CA: EE6221 Robotics & Intelligent Sensors

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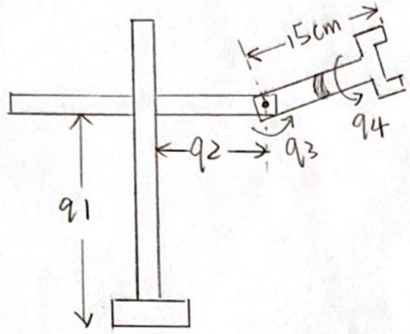
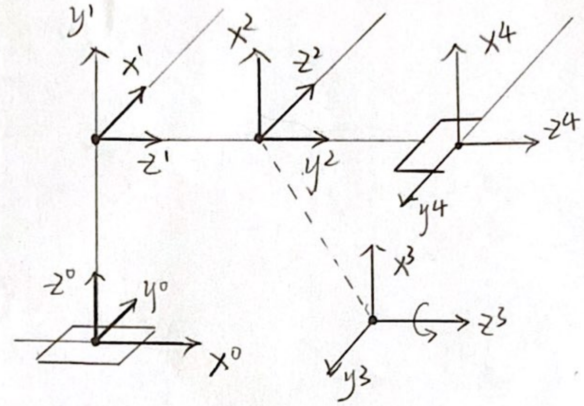




(a) D-H parameters:

Axis	θ	d	a	α
1	$\frac{\pi}{2}$	q_1	0	$\frac{\pi}{2}$
2	$\frac{\pi}{2}$	q_2	0	$\frac{\pi}{2}$
3	q_3	0	0	$-\frac{\pi}{2}$
4	q_4	15cm	0	0

(b) $T_{base}^{tool} = T_0^1 T_1^2 T_2^3 T_3^4$.

$$T_0^1 = \begin{bmatrix} \cos\frac{\pi}{2} & -\cos\frac{\pi}{2}\sin\frac{\pi}{2} & \sin\frac{\pi}{2}\sin\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2}\cos\frac{\pi}{2} & -\sin\frac{\pi}{2}\cos\frac{\pi}{2} & 0 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$T_1^2 = \begin{bmatrix} \cos\frac{\pi}{2} & -\cos\frac{\pi}{2}\sin\frac{\pi}{2} & \sin\frac{\pi}{2}\sin\frac{\pi}{2} & 0 \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2}\cos\frac{\pi}{2} & -\sin\frac{\pi}{2}\cos\frac{\pi}{2} & 0 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$T_2^3 = \begin{bmatrix} \cos q_3 & -\cos(-\frac{\pi}{2})\sin q_3 & \sin(-\frac{\pi}{2})\sin q_3 & 0 \\ \sin q_3 & \cos(-\frac{\pi}{2})\cos q_3 & -\sin(-\frac{\pi}{2})\cos q_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos q_3 & 0 & -\sin q_3 & 0 \\ \sin q_3 & 0 & \cos q_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$T_3^4 = \begin{bmatrix} \cos q_4 & -\cos 0 \sin q_4 & \sin 0 \sin q_4 & 0 \\ \sin q_4 & \cos 0 \cos q_4 & -\sin 0 \cos q_4 & 0 \\ 0 & \sin 0 & \cos 0 & 15cm \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos q_4 & -\sin q_4 & 0 & 0 \\ \sin q_4 & \cos q_4 & 0 & 0 \\ 0 & 0 & 1 & 15cm \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$T_{base}^{total} = T_0^1 T_1^2 T_2^3 T_3^4$$

$$= \begin{bmatrix} sq_3cq_4 & -sq_3sq_4 & cq_3 & 15cq_3+q_2 \\ -sq_4 & -cq_4 & 0 & 1 \\ cq_3cq_4 & -cq_3sq_4 & -sq_3 & -15sq_3+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $P = [15\cos q_3 \,,\, 1 \,,\, -15\sin q_3 + q_1]^T$,

$$R = \begin{bmatrix} \sin q_3 \cos q_4 & -\sin q_3 \sin q_4 & \cos q_3 \\ -\sin q_4 & -\cos q_4 & 0 \\ \cos q_3 \cos q_4 & -\cos q_3 \sin q_4 & -\sin q_3 \end{bmatrix}.$$

