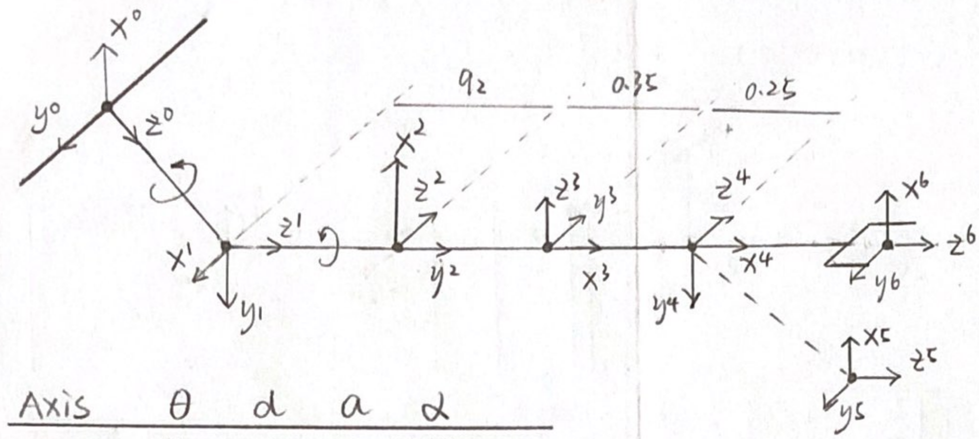
Q1. (a)



Axis	θ	d	a	α
1	q_1	0.5	0	45°
2	0	q_2	0	90°
3	q_3	0	0.35	90°
4	q_4	0	0.25	-90°
5	q_5	0	0	-90°
6	q_6	0.3	0	0

Q2. (a)

$$(1+\cos^2(q_1))\ddot{x} + 2\sin(q_1)\cos(q_1)\ddot{y} - 5\dot{q}_1\dot{q}_3\cos(q_1) + 2\dot{q}_1^2\sin(q_1) = u_x$$
$$2\sin(q_1)\cos(q_1)\ddot{x} + (1+\sin^2(q_1))\ddot{y} + 5\dot{q}_1\dot{q}_3\sin(q_1) - 2\dot{q}_1^2\cos(q_1) = u_y$$
$$10\ddot{z} + 98q_2 = u_z$$

$$\begin{bmatrix} 1+\cos^2(q_1) & 2\sin(q_1)\cos(q_1) & 0 \\ 2\sin(q_1)\cos(q_1) & 1+\sin^2(q_1) & 0 \\ 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} -5\dot{q}_1\dot{q}_3\cos(q_1)+2\dot{q}_1^2\sin(q_1) \\ 5\dot{q}_1\dot{q}_3\sin(q_1)-2\dot{q}_1^2\cos(q_1) \\ 98q_2 \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \alpha v + \beta. \quad v = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}.$$

position control:

$v_1 = \ddot{x}_d + k_{1v}\dot{e}_1 + k_{1p}e_1$,

$e_1 = x_d - x$,

$\Rightarrow$ $\ddot{e}_1 + k_{1v}\dot{e}_1 + k_{1p}e = 0$.

$s^2 + k_{1v}s + k_{1p} = 0$.

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$.

choose $\xi = 1$, $\omega_n = 0.5\omega_{res1} = 0.5\times6\times2\pi$

$k_{1v} = 2\xi\omega_n = 12\pi$, $k_{1p} = \omega_n^2 = 36\pi^2$.

$v_2 = \ddot{y}_d + k_{2v}\dot{e}_2 + k_{2p}e_2$,

$e_2 = y_d - y$.

$\Rightarrow$ $\ddot{e}_2 + k_{2v}\dot{e}_2 + k_{2p}e_2 = 0$.

$s^2 + k_{2v}s + k_{2p} = 0$.

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$.

choose $\xi = 1$, $\omega_n = 0.5\omega_{res} = 12\pi$.

