Q1

	1	2	3	S	ui	
1	$\boxed{20}$ 1	$\boxed{24}$ 55	$\boxed{24}$ 5	60	4	4
2	$\boxed{18}$ 3	$\boxed{23}$ 3	$\boxed{20}$ 70	70	0	2
3	$\boxed{20}$ 40	$\boxed{25}$ 10	$\boxed{26}$ 1	50	5	5
D	40	65	75	180		
vi	15	20	20			
	2	1	4			

	2	3	S	
1	$\boxed{24}$	$\boxed{24}$	60	0
2	$\boxed{23}$	$\boxed{20}$ 70	70	3
3	$\boxed{25}$	$\boxed{26}$	10	1
D	65	75	140	
	1	4		

	2	3	S	
1	$\boxed{24}$	$\boxed{24}$ 5	60	0
3	$\boxed{25}$	$\boxed{26}$	10	1
D	65	5	70	
	1	2		

	2	S
1	$\boxed{24}$	55
3	$\boxed{25}$	10
D	65	

(a) $\min.\ 20x_{11} + 24x_{12} + 24x_{13} + 18x_{21} + 23x_{22} + 20x_{23} + 20x_{31} + 25x_{32} + 26x_{33}$

s.t.
$$x_{11} + x_{12} + x_{13} = 60$$
$$x_{21} + x_{22} + x_{23} = 70$$
$$x_{31} + x_{32} + x_{33} = 50$$
$$x_{11} + x_{21} + x_{31} = 40$$
$$x_{12} + x_{22} + x_{32} = 65$$
$$x_{13} + x_{23} + x_{33} = 75$$

(b) $f_{min} = 24\times55 + 24\times5 + 20\times70 + 20\times40 + 25\times10 = 3890.$

(c) $26 + \Delta - 5 - 20 \geq 0$

$\Delta \geq -1.$

Q2

(a) length x_1 , breadth x_2 , height $\frac{x_1}{2.5} = \frac{2}{5}x_1$

min. $20(x_1x_2 + \frac{2}{5}x_1^2 + \frac{2}{5}x_1x_2) = (\frac{4}{5}x_1^2 + \frac{14}{5}x_1x_2)\times 10 = 8x_1^2 + 28x_1x_2$

s.t. $\frac{2}{5}x_1^2x_2 = 98$

$x_1, x_2 > 0$

$L = 8x_1^2 + 28x_1x_2 + \lambda(\frac{2}{5}x_1^2x_2 - 98)$

$\frac{\partial L}{\partial x_1} = 16x_1 + 28x_2 + \frac{4}{5}\lambda x_1x_2 = 0.$

$\frac{\partial L}{\partial x_2} = 28x_1 + \frac{2}{5}\lambda x_1^2 = 0.$ $\quad \frac{2}{5}\lambda x_1^2 = -28x_1$,

$\frac{\partial L}{\partial \lambda} = \frac{2}{5}x_1^2x_2 - 98 = 0,$ $\quad \frac{2}{5}x_1^2x_2 = 98,$ $\quad x_1^2x_2 = 245.$

$16x_1^2 + 28x_1x_2 + \frac{4}{5}\lambda x_1^2x_2 = 0$

$16x_1^2 + 28x_1x_2 - 56x_1x_2 = 16x_1^2 - 28x_1x_2 = 0$

$16x_1^3 - 28x_1^2x_2 = 0$

$16x_1^3 = 28\times 245$

$\Rightarrow x_1 = 7.54,\quad x_2 = \frac{245}{x^2} = 4.31$

$\lambda = \frac{-5}{2}\frac{1}{x_1} = -0.331$

$\nabla h^T Y = [\frac{4}{5}x_1x_2 \quad \frac{2}{5}x_1^2]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [25.99 \quad 22.74]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 25.99y_1 + 22.74y_2 = 0$

