Q1. (a) (i) min. $z = 3x_1 + 2x_2 + 0\cdot x_3 + M\cdot \bar{x}_4$

s.t. $-2x_1 + 3x_2 + x_3 = 3$

$4x_1 + 8x_2 + \bar{x}_4 = 5$

$x_1, x_2, x_3, \bar{x}_4 \geq 0$

$X = [x_1 \ x_2 \ x_3 \ \bar{x}_4]^T$, $C = [3 \ 2 \ 0 \ M]^T$. $A = \begin{bmatrix} -2 & 3 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{bmatrix}$. $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. $C_0 = \begin{bmatrix} 0 \\ M \end{bmatrix}$.

$C^T - C_0^T A = [3 \ 2 \ 0 \ M] - [4M \ 8M \ 0 \ M] = [3-4M \ \ 2-8M \ \ 0 \ \ 0]$

$-C_0^T B = -5M.$

	x_1 A^*	x_2	x_3	$\bar{x}_4$	
x_3	-2	3	1	0	3
$\bar{x}_4$	4	8^*	0	1	5
	3	2	0	0	0
	-4	-8	0	0	-5

				S	
x_3	-3.5	0	1	$-\frac{3}{8}$	$\frac{9}{8}$
x_2	0.5	1	0	$\frac{1}{8}$	$\frac{5}{8}$
	2	0	0	$-\frac{1}{4}$	$-\frac{5}{4}$
	0	0	0	1	0

$x_2^* = \frac{5}{8}$, $Z_{min} = 2\times\frac{5}{8} = 1.25$

(ii) $C_1 = 3$, $C_1^* = 3 + \Delta$

$y^T A^* - C_1^* \geq 0$

$[0 \ -\frac{1}{4}]\begin{bmatrix} -2 \\ 4 \end{bmatrix} - (3+\Delta) = -4 - \Delta \geq 0$

$\Delta \leq -4$

$C_1^* \leq -1$

(iii)

(b) (i) min. $f = 6x_1^2 + 4x_2^2 + x_3^2$

s.t. $24x_1 + 24x_2 - 360 = 0$

$x_3 - 1 = 0$

$L = 6x_1^2 + 4x_2^2 + x_3^2 + \lambda_1(24x_1 + 24x_2 - 360) + \lambda_2(x_3 - 1)$

$\frac{\partial L}{\partial x_1} = 12x_1 + 24\lambda_1 = 0$, $x_1 = -2\lambda_1$ $3x_1 = 2x_2$, $x_2 = \frac{3}{2}x_1$

$\frac{\partial L}{\partial x_2} = 8x_2 + 24\lambda_1 = 0$, $x_2 = -3\lambda_1$ $\frac{5}{2}x_1 = 15$, $x_1 = 6$

$x_2 = 9$

$\frac{\partial L}{\partial x_3} = 2x_3 + \lambda_2 = 0$, $x_3 = -\frac{1}{2}\lambda_2$

$\lambda_1 = -3$

$\frac{\partial L}{\partial \lambda_1} = 24x_1 + 24x_2 - 360 = 0$, $x_1 + x_2 = \frac{360}{24} = 15$

$\frac{\partial L}{\partial \lambda_2} = x_3 - 1 = 0$, $x_3 = 1$, $\lambda_2 = -2$.

$\Rightarrow x_1^* = 6$, $x_2^* = 9$, $x_3^* = 1$, $\lambda_1^* = -3$, $\lambda_2^* = -2$.

$f_{min} = 6 \times 6^2 + 4 \times 9^2 + 1 = 6 \times 36 + 4 \times 81 + 1 = 541$.

$\nabla h_1^T Y = 0$, $\begin{bmatrix} 24 & 24 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 24(y_1 + y_2) = 0$, $y_1 = -y_2$

$\nabla h_2^T Y = 0$. $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = y_3 = 0$.

$$Y^T \nabla^2 L_{xx} Y = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 12 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= \begin{bmatrix} 12y_1 & 8y_2 & 2y_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = 12y_1^2 + 8y_1^2 + 2y_3^2 = 20y_1^2 > 0.$$

$\Rightarrow$ optimal

AY 2014 - 2015

Q4. (a) (i) state space : $S = \{0, 1, 2, 3\}$.

(ii) α_k's are i.i.d. Possion r.v. with $\lambda = 3$:

$$P(\alpha_k = k) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$P_{00} = P(\alpha_k \geq 3) = 1 - 8.5e^{-3}$.

$P_{01} = P(\alpha_k = 2) = \frac{e^{-3}(3)^2}{2} = \frac{9}{2}e^{-3}$.

$P_{02} = P(\alpha_k = 1) = e^{-3}(3)^1 = 3e^{-3}$.