$k_{2v} = 2\xi\omega_n = 24\pi$, $k_{2p} = \omega_n^2 = 144\pi^2$.

force control:

$f = k_e(z - z_e) = 100(z-1)$, $\ddot{f} = 100\ddot{z}$, $v_3 = \ddot{z} = \frac{\ddot{f}}{100}$.

let $e_f = f_d - f$, $v_3 = \frac{1}{100}(\ddot{f}_d + k_{3v}\dot{e}_f + k_{3p}e_f)$, $\ddot{e}_f + k_{3v}\dot{e}_f + k_{3p}e_f = 0$.

choose $\xi = 1$, $\omega_n = 0.5\omega_{res} = 18\pi$, $k_{3v} = 2\xi\omega_n = 36\pi$, $k_{3p} = \omega_n^2 = 324\pi^2$.

Q3. (a)

For castor wheel 1:

$\alpha = 135° = \frac{3}{4}\pi$, $\beta = \beta_1(t)$, $l = 0.3\sqrt{2}$, $r = 0.12$, $d = 0.25$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - \dot{\varphi}_{c1} r = 0$,

  $j(\beta) = [\sin(\frac{3}{4}\pi + \beta_1(t)) \quad -\cos(\frac{3}{4}\pi + \beta_1(t)) \quad -0.3\sqrt{2}\cos(\beta_1(t))]$;

- Sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I + d\dot{\beta} = 0$.

  $\Rightarrow c(\beta) R(\theta) \dot{\xi}_I + 0.25\dot{\beta}_1(t) = 0$.

  where $c(\beta) = [\cos(\frac{3}{4}\pi + \beta_1(t)) \quad \sin(\frac{3}{4}\pi + \beta_1(t)) \quad 0.3\sqrt{2}\sin(\beta_1(t))]$.

For castor wheel 2:

$\alpha = -135° = -\frac{3}{4}\pi$, $\beta = \beta_2(t)$, $l = 0.3\sqrt{2}$, $r = 0.12$, $d = 0.25$.

- rolling constraint: $j(\beta) R(\theta) \dot{\xi}_I - 0.12\dot{\varphi}_{c2} = 0$.

  where $j(\beta) = [\sin(-\frac{3}{4}\pi + \beta_2(t)) \quad -\cos(-\frac{3}{4}\pi + \beta_2(t)) \quad -0.3\sqrt{2}\cos(\beta_2(t))]$.

- Sliding constraint: $c(\beta) R(\theta) \dot{\xi}_I + 0.25\dot{\beta}_2(t) = 0$.

  where $c(\beta) = [\cos(-\frac{3}{4}\pi + \beta_2(t)) \quad \sin(-\frac{3}{4}\pi + \beta_2(t)) \quad 0.3\sqrt{2}\sin(\beta_2(t))]$.

For Swedish wheel 1:

$\alpha = 45° = \frac{\pi}{4}$, $\beta = 45° = \frac{\pi}{4}$, $\gamma = 120° = \frac{2}{3}\pi$, $r = 0.15$, $l = 0.35\sqrt{2}$, $r_{sw} = 0.03$

- rolling constraint: $j R(\theta) \dot{\xi}_I - 0.15\dot{\varphi}_{s1}\cos\frac{2}{3}\pi = 0$

  where $j = [\sin(\frac{\pi}{4} + \frac{\pi}{4} + \frac{2}{3}\pi) \quad -\cos(\frac{\pi}{4} + \frac{\pi}{4} + \frac{2}{3}\pi) \quad -0.35\sqrt{2}\cos(\frac{\pi}{4} + \frac{2}{3}\pi)]$.

- sliding constraint: $C R(\theta) \dot{\xi}_I - 0.15\dot{\varphi}_{s1}\sin(\frac{2}{3}\pi) - 0.03\dot{\varphi}_{sw1} = 0$,

  where $C = [\cos(\frac{\pi}{4} + \frac{\pi}{4} + \frac{2}{3}\pi) \quad \sin(\frac{\pi}{4} + \frac{\pi}{4} + \frac{2}{3}\pi) \quad 0.35\sqrt{2}\sin(\frac{\pi}{4} + \frac{2}{3}\pi)]$.

