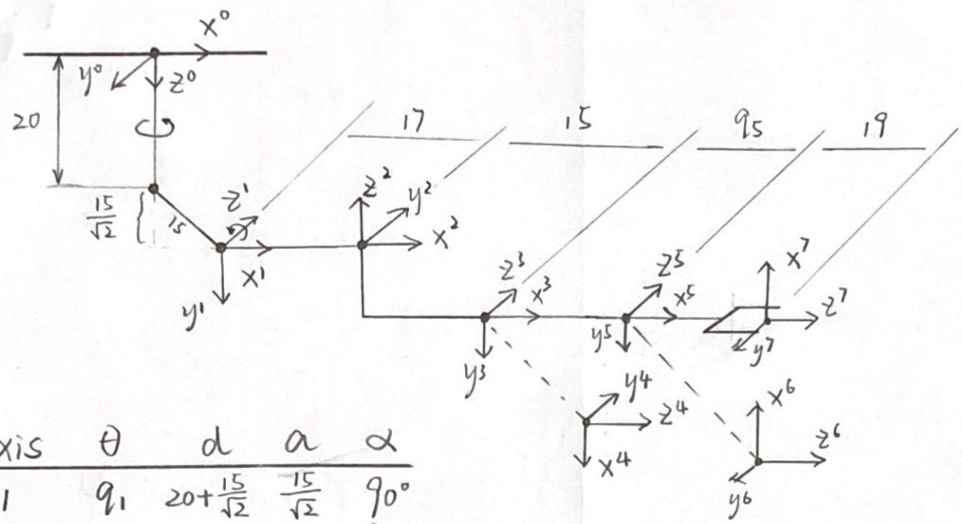
Q1 (a)



Axis	θ	d	a	α
1	q_1	$20+\frac{15}{\sqrt{2}}$	$\frac{15}{\sqrt{2}}$	$90°$
2	q_2	0	17	$90°$
3	q_3	-11	15	$-90°$
4	q_4	0	0	$-90°$
5	0	q_5	0	$90°$
6	q_6	0	0	$-90°$
7	q_7	19	0	0

Q2 (a) $J^{-T}(\theta)M(\theta)J^{-1}(\theta)\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} - J^{-T}(\theta)M(\theta)J^{-1}(\theta)\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} + J^{-T}(\theta)V(\theta,\dot{\theta}) = \begin{bmatrix}u_1\\ u_2\end{bmatrix}$

$V = \begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}v_1\\ v_2\end{bmatrix}$, $\ddot{x} = v_1$, let $f = k_e(x - x_e) = 25(x-0.3)$, $\ddot{f} = 25\ddot{x}$

$\Rightarrow v_1 = \frac{\ddot{f}}{25}$, $e_f = f_d - f$, $v_1 = \frac{1}{25}(\ddot{f}_d + k_{1v}\dot{e}_f + k_{1p}e_f)$

$\ddot{e}_f + k_{1v}\dot{e}_f + k_{1p}e_f = 0$, $s^2 + k_{1v}s + k_{1p} = 0$, $s^2 + 2\xi w_n s + w_n^2 = 0$.

$\xi = 1.2$, $w_n = 0.5 w_{res1} = 0.5\times10\times2\pi = 10\pi$, $\therefore k_{1v} = 2\xi w_n = 24\pi$, $k_{1p} = 100\pi^2$

$v_2 = \ddot{y}_d + k_{2v}\dot{e} + k_{2p}e$, where $e = y_d - y$. $\ddot{e} + k_{2v}\dot{e} + k_{2p}e = 0$.

$s^2 + k_{2v}s + k_{2p} = 0$, $k_{2v} = 2\xi w_n = 2\times1.2\times20\times2\pi = 96\pi$, $k_{2p} = w_n^2 = 1600\pi^2$.

(b) $v_2 = \ddot{y}_d + k_{2v}\dot{e} + k_{2p}e + k_{2i}\int e\,dt$.

$\ddot{e} + k_{2v}\dot{e} + k_{2p}e + k_{2i}\int e\,dt = 0$, $s^2 + k_{2v}s + k_{2p} + \frac{1}{s}k_i = 0$.

