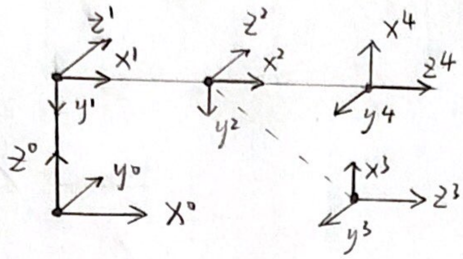
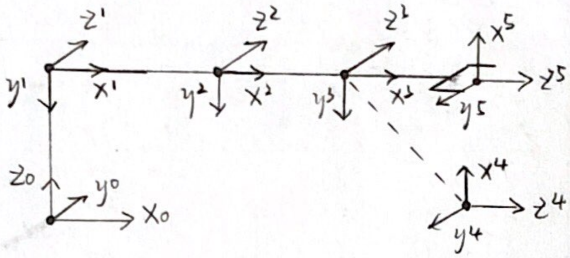
Slide P47



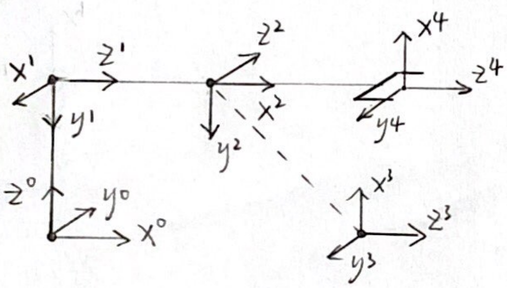
Axis	θ	d	a	α
1	θ_1	d_1	0	$-90°$
2	θ_2	0	a_2	0
3	θ_3	0	0	$-90°$
4	θ_4	d_4	0	0

Slide P49



Axis	θ	d	a	α
1	θ_1	d_1	0	$-90°$
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	0	0	$-90°$
5	θ_5	d_5	0	0

CA1



Axis	θ	d	a	α
1	$-90°$	d_1	0	$-90°$
2	$-90°$	d_2	0	$90°$
3	θ_3	0	0	$-90°$
4	θ_4	0.15	0	0

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_0^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

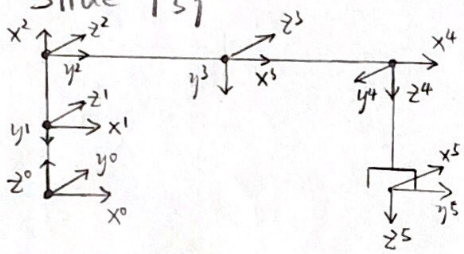
$$T_3^4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_2^3 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_3^4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_0^1 T_1^2 = \begin{bmatrix} 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_0^3 = \begin{bmatrix} S_3 & 0 & C_3 & d_2 \\ 0 & -1 & 0 & 0 \\ C_3 & 0 & -S_3 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$T_{base}^{tool} = T_0^4 = \left[\begin{array}{ccc|c} S_3C_4 & -S_3S_4 & C_3 & 0.15C_3 + d_2 \\ -S_4 & -C_4 & 0 & 0 \\ C_3C_4 & -C_3S_4 & -S_3 & -0.15S_3 + d_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right].$$

Slide P59



Axis	θ	d	a	α
1	θ_1	d_1	0	$-90°$
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	θ_4	0	a_4	$-90°$
5	θ_5	d_5	0	0

$$T_0^1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_1^2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_2^3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

CA1 Continuous

$$w = \begin{bmatrix} w^1 \\ w^2 \end{bmatrix} = \begin{bmatrix} q \\ [\exp(\frac{q_n}{\pi})]r^3 \end{bmatrix}.$$

$w_1 = 0.15C_3 + d_2, \quad w_2 = 0, \quad w_3 = -0.15S_3 + d_1$

$w_4 = e^{\frac{q_4}{\pi}} C_3, \quad w_5 = 0, \quad w_6 = -e^{\frac{q_4}{\pi}} S_3$

