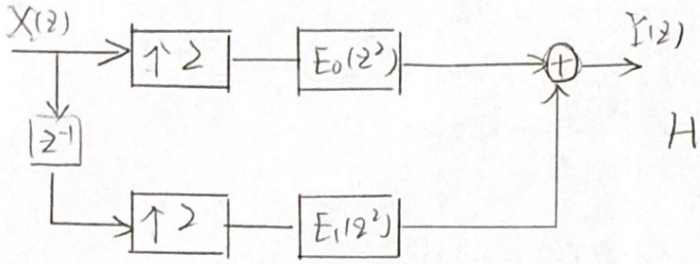
1. (a) $H(z) = \dfrac{1-2z^{-1}}{1+3z^{-1}} = E_0(z^2) + z^{-1}E_1(z^2)$, $\cancel{E_0(z) = \dfrac{1}{1+3z^{-1}}}$

$\cancel{E_1(z) = \dfrac{-2}{1+3z^{-1}}}$



$$H(z) = \frac{(1-2z^{-1})(1-3z^{-1})}{(1+3z^{-1})(1-3z^{-1})}$$

$$= \frac{1-5z^{-1}+6z^{-2}}{1-9z^{-2}}$$

$$= \left(\frac{1+6z^{-2}}{1-9z^{-2}}\right) + z^{-1}\left(\frac{-5}{1-9z^{-2}}\right)$$

$$= E_0(z^2) + z^{-1}E_1(z^2)$$

$\Rightarrow E_0(z) = \dfrac{1+6z^{-1}}{1-9z^{-1}}$

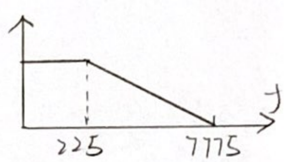
$E_1(z) = \dfrac{-5}{1-9z^{-1}}$

(b) $D = 48$, $F_T = 24\,\text{kHz}$, $f_p = 225\,\text{Hz}$, $\delta_p = 0.01$, $\delta_s = 0.001$

Stage 1: $F_T = 24\,\text{kHz}$, $f_1 = \frac{F_T}{3} = 8\,\text{kHz}$. $f_s = f_1 - f_p = 7775\,\text{Hz}$. $f_p = 225\,\text{Hz}$.

$\Delta f = (f_s - f_p)/F_T = 0.314583$. $\delta_{p1} = \frac{1}{2}\delta_p = 0.005$, $\delta_{s1} = 0.001$.

$N_1 = \dfrac{-10\log_{10}(\delta_{p1}\delta_{s1}) - 13}{14.6(\omega_s - \omega_p)/2\pi} = 8.7113 \approx 9$.



length $L_1 = N_1 + 1 = 10$.

$M_1 = \frac{N_1+1}{2} \times F_T = 120 \times 10^3$ multiplications / second.

$A_1 = N_1 \times F_T = 216 \times 10^3$ additions/second.

Stage 2: $F_T = f_1 = 8\,\text{kHz}$, $f_s = \frac{F_T}{2D} = 250\,\text{Hz}$, $f_p = 225\,\text{Hz}$.

$\Delta f = (250-225)/8k = 3.125 \times 10^{-3}$.

$N_2 = 877$

$M_2 = \frac{N_2+1}{2} \times F_T = \frac{878}{2} \times 8k = 3512 \times 10^3$ mul/sec.

$A_2 = N_2 \times F_T = 7016 \times 10^3$ add/sec.

$M = M_1 + M_2 = 3632 \times 10^3$

$A = A_1 + A_2 = 7232 \times 10^3$

2. (a) $H(z) = 1 + a5z^{-1}$

$$X_0(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_0\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_0\left(\frac{w-2\pi}{2}\right)\right]$$

$$X_1(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_1\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_1\left(\frac{w-2\pi}{2}\right)\right]$$

$$\hat{X}(w) = X_0(2w)G_0(w) + X_1(2w)G_1(w)$$

$$= \frac{1}{2}\left[X(w)H_0(w) + X(w-\pi)H_0(w-\pi)\right]G_0(w)$$

$$+ \frac{1}{2}\left[X(w)H_1(w) + X(w-\pi)H_1(w-\pi)\right]G_1(w)$$

$$= \frac{1}{2}\left[H_0(w)G_0(w) + H_1(w)G_1(w)\right]X(w)$$

$$+ \frac{1}{2}\left[H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w)\right]X(w-\pi)$$

