EE6401 Advanced Digital Signal Processing

Assignment for Part II

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Q1:

Consider the autocorrelation matrix of $x[n]$, $\Gamma_M = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$ and the cross-correlation vector between $x[n]$ and $d[n]$ is $\gamma_d = \begin{bmatrix} 0.25 \\ -0.25 \end{bmatrix}$.

1. Find $h_{opt} = \begin{bmatrix} h_{opt}[0] \\ h_{opt}[1] \end{bmatrix}$ directly.

2. Let $\mu = 0.1$ and $h_M[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, use the LMS algorithm to find h_{opt}.

3. Let $\mu = 0.35$ and $h_M[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, use the LMS algorithm to find h_{opt}.

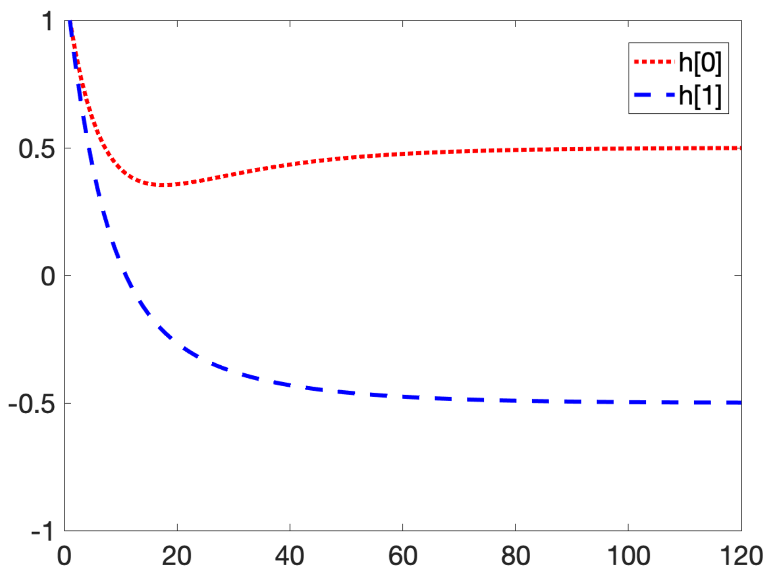
Solution:

$$\Gamma_M \times h_{opt} = \gamma_d$$

$$h_{opt} = \Gamma_M^{-1}\gamma_d = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.25 \\ -0.25 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

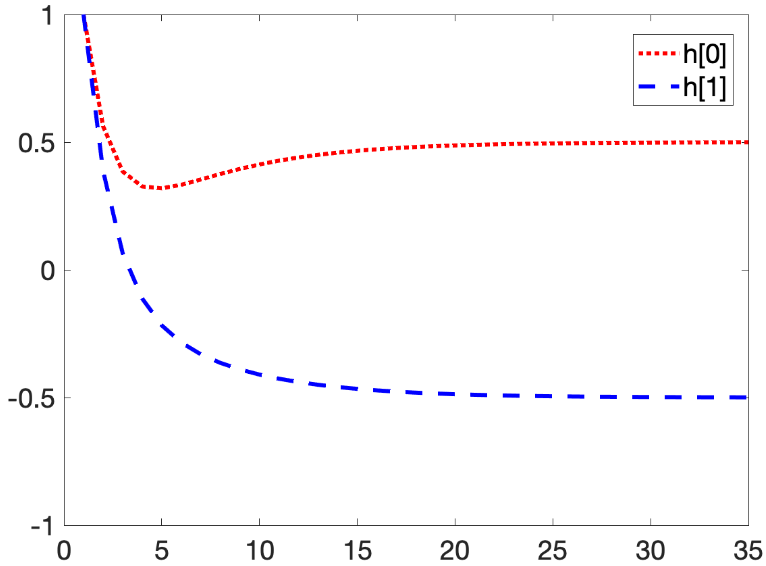
Consider $\mu = 0.1$:

$$h_{opt} = \begin{bmatrix} h_{opt}[0] \\ h_{opt}[1] \end{bmatrix} = \begin{bmatrix} 0.4991 \\ -0.4991 \end{bmatrix}$$



Consider $\mu = 0.35$:

$$h_{opt} = \begin{bmatrix} h_{opt}[0] \\ h_{opt}[1] \end{bmatrix} = \begin{bmatrix} 0.4993 \\ -0.4993 \end{bmatrix}$$