For Swedish wheel 2:

$\alpha = -45° = -\frac{\pi}{4}$, $\beta = \frac{\pi}{4}$, $\gamma = 60° = \frac{\pi}{3}$, $r = 0.15$, $l = 0.35\sqrt{2}$, $r_{sw} = 0.03$

- rolling constraint: $j R(\theta) \dot{\xi}_I - 0.15\dot{\varphi}_{s2}\cos\frac{\pi}{3} = 0$.

  where $j = [\sin(-\frac{\pi}{4} + \frac{\pi}{4} + \frac{\pi}{3}) \quad -\cos(-\frac{\pi}{4} + \frac{\pi}{4} + \frac{\pi}{3}) \quad -0.35\sqrt{2}\cos(\frac{\pi}{4} + \frac{\pi}{3})]$

- sliding constraint: $c R(\theta) \dot{\xi}_I - 0.15\dot{\varphi}_{s2}\sin(\frac{\pi}{3}) - 0.03\dot{\varphi}_{sw2} = 0$

  where $c = [\cos(-\frac{\pi}{4} + \frac{\pi}{4} + \frac{\pi}{3}) \quad \sin(-\frac{\pi}{4} + \frac{\pi}{4} + \frac{\pi}{3}) \quad 0.35\sqrt{2}\sin(\frac{\pi}{4} + \frac{\pi}{3})]$.

(b) $x = \cos q_1 \sin q_2 (q_3 + 0.5) - 0.2 \sin q_1$.

$y = \sin q_1 \sin q_2 (q_3 + 0.5) + 0.2 \cos q_1$.

$z = \cos q_2 (q_3 + 0.5)$.

$$x^2 + y^2 = \sin^2 q_2 (q_3 + 0.5)^2 - 0.4 \sin q_1 \sin q_2 \cos q_1 (q_3 + 0.5) + 0.4 \sin q_1 \sin q_2 \cos q_1 (q_3 + 0.5) + 0.04$$

$$= \sin^2 q_2 (q_3 + 0.5)^2 + 0.04$$

$$= \sin^2 q_2 \cdot q_3^2 + \sin^2 q_2 \cdot q_3 + 0.25 \sin^2 q_2 + 0.04.$$

$$z^2 = \cos^2 q_2 \cdot q_3^2 + 0.25 \cos^2 q_2 + \cos^2 q_2 \cdot q_3.$$

$$x^2 + y^2 + z^2 = q_3^2 + q_3 + 0.29.$$

$$\Rightarrow x^2 + y^2 + z^2 - 0.04 = q_3^2 + q_3 + 0.25 = (q_3 + 0.5)^2.$$

$$q_3 = \pm \sqrt{x^2 + y^2 + z^2 - 0.04} - 0.5. \quad ①$$

$$z = \cos q_2 (q_3 + 0.5) \Rightarrow q_2 = \pm \arccos\left(\frac{z}{q_3 + 0.5}\right), \quad ②$$

with known q_2 and q_3, q_1 can be calculated by x or y.

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2017-2018

EE6221 – ROBOTICS AND INTELLIGENT SENSORS

April/May 2018 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer ALL 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. A robotic manipulator is mounted on a slanted wall as shown in Figure 1.

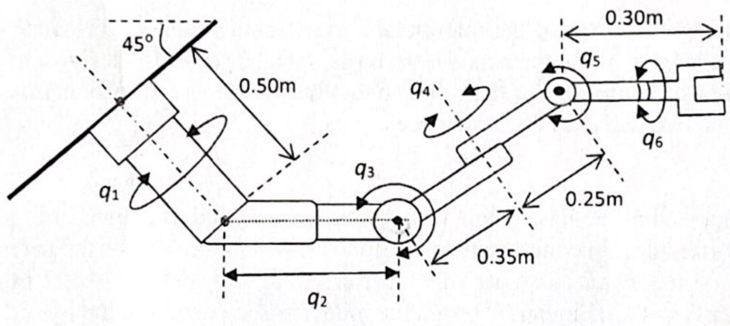


Figure 1

(a) Use the Denavit-Hartenberg (D-H) algorithm to obtain the link coordinate diagram. Derive the kinematic parameters of the robot.

(15 Marks)

(b) Assuming that a reference frame is assigned as shown in Figure 2 on page 2 and the arm matrix of the robot manipulator in part 1(a) has been derived as $T(q)_{base}^{tool}$, obtain the transformation matrix of the tool frame with respect to the reference frame.