$|X^4|^2 = |Y^4|^2 = |Z^4|^2 = 1$.

$$w(q) = [15\cos q_3 + q_2 \,,\, 1 \,,\, -15\sin q_3 + q_1 \,,\, e^{\frac{q_4}{\pi}}\cos q_3 ,\, 0 ,\, -e^{\frac{q_4}{\pi}}\sin q_3]^T .$$
$$= [w_1 \,,\, w_2 \,,\, w_3 \,,\, w_4 \,,\, w_5 \,,\, w_6]^T$$

(c) $\tan q_3 = \dfrac{\sin q_3}{\cos q_3} = -\dfrac{w_6}{w_4}$. $\Rightarrow$ $q_3 = \arctan2(-\dfrac{w_6}{w_4})$.

$w_4 \cos q_3 - w_6 \sin q_3 = e^{\frac{q_4}{\pi}}(\cos^2 q_3 + \sin^2 q_3) = e^{\frac{q_4}{\pi}}$.

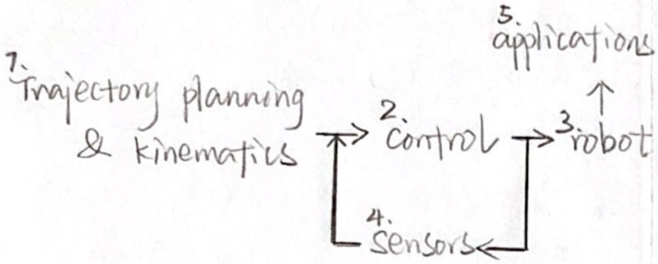
$\Rightarrow$ $q_4 = \pi \cdot \ln(w_4\cos q_3 - w_6 \sin q_3)$.

$q_2 = w_1 - 15 \cdot \cos q_3$.

$q_1 = w_3 + 15 \cdot \sin q_3$.

Outline:

1. Basis and applications
2. Kinematics
3. Trajectory planning
4. Robot control
5. Mobile robot



Classification:

1. <u>Drive technologies:</u>
   ① electric ; ② hydraulic
2. <u>Work-envelope geometries:</u>
   major axes: the first 3 joints
   ① revolute joint (R)
   ② prismatic joint (P)
   * Cartesian-coordinate robot (PPP) ⇨ rectangular box
   * Clindrical-coordinate robot (RPP) ⇨ the volume between two concentric cylinders
   * Spherical-coordinate robot (RRP) ⇨ the volume between two concentric spheres
   * SCARA (RRP) ⇨ the volume between two concentric cylinders
   * Articulated-coordinate robot (RRR) ⇨ complex work envelope

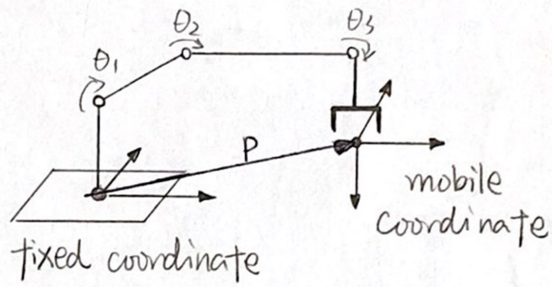
3. <u>Motion control methods:</u>
   ① point-to-point motion
   ② continuous-path motion

Specifications:

1. # of axes
2. capacity and speed
3. reach and stroke
   可行域 可操作域
4. tool orientation
   yaw — 偏转角
   pitch — 俯仰角
   roll — 滚动角
5. repeatability, accuracy, precisio
   可重复性 准确度 精度

Direct Kinematics:

Given: joint variable
want: position & orientation

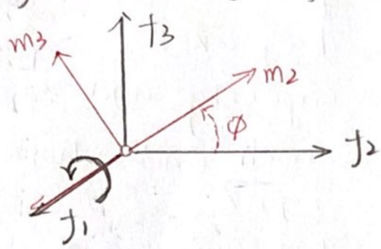


$$\begin{bmatrix} P_1^f \\ P_2^f \\ P_3^f \end{bmatrix} = \begin{bmatrix} f^1\cdot m^1 & f^1\cdot m_2 & f^1\cdot m_3 \\ f^2\cdot m^1 & f^2\cdot m_2 & f^2\cdot m_3 \\ f^3\cdot m^1 & f^3\cdot m_2 & f^3\cdot m_3 \end{bmatrix} \begin{bmatrix} P_1^m \\ P_2^m \\ P_3^m \end{bmatrix}$$

★ Fundamental rotation matrix：
基础旋转矩阵

定义： mobile coordinate M is obtained from fixed coordinate F

① rotation of M about f^1 by ϕ：



$$R_1(\phi) = \begin{bmatrix} f^1\cdot m^1 & f^1\cdot m^2 & f^1\cdot m^3 \\ f^2\cdot m^1 & f^2\cdot m^2 & f^2\cdot m^3 \\ f^3\cdot m^1 & f^3\cdot m^2 & f^3\cdot m^3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}.$$

② rotation of M about f^2 by ϕ：

$$R_2(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$$

③ rotation of M about f^3 by ϕ：

$$R_3(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

★ Inverse rotation transformation：

$$R^{-1} = R^T$$

Example 2.3： $q^F = [3, 4, 0]^T$.

$$q^M = R_1^{-1}(\tfrac{\pi}{3})\cdot q^F = R_1^T(\tfrac{\pi}{3})\cdot q^F$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}$$

$$= [3 \;\; 2 \;\; -2\sqrt{3}]^T.$$

★ Composite rotation：

① If M is to be rotated by ϕ about F, then premultiply R by $R_k(\phi)$ 左乘；

② If M is to be rotated by ϕ about its own axis, then postmultiply R by $R_k(\phi)$ 右乘.

★ Yaw-Pitch-Roll Transformation：
偏转角-俯仰角-滚动角变换

Example 2.5：

Suppose we rotate the tool about the fixed axes starting with a yaw of $\frac{\pi}{2}$, followed by a pitch of $-\frac{\pi}{2}$. finally, a roll of $\frac{\pi}{2}$. What is the resulting composite rotation matrix?

$$R = R_3(\tfrac{\pi}{2})\, R_2(-\tfrac{\pi}{2})\, R_1(\tfrac{\pi}{2})$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

★ Homogeneous coordinate
齐次坐标

homogeneous transformation matrix T
齐次变换矩阵

$$T = \left[\begin{array}{ccc|c} & R & & P \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \in \mathbb{R}^{4\times4}$$

Inverse translation：

$$Tran^{-1}(p) = Tran(-p)$$

Example 2.6 :

Suppose $q^M = [0, 0, 10, 1]^T$. The mobile M is translated 5 units along f^1 and -3 units along f^2.

$$q^F = Tran(p) \cdot q^M = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ 10 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} 5 \\ -3 \\ 10 \\ \hline 1 \end{array}\right]$$

Inverse rotation :

the inverse of the fundamental homogeneous rotation matrix $Rot(\phi, k)$ always exists and is given by :

$$Rot^{-1}(\phi, k) = Rot(-\phi, k) = Rot^T(\phi, k)$$

Example 2.7 :

let $F = \{f^1, f^2, f^3\}$ and $M = \{m^1, m^2, m^3\}$ be two initially coincident fixed and mobile orthonormal coordinate frames. Suppose we translate M along f^2 by 3 units and then rotate M about f^3 by π radians.

