HW2

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Environment

Python == 3.11.0

opencv-python == 4.12.0.88

numpy == 2.2.6

open3d == 0.19.0

pandas == 2.3.2

scipy == 1.16.2

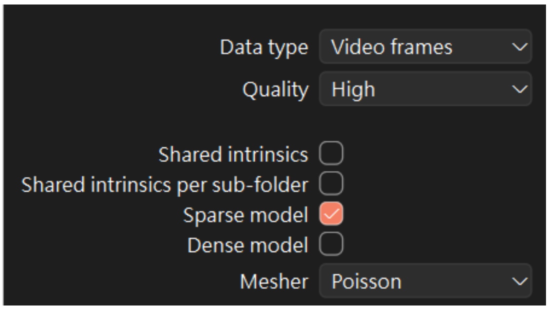
tqdm == 4.67.1

Problem1

1-1

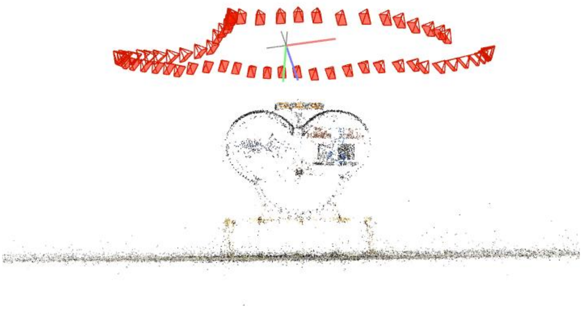
使用 splitvideo.py 將影片每 1 秒擷取 5 frame，輸入到 colmap 用 automatic reconstruction

設定:



Data type: Video frames
Quality: High
Shared intrinsics □
Shared intrinsics per sub-folder □
Sparse model ☑
Dense model □
Mesher: Poisson

result:

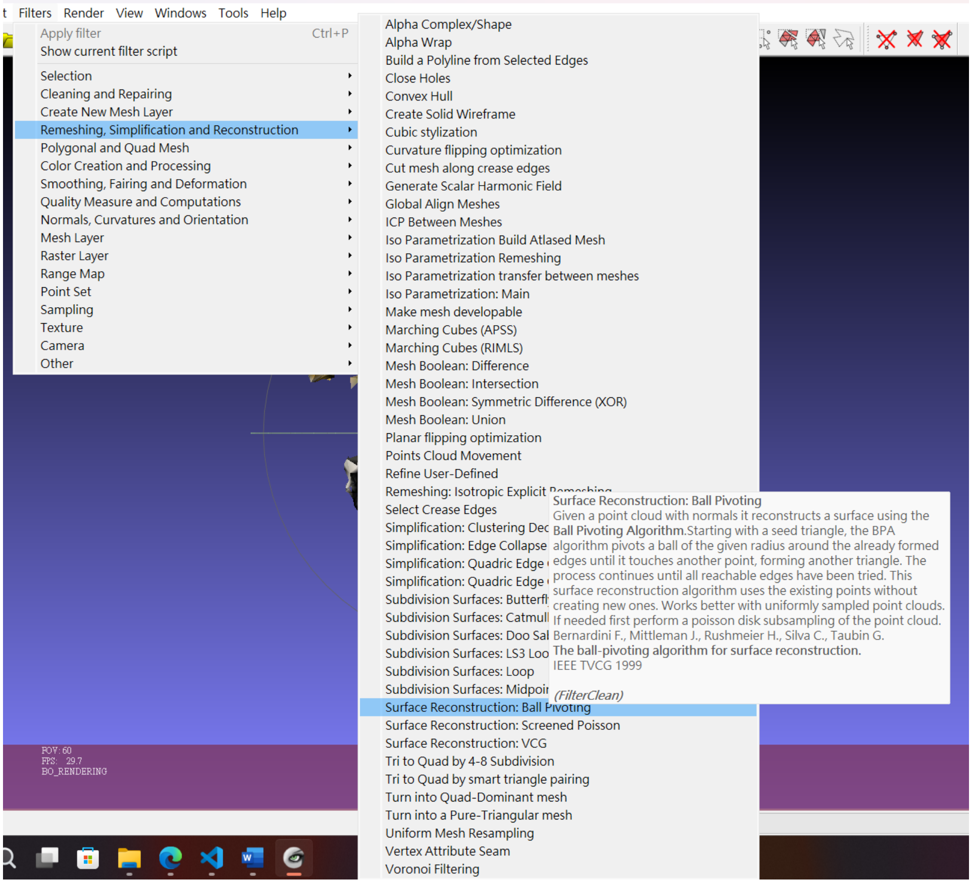


1-2

用 meshlab 重建

1.把多餘的點手動清除

2.使用 Ball Pivoting 重建 surface


t Filters Render View Windows Tools Help
Apply filter Ctrl+P
Show current filter script
Selection
Cleaning and Repairing
Create New Mesh Layer
Remeshing, Simplification and Reconstruction
Polygonal and Quad Mesh
Color Creation and Processing
Smoothing, Fairing and Deformation
Quality Measure and Computations
Normals, Curvatures and Orientation
Mesh Layer
Raster Layer
Range Map
Point Set
Sampling
Texture
Camera
Other
Alpha Complex/Shape
Alpha Wrap
Build a Polyline from Selected Edges
Close Holes
Convex Hull
Create Solid Wireframe
Cubic stylization
Curvature flipping optimization
Cut mesh along crease edges
Generate Scalar Harmonic Field
Global Align Meshes
ICP Between Meshes
Iso Parametrization Build Atlased Mesh
Iso Parametrization Remeshing
Iso Parametrization transfer between meshes
Iso Parametrization: Main
Make mesh developable
Marching Cubes (APSS)
Marching Cubes (RIMLS)
Mesh Boolean: Difference
Mesh Boolean: Intersection
Mesh Boolean: Symmetric Difference (XOR)
Mesh Boolean: Union
Planar flipping optimization
Points Cloud Movement
Refine User-Defined
Remeshing: Isotropic Explicit Remeshing
Select Crease Edges
Simplification: Clustering Dec
Simplification: Edge Collapse
Simplification: Quadric Edge
Simplification: Quadric Edge
Subdivision Surfaces: Butterfl
Subdivision Surfaces: Catmull
Subdivision Surfaces: Doo Sal
Subdivision Surfaces: LS3 Loo
Subdivision Surfaces: Loop
Subdivision Surfaces: Midpoi
Surface Reconstruction: Ball Pivoting
Surface Reconstruction: Screened Poisson
Surface Reconstruction: VCG
Tri to Quad by 4-8 Subdivision
Tri to Quad by smart triangle pairing
Turn into Quad-Dominant mesh
Turn into a Pure-Triangular mesh
Uniform Mesh Resampling
Vertex Attribute Seam
Voronoi Filtering
FOV: 60
FPS: 29.7
BO_RENDERING

