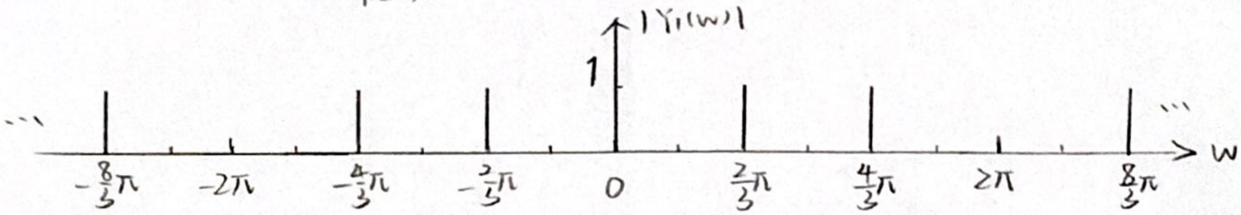
1. (a) $x[n] = 2\cos(\frac{\pi}{3}n) = e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}$

$y[n] = x[qn] = e^{j\frac{q}{3}\pi n} + e^{-j\frac{q}{3}\pi n}$

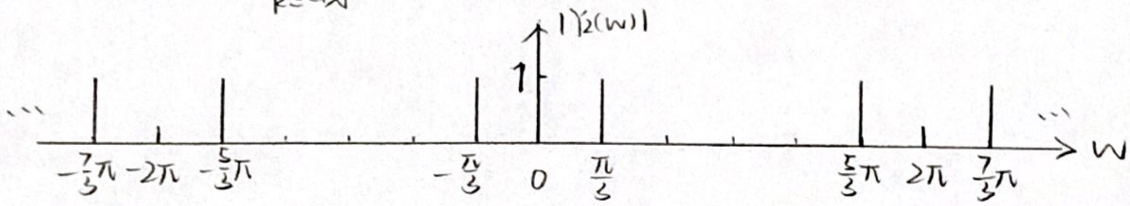
$Y(w) = \sum_{n=-\infty}^{\infty} y[n] e^{-jwn} = \sum_{n=-\infty}^{\infty} (e^{j\frac{q}{3}\pi n} + e^{-j\frac{q}{3}\pi n}) e^{-jwn}$

$= 2\pi \sum_{k=-\infty}^{\infty} (\delta(w - \frac{q}{3}\pi - 2\pi k) + \delta(w + \frac{q}{3}\pi - 2\pi k))$

$q=2$: $Y_1(w) = 2\pi \sum_{k=-\infty}^{\infty} (\delta(w - \frac{2}{3}\pi - 2\pi k) + \delta(w + \frac{2}{3}\pi - 2\pi k))$



$q=5$: $Y_2(w) = 2\pi \sum_{k=-\infty}^{\infty} (\delta(w - \frac{5}{3}\pi - 2\pi k) + \delta(w + \frac{5}{3}\pi - 2\pi k))$



DTFT is periodic and symmetric.

(b) $N = 97$, $F_T = 4KHz$

(i) direct form: length $N = 97$

for non-linear phase: $M = 97 \times 4KHz$

$A = (97-1) \times 4KHz$

for linear phase (symmetric): $\frac{(97-1)+1}{2} \times 4KHz$

$A = (97-1) \times 4KHz$

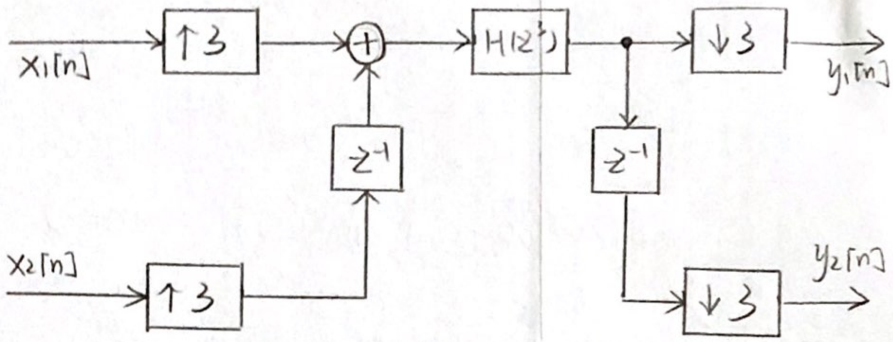
(ii) FFT:

$2^6 < 97-1 < 2^7$, choose $N = 2^7 = 128$.