$s^3 + k_{2v}s^2 + k_{2p}s + k_i = 0$.

(c)

Q3. (a)

For steered standard wheel:

$\alpha = 0,\ \beta = \beta(t),\ r = 0.045\,m,\ l = 0.31\,m,\ \dot{\varphi} = \dot{\varphi}_{ss}$.

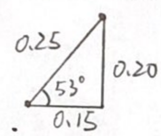
- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 0.045\,\dot{\varphi}_{ss} = 0$.
  where $j(\beta) = [\sin(\beta(t)) \quad -\cos(\beta(t)) \quad -0.31\cos(\beta(t))]$.
- Sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I = 0$,
  where $c(\beta) = [\cos(\beta(t)) \quad \sin(\beta(t)) \quad 0.31\sin(\beta(t))]$.

For the castor wheel:

$\alpha = \pi,\ \beta = \beta(t),\ r = 0.04\,m,\ l = 0.19\,m,\ \dot{\varphi} = \dot{\varphi}_c$. $d = 0.17 - 0.04 = 0.13\,m$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 0.04\,\dot{\varphi}_c = 0$.
  where $j(\beta) = [\sin(\pi + \beta(t)) \quad -\cos(\pi + \beta(t)) \quad -0.19\cos(\beta(t))]$.
- Sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I + 0.13\,\dot{\beta}(t) = 0$,
  where $c(\beta) = [\cos(\pi + \beta(t)) \quad \sin(\pi + \beta(t)) \quad 0.19\sin(\beta(t))]$.

For the fixed standard wheel 1:



$\alpha = 53°,\ \beta = 37°,\ r = 0.045\,m,\ l = 0.25\,m,\ \dot{\varphi} = \dot{\varphi}_{f_1}$.

- rolling constraint: $j R(\theta) \dot{\xi}_I - 0.045\,\dot{\varphi}_{f_1} = 0$,
  where $j = [\sin\pi \quad -\cos\pi \quad -0.25\cos 37°]$.
- Sliding constraint: $c R(\theta) \dot{\xi}_I = 0$.
  where $c = [\cos\pi \quad \sin\pi \quad 0.25\sin 37°]$.

For the fixed standard wheel 2:

$\alpha = -53°,\ \beta = 37°,\ r = 0.045\,m,\ l = 0.25\,m,\ \dot{\varphi} = \dot{\varphi}_{f_2}$.

- rolling constraint: $j R(\theta) \dot{\xi}_I - 0.045\,\dot{\varphi}_{f_2} = 0$.
  where $j = [\sin(-53° + 37°) \quad -\cos(-53° + 37°) \quad -0.25\cos 37°]$.
- sliding constraint: $c R(\theta) \dot{\xi}_I = 0$.
  where $c = [\cos(-53° + 37°) \quad \sin(-53° + 37°) \quad 0.25\sin 37°]$.

(b) $x = 0.1 + 0.3\cos q_1 \cos(q_2+q_3) + 0.2\cos q_1 \cos q_2$.

$y = 0.1 + 0.3\sin q_1 \cos(q_2+q_3) + 0.2\sin q_1 \cos q_2$.

$z = 0.2 + 0.3\sin(q_2+q_3) + 0.2\sin q_2$

when $q_1 = q_2 = q_3 = 0$:

$x = 0.1 + 0.3 + 0.2 = 0.6$.

$y = 0.1 + 0 + 0 = 0.1$.

$z = 0.2 + 0 + 0 = 0.2$. $\quad (x_0, y_0, z_0) = (0.6, 0.1, 0.2)$.

$(x-0.1)^2 = 0.09\cos^2 q_1 \cos^2(q_2+q_3) + 0.04\cos^2 q_1 \cos^2 q_2 + 0.12\cos^2 q_1 \cos q_2 \cos(q_2+q_3)$.

$(y-0.1)^2 = 0.09\sin^2 q_1 \cos^2(q_2+q_3) + 0.04\sin^2 q_1 \cos^2 q_2 + 0.12\sin^2 q_1 \cos q_2 \cos(q_2+q_3)$.

