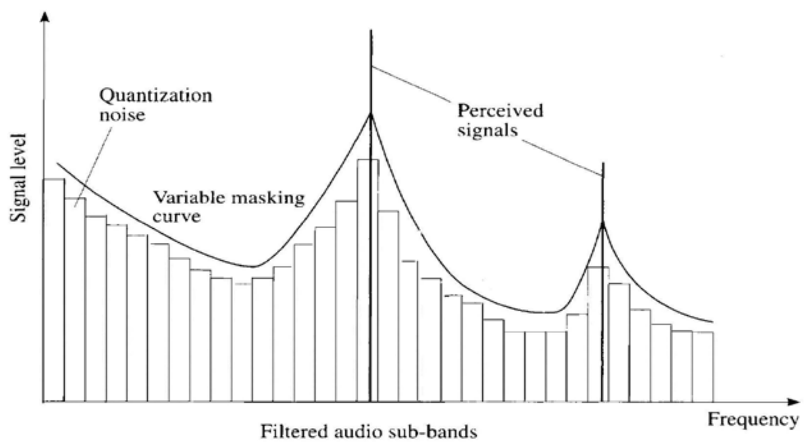
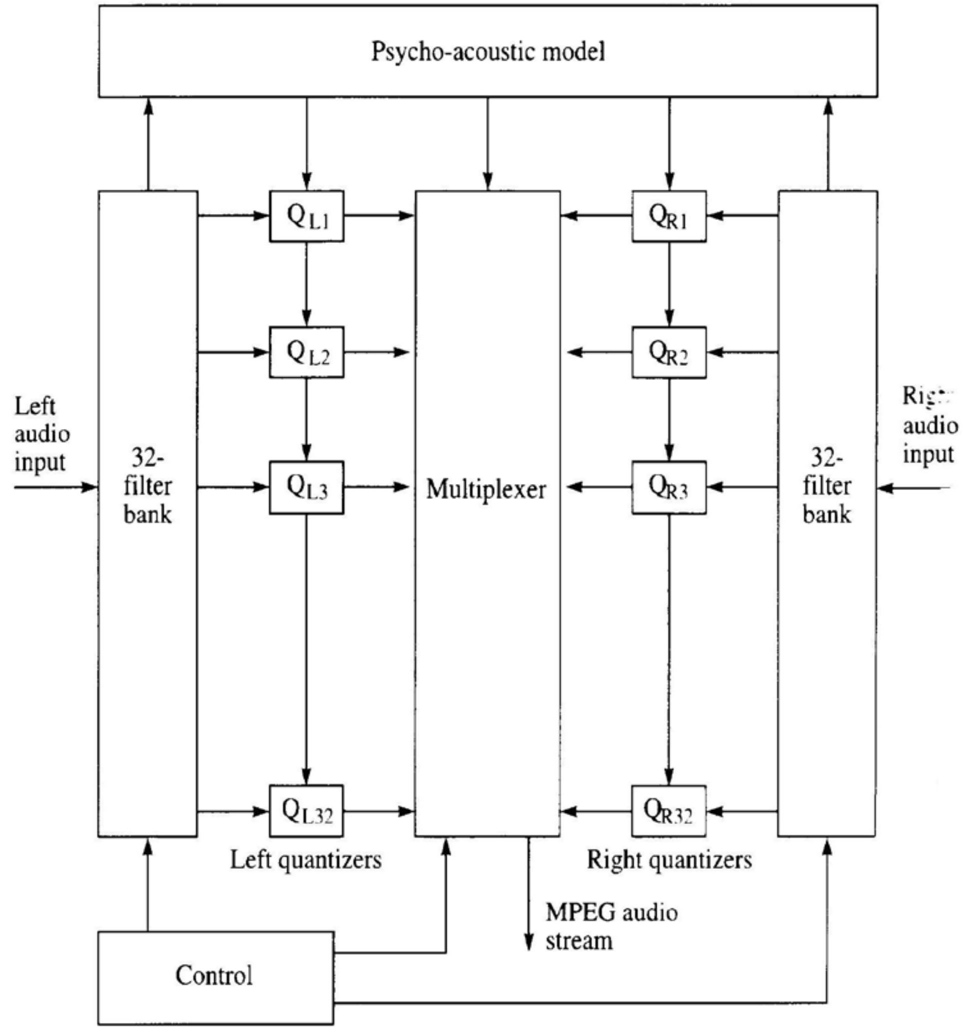
Applications

- Filter Banks
- Quadrature Mirror Filters and Implementation
- Subband Coding of Speech Signals

Why Filter Banks?

- Different characteristics are for different sub-bands (see figure below)
- Using suitable processing techniques (**i.e., code with** no. of bits) for different sub-bands
- See the encoder for 32 sub-bands



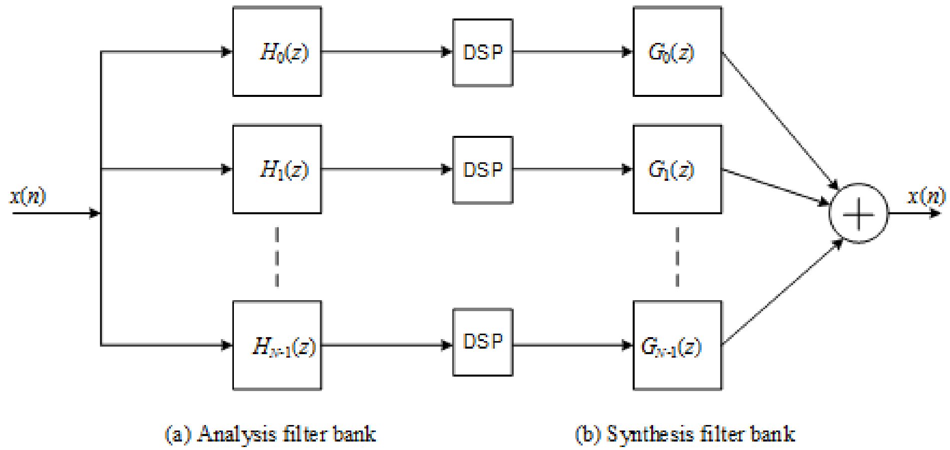


MPEG audio encoder

Digital Filter Banks

- An analysis filter bank consists of a set of parallel filters with system function $\{H_i(k)\}$ (Figure 9.1(a)) to split the signal into a number of subbands.

Figure 9.1 A digital filter bank

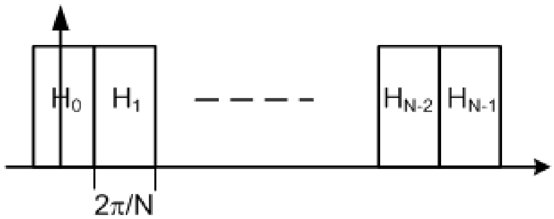


- A synthesis filter bank consists of a set of parallel filters with system function $\{G_i(k)\}$ (Figure 9.1(b))
- It is important to find an optimum method to divide the signal into different subbands so that they can be combined to restore the original signal **with distortions as small as** possible.
- One important issue is to minimize the effects of **non-ideal transitional bands** of filters.

Digital Filter Banks

- These filter banks are used for performing spectrum analysis and signal synthesis.

- The frequency response characteristics of filter k can be derived from



$$H_k(\omega) = H_0(\omega - \frac{2\pi k}{N}), \qquad k = 0,1,\ldots,N-1 \tag{9.1}$$

where $H_0(\omega)$ is the spectrum of prototype filter required by the applications.

- The time domain impulse response of these filters are expressed by

$$h_k(n) = h_0(n)e^{j2\pi nk/N}, \qquad k = 0,1,\ldots,N-1 \tag{9.2}$$

where $h_0(n)$ is the impulse response of prototype filter required by the applications

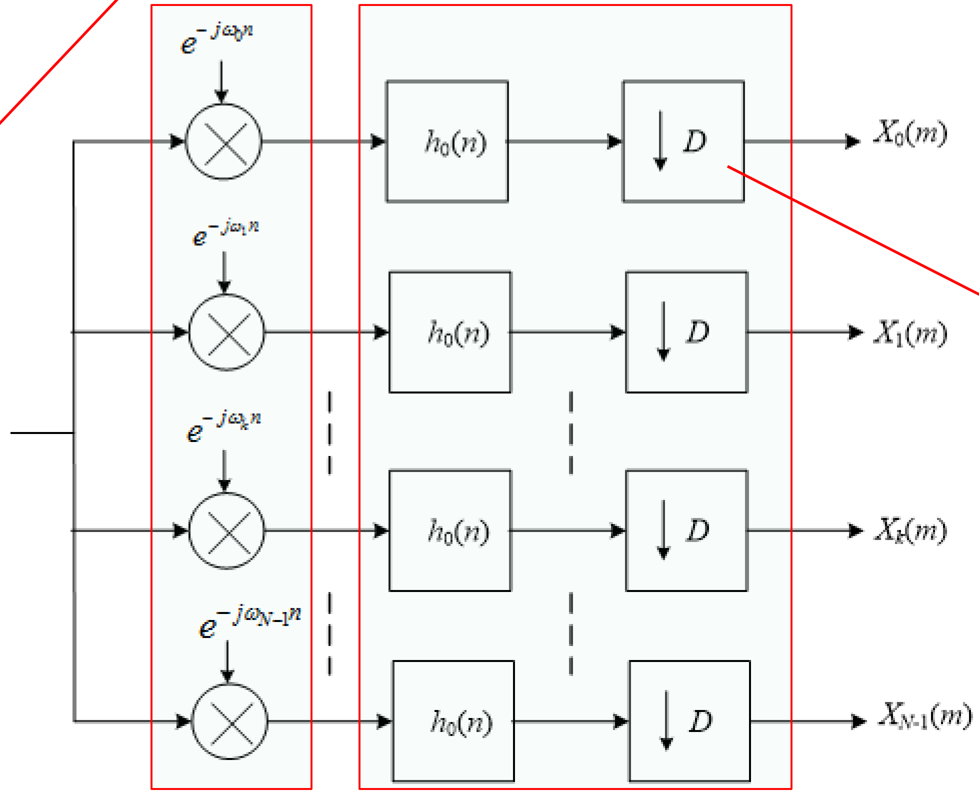
Digital Filter Banks

- The output of the lowpass filter is relatively narrow in bandwidth, the signal is decimated by a factor of $D \le N$, which is expressed by

$$X_k(m) = \sum_n h_0(mD-n)\boxed{e^{-j2\pi nk/N} x(n),} \qquad k = 0,1,\ldots,N-1; \quad m = 0,1\ldots$$

where $\{X_k(m)\}$ are the DFT of kth band ouput.

