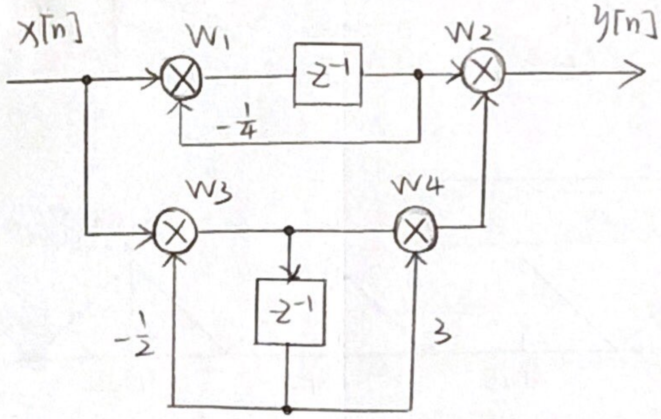
7. (a)



$W_1 = X(z) - \frac{1}{4} z^{-1} W_1$, $X(z) = (1+\frac{1}{4}z^{-1}) W_1$, $W_1 = \frac{1}{1+\frac{1}{4}z^{-1}} X(z)$,

$W_2 = Y(z) = z^{-1} W_1 + W_4$,

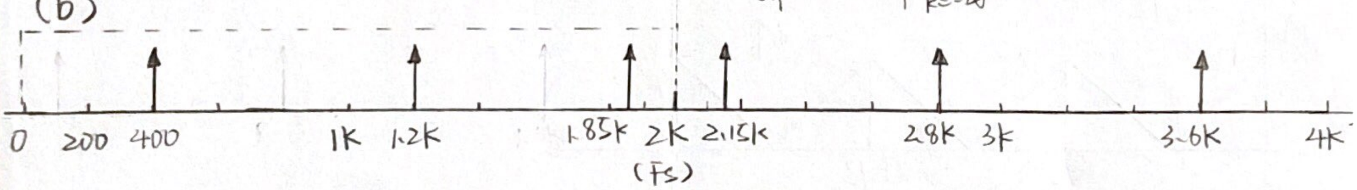
$W_3 = X(z) - \frac{1}{2} z^{-1} W_3$, $X(z) = (1+\frac{1}{2}z^{-1}) W_3$, $W_3 = \frac{1}{1+\frac{1}{2}z^{-1}} X(z)$,

$$W_4 = 3z^{-1} W_3 + W_3 = (1+3z^{-1}) W_3 = \frac{1+3z^{-1}}{1+\frac{1}{2}z^{-1}} X(z)$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1+\frac{1}{4}z^{-1}} + \frac{1+3z^{-1}}{1+\frac{1}{2}z^{-1}} .$$

(b)

$$G_p(\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} G_a(\Omega - k\Omega_T)$$



(i) freq components in $y(t)$: 400Hz, 1.2kHz, 1.85kHz.

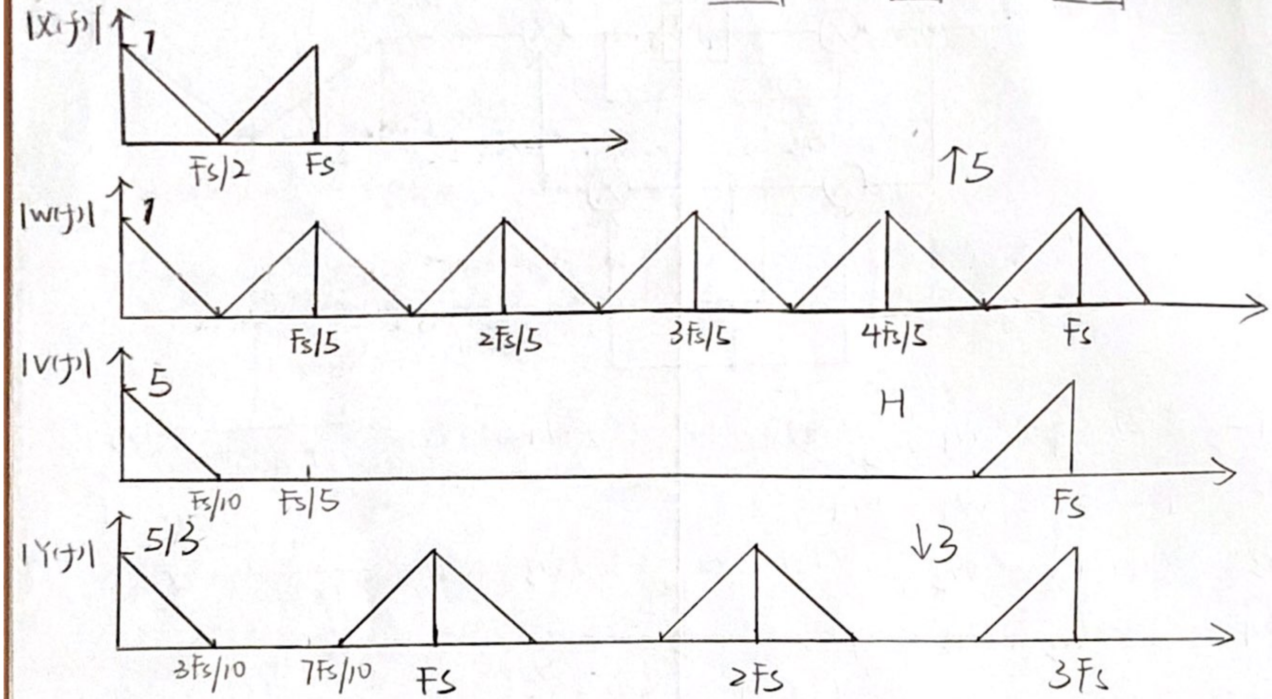
(ii) $x(t)$ has a 2.15kHz component while $y(t)$ doesn't.