Code:

```
autocorrelation_matrix=[1 0.5; 0.5 1];
crosscorrelation_vector=[0.25; -0.25];
hm_0=[1;1];
step_size=0.35;
I=eye(2);
output=zeros(4,1);
output(1,1)=hm_0(1);
output(2,1)=hm_0(2);
max_ite=1000;
min_err=0.001;
for i=2:max_ite
    h_pre = [output(1,i-1); output(2,i-1)];%M=2
    g = 2*(autocorrelation_matrix * h_pre -
crosscorrelation_vector);
    h = (I - step_size * autocorrelation_matrix) * h_pre +
(step_size * crosscorrelation_vector);
    output(1,i)=h(1);
    output(2,i)=h(2);
    output(3,i)=g(1);
    output(4,i)=g(2);
    if (g(1)<min_err && g(2)<min_err)
       break;
    end
end
```

```
x = (1: size(output,2));
y1=output(1,:);
y2=output(2,:);
plot(x, y1);
hold on
plot(x, y2);
ylim([-1 1]);
```

Q2:

Consider the primary signal $d[n] = \cos(2\pi\omega_0 n) + y_0[n]$ where the noise $y_0[n]$ is the output of the output of the $F_0(z) = 0.5 + 0.5z^{-1}$ when excited with a white noise sequence $w[n]$.

Consider an input signal x[n]=w[n].

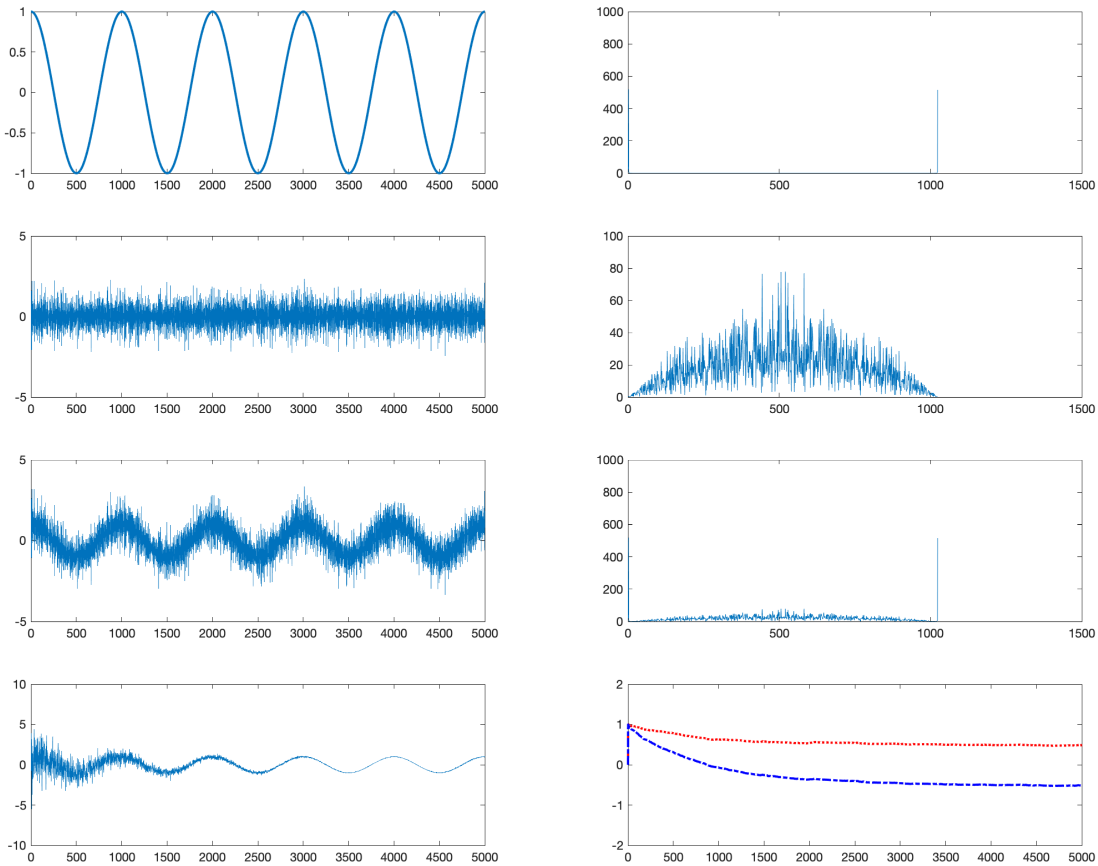
1. Let $\mu = 0.00125$ and $h_M[0] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, use the LMS algorithm to find h_{opt}.

2. Plot the primary signal $d[n]$.

3. Plot $e[n] = d[n] - y[n]$ and the desired wave signal.

4. Plot the evolution of the two filter parameters.

Solution:



The results are shown in the figures above.

With unknown Γ_M and γ_d, we can use the stochastics-gradient-descent algorithm to find gradient $g(n) = -2e[n]X_M^*(n)$.

$$h_M(n+1) = h_M(n) - \frac{1}{2}\Delta(n)g(n) = h_M(n) + \mu e[n]X_M^*(n)$$

Where $e[n] = d[n] - \hat{d}[n]$.

