1. (a) $\Delta f = (f_s - f_p)/F_T = 2.5/2500 = 10^{-3}$,

$$N = \frac{-10\log(\delta_p\delta_s) - 13}{14.6 \times (\omega_s - \omega_p)/2\pi} = 1849.32 \approx 1849.$$

length $L = N + 1 = 1850$.

(b) Option #1: $D_1 = 25$, $D_2 = 4$:

Stage 1: $f_p = 10Hz$, $F_T = 2500Hz$,

$f_1 = \frac{F_T}{25} = 100\ Hz$. $f_s = f_1 - f_p = 90Hz$,

$\Delta f = (f_s - f_p)/F_T = \frac{80}{2500} = 0.032$,

$\delta_{p1} = \frac{1}{2}\delta_p = 0.05$, $\delta_{s1} = \delta_s = 0.001$,

$N_1 = 64.23 \approx 64$, length $L_1 = 65$.

Stage 2: $F_T = f_1 = 100Hz$, $f_p = 10Hz$,

$f_s = 12.5\ Hz$, $\Delta f = 2.5/100 = 0.025$,

$N_2 = 82.21 \approx 82$, $L_2 = 83$.

Option #2: $D_1 = 4$, $D_2 = 25$:

Stage 1: $F_T = 2500\ Hz$,

$f_p = 10Hz$, $f_1 = \frac{F_T}{4} = 625\ Hz$

$f_s = f_1 - f_p = 615\ Hz$.

$\Delta f = (615 - 10)/2500 = 0.242$,

$N_1 = 8.49 \approx 9$, $L_1 = 10$

Stage 2: $F_T = f_1 = 625\ Hz$,

$f_p = 10\ Hz$, $f_s = 12.5\ Hz$,

$\Delta f = (2.5/625) = 4\times10^{-3}$.

$N_2 = 513.87 \approx 514$, $L_2 = 515$.

(c) Option #1:

$\frac{N_1+1}{2} \times F_T = \frac{65}{2} \times 2500 = 81250$ mul/sec,

$N_1 \times F_T = 160000$ add/sec,

$\frac{N_2+1}{2} \times F_{T2} = \frac{83}{2} \times 100 = 4150$ mul/se,

$N_2 \times F_{T2} = 82 \times 100 = 8200$ add/sec.

$\Rightarrow$ $M = 85400$ mul/sec.

$A = 168200$ add/sec.

Option #2:

$\frac{N_1+1}{2} \times F_T = \frac{10}{2} \times 2500 = 12500$ mul/sec

$N_1 \times F_T = 9 \times 2500 = 22500$ add/sec

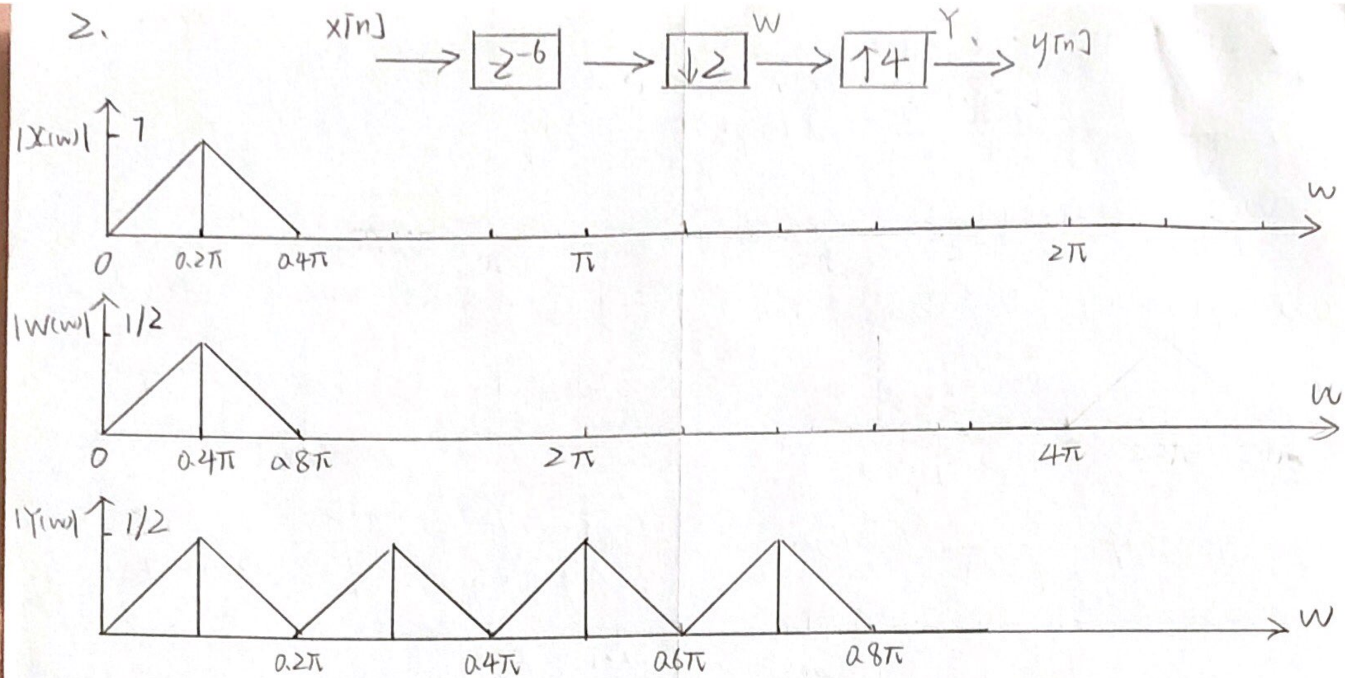
$\frac{N_2+1}{2} \times F_{T1} = \frac{515}{2} \times 625 = 160938$.