Filter input



Reducing the sampling rate

Figure 9.2 (a) Analysis part of a uniform DFT filter bank

Digital Filter Banks

- The synthesis process can be expressed into

$$v(n)=\frac{1}{N}\sum_{k=0}^{N-1} e^{j2\pi nk/N}\left[\sum_{m} Y_k(m)g_0(n-mD)\right]$$

$$=\sum_{m} g_0(n-mD)\left[\frac{1}{N}\sum_{k=0}^{N-1} Y_k(m)e^{j2\pi nk/N}\right]=\sum_{m} g_0(n-mD)y_n(m)$$

Increasing the sampling rate

Shift the baseband output to be bandpass output

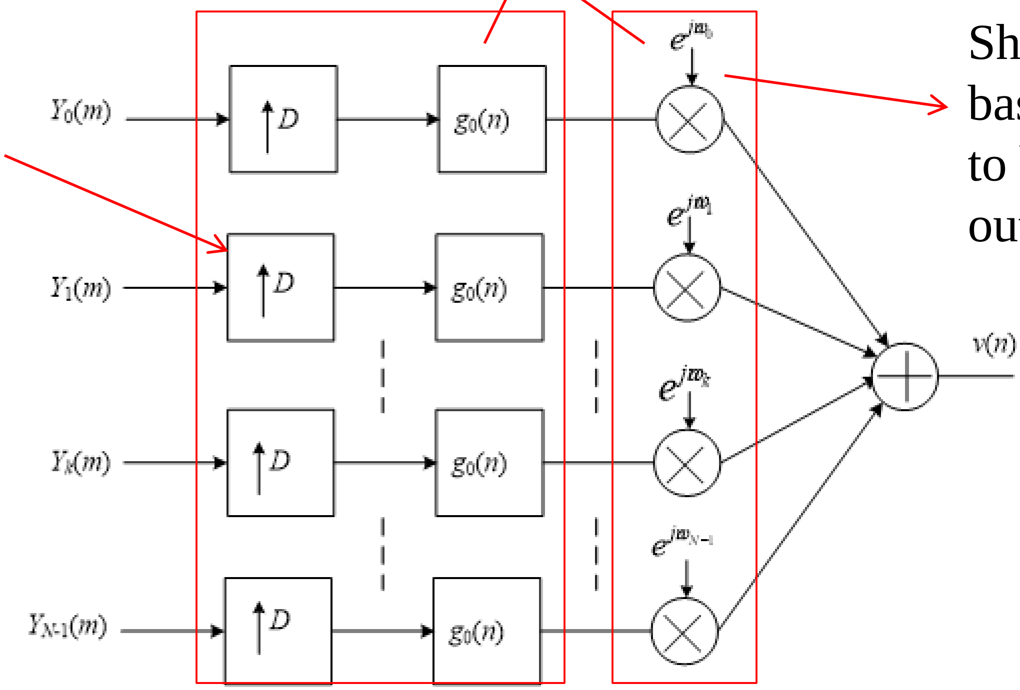
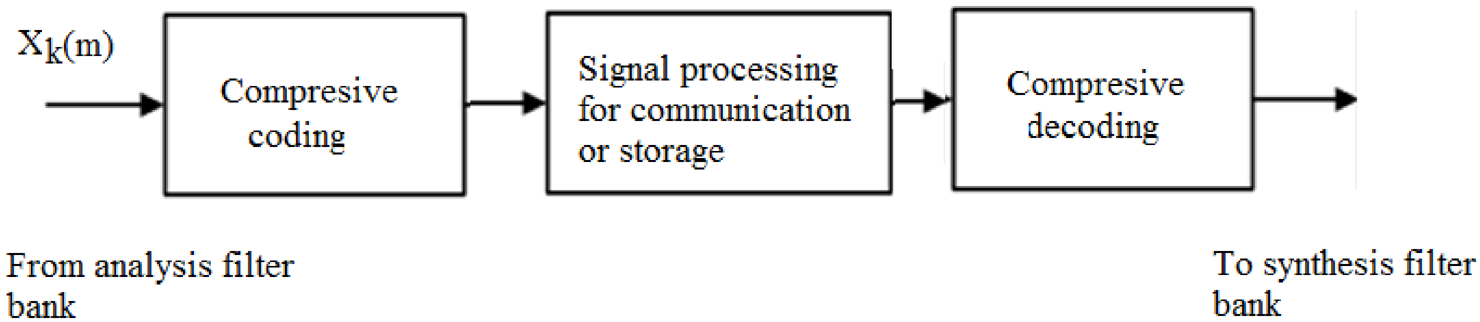


Figure 9.2 (b) Synthesis part of a uniform DFT filter bank, where $\omega_k = 2\pi k/N$

Digital Filter Banks

- In practical application, the outputs of these branches from the analysis process are further processed according to the specific requirement of the applications, for example audio or image coding/decoding and communications.



- You will learn in the subject of audio, image processing and digital communications. Some are done in your handphone and computer.
- The concept of **critically sampling** is that the output sampling frequency is the same as 2 times of the signal bandwidth.
- The filter bank is known as **oversampled** if the output sampling frequency of the filter bank is larger than 2 times of the signal bandwidth.

Digital Filter Banks

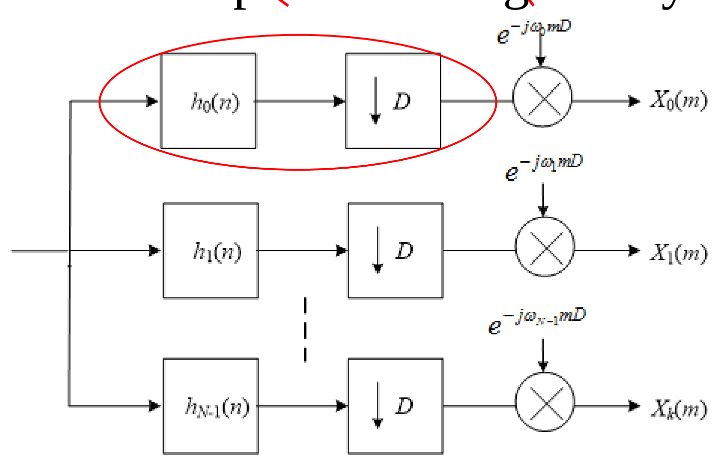
- In the above equation $1/N$ is a normalization factor, $\{y_n(m)\}$ represents samples of the inverse DFT sequence corresponding to $\{Y_k(m)\}$, $\{g_0(n)\}$ is the impulse response of the interpolation filter.
- $\{Y_k(m)\}$ is often the modified version of $\{X_k(m)\}$ in many applications
- **These arrangements in previous pages are not computational efficient**
- An alternative realization of the analysis filter bank is described by

$$X_k(m) = \left[\sum_n x(n)h_0(mD-n)e^{j2\pi k(mD-n)/N} \right] e^{-j2\pi kmD/N} \quad (9.3)$$

where the impulse response of the bandpass filter is given by

$$h_k(n) = h_0(n)e^{j2\pi kn/N},$$
$$k = 0,1,\ldots,N-1$$

- Compared to the structure in Fig. 9.2 (a). here decimation before frequency shifting

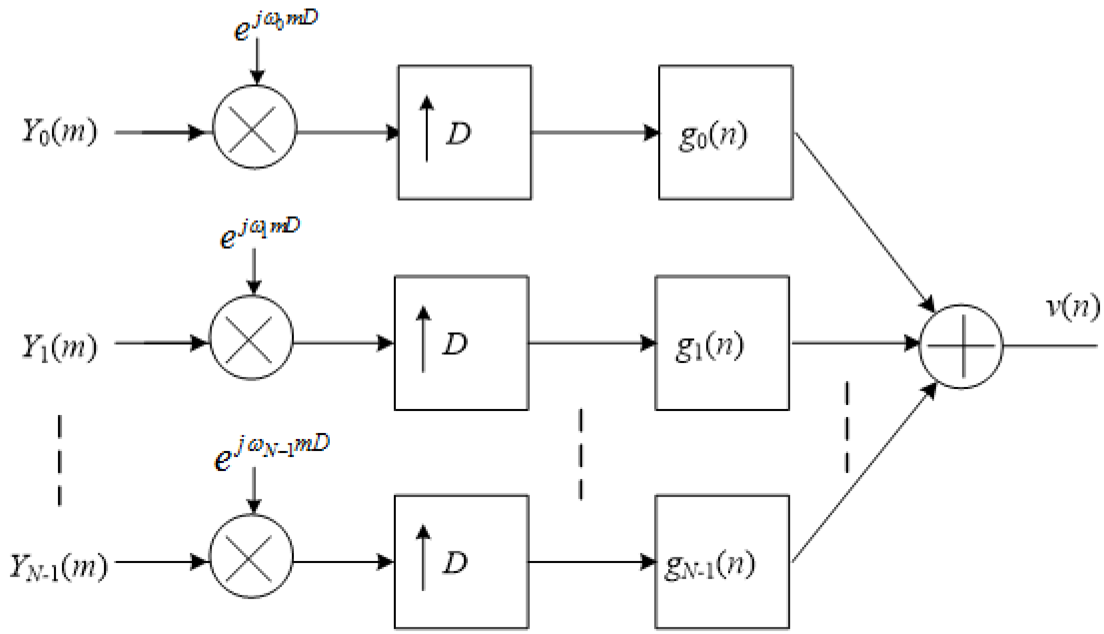


(a) Analysis

Digital Filter Banks

- Similarly, for synthesizing, we have

- Compared to the structure in Fig. 9.2(b), here frequency shifting before interpolation



