preferred.

(5 marks)

5. A company is considering whether it should tender for two service contracts (MS1 and MS2) on offer from a government department. The company has three options:

投标

(I) tender for MS1 only;

(II) tender for MS2 only;

(III) tender for both MS1 and MS2.

If a tender is submitted, the company will incur an additional cost. This cost will have to be entirely recouped from the contract price. Unsuccessful tenders mean that the company will make a loss.

The cost of tendering for contract MS1 only is $50,000. If the tender is successful, the incurred component supply cost will be $18,000.

The cost of tendering for contract MS2 only is $14,000. If the tender is successful, the incurred component supply cost will be $12,000.

The cost of tendering for both contracts MS1 and MS2 is $55,000. If the tender is successful, the incurred component supply cost will be $24,000.

For each contract, possible tender prices are shown in Table 1. In addition, subjective assessments are made of the probability of winning the contract with a particular tender price as shown in Table 1. Note that the company can only submit one tender for one contract.

Option	Possible tender price ($)	Probability of winning contracts
MS1 only	130,000	0.20
	115,000	0.85
MS2 only	70,000	0.15
	65,000	0.80
	60,000	0.95
MS1 and MS2	190,000	0.05
	140,000	0.65

Table 1

In the event that the company tenders for both MS1 and MS2, it will either win both contracts at the price shown above or win no contract.

Note: Question No. 5 continues on Page 5

(a) What is your suggestion to the company and why?

(10 marks)

(b) For a commission of $20,000, a consultant can ensure that the company's tender of $60,000 for contract MS2 only is guaranteed to be successful. Should the company engage the consultant's service or not and why?

(5 marks)

Appendix A

M/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \rho^k(1-\rho), \qquad k \geq 1 \\
L &= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \\
Q &= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \\
W &= \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \\
D &= D = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}
\end{aligned}$$

M/M/1/N Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= \left(\sum_{k=0}^{N} \rho^k\right)^{-1} = \frac{1-\rho}{1-\rho^{N+1}} \\
\pi_k &= \rho^k \pi_0 = \frac{\rho^k(1-\rho)}{1-\rho^{N+1}}, \qquad 0 \leq k \leq N \\
L &= \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
Q &= \frac{\rho^2[1-\rho^N - N\rho^{N-1}(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
W &= \frac{1-\rho^N - N\rho^N(1-\rho)}{\mu(1-\rho)(1-\rho^{N+1})} \\
D &= \frac{\rho[1-\rho^N - N\rho^{N-1}(1-\rho)]}{\mu(1-\rho)(1-\rho^{N+1})}
\end{aligned}$$

M/M/m Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{\lambda}{m\mu} \\
\pi_0 &= \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!}\right]^{-1} \\
\pi_k &= \pi_0 \begin{cases} \dfrac{(m\rho)^k}{k!}, & 0 \le k \le m-1 \\ \\ \dfrac{m^m \rho^k}{m!}, & k \ge m \end{cases} \\
L &= \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} \\
Q &= \sum_{k=m}^{\infty} (k-m)\pi_k = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} \\
W &= \frac{L}{\lambda} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2} + \frac{1}{\mu} \\
D &= W - \frac{1}{\mu} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2}
\end{aligned}
$$

M^b/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{b\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \begin{cases} \left(\dfrac{\lambda+\mu}{\mu}\right)^{k-1} \dfrac{\lambda}{\mu}\pi_0 & 1 \le k \le b \\ \\ \dfrac{\lambda+\mu}{\mu}\pi_{k-1} - \dfrac{\lambda}{\mu}\pi_{k-b-1} & k \ge b+1 \end{cases} \\
L &= \frac{\rho(1+b)}{2(1-\rho)} \\
Q &= L - \rho = \frac{\rho(b-1+2\rho)}{2(1-\rho)} \\
W &= \frac{L}{\lambda b} = \frac{1+b}{2\mu(1-\rho)} \\
D &= W - \frac{1}{\mu} = \frac{b+2\rho-1}{2\mu(1-\rho)}
\end{aligned}
$$

End of Paper