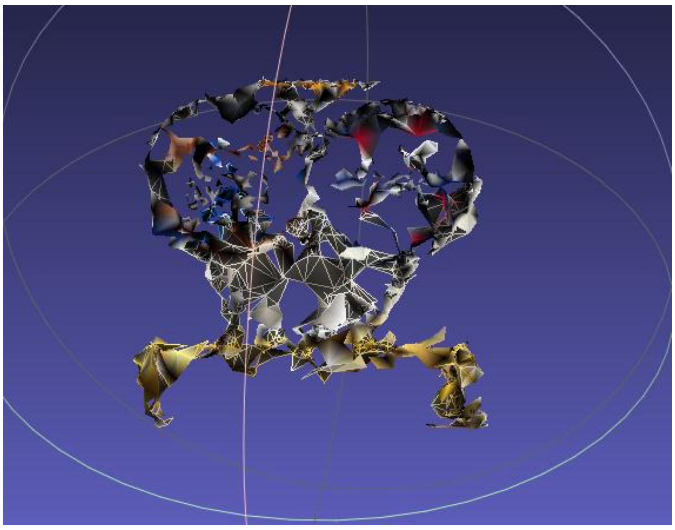
Surface Reconstruction: Ball Pivoting

Given a point cloud with normals it reconstructs a surface using the **Ball Pivoting Algorithm.**Starting with a seed triangle, the BPA algorithm pivots a ball of the given radius around the already formed edges until it touches another point, forming another triangle. The process continues until all reachable edges have been tried. This surface reconstruction algorithm uses the existing points without creating new ones. Works better with uniformly sampled point clouds. If needed first perform a poisson disk subsampling of the point cloud.

Bernardini F., Mittleman J., Rushmeier H., Silva C., Taubin G.
The ball-pivoting algorithm for surface reconstruction.
IEEE TVCG 1999

(FilterClean)

result:



觀察:sparse model 的點較為稀疏，重建有一定難度

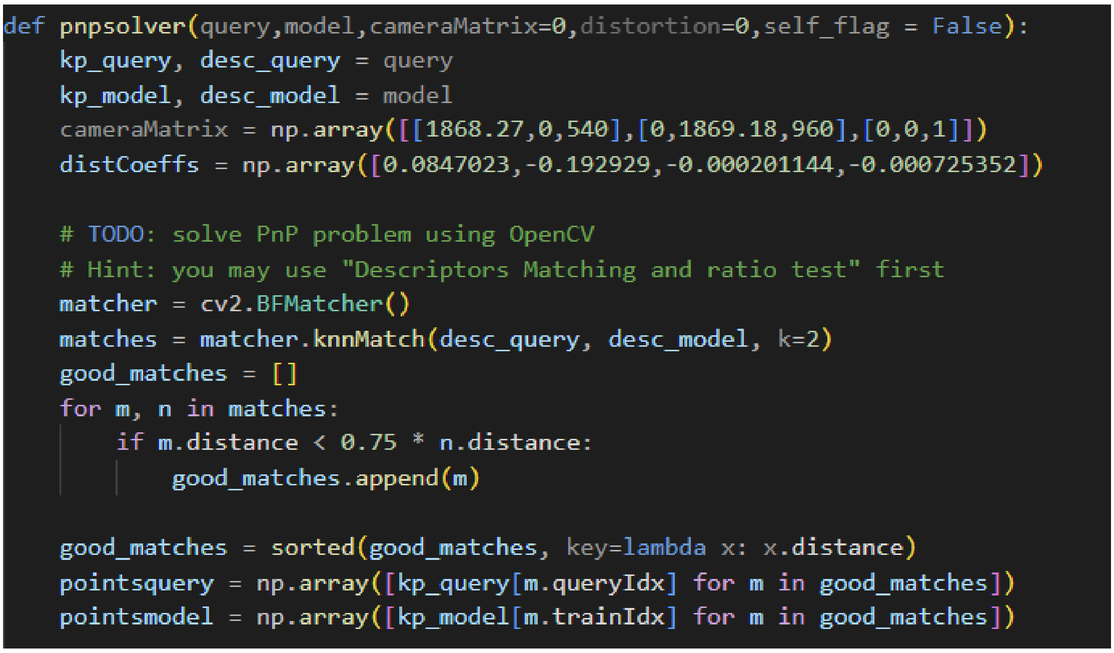
影片連結:https://www.youtube.com/watch?v=kS03sSNckk0

Problem2

使用 2d3dmathcing.py 完成 2-1 與 2-2

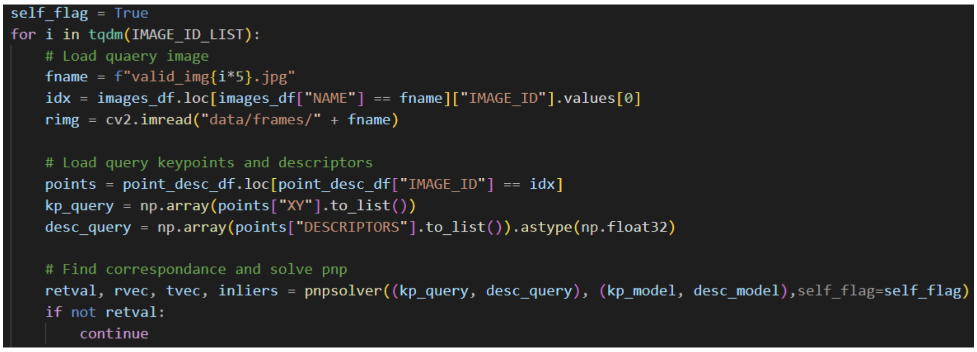
2-1

Step1:算 world to camera [R|t]

