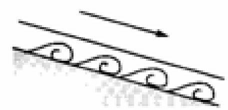
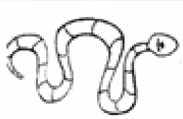
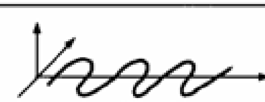
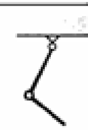
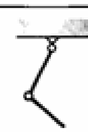
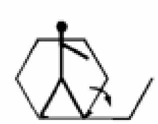
Mobile Robots

1. Locomotion

Locomotion is the process of causing an autonomous robot to move. In order to produce motion, forces must be applied to the vehicle.

1.1 Locomotion in Nature

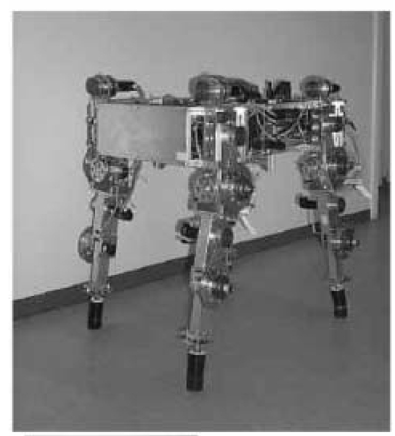
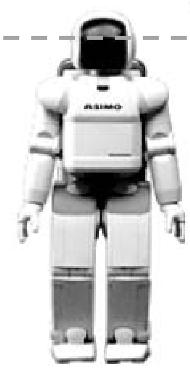
Biological systems succeed in moving through a wide variety of harsh environments.

Type of motion		Resistance to motion	Basic kinematics of motion	
Flow in a Channel		Hydrodynamic forces	Eddies	
Crawl		Friction forces	Longitudinal vibration	
Sliding		Friction forces	Transverse vibration	
Running		Loss of kinetic energy	Oscillatory movement of a multi-link pendulum	
Jumping		Loss of kinetic energy	Oscillatory movement of a multi-link pendulum	
Walking		Gravitational forces	Rolling of a polygon (see figure 2.2)	

1.2 Locomotion in Robots

- Many locomotion concepts in robotics are inspired by nature.
- Most natural locomotion concepts are difficult to imitate technically.
- Rolling, which is NOT found in nature, is most efficient.



1.3 Wheeled Mobile Robots (WMR)

The wheel has been by far the most popular locomotion mechanism in mobile robotics and in man-made vehicles in general.

Yamabico

MagellanPro

Sojourner

ATRV-2

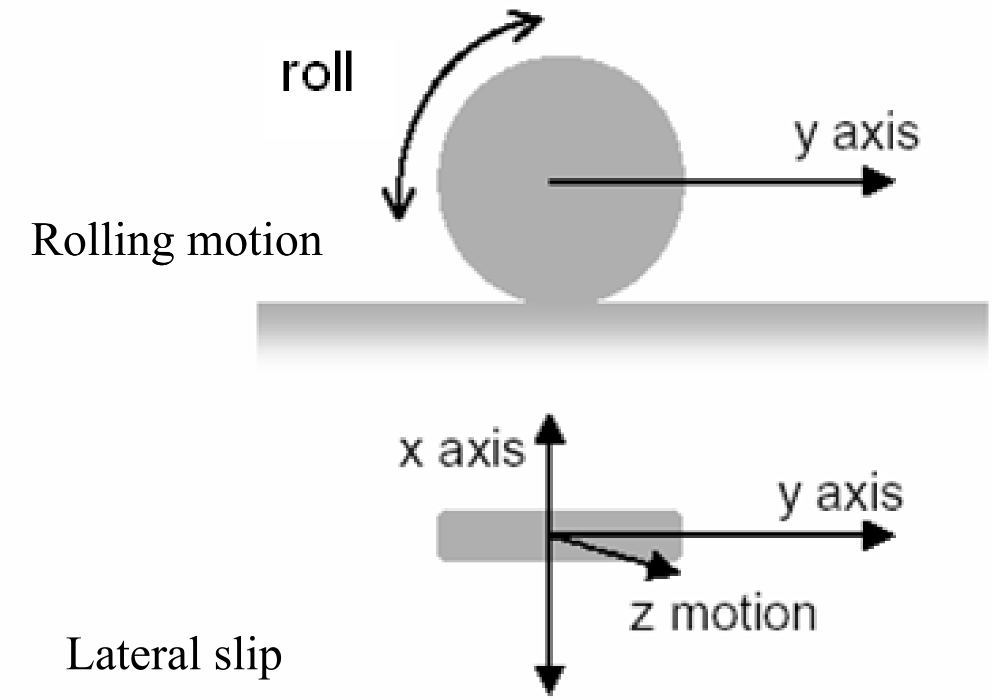
Hilare 2-Bis

Koy

Stability is guaranteed with 3 wheels, and improved with four.

There are many possibilities of wheel configurations when consider possible techniques for mobile robot locomotion.

1.3.1 Wheels

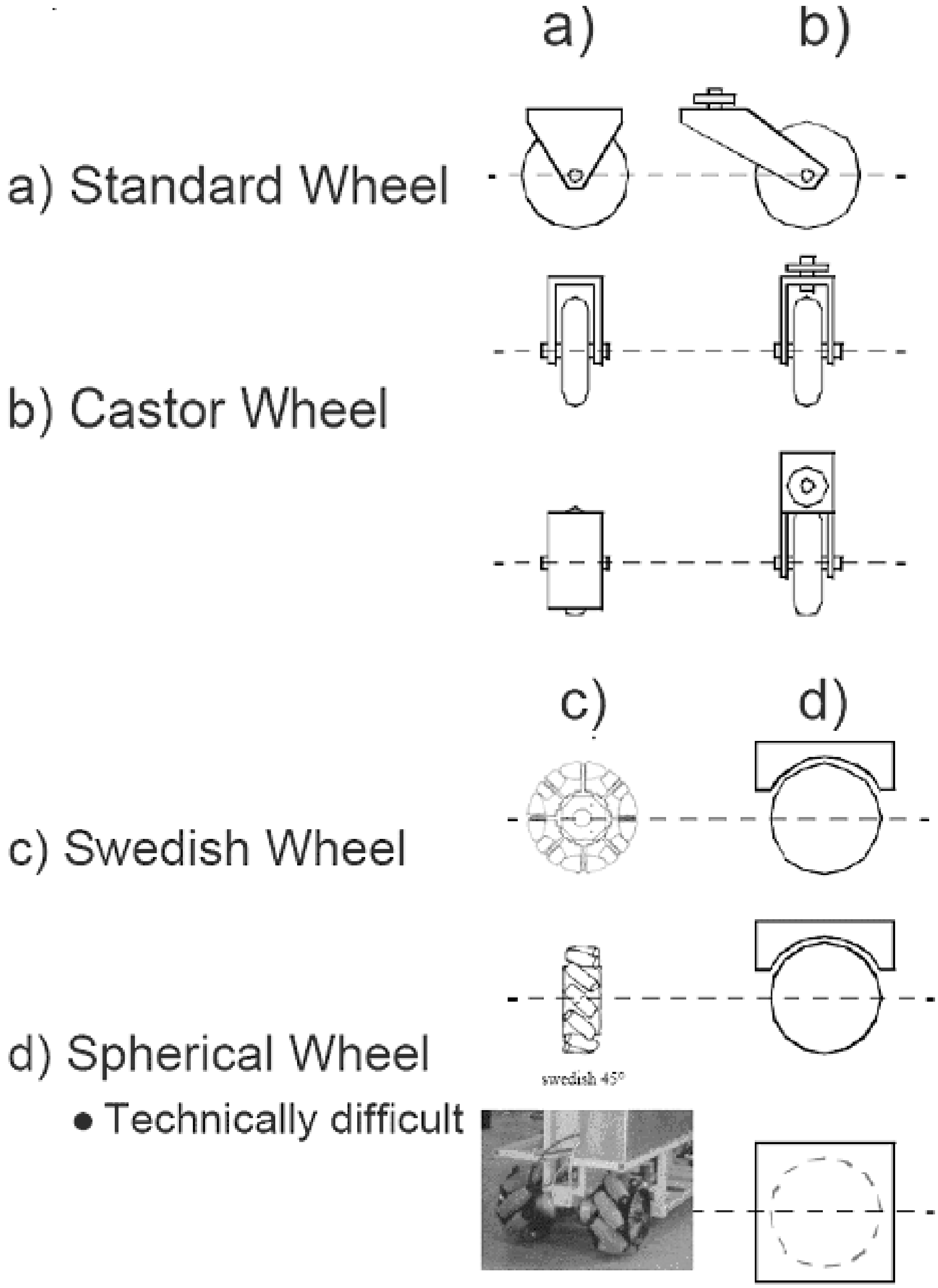


Four Basic Wheel types

There are four major wheel classes that differ widely in their kinematics.

