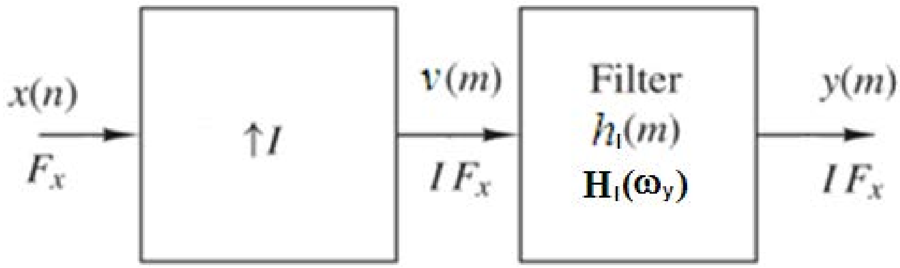
Multistage Implementation

Multistage Implementation



- The processing system becomes too complex to implement due to:
  - the no. of filter coefficients may have to be very large
  - the ratio of *I*/*D* and/or the filter requirements
- Let us assume $I >> 1$ and $I = I_1 \times I_2 \ldots I_L$. The interpolation by a factor of *I* is accomplished by *L* stages of upsampling and filtering as shown in Figure 6.1. The sampling frequency at the output of the *i*th stage is $F_{L-i} = F_{L+1-i}\, I_{L+1-i}$, $i = 1, 2, \ldots L$.

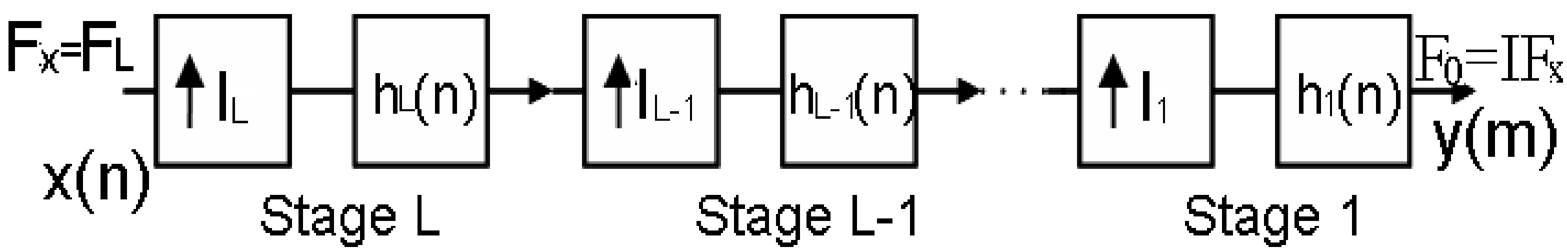


Figure 6.1 Multi-stage implementation of interpolation

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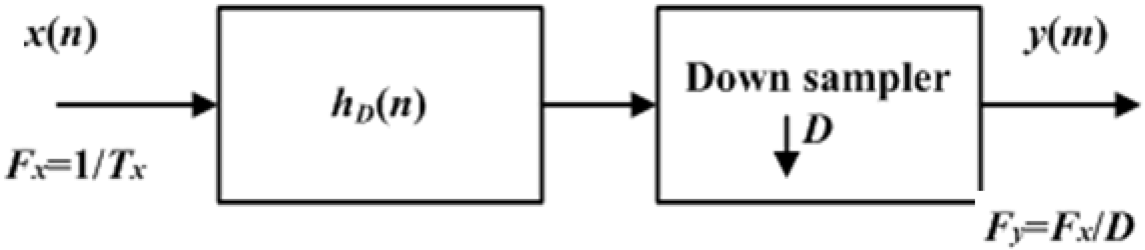
- Similarly, decimation by a factor $D >> 1$ and $D = D_1 \times D_2 \times \ldots \times D_J$ is realized by cascading J stages of filtering and downsampling.
- The sampling frequency at the output of the ith stage is $F_i = F_{i-1}/D$, $i=1,2,\ldots J$.



- To ensure that no aliasing occurs in the entire process, we have to design each filter stage within the frequency band of interest.