$y_1 = -0.875y_2.$

$Y^T \nabla^2 L_{xx} Y = [y_1 \quad y_2]\begin{bmatrix} 16+\frac{4}{5}\lambda x_2 & 28+\frac{4}{5}\lambda x_1 \\ 28+\frac{4}{5}\lambda x_1 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [y_1 \quad y_2]\begin{bmatrix} 14.86 & 26 \\ 26 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [14.86y_1 + 26y_2 \quad 26y_1]\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 14.86y_1^2 + 26y_1y_1 + 26y_1y_2$

$= 14.86y_1^2 + 52y_1y_2$

$= -30.64y_1^2$

Q4. (a) 0: baking
1: preparation
2: repairing

$q_{00} = -(b+f),\ q_{01} = b,\ q_{02} = f$
$q_{10} = p,\ q_{11} = -p,\ q_{12} = 0$
$q_{20} = r,\ q_{21} = 0\quad q_{22} = -r$

$$Q = \begin{bmatrix} -(b+f) & b & f \\ p & -p & 0 \\ r & 0 & -r \end{bmatrix}$$

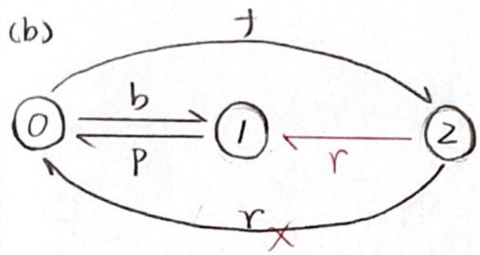
$\pi Q = 0.\quad \sum_j \pi_j = 1.$

$$[\pi_0\ \pi_1\ \pi_2]\,Q \Rightarrow \begin{cases} -(b+f)\pi_0 + p\pi_1 + r\pi_2 = 0 \\ b\pi_0 - p\pi_1 = 0 \\ f\pi_0 - r\pi_2 = 0 \end{cases}$$

(c) average production rate:

$$\pi_0 b = \frac{bpr}{pr+br+fp}$$

(b)



$q_{00} = -(b+f),\ q_{01} = b,\quad q_{02} = f$
$q_{10} = p,\ q_{11} = -p\quad q_{12} = 0$
$q_{20} = 0,\ q_{21} = r,\quad q_{22} = -r$

$$Q = \begin{bmatrix} -(b+f) & b & f \\ p & -p & 0 \\ 0 & r & -r \end{bmatrix}$$

$(b+f)\pi_0 =$

$\pi_1 = \frac{b}{p}\pi_0$
$\pi_2 = \frac{f}{r}\pi_0$

$\pi_0 + \pi_1 + \pi_2 = (1 + \frac{b}{p} + \frac{f}{r})\pi_0 = 1$

$$\pi_0 = \frac{pr}{pr+br+fp}$$

$$\pi_1 = \frac{br}{pr+br+fp}$$

$$\pi_2 = \frac{fp}{pr+br+fp}$$

$\pi Q = 0$

$$\begin{cases} -(b+f)\pi_0 + p\pi_1 = 0 \\ b\pi_0 - p\pi_1 + r\pi_2 = 0 \\ f\pi_0 - r\pi_2 = 0 \end{cases}$$

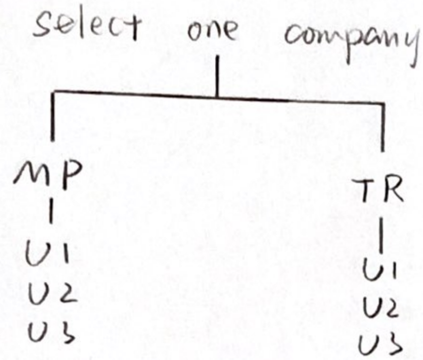
$\pi_2 = \frac{f}{r}\pi_0,\quad \pi_1 = \frac{b+f}{p}\pi_0$

$\pi_0 + \frac{f}{r}\pi_0 + \frac{b+f}{p}\pi_0 = 1$

$(1 + \frac{f}{r} + \frac{b+f}{p})\pi_0 = 1$

$$\pi_0 = \frac{pr}{pr+fp+br+fr}$$

Q5. (a)