$y(t)$ has a 1.85kHz component while $x(t)$ doesn't.

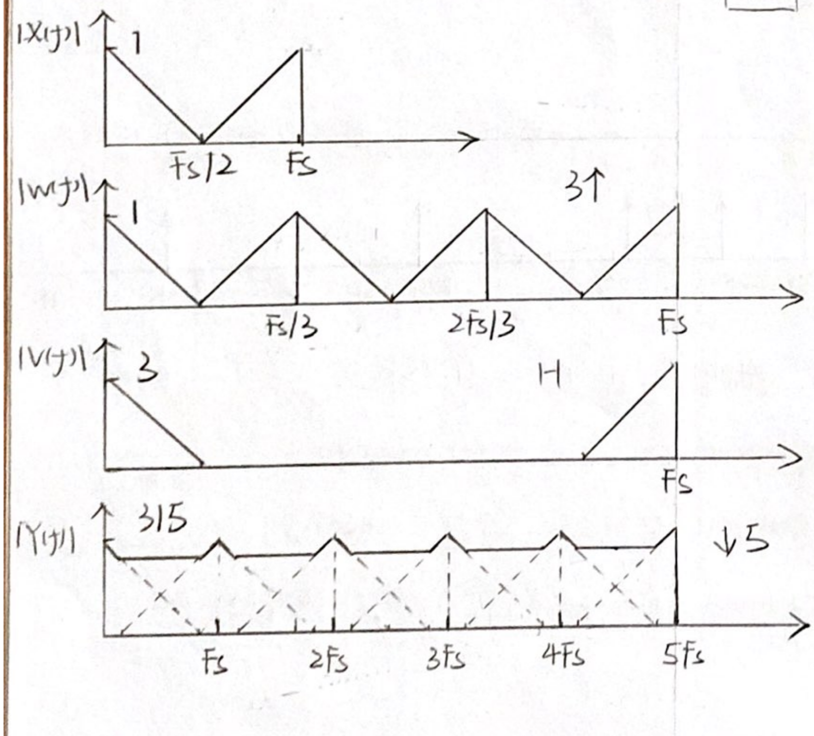
This is due to the symmetric and periodic properties of the signals.

2.

$$X \rightarrow \boxed{\uparrow 5} \xrightarrow{W} \boxed{H} \xrightarrow{V} \boxed{\downarrow 3} \rightarrow Y$$



$$X \rightarrow \boxed{\uparrow 3} \xrightarrow{W} \boxed{H} \xrightarrow{V} \boxed{\downarrow 5} \rightarrow Y$$