$\Rightarrow$ $H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w) = 0$

$G_0(w) = H_1(w-\pi)$, $G_0(z) = H_1(-z)$;

$G_1(w) = -H_0(w-\pi)$, $G_1(z) = -H_0(-z)$;

$H_1(z) = H_0(-z)$

$\Rightarrow$ $H_0(z) = H(z) = 1 + a5z^{-1}$,

$H_1(z) = H_0(-z) = 1 - a5z^{-1}$,

$G_0(z) = H_1(-z) = 1 + a5z^{-1}$,

$G_1(z) = -H_0(-z) = -1 + a5z^{-1}$

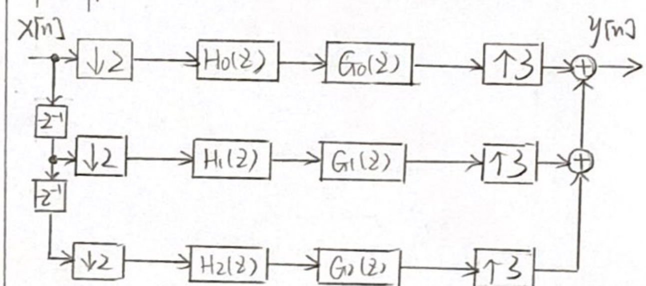
(ii) $h_0[n] = 1 + a5h_0[n-1]$,

$h_1[n] = 1 - a5h_1[n-1]$,

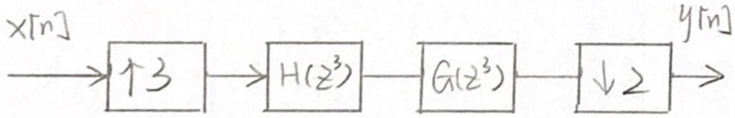
$g_0[n] = 1 + a5g_0[n-1]$,

$g_1[n] = -1 + a5g_1[n-1]$.

polyphase:



(b) direct form: $\frac{I}{D} = \frac{3}{2}$



By using the polyphase structure, the problems of processing zero-value data for the up-samplers and ignoring the filtered output by the down-samplers are avoided. Also, filtering after decimation and filtering befor interpolation guarantees that the subfilters are working at the lowest freq