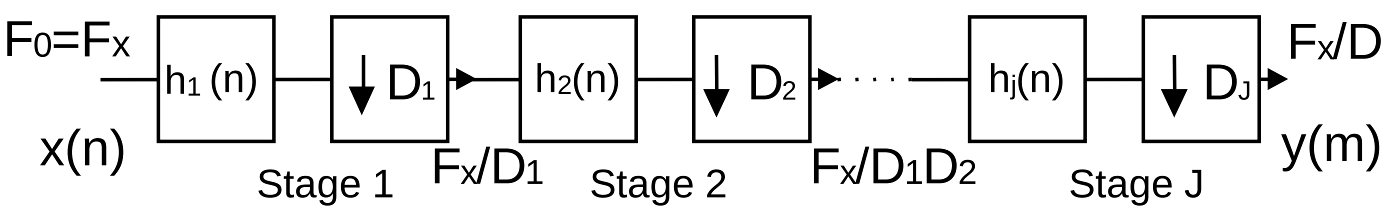


Figure 6.2 Multi-stage implementation of decimation

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- We define the required passband and stopband in the **OVERALL** system

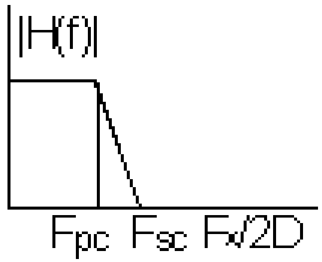

Figure 6.3

$$\begin{cases} \text{Passband} & : 0 \le F \le F_{pc} \\ \text{Transation band} & : F_{pc} \le F \le F_{sc} \end{cases} \qquad (6.1)$$

$$\text{where } F_{sc} \le F_x / 2D$$

where F_{pc} and F_{sc} are the passband and the stopband frequencies.

For a general FIR filter design, the stopband frequency may not be larger than one half of the sampling frequency, i.e., $F_{sc} < 0.5F_T$, where F_T is the sampling frequency of the input signal.

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- Otherwise, aliasing will occur to produce distortion (see the figures below) due to the symmetric property of the filter frequency response.

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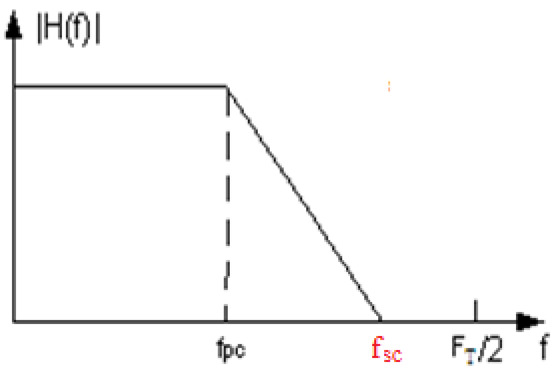
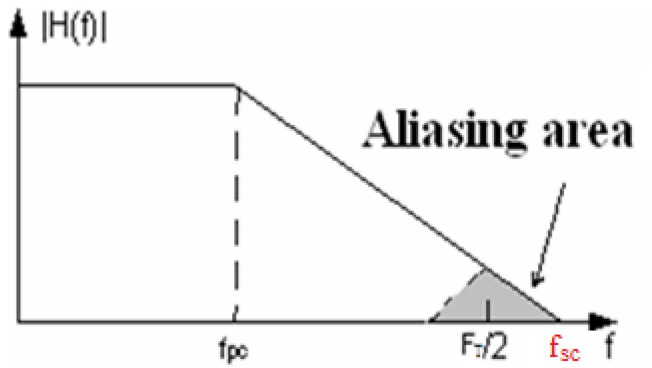
$$N \cong \frac{-10\log_{10}(\delta_p\delta_s)-13}{14.6(\omega_s-\omega_p)/2\pi} = \frac{-10\log_{10}(\delta_p\delta_s)-13}{14.6(f_{sc}-f_{pc})/F_T}$$



(a) Stopband frequency is smaller than one half of the input sampling frequency



(b) Stopband frequency is larger than one half of the input sampling frequency

- When $f_{sc} > 0.5F_T$, however, the transition bandwidth is increased, which leads to the reduction of filter order.
- Also the reduction of the input sampling frequency leads to the reduction of the filter order.

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- For decimation, the aliasing in the passband is avoided by selecting the frequency bands of each filter stage as follows:

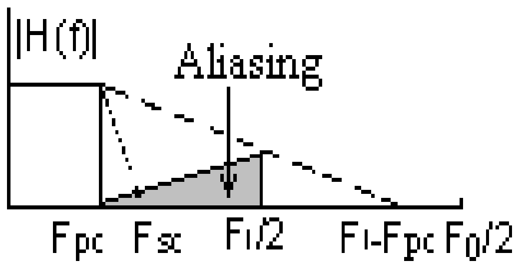
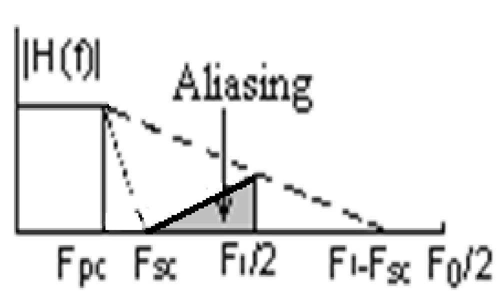


