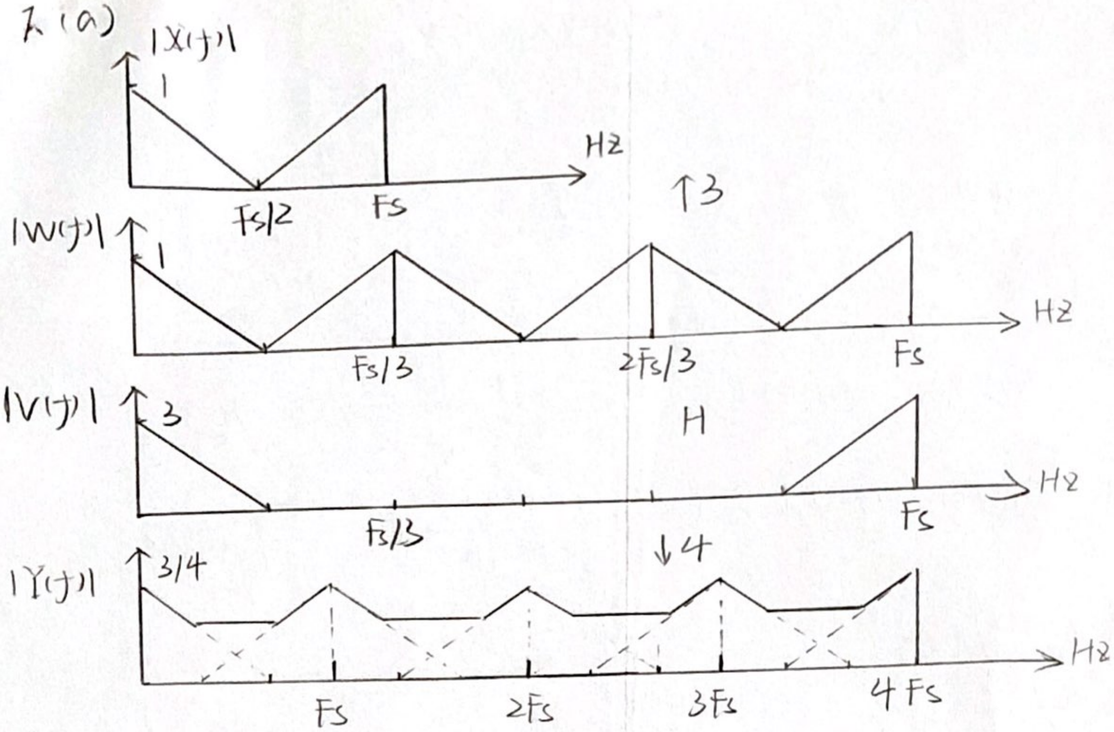
2. (a)



(b) polyphase structure:

the subfilters are operating at the lowest sampling frequencies in the whole system.

multi-stage implementation:

expand the trainsition bandwidth and reduce the order N and thus reduce the computational complexity.

avoid the problem of processing zero value data by the up-samplers and ignoring the filtered output by the down-samplers.

2. (a) $W_1(z) = X(z) - \frac{1}{3} z^{-1} W_1(z)$.

$$W_1(z) = \frac{X(z)}{1+\frac{1}{3}z^{-1}};$$

$W_2(z) = X(z) - \frac{1}{2} z^{-1} W_2(z)$,

$$W_2(z) = \frac{X(z)}{1+\frac{1}{2}z^{-1}};$$

$$W_3(z) = W_2(z) + 2z^{-1} W_2(z) = \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}} X(z)$$

$$Y(z) = W_3(z) + z^{-1} W_1(z) = \left(\frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}} + \frac{z^{-1}}{1+\frac{1}{3}z^{-1}}\right) X(z)$$

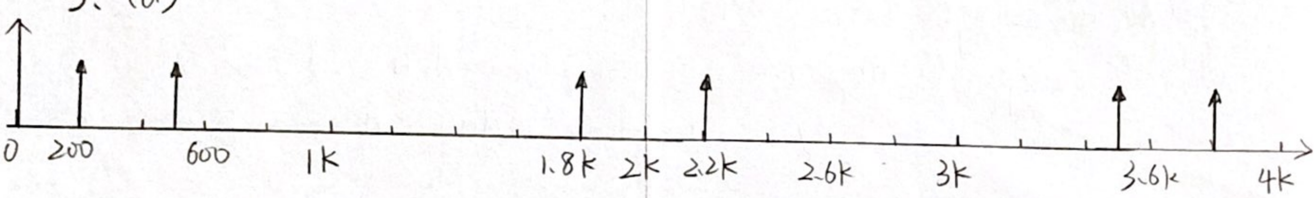
$$H(z) = \frac{Y(z)}{X(z)}$$

(b) $x_1[n] = x[\frac{n}{5}]$.

