Q3. (a)

For the steered standard wheel:

$\alpha = 0$, $\beta = -\beta$, $l = 55cm$, $r = 12cm$, $\dot{\varphi} = \dot{\varphi}_{ss}$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 12 \dot{\varphi}_{ss} = 0$.
  where $j(\beta) = [\sin(-\beta) \quad -\cos(-\beta) \quad -55\cos(-\beta)]$.
- sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I = 0$.
  where $c(\beta) = [\cos(-\beta) \quad \sin(-\beta) \quad 55\sin(-\beta)]$.

For the fixed standard wheel:

$\alpha = 0$, $\beta = \frac{\pi}{2}$, $l = 0$, $r = 12cm$, $\varphi = \dot{\varphi}_{fs}$

- rolling constraint: $j R(\theta) \dot{\xi}_I - 12 \dot{\varphi}_{fs} = 0$.
  where $j = [\sin(\frac{\pi}{2}) \quad -\cos(\frac{\pi}{2}) \quad 0]$.
- sliding constraint: $c R(\theta) \dot{\xi}_I = 0$,
  where $c = [\cos(\frac{\pi}{2}) \quad \sin(\frac{\pi}{2}) \quad 0]$.

For castor wheel 1:

$\alpha = \frac{\pi}{4}$, $\beta = \beta_1(t)$, $l = 35\sqrt{2}$, $r = 10$, $\dot{\varphi} = \dot{\varphi}_{c1}$, $d = 15$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 10 \dot{\varphi}_{c1} = 0$.
  where $j(\beta) = [\sin(\frac{\pi}{4} + \beta_1(t)) \quad -\cos(\frac{\pi}{4} + \beta_1(t)) \quad -35\sqrt{2}\cos(\beta_1(t))]$.
- sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I + 15 \dot{\beta}_1(t) = 0$.
  where $c(\beta) = [\cos(\frac{\pi}{4} + \beta_1(t)) \quad \sin(\frac{\pi}{4} + \beta_1(t)) \quad 35\sqrt{2}\sin(\beta_1(t))]$.

For castor wheel 2:

$\alpha = -\frac{\pi}{4}$, $\beta = \beta_2(t)$, $l = 35\sqrt{2}$, $r = 10$, $\varphi = \dot{\varphi}_{c2}$, $d = 15$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 10 \dot{\varphi}_{c2} = 0$.
  where $j(\beta) = [\sin(-\frac{\pi}{4} + \beta_2(t)) \quad -\cos(-\frac{\pi}{4} + \beta_2(t)) \quad -35\sqrt{2}\cos(\beta_2(t))]$.
- sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I + 15 \dot{\beta}_2(t) = 0$.
  where $c(\beta) = [\cos(-\frac{\pi}{4} + \beta_2(t)) \quad \sin(-\frac{\pi}{4} + \beta_2(t)) \quad 35\sqrt{2}\sin(\beta_2(t))]$.

Q4. (a)

For $F_a \to F_b$: points O_1, O_2, O_3, O_4 are used.

$$m_{jb} = \alpha_j H m_{ja}, \quad \alpha_j = \frac{z_{ja}}{z_{jb}}, \quad H = R + \frac{x}{d_a} n_a^T,$$

$$\frac{H}{h_{33}} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{bmatrix} \xRightarrow{\text{unwarp}} h = [h_1\ h_2\ h_3\ h_4\ h_5\ h_6\ h_7\ h_8]^T.$$

Define M_j:

$$m_{ia} = [x_{ia}, y_{ia}, 1]^T, \quad m_{ib} = [x_{ib}, y_{ib}, 1]^T.$$

$$M_j = \begin{bmatrix} x_{ia} & y_{ia} & 1 & 0 & 0 & 0 & -x_{ia}x_{ib} & -y_{ia}x_{ib} \\ 0 & 0 & 0 & x_{ia} & y_{ia} & 1 & -x_{ia}y_{ib} & -y_{ia}y_{ib} \end{bmatrix},$$

$$M_j h = m_j \xRightarrow{\text{stack}} M h = m.$$

Find vector h by using linea algebra, then wrap h back into matrix will give $\frac{H}{h_{33}}$.