Figure 6.4

$$\begin{aligned} &\text{Passband} && : 0 \le F \le F_{pc} \\ &\text{Transation band} && : F_{pc} \le F \le F_i - F_{pc} \\ &\text{Stopband} && : F_i - F_{pc} \le F \le F_{i-1}/2 \end{aligned} \quad (6.2)$$

- Or avoiding aliasing in the transition band

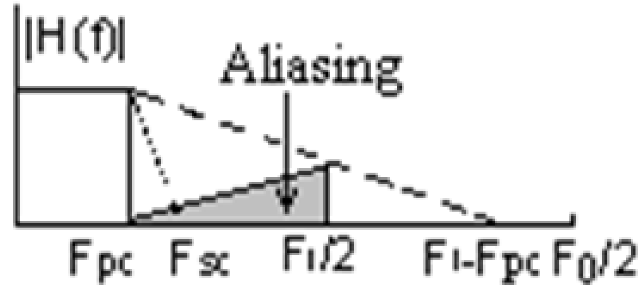


$$\begin{aligned} &\text{Passband} && : 0 \le F \le F_{sc} \\ &\text{Transation band} && : F_{sc} \le F \le F_i - F_{sc} \\ &\text{Stopband} && : F_i - F_{sc} \le F \le F_{i-1}/2 \end{aligned}$$

Multistage Implementation

- For example, in the first stage we have $F_1=F_x/D_1$, and the filter is designed to have the following frequency bands:

Passband	$: 0 \le F \le F_{pc}$
Transation band	$: F_{pc} \le F \le F_1 - F_{pc}$
Stopband	$: F_1 - F_{pc} \le F \le F_0/2$



- After down-sampling by D_1, there will be aliasing from the signal components that fall into the filter transition band (for the first case), but the aliasing occur at frequencies above F_{pc} only. Thus, there is no aliasing in the passband.
- In the second case, no aliasing from the signal components that fall into the filter transition band, but the aliasing occur at frequencies above F_{sc}.
- It is up to application requirements to decide if the aliasing is allowed in the transitional band.

Example: Consider an audio-band signal with a nominal bandwidth of 4 *k*Hz that is sampled at a rate of 8 *k*Hz. Suppose that we wish to isolate the frequency components below 75 Hz with a filter that has a passband and a transition band.

Solution: From the specification we have F_{pc}= 75 Hz and F_{sc}= 80 Hz, (with 5 Hz as transitional band). Decimation rate $D= F_T/(2F_{sc})= 50$. Assuming that the filter has a passband ripple $\delta_1=10^{-2}$ and a stopband ripple $\delta_2=10^{-4}$.

a) We use the formula to estimate the filter length N

$$N=\frac{-10\log_{10}\delta_1\delta_2-13}{14.6\Delta f}+1 \qquad (6.3)$$

where the normalized width of transitional region $\Delta f = (F_{sc}-F_{pc})/F_T$ $=1/1600$. The length of the filter ***N*=5152**, which is impossible for any practical applications.

b) We design a two-stage decimation process with $D_1=25$ and $D_2=2$.

c) The first stage. $F_1=8000/25 = 320$ Hz, is the passband frequency, transition band: $75 < F \leq 245$ (=320-75), and $\Delta f =(245-75)/8000$, $\delta_{11}=\delta_1/2$, $\delta_{21}= \delta_2$.

Multistage Implementation

d) The passband ripple is reduced by a factor of 2 so that the total pass band ripple in the cascade of the two filters does not exceed δ_1. The stopband ripple is maintained at δ_2 in both stages. Using the Kaiser formula we have $N_1 = 170$.