Find m^1 with respect to F after the composite transformation.

$$T = Rot(\pi, 3)\, Tran(3 \cdot f^2)\, I$$

$$= \left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right].$$

$$T m^1 = \left[\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ \hline 1 \end{array}\right] = \left[\begin{array}{c} -1 \\ -3 \\ 0 \\ \hline 1 \end{array}\right].$$

★ Inverse Homogeneous Transformation :

$$T^{-1} = \left[\begin{array}{c|c} R^T & -R^T p \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Example 2.9 :

Suppose the homogeneous coordinate transformation matrix which maps M into F is :

$$T = \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Find the homogeneous coordinate transformation which maps F into M and use it to find f^2 with respect to M.

$$T^{-1} = \left[\begin{array}{c|c} R^T & -R^T p \\ \hline 0\ 0\ 0 & 1 \end{array}\right].$$

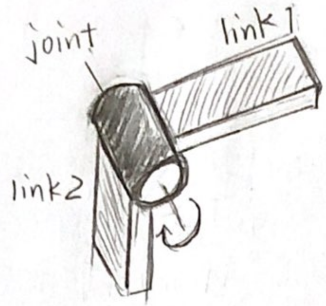
$$-R^T \cdot p = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$$

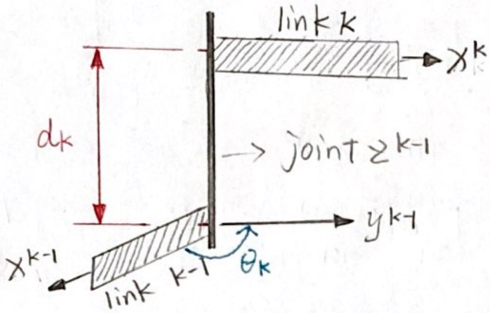
$$\Rightarrow T^{-1} = \left[\begin{array}{ccc|c} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T^{-1} f^2 = [-1, 0, -2, 1]^T.$$

★ Link Coordinates

Kinematic parameters:



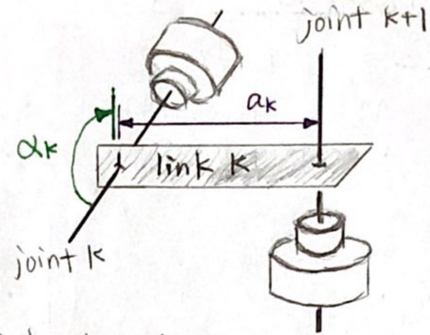


- joint angle θ_k (angle between links)
- joint distance d_k (distance between links)

Note:

for revolute joints, θ_k is variable and d_k is fixed;

for prismatic joints, θ_k is fixed and d_k is variable.

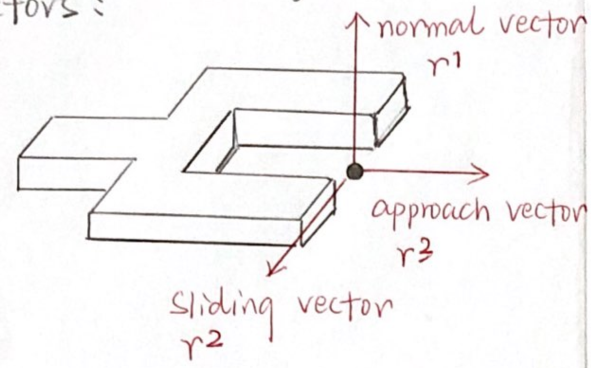


- link length a_k (distance between joints)
- link twist angle α_k (angle between axes)