2. (a) $H(z) = 1 + 0.5z^{-1} = H_0(z)$

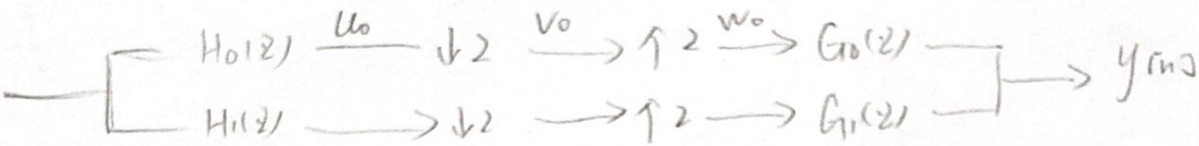
$H_1(z) = H_0(-z) = 1 - 0.5z^{-1}$

$G_0(z) = H_1(-z) = 1 + 0.5z^{-1}$

$G_1(z) = -H_0(-z) = -1 + 0.5z^{-1}$

$H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$

$H_0(z)G_0(z) + H_1(z)G_1(z) = 2z^{-1}$



$u_0[n] = x[n] + 0.5x[n-1]$

$u_1[n] = x[n] - 0.5x[n-1]$

$v_0[n] = u_0[2n]$ $\qquad$ $v_1[n] = u_1[2n]$

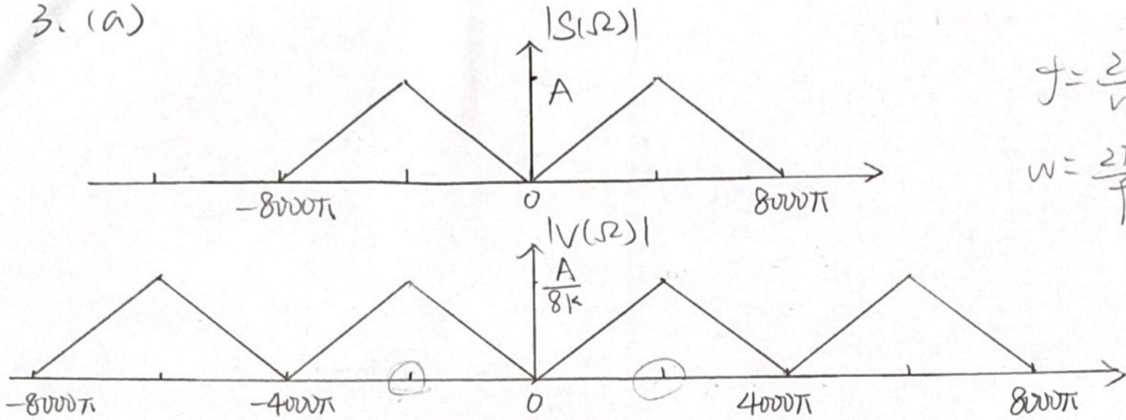
$$w_0[n] = \begin{cases} v_0[\frac{n}{2}] \\ 0 \end{cases}$$

$y_0[n] = w_0[n] + 0.5w_1[n]$

$y_1[n] = -w_0[n] + 0.5w_1[n]$

$\hat{x}[n] = y_0[n] + y_1[n]$

3. (a)



$f = \frac{2\pi}{w}$

$w = \frac{2\pi}{f} = 2\pi T$

(b) $y[n] = v[n] + 0.5v[n-1] + 0.5v[n+1]$.

$$Y(z) = V(z) + 0.5z^{-1}V(z) + 0.5zV(z)$$
$$= (0.5z + 0.5z^{-1} + 1)V(z).$$

$$\frac{Y(z)}{V(z)} = \frac{1}{0.5z + 1 + 0.5z^{-1}}$$

4. (a) ARMA: $x[n] = 1.6x[n-1] - 0.63x[n-2] + w[n] + 0.9w[n-1]$

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1+\sum_{k=1}^{p} a_k z^{-k}} = \frac{1+0.9z^{-1}}{1-1.6z^{-1}+0.63z^{-2}}$$

(b) $B(z) = 1+0.9z^{-1} = 0 \Rightarrow$ zero: $z_1 = -0.9$

$A(z) = 1-1.6z^{-1}+0.63z^{-2} = 0$

$(1-0.7z^{-1})(1-0.9z^{-1}) = 0 \Rightarrow$ poles: $p_1 = 0.7$, $p_2 = 0.9$

(c) $\Gamma_{xx}(z) = \sigma_w^2 H(z)H(z^{-1})$

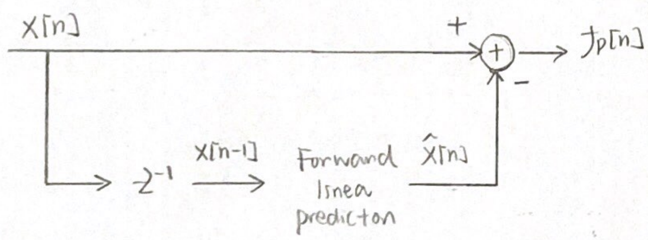
$$= \frac{\sigma_w^2(1+0.9z^{-1})(1+0.9z)}{(1-1.6z^{-1}+0.63z^{-2})(1-1.6z+0.63z^{2})},$$

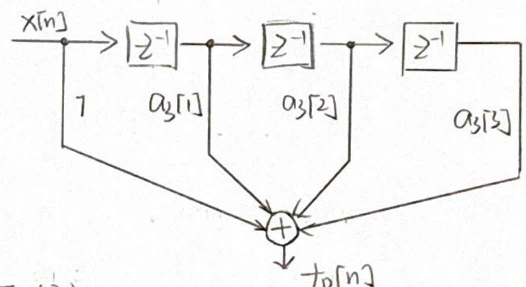
where σ_w is the variance of the white noise process.