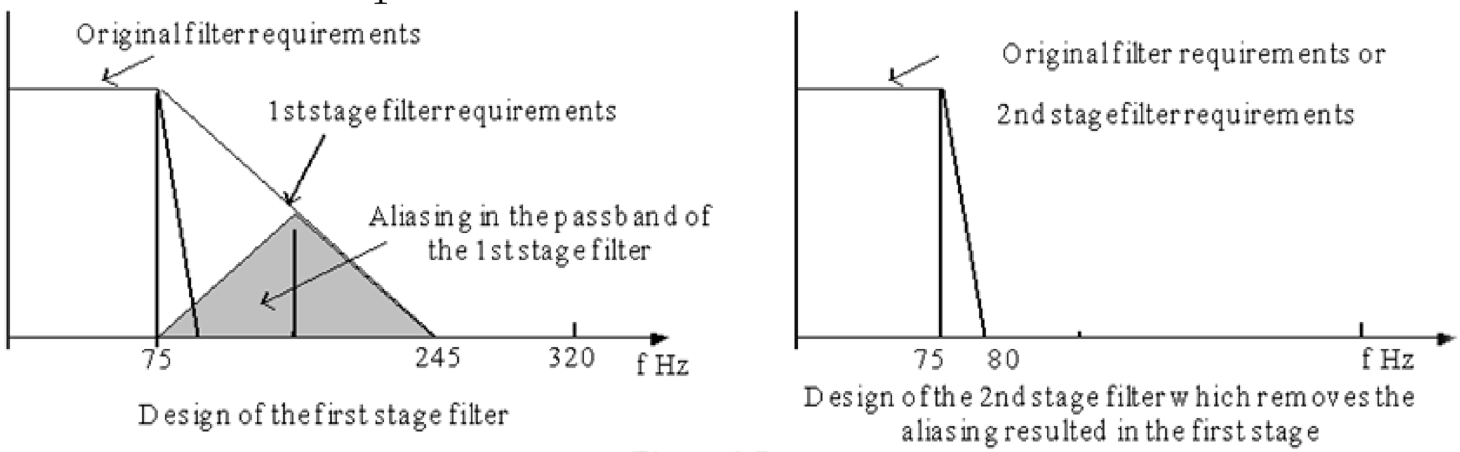


Figure 6.5

e) The second stage. F_2=F_1/2= 160 Hz, transition band: 75 < F ≤ 80, and Δf =(80-75)/320, δ_{11}=δ_1/2, δ_{21}= δ_2. The estimate of the length N_2=220.

f) Therefore, the total length is $\mathbf{N_1 + N_2 = 390}$, which is reduced by a factor of 13 compared to the single stage design.

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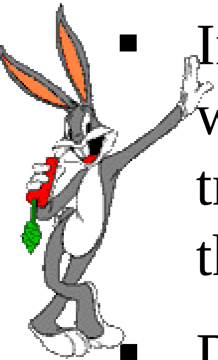
The deduction in the filter length is achieved by



- Increasing Δf that appears in the denominator of (6.3) by widening the width of the transition band. For example, the transitional bandwidth of the 1st stage is about 33 times as that of the original transitional bandwidth;
- Decreasing the input sampling frequency of the following stages (except 1st stage). For example, the input sampling frequency of the 2nd stage is reduced by 25 times compared to that of the original input sampling frequency.
- The order of the filter N is proportional to the input sampling frequency and inversely proportional to the transition bandwidth;
- In general, the effectiveness of reducing computational complexity by using multi-stage systems decreases with the increase of the number of stages. Excessive number of stages is not recommended.

Multistage Implementation

- For multistage interpolator, the sampling rate at the output of the ith stage is

$$F_{i-1} = I_i F_i, \quad i = J, J-1, \ldots, 1$$

and the output rate is $F_o = IF_J$ when the input sampling rate is F_J. The corresponding frequency band specifications are

$$\text{Passband:} \qquad 0 \le F \le F_{pc}$$

$$\text{Transitional band} \quad F_{pc} \le F \le F_i - F_{pc}$$

- **Example:** Let us reverse the filtering problem previously considered in the decimation process. We assume the input signal having a passband $0 \le F \le 75$ and a transitional band $75 \le F \le 80$. We wish to interpolate by a factor of 50.

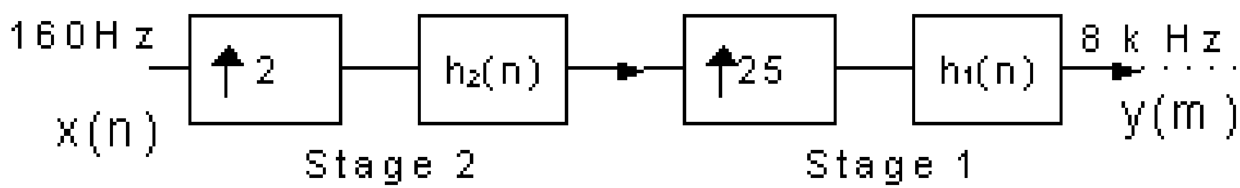


Figure 6.7

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Solution:

- Selecting $I_1 = 25$ and $I_2=2$. The output sampling frequency is 8 *k*Hz. The input sampling frequency at the 1st stage is 8000/25=320 Hz and the input sampling frequency at the second stage is 160 Hz.
- For stage 1, the transitional band is (320-75-75)/8000 and the second stage, the transitional band is (80-75)/320. Putting the transitional bandwidth and the ripples specifications into consideration, the number of coefficients can be calculated.
- The computational complexity for each stage is obtained by

$$(N+1)/2 \times F_i \text{ multiplications per second}$$

$$N \times F_i \text{ additions per second}$$

where N is the filter order and F_i is the stage sampling frequency

- **Example: Demonstration** 

Main requirements for Multistage Implementation

- Correctly specify the filters used for each stage
- Estimates the required computational complexity based on the estimated no. of filter coefficients.
- Calculate the required number of multiplications and additions per seconds.