Note: Question No. 1 continues on page 2

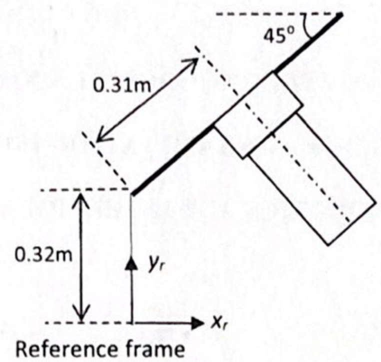


Figure 2

(5 Marks)

2. The dynamic model of a robot manipulator with three joint variables q_1, q_2, q_3 and three control inputs u_x, u_y, u_z is expressed in Cartesian coordinates (x, y, z) as follows:

$$(1 + cos^2(q_1))\ddot{x} + 2\,sin(q_1)\,cos(q_1)\,\ddot{y} - 5\dot{q}_1\dot{q}_3\,cos(q_1) + 2\dot{q}_1^2\,sin(q_1) = u_x$$
$$2\,sin(q_1)\,cos(q_1)\,\ddot{x} + (1 + sin^2(q_1))\ddot{y} + 5\dot{q}_1\dot{q}_3\,sin(q_1) - 2\dot{q}_1^2\,cos(q_1) = u_y$$
$$10\ddot{z} + 98q_2 = u_z$$

The system possesses unmodelled resonances at 6 Hz, 12 Hz and 18 Hz.

(a) If the end effector of the manipulator is in contact with a workpiece such that the contact force along the z-axis is given as $f = 100(z - 1)$, design a hybrid position and force controller for the robot so that the system is critically damped and should not be excited at all the resonances.

(12 Marks)

(b) Suppose that the base frame of the robot is assigned as shown in Figure 3 and the end effector is in contact with a frictionless workpiece. The surface of the workpiece can be modelled as an arc of a quarter circle with radius $r = 0.2$ m and centroid $(y_c, z_c) = (1, 0)$ meters. Design the model based portion of the hybrid position and force controller to perform a contact task on the workpiece.

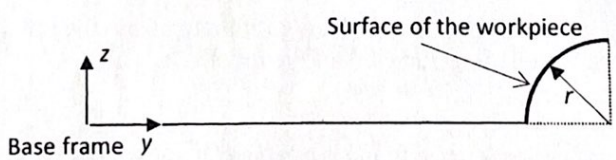
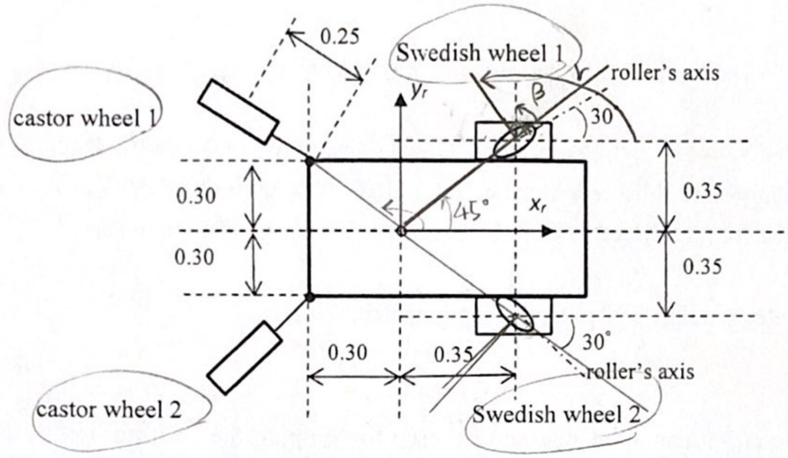


Figure 3

(8 Marks)

3. (a) A mobile robot with a local reference frame (x_r, y_r) is shown in Figure 4. There are two castor wheels and two Swedish wheels. The radius of each castor wheel is 0.12 m and the radius of each Swedish wheel is 0.15 m. The roller's axis of the Swedish wheel is shown in Figure 4 and the radius of each roller is 0.03 m. The rotational velocities of the castor wheels, Swedish wheels, and the rollers are denoted by $\dot{\varphi}_{c1}$, $\dot{\varphi}_{c2}$, $\dot{\varphi}_{S1}$, $\dot{\varphi}_{S2}$, $\dot{\varphi}_{sr1}$, and $\dot{\varphi}_{sr2}$, respectively. Derive the rolling and sliding constraints of the mobile robot.