For $F_b \to F_c$: points O_2, O_3, O_4, O_5 are used.

$$m_{jc} = \alpha_j H m_{jb}$$

similar as above.

(b) No. at least 4 feature points must be matched from image a and image c. These 4 points must be coplanar and noncolinear. One more matched feature point and scaled factor ~~is~~ are needed.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April/May 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 6 pages.
2. Answer ALL 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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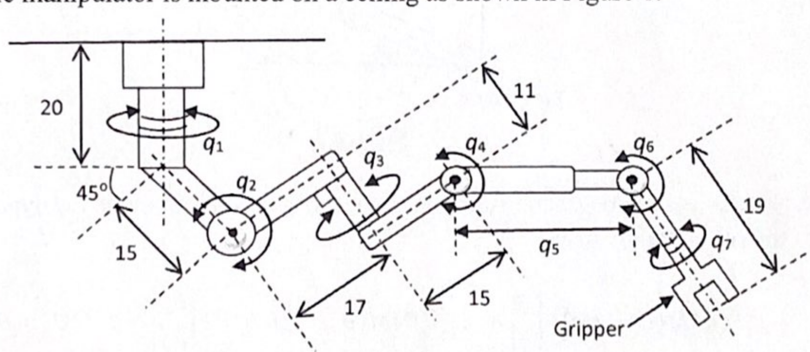
1. A robotic manipulator is mounted on a ceiling as shown in Figure 1.



Note: All units are in centimeter (cm).

Figure 1

(a) Use Denavit-Hartenberg (D-H) algorithm to obtain the link coordinate diagram. Derive the kinematic parameters of the robot.

(15 Marks)

Note: Question No. 1 continues on page 2

(b) Suppose that the arm matrix of the robot manipulator in part 1(a) has been derived as $T(q)_{base}^{tool}$ and the robot end effector or gripper is now grasping an object as shown in Figure 2. Briefly discuss how the position and orientation of the tip of the object can be determined. Discuss the sources of errors in deriving the forward kinematic equations.

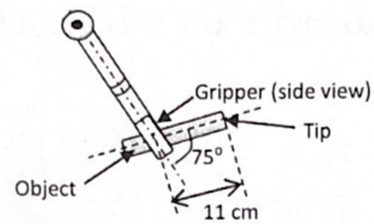


Figure 2

(5 Marks)

2. A two-link planar robot with joint variables θ_1 and θ_2 is shown in Figure 3.

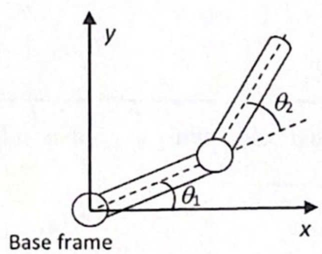


Figure 3

When the robot end effector is not in contact with the environment, the dynamic equations of the robot are given as:

$$J^{-T}(\theta)M(\theta)J^{-1}(\theta)\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} - J^{-T}(\theta)M(\theta)J^{-1}(\theta)\dot{J}(\theta)\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} + J^{-T}(\theta)v(\theta,\dot{\theta}) = \begin{bmatrix}u_1\\ u_2\end{bmatrix}$$

where

$$M(\theta) = \begin{bmatrix} 5 & 3+2\cos\theta_2 \\ 3+2\cos\theta_2 & 3 \end{bmatrix},\quad J(\theta) = \begin{bmatrix} -l_1\sin(\theta_1) - l_1\sin(\theta_1+\theta_2) & l_2\sin(\theta_1+\theta_2) \\ l_1\cos(\theta_1) + l_1\cos(\theta_1+\theta_2) & l_2\cos(\theta_1+\theta_2) \end{bmatrix},$$

$$v(\theta,\dot{\theta}) = \begin{bmatrix} -(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin\theta_2 + 50\cos\theta_1 + 25\cos(\theta_1+\theta_2) \\ 2\dot{\theta}_1^2\sin\theta_2 + 10\cos(\theta_1+\theta_2) \end{bmatrix},\qquad \begin{bmatrix}u_1\\ u_2\end{bmatrix} = J^{-T}(\theta)\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix},$$

Note: Question No. 2 continues on page 3

x and y denote the position of the end-effector of the robot in Cartesian space, $l_1 = 0.35$ m, $l_2 = 0.20$ m are the link lengths, τ_1 and τ_2 are the joint torques of links 1 and 2, respectively. The system possesses unmodelled resonances at 10 Hz, 20 Hz, and 30 Hz.