$x_2[n] = x_1[20n] = x[4n]$,

$y[n] = x_2[\frac{n}{4}] = x[n]$.

3. (a)



$x_a(t)$ can be recovered from $x[n]$ without any distortion if a cutoff frequency w_c between 2.2k ~ 3.8kHz is applied.

(b) Stage 1: $F_T = 48kHz$,

$F_1 = \frac{48k}{16} = 3kHz$, $F_s = F_1 - 1k = 2kHz$,

$F_p = 460 Hz$, $\Delta f = (F_s - F_p)/F_T = 0.03208$.

$$N_1 = \frac{-10\log_{10}(0.005 \times 0.001) - 13}{14.6 \times 0.03208} = 85.425 \approx 85$$

length $L_1 = N_1 + 1 = 86$

$\frac{N_1+1}{2} \times F_T = \frac{85+1}{2} \times 48k$ mul/sec

$N_1 \times F_T = 85 \times 48k$ add/sec

Stage 2: $F_T = F_1 = 3kHz$.

$F_s = \frac{F_T}{2D} = 500 Hz$

$F_p = 460 Hz$,

$\Delta f = (F_s - F_p)/F_T = \frac{40}{3k} = \frac{1}{75}$

$$N_2 = \frac{-10\log_{10}(0.005 \times 0.001) - 13}{14.6 \times \frac{1}{75}} \approx 206,$$

length $L_2 = N_2 + 1 = 207$.

$\frac{N_2+1}{2} \times F_T = \frac{206+1}{2} \times 3k$ mul/sec

$N \times F_T = 206 \times 3k$ add/sec

4. (a) $x[n] - 1.25\,x[n-1] + 1.25\,x[n-2] = w[n]$

(b) $$H(z) = \frac{X(z)}{W(z)} = \frac{1}{1 - 1.25z^{-1} + 1.25z^{-2}}$$

(c) $$\Gamma_{xx}(z) = H(z)H(z^{-1})\sigma_w^2 = \frac{0.81}{(1-1.25z^{-1}+1.25z^{-2})(1-1.25z+1.25z^{2})}$$

(d)
$$r_{xx}[m] = \begin{cases} -1.25\,r_{xx}[m-1] + 1.25\,r_{xx}[m-2], & m>0 \\ -1.25\,r_{xx}[m-1] + 1.25\,r_{xx}[m-2] + \sigma_w^2, & m=0 \\ r_{xx}[-m], & m<0 \end{cases}$$

$r_{xx}[0] = -1.25\,r_{xx}[1] + 1.25\,r_{xx}[2] + \sigma_w^2$

$r_{xx}[1] = -1.25\,r_{xx}[0] + 1.25\,r_{xx}[1]$, $1.25\,r_{xx}[0] = 0.25\,r_{xx}[1]$, $r_{xx}[1] = 5\,r_{xx}[0]$

$r_{xx}[2] = -1.25\,r_{xx}[1] + 1.25\,r_{xx}[0] = -5\,r_{xx}[0]$.

$r_{xx}[0] = -6.25\,r_{xx}[0] - 6.25\,r_{xx}[0] + 0.81$, $13.5\,r_{xx}[0] = 0.81$, $r_{xx}[0] = 0.06$

5. (a) Two basic process:

① A filtering process designed to produce an output in response to a sequence of input data.

② An adaptive process designed to adjust the set of coefficients used in the filtering process.