(b) Synthesis

- The output is

$$v(n) = \frac{1}{N} \sum_{k=0}^{N-1} \left\{ \sum_{m} [Y_k(m) e^{j2\pi kmD/N}] g_k(n - mD) \right\}$$

$$g_k(n) = g_0(n) e^{j2\pi nk/N}$$

Digital Filter Banks

- For analysis bank filter, the ***critically sampled*** filter bank is achieved by $N = D$ polyphase filters with impulse response

$$p_k(n) = h_0(nN - k), \qquad k = 0,1,\ldots,N-1 \tag{9.4}$$

- The corresponding set of decimated input sequences are

$$x_k(n) = x(nN + k), \qquad k = 0,1,\ldots,N-1 \tag{9.5}$$

- Putting (9.4) and (9.5) into (9.3), we have

$$X_k(m) = \left[\sum_n x(n)h_0(mD-n)e^{j2\pi k(mD-n)/N}\right]e^{-j2\pi kmD/N} \tag{9.3}$$

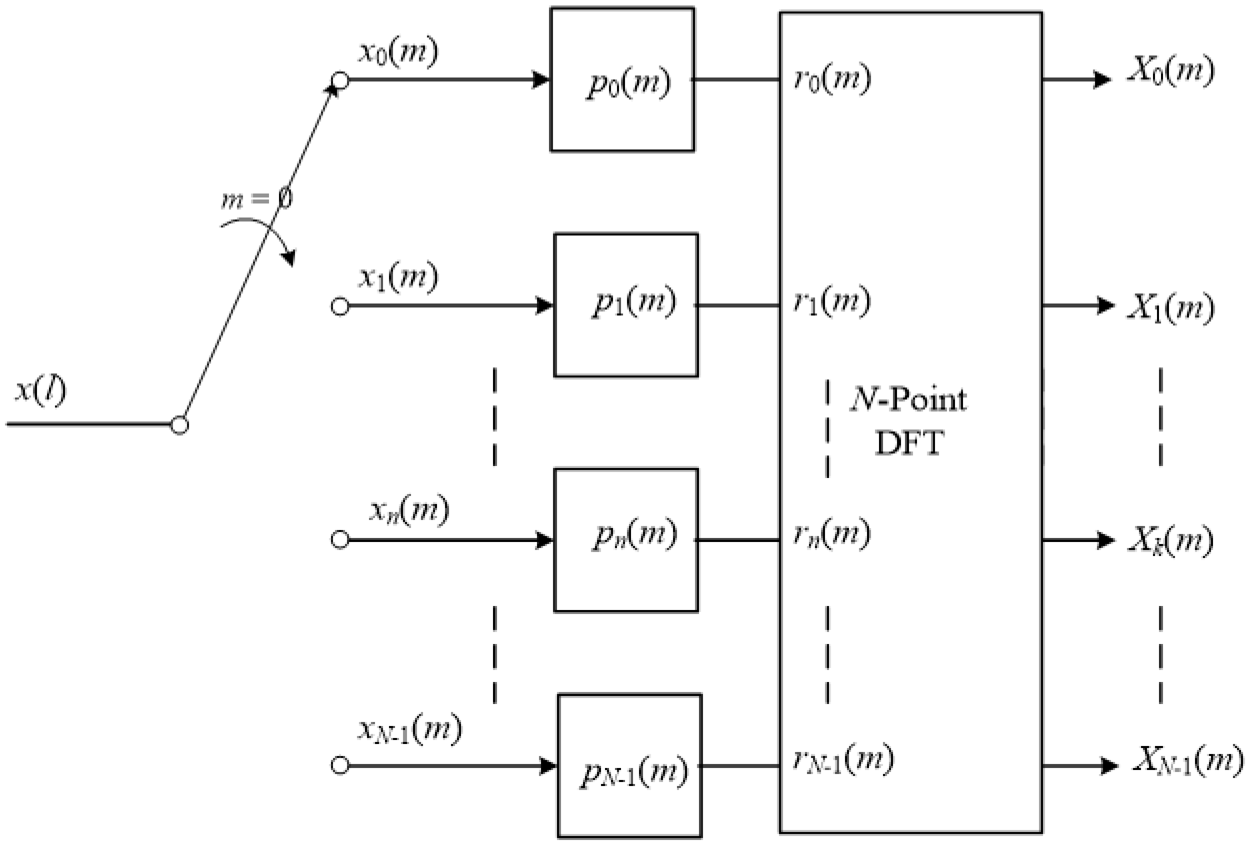
$$X_k(m) = \sum_{n=0}^{N-1}\left[\sum_l p_n(l)x_n(m-l)\right]e^{-j2\pi nk/N}, \qquad k = 0,1,\ldots,D-1 \tag{9.6}$$

- The inner sum is the convolution of $\{p_n(l)\}$ with $x_n(l)\}$ and the outer sum is the N-point DFT of the filter outputs.

Digital Filter Banks

- The filter structure is in the figure below.

$$X_k(m) = \sum_{n=0}^{N-1} \left[\sum_{l} p_n(l) x_n(m-l) \right] e^{-j2\pi nk/N}$$

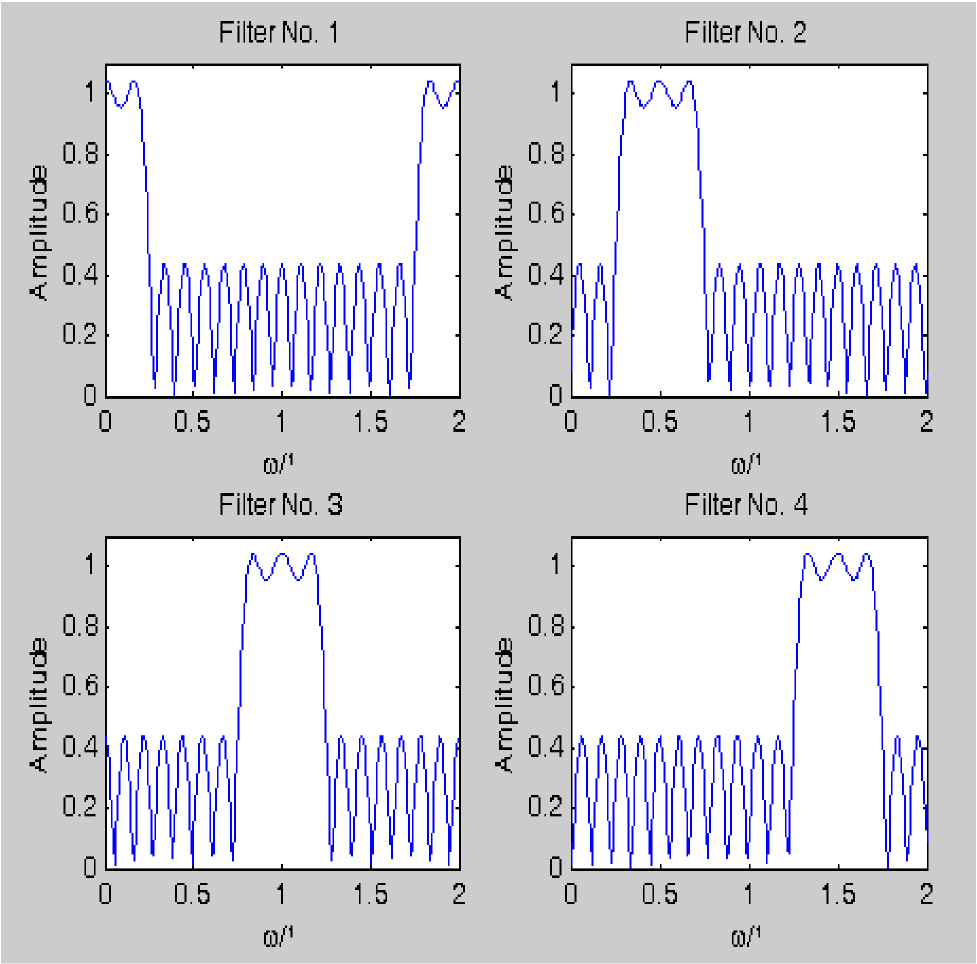


Digital filter bank structure for the computation of equation (9.6)

Digital Filter Banks

Example: MATLAB CODE

```
% Design the prototype LPF
b = remez(20, [0 0.2 0.25 1], [1 1 0 0], [10 1]);
% b contains the filter coefficients
w = 0:2*pi/255:2*pi;
n = 0:20;
for k = 1:4;
c = exp(2*pi*(k-1)*n*i/4) ;
% c contains the transform matrix
FB = b.*c;
% FB contains the DFT outputs
HB(k, :) = freqz(FB, 1, w);
end
```



Responses of polyphase filters for $D = 4$

Digital Filter Banks

- For synthesis bank filter, N polyphase filters are used for interpolation
- The corresponding set of output signals is

$$q_k(n) = g_0(nN+k), \qquad k = 0,1,\ldots,N-1$$

- The output of the lth polyphase filter becomes

$$v_k(n) = v(nN+k), \qquad k = 0,1,\ldots,N-1 \tag{9.7}$$

- The term in bracket is the N-point IDFT of $\{Y_k(m)\}$, which is denoted as $\{y_l(m)\}$, hence