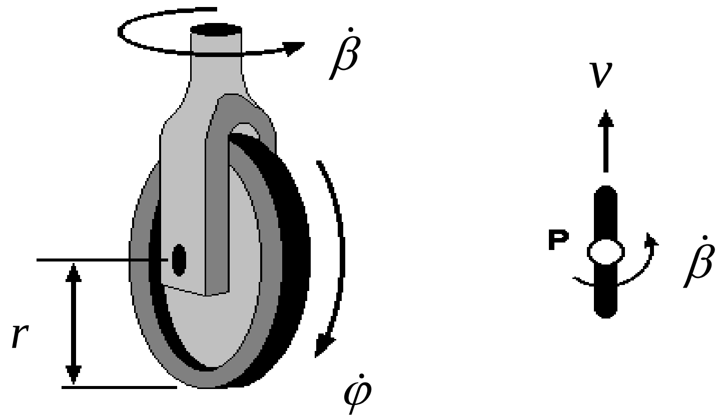
- Standard wheel
- Castor wheel
- Swedish wheel
- Ball or spherical wheel

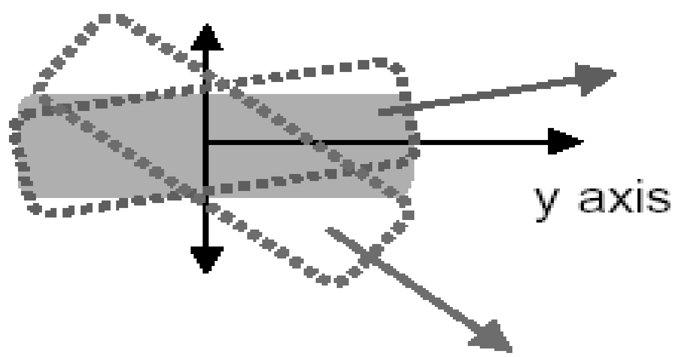
The choice of wheel type has a large effect on the overall kinematics of the mobile robot.



Steered wheel

The orientation of the rotation axis can be controlled





- Wheel parameters:
  - r = wheel radius
  - v = wheel linear velocity
  - $\dot{\varphi}$ = wheel angular velocity
  - $\dot{\beta}$ = steering velocity

1.3.2 Wheel configurations

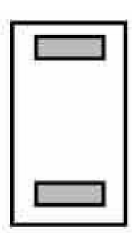
The choice of wheel types is strongly linked to the choice of wheel arrangements or wheel configurations.

Three issues: *Stability*, *Maneuverability* and *Controllability*

- Stability is guaranteed with 3 wheels, improved with four.
- Tradeoff between Maneuverability and Controllability
  Combining actuation and steering on one wheel increases complexity

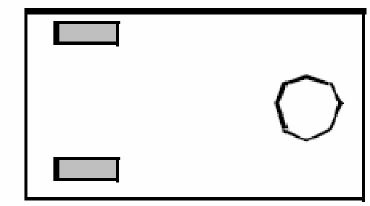
Two-wheel configurations

One steering wheel and one traction wheel

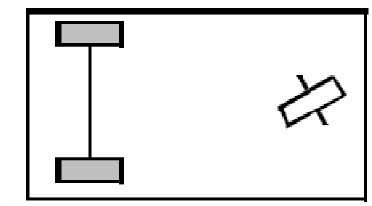


Two-wheel differential drive (independent actuators for each wheel)

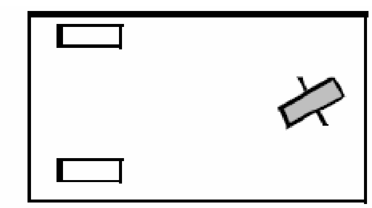
Three-wheel configurations



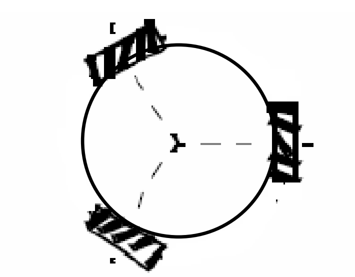
Two-wheel differential drive and one unpowered omnidirectional wheel



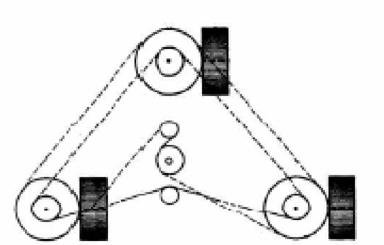
Two connected traction wheels and one steering wheel



Two free wheels and one steered traction wheel

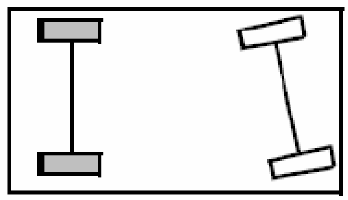


Three Swedish wheels: omni-directional movement

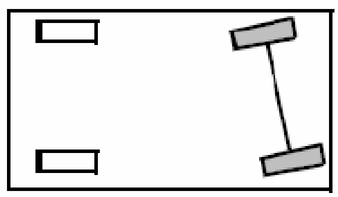


Three synchronously driven and steered wheels: orientation not controllable

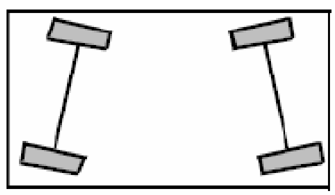
Four-wheel configurations



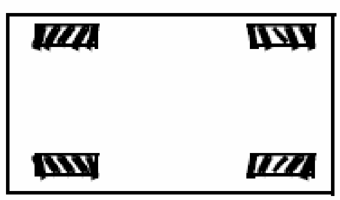
Two connected traction wheels and two steering wheels



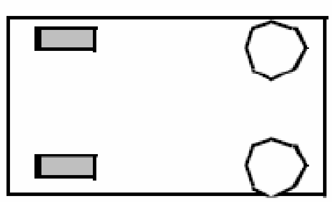
Two free wheels and two connected steered traction wheels