$N_2 \times F_{T1} = 514 \times 625 = 321250$.

$\Rightarrow$ $M = 173438$ mul/sec.

$A = 343750$ add/sec.

2.



$$y[n] = \frac{D}{I} x[n\frac{D}{I}] = \frac{1}{2} x[\frac{n}{2}]$$

avoid the problem of processing zero value data by the up-samplers and ignoring the filtered output by the down-samplers.

multistage: the subfilters are operating at the lowest frequencies in the system.

3. (a) $H_0(z) = a + bz^{-1}$, $H_1(z) = c + dz^{-1}$

$$X_0(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_0\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_0\left(\frac{w-2\pi}{2}\right)\right].$$

$$X_1(w) = \frac{1}{2}\left[X\left(\frac{w}{2}\right)H_1\left(\frac{w}{2}\right) + X\left(\frac{w-2\pi}{2}\right)H_1\left(\frac{w-2\pi}{2}\right)\right].$$

$$\begin{aligned}\hat{X}(w) &= X_0(2w)G_0(w) + X_1(2w)G_1(w)\\ &= \frac{1}{2}\left[X(w)H_0(w) + X(w-\pi)H_0(w-\pi)\right]G_0(w)\\ &\quad + \frac{1}{2}\left[X(w)H_1(w) + X(w-\pi)H_1(w-\pi)\right]G_1(w)\\ &= \frac{1}{2}\left[H_0(w)G_0(w) + H_1(w)G_1(w)\right]X(w)\\ &\quad + \frac{1}{2}\left[H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w)\right]X(w-\pi)\end{aligned}$$

$\Rightarrow$ $H_0(w-\pi)G_0(w) + H_1(w-\pi)G_1(w) = 0$

select $G_0(w) = H_1(w-\pi) \Rightarrow G_0(z) = H_1(-z)$.

$G_1(w) = -H_0(w-\pi) \Rightarrow G_1(z) = -H_0(-z)$.

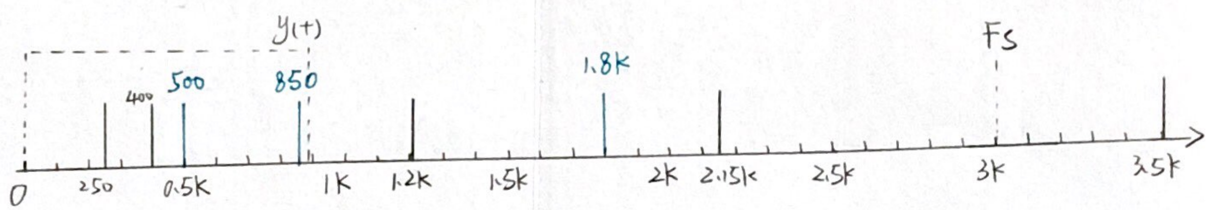
$H_1(w-\pi) = H_0(w) \Rightarrow H_1(-z) = H_0(z)$.

$\Rightarrow$ $H_0(-z)G_0(z) + H_1(-z)G_1(z) = 0$

$H_0(z) = a + bz^{-1}$, $G_0(z) = H_1(-z) = c - dz^{-1}$.

$H_1(z) = c + dz^{-1}$, $G_1(z) = -H_0(-z) = -a + bz^{-1}$.

(b)



frequency components in $y(t)$: 250Hz, 400Hz, 500Hz, 850Hz

$$G_p(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} G_a(\Omega - k\Omega_T)$$

4. (a) $x[n] - 1.6x[n-1] + 0.63x[n-2] = w[n] + 0.9w[n-1]$,

$$H(z) = \frac{1+0.9z^{-1}}{1-1.6z^{-1}+0.63z^{-2}} = \frac{z^2+0.9z}{z^2-1.6z+0.63}$$

(b) poles: $z^2-1.6z+0.63=0$. $p_1 = 0.7$, $p_2 = 0.9$

zeros: $z^2+0.9z=0$, $z_1=0$, $z_2=-0.9$.

all the poles are inside the unit circle, hence the system is stable.