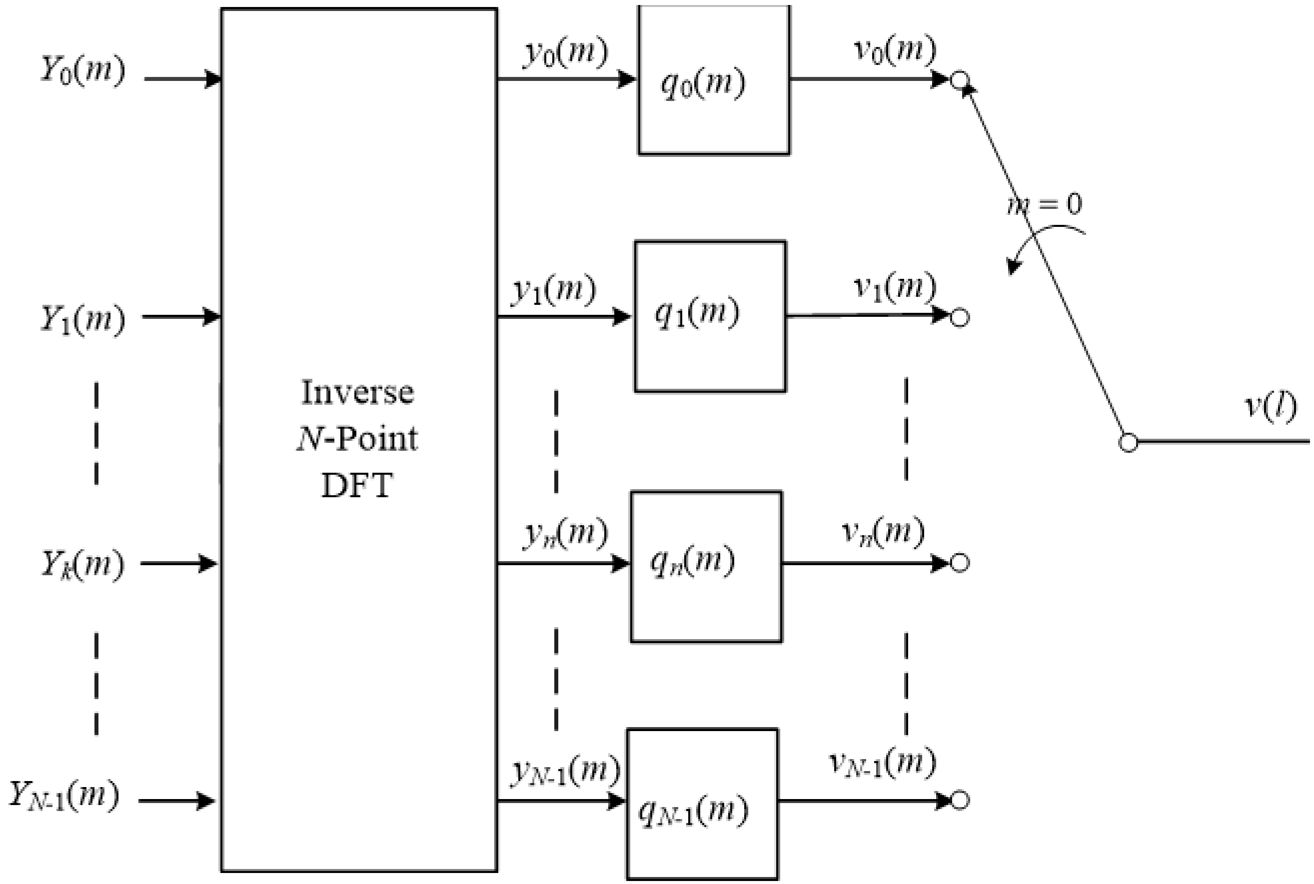
$$v_l(n) = \sum_m q_l(n-m)\left[\frac{1}{N}\sum_{k=0}^{N-1} Y_k(m)e^{j2\pi kl/N}\right], \qquad l = 0,1,\ldots,D-1 \tag{9.8}$$

$$v_l(n) = \sum_m q_l(n-m)y_l(m)$$

Digital Filter Banks

- The structure is shown in the figure below

$$v_l(n) = \sum_m q_l(n-m)\left[\frac{1}{N}\sum_{k=0}^{N-1} Y_k(m)e^{j2\pi kl/N}\right]$$

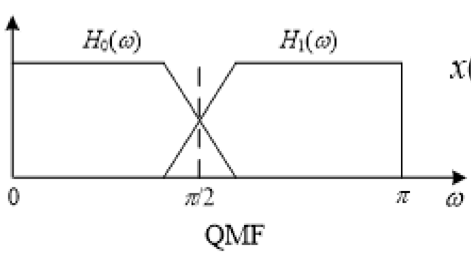


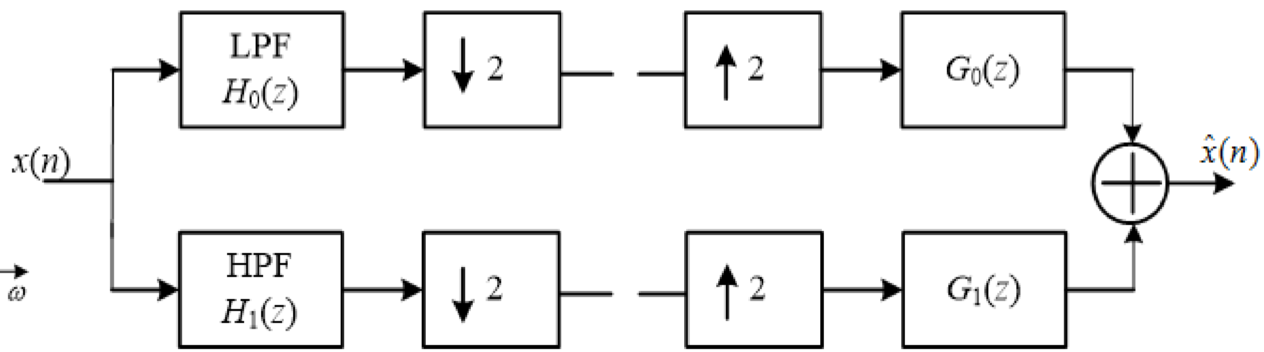
Digital filter bank structure for the computation of equation (9.8)

Quadrature Mirror Filters

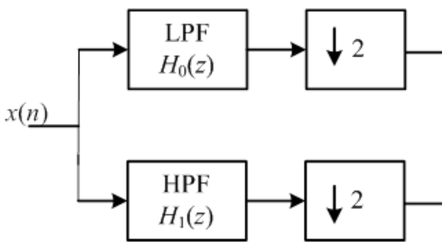
Quadrature Mirror Filters

- Design of multirate filters is particularly important in achieving good performance. Aliasing and imaging resulting from decimation and interpolation process must be negligible.
- A practical solution to the aliasing problem is to use *quadrature mirror filters* (QMF) (Figure below). The basic building block is the two channel QMF bank and the frequency response of the filters are in the figure below.
- The lowpass and highpass filters in the analysis section have impulse response $h_0(n)$ and $h_1(n)$, respectively.
- Similarly, the lowpass and highpass filters in the synthesis section have impulse response $g_0(n)$ and $g_1(n)$.





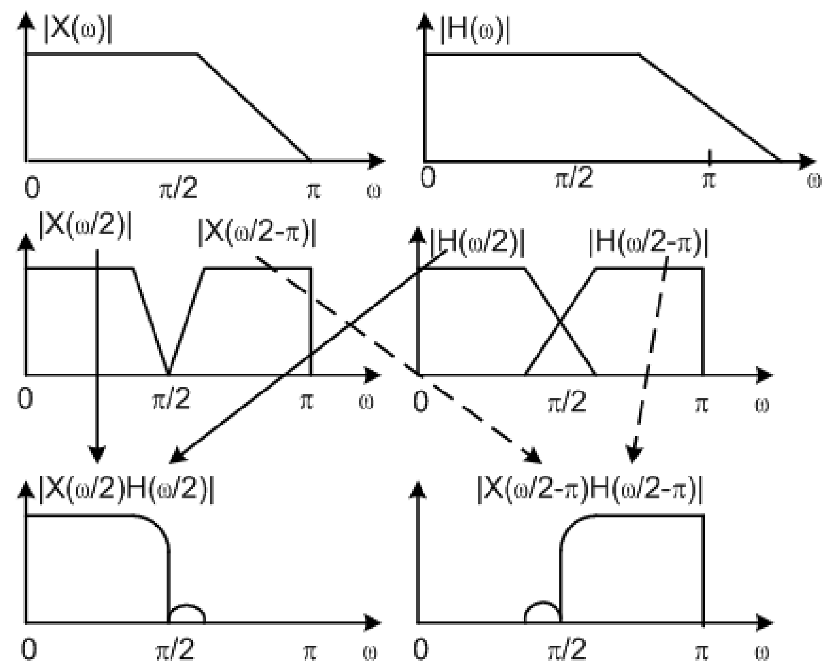
- The Fourier transforms of the signals at the outputs of two decimators are :



$$X_{a0}(\omega)=\frac{1}{2}\left[X\left(\frac{\omega}{2}\right)H_0\left(\frac{\omega}{2}\right)+X\left(\frac{\omega-2\pi}{2}\right)H_0\left(\frac{\omega-2\pi}{2}\right)\right]$$

$$X_{a1}(\omega)=\frac{1}{2}\left[X\left(\frac{\omega}{2}\right)H_1\left(\frac{\omega}{2}\right)+X\left(\frac{\omega-2\pi}{2}\right)H_1\left(\frac{\omega-2\pi}{2}\right)\right] \qquad (10.1)$$

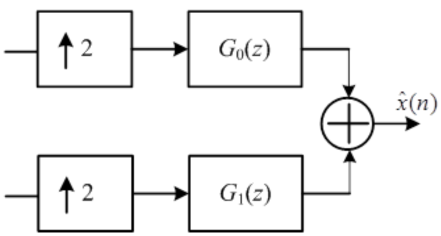
- Note that in the output of each branch, there is some aliasing components, which is the second term of the outputs.
- This is due to the consequence of filter design which has to use a non-zero transitional band.