(b) $e[n] = y[n] - \hat{w}_2[n]$, $y[n] = x[n] + w_1[n] + w_2[n]$,

$\hat{w}_2[n] = \sum_{k=0}^{M-1} h[k]\, v[n-k]$, $v[n] = v_2[n] + w_3[n]$,

$v_2[n] = \sum_{l=0}^{\infty} h[l]\, w_2[n-l]$, normal equation:

$$r_{vy}[l] = \sum_{k=0}^{M-1} h[k]\, r_{vv}[l-k]$$

$$\Gamma_M h_M = r_y \,,\quad h_{opt} = \Gamma_M^{-1} r_y$$

$$\mathcal{E}_M = \sigma_d^2 - \sum_{k=0}^{M-1} h_{opt}[n]\, r_{vy}^*[l-n]$$

$$h_M[n+1] = h_M[n] + \frac{1}{2}\Delta[n]\, S[n]$$

(c) $h_M[n+1] = h_M[n] + \mu e[n] X_M^*[n]$

$$
\begin{aligned}
E[h_M[n+1]] &= E[\, h_M[n] + \mu e[n] X_M^*[n]\,] \\
&= E[h_M[n]] + \mu E[e[n] X_M^*[n]] \\
&= \bar{h}_M[n] + \mu(r_y - \Gamma_M \bar{h}_M[n]) \\
&= (I - \mu \Gamma_M)\bar{h}_M[n] + \mu r_y
\end{aligned}
$$

$\Gamma_M = U \Lambda U^H$, $\bar{h}^{\circ}(n,k) = C(1-\mu\lambda_k)^n u[n]$

$$|1-\mu\lambda_k| < 1 \,,\quad 0 < \mu < \frac{2}{\lambda_k}$$

$$\Rightarrow \bar{h}^{\circ}(n,k) = C(1-\frac{\lambda_{min}}{\lambda_{max}})^n u[n]$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2013-2014

EE6401 – ADVANCED DIGITAL SIGNAL PROCESSING

November / December 2013 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.

2. Answer ALL questions.

3. All questions carry equal marks.

4. This is a closed-book examination.

5. Unless specifically stated, all symbols have their usual meanings.

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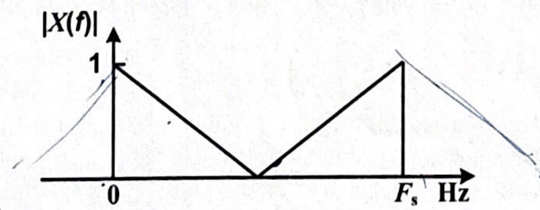
1. (a) Figure 1 shows a sampling rate converter with a decimation factor D = 4, interpolation factor L = 3, and the spectrum, $|X(f)|$, of the input signal $x(n)$. With reasonable assumptions on the specification of filter H, sketch the spectra of the signals at the points W, V and $y(m)$ indicated in Figure 1.



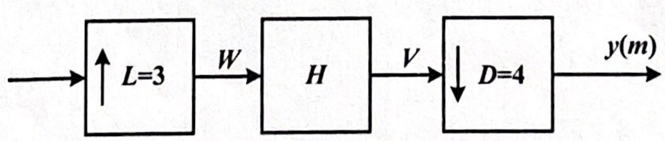


Figure 1. Block diagram of a sampling rate conversion system.

(12 Marks)

Note: Question No. 1 continues on page 2.

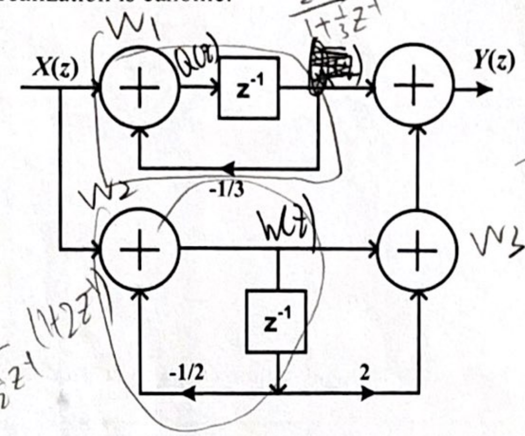
multi-stage
polyphase

(b) It is necessary to minimize the computational complexity of a multirate system. Discuss the principles of two main methods you have learnt, and state the complexity reduction of each method.

(8 Marks)

2. (a) Determine the transfer function of the system shown in Figure 2 and decide if this system realization is canonic.

$\frac{z^{-1}}{1+\frac{1}{3}z^{-1}}$



$\frac{z^{-1}}{1+\frac{1}{3}z^{-1}} + \frac{1+2z^{-1}}{1+\frac{1}{2}z^{-1}}$

$\frac{1}{1+\frac{1}{2}z^{-1}}(1+2z^{-1})$

Figure 2. Block diagram of an IIR filter.