$\tan\theta_3 = \frac{S_3}{C_3} = -\frac{w_6}{w_4}. \quad \theta_3 = \arctan 2(-\frac{w_6}{w_4}).$

$d_2 = w_1 - 0.15C_3, \quad d_1 = w_3 + 0.15S_3,$

$$w_4C_3 - w_6S_3 = e^{\frac{q_4}{\pi}} C_3^2 + e^{\frac{q_4}{\pi}} S_3^2 = e^{\frac{q_4}{\pi}}.$$

$$\Rightarrow q_4 = \pi \ln(w_4C_3 - w_6S_3)$$

Differential Motion and Statics

$$x = w(q) = \begin{bmatrix} p \\ \exp(\frac{q_n}{\pi}) r^3 \end{bmatrix}.$$

$$\dot{x} = V(q)\dot{q}. \quad V_{kj}(q) = \frac{\partial w_k(q)}{\partial q_j}$$

$$V(q) = \begin{bmatrix} \frac{\partial w(q)}{\partial q_1} & \frac{\partial w(q)}{\partial q_2} & \cdots & \frac{\partial w(q)}{\partial q_j} \end{bmatrix}$$ Jacobian matrix

Example 5.2

$$w(q) = \begin{bmatrix} a_1 C_1 + a_2 C_{1-2} \\ a_1 S_1 + a_2 S_{1-2} \\ d_1 - q_3 - d_4 \\ 0 \\ 0 \\ -\exp(\frac{q_4}{\pi}) \end{bmatrix}.$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -a_1 S_1 - a_2 S_{1-2} & a_2 S_{1-2} & 0 & 0 \\ a_1 C_1 + a_2 C_{1-2} & -a_2 C_{1-2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\pi}\exp(\frac{q_4}{\pi}) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}.$$

$$\dot{x} = V(q)\dot{q} \Rightarrow \dot{q} = V^{+}(q)\dot{x},$$

$$V^{+}(q) = \begin{cases} V^T[V^T V]^{-1}, & m \le n \\ V^{-1}, & m = n \\ [V^T V]^{-1} V^T, & m \ge n \end{cases}$$

Robot Control

non-linear system:

$$m\ddot{x} + b\dot{x} + kx^3 = u$$

$\Downarrow$ control law partitioning

design controller $u = \alpha v + \beta$:

$$m\ddot{x} + b\dot{x} + kx^3 = \alpha v + \beta.$$

v is input. choose α, β as:

$$\begin{cases} \alpha = m \\ \beta = b\dot{x} + kx^3 \end{cases} \Rightarrow \ddot{x} = v$$

design control law:

$$v = \ddot{x}_d + k_v\dot{e} + k_p e.$$

$$e = x_d - x.$$

x_d is desired position

$$\Rightarrow \quad \ddot{x} = \ddot{x}_d + k_v\dot{e} + k_p e.$$

$$\underline{\ddot{e} + k_v\dot{e} + k_p e = 0.}$$

$$s^2 + k_v s + k_p = 0.$$

---

performance Specification:

二阶系统: $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\xi \geq 1$ is the damping ratio

$\omega_n \leq 0.5\,\omega_{res}$ is undamped natural frequency

$$k_v = 2\xi\omega_n. \quad k_p = \omega_n^2$$

---

disturbance: $\underset{\sim\sim}{m\ddot{x} = mv - d}$

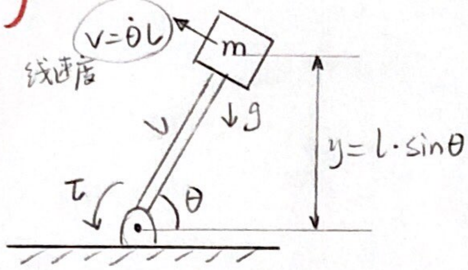
$$m\ddot{x} + b\dot{x} + kx^3 + d = u, \quad \ddot{e} + k_v\dot{e} + k_p e = \frac{d}{m}. \quad \frac{E(s)}{D(s)} = \frac{1}{m(s^2 + k_v s + k_p)}$$