Four connected steered traction wheels



Four Swedish wheels

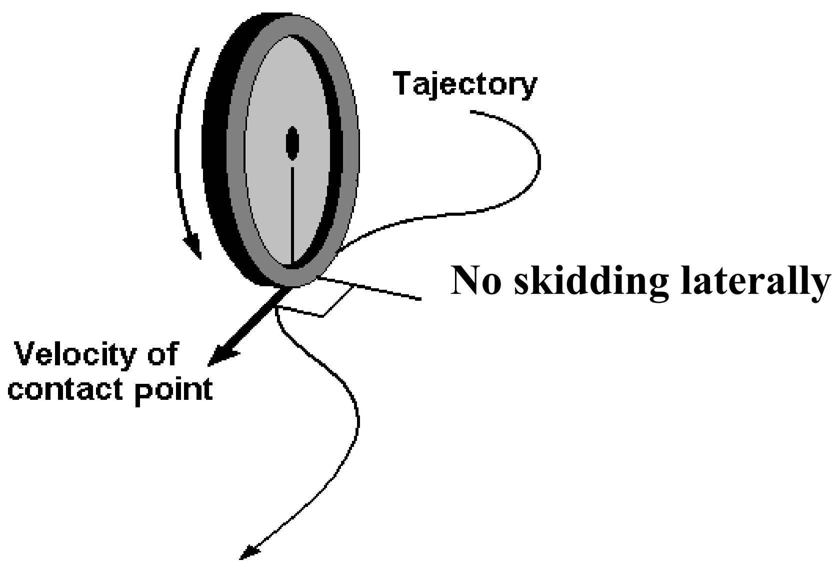


Two traction wheels and two omnidirectional wheels

2. Mobile Robot Kinematics

Mobile robot kinematics is the most basic study of how mobile robot systems behave. The process of understanding the motions of a mobile robot begins with the process of describing the contribution of each wheel provides for motion.

Each wheel has a role in enabling the whole robot to **move**. By the same token, each wheel also imposes **constraints** on the robot's motion; for example, refusing to skid laterally.



Basic Assumptions:

1. The robot is built from rigid mechanisms.

2. No slip occurs in the orthogonal direction of rolling (non-slipping).

3. No translational slip occurs between the wheel and the floor (pure rolling).

4. All steering axes are perpendicular to the floor.

2.1 Kinematic Models and Constraints

Kinematic models describe how the mobile robot as a whole moves as a function of its geometry and individual wheel behavior.

Deriving a model for the whole robot's motion is a bottom-up process.

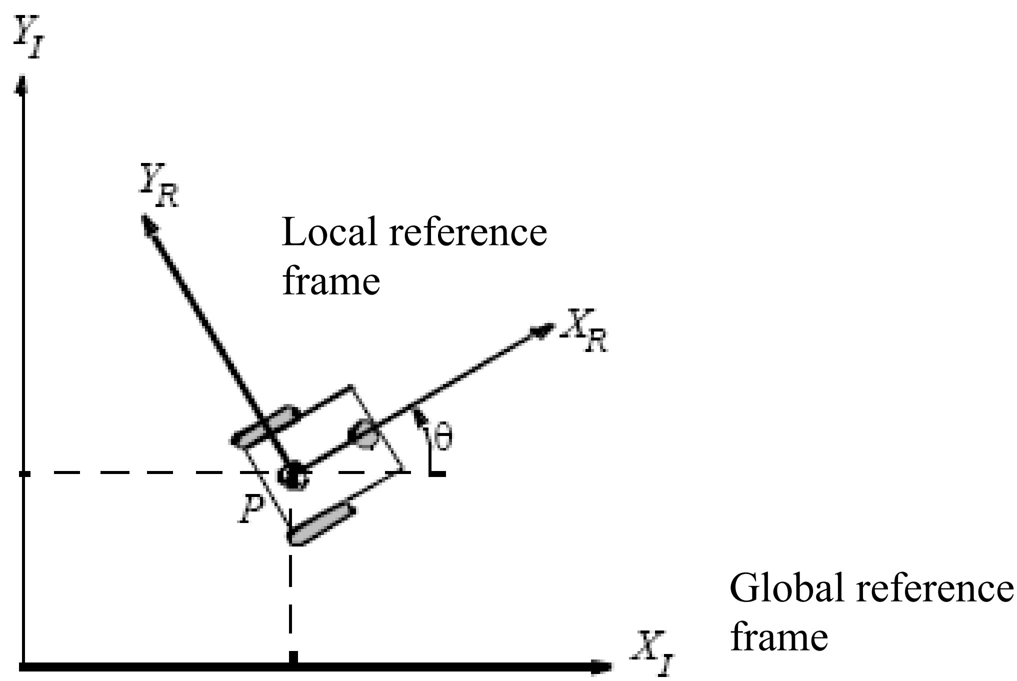
- Each *individual* wheel contributes to the robot's motion and, at the same time, imposes constraints on robot motion.
- Wheels are tied together based on robot *chassis geometry*.

2.1.1 Position and Coordinate Frames

The mobile robot is modeled as a rigid body on wheels, operating on a **horizontal plane**. The total dimensionality of this robot chassis on the plane is three,

- two for position in the plane and
- one for orientation along the vertical axis, which is orthogonal to the plane.

In order to specify the position of the robot on the plane we establish a relationship between the **global reference frame** of the plane and the **local reference frame** of the robot.



Global reference frame: $\{\boldsymbol{X_I}, \boldsymbol{Y_I}\}$

Local reference frame: $\{\boldsymbol{X_R}, \boldsymbol{Y_R}\}$

To specify the position of the robot, choose a point *P* on the robot chassis as its position reference point. The position of *P* in the **global reference frame** is specified by coordinates ***x*** and ***y***, and the angular difference between the global and local reference frames is given by $\boldsymbol{\theta}$.

That is,

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

To describe robot motion in terms of component motions, it will be necessary to *map motion along the axes of the global reference frame to motion along the axes of the robot's local reference frame*. Of course, the mapping is a function of the current pose of the robot. This mapping is accomplished using the orthogonal rotation matrix:

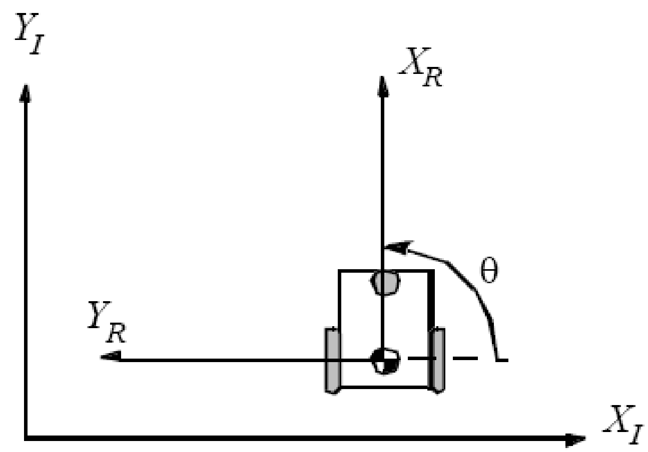
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix can be used to map motion in the **global reference frame** to motion in terms of the **local reference frame**. That is,

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

Example 1

Consider the robot in the following configuration.



Since $\theta = \frac{\pi}{2}$, the instantaneous rotation matrix *R* is

$$R(\frac{\pi}{2}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Given some velocity in the global reference frame we can compute the components of motion along this robot's local axes

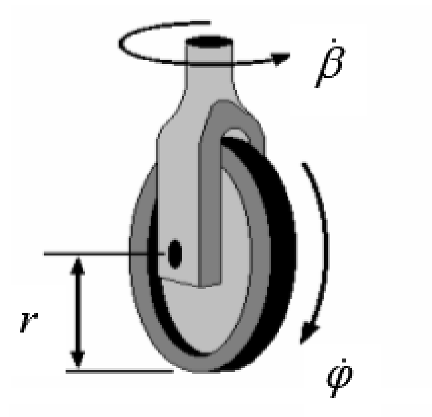
$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

That is, motion along $\boldsymbol{X_R}$ is equal to $\dot{y}$ and motion along is $\boldsymbol{Y_R}$ is $-\dot{x}$.

2.1.2 Forward kinematic models

Forward kinematics of the mobile robot:

How does the robot move, given its geometry and the speeds of its wheels?