$P_{03} = P(\alpha_k = 0) = e^{-3}$.

$P_{10} = P(\alpha_k \geq 1) = 1 - 8.5e^{-3}$.

$P_{11} = P(\alpha_k = 2) = \frac{9}{2}e^{-3}$.

$P_{12} = P(\alpha_k = 1) = 3e^{-3}$.

$P_{13} = P(\alpha_k = 0) = e^{-3}$.

$P_{20} = P(\alpha_k \geq 2) = 1 - 4e^{-3}$.

$P_{21} = P(\alpha_k = 1) = 3e^{-3}$.

$P_{22} = P(\alpha_k = 0) = e^{-3}$.

$P_{23} = 0$.

$P_{30} = P(\alpha_k \geq 3) = 1 - 8.5e^{-3}$.

$P_{31} = P(\alpha_k = 2) = \frac{9}{2}e^{-3}$.

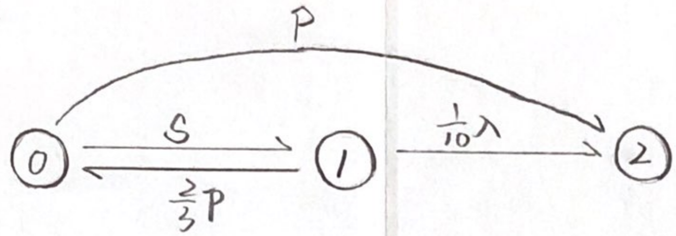
$P_{32} = P(\alpha_k = 1) = 3e^{-3}$.

$P_{33} = P(\alpha_k = 0) = e^{-3}$

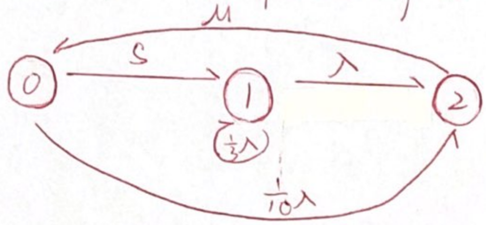
TPM:

$$\begin{matrix} 1-8.5e^{-3} & 4.5e^{-3} & 3e^{-3} & e^{-3} \\ 1-8.5e^{-3} & 4.5e^{-3} & 3e^{-3} & e^{-3} \\ 1-4e^{-3} & 3e^{-3} & & \end{matrix}$$

Q5. (a)



0: set-up 1: processing 2: repairing



NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2014-2015

EE6204 – SYSTEMS ANALYSIS

April / May 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises FIVE (5) pages.
2. Answer ALL questions.
3. Marks for each question are as indicated.

Question 1 (30 Marks)

1. (a) (i) Solve the following Linear Programming (LP) problem:

Minimize $z = 3x_1 + 2x_2$

Subject to: $-2x_1 + 3x_2 \leq 3$

$4x_1 + 8x_2 = 5$

$x_1, x_2 \geq 0$

(10 Marks)

(ii) If the cost function of the above LP becomes $z = (3 + \Delta)x_1 + 2x_2$, find the range of Δ for the obtained solution in part (i) to remain optimal.

(3 Marks)

(iii) If the LP is of the maximization type, do we negate the cost coefficient for an artificial variable in Phase I in the Two-phase method by replacing M with -M? Provide your explanation.

(2 Marks)

(b) (i) Find the optimal solution to the following constrained nonlinear optimization problem using the Lagrange multiplier method.

Minimize $f(X) = 6(x_1)^2 + 4(x_2)^2 + (x_3)^2$

Subject to: $24x_1 + 24x_2 = 360$

$x_3 = 1$

(10 Marks)

Note: Question No. 1 continues on page 2

(ii) Show that the optimal Lagrange multiplier, $(\lambda_i)^*$ associated with constraint $g(x) = b_i$, can be interpreted as the rate of change in optimal value per unit increase in the right-hand side b_i.

(3 Marks)

(iii) Give two limitations of the Language multiplier method.

(2 Marks)

Question 2 (15 Marks)

2. A company is planning to open a new branch at one of three candidate locations: LA, LB, LC. The main selection criteria for the location are Population (P), Average Income (I) and Mobility (M). The pairwise comparison matrices are shown below:

Criterion

	P	I	M
P	1	2	0.5
I	0.5	1	0.25
M	2	4	1

Population (P)

	LA	LB	LC
LA	1	1	0.25
LB	1	1	0.25
LC	4	4	1

Average Income (I)

	LA	LB	LC
LA	1	2	0.8
LB	0.5	1	0.4
LC	1.25	2.5	1

Mobility (M)

	LA	LB	LC
LA	1	0.5	0.25
LB	2	1	0.5
LC	4	2	1

(a) Is the matrix Criterion consistent? Give your detailed explanation.