(a) If the end effector is now in contact with a vertical and frictionless wall that is located at a distance of 0.3 m along the x-axis of the base frame and the stiffness of the wall is 25 N/m, design a hybrid position and force controller for the robot to perform the contact task by moving on the wall while exerting a desired contact force. The system should be overdamped with a damping ratio of 1.2 and should not excite the resonances.

$\xi = 1.2$

(10 Marks)

(b) If a constant disturbance force is acting on the end effector along vertical y-axis, derive the error equations of the closed-loop system. What controller should be used in this case to achieve the same performance specifications in part 2(a)?

(5 Marks)

(c) If the vertical wall with the same stiffness as in part 2(a) is now located at a distance of 0.5 m along the x-axis and friction is present on the surface, discuss the problems associated with the implementation of the hybrid position and force controller.

(5 Marks)

3. (a) A mobile platform with one steered standard wheel, one castor wheel and two fixed standard wheels is shown in Figure 4. The radii of all standard wheels are 0.045 m and the radius of the castor wheel is 0.040 m. A local reference frame is assigned as shown in Figure 4 and the steered angle of the steered standard wheel is defined as shown in the figure. The rotational velocities of the steered standard wheel, castor wheel and fixed standard wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_c$, $\dot{\varphi}_{f1}$, and $\dot{\varphi}_{f2}$, respectively. Derive the rolling and sliding constraints of the mobile platform.

(10 Marks)

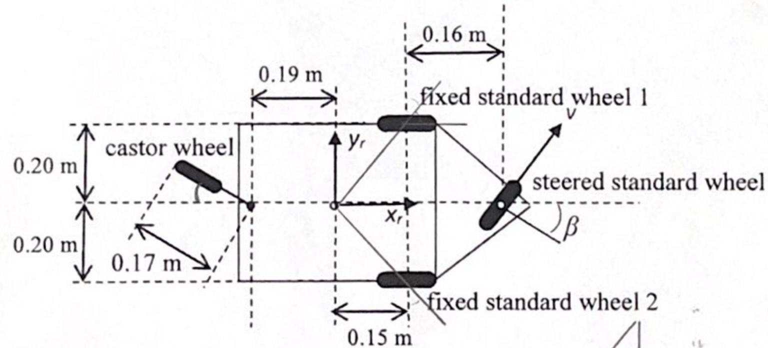


Figure 4

Note: Question No. 3 continues on page 4

(b) A robot manipulator with 4 joints q_1, q_2, q_3 and q_4 is now mounted on the mobile platform to form a mobile manipulator system. The forward kinematics equations of the robot with respect to its base are given in terms of the first 3 joint variables as:

$$x = 0.1 + 0.3\cos(q_1)\cos(q_2 + q_3) + 0.2\cos(q_1)\cos(q_2)$$

$$y = 0.1 + 0.3\sin(q_1)\cos(q_2 + q_3) + 0.2\sin(q_1)\cos(q_2)$$

$$z = 0.2 + 0.3\sin(q_2 + q_3) + 0.2\sin(q_2)$$

Find the position of the manipulator when $q_1 = 0$, $q_2 = 0$ and $q_3 = 0$. Solve the inverse kinematics problem to express the joint angles q_1, q_2 and q_3 in terms of x, y and z. If the approach vector of the robot is denoted as r_3, discuss whether arbitrary position and orientation can be specified for the mobile manipulator system.

(10 Marks)

4. As shown in Figure 5, six feature points O_1, O_2, O_3, O_4, O_5, and O_6 are on three different lines in the same plane. A moving camera takes two images of the points at two poses. Two coordinate frames represented by F_a and F_b are attached to the projection centre of the camera at the two poses, respectively. Let H denote the Euclidean homography matrix from F_a to F_b and let h_{33} denote the third row third column element of the matrix H.