- If $X_{s0}(\omega)$ and $X_{s1}(\omega)$ represent two inputs to the synthesis section, the output is simply

$$\hat{X}(\omega) = X_{s_0}(2\omega)G_0(\omega) + X_{s_1}(2\omega)G_1(\omega)$$

- Let $X_{a0}(\omega)=X_{s0}(\omega)$ and $X_{a1}(\omega)=X_{s1}(\omega)$. By using (10.1), we have



Synthesis Section

$$\hat{X}(\omega) = \frac{1}{2}[H_0(\omega)G_0(\omega) + H_1(\omega)G_1(\omega)]X(\omega)$$
$$+\frac{1}{2}[H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega)]X(\omega-\pi)$$

- The first term is the desired output from the QMF bank. The second term represents the effect of aliasing. To eliminate the aliasing, we need

$$H_0(\omega-\pi)G_0(\omega) + H_1(\omega-\pi)G_1(\omega) = 0$$

Quadrature Mirror Filters

- The condition can be simply satisfied by selecting $G_0(\omega)$ and $G_1(\omega)$ as

$$G_0(\omega) = H_1(\omega - \pi) \qquad G_1(\omega) = -H_0(\omega - \pi) \tag{10.5}$$

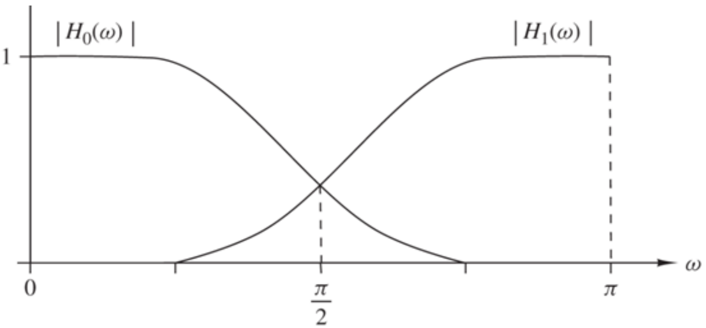
- If assuming that $H_0(\omega)$ is a lowpass filter and $H_1(\omega)$ is a mirror image highpass filter, we may express $H_0(\omega)$ and $H_1(\omega)$ as:

$$H_0(\omega) = H(\omega) \qquad H_1(\omega) = H(\omega - \pi)$$

where $H(\omega)$ is the frequency response of a low pass filter. In the time domain, the corresponding relations are:

$$h_0(n) = h(n) \qquad h_1(n) = (-1)^n h(n) \tag{10.6}$$

- As a consequence, $H_0(\omega)$ and $H_1(\omega)$ have mirror-image symmetry about the frequency $\omega = \pi/2$, as shown in the Figure.



Mirror image characteristics of the analysis filters $H_0(\omega)$ and $H_1(\omega)$.

Quadrature Mirror Filters

- To be consistent with the constraint (10.5), we select the lowpass filter $G_0(\omega)$ and $G_1(\omega)$ as

$$G_0(\omega) = 2H(\omega) \qquad\qquad G_1(\omega) = -2H(\omega - \pi)$$

- In the time domain, the relations become

$$g_0(n) = 2h(n) \qquad\qquad g_1(n) = -2(-1)^n h(n)$$

- The scaling factor 2 corresponds to the interpolation factor that used to normalize the overall frequency response of the QMF.
  - With the above arrangements, the component due to aliasing vanishes.
  - Thus the aliasing resulting from decimation in the analysis section is ***perfectly canceled*** by the image signal spectrum that arise due to interpolation.
  - The two channel QMF behaves as a linear, time-invariant system.

$$X_{a0}(\omega)=\frac{1}{2}\left[X\left(\frac{\omega}{2}\right)H_0\left(\frac{\omega}{2}\right)+X\left(\frac{\omega-2\pi}{2}\right)H_0\left(\frac{\omega-2\pi}{2}\right)\right]$$

$$X_{a1}(\omega)=\frac{1}{2}\left[X\left(\frac{\omega}{2}\right)H_1\left(\frac{\omega}{2}\right)+X\left(\frac{\omega-2\pi}{2}\right)H_1\left(\frac{\omega-2\pi}{2}\right)\right]$$

Quadrature Mirror Filters

- If we put $H_0(\omega)$, $H_1(\omega)$, $G_0(\omega)$ and $G_1(\omega)$ into the first term of (10.1)

$$\hat{X}(\omega)=[H^2(\omega)-H^2(\omega-\pi)]X(\omega)$$

where $H(\omega)$ is a frequency response of a lowpass filter and ideally we need

$$|H^2(\omega)-H^2(\omega-\pi)|=1 \quad \text{for all } \omega$$

- If $H(\omega)$ has a linear phase, it can be expressed by

$$H(\omega)=H_r(\omega)e^{-j\omega(N-1)/2}$$

where N is the filter length, then

$$H^2(\omega)=H_r^2(\omega)e^{-j\omega(N-1)}=|H_r(\omega)|^2 e^{-j\omega(N-1)}$$

and

$$H^2(\omega-\pi)=H_r^2(\omega-\pi)e^{-j(\omega-\pi)(N-1)}=(-1)^{N-1}|H_r(\omega-\pi)|^2 e^{-j\omega(N-1)}$$

Quadrature Mirror Filters

- Therefore the overall transfer function of the two-channel QMF which uses linear phase FIR filter is

$$\frac{\hat{X}(\omega)}{X(\omega)} = \left[|H(\omega)|^2 - (-1)^{N-1} |H(\omega-\pi)|^2 \right] e^{-j\omega(N-1)} = A(\omega) e^{-j\omega(N-1)}$$

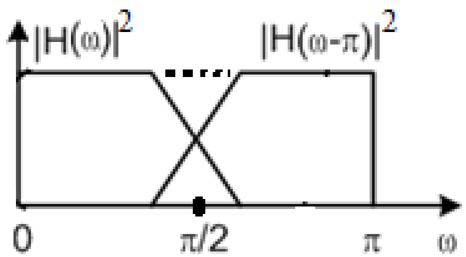
- It has a delay of N-1 samples and a magnitude characteristic

$$A(\omega) = |H(\omega)|^2 - (-1)^{N-1} |H(\omega-\pi)|^2$$

- When N is odd, $A(\pi/2) = 0$, which is an undesirable property for a QMF design. When N is even,

$$A(\omega) = |H(\omega)|^2 + |H(\omega-\pi)|^2$$

which avoids the problem of a zero at $\omega = \pi/2$



Quadrature Mirror Filters

- The condition $A(\omega) = 1$ cannot be met because any non-trivial linear-phase FIR filter $H(\omega)$ will introduce some amplitude distortion.
- The amount of distortion can be minimized by optimize the FIR filter coefficients, for example, one minimization method defines

$$J = w\int_{\omega_s}^{\pi} |H(\omega)|^2 \, d\omega + (1-w)\int_0^{\pi/2} [A(\omega)-1]^2 \, d\omega$$

where $0 < w < 1$, is a weighting factor to control the tradeoff between stopband energy and the flatness of $A(\omega)$.

- For two channel QMFs, the use of halfband filters can eliminate completely both amplitude and phase as well as canceling aliasing distortion.

A Simple PR Quadrature Mirror Filter

Example:

- Consider a two-channel QMF bank with analysis/synthesis filters

$$H_0(z)=\frac{1}{\sqrt{2}}(1+z^{-1}), \qquad H_1(z)=\frac{1}{\sqrt{2}}(1-z^{-1})$$

$$G_0(z)=\frac{1}{\sqrt{2}}(1+z^{-1}), \qquad G_1(z)=\frac{-1}{\sqrt{2}}(1-z^{-1})$$

- It can be easily verify that

$$H_0(-z)G_0(z)+H_1(-z)G_1(z)=0$$

$$H_0(z)G_0(z)+H_1(z)G_1(z)=2z^{-1}$$

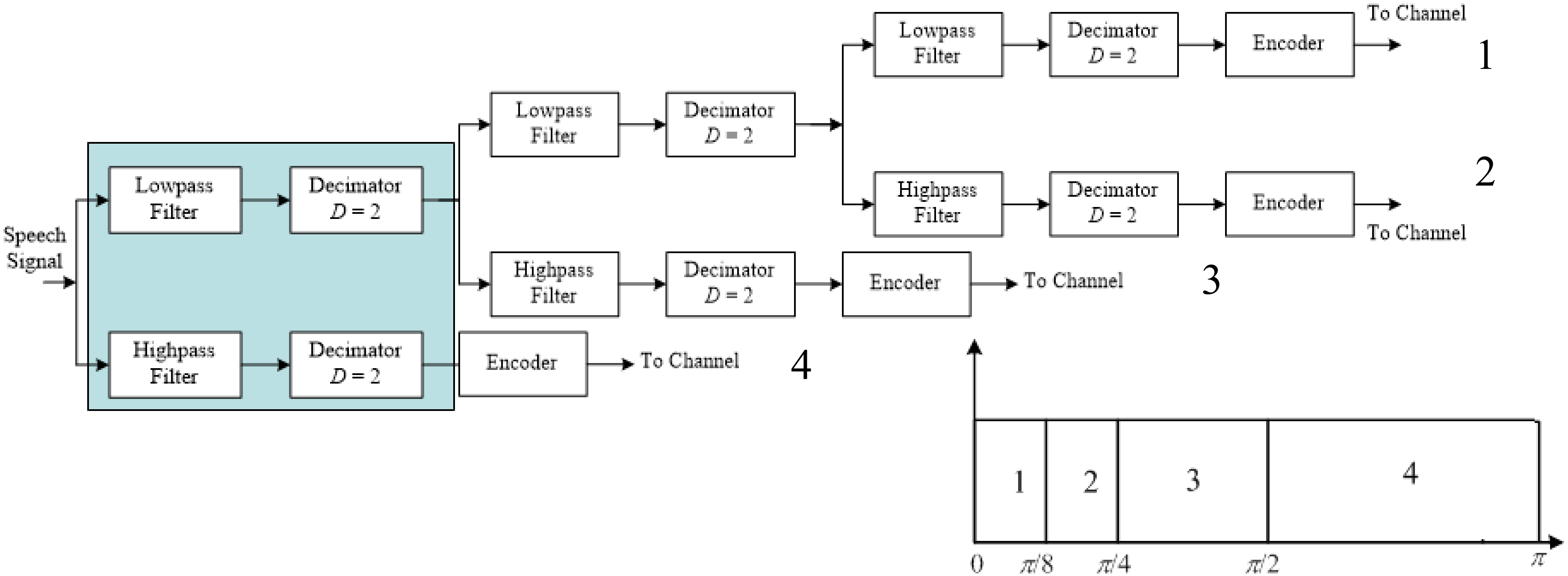
- In the time domain, the outputs of the analysis filters are

$$u_0[n]=\frac{1}{\sqrt{2}}(x[n]+x[n-1]), \qquad u_1[n]=\frac{1}{\sqrt{2}}(x[n]-x[n-1])$$

Subband Coding of Speech Signals

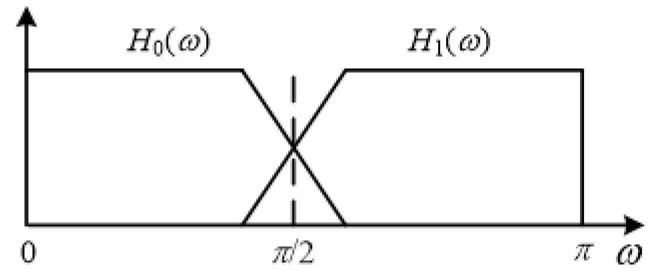
Subband Coding of Speech Signals

- Most of speech energy is contained in the lower frequencies so that better accuracy (using more bits for coding) is required
- The speech signal is divided into several bands and each is encoded separately with different accuracy.