$(x-0.1)^2 + (y-0.1)^2 = 0.09\cos^2(q_2+q_3) + 0.04\cos^2 q_2 + 0.12\cos q_2 \cos(q_2+q_3)$.

$(z-0.2)^2 = 0.09\sin^2(q_2+q_3) + 0.04\sin^2 q_2 + 0.12\sin q_2 \sin(q_2+q_3)$

$$(x-0.1)^2 + (y-0.1)^2 + (z-0.2)^2 = 0.09 + 0.04 + 0.12\cos(q_2+q_3-q_2)$$
$$= 0.09 + 0.04 + 0.12\cos q_3$$

$\Rightarrow \quad q_3 = \pm \frac{1}{0.12}\arccos\left((x-0.1)^2 + (y-0.1)^2 + (z-0.2)^2 - 0.13\right)$

with q_3, q_2 can be calculated by z;

with q_2 and q_3, q_1 can be calculated by x or y.

Q4 (a) Pixel coordinates: O_{ia}, O_{ib}, $i = 2, 3, 5, 6$.

Normalized coordinates: m_{ia}, m_{ib}, $i = 2, 3, 5, 6$

$m_{ia} = A^{-1} O_{ia}$, $m_{ib} = A^{-1} O_{ib}$.

$$m_{ib} = \frac{\bar{m}_{ib}}{z_{ib}} = \cdots = \frac{z_{ia}}{z_{ib}} \left(R + \frac{x}{d_{ia}} n_{ia}^T\right) m_{ia} = \alpha_i H m_{ia}$$

$\Rightarrow$ $A^{-1} O_{ib} = \alpha_i H A^{-1} O_{ia}$. $\frac{H}{h_{33}} = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \\ h_7 & h_8 & 1 \end{bmatrix}$.

$\Rightarrow$ unwrap: $h = [h_1 \ h_2 \ h_3 \ h_4 \ h_5 \ h_6 \ h_7 \ h_8]^T$.

Define M_j:

$m_{ia} = [x_{ia} \ y_{ia} \ 1]^T$, $m_{ib} = [x_{ib} \ y_{ib} \ 1]^T$

$$M_j = \begin{bmatrix} x_{ia} & y_{ia} & 1 & 0 & 0 & 0 & -x_{ia}x_{ib} & -y_{ia}x_{ib} \\ 0 & 0 & 0 & x_{ia} & y_{ia} & 1 & -x_{ia}y_{ib} & -y_{ia}y_{ib} \end{bmatrix}$$

Concatenate together: $Mh = m$

Find vector h by linear algebra, then wrap h back into matrix. this will give the $\frac{H}{h_{33}}$.

(b) No, at least 4 coplanar and noncolinear feature points must be matched in two images.

(c) (O_1, O_2, O_3, O_4), (O_1, O_2, O_3, O_5), (O_1, O_2, O_3, O_6)
(O_2, O_3, O_4, O_5), (O_2, O_3, O_4, O_6), (O_3, O_4, O_5, O_6)

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2015-2016

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April/May 2016 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 7 pages.
2. Answer ALL 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. A redundant robotic manipulator with six revolute joints and one prismatic joint is shown in Figure 1.

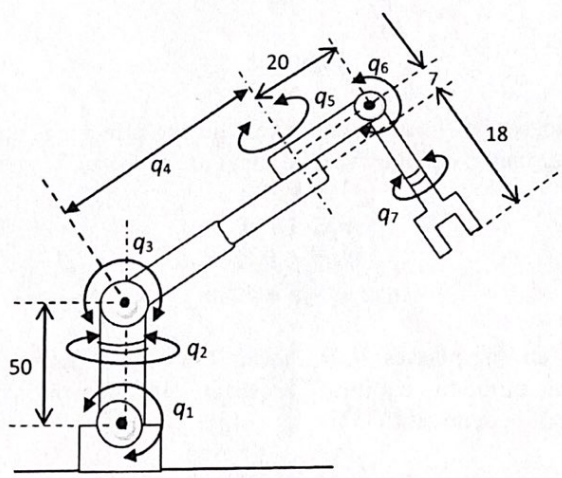

Figure 1

Note: All units are in centimeter (cm).