	MP	TR
MP	1	1/5
TR	5	1
sum	6	1.2

MP	TR	priority
0.1667	0.1667	0.1667
0.8333	0.8333	0.8333

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} 0.1667 + \begin{bmatrix} 0.2 \\ 1 \end{bmatrix} 0.8333 = \begin{bmatrix} 0.3334 \\ 1.6668 \end{bmatrix}.$$

MP : $0.3334 \div 0.1667 = 2$

TR : $1.6668 \div 0.8333 = 2$

$$\lambda_{max} = \frac{1}{2} \times (2+2) = 2$$

$$CI = \frac{\lambda_{max} - n}{n-1} = \frac{2-2}{2-1} = 0$$

A_{MP} :

	U1	U2	U3
U1	1	0.5	0.2
U2	2	1	0.5
U3	5	2	1
sum	8	3.5	1.7

			priority
0.125	0.143	0.118	0.129
0.25	0.286	0.294	0.277
0.625	0.571	0.588	0.595

A_{TR} :

	U1	U2	U3
U1	1	2	3
U2	0.5	1	1.5
U3	0.333	0.666	1
sum	1.833	3.666	5.5

0.545	0.545	0.545	0.545
0.273	0.273	0.273	0.273
0.182	0.182	0.182	0.182

$$\begin{bmatrix} 0.129 & 0.545 \\ 0.277 & 0.273 \\ 0.595 & 0.182 \end{bmatrix} \begin{bmatrix} 0.1667 \\ 0.8333 \end{bmatrix} = \begin{bmatrix} 0.4757 \\ 0.2737 \\ 0.2508 \end{bmatrix} \checkmark$$

U1 should be selected

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2018-2019

EE6204 – SYSTEMS ANALYSIS

April / May 2019 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A company makes a product for three retail stores. The company has three factories to make the product. The amount of the product required for each retail store and the unit cost ($/box) in shipping the product from the three factories to the retail stores are given in Table 1.

Shipping cost ($) per box

		Retail Stores 1	2	3	Supply (box)
Factory	1	20	24	24	60
	2	18	23	20	70
	3	20	25	26	50
Demand (box)		40	65	75	

Table 1

Note: Question No. 1 continues on page 2.

The company would like to minimize the total shipping cost.

(a) Formulate the above problem as a balanced transportation problem.

(3 Marks)

(b) Use Vogel's method to construct an initial basic feasible solution and apply the transportation algorithm to obtain an optimal solution. Show all your calculations.

(15 Marks)

(c) If the unit shipping cost of shipping the product from factory 3 to retail store 3 is changed by $\$\Delta$/box, determine the range of values of Δ for which the current basic feasible solution in part 1(b) remains optimal.

(2 Marks)

2. You are required to design a closed rectangular container to carry 98 cm^3 of a certain type of powder. The length of the container must be 2.5 times that of the height. The container is to be made of a special material costing $\$10/cm^2$. The aim is to design the container with the least material cost possible.

(a) Formulate a nonlinear program for the above problem. You may let the length of the container be x_1 (cm) and the breadth be x_2 (cm).

(5 Marks)

(b) Apply the method of Lagrange multipliers to the nonlinear program in part 2(a) to determine the dimensions of the container (in cm) as well as the optimum material cost. Show all your steps.

(15 Marks)

3. A car dealer sells a brand of expensive cars imported from overseas. The shop carries only 1 car in the shop at any time due to space and cost limitations. An import order for one car will be placed at the end of each week if there is no car left in the shop at the end of the week. The ordered car will take one whole week to arrive. Namely, if an order is placed at the end of this week, then the new car will arrive in the shop for sale at the beginning of the week after next week.

For $k = 1, 2, \ldots$, let

A_k be the demand for the car in the k^{th} week,

B_k be the number of car unsold at the end of the k^{th} week.

Note: Question No. 3 continues on page 3.