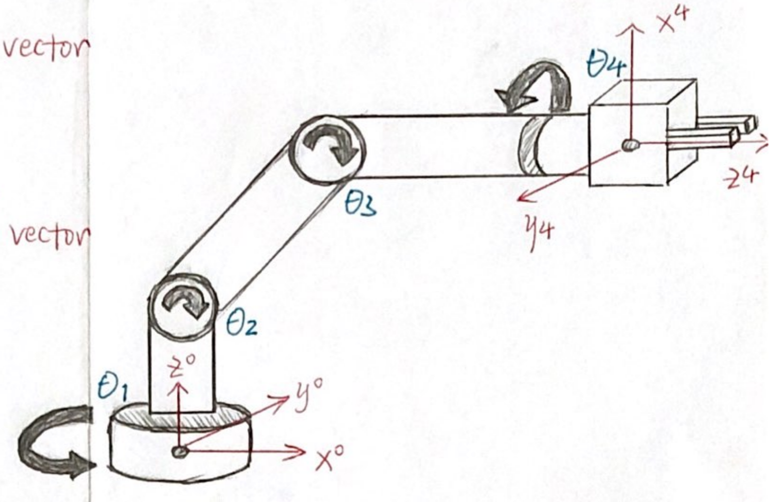
Kinematic parameters

parameter	symbol	joint(R)	joint(P)
joint angle	θ_k	variable	fixed
joint distance	d_k	fixed	variable
link length	a_k	fixed	fixed
link twist angle	α_k	fixed	fixed

★ normal, sliding, approach vectors :

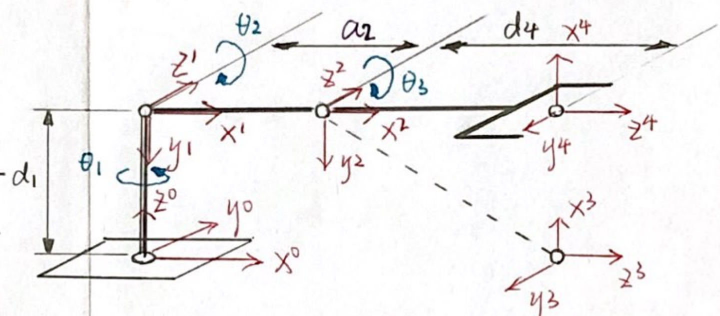


orientation $R = [r^1, r^2, r^3]$.
the origin of $\{r^1, r^2, r^3\}$ frame is placed at the tool tip.

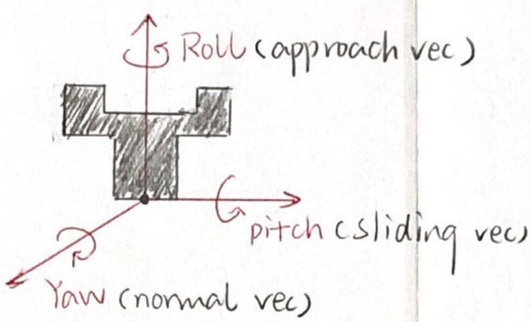


★ D-H representation :

axis	θ	d	a	α
1	θ_1	d_1	0	$-\frac{\pi}{2}$
2	θ_2	0	a_2	0
3	θ_3	0	0	$-\frac{\pi}{2}$
4	θ_4	d_4	0	0



YPR(θ)



$$w = \begin{bmatrix} w^1 \\ w^2 \end{bmatrix} = \begin{bmatrix} p \\ \exp\left[\frac{q_n}{\pi}\right] r^3 \end{bmatrix}$$

Example: four-axis SCARA robot:

$r^3 = -i^3$.

$w = [p_1, p_2, p_3, 0, 0, -\exp\frac{q_4}{\pi}]^T$

Summary

1. Apply D-H Representation (page 46).

2. Define the four kinematic parameters (see page 47).

3. Find Link-Coordinate Transformation (page 54).

$$T_{k-1}^{k} = \begin{pmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_{base}^{tool}(q) = T_0^1(q_1)T_1^2(q_2)\cdots T_{n-1}^{n}(q_n) = T_0^n(q\ )$$

It is often helpful to *partition* the problem at wrist.

$$T_{base}^{tool}(q) = T_{base}^{wrist}(q_1, q_2, q_3)T_{wrist}^{tool}(q_4, q_5, \ldots, q_n)$$

4. Find the Arm Equation (page 56).

$$T_{base}^{tool}(q) = \begin{pmatrix} & R(q) & & p(q) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- 3x3 matrix $R(q)$ specifies the *orientation* of the tool, while the 3x1 matrix
- $p(q)$ specifies the *position* of the tool tip.