Note: Question No. 1 continues on page 2

(a) Use Denavit-Hartenberg (D-H) algorithm to obtain the link coordinate diagram. Derive the kinematic parameters of the robot.

(14 Marks)

(b) If the Jacobian matrix of the robot manipulator is denoted as $V(q)$, formulate a resolved motion rate control problem. Modify the resolved motion rate control method to include an additional objective of obstacle avoidance.

Note: Calculation or derivation of $V(q)$ is not required.

(6 Marks)

2. A Cartesian robot is mounted on a floor at a distance away from a wall. The base coordinate frame of the robot and the static positions of the floor and the wall when the end effector of the robot is not in contact with them are shown in Figure 2.

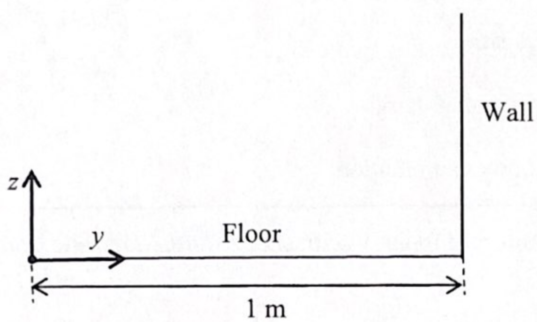

Figure 2

The robot has 3 degrees of freedom and when the end effector is not in contact with the wall or floor, the dynamic equations of the robot are given as:

$$m_x\ddot{x} + b_x\dot{x} = u_x$$
$$m_y\ddot{y} + b_y\dot{y} = u_y$$
$$m_z\ddot{z} + b_z\dot{z} + 9.8m_z = u_z$$

where m_x, m_y, m_z are the masses, u_x, u_y, u_z are the control inputs and b_x, b_y, b_z are the friction coefficients of the three joints. The stiffness of the wall is denoted as k_w and the stiffness of the floor is denoted as k_f.

(a) Suppose that the end effector is now in contact with the wall, design a hybrid position and force controller for the robot to perform the contact task by moving on the wall while exerting a desired contact force. The system should be critically damped and should not excite resonances at 2.869 Hz and 5.738 Hz.

(10 Marks)

Note: Question No. 2 continues on page 3

(b) Suppose that the controller designed in part 2(a) is now used for another contact task where the end effector is in contact with the floor, derive the error equations of the closed-loop system.

(7 Marks)

(c) If the wall is rigid, how will you implement the hybrid position and force controller designed in part 2(a)?

(3 Marks)

3. (a) A mobile robot with one steered standard wheel, one fixed standard wheel and two identical castor wheels is shown in Figure 3. A local reference frame is attached to the center of the fixed standard wheel as shown in the figure. The radius of the standard wheel is 12 cm and the radius of the castor wheel is 10 cm. The steered angle of the steered standard wheel is denoted by β as shown in Figure 3. The rotational velocities of the steered standard wheel, fixed standard wheel and castor wheels are denoted by $\dot{\varphi}_{ss}$, $\dot{\varphi}_{fs}$, $\dot{\varphi}_{c1}$ and $\dot{\varphi}_{c2}$, respectively. Derive the rolling and sliding constraints of each wheel.

(10 Marks)

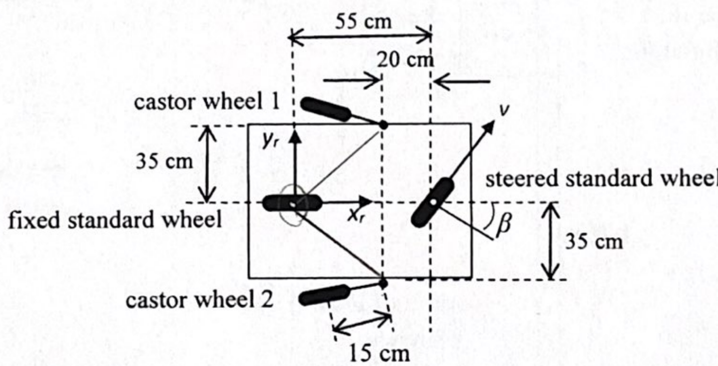

Figure 3

(b) The mobile robot in part 3(a) is now used for transporting parts in an automated plant. A robot manipulator with six degrees of freedom is used to pick up a metal rectangular part which is attached to a magnetic board as shown in Figure 4 and then place it on the mobile robot. The joint angle of the first joint of the robot manipulator is indicated as shown in the figure. The length of the rectangular part is 25 cm and the base is a square with a side length of 12 cm.