$s = 0 \Rightarrow$ steady state error: $e_{ss} = \dfrac{d}{mk_p}$, d大，干扰大 k_p大，干扰小

加入积分项: $v = \ddot{x}_d + k_v\dot{e} + k_p e + k_i\int e\,dt$. $\underline{\ddot{e} + k_v\dot{e} + k_p e + k_i\int e\,dt = \frac{d}{m}}$.

$$\frac{E(s)}{D(s)} = \frac{s}{m(s^3 + k_v s^2 + k_p s + k_i)}. \quad \text{steady state: } e_{ss} = 0$$

Dynamics



<u>single-link robot:</u>

动能： $K=\frac{1}{2}mv^2=\frac{1}{2}ml^2\dot{\theta}^2$

势能： $P=mgy=mgl\cdot\sin\theta$

Lagrangian： $L=K-P$

dynamic model: $\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}}=\tau+\frac{\partial L}{\partial\theta}$

$\frac{d}{dt}$(动量) = 力

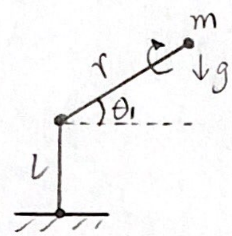
$L=K-P=\frac{1}{2}ml^2\dot{\theta}^2-mgl\cdot\sin\theta$.

$\frac{\partial L}{\partial\dot{\theta}}=ml^2\dot{\theta}$, $\frac{\partial L}{\partial\theta}=-mgl\cdot\cos\theta$.

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}}=\tau+\frac{\partial L}{\partial\theta}$$

$$ml^2\ddot{\theta}+mgl\cdot\cos\theta=\tau$$

<u>two-link polar robot:</u>



$K=\frac{1}{2}mr^2\dot{\theta}_1^2+\frac{1}{2}m\dot{r}^2$,

$P=mg\cdot\sin\theta_1$.

$L=\frac{1}{2}mr^2\dot{\theta}_1^2+\frac{1}{2}m\dot{r}^2-mgr\sin\theta_1$.

$\theta=\begin{bmatrix}\theta_1\\ r\end{bmatrix}$, $\dot{\theta}=\begin{bmatrix}\dot{\theta}_1\\ \dot{r}\end{bmatrix}$, $\tau=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$.

$$\frac{\partial L}{\partial\dot{\theta}}=\begin{bmatrix}\partial L/\partial\dot{\theta}_1\\ \partial L/\partial\dot{r}\end{bmatrix}=\begin{bmatrix}mr^2\dot{\theta}_1\\ m\dot{r}\end{bmatrix}.$$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}}=\begin{bmatrix}mr^2\ddot{\theta}_1+2mr\dot{r}\dot{\theta}_1\\ m\ddot{r}\end{bmatrix}.\quad \frac{\partial L}{\partial\theta}=\begin{bmatrix}\partial L/\partial\theta_1\\ \partial L/\partial r\end{bmatrix}=\begin{bmatrix}-mgr\cdot\cos\theta_1\\ mr^2\dot{\theta}_1^2-mg\cdot\sin\theta_1\end{bmatrix}.$$

$$\Rightarrow\begin{cases}mr^2\ddot{\theta}_1+2mr\dot{r}\dot{\theta}_1+mgr\cdot\cos\theta_1=\tau_1\\ m\ddot{r}-mr^2\dot{\theta}_1+mg\cdot\sin\theta_1=\tau_2\end{cases}$$

$$\Rightarrow M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+g(\theta)=\tau.$$

$$\begin{bmatrix}mr^2 & 0\\ 0 & m\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{r}\end{bmatrix}+\begin{bmatrix}2mr\dot{r}\dot{\theta}_1\\ -mr^2\dot{\theta}_1\end{bmatrix}+\begin{bmatrix}mgr\cdot\cos\theta_1\\ mg\cdot\sin\theta_1\end{bmatrix}=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

non-linear control

$ml^2\ddot{\theta}+mgl\cdot\cos\theta=\tau$.