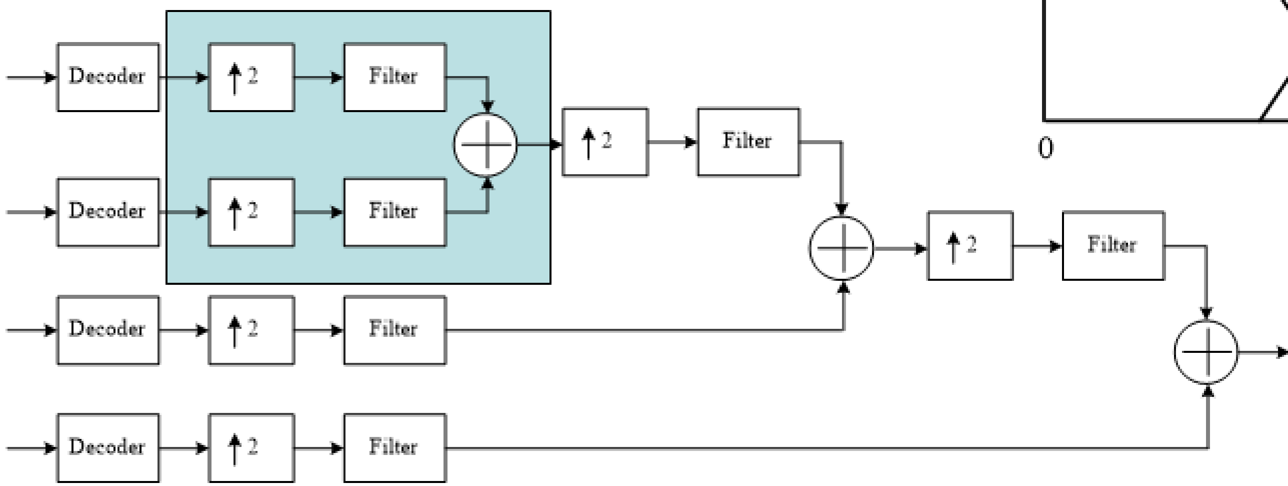


Subband Coding of Speech Signals

- A practical solution is to use quadrature mirror filters (QMF) with the filter characteristics shown in the Figure.

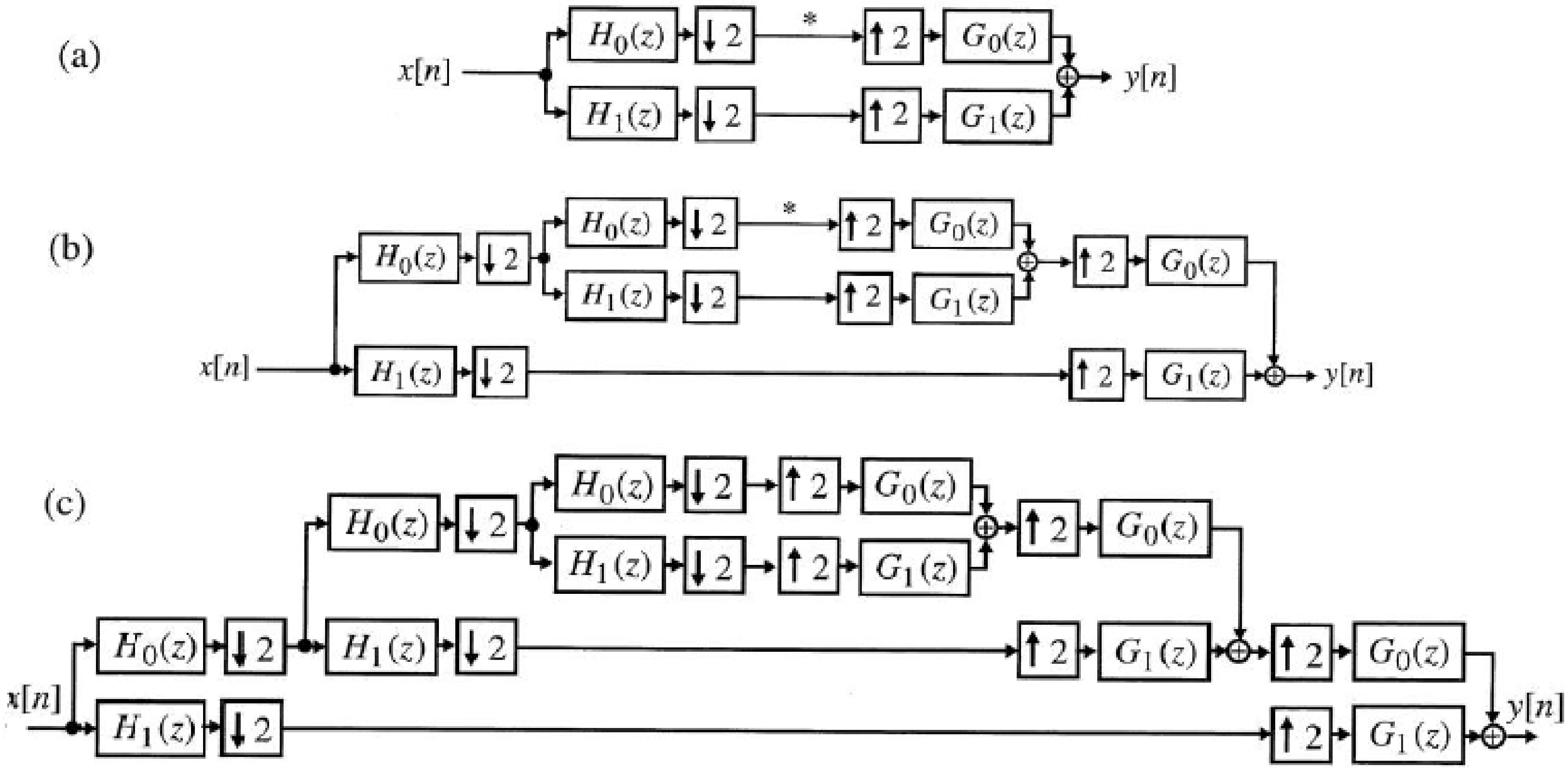


QMF



Synthesis of subband-encoded signals

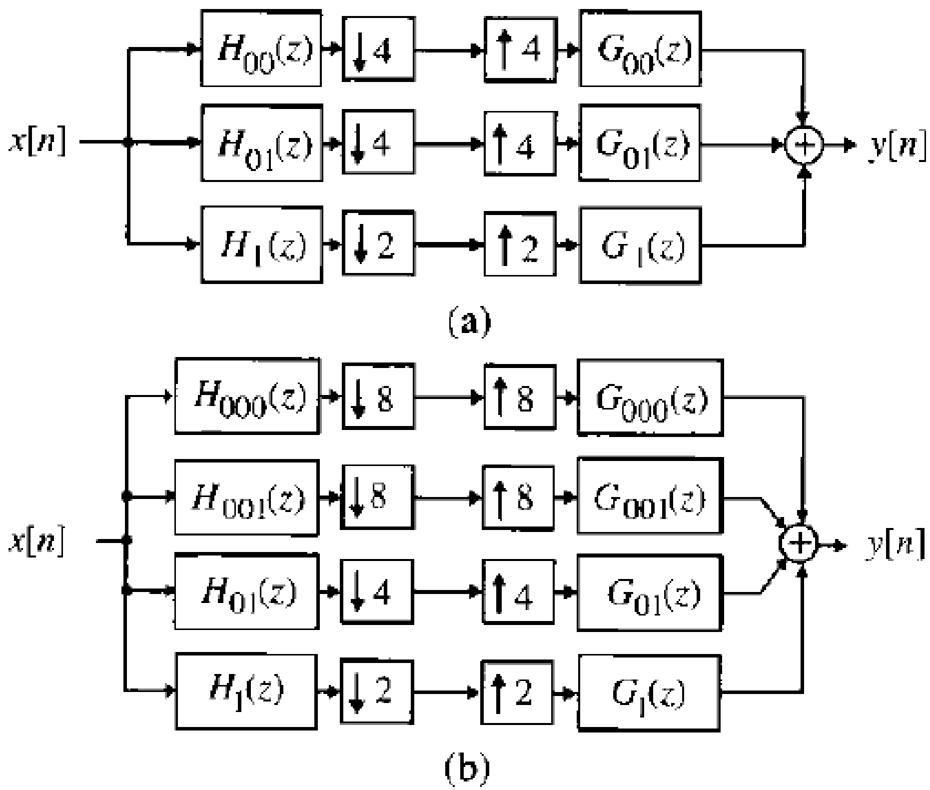
- The synthesis process for subband coding process is a reverse process of encoding process.
- **We just need to design the shaded parts for analysis and synthesis process**



(a) A two-channel QMF bank, (b) a three-channel QMF bank derived from the two-channel QMF bank and (c) a four-channel QMF bank derived from the three-channel QMF bank.