Code:

```
t=0:pi/500:10*pi;
x=cos(t);
x=x(:);
sz=size(x,1);
subplot(4,2,1);
title('the desired cos wave signal');
plot(x);

xf=fft(x,1024);
subplot(4,2,2);
title('the desired cos wave signal in freq domain');
plot(abs(xf));

%white noise
z=tf('z',1);
F0=0.5-0.5*z^-1;
w=randn(size(x));
noise=lsim(F0,w);
subplot(4,2,3);
title('white noise');
plot(noise);

%noise in freq domain
wf=fft(noise,1024);
subplot(4,2,4);
title('white noise in freq domain');
plot(abs(wf));

%the primal signal
d=x+noise;
subplot(4,2,5);
title('primal signal');
plot(d);

%the primal signal in freq domain
df=fft(d,1024);
```

```
subplot(4,2,6);
title('primal signal in freq domain');
plot(abs(df));

%step size
mu=0.00125;
%orignal coefficients vector
hm_0=[1;1];
[hm,ee] = LMS(mu, hm_0, w, d);

%en = dn-ee
en=d-ee;
subplot(4,2,7);
title('estimated signal');
plot(en);

subplot(4,2,8);
title('filter coefficients');
plot((1:1:5001),hm(1,:));
hold on;
plot((1:1:5001),hm(2,:));

%using stochastic gradient descent algorithm
function [hm,ee]=LMS(mu, h0, x, d)
%input signal
N=length(x);
x=x(:);
d=d(:);

%filter coefficient
M=size(h0, 1);
hm=zeros(2,1);
ee=zeros(size(x));

for n=M:N
    arr=x(n:-1:n-M+1);
    e(n)=d(n)-h0'*arr;
    h0=h0+mu*e(n)*arr;
    hm(1,n)=h0(1);
    hm(2,n)=h0(2);
    ee(n)=h0'*arr;
end

end
```

Q3:

1. Let $\Gamma_{ss}(z) = \frac{1}{(1-0.6z^{-1})(1-0.6z)}$, $\Gamma_{ww}(z) = 1$, and $\Gamma_{xx}(z) = \Gamma_{xx}(z) + \Gamma_{ww}(z)$. Factorize $\Gamma_{xx}(z)$ as $\Gamma_{xx}(z) = G(z)G(z^{-1})$, where $G(z)$ is the minimum phase causal and stable filter.

2. Plot the power spectrum density function $\Gamma_{xx}(f)$.

3. Consider an aut-oregessive AR(2) random process $\mathrm{v}[n]$, having auto-correlation function $\mathrm{r}_v[m]$, with $\mathrm{r}_v[0] = 0.5196$, $\mathrm{r}_v[1] = 0.4171$ and $\mathrm{r}_v[2] = 0.1654$. Plot the power spectrum density function $\Gamma_{vv}(f)$ using the Yule-Walker method.

4. Compare the plots of $\Gamma_{xx}(f)$ and $\Gamma_{vv}(f)$ and briefly comment on them.

Solution:

$$\Gamma_{xx}(z) = \Gamma_{ss}(z) + \Gamma_{ww}(z) = \frac{1}{(1-0.6z^{-1})(1-0.6z)} + 1$$

$$= \frac{(1-0.6z^{-1})(1-0.6z)+1}{(1-0.6z^{-1})(1-0.6z)} = \frac{2.36 - 0.6(z^{-1}+z)}{(1-0.6z^{-1})(1-0.6z)}$$

Numerator: $2.36 - 0.6(z^{-1}+z) = b(1-az^{-1})(1-az) = b(1+a^2 - a(z^{-1}+z))$

$$b(1+a^2) = 2.36$$

$$ba = 0.6$$

$$b = \frac{0.6}{a}$$

$$\frac{0.6}{a} + 0.6a = 2.36$$

$$0.6a^2 - 2.36a + 0.6 = 0$$

$$a = 0.2732, b = \frac{0.6}{0.2732} = 2.1962$$

$$\Rightarrow \Gamma_{xx}(z) = \frac{2.1962(1-0.2732z^{-1})(1-0.2732z)}{(1-0.6z^{-1})(1-0.6z)} = G(z)G(z^{-1})$$

$$\Rightarrow G(z) = \frac{\sqrt{2.1962}(1-0.2732z^{-1})}{(1-0.6z^{-1})}$$

$$\Rightarrow G(z^{-1}) = \frac{\sqrt{2.1962}(1-0.2732z)}{(1-0.6z)}$$

$$\Gamma_{xx}(f) = \frac{2.1962\left(1-0.2732e^{-j2\pi f}\right)\left(1-0.2732e^{j2\pi f}\right)}{(1-0.6e^{-j2\pi f})(1-0.6e^{j2\pi f})}$$