Assume that A_k's are independent and identical Poisson random variables with parameter $\lambda = 1$.

(a) Is $\{B_k\}$ a Markov chain? Give reasons.

(3 Marks)

(b) Formulate this process as a Discrete Time Markov Chain, identify a suitable state space and write down the state space S.

[Hint: Consider the number of cars over two consecutive weeks.]

(5 Marks)

(c) Derive the Transition Probability Matrix (TPM) P and determine the steady-state probabilities.

(9 Marks)

(d) Comment on the operation of the shop and give recommendations to the shop owner to improve the operation.

(3 Marks)

4. A bread making machine can bake one loaf of bread at a time. After the machine finishes baking a loaf, preparation (such as cleaning and greasing the tin, and putting all ingredients into the tin for the next loaf of bread) is needed before baking a new loaf of bread. If the machine breaks down during the baking process, a serviceman will immediately start repairing the machine. After repair, the chef will throw the half-baked bread away and prepare the baking of a new loaf. It is assumed that the machine does not fail during the preparation process before baking the next loaf of bread.

Assume that the baking time, preparation time, repair time and time to failure are all independent and exponentially distributed random processes with rates b, p, r and f, respectively.

(a) Model this as a continuous-time Markov chain and identify a suitable state space S.

(3 Marks)

Note: Question No. 4 continues on page 4.

(b) Draw the state transition diagram for the process. Write down the rate balance equations and find the steady-state probabilities.

(9 Marks)

(c) Find the average production rate of the baking machine.

(4 Marks)

(d) Suppose that, while waiting for the repair to be completed, the chef takes out the tin and prepares the baking of a new loaf, so that immediately after repair, baking can start. How the state transition diagram of part 4(b) needs to be changed? Without any calculation, how do you expect the production rate of the baking machine to be affected? Give reasons for your answer.

(4 Marks)

5. The company M produces a component (C1) which is required by three other companies (U1, U2 and U3) for their products. As these three companies are competitors and the company M has to select one of them for long term business. The main criteria for selection are its market potential (MP) and its track record (TR). The company M views MP is strongly more important than TR with the following comparison matrix:

$$A = \begin{array}{c|cc} & MP & TR \\ \hline MP & 1 & 1/5 \\ TR & 5 & 1 \end{array}$$

In the company M's view, the standing of the three companies (U1, U2 and U3) from the standpoint of the two criteria MP and TR is summarized in the following comparison matrices:

$$A_{MP} = \begin{array}{c|ccc} & U1 & U2 & U3 \\ \hline U1 & 1 & 1/2 & 1/5 \\ U2 & 2 & 1 & 1/2 \\ U3 & 5 & 2 & 1 \end{array}, \qquad A_{TR} = \begin{array}{c|ccc} & U1 & U2 & U3 \\ \hline U1 & 1 & 2 & 3 \\ U2 & 1/2 & 1 & 3/2 \\ U3 & 1/3 & 2/3 & 1 \end{array}$$

(a) Draw an AHP (Analytic Hierarchy Process) structure to show the decision process. Mark the obtained values in the AHP structure. Show all calculations clearly and state your recommendation for the selection of a company as the long term business partner.

(12 Marks)

Note: Question No. 5 continues on page 5.

(b) What can you say about the consistency of matrix A_{MP}?

(3 Marks)

(c) The company M produces a second component (C2) and decides to bundle C1 and C2 together as supplies to one of U1, U2 and U3 as all of them require C2 as well. Suppose that the importance weights of C1 and C2 are equal with 0.5 each. For C2, the company M assigns a weight of approximately 60% to MP and 40% to TR, respectively, and then ranks U1, U2 and U3 from the stand-point of MP and TR as shown in Table 2.

	Percent weight estimates for C2		
Criterion	U1	U2	U3
MP	20	30	50
TR	50	20	30

Table 2

Design the structure of AHP for these two hierarchies of criteria. Mark the available values in the AHP structure.

(5 Marks)

END OF PAPER