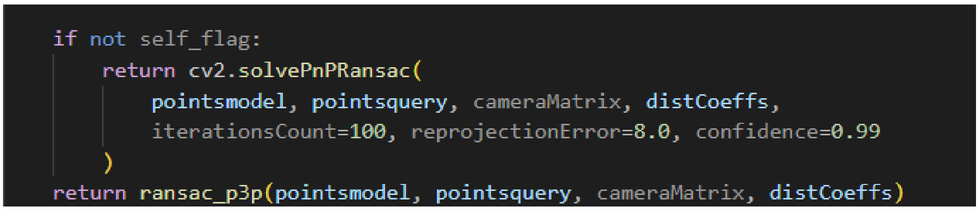


```python
def pnpsolver(query,model,cameraMatrix=0,distortion=0,self_flag = False):
    kp_query, desc_query = query
    kp_model, desc_model = model
    cameraMatrix = np.array([[1868.27,0,540],[0,1869.18,960],[0,0,1]])
    distCoeffs = np.array([0.0847023,-0.192929,-0.000201144,-0.000725352])

    # TODO: solve PnP problem using OpenCV
    # Hint: you may use "Descriptors Matching and ratio test" first
    matcher = cv2.BFMatcher()
    matches = matcher.knnMatch(desc_query, desc_model, k=2)
    good_matches = []
    for m, n in matches:
        if m.distance < 0.75 * n.distance:
            good_matches.append(m)

    good_matches = sorted(good_matches, key=lambda x: x.distance)
    pointsquery = np.array([kp_query[m.queryIdx] for m in good_matches])
    pointsmodel = np.array([kp_model[m.trainIdx] for m in good_matches])
```

用 hw1 的 match 方法選出 match 的 2D 點與 3D 點



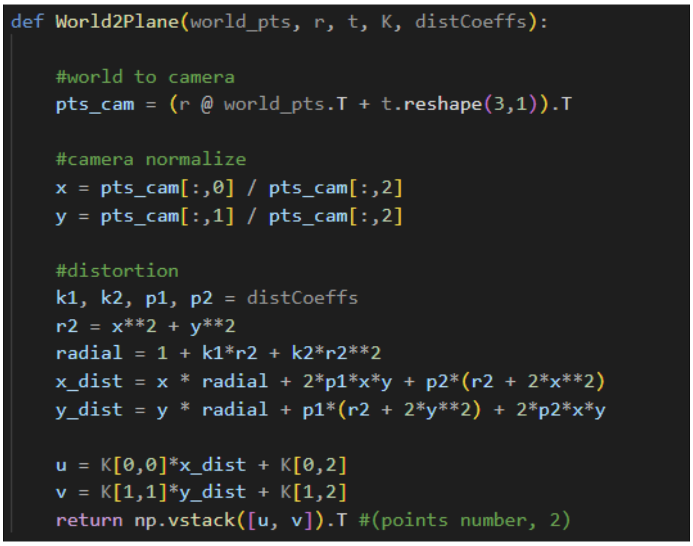
```python
self_flag = True
for i in tqdm(IMAGE_ID_LIST):
    # Load quaery image
    fname = f"valid_img{i*5}.jpg"
    idx = images_df.loc[images_df["NAME"] == fname]["IMAGE_ID"].values[0]
    rimg = cv2.imread("data/frames/" + fname)

    # Load query keypoints and descriptors
    points = point_desc_df.loc[point_desc_df["IMAGE_ID"] == idx]
    kp_query = np.array(points["XY"].to_list())
    desc_query = np.array(points["DESCRIPTORS"].to_list()).astype(np.float32)

    # Find correspondance and solve pnp
    retval, rvec, tvec, inliers = pnpsolver((kp_query, desc_query), (kp_model, desc_model),self_flag=self_flag)
    if not retval:
        continue
```



```python
    if not self_flag:
        return cv2.solvePnPRansac(
            pointsmodel, pointsquery, cameraMatrix, distCoeffs,
            iterationsCount=100, reprojectionError=8.0, confidence=0.99
        )
    return ransac_p3p(pointsmodel, pointsquery, cameraMatrix, distCoeffs)
```

用 self_flag 決定用自己的 p3p_ransac 還是 opencv 的 solvePnPRansac function 解出[R|t]

自己實作 p3p(distortion 部分用 chatGPT 得出):



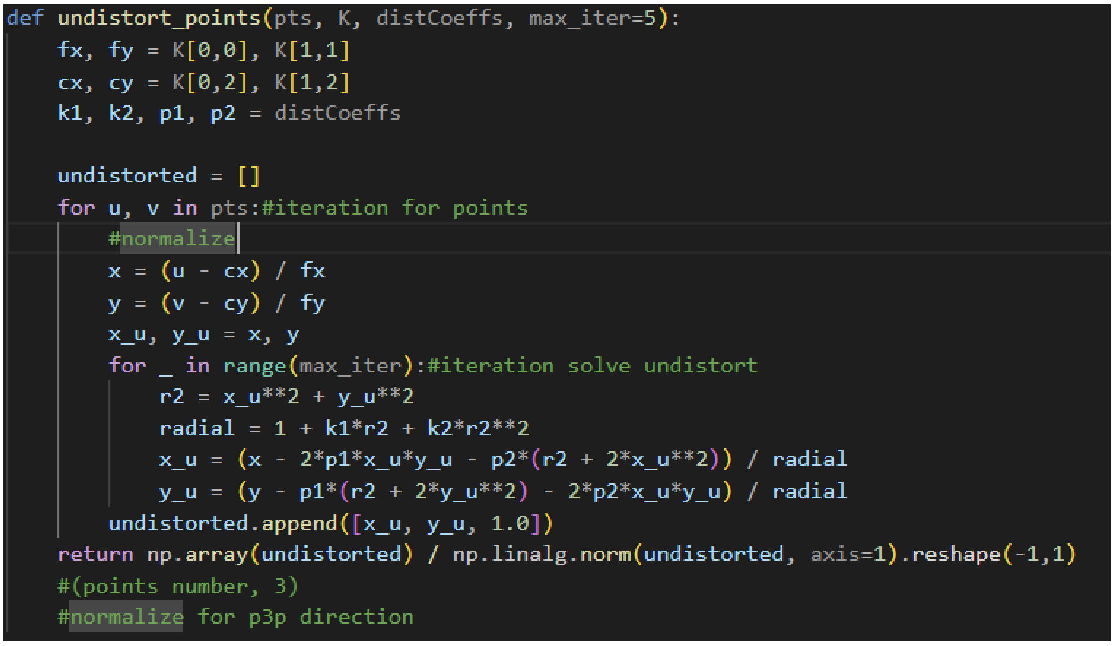
```
def World2Plane(world_pts, r, t, K, distCoeffs):

    #world to camera
    pts_cam = (r @ world_pts.T + t.reshape(3,1)).T

    #camera normalize
    x = pts_cam[:,0] / pts_cam[:,2]
    y = pts_cam[:,1] / pts_cam[:,2]

    #distortion
    k1, k2, p1, p2 = distCoeffs
    r2 = x**2 + y**2
    radial = 1 + k1*r2 + k2*r2**2
    x_dist = x * radial + 2*p1*x*y + p2*(r2 + 2*x**2)
    y_dist = y * radial + p1*(r2 + 2*y**2) + 2*p2*x*y

    u = K[0,0]*x_dist + K[0,2]
    v = K[1,1]*y_dist + K[1,2]
    return np.vstack([u, v]).T #(points number, 2)
```