(c) $$\Gamma_{xx}(z) = \sigma_w^2 H(z)H(z^{-1}) = \frac{(1+0.9z^{-1})(1+0.9z)}{(1-1.6z^{-1}+0.63z^{-2})(1-1.6z+0.63z^2)}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2012-2013

EE6401 – ADVANCED DIGITAL SIGNAL PROCESSING

November / December 2012

Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A decimation system has the following specifications:

    Pass band frequency $f_p = 10\text{Hz}$
    Stop band frequency $f_s = 12.5\text{Hz}$
    Pass band ripple $\delta_p = 0.1$
    Stop band ripple $\delta_s = 0.001$
    Sampling frequency $F = 2{,}500\text{Hz}$
    Decimation factor $D = 100.$

    Assume a finite impulse response (FIR) filter is used and aliasing is allowed in the transition band.

    (a) Find the number of filter coefficients needed by the one-stage design approach.

    (5 Marks)

Note: Question No. 1 continues on page 2.

(b) Find the total number of filter coefficients needed by the two-stage design approach. The results should be optimized for multiplication requirements between the options: D_1=25, D_2=4, and D_1=4, D_2=25, where D_1 and D_2 are the decimation factors used in the two-stage design approach. In your solution, the transitional bandwidth of the filter used at each stage should be clearly specified.

(10 Marks)

(c) Calculate the numbers of additions and multiplications per second needed for the two-stage design approach.

(5 Marks)

2. (a) Develop a time-domain expression for the output $y[n]$ as a function of input $x[n]$ according to the system structure shown in Figure 1(a).

(4 Marks)

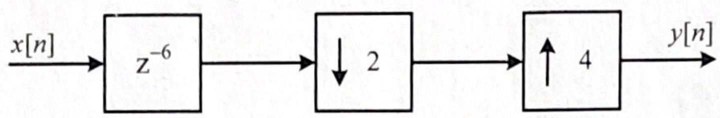

Figure 1(a)

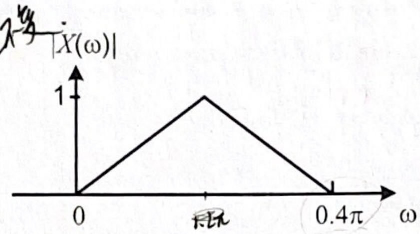

Figure 1(b)

(b) If $x[n]$ has an amplitude spectrum $|X(\omega)|$, as shown in Figure 1(b), plot the amplitude spectra at the output of the down-sampler and up-sampler.

(10 Marks)

(c) Discuss the principles that are used by the polyphase method and multi-stage method to minimize the computation complexity of the multi-rate systems.

(6 Marks)

3. (a) The transfer functions of the lowpass and highpass analysis filters of a two-channel quadrature mirror filter (QMF) bank are given by

$$H_0(z) = a + bz^{-1} \quad \text{and} \quad H_1(z) = c + dz^{-1}.$$

Determine the expression for the transfer functions of the lowpass and highpass synthesis filters $G_0(z)$ and $G_1(z)$, so that the two-channel QMF filter bank is a perfect reconstruction system.

(12 Marks)

(b) An analogue signal $x(t)$ contains sinusoidal components of frequencies 250 Hz, 400 Hz, 1.2 kHz, 2.15 kHz and 3.5 kHz. The signal $x(t)$ is sampled at a frequency of 3.0 kHz. The sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 875 Hz. The filter is a reconstruction filter used to generate an analogue signal $y(t)$. Find the frequency components present in the reconstructed signal $y(t)$.

(8 Marks)

4. Consider the Auto Regressive Moving Average (ARMA) process generated by the difference equation

$$x[n] = 1.6x[n-1] - 0.63x[n-2] + w[n] + 0.9w[n-1]$$

where $w[n]$ is the white noise with variance σ_w^2.

(a) Determine the system function of the whitening filter.

(8 Marks)

(b) Calculate the poles and the zeros of the system function. Is the system stable?

(4 Marks)

(c) Determine the power spectrum density of $x[n]$.

(8 Marks)

5. Consider the following periodic continuous-time signal with parametric representation given by

$$x(t) = \delta(t-1.2) + 1.5\delta(t-3.6), t \in [0,4].$$

(a) Is the signal $x(t)$ bandlimited?

(2 Marks)

(b) What is the rate of innovation of $x(t)$?

(2 Marks)

Note: Question No. 5 continues on page 4.

(c) What is the critical sampling rate to sample the signal $x(t)$?

(2 Marks)

(d) What is the sufficient number of samples for perfect reconstruction of $x(t)$?

(2 Marks)

(e) Find the annihilating filter such that the signal parameters can be recovered from the spectral values.

(12 Marks)

END OF PAPER