$M = \frac{N}{2} \times \log_2 N \times 4 = 1792$ real multiplications.

$N \times \log_2 N \times 2 = 1792$ real additions.

2. (a)



$$Y_1(z) = [X_1(z^3) + z^{-1} X_2(z^3)] H(z^3)$$

$$Y_2(z) = z^{-1} [X_1(z^3) + z^{-1} X_2(z^3)] H(z^3)$$

2. (a) $W_1(z) = X_1(z^3) + z^{-1} X_2(z^3)$.

$Y_1(z^3) = [X_1(z^3) + z^{-1} X_2(z^3)] H(z^3)$,

$Y_2(z^3) = z^{-1} [X_1(z^3) + z^{-1} X_2(z^3)] H(z^3)$.

$$v_1[n] = \begin{cases} x_1[n/3], & n = 0, \pm 3, \pm 6, \cdots \\ 0, & \text{otherwise} \end{cases}$$

$$v_2[n] = \begin{cases} x_2[n/3], & n = 0, \pm 3, \pm 6, \cdots \\ 0, & \text{otherwise} \end{cases}$$

$$v_3[n] = v_1[n] + v_2[n-1] = \begin{cases} x_1[\frac{n}{3}], & n = 0, \pm 3, \pm 6, \cdots \\ x_2[\frac{n}{3}], & n = 1, \pm 3+1, \pm 6+1, \cdots \\ 0, & \text{otherwise} \end{cases}$$

$y_1[n] = v_3'[n]$

$y_2[n] = v_3'[n-1]$

$$v_3'[n] = \sum_{k=0}^{M-1} h[3k] v[3n-3k]$$

$$= \sum_{k=0}^{M-1} h[3k] \left(x_1[n-k] + x_2[n-k-1] \right)$$

3、(a) $H(z) = p(z^3)a(z)$, $p(z^3) = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 4 & -2 \\ 2 & -1 & 2 \end{bmatrix}$

$$Q(z^3) = 3 \times [p(z^3)]^{-1} = \begin{bmatrix} 6 & 12 & 3 \\ -3 & 12 & -6 \\ 6 & -3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} G_0(z) \\ G_1(z) \\ G_2(z) \end{bmatrix} = z^{-2}Q(z^3)a(z^{-1}) = z^{-2}\begin{bmatrix} 6 & 12 & 3 \\ -3 & 12 & -6 \\ 6 & -3 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix}$$

$$= \begin{bmatrix} 6z^{-2} + 12z^{-1} + 3 \\ -3z^{-2} + 12z^{-1} - 6 \\ 6z^{2} - 3z^{-1} + 6 \end{bmatrix}$$

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \end{bmatrix} = \begin{bmatrix} 2z^{-2} - z^{-1} + 2 \\ 4z^{-2} + 4z^{-1} - 1 \\ z^{-2} - 2z^{-1} + 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 4 & 4 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \end{bmatrix}$$

$$\Rightarrow p(z^3) = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 4 & 4 \\ 2 & -2 & 1 \end{bmatrix}, \quad [p(z^3)]^{-1} = \begin{bmatrix} 4 & -1 & -4 \\ 3 & -\frac{2}{3} & -\frac{10}{3} \\ -2 & \frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

$$Q(z^3) = 3[p(z^3)]^{-1} = \begin{bmatrix} 12 & -3 & -12 \\ 3 & -2 & -10 \\ -6 & 2 & 7 \end{bmatrix}.$$

$$\begin{bmatrix} Q_0(z) \\ Q_1(z) \\ Q_2(z) \end{bmatrix} = z^{-2}Q(z^3)a(z^{-1}) = \begin{bmatrix} 12 & -3 & -12 \\ 3 & -2 & -10 \\ -6 & 2 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ z \\ z^2 \end{bmatrix}$$

$$= \begin{bmatrix} 12z^{-2} - 3z^{-1} - 12 \\ 3z^{-2} - 2z^{-1} - 10 \\ -6z^{-2} + 2z^{-1} - 7 \end{bmatrix}$$