$\tau=\alpha v+\beta$,

$v=\ddot{\theta}$.

$$\Rightarrow\begin{cases}\alpha=ml^2\\ \beta=mgl\cdot\cos\theta\end{cases}$$

---

design $v=\ddot{\theta}_d+k_v\dot{e}+k_pe$ $\Rightarrow$ $\ddot{e}+k_v\dot{e}+k_pe=0$.

<u>MIMO system:</u> $M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+g(\theta)=\tau=\alpha v+\beta$.

$\Rightarrow\begin{cases}\alpha=M(\theta)\\ \beta=C(\theta,\dot{\theta})\dot{\theta}+g(\theta)\end{cases}$, design $v=\ddot{\theta}_d+K_v\dot{E}+K_pE$ $\Rightarrow$ $\ddot{E}+K_v\dot{E}+K_pE=0$

<u>PID:</u> $v=\ddot{\theta}_d+K_v\dot{E}+K_pE+K_I\int E\,dt$, $\ddot{E}+K_v\dot{E}+K_pE+K_I\int E\,dt=0$

Example:

2-link polar robot, $m = 2kg$, $\omega_{res} = 12\ rad \cdot s^{-1}$

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = \tau.$$

$$\begin{bmatrix} mr^2 & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2mr\dot{r}\dot{\theta}_1 \\ -mr\dot{\theta}_1^2 \end{bmatrix} + \begin{bmatrix} mgr\cos\theta_1 \\ mg\sin\theta_1 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}.$$

$$M(\theta) = \begin{bmatrix} 2r^2 & 0 \\ 0 & 2 \end{bmatrix}. \quad C(\theta,\dot{\theta})\dot{\theta} = \begin{bmatrix} 4r\dot{r}\dot{\theta}_1 \\ -2r\dot{\theta}_1^2 \end{bmatrix}. \quad g(\theta) = \begin{bmatrix} 19.6\,r\cos\theta_1 \\ 19.6\sin\theta_1 \end{bmatrix}.$$

Controller: $V = \alpha V + \beta$, $\alpha = M(\theta) = \begin{bmatrix} 2r^2 & 0 \\ 0 & 2 \end{bmatrix}.$

$$\beta = \begin{bmatrix} 4r\dot{r}\dot{\theta}_1 + 19.6\,r\cos\theta_1 \\ -2r\dot{\theta}_1^2 + 19.6\sin\theta_1 \end{bmatrix}.$$

$$\Rightarrow \ddot{\theta} = V. \quad \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad V = \ddot{\theta}_d + K_v\dot{E} + K_pE.$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_{1d} \\ \ddot{r}_d \end{bmatrix} + \begin{bmatrix} k_{v1} & 0 \\ 0 & k_{v2} \end{bmatrix}\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} k_{p1} & 0 \\ 0 & k_{p2} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$

$$\Rightarrow \begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} + \begin{bmatrix} k_{v1} & 0 \\ 0 & k_{v2} \end{bmatrix}\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} k_{p1} & 0 \\ 0 & k_{p2} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = 0.$$

or $\ddot{e}_i + k_{vi}\dot{e} + k_{pi}e = 0$, $i = 1,2$

$$\Rightarrow \quad s^2 + k_{vi}s + k_{pi} = 0.$$

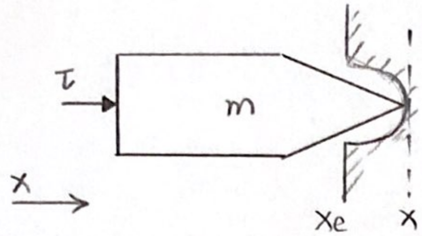
$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0,$$

$$\xi \geq 1, \quad \omega_n \leq 0.5\,\omega_{res} = 0.5 \times 12$$

$$\Rightarrow \quad s^2 + 12s + 36 = 0$$

$$\Rightarrow \quad k_v = 12, \quad k_p = 36$$

Force Control



let $e_f = f_d - f$.