More formally, forward kinematic model is to establish the *robot speed* $\dot{\xi}_I = [\,\dot{x}, \dot{y}, \dot{\theta}\,]^T$ as a function of the *wheel speeds* $\dot{\varphi}_1, \dot{\varphi}_2, \ldots, \dot{\varphi}_N$, *steering speeds* $\dot{\beta}_1, \dot{\beta}_2, \ldots, \dot{\beta}_M$ and the *geometric parameters* (e.g. radius of wheels, distance between wheels) of the robot. That is.

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dot{\varphi}_2, \ldots, \dot{\varphi}_N, \dot{\beta}_1, \dot{\beta}_2, \ldots, \dot{\beta}_M)$$

Forward kinematic model describes the motion of mobile robot in *global reference frame* as a function of wheel velocities. Recall that

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

We can first express the motion of the robot in *local reference frame*, and then transform it to *global reference frame* using:

$$\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$$

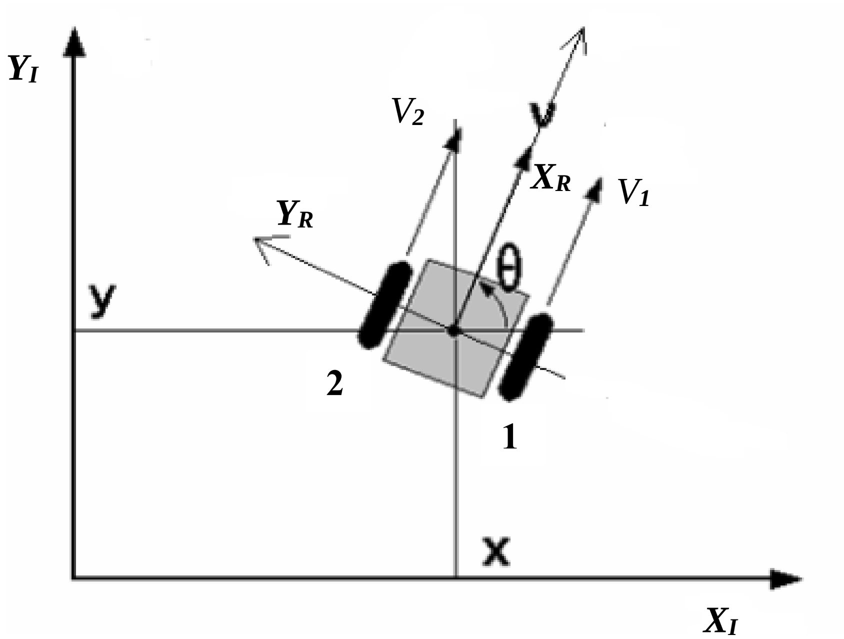
Differential Drive Robot

A differential drive robot has two wheels, each with diameter *r*. Given a point *P* centered between the two drive wheels, each wheel is a distance *l* from *P*.

The forward kinematic problem is thus to establish the robot speed $\dot{\xi}_I = [\dot{x}, \dot{y}, \dot{\theta}]^T$ as a function of the two wheel speeds $\dot{\varphi}_1, \dot{\varphi}_2$, and the geometric parameters *r*. *l* of the robot.

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(\dot{\varphi}_1, \dot{\varphi}_2)$$

We can compute the robot's motion in the global reference frame from motion in its local reference frame.



First, we express the velocities of the robot in its local reference frame,

$$v_x = \frac{1}{2}(v_1 + v_2)$$

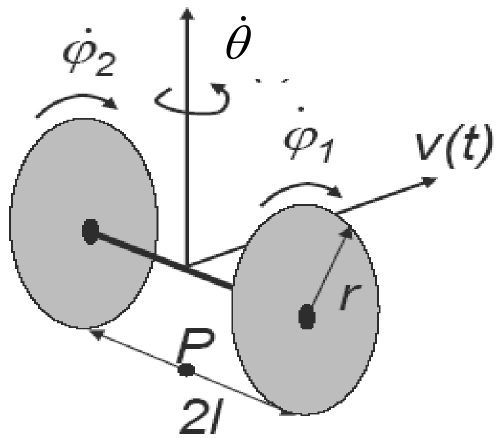
$$v_y = 0$$

$$\dot{\theta} = \frac{v_1}{2l} - \frac{v_2}{2l}$$

where

$v_1 = \dot{\varphi}_1 r$

$v_2 = \dot{\varphi}_2 r$



Using the above equations, the robot speed in local reference frame $\dot{\xi}_R$ can be expressed as a function of the two wheel speeds $\dot{\varphi}_1, \dot{\varphi}_2$, and the geometric parameters *r*. *l* of the robot as:

$$\dot{\xi}_R = \begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}_2}{2l} \end{bmatrix}$$

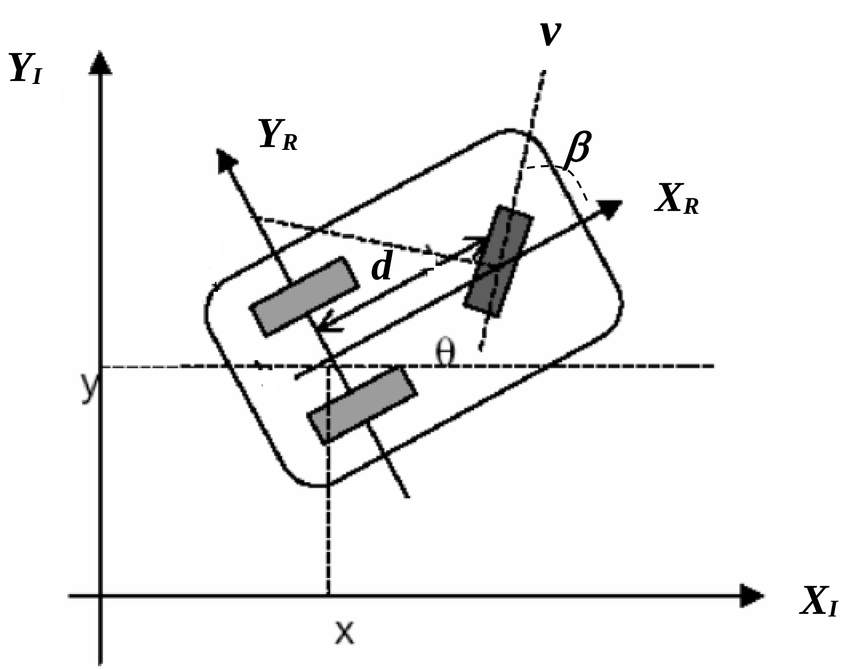
We can now compute the robot’s motion in the global reference frame as

$$\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} - \frac{r\dot{\varphi}}{2l} \end{bmatrix}$$

Tricycle

The tricycle has three wheels and steering and power are provided through the front wheel. The control variables are the steering direction β and the angular velocity of steering wheel $\dot{\varphi}$.