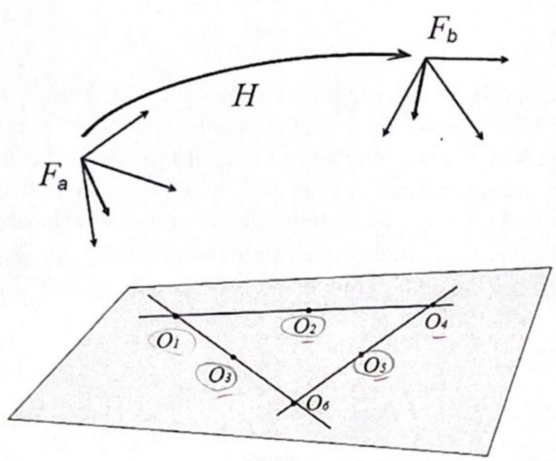


Figure 5

Note: Question No. 4 continues on page 5

(a) Assume that four feature points O_2, O_3, O_5, and O_6 can be detected in the images taken at the poses attached to F_a and F_b. Their corresponding normalized coordinates in F_a and F_b are given by

$$m_{2a} = [a_1, a_2, 1]^T, \; m_{3a} = [a_3, a_4, 1]^T, \; m_{5a} = [a_5, a_6, 1]^T, \; m_{6a} = [a_7, a_8, 1]^T$$

and

$$m_{2b} = [b_1, b_2, 1]^T, \; m_{3b} = [b_3, b_4, 1]^T, \; m_{5b} = [b_5, b_6, 1]^T, \; m_{6b} = [b_7, b_8, 1]^T,$$

respectively. Find $\dfrac{H}{h_{33}}$.

(12 Marks)

(b) Assume that only four feature points O_1, O_2, O_3, and O_5 can be detected in the image taken at the pose attached to F_a and only four feature points O_2, O_3, O_4, and O_5 can be detected in the image taken at the pose attached to F_b. Can $\dfrac{H}{h_{33}}$ be calculated? Justify your answer.

(4 Marks)

(c) Assume that all six feature points O_1, O_2, O_3, O_4, O_5, and O_6 can be detected in the images taken at the pose attached to F_a and at the pose attached to F_b. List all combinations of feature points that can be used to calculate $\dfrac{H}{h_{33}}$. Justify your answers.

(4 Marks)

5. Three sensors are used to measure the state x_k modelled by $x_{k+1} = x_k$. Let y_{1k}, y_{2k} and y_{3k} represent the outputs of the three sensors modelled by

$$y_{1k} = x_k + v_{1k}$$
$$y_{2k} = x_k + v_{2k}$$
$$y_{3k} = x_k + v_{3k}$$

where v_{1k}, v_{2k} and v_{3k} are zero mean Gaussian noises with variances given by σ^2, σ^2 and $4\sigma^2$, respectively.

A predictor is designed as below to estimate x_k:

$$\hat{x}_{k+1} = \hat{x}_k + M_k(y_{1k} - \hat{x}_k) + M_k(y_{2k} - \hat{x}_k) + M_k(y_{3k} - \hat{x}_k)$$

where $\hat{x}_k$ represents the estimate of x_k and M_k represents the predictor gain.

Let the estimation error be denoted as $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error and the noise terms v_{1k}, v_{2k} and v_{3k} are all uncorrelated, and that $E[\tilde{x}_{k+1}] = 0$. Let the estimation error variance be $p_{k+1} = E[\tilde{x}_{k+1}^2]$.

(a) Find the update equation for p_{k+1} and derive the update law for M_k to minimize p_{k+1}.

(15 Marks)

(b) Assume that the initial value of the estimation error variance is equal to σ^2 (i.e., $p_1 = \sigma^2$). Find the value of k, if any, so that p_{k+1} becomes less than $\frac{\sigma^2}{10}$. Show details of your calculation to justify your answer.

(5 Marks)

END OF PAPER