4. (a) $x[n] = s[n] + w[n]$, $H(z) = \frac{1}{1-0.4z^{-1}}$

$$\Gamma_{ss}(z) = H(z)H(z^{-1})\sigma_v^2$$
$$= \frac{0.64}{(1-0.4z^{-1})(1-0.4z)}$$

$$\Rightarrow \Gamma_{ss}(z) = \frac{0.64}{1+0.16-0.4(z+z^{-1})} = \frac{0.64}{1.16-0.4(z+z^{-1})}$$

(b) $\Gamma_{ss}(z) = \frac{0.64}{0.84}\frac{1}{1-0.4z^{-1}} = \frac{16}{21}\frac{1}{1-0.4z^{-1}}$,

$r_{ss}[m] = \frac{16}{21}(0.4)^{|m|}$, $r_{xx}[m] = r_{ss}[m] + r_{ww}[m]$

$$= \frac{16}{21}(0.4)^{|m|} + \delta[m]$$

(c)
$$\begin{bmatrix} r_{xx}[0] & r_{xx}[1] \\ r_{xx}[-1] & r_{xx}[0] \end{bmatrix}\begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} r_{ss}[0] \\ r_{ss}[1] \end{bmatrix}$$

$$\begin{bmatrix} \frac{37}{21} & \frac{32}{105} \\ \frac{32}{105} & \frac{37}{21} \end{bmatrix}\begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} \frac{16}{21} \\ \frac{32}{105} \end{bmatrix}.$$

$$\begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} 0.5851 & -0.1012 \\ -0.1012 & 0.5851 \end{bmatrix}\begin{bmatrix} \frac{16}{21} \\ \frac{32}{105} \end{bmatrix} = \begin{bmatrix} 0.4149 \\ 0.1012 \end{bmatrix}$$

$$\varepsilon_M^h = r_{ss}[0] - [h[0] \;\; h[1]]\begin{bmatrix} r_{ss}[0] \\ r_{ss}[1] \end{bmatrix}$$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

EE6401 – ADVANCED DIGITAL SIGNAL PROCESSING

November/December 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. (a) Plot the discrete-time Fourier transform (DTFT) of the signals

$$x[n] = 2\cos(\frac{\pi n}{3})$$

and $y[n] = x[qn]$, for $q = 2$ and $q = 5$. Compare these DTFTs and explain the relations of the frequency components obtained for these cases.

(10 Marks)

(b) The length of a linear phase finite impulse response (FIR) filter is 97. The filter has an input sequence sampled at 4 kHz. Find the required number of real multiplications and real additions per second, respectively, when the system is implemented with

(i) a direct form structure, and

(ii) convolution property by using the fast Fourier transform (FFT).

When the convolution property is used, you may assume that only one convolution is performed in each second and the length of FFT algorithm is a power of 2.

(10 Marks)

2. (a) An efficient implementation of two separate single-input, single-output linear time invariant system with an identical transfer function is shown in Figure 1. Show that the system is time-invariant and determine the transfer function from each input to each output.

(14 Marks)

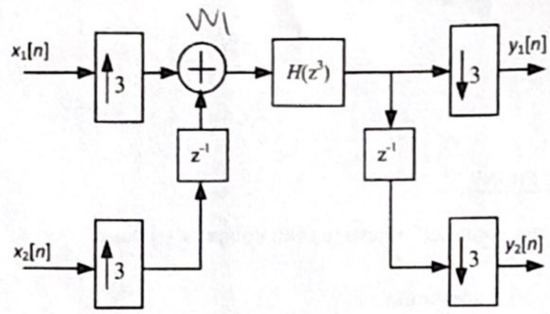

Figure 1

(b) If $y_1[n]$ is connected to $x_2[n]$ in Figure 1, show that the system is also time-invariant and determine its transfer function.