For the aut-oregessive AR(2) random process $\mathrm{v}[n]$, by using the Yule-Walker method, we have the following relationship of PSD:

$$R_x^{AR}(\theta, f) = \frac{\hat{\sigma}_w^2}{|1+\sum_{k=1}^{p}\hat{a}_k e^{-j2\pi fk}|^2}$$

$$\begin{bmatrix} r_v[0] & r_v[-1] \\ r_v[1] & r_v[0] \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = -\begin{bmatrix} r_v[1] \\ r_v[2] \end{bmatrix}$$

$$\begin{bmatrix} 0.5196 & 0.4171 \\ 0.4171 & 0.5196 \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = -\begin{bmatrix} 0.4171 \\ 0.1654 \end{bmatrix}$$

Thus $\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = \begin{bmatrix} -1.5387 \\ 0.9169 \end{bmatrix}$.

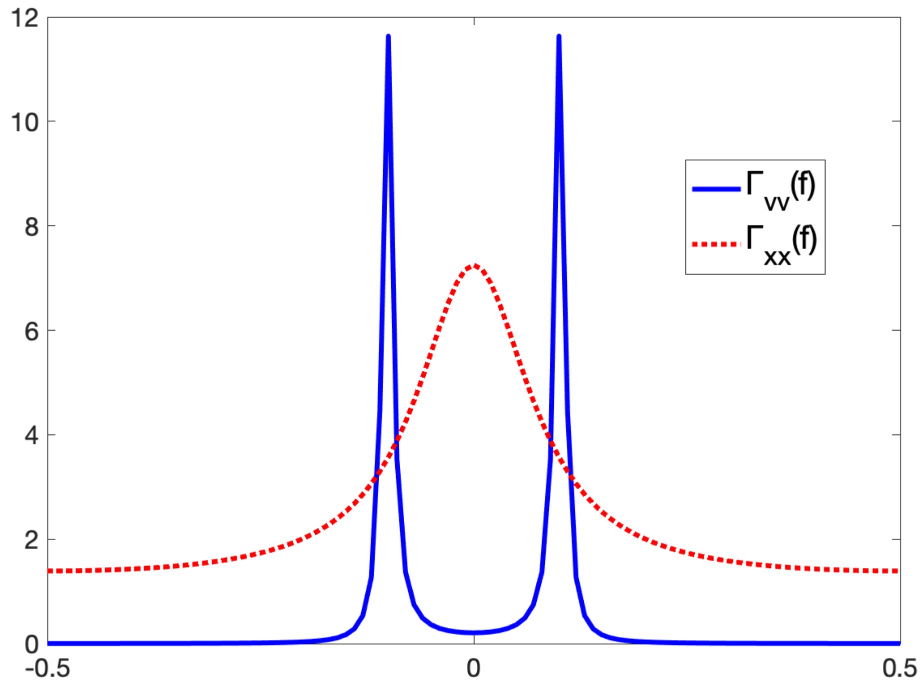
As for $\hat{\sigma}_w^2$, we have

$$\hat{\sigma}_w^2 = r_v[0] + [r_v[1] \quad r_v[2]] \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} = [0.4171 \quad 0.1654] \begin{bmatrix} -1.5387 \\ 0.9169 \end{bmatrix} = 0.02946$$

Finally, the PSD is obtained as follows:

$$\Gamma_{vv}(f) = R_x^{AR}(\theta, f) = \frac{0.02946}{|1 + (-1.5387e^{-j2\pi f}) + (0.9169e^{-j4\pi f})|^2}$$

The results are plotted as follows:



As we can see from the figure, $\Gamma_{vv}(f)$ has two peaks while $\Gamma_{xx}(f)$ has only one peak. This error could be introduced by the projection process of $\Gamma_{xx}(z)$ by letting z=$e^{j2\pi f}$.

Code:

```
f=-0.5:0.01:0.5;
R=zeros(size(f));
Gamma=zeros(size(f));
for i=1:101
    M=0.02946;
    N=(abs(1+(-1.5387*exp(-1i*2*pi*f(i)))+(0.9169*exp(-
1i*4*pi*f(i)))))^2;
    R(i)=M/N;
```

```
    M2=2.1962*((abs(1-0.2732*exp(-1i*2*pi*f(i))))^2);
    N2=(abs(1-0.6*exp(-1i*2*pi*f(i))))^2;
    Gamma(i)=M2/N2;
end

plot(f,R);
hold on;
plot(f,Gamma);
```