(3 Marks)

Note: Question No. 2 continues on page 3

(b) Compute the priority of each criterion (P, I, and M) based on its contribution to the overall goal of selecting the best location. Compute the consistency ratio and comment on its acceptability. For the 3 criteria, the consistency index of a randomly generated pairwise comparison matrix (RI) is 0.58.

(7 Marks)

(c) Determine the overall priority for each location, and select the best one.

(5 Marks)

Question 3 (15 Marks)

3. Jack has an invention which is likely to generate a commercial value. Two companies, A and B, are interested in his invention. Company A would like to give a lump sum payment of \$250,000. Company B would like to make a payment, with the value at 40% of the actual market profit earned in the first year. It is estimated that the market profit can be either \$1,000,000 in a good year, or \$600,000 in a normal year or \$200,000 in a bad year. There is 30% chance for him to have a good year, 50% chance for him to have a normal year, and 20% chance for him to have a bad year. But, if Jack spends \$100,000 on advertisement from his own pocket, the chance for him to have a good year will increase to 40%, the chance for him to have a normal year will remain at 50%, and the chance for him to have a bad year will decrease to 10%.

(a) Draw a decision tree of this problem.

(2 Marks)

(b) Which company should Jack sell his invention to? Provide your justification.

(5 Marks)

(c) Assume that the utility function for the aforementioned cases is defined as:

$$U(x) = \sqrt{x + 20000} \,,$$

where x denotes the payment received by Jack. Determine the utility of each offer and decide which offer to accept.

(5 Marks)

(d) Following the setup in (c), is Jack risk neutral, risk averse or risk taking on the offer that he accepts?

(3 Marks)

Question 4 (20 Marks)

4. A shop sells a particular brand of watches and keeps at most 3 pieces of the watch in the shop. New watches will be ordered only when there is only one watch left or when all 3 watches are sold out. Every day right before closing at 9 pm, the shop assistant will check the number of watches left. He will let the supplier know if new watches have to be delivered to the shop before the shop opens at 10 am.

Let

$\alpha_k =$ demand for the watch during the kth day, $k = 1, 2, ...$

$\beta_k =$ the number of watches left unsold at the end of the kth day, $k = 1, 2, ...$

Suppose that α_k's are independent random variables and have identical Poisson distribution with the average daily demand of $\lambda = 3$. Let $\beta_0 = 3$ be the initial number of watches available for sale. Assume that $\{\beta_k\}$ forms a discrete-time Markov chain (DTMC) with transition probability matrix $P = [p_{ij}]$.

(a) (i) Write down the state space for $\{\beta_k\}$.

(ii) Compute the transition probability matrix P.

(13 Marks)

(b) Define π_0, π_1, π_2 and π_3 the *steady-state probabilities* of $\{\beta_k\}$. Write down a set of equations so that π_0, π_1, π_2 and π_3 can be computed. Do not solve these equations.

(3 Marks)

(c) All watches were sold out on Wednesday. What is the probability that there is only one watch left when the shop closes on Friday (2 days later)?

(4 Marks)

Question 5 (20 Marks)

5. A machine is capable of producing 3 different types of parts when it is properly set up with the appropriate tools for operation. After a part has been processed, the next part can be any of the 3 types with equal probability. If the next part is of the same type as the previous one, then the machine can immediately start processing the new part, without the need for setting up and changing tools. However, if the next part is of a different type, then the machine must be set up with proper tools, before processing can begin.

Note: Question No. 5 continues on page 5

The processing time for the 3 types of parts are the same and are known to obey the exponential distribution with processing rate p.

The set-up time (including changing tools) also obeys the exponential distribution with set-up rate s, and is the same for all 3 types of parts.

The machine can fail when processing a part. The time to failure obeys an exponential distribution and has a failure rate λ, and is the same for all 3 types of parts.

The machine can also fail during set-up and changing tools, but with a much lower failure rate of $\lambda / 10$.

When the machine fails, repair is immediately carried out, and the repair time obeys the exponential distribution with repair rate μ.

After the machine has been repaired, it discards the unfinished part and restarts with a new part.

All processing time, set-up time, time to failure, time to repair are mutually independent.

The system can be modeled as a continuous-time Markov chain (CTMC) with the following states:

State 0:	the machine is being set up with proper tools
State 1:	the machine is processing a part
State 2:	the machine failed and is under repair

(a) Draw the state transition (rate) diagram.

(8 Marks)

(b) Given the various rates as follows:

$$s = 2, \qquad p = 5, \qquad \lambda = 0.1, \qquad \mu = 2$$

Find the steady-state probabilities and the average production rate.

(9 Marks)

(c) Upon each visit to each of the 3 states, how long on average will the system remain in that state?

(3 Marks)

END OF PAPER