(6 Marks)

3. (a) The analysis filters of a three-channel QMF bank are given by

$$[H_0(z) \quad H_1(z) \quad H_2(z)] = [z^{-2} \; z^{-1} \; 1]\begin{bmatrix} 2 & 4 & 1 \\ -1 & 4 & -2 \\ 2 & -1 & 2 \end{bmatrix}$$

Determine the synthesis filters for implementing a perfect reconstruction filter bank.

(14 Marks)

(b) List the parameters that must be used to generate the function $h[n]$ of a low-pass finite impulse response filter and discuss their impacts on the imperfection of the corresponding frequency response.

(6 Marks)

4. Consider an input signal $x[n]=s[n]+w[n]$ whereby $s[n]$ is an AR(1) process satisfying the following difference equation

$$s[n]-0.4s[n-1]=v[n]$$

where $v[n]$ is a white noise random process with variance $\sigma_v^2=0.64$ and statistically independent of the white noise random process $\{w[n]\}$ with variance $\sigma_w^2=1$.

(a) Determine the power spectral density of the random process $s[n]$.

(8 Marks)

(b) Determine the autocorrelation functions of $s[n]$ and $x[n]$.

(6 Marks)

(c) Design an FIR Wiener filter of length M = 2 which estimates $s[n]$.

(6 Marks)

5. (a) Describe the four basic classes of adaptive filtering applications depicted in Figure 2 on page 4.

(8 Marks)

(b) Determine the complete system equations for the application illustrated in Figure 2(a) on page 4.

(6 Marks)

(c) Describe an algorithm which allows to find the adaptive filter coefficients.

(6 Marks)

Note: Question No. 5 continues on page 4

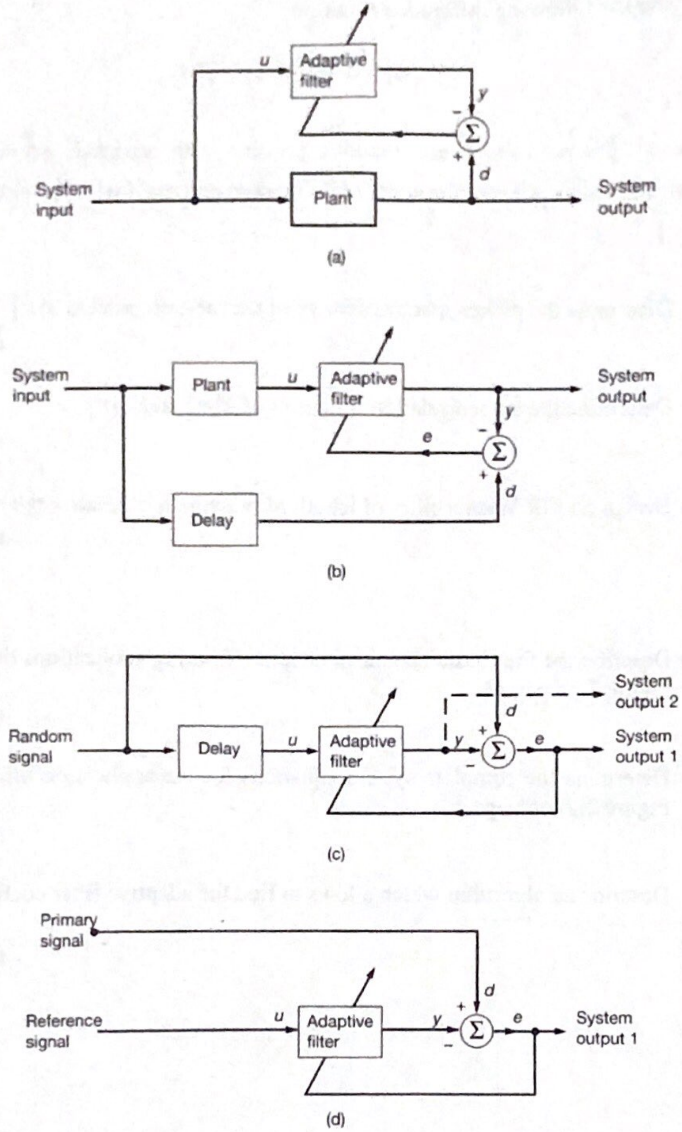

Figure 2

END OF PAPER