World2Plane:

1.把 world 座標轉乘 camera 座標

2.camera 座標標準化

3.用 k1,k2,p1,p2 與 instinct matrix 得到 distortion 後的 plane 座標

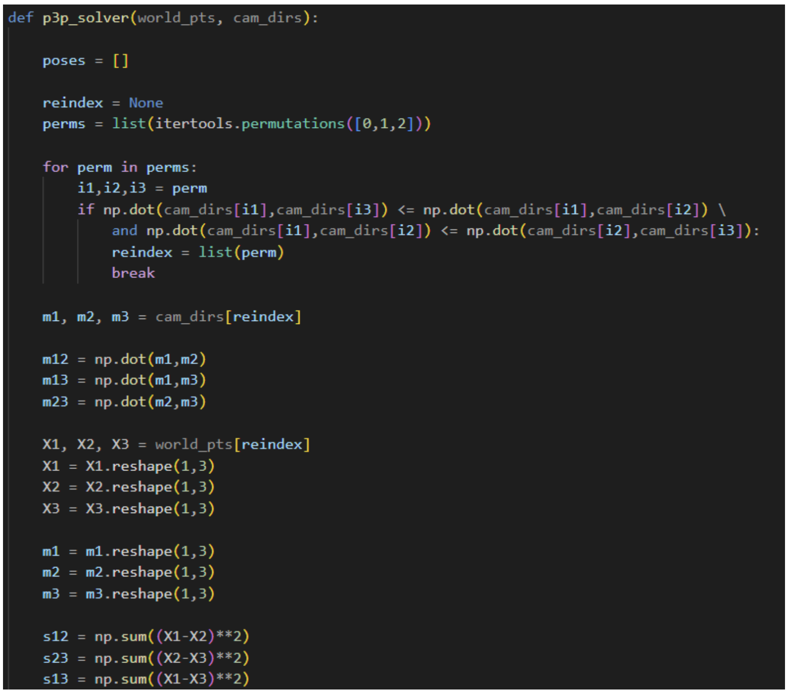


```
def undistort_points(pts, K, distCoeffs, max_iter=5):
    fx, fy = K[0,0], K[1,1]
    cx, cy = K[0,2], K[1,2]
    k1, k2, p1, p2 = distCoeffs

    undistorted = []
    for u, v in pts:#iteration for points
        #normalize
        x = (u - cx) / fx
        y = (v - cy) / fy
        x_u, y_u = x, y
        for _ in range(max_iter):#iteration solve undistort
            r2 = x_u**2 + y_u**2
            radial = 1 + k1*r2 + k2*r2**2
            x_u = (x - 2*p1*x_u*y_u - p2*(r2 + 2*x_u**2)) / radial
            y_u = (y - p1*(r2 + 2*y_u**2) - 2*p2*x_u*y_u) / radial
        undistorted.append([x_u, y_u, 1.0])
    return np.array(undistorted) / np.linalg.norm(undistorted, axis=1).reshape(-1,1)
    #(points number, 3)
    #normalize for p3p direction
```

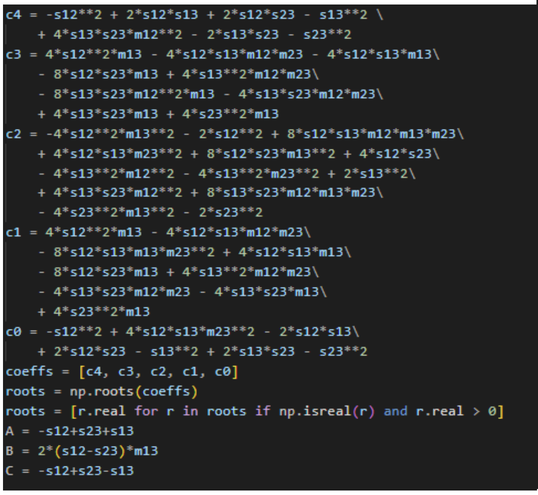
undistort_points:

1.用 iteration 的方式估計 undistorted 的 camera 座標

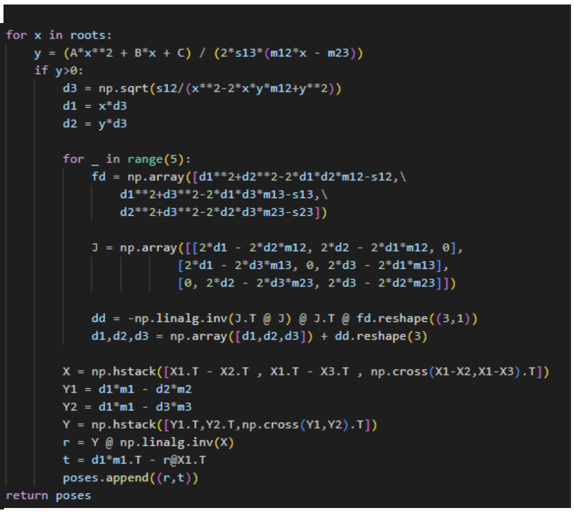
2.將 camera 座標 normalize 成單位方向，p3p 要用



```
def p3p_solver(world_pts, cam_dirs):

    poses = []

    reindex = None
    perms = list(itertools.permutations([0,1,2]))

    for perm in perms:
        i1,i2,i3 = perm
        if np.dot(cam_dirs[i1],cam_dirs[i3]) <= np.dot(cam_dirs[i1],cam_dirs[i2]) \
            and np.dot(cam_dirs[i1],cam_dirs[i2]) <= np.dot(cam_dirs[i2],cam_dirs[i3]):
            reindex = list(perm)
            break

    m1, m2, m3 = cam_dirs[reindex]

    m12 = np.dot(m1,m2)
    m13 = np.dot(m1,m3)
    m23 = np.dot(m2,m3)

    X1, X2, X3 = world_pts[reindex]
    X1 = X1.reshape(1,3)
    X2 = X2.reshape(1,3)
    X3 = X3.reshape(1,3)

    m1 = m1.reshape(1,3)
    m2 = m2.reshape(1,3)
    m3 = m3.reshape(1,3)

    s12 = np.sum((X1-X2)**2)
    s23 = np.sum((X2-X3)**2)
    s13 = np.sum((X1-X3)**2)
```



```
c4 = -s12**2 + 2*s12*s13 + 2*s12*s23 - s13**2 \
    + 4*s13*s23*m12**2 - 2*s13*s23 - s23**2
c3 = 4*s12**2*m13 - 4*s12*s13*m12*m23 - 4*s12*s13*m13\
    - 8*s12*s23*m13 + 4*s13**2*m12*m23\
    - 8*s13*s23*m12**2*m13 - 4*s13*s23*m12*m23\
    + 4*s13*s23*m13 + 4*s23**2*m13
c2 = -4*s12**2*m13**2 - 2*s12**2 + 8*s12*s13*m12*m13*m23\
    + 4*s12*s13*m23**2 + 8*s12*s23*m13**2 + 4*s12*s23\
    - 4*s13**2*m12**2 - 4*s13**2*m23**2 + 2*s13**2\
    + 4*s13*s23*m12**2 + 8*s13*s23*m12*m13*m23\
    - 4*s23**2*m13**2 - 2*s23**2
c1 = 4*s12**2*m13 - 4*s12*s13*m12*m23\
    - 8*s12*s13*m13*m23**2 + 4*s12*s13*m13\
    - 8*s12*s23*m13 + 4*s13**2*m12*m23\
    - 4*s13*s23*m12*m23 - 4*s13*s23*m13\
    + 4*s23**2*m13
c0 = -s12**2 + 4*s12*s13*m23**2 - 2*s12*s13\
    + 2*s12*s23 - s13**2 + 2*s13*s23 - s23**2
coeffs = [c4, c3, c2, c1, c0]
roots = np.roots(coeffs)
roots = [r.real for r in roots if np.isreal(r) and r.real > 0]
A = -s12+s23+s13
B = 2*(s12-s23)*m13
C = -s12+s23-s13
```



```
for x in roots:
    y = (A*x**2 + B*x + C) / (2*s13*(m12*x - m23))
    if y>0:
        d3 = np.sqrt(s12/(x**2-2*x*y*m12+y**2))
        d1 = x*d3
        d2 = y*d3

        for _ in range(5):
            fd = np.array([d1**2+d2**2-2*d1*d2*m12-s12,\
                d1**2+d3**2-2*d1*d3*m13-s13,\
                d2**2+d3**2-2*d2*d3*m23-s23])

            J = np.array([[2*d1 - 2*d2*m12, 2*d2 - 2*d1*m12, 0],
                          [2*d1 - 2*d3*m13, 0, 2*d3 - 2*d1*m13],
                          [0, 2*d2 - 2*d3*m23, 2*d3 - 2*d2*m23]])

            dd = -np.linalg.inv(J.T @ J) @ J.T @ fd.reshape((3,1))
            d1,d2,d3 = np.array([d1,d2,d3]) + dd.reshape(3)

        X = np.hstack([X1.T - X2.T , X1.T - X3.T , np.cross(X1-X2,X1-X3).T])
        Y1 = d1*m1 - d2*m2
        Y2 = d1*m1 - d3*m3
        Y = np.hstack([Y1.T,Y2.T,np.cross(Y1,Y2).T])
        r = Y @ np.linalg.inv(X)
        t = d1*m1.T - r@X1.T
        poses.append((r,t))
return poses
```

p3p_solver:

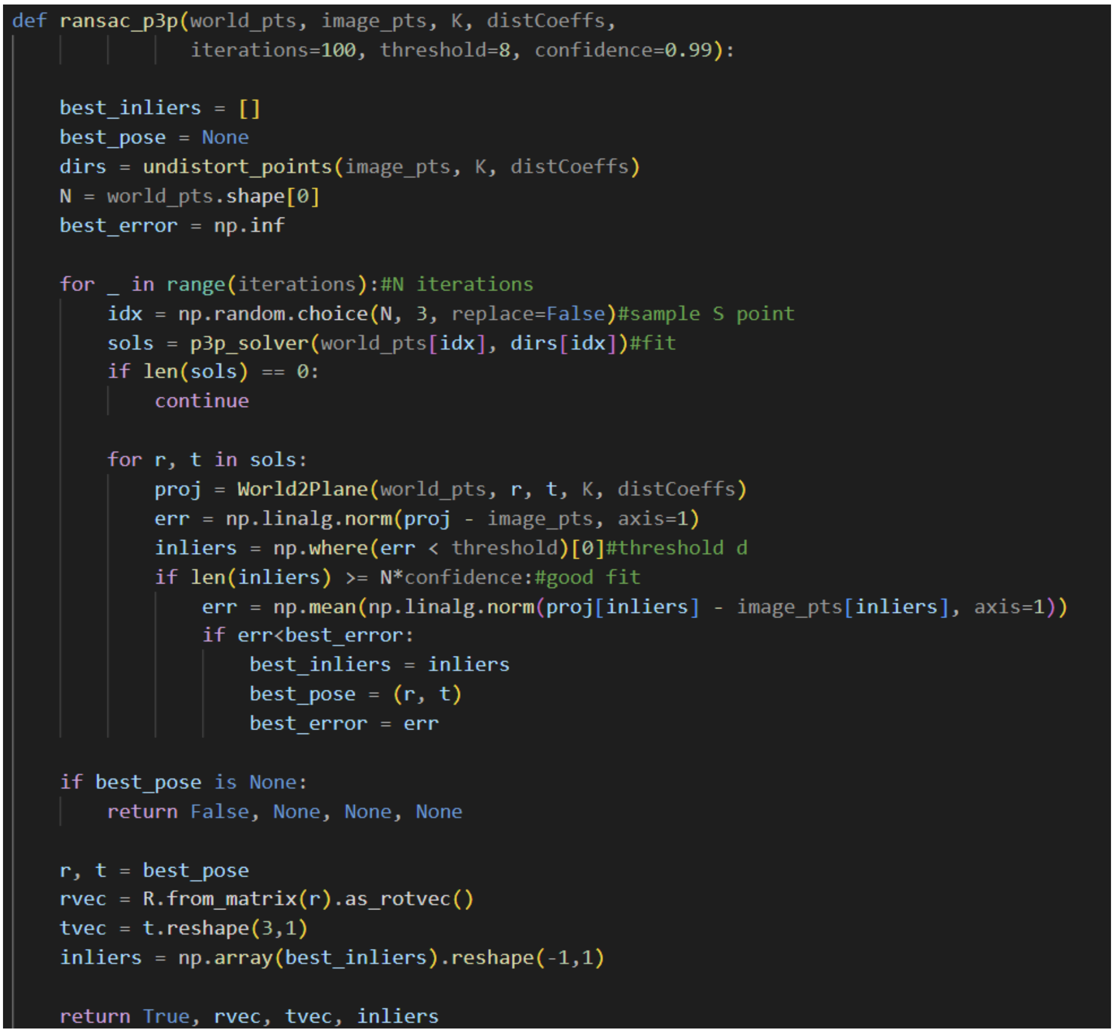
根據 P3P Made Easy

1.使用 world points+對應 camera dir，求出對應 c4,c3,c2,c1,c0 後求根 x

2.用 x 求出 y 與三個點的深度 d1,d2,d3

3. 用 Gauss-Newton method 來 refine 深度 d1,d2,d3 (由 gemini 得出)