Filter Banks with Unequal Passband Widths

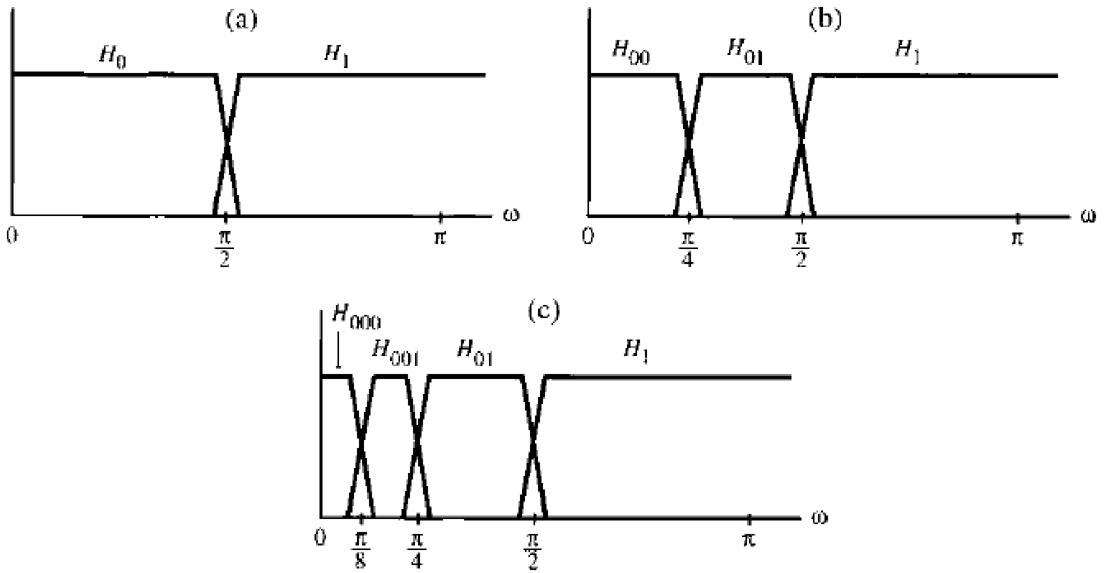
- Inserting another two channel QMF bank in the top channel at the position marked by *;



- Figure shows the development of iteratively deriving a three (a) and four channel (b) unequal passband width filter banks.

Filter Banks with Unequal Passband Widths

- The bandwidths of a two-channel, three-channel and four-channel are shown below



- The equivalent representation of the three-channel filter bank is

$$H_{00}(z) = H_0(z)H_0(z^2), \quad H_{01}(z) = H_0(z)H_1(z^2), \; H_1(z)$$

$$G_{00}(z) = G_0(z)G_0(z^2), \quad G_{01}(z) = G_0(z)G_1(z^2), \; G_1(z)$$

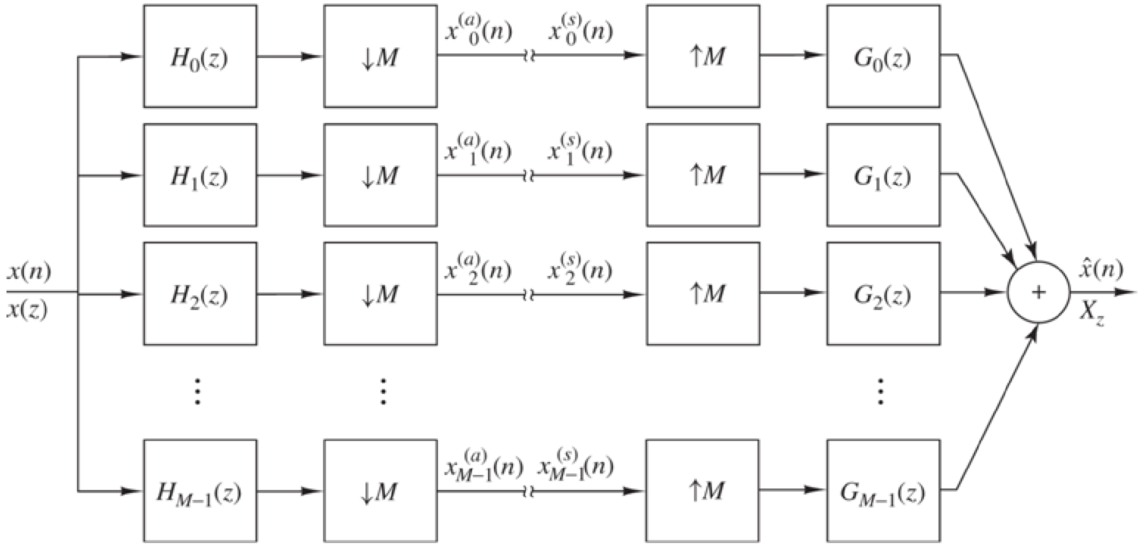
Filter Banks with Unequal Passband Widths

- Similarly, the four channel filter banks can be developed and its equivalent representation is

$$H_{000}(z) = H_0(z)H_0(z^2)H_0(z^4), \quad H_{001}(z) = H_0(z)H_0(z^2)H_1(z^4)$$
$$H_{01}(z) = H_0(z)H_1(z^2), \quad H_1(z),$$
$$G_{000}(z) = G_0(z)G_0(z^2)G_0(z^4), \quad G_{001}(z) = G_0(z)G_0(z^2)G_1(z^4),$$
$$G_{01}(z) = G_0(z)G_1(z^2), \quad G_1(z),$$

M-Channel QMF Bank

- We can have *M*-channel QMF bank shown in the Figure
- $x_k^{(a)}(n), 0 \le k \le M-1$ are the outputs of analysis filter
- $x_k^{(s)}(n), 0 \le k \le M-1$ are the inputs to the synthesis filter



An *M*-channel QMF bank.

- Although the math derivation is more complicated, the perfect reconstruction concept is still valid.
- Please refer to Proakis' book (4th edition) page 761 for more information.

Polyphase Form of *M*-Channel QMF Bank

- By using the polyphase filters, the kth filter $H_k(z)$ is represented by

$$H_k(z) = \sum_{m=0}^{M-1} z^{-m} P_{km}(z) \qquad 0 \le k \le M-1$$

- The equations for the M polyphase filters in matrix form is

$$\mathbf{H}(z) = \mathbf{P}(z^M)\mathbf{a}(z) \qquad (11.12.11)$$

where

$$\mathbf{H}(z) = [H_0(z)\ H_1(z) \cdots H_{M-1}(z)]^T$$

$$\mathbf{a}(z) = [1\ \ z^{-1}\ z^{-2} \cdots z^{-(M-1)}]^T$$

and

$$\mathbf{P}(z) = \begin{bmatrix} P_{00}(z) & P_{01}(z) & \cdots & P_{0(M-1)}(z) \\ P_{10}(z) & P_{11}(z) & \cdots & P_{1(M-1)}(z) \\ \vdots & \vdots & \vdots & \vdots \\ P_{(M-1)0}(z) & P_{(M-1)1}(z) & \cdots & P_{(M-1)(M-1)}(z) \end{bmatrix}$$

Polyphase Form of *M*-Channel QMF Bank

- Figure 11.12.2 (a) below shows the analysis filter bank and (b) is obtained by applying the noble identity to (a).

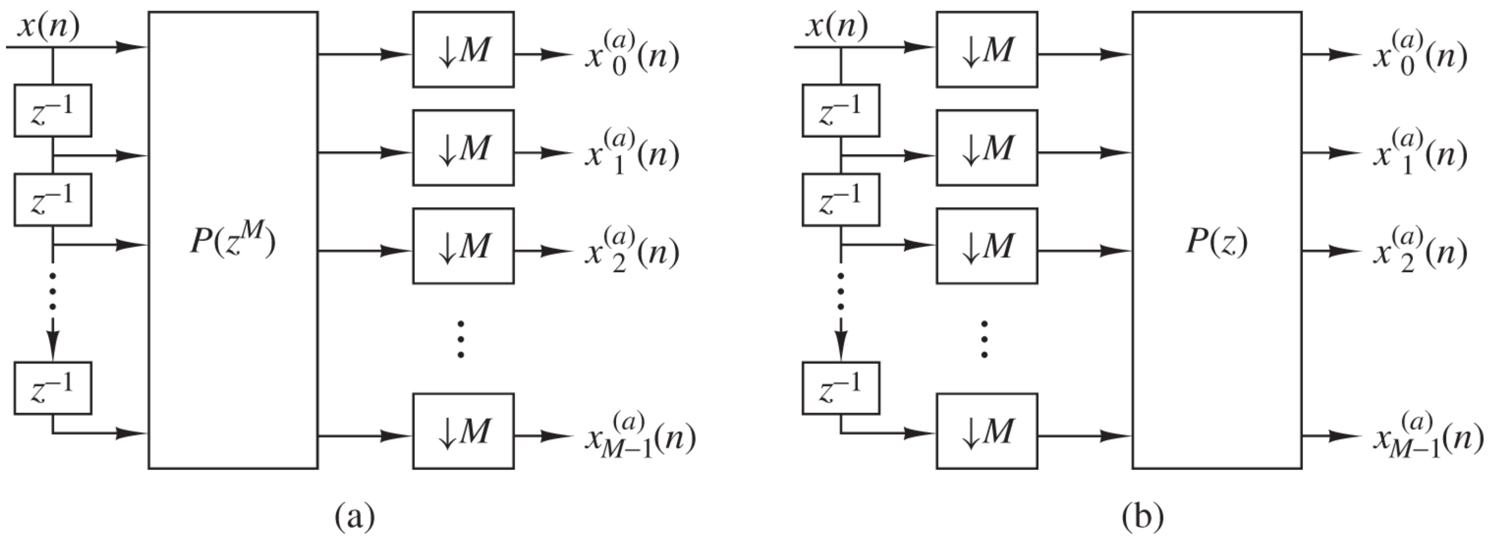


Figure 11.12.2 Polyphase structure of the analysis section of an M-channel QMF bank (a) before and (b) after applying the first noble identity.