force servo controller:

$$v = \frac{1}{k_e}(\ddot{f}_d + k_v \dot{e}_f + k_p e_f)$$

$$\ddot{e}_f + k_v \dot{e}_f + k_p e_f = 0$$

$$m\ddot{x} + f = \tau.$$

$$f = k_e(x - x_e), \quad \tau = \alpha v + \beta$$

$$\Rightarrow \quad m\ddot{x} + f = \alpha v + \beta$$

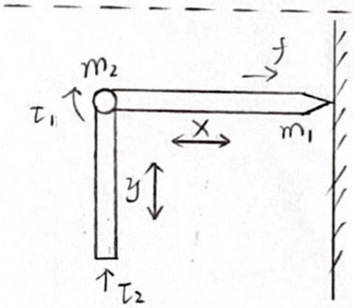
$$\begin{cases} \alpha = m \\ \beta = f \end{cases}$$

$$\Rightarrow \quad \ddot{x} = v, \quad \ddot{f} = k_e \ddot{x}, \quad \frac{\ddot{f}}{k_e} = \ddot{x} = v.$$

k_e: environmental stiffness

x_e: static location

Hybrid Position/Force Control



$x \rightarrow$ force control

$y \rightarrow$ motion control

$$m_1\ddot{x} + f = \tau_1.$$

$$(m_1 + m_2)\ddot{y} + (m_1 + m_2)g = \tau_2.$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_1 + m_2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ (m_1+m_2)g \end{bmatrix} + \begin{bmatrix} f \\ 0 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}. \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \alpha v + \beta.$$

$$\Rightarrow \quad \ddot{X} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \qquad \alpha = \begin{bmatrix} m_1 & 0 \\ 0 & m_1+m_2 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0 \\ (m_1+m_2)g \end{bmatrix} + \begin{bmatrix} f \\ 0 \end{bmatrix}$$

$\ddot{x} = v_1$ (force controller)

$\ddot{y} = v_2$ (motion controller)

position: $v_2 = \ddot{x}_d + k_{2v}\dot{e} + k_{2p}e$

$$\ddot{e} + k_{2v}\dot{e} + k_{2p}e = 0$$

force: $f = k_e(x - x_e)$, $\ddot{f} = k_e\ddot{x}$

$$\Rightarrow \quad v_1 = \ddot{x} = \frac{\ddot{f}}{k_e}, \quad e_f = f_d - f,$$

$$\Rightarrow \quad v_1 = \frac{1}{k_e}(\ddot{f}_d + k_{1v}\dot{e}_f + k_{1p}e_f)$$

$$\ddot{e}_f + k_{1v}\dot{e}_f + k_{1p}e_f = 0$$

In general:

$$M_x\ddot{X} + C_x + g_x + f_e = \tau$$

$$\tau = \alpha v + \beta.$$

$$\alpha = M_x,$$

$$\beta = C_x + g_x + f_e$$

$$\Rightarrow \quad M_x\ddot{X} + C_x + g_x + f_e = \alpha v + \beta$$

$$\ddot{X} = v = \begin{bmatrix} x_T \\ x_N \end{bmatrix}, \quad x_T = \begin{bmatrix} x \\ z \end{bmatrix}, \quad x_N = y$$

$$\begin{cases} \ddot{x}_T = F_T \text{ (motion control)} \\ \ddot{x}_N = F_N \text{ (force control)} \end{cases}$$

motion control:

$$F_T = \ddot{X}_{Td} + K_{Tv}\dot{E}_T + K_{Tp}E_T$$

$$\ddot{E}_T + K_{Tv}\dot{E}_T + K_{Tp}E_T = 0$$

force control: $f_e = K_e(x_N - x_e)$, $\ddot{f}_e = K_e\ddot{x}_N$.