Note: Question No. 3 continues on page 4

The mobile robot is parked stationary at a distance away from the base of the manipulator as shown in Figure 4. A point A is located next to the rectangular part as shown in the figure. The origin of the base of robot manipulator, the origin of the local reference frame of mobile robot, the centroid of the rectangular block and point A are all in the same plane. Determine the transformation matrix T_{base}^{pick} for the robot manipulator to pick up the rectangular part by grasping on the side of the square base and the transformation matrix T_{base}^{place} for the robot manipulator to place it on the mobile robot so that the center of the square base of the rectangular part is aligned with the origin of the reference frame of the mobile robot. If the mobile robot moves immediately after the rectangular part is being placed on it, discuss the effects on the specification of the transformation matrix T_{base}^{place}.

(10 Marks)

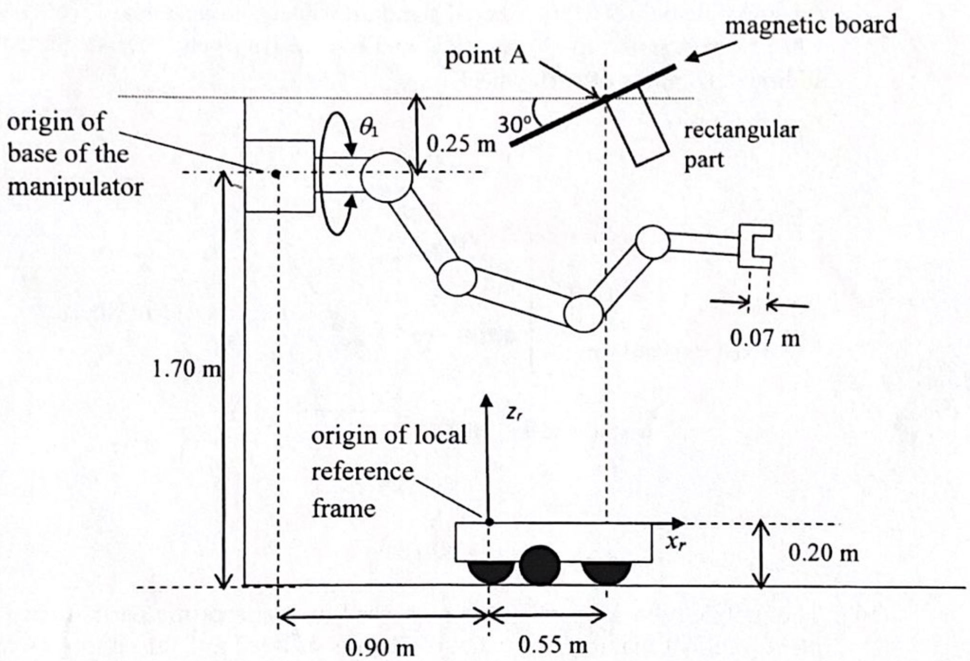

Figure 4

4. A moving camera takes three images of a fixed object at three poses. Three coordinate frames represented by F_a, F_b, and F_c are attached to the projection centre of the camera at the three poses, respectively, as shown in Figure 5. Five coplanar feature points on the fixed object have been selected and labeled by O_1, O_2, O_3, O_4, and O_5, with any four of them non-collinear.