3. (a) $W_1(z^2) = z^{-1}X_1(z^2) + X_2(z^2)$,

$Y_1(z) = z^{-1}W_1(z) = z^{-2}X_1(z) + z^{-1}X_2(z)$

$Y_2(z) = W_1(z) = z^{-1}X_1(z) + X_2(z)$

(b) $H(z) = \sum_{k=0}^{D-1} z^{-k} H_k(z^D)$

(i) $H(z) = \sum_{n=0}^{N-1} h[n] z^{-n}$,

$$H(z) = \sum_{k=0}^{D-1} z^{-k} H_k(z^D)$$

$$= \sum_n h[Dn] z^{-Dn} + \sum_n h[Dn+1] z^{-(Dn+1)}$$

$$+ \cdots$$

$$= \sum_{k=0}^{D-1} z^{-k} \sum_n h[Dn+k] (z^D)^{-n},$$

$$\Rightarrow H_k(z) = \sum_n h[Dn+k] z^{-n}.$$

(ii) $H(z) = \sum_{n=0}^{\infty} 8^n z^{-n}$

$$H_0(z) = \sum_{n=0}^{\infty} 8^{2n} z^{-n} = \frac{1}{1-64z^{-1}}$$

$$H_1(z) = \sum_{n=0}^{\infty} 8^{2n+1} z^{-n} = \frac{8}{1-64z^{-1}}$$

Example: Q&3

Consider $H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$

(a) $H(z) = H_0(z^2) + z^{-1} H_1(z^2)$:

$$H(z) = \sum_n h[2n] z^{-2n} + \sum_n h[2n+1] z^{-(2n+1)}$$

$$= \sum_n h[2n] (z^2)^{-n} + \sum_n h[2n+1] (z^2)^{-n} \cdot z^{-1}$$

$$= H_0(z^2) + z^{-1} H_1(z^2).$$

$$\Rightarrow H_0(z) = \sum_n h[2n] z^{-n}.$$

$$H_1(z) = \sum_n h[2n+1] z^{-n}.$$

(b) $H(z) = \sum_{k=0}^{D-1} z^{-k} H_k(z^D)$:

$$H(z) = \sum_n h[Dn] z^{-Dn} + \sum_n h[Dn+1] z^{-(Dn+1)}$$

$$+ \cdots$$

$$= \sum_{k=0}^{D-1} z^{-k} \sum_n h[Dn+k] (z^D)^{-n}.$$

$$\Rightarrow H_k(z) = \sum_n h[Dn+k] z^{-n};$$

(c) $H(z) = \frac{1}{1-az^{-1}} = \sum_{n=0}^{\infty} a^n z^{-n}$

$$H_0(z) = \sum_{n=0}^{\infty} a^{2n} z^{-n} = \frac{1}{1-a^2 z^{-1}}$$

$$H_1(z) = \sum_{n=0}^{\infty} a^{2n+1} z^{-n} = \frac{a}{1-a^2 z^{-1}}$$

4. (a) $x[n] - 0.5x[n-1] + 0.06x[n-2] = w[n] + 0.8w[n-1]$

$$H(z) = \frac{1+0.8z^{-1}}{1-0.5z^{-1}+0.06z^{-2}} = \frac{z^2+0.8z}{z^2-0.5z+0.06}$$

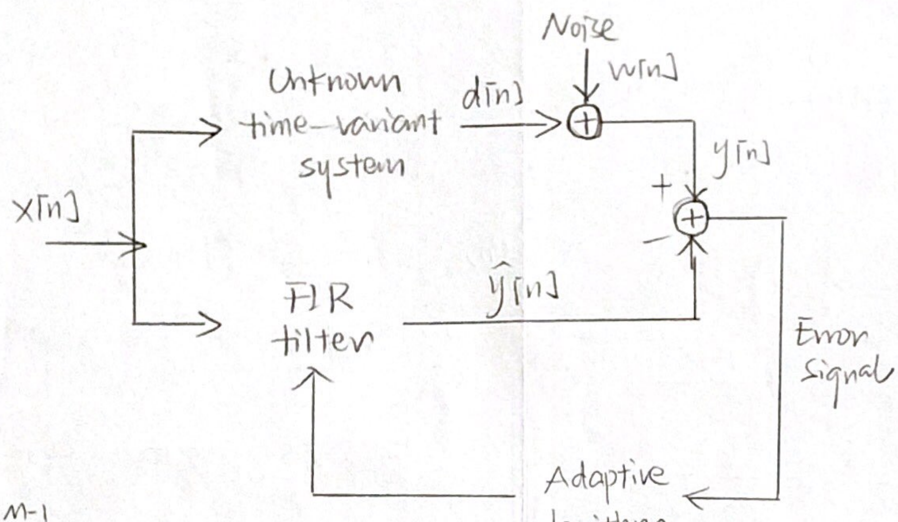
zeros: $z^2+0.8z=0$, $z_1=-0.8$, $z_2=0$

poles: $z^2-0.5z+0.06=0$, $p_1=0.3$, $p_2=0.2$

(b) Since all the poles are inside the unit circle, the system is stable.