5. (a) one-step forward: $\hat{x}[n] = -\sum_{k=1}^{p} a_p[k]x[n-k]$

$p=3$: $\hat{x}[n] = -a_3[1]x[n-1] - a_3[2]x[n-2] - a_3[3]x[n-3]$

(b) $f_p[n] = x[n] - \hat{x}[n] = x[n] + \sum_{k=1}^{p} a_p[k]x[n-k]$





(c) $F_p(z) = A_p(z)X(z) \Rightarrow A_p(z) = \frac{F_p(z)}{X(z)} = \frac{F_p(z)}{F_0(z)}$

$$= 1+\sum_{k=1}^{p} a_p[k]z^{-k}.$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

EE6401 – ADVANCED DIGITAL SIGNAL PROCESSING

November/December 2018

Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 3 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

$H(z) = E_0(z^2) + z^{-1} E_1(z^2)$

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1. (a) Consider a filter transfer function

$$H(z) = \frac{1 - 2z^{-1}}{1 + 3z^{-1}}$$

$H_0(z) = \frac{1}{1+3z^{-1}}$

$H_1(z) = z^{-1} \frac{-2}{1+3z^{-1}}$

Find the poly-phase implementation of a multi-rate system for a decimation factor $M = 2$.

(6 Marks)

(b) An audio signal $x[n]$ with a sampling frequency of 24 kHz need to be decimated by a factor of 48. The highest frequency of interest after decimation is 225 Hz. Assume that an overall passband ripple is $\delta_p = 0.01$ and stopband ripple is $\delta_s = 0.001$. Design an efficient decimator of two stages whose decimation factors are 3 and 16 respectively, and calculate the required computational complexity in terms of the number of additions and multiplications per second.

(14 Marks)

2. (a) Consider a two-channel quadrature mirror filter (QMF) bank with a prototype filter transfer function given by

$$H(z)= 1+0.5z^{-1}$$

(i) Find the filters used for analysis and synthesis filter banks to obtain an alias-free and perfect reconstruction system;

(ii) Find the time domain expression of the above QMF system.

(10 Marks)

(b) A sampling rate converter is needed to change its input sampling frequency by 3/2 times. To minimize the computational costs, *derive* a polyphase structure in its *most efficient* form, and *explain* why the computational complexity is reduced by the polyphase implementation compared to the implementation directly using a finite impulse response (FIR) filter.

(10 Marks)

3. Suppose that we would like to process a speech segment $s(t)$ that has a bandwidth of 4*k*Hz and is sampled at a rate of 8 *k*Hz. The following system in Figure 1 is used to process the speech segment.

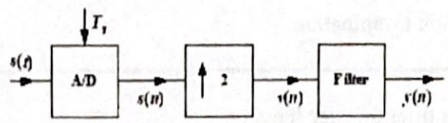

Figure 1

The speech signal has a spectrum shown below in Figure 2.

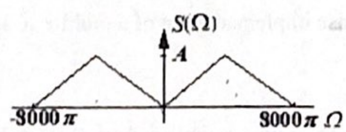

Figure 2

(a) Find the spectrum of $v(n)$.

(6 Marks)

(b) If the discrete-time filter is described by the difference equation

$$y(n) = v(n) + 0.5[v(n - 1) + v(n + 1)]$$

find the frequency response of the filter.

(6 Marks)

(c) Discuss the filter effect on signal $v(n)$.

(8 Marks)

4. Consider the ARMA process generated by the difference equation

$$x[n] = 1.6x[n-1] - 0.63x[n-2] + w[n] + 0.9w[n-1]$$

(a) Determine the system function of the whitening filter. (8 Marks)

(b) Calculate the poles and the zeros of the system function. (4 Marks)

(c) Determine the power density spectrum. (8 Marks)

5. Consider a stationary random process $x[n]$.

(a) Give the expression for a one step forward linear predictor of order $p = 3$

$\hat{x}[n] = -\sum_{k=1}^{p} a_p[k]x[n-k] = -a_3[1]x[n-1] - a_3[2]x[n-2] - a_3[3]x[n-3]$

(6 Marks)

(b) Draw the system diagram for forward linear prediction error. (6 Marks)

(c) Determine the system function in terms of the Z-transform of the forward linear prediction error. (8 Marks)

END OF PAPER