We first express the velocities of the robot in its local reference frame as

$$v_x = v\cos\beta$$

$$v_y = 0$$

$$\dot{\theta} = \frac{v\sin\beta}{d}$$

where

$$v = \dot{\varphi}r$$

Therefore, the robot speed is expressed in local reference frame as:

$$\dot{\xi}_R = \begin{bmatrix} v_x \\ v_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}\ r\cos\beta \\ 0 \\ \dfrac{\dot{\varphi}\ r\sin\beta}{d} \end{bmatrix}$$

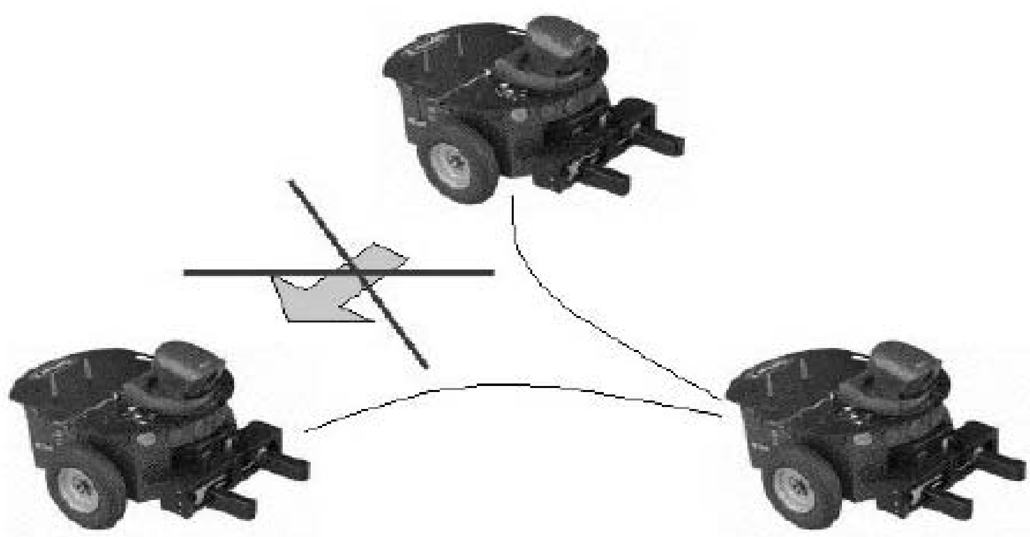
The robot's motion in the global reference frame is

$$\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\varphi}\ r\cos\beta \\ 0 \\ \dfrac{\dot{\varphi}\ r\sin\beta}{d} \end{bmatrix}$$

2.1.3 Wheel kinematic constraints

Some mobile robots can move in some directions and not other directions.



An important step in kinematic modeling of the robot is to express constraints on the motions of individual wheels. The motions of individual wheels can later be combined to compute the motion of the robot as a whole.

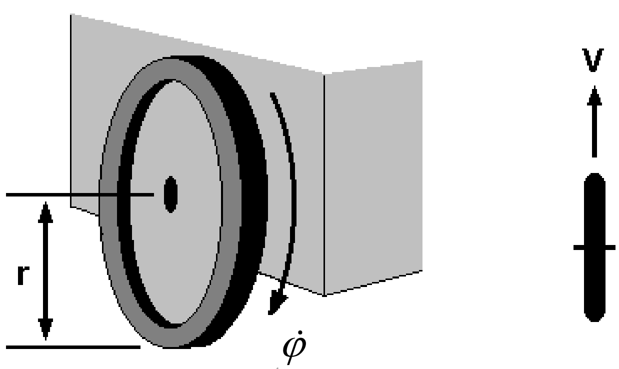
As discussed, there are four basic wheel types with widely varying kinematic properties. Therefore, we begin by presenting sets of constraints specific to each wheel type.

Two constraints for every wheel type will be shown.

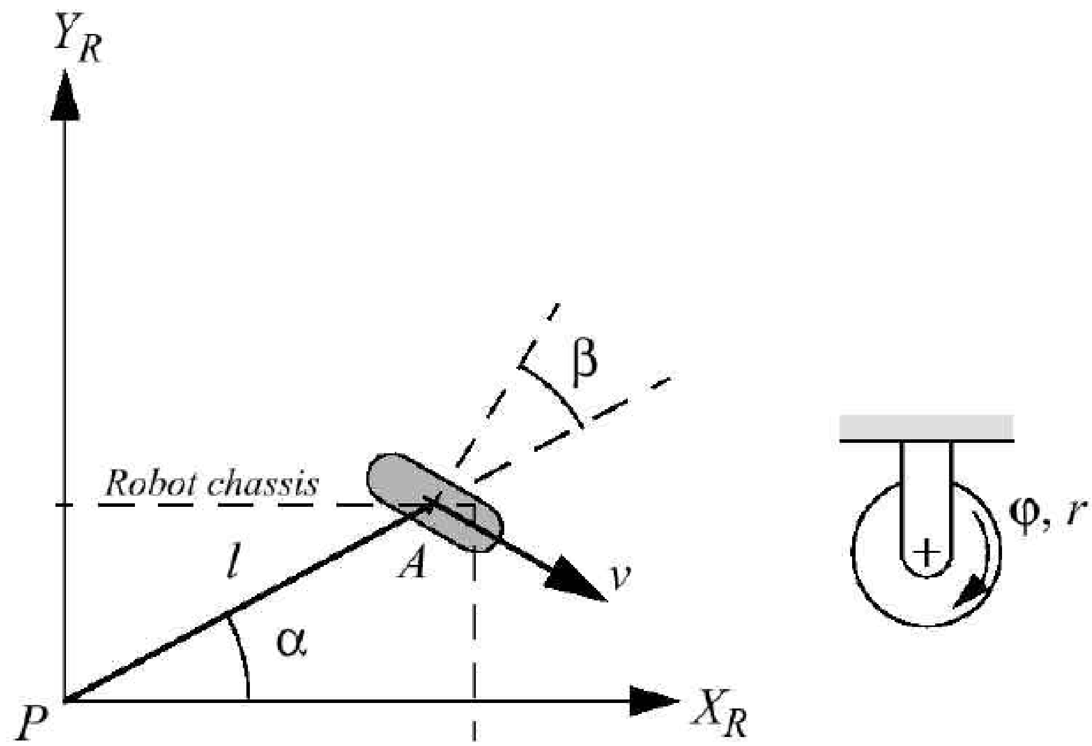
- The first constraint enforces the concept of rolling contact — *that the wheel must roll when motion takes place in the appropriate direction.*
- The second constraint enforces the concept of no lateral slippage — *that the wheel must not slide orthogonal to the wheel plane.*

Fixed standard wheel

The *fixed* standard wheel has *no vertical axis of rotation for steering*. Its angle to the chassis is thus fixed, and it is limited to motion back and forth along the wheel plane and rotation around its contact point with the ground plane.



Consider a fixed standard wheel *A* and indicates its position pose relative to the robot's local reference frame **{*X_R*, *Y_R*}.**



The position of ***A*** is expressed in polar coordinates by *distance* ***1*** and *angle* $\boldsymbol{\alpha}$.

The *angle of the wheel plane relative to the chassis* is denoted by $\boldsymbol{\beta}$, which is fixed since the fixed standard wheel is not steerable.

The wheel, which has radius ***r***, can spin over time, and so its rotational position around its horizontal axle is a function of time ***t***: $\boldsymbol{\varphi(t)}$.

Let $\dot{\xi}_R = [\,\dot{x}_r, \dot{y}_r, \dot{\theta}\,]^T$ be the velocities in local coordinate frame and recall that

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

The rolling constraint for this wheel enforces that all motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel spin so that there is pure rolling at the contact point. That is,

$$\begin{aligned} v = \dot{\varphi}\; r &= \dot{x}_r \sin(\alpha+\beta) - \dot{y}_r \cos(\alpha+\beta) - \dot{\theta}\; l\cos\beta \\ &= \begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta \end{bmatrix} \begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta} \end{bmatrix} \end{aligned}$$

The above *rolling constraint* can be written as:

$$\begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta \end{bmatrix} R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

or

$$j\;\; R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

where $j = \begin{bmatrix} \sin(\alpha+\beta) & -\cos(\alpha+\beta) & -l\cos\beta \end{bmatrix}$.