4.用 d1,d2,d3 與其他參數算出 world to camera 的 R,t



```
def ransac_p3p(world_pts, image_pts, K, distCoeffs,
               iterations=100, threshold=8, confidence=0.99):

    best_inliers = []
    best_pose = None
    dirs = undistort_points(image_pts, K, distCoeffs)
    N = world_pts.shape[0]
    best_error = np.inf

    for _ in range(iterations):#N iterations
        idx = np.random.choice(N, 3, replace=False)#sample S point
        sols = p3p_solver(world_pts[idx], dirs[idx])#fit
        if len(sols) == 0:
            continue

        for r, t in sols:
            proj = World2Plane(world_pts, r, t, K, distCoeffs)
            err = np.linalg.norm(proj - image_pts, axis=1)
            inliers = np.where(err < threshold)[0]#threshold d
            if len(inliers) >= N*confidence:#good fit
                err = np.mean(np.linalg.norm(proj[inliers] - image_pts[inliers], axis=1))
                if err<best_error:
                    best_inliers = inliers
                    best_pose = (r, t)
                    best_error = err

    if best_pose is None:
        return False, None, None, None

    r, t = best_pose
    rvec = R.from_matrix(r).as_rotvec()
    tvec = t.reshape(3,1)
    inliers = np.array(best_inliers).reshape(-1,1)

    return True, rvec, tvec, inliers
```

ransac_p3p:

實作 ransac

在多次 iteraion 中

1.每次隨機選 3 組點用 p3p_solver 去 fit

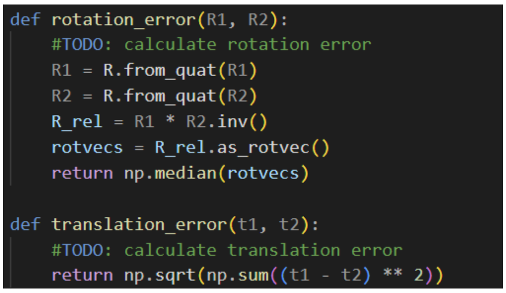
2.fit 結果用來把 world 座標用 World2Plane 得到 plane 座標

3.算有多少點誤差在 threshold 內

4.如果在 threshold 內的 inliner 數量>總數*confidence，則是 good fit

回傳最好的 fit 結果

Step2:error 計算



```
def rotation_error(R1, R2):
    #TODO: calculate rotation error
    R1 = R.from_quat(R1)
    R2 = R.from_quat(R2)
    R_rel = R1 * R2.inv()
    rotvecs = R_rel.as_rotvec()
    return np.median(rotvecs)

def translation_error(t1, t2):
    #TODO: calculate translation error
    return np.sqrt(np.sum((t1 - t2) ** 2))
```

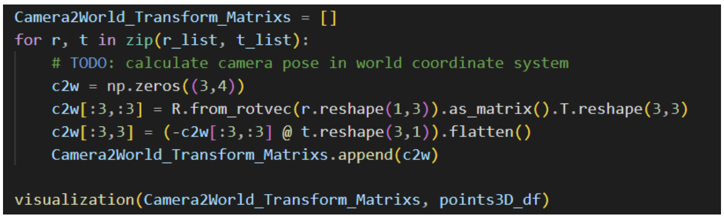
rotation error:

把 R1 與 R2 的 inverse 相乘後轉乘 rvec 形式，取中位數

translation error:

算 t1,t2 的 2-norm distance

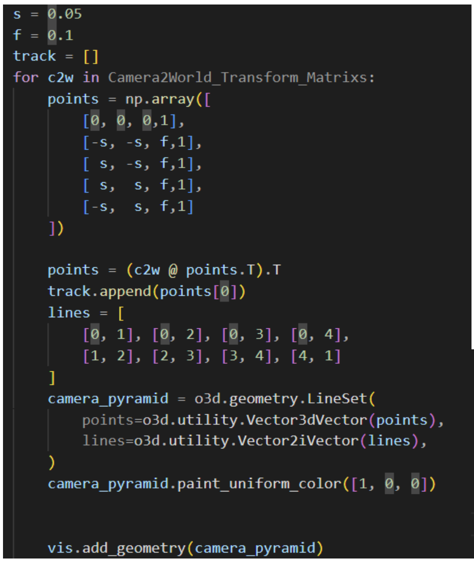
Step3:畫相機位置與軌跡



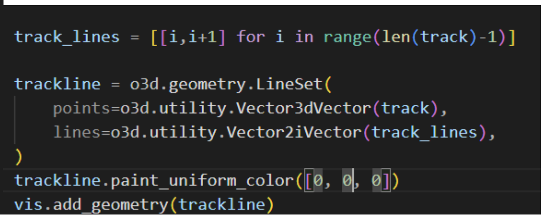
```
Camera2World_Transform_Matrixs = []
for r, t in zip(r_list, t_list):
    # TODO: calculate camera pose in world coordinate system
    c2w = np.zeros((3,4))
    c2w[:3,:3] = R.from_rotvec(r.reshape(1,3)).as_matrix().T.reshape(3,3)
    c2w[:3,3] = (-c2w[:3,:3] @ t.reshape(3,1)).flatten()
    Camera2World_Transform_Matrixs.append(c2w)

visualization(Camera2World_Transform_Matrixs, points3D_df)
```

先把 world_to_camera matrix 轉成 camera_to_world matrix

Xw = R^{-1} * Xc - R^{-1} * t



```
s = 0.05
f = 0.1
track = []
for c2w in Camera2World_Transform_Matrixs:
    points = np.array([
        [0, 0, 0,1],
        [-s, -s, f,1],
        [ s, -s, f,1],
        [ s,  s, f,1],
        [-s,  s, f,1]
    ])

    points = (c2w @ points.T).T
    track.append(points[0])
    lines = [
        [0, 1], [0, 2], [0, 3], [0, 4],
        [1, 2], [2, 3], [3, 4], [4, 1]
    ]
    camera_pyramid = o3d.geometry.LineSet(
        points=o3d.utility.Vector3dVector(points),
        lines=o3d.utility.Vector2iVector(lines),
    )
    camera_pyramid.paint_uniform_color([1, 0, 0])


    vis.add_geometry(camera_pyramid)
```