- We use a transformed form for the polyphase representation of the synthesis section, which is given

$$G_k(z) = \sum_{m=0}^{M-1} z^{-(M-1-m)} Q_{km}(z^M) \qquad 0 \le k \le M-1 \qquad (11.12.14)$$

Polyphase Form of *M*-Channel QMF Bank

- In matrix form,

$$\mathbf{G}(z) = z^{-(M-1)}\mathbf{Q}(z^M)\mathbf{a}(z^{-1})$$

$$\mathbf{G}(z) = [G_0(z)\ G_1(z) \cdots G_{M-1}(z)]^T$$

$$\mathbf{a}(z^{-1}) = [1\ \ z^1\ z^2 \cdots z^{(M-1)}]^T$$

$$\mathbf{Q}(z) = \begin{bmatrix} Q_{00}(z) & Q_{01}(z) & \cdots & Q_{0M-1}(z) \\ Q_{10}(z) & Q_{11}(z) & \cdots & Q_{1M-1}(z) \\ \vdots & \vdots & \vdots & \vdots \\ Q_{M-10}(z) & Q_{M-11}(z) & \cdots & Q_{M-1M-1}(z) \end{bmatrix}$$

Polyphase Form of *M*-Channel QMF Bank

- The synthesis section is shown in Figure 11.12.3.

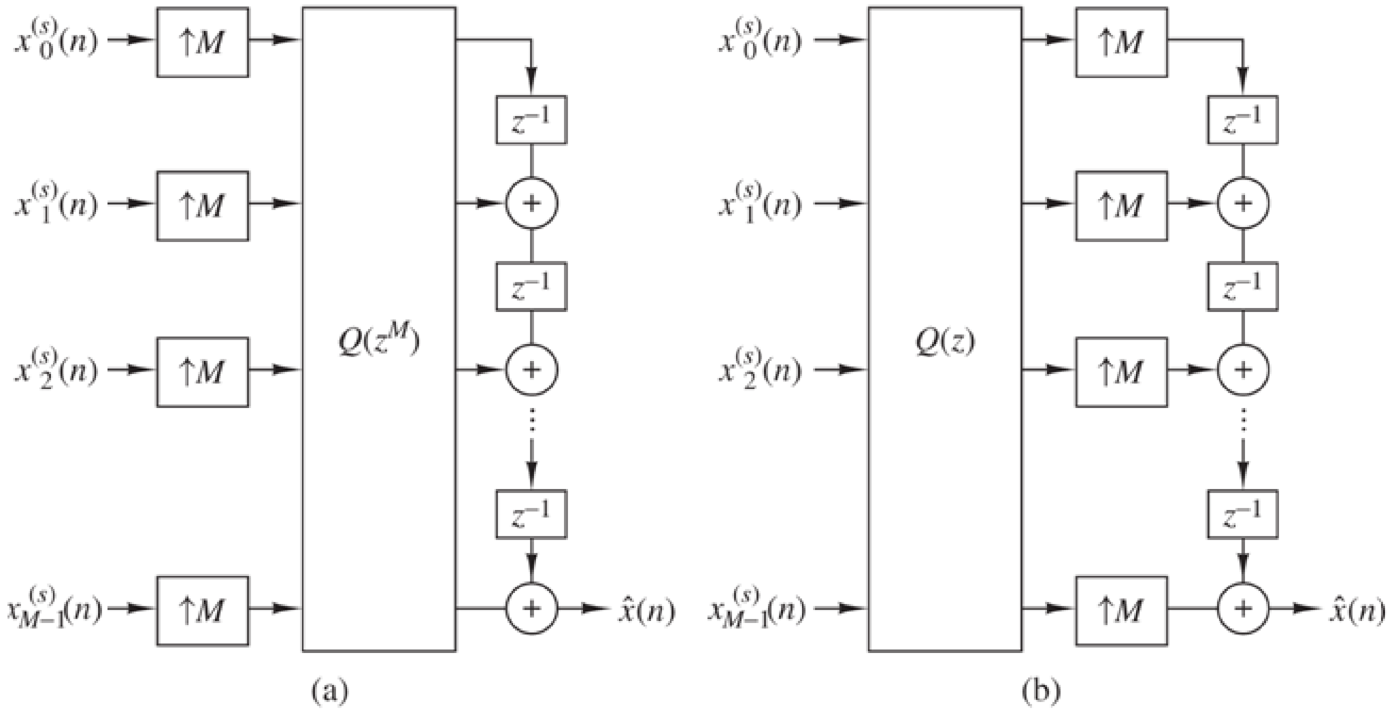


Figure 11.12.3 Polyphase structure of the synthesis section of an M-channel QMF bank (a) before and (b) after applying the first noble identity.

Polyphase Form of M-Channel QMF Bank

- The complete M channel QMF filter bank is given in 11.12.4.

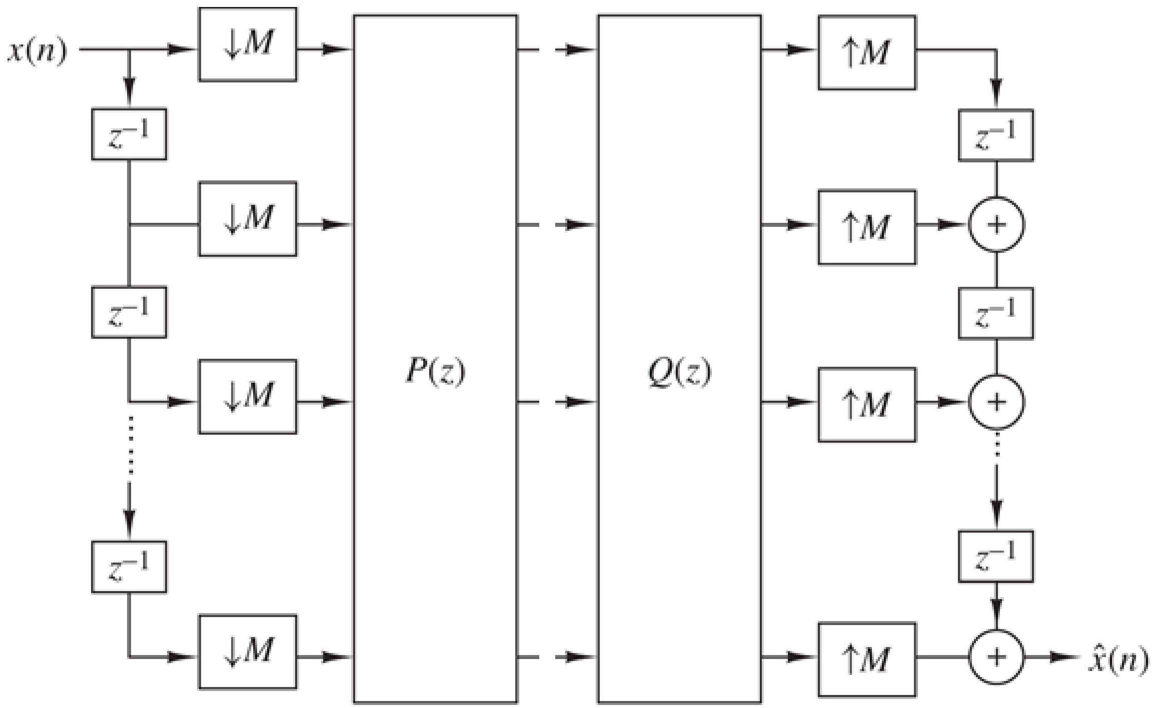


Figure 11.12.4 Polyphase realization of the M-channel QMF bank.

- The perfect reconstruction condition for the M channel QMF filter bank is

$$\mathbf{Q}(z)\mathbf{P}(z) = \mathbf{C}z^{-k}\mathbf{I}$$

- $\mathbf{I}$ is an $M \times M$ identity matrix

Polyphase Form of *M*-Channel QMF Bank

- If $\mathbf{P}(z)$ is known, we have

$$\mathbf{Q}(z) = \mathbf{C}z^{-k}\left[\mathbf{P}(z)\right]^{-1}$$

EXAMPLE:

Suppose the polyphase matrix for a 3-channel perfect reconstruction FIR QMF bank is

$$\mathbf{P}(z^3) = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Determine the analysis and synthesis filters in the QMF bank.

Solution:

The analysis filters are given by (11.12.11) as

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \end{bmatrix} = \begin{bmatrix} 1+z^{-1}+2z^{-2} \\ 2+3z^{-1}+z^{-2} \\ 1+2z^{-1}+z^{-2} \end{bmatrix}$$

Polyphase Form of *M*-Channel QMF Bank

- The inverse of $\mathbf{P}(z^3)$ is

$$\left[\mathbf{P}(z^3)\right]^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 3 & -5 \\ -1 & -1 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

- With a scaling factor of 2, we have

$$\mathbf{Q}(z^3) = 2\left[\mathbf{P}(z^3)\right]^{-1}$$

- By applying (11.12.15), the synthesis filter is obtained by

$$\begin{bmatrix} G_0(z) \\ G_1(z) \\ G_2(z) \end{bmatrix} = z^{-(M-1)}\mathbf{Q}(z^M)\mathbf{a}(z^{-1})$$

$$= z^{-2}\begin{bmatrix} 1 & 3 & -5 \\ -1 & -1 & 3 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ z^1 \\ z^2 \end{bmatrix} = \begin{bmatrix} -5+3z^{-1}+z^{-2} \\ 3-z^{-1}-z^{-2} \\ 1-z^{-1}+z^{-2} \end{bmatrix}$$

Demonstration  

Summary

- Quadrature mirror filter in filter banks
- DFT filter bank structure
- M-Channel filter bank structure