The sliding constraint for this wheel enforces that the component of the wheel's motion orthogonal to the wheel plane must be zero. That is,

$$\dot{x}_r\, cos(\alpha+\beta) + \dot{y}_r\, sin(\alpha+\beta) + \dot{\theta}\ l\, sin\,\beta = 0$$

The above *sliding constraint* can be written as:

$$\left[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad l\, sin\,\beta\right] R(\theta)\dot{\xi}_I = 0$$

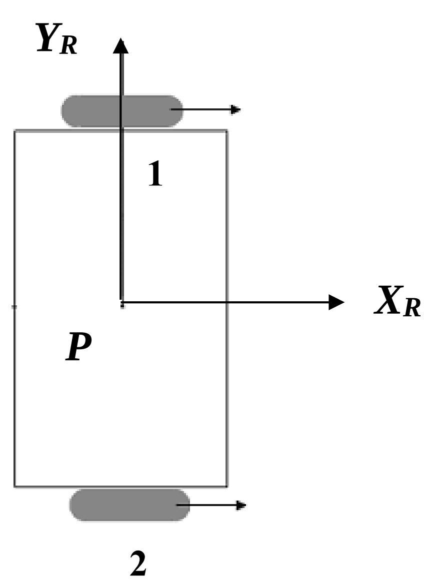
or

$$c\ R(\theta)\dot{\xi}_I = 0$$

where $c = \left[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad l\, sin\,\beta\right]$.

Example 3:

Consider the following differential drive robot with two wheels. Derive the rolling and sliding constraints for wheel 1.



Note that wheel 1 is in a position such that $\alpha = \frac{\pi}{2}$, $\beta =$ 0. This would place the contact point of the wheel on X_R with the plane of the wheel oriented parallel to Y_R.

If θ = 0, then the rolling constraint for wheel 1 reduces to

$$[1 \quad 0 \quad l] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} - r\dot{\varphi} = 0$$

$$\Rightarrow \quad [1 \quad 0 \quad l] \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} - r\dot{\varphi} = 0$$

$$\Rightarrow \quad \dot{x} = r\dot{\varphi}$$

The sliding constraint for wheel A is

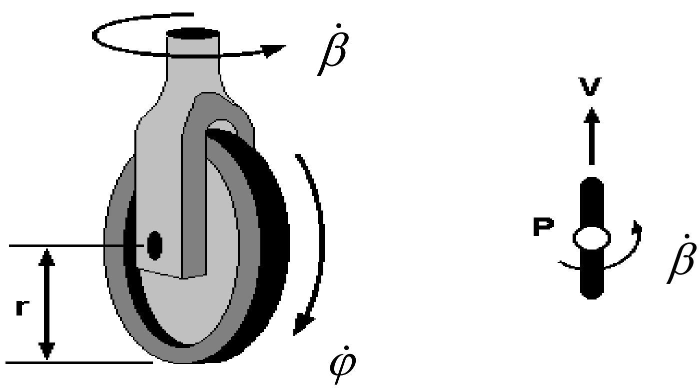
$$[0 \quad 1 \quad 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

$$\Rightarrow \quad [0 \quad 1 \quad 0] \begin{bmatrix} \dot{x} \\ \dot{y} \\ 0 \end{bmatrix} = 0$$

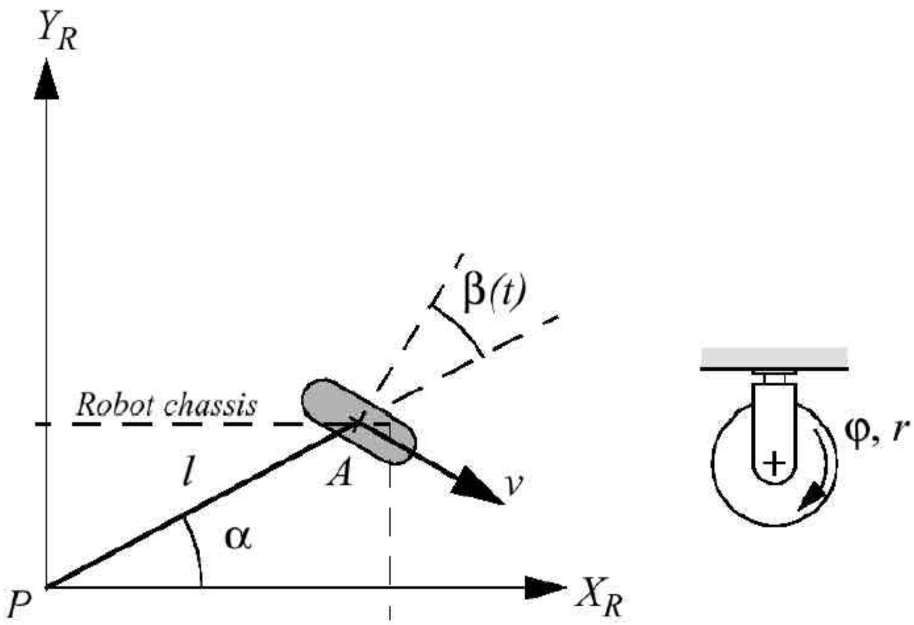
$$\Rightarrow \quad \dot{y} = 0$$

This constrains the component of motion along Y_I to be zero and since Y_I and Y_R are parallel in this example, the ***wheel is constrained from sliding sideways***, as expected.

Steered standard wheel



The steered standard wheel differs from the fixed standard wheel only in that there is an additional degree of freedom: the wheel may rotate around a vertical axis passing through the center of the wheel and the ground contact point.



The equations of position for the steered standard wheel are identical to that of the fixed standard wheel with one exception. *The orientation of the wheel to the robot chassis is no longer a single fixed value*, $\boldsymbol{\beta}$ but instead varies as a function of time: $\boldsymbol{\beta(t)}$.

The rolling and sliding constraints are:

$$\left[sin(\alpha+\beta) \quad -cos(\alpha+\beta) \quad -l\cos\beta \right] R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$\left[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad l\sin\beta \right] R(\theta)\dot{\xi}_I = 0$$