(c) $\Gamma_{xx}(z) = \sigma_w^2 H(z)H(z^{-1}) = \dfrac{0.25(1+0.8z^{-1})(1+0.8z)}{(1-0.5z^{-1}+0.06z^{-2})(1-0.5z+0.06z^2)}$

5.



$\hat{y}[n] = \sum_{k=0}^{M-1} h[k]x[n-k]$

$e[n] = y[n] - \hat{y}[n]$, $y[n] = d[n] + w[n]$

$\mathcal{E}_M = E[|e[n]|^2] \Rightarrow r_{yx}[l] = \sum_{k=0}^{M-1} h[k]r_{xx}[l-k] \Rightarrow \Gamma_M h_M = r_y$

$h_M[n+1] = h_M[n] + \frac{1}{2}\mu S[n]$, $S[n] = -g[n]$, $g[n] = \dfrac{d\mathcal{E}_M}{d h_M} = 2[\Gamma_M h_M - r_y]$

$\Rightarrow h_M[n+1] = h_M[n] - \mu(\Gamma_M h_M - r_y)$

$E[h_M[n+1]] = E[h_M[n] + \mu e[n]X_M^*[n]]$

$= \bar{h}_M[n] + \mu E[e[n]X_M^*[n]]$

$= \bar{h}_M[n] + \mu(r_d - \Gamma_M \bar{h}_M)$ orthogonal property

$= (I - \mu\Gamma_M)\bar{h}_M[n] + \mu r_d$

$\bar{h}^\circ(n,k) = C(1-\mu\lambda_k)^n u[n] \Rightarrow |1-\mu\lambda_k| < 1$, $0 < \mu < \dfrac{2}{\lambda_k}$

$\bar{h}^\circ(n,k) = C(1-\dfrac{\lambda_{min}}{\lambda_{max}})u[n]$

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2014-2015

EE6401 – ADVANCED DIGITAL SIGNAL PROCESSING

November / December 2014 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.

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1. (a) Determine the system transfer function shown in Figure 1.

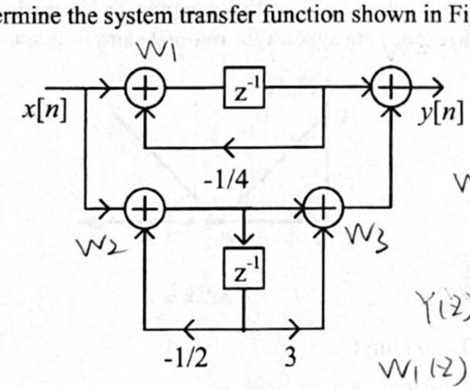

Figure 1

(10 Marks)

$W_1(z) = X(z) - \frac{1}{4} z^{-1} W_1(z)$

$W_2(z) = X(z) - \frac{1}{2} z^{-1} W_2(z)$

$W_3(z) = W_2(z) + 3z^{-1} W_2(z)$

$= (1+3z^{-1}) W_2(z)$

$Y(z) = z^{-1} W_1(z) + W_3(z)$

$W_1(z) = \frac{X(z)}{1+\frac{1}{4}z^{-1}}$

$W_3(z) = \frac{1+3z^{-1}}{1+\frac{1}{2}z^{-1}} X(z)$

(b) An analogue signal $x(t)$ contains sinusoidal components at the frequencies of 400 Hz, 1.2 kHz and 2.15 kHz. The signal $x(t)$ is sampled at a frequency F_s=4.0 kHz to obtain a discrete sequence $x[n]$.

Note: Question No. 1 continues on page 2

(i) If $y(t)$ is reconstructed from $x[n]$, where a low pass filter whose cut off frequency is $0.5F_s$ is used, find the frequency components present in $y(t)$?

(ii) Compare $y(t)$ and $x(t)$ and explain their differences.

(10 Marks)

2. (a) The magnitude of a 256-point discrete Fourier transform, $X[k]$, of a discrete signal sequence, $x[n]$, is given by

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}$$

$$|X[k]| = \begin{cases} 25 & k = 5,122 \\ 70 & k = 7,120 \\ 28 & k = 9,118 \\ 70 & k = 18,109 \\ 0 & \text{elsewhere} \end{cases}$$

It is also known that the sampling frequency of $x[n]$ is 128 Hz. Estimate the frequencies that are possibly in $x[n]$ based on the given $|X[k]|$. What is the accuracy of the estimated frequencies?