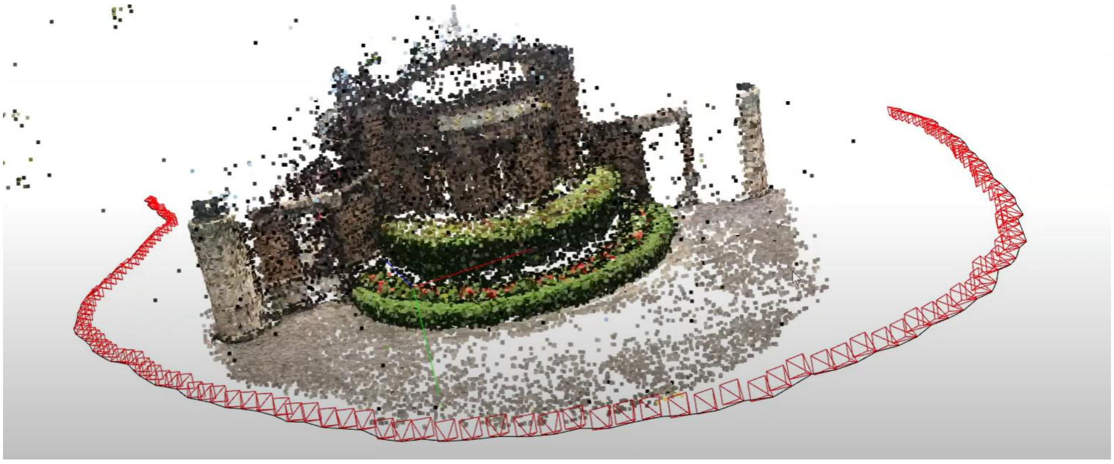


```
track_lines = [[i,i+1] for i in range(len(track)-1)]

trackline = o3d.geometry.LineSet(
    points=o3d.utility.Vector3dVector(track),
    lines=o3d.utility.Vector2iVector(track_lines),
)
trackline.paint_uniform_color([0, 0, 0])
vis.add_geometry(trackline)
```

visualization:

用 camera_to_world matrix 轉換角錐點(points)位置後，用 points 與 lines 畫一個四角椎 camera_pyramid，角錐頂點則一個一個連成線(trackline) result:

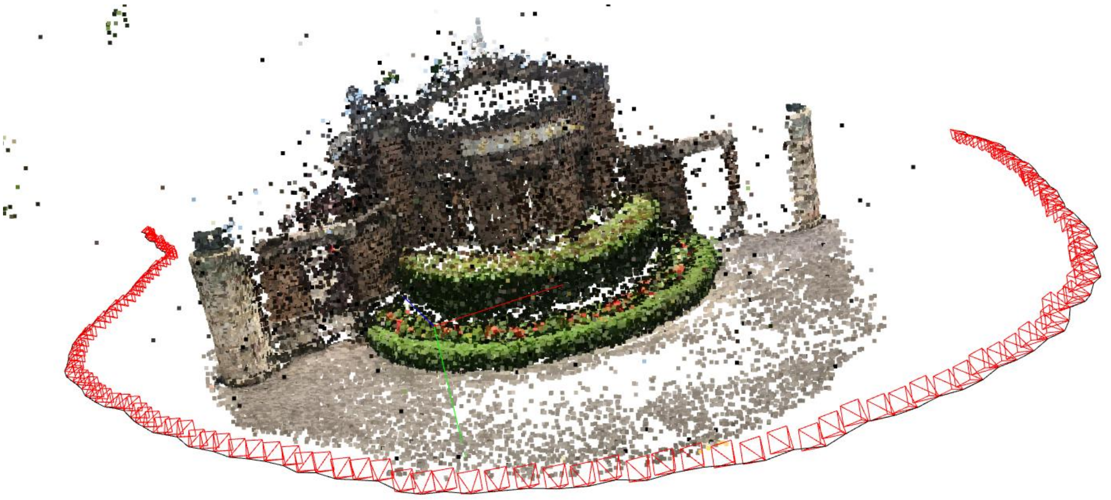
pnp+ransac with opencv:



rotation error: 8.942008584830542e-07

translation error: 0.00016467364717986384

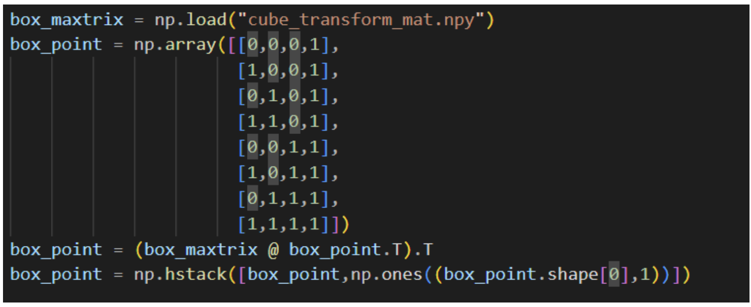
手刻 p3p+ransac:



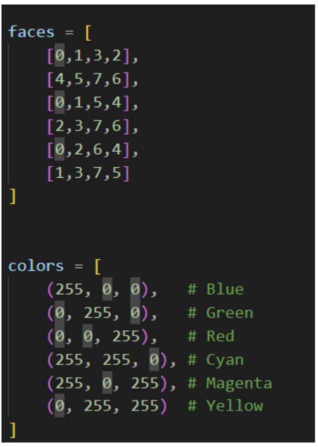
rotation error: 5.550656777684412e-06

translation error: 0.0019143133395420225

2-2



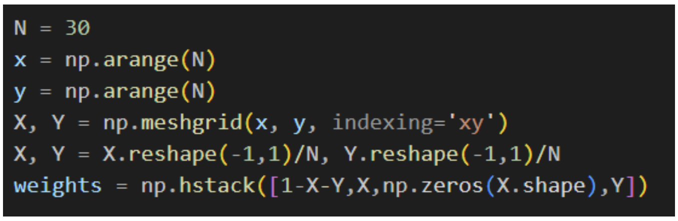
```
box_maxtrix = np.load("cube_transform_mat.npy")
box_point = np.array([[0,0,0,1],
                      [1,0,0,1],
                      [0,1,0,1],
                      [1,1,0,1],
                      [0,0,1,1],
                      [1,0,1,1],
                      [0,1,1,1],
                      [1,1,1,1]])
box_point = (box_maxtrix @ box_point.T).T
box_point = np.hstack([box_point,np.ones((box_point.shape[0],1))])
```