$$F_N = \ddot{x}_N = K_e^{-1}\ddot{f}_e = K_e^{-1}(\ddot{f}_{ed} + K_{Nv}\dot{E}_N + K_{Np}E_N)$$

$$\ddot{E}_N + K_{Nv}\dot{E}_N + K_{Np}E = 0$$

Exercise – Control

1. Design the α, β partitioned controllers for the following nonlinear systems:

a) $3\theta\ddot{\theta} + 5\dot{\theta}^2 - 2\cos(\theta) = \tau$.

$\tau = \alpha v + \beta$, $\alpha = 3\theta$, $\beta = 5\dot{\theta}^2 - 2\cos(\theta)$.

b) $\ddot{\theta} + 2\cos(\theta) + \sqrt{3}\theta\ddot{\theta} + \dot{\theta} + \sqrt{2}\theta = \tau$.

$\tau = \alpha v + \beta$, $\alpha = (1+\sqrt{3}\theta)$, $\beta = \dot{\theta} + \sqrt{2}\theta + 2\cos(\theta)$.

2. The dynamic equations of a three-link robot is given as:

$$m_2 r^2 \ddot{\theta} + 2 m_2 r \dot{r} \dot{\theta} = \tau_1$$
$$(m_1 + m_2)\ddot{h} + (m_1 + m_2) g h = \tau_2$$
$$m_2 \ddot{r} - m_2 r \dot{\theta}^2 = \tau_3$$

Design a PD computed torque control law in joint space for the robot. $\xi = 1$. $\omega_{res} = 14.5\,Hz$.

$$q = \begin{bmatrix} \theta \\ h \\ r \end{bmatrix}, \quad \ddot{q} = \begin{bmatrix} \ddot{\theta} \\ \ddot{h} \\ \ddot{r} \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$$\begin{bmatrix} m_2 r^2 & 0 & 0 \\ 0 & m_1+m_2 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{h} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2 m_2 r \dot{r} \dot{\theta} \\ 0 \\ -m_2 r \dot{\theta}^2 \end{bmatrix} + \begin{bmatrix} 0 \\ (m_1+m_2) g h \\ 0 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

$\Rightarrow$ $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$

$\tau = \alpha v + \beta$.

$\alpha = M(q)$. $\beta = C + g$. $v = \ddot{q}$.

$\ddot{\theta} = v_1 = \ddot{\theta}_d + k_{v1}\dot{e}_1 + k_{p1} e_1$.

$\ddot{h} = v_2 = \ddot{h}_d + k_{v2}\dot{e}_2 + k_{p2} e_2$.

$\ddot{r} = v_3 = \ddot{r}_d + k_{v3}\dot{e}_3 + k_{p3} e_3$.

$\Rightarrow$ $\ddot{e}_1 + k_{1v}\dot{e}_1 + k_{1p} e_1 = 0$.

$\ddot{e}_2 + k_{2v}\dot{e}_2 + k_{2p} e_2 = 0$.

$\ddot{e}_3 + k_{3v}\dot{e}_3 + k_{3p} e_3 = 0$.

$s^2 + k_{iv} s + k_{ip} = 0$. $i = 1,2,3$

$s^2 + 2\xi_i \omega_n s + \omega_n^2 = 0$.

$\omega_n \le 0.5\,\omega_{res} = 0.5 \times 2\pi \times 14.5 = 14.5\pi$.

$k_{iv} = 2 \times 1 \times 14.5\pi = 29\pi$.

$k_{ip} = \omega_n^2 = (14.5\pi)^2$.