(10 Marks)



Note: all lengths are in meters

Figure 4

(b) A manipulator with 3 joint variables q_1, q_2 and q_3 is mounted on the mobile robot. The forward kinematic equations of the manipulator are given as

$$x = \cos(q_1)\sin(q_2)(q_3 + 0.5) - 0.2\sin(q_1)$$
$$y = \sin(q_1)\sin(q_2)(q_3 + 0.5) + 0.2cos(q_1)$$
$$z = \cos(q_2)q_3 + 0.5\cos(q_2)$$

Solve the inverse kinematics problem to express the joint angles q_1, q_2 and q_3 in terms of x, y and z.

(10 Marks)

4. A moving camera at poses P_a and P_b takes two images of a stationary object, respectively. Assume that four coplanar and non-collinear feature points are detected on these images. The pixel coordinates of the four feature points at poses P_a and P_b are denoted as

$$p_{1a}=[a_1,a_2,1]^T,\quad p_{2a}=[a_3,a_4,1]^T,\quad p_{3a}=[a_5,a_6,1]^T,\quad p_{4a}=[a_7,a_8,1]^T$$

and

$$p_{1b}=[b_1,b_2,1]^T,\quad p_{2b}=[b_3,b_4,1]^T,\quad p_{3b}=[b_5,b_6,1]^T,\quad p_{4b}=[b_7,b_8,1]^T,$$

respectively. Let H denote the 3 × 3 Euclidean homography matrix from P_a to P_b and let h_{33} denote the third row third column element of the matrix H. Let α_i, $i=1,2,3,4$ denote the four depth ratios and A denote the camera calibration matrix.

(a) Determine the scaled homography matrix $\dfrac{H}{h_{33}}$.

(10 Marks)

(b) Describe a method that can be used to compute the rotation matrix and scaled translation vector from the scaled homography matrix.

(4 Marks)

(c) We aim to control the camera to move from the current pose P to a desired fixed pose P^*. Assume that four fixed coplanar and non-collinear feature points can be detected during the whole control process. A 6 × 1 error term $e(t)$ is defined based on three of the feature points to represent the mismatch between the poses P and P^*. The error term is governed by the dynamical equation $\dot{e}(t)=L(t)\xi(t)$ where $\xi(t)$ is the 6 × 1 velocity control input to the camera and $L(t)$ is a 6 × 6 matrix. Assume that $L(t)$ is nonsingular during the control process. Design a control law for $\xi(t)$ so that the error term $e(t)$ goes to zero eventually, and prove that your design works.

(6 Marks)

5. A sensing system consisting of multiple sensors is used to achieve accurate temperature measurement of an indoor environment. The temperature x is modelled by $x_{k+1} = x_k + w_k$ where w_k represents noise and disturbance. N sensors with outputs z_{ik}, $i = 1, 2, 3, \cdots, N$ are used to measure the temperature. The sensors are modelled by $z_{ik} = x_k + v_{ik}$ where v_{ik} with $k = 1, 2, 3, \cdots, N$ represent sensor noises. The signals w_k and v_{ik}, $i = 1, 2, 3, \cdots, N$ are all zero mean Gaussians with variances given by σ_w^2 and σ^2, respectively.

Denote $\hat{x}_{k+1}$ as the estimation of x_{k+1}. Let the temperature estimation error be defined as $\tilde{x}_{k+1} = x_{k+1} - \hat{x}_{k+1}$. Assume that the estimation error $\tilde{x}_{k+1}$ and the noise terms w_k, v_{1k}, v_{2k}, $v_{3k}, \ldots,$ $v_{(N-1)k}$, and v_{Nk} are all uncorrelated, and that the expectation value $E[\tilde{x}_{k+1}] = 0$.

A fusion-based estimator is proposed to estimate the temperature:

$$\hat{x}_{k+1} = \hat{x}_k + \sum_{1}^{N} K_{ik}\left(z_{ik} - \hat{x}_k\right)$$

where K_{ik}, $i = 1, 2, 3, \cdots, N$, are the gains.

(a) Derive the difference equations that govern the behaviours of the estimation error $\tilde{x}_{k+1}$ and the estimation error variance p_{k+1}, respectively.

(12 Marks)

(b) Determine the update laws for K_{ik}, $i = 1, 2, 3, \cdots, N$, to minimize the estimation error variance, and determine the update law for the estimation error variance p_{k+1} to obtain the optimal estimation.

(8 Marks)

END OF PAPER