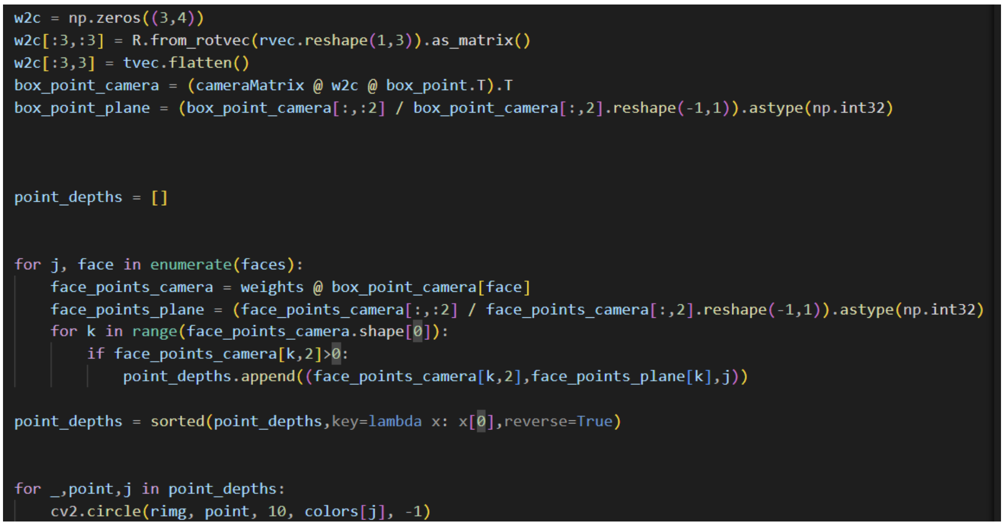
```
faces = [
    [0,1,3,2],
    [4,5,7,6],
    [0,1,5,4],
    [2,3,7,6],
    [0,2,6,4],
    [1,3,7,5]
]


colors = [
    (255, 0, 0),    # Blue
    (0, 255, 0),    # Green
    (0, 0, 255),    # Red
    (255, 255, 0),  # Cyan
    (255, 0, 255),  # Magenta
    (0, 255, 255)   # Yellow
]
```

將 box point 的 8 個點，每面需要的點與顏色設定好

8 個點為 1*1*1 的立方體乘上 cube_transform_mat



```
N = 30
x = np.arange(N)
y = np.arange(N)
X, Y = np.meshgrid(x, y, indexing='xy')
X, Y = X.reshape(-1,1)/N, Y.reshape(-1,1)/N
weights = np.hstack([1-X-Y,X,np.zeros(X.shape),Y])
```



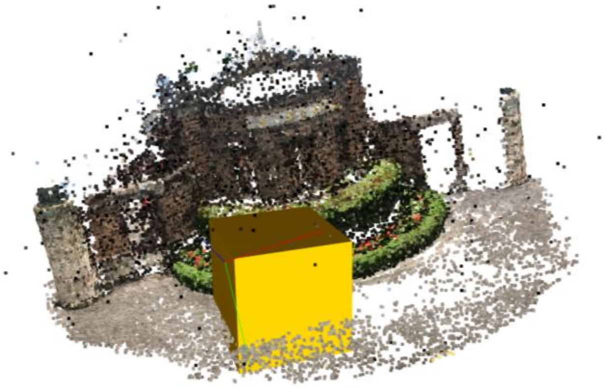
```
w2c = np.zeros((3,4))
w2c[:3,:3] = R.from_rotvec(rvec.reshape(1,3)).as_matrix()
w2c[:3,3] = tvec.flatten()
box_point_camera = (cameraMatrix @ w2c @ box_point.T).T
box_point_plane = (box_point_camera[:,:2] / box_point_camera[:,2].reshape(-1,1)).astype(np.int32)



point_depths = []


for j, face in enumerate(faces):
    face_points_camera = weights @ box_point_camera[face]
    face_points_plane = (face_points_camera[:,:2] / face_points_camera[:,2].reshape(-1,1)).astype(np.int32)
    for k in range(face_points_camera.shape[0]):
        if face_points_camera[k,2]>0:
            point_depths.append((face_points_camera[k,2],face_points_plane[k],j))

point_depths = sorted(point_depths,key=lambda x: x[0],reverse=True)


for _,point,j in point_depths:
    cv2.circle(rimg, point, 10, colors[j], -1)
```

1.用 world_to_camera matrix 與 instinct matrix of camera 把 box point 的點投射到 plane 上

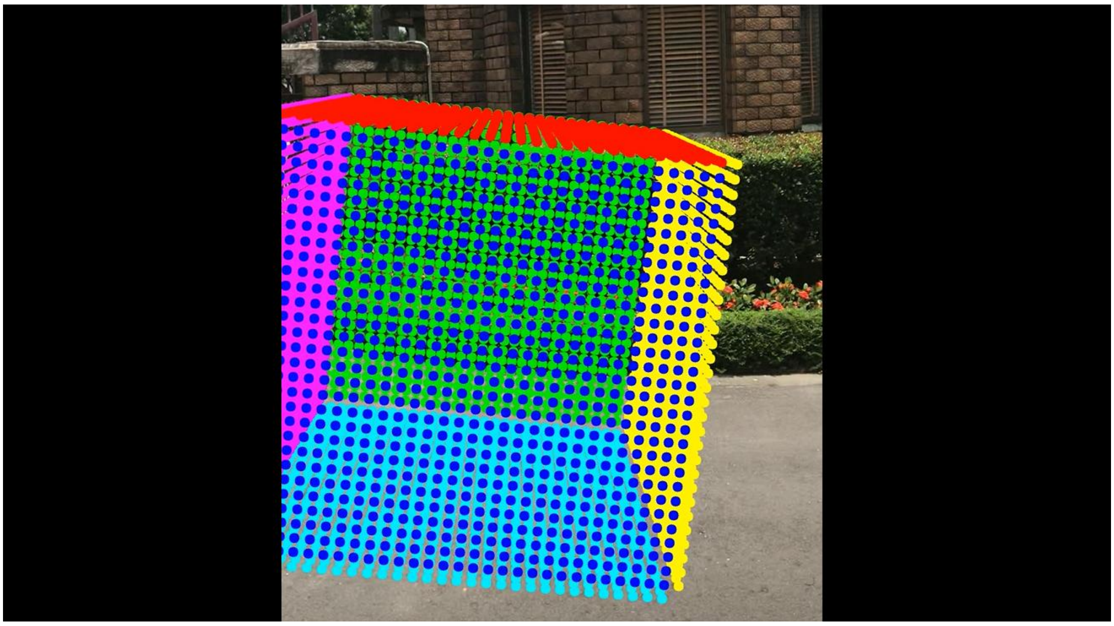
2.接著用 meshgrid 得出每個面的點所需要的四頂點 weights，將每一面的平行四邊形分成 30*30 個點

3.根據點的深度做排序，從深度高到低一點一點畫上去，並排除深度<0(在相機後面)的點

方塊位置:



result(影片截圖):



影片連結: https://youtu.be/XtT7NHUgbVY