(12 Marks)

? (b) Assume $x(n)$ is the input sequence of a system that contains a CASCADE of an up-sampler by a factor of 5, a down-sampler by a factor of 20 and an up-sampler by a factor of 4. Find the output, $y(n)$, of this system.

(8 Marks)

3. (a) An analog signal, $x_a(t)$, is sampled at a sampling frequency of 4.0 kHz to obtain a discrete-time signal $x(n)$. This analog signal contains sinusoidal waves with frequencies 200 Hz, 500 Hz, 1.8 kHz, and 2.2 kHz that have the same non-zero amplitude. Plot the spectrum of $x(n)$ and comment if $x_a(t)$ can be recovered from $x(n)$ without any distortion.

(8 Marks)

Note: Question No. 3 continues on page 3.

(b) The sampling frequency of a digital signal $x(n)$ is to be reduced by decimation, from 48 kHz to 1 kHz. The highest frequency of the output signal after decimation is 460 Hz. Assume that an FIR filter is to be used, with an overall passband ripple, $\delta_p = 0.01$, and stopband ripple, $\delta_s = 0.001$. Distortion in the transitional band is not allowed in this design. Based on the above given information,

(i) suggest a suitable stopband frequency and design an efficient decimator with the structures shown in Figure 3,

(ii) calculate the computational complexity in terms of the number of additions and multiplications per second.

$F_S = \frac{F_T}{2D}$

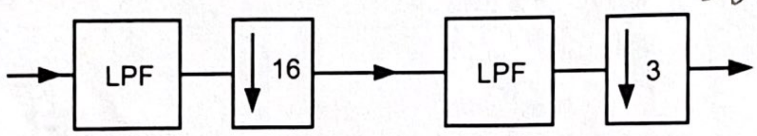


Figure 3. Block diagram of a two-stage decimator.

(12 Marks)

4. Consider a system whose input is white noise with variance $\sigma_w = 0.9$ and output is an Auto-Regressive AR random process $x(n)$ with mean $\mu_x = 0$ and variance $\sigma_x = 0.2$ and which can be characterized by the prediction coefficients $a_2(1) = -1.25$ and $a_2(2) = 1.25$.

(a) Determine the difference equation for the system.

(4 Marks)

(b) Determine the system function.

(4 Marks)

(c) Determine the power spectral density of the random process $x(n)$.

(4 Marks)

(d) Determine the auto-correlation function of the random process $x(n)$.

(8 Marks)

5. Consider using an adaptive filtering algorithm to implement a dual input, single weight adaptive noise canceller illustrated in Figure 4, where the additive noise sequences $w_1(n), w_2(n), w_3(n)$ are assumed to be mutually uncorrelated and zero mean.

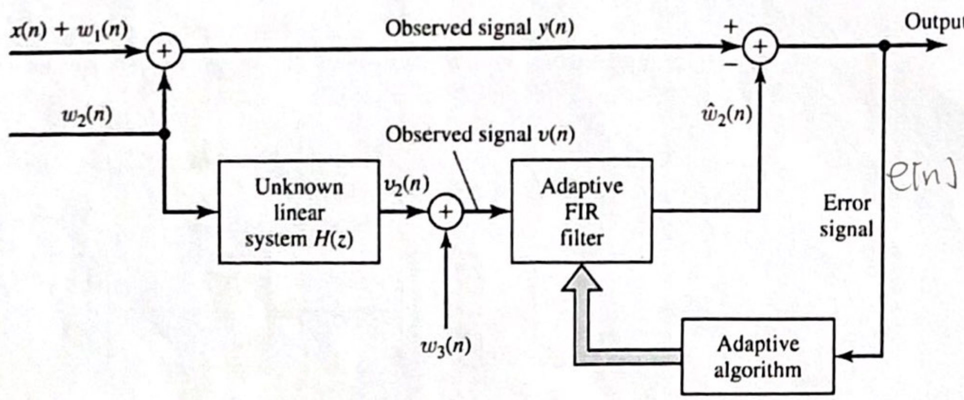


Figure 4. An adaptive noise cancelling system.

(a) Describe the two basic processes in a linear adaptive filtering algorithm.
(6 Marks)

(b) Derive the equations that define the adaptive noise cancelling system given in Figure 4.
(10 Marks)

(c) Consider a white noise sequence of zero mean and variance σ^2 as the input to the least-mean-square (LMS) algorithm, what is the condition for convergence in the mean-square sense?
(4 Marks)

END OF PAPER