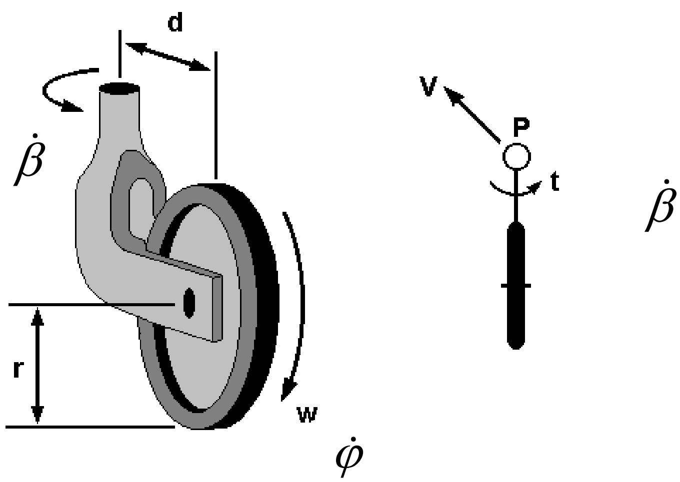
or

$$j(\beta)R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

$$c(\beta)R(\theta)\dot{\xi}_I = 0$$

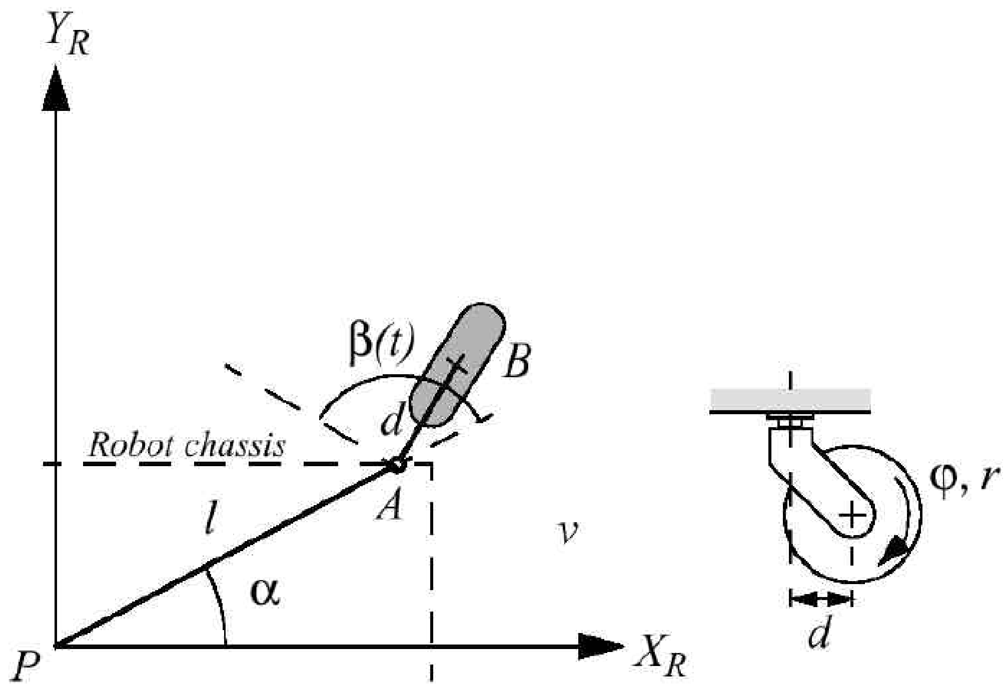
These constraints are identical to those of the fixed standard wheel because, unlike $\dot{\varphi}$, $\dot{\beta}$ *does not have a direct impact on the instantaneous* ***motion constraints*** *of a robot*. It is only by integrating over time that changes in steering angle can affect the *mobility* of a vehicle.

Castor wheel



Castor wheels are *able to steer around a vertical axis*.

However, unlike the steered standard wheel, the vertical *axis of rotation* in a castor wheel *does not pass through the ground contact point*.



The formal specification of the castor wheel's position requires an additional parameter.

The wheel *contact point is now at position B*, which is connected by a rigid rod *AB* of fixed length *d* to point *A* fixes the location of the vertical axis about which *B* steers, and this point *A* has a position specified in the robot's reference frame.

We assume that the plane of the wheel is aligned with *AB* at all times. Similar to the steered standard wheel, the castor wheel has two parameters that vary as a function of time. $\varphi(t)$ represents the wheel spin over time as before. $\beta(t)$ denotes the steering angle and orientation of *AB* over time.

For the castor wheel, the rolling constraint is identical to the standard wheel because the offset axis plays no role during motion that is *aligned with the wheel plane*:

$$\left[sin(\alpha+\beta) \quad -cos(\alpha+\beta) \quad -l\cos\beta\right]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

or

$$j(\beta)R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

The castor geometry does, however, have significant impact on the sliding constraint. The critical issue is that the lateral force on the wheel occurs at point A because this is the attachment point of the wheel to the chassis.

Because of the offset ground contact point relative to A , the constraint that there be zero lateral movement would be wrong. Instead, the constraint is much like a rolling constraint, in that appropriate rotation of the vertical axis must take place:

$$\left[cos(\alpha+\beta) \quad sin(\alpha+\beta) \quad l\sin\beta\right]R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

or

$$c(\beta)R(\theta)\dot{\xi}_I + d\dot{\beta} = 0$$

In the above equation, any motion orthogonal to the wheel plane must be balanced by an equivalent and opposite amount of castor steering motion.

In a steered standard wheel, the steering action does not by itself cause a movement of the robot chassis. But in a castor wheel the steering action itself moves the robot chassis because of the offset between the ground contact point and the vertical axis of rotation.

It can be surmised from the above equations that:

Given any robot chassis motion ξ_I, there exists some value for spin speed $\dot{\varphi}$ and steering speed $\dot{\beta}$ such that the constraints are met.

Therefore, a robot with only castor wheels can move with any velocity in the space of possible robot motions. We term such systems ***omnidirectional***.

A real-world example of such a system is the five-castor wheel office chair.

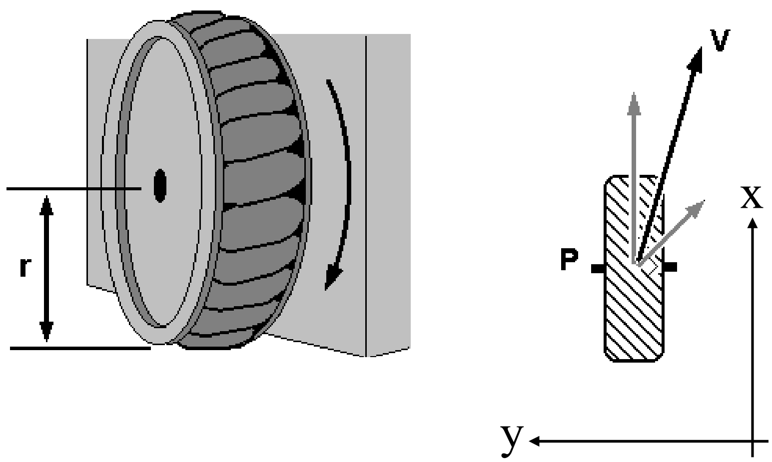

Assuming that all joints are able to move freely, you may select any motion vector on the plane for the chair and push it by hand. Its castor wheels will spin and steer as needed to achieve that motion without contact point sliding.

By the same token, if each of the chair's castor wheels housed two motors, one for spinning and one for steering, then a control system would be able to move the chair along any trajectory in the plane.

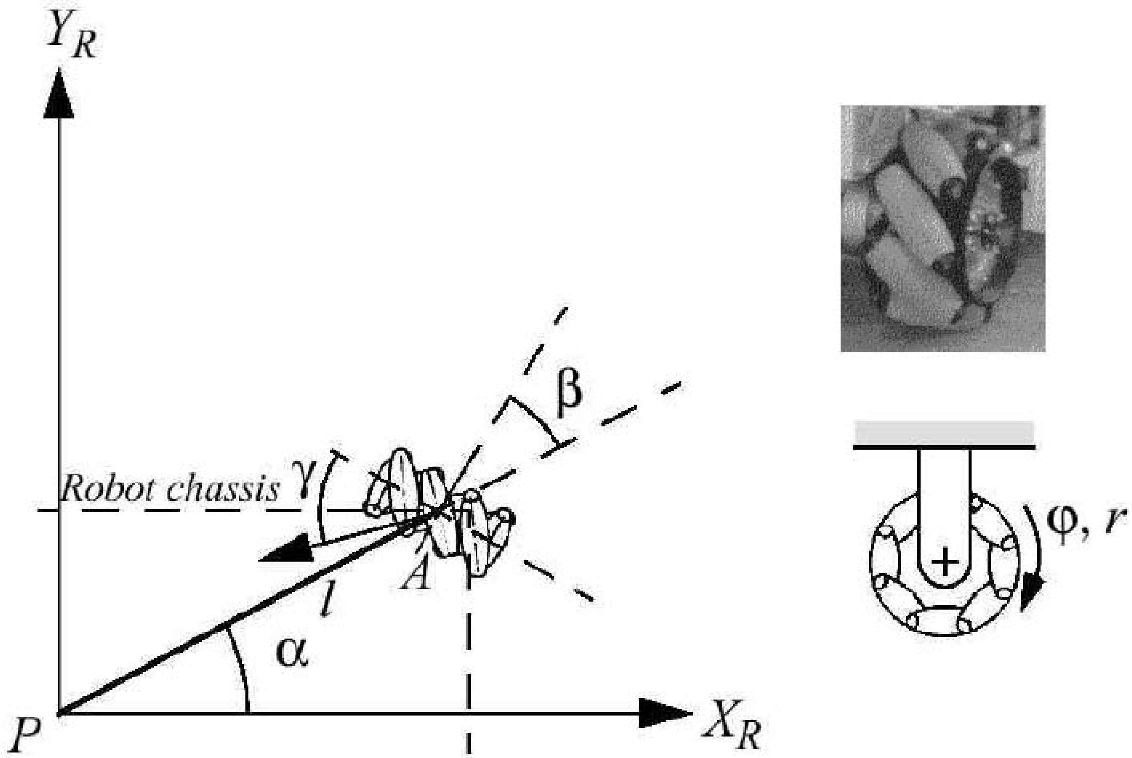
Thus, although the kinematics of castor wheels is somewhat complex, such wheels do not impose any real constraints on the kinematics of robot chassis.