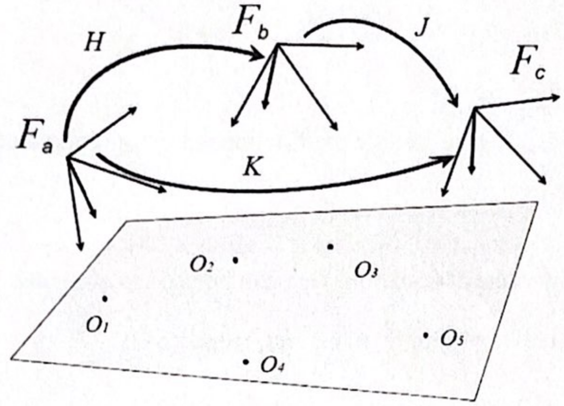

Figure 5

The five feature points can all be detected in the image taken at the pose attached to F_b. Their corresponding normalized coordinates in F_b are given by

$$m_{1b} = [b_{1x}, b_{1y}, 1]^T, \quad m_{2b} = [b_{2x}, b_{2y}, 1]^T, \quad m_{3b} = [b_{3x}, b_{3y}, 1]^T$$
$$m_{4b} = [b_{4x}, b_{4y}, 1]^T, \quad m_{5b} = [b_{5x}, b_{5y}, 1]^T.$$

Feature points O_1, O_2, O_3, and O_4 can be detected in the image taken at the pose attached to F_a. Their corresponding normalized coordinates in F_a are given by

$$m_{1a} = [a_{1x}, a_{1y}, 1]^T, \quad m_{2a} = [a_{2x}, a_{2y}, 1]^T,$$
$$m_{3a} = [a_{3x}, a_{3y}, 1]^T, \quad m_{4a} = [a_{4x}, a_{4y}, 1]^T.$$

Feature points O_2, O_3, O_4, and O_5 can be detected in the image taken at the pose attached to F_c. Their corresponding normalized coordinates in F_b are given by

$$m_{2c} = [c_{2x}, c_{2y}, 1]^T, \quad m_{3c} = [c_{3x}, c_{3y}, 1]^T$$
$$m_{4c} = [c_{4x}, c_{4y}, 1]^T, \quad m_{5c} = [c_{5x}, c_{5y}, 1]^T.$$

Note: Question No. 4 continues on page 6.

Let $H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$ denote the Euclidean homography matrix from F_a to F_b.

Let $J = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix}$ denote the Euclidean homography matrix from F_b to F_c.

Let $K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$ denote the Euclidean homography matrix from F_a to F_c.

(a) Derive two sets of equations that can be used to compute $\dfrac{H}{h_{33}}$ and $\dfrac{J}{j_{33}}$ (i.e., scaled homography matrices), respectively.

(14 Marks)

(b) Can $\dfrac{K}{k_{33}}$ be obtained? If yes, explain how to calculate it? If no, explain why it cannot be obtained.

(6 Marks)

5. Two sensors are used to measure a parameter x_k that can be modelled by $x_{k+1} = x_k$. The outputs of the two sensors are represented by z_{1k} and z_{2k}, respectively, and they are modelled by

$$z_{1k} = c_1 x_k + v_{1k}$$
$$z_{2k} = c_2 x_k + v_{2k}$$

where v_{1k} and v_{2k} are zero mean Gaussian noises with variances given by σ_1^2 and σ_2^2, respectively, and c_1 and c_2 are positive parameters of the sensors.

Denoting $\hat{x}_k$ as the estimate of x_k, a predictor is designed as:

$$\hat{x}_{k+1} = \hat{x}_k + L_{1k}\left(z_{1k} - c_1\hat{x}_k\right) + L_{2k}\left(z_{2k} - c_2\hat{x}_k\right)$$

Let the estimation error be denoted as $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error and the noise terms v_{1k} and v_{2k} are uncorrelated, and that $E\left[\tilde{x}_{k+1}\right] = 0$. Let the estimation error variance be $p_{k+1} = E\left[\tilde{x}_{k+1}^2\right]$.

(a) Derive the update equation for the estimation error variance p_{k+1}.

(6 Marks)

(b) Derive the update laws for L_{1k} and L_{2k} to minimize the estimation error variance p_{k+1}.

(8 Marks)

(c) Usually, the estimation error variance is dependent on the variances of noises. For the designs in part 5(a) and part 5(b), select values for the parameters of the sensors c_1 and c_2 so that p_{k+1} is independent of σ_1^2 and σ_2^2, if such values exist.

(6 Marks)

END OF PAPER