(8 Marks)

(b) For the input signal having the spectrum in Figure 2, plot the signals and their corresponding spectra for rational sampling rate conversion by

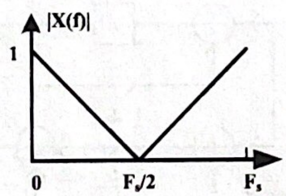

Figure 2

(i) $I/D = 5/3$, and

(ii) $I/D = 3/5$,

where I and D are the interpolation and decimation factors, respectively, and assume that the input signal $x[n]$ is critically sampled and the filter used in the operation is properly designed.

(12 Marks)

3. (a) Figure 3 shows the implementation a system that has two inputs and two outputs. Determine the transfer functions between any pair of input and output, and comment the functionality of these transfer functions.

(6 Marks)

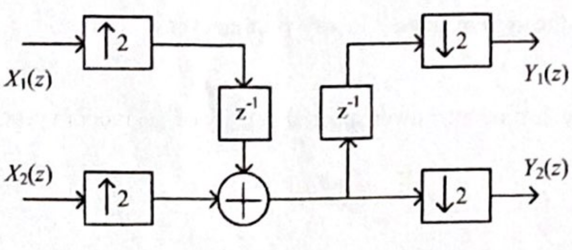

Figure 3

(b) A system transfer function $H(z)$ can be decomposed into a D-branch polyphase structure expressed by

$$H(z) = \sum_{k=0}^{D-1} z^{-k} H_k(z^D)$$

where $H_k(z)$ is the kth branch transfer function.

(i) Based on the definition

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n},$$

determine the expression of $H_k(z)$.

$H(z) = \sum_n h[nD] z^{-nD} + \sum_n h[nD+1] z^{-(nD+1)} + \cdots + \sum_n h[nD+D-1] z^{-(nD+D-1)} = \sum_{k=0}^{D-1} z^{-k} \sum_n h[nD+k] (z^D)^{-n}$

(ii) Given a system transfer function

$$H(z) = \frac{1}{1-8z^{-1}}$$

$H(z) = \sum_n a^{2n} z^{-n} + z^{-1} \sum_n a^{2n+1} z^{-n}$

determine $H_k(z)$ for D=2.

$H_1(z) = \sum_n 8^{2n} z^{-n} = \frac{1}{1-64z^{-1}}$

$H_2(z) = \sum_n 8^{2n+1} z^{-n} = \frac{8}{1-64z^{-1}}$

$\Rightarrow H_k(z) = \sum_n h[nD+k] z^{-n}$

(14 Marks)

4. (a) Consider the ARMA random process $x[n]$ generated by the difference equation

$$x[n] = 0.5x[n-1] - 0.06x[n-2] + w[n] + 0.8w[n-1]$$

where $w[n]$ is white noise with variance 0.5.

(i) Determine the system function.

(5 Marks)

Note: Question No. 4 continues on page 4

(ii) Calculate the poles and the zeros of the system function.

(6 Marks)

(b) Is the system stable? Justify your answer.

(4 Marks)

(c) Determine the power spectral density of the random process.

(5 Marks)

5. (a) What is the main function of the adaptive system illustrated in Figure 4?

(4 Marks)

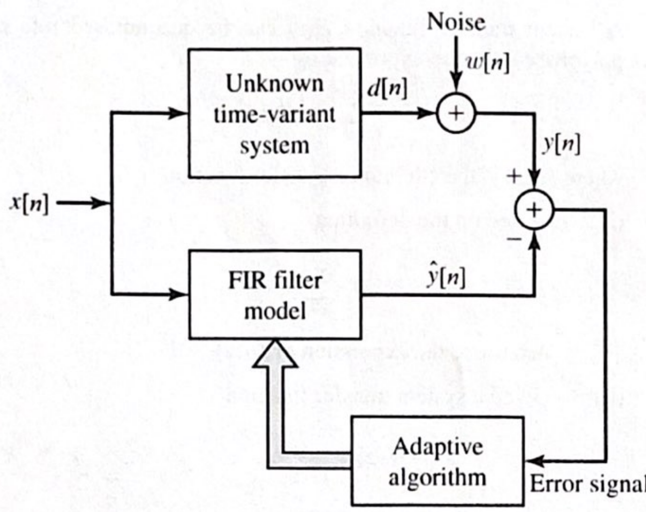

Figure 4

(b) Write out the equations for the adaptive system.

(10 Marks)

(c) Name and describe in detail one Adaptive Algorithm which can be applied onto this system.

(6 Marks)

END OF PAPER