Swedish wheel

Swedish wheels have no vertical axis of rotation, yet are able to move *omnidirectionally* like the castor wheel. This is possible by *adding a degree of freedom to the fixed standard wheel*.



Swedish wheels consist of ***a fixed standard wheel with rollers*** *attached to the wheel perimeter with axes that are antiparallel to the main axis of the fixed wheel component*.



The pose of a Swedish wheel is expressed exactly as in a fixed standard wheel, with the addition of a term, γ, representing the angle between the main wheel plane and the axis of rotation of the small circumferential rollers.

The motion vectors of the principal axis and the roller axes can spin clockwise or counterclockwise. One can combine any vector along one axis with any vector along the other axis. Hence any desired direction of motion is achievable by choosing the appropriate two vectors.

The instantaneous constraint is due to the specific orientation of the small rollers. The axis around which these rollers spin is a zero component of velocity at the contact point. That is, moving in that direction without spinning the main axis is not possible without sliding. The motion constraint that is derived looks identical to the rolling constraint for the fixed standard wheel in equation (3.12) except that the formula is modified by adding **γ** such that the effective direction along which the rolling constraint holds is along this zero component rather than along the wheel plane:

$$[sin(\alpha+\beta+\gamma) \quad -cos(\alpha+\beta+\gamma) \quad -l\,cos(\beta+\gamma)]R(\theta)\dot{\xi}_I$$
$$-r\dot{\varphi}\,cos\,\gamma = 0$$

$$j\ \ R(\theta)\dot{\xi}_I - r\,cos\,\gamma\ \ \dot{\varphi} = 0$$

Orthogonal to this direction the *motion is not constrained* because of the free rotation $\dot{\varphi}_{sw}$ of the small rollers

$$[cos(\alpha+\beta+\gamma) \quad sin(\alpha+\beta+\gamma) \quad l\,sin(\beta+\gamma)]R(\theta)\dot{\xi}_I$$
$$-r\dot{\varphi}\,sin\,\gamma - r_{sw}\dot{\varphi}_{sw} = 0$$

where r_{sw} is the radius of the small rollers.

3.1.4 Robot kinematic constraints

We now consider a general mobile robot with N wheels of the above described categories. We use the following subscripts to identify quantities relative to these 4 classes:

- f for fixed standard wheel,
- s for steerable standard wheel,
- c for castor wheels, and
- sw for Swedish wheels.

For example, the numbers of wheels of each type are denoted N_f , N_s,, N_c,, N_{sw}, φ_f, φ_s, φ_c, φ_{sw} denote the rotation angles of the wheels, and β_s, β_c denote the steering angles of the wheels.

Combining the wheel constraints imposes the overall constraints for the vehicle.

The *rolling constraints* of all wheels can now be collected in the following general expressions in matrix form:

$$J_1(\beta_s, \beta_c)R(\theta)\dot{\xi}_I + J_2\dot{\varphi} = 0$$

This expression bears a strong resemblance to the rolling constraint of a single wheel, but substitutes matrices in lieu of single values, thus taking into account all wheels. J_2 is a constant diagonal matrix and J_1 denotes a matrix with projections for all wheels to their motions along their individual wheel planes.

In summary, this equation represents the rolling constraint that all wheels must spin around their horizontal axis an appropriate amount based on their motions along the wheel plane so that rolling occurs at the ground contact point.

We use the same technique to collect the sliding constraints of all standard wheels into a single expression

$$C(\beta_s, \beta_c)R(\theta)\dot{\xi}_I + D\dot{\beta} = 0$$

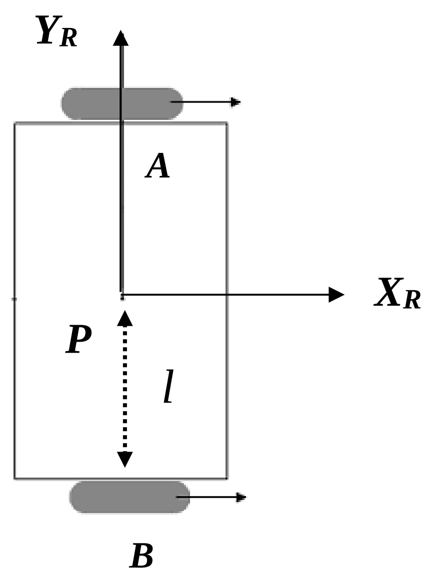
For a vehicle with only standard wheels (fixed or steered), the above equation reduces to:

$$C(\beta_s)R(\theta)\dot{\xi}_I = 0$$

The above equation is a constraint over all standard wheels that their components of motion orthogonal to their wheel planes must be zero. This sliding constraint over all standard wheels has the most significant impact on defining the overall maneuverability of the robot chassis.

Example 4:

Consider the following robot with two standard fixed wheels as in example 3.



Note that wheel 1 is in a position such that $\alpha = \frac{\pi}{2}$, $\beta =$ 0 and wheel 2 is in a position such that $\alpha = -\frac{\pi}{2}$, $\beta = \pi$.

Note the value of β of the wheel 2 is necessary to ensure that positive spin causes motion in the $+X_R$ direction.

The rolling constraint for standard fixed wheels is given as:

$$\left[\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta\right]R(\theta)\dot{\xi}_I - r\dot{\varphi} = 0$$

Hence, we have

wheel A: $\left[1 \quad 0 \quad -l\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}_{fA} = 0$

wheel B: $\left[1 \quad 0 \quad l\right]R(\theta)\dot{\xi}_I - r\dot{\varphi}_{fB} = 0$

Combining the above equations yield

$$\underbrace{\begin{bmatrix} 1 & 0 & -l \\ 1 & 0 & l \end{bmatrix}}_{J_1} R(\theta)\dot{\xi}_I - \underbrace{\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}}_{J_2} \begin{bmatrix} \dot{\varphi}_{fA} \\ \dot{\varphi}_{fB} \end{bmatrix} = 0$$

The *sliding constraint* is given as:

$$\left[\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta\right]R(\theta)\dot{\xi}_I = 0$$

Hence, we have

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{C} R(\theta)\dot{\xi}_I = 0$$

These equations relate robot motion to the rolling and sliding constraints and the wheel spin speed of the robot's wheels. Fusing these two equations yields the following expression:

$$\begin{bmatrix} J_1 \\ C_1 \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} J_2\dot{\varphi} \\ 0 \end{bmatrix} = 0$$

where $\dot{\varphi} = \begin{bmatrix} \dot{\varphi}_{fA} \\ \dot{\varphi}_{fB} \end{bmatrix}$.

Hence

$$\begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix} R(\theta)\dot{\xi}_I = \begin{bmatrix} r\dot{\varphi}_{fA} \\ r\dot{\varphi}_{fB} \\ 0 \end{bmatrix}$$

Inverting the above equation yields the kinematic equation of the mobile robot:

$$\dot{\xi}_I = R^T(\theta) \begin{bmatrix} 1 & 0 & l \\ 1 & 0 & -l \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} r\dot{\varphi}_{fA} \\ r\dot{\varphi}_{fB} \\ 0 \end{bmatrix}$$