No.: Chan JianPeng   Date:

Given that $V(\vec{r}) = \frac{V_0}{2}\frac{z^2 - s^2/2}{d^2}$, in spherical coordinates,

in cartesian coordinates, it is written as

$$V(x,y,z) = \frac{V_0}{2}\left(\frac{z^2 - \frac{(x^2+y^2)}{2}}{d^2}\right)$$

$$\vec{E} = -\nabla \vec{V}$$

$$= -\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}\left[\frac{V_0}{2}\left(\frac{z^2 - \left(\frac{x^2+y^2}{2}\right)}{d^2}\right)\right]$$

$$= \begin{bmatrix} \frac{V_0}{2}\left(\frac{x}{d^2}\right) \\ \frac{V_0}{2}\left(\frac{y}{d^2}\right) \\ -\frac{V_0}{2}\left(\frac{2z}{d^2}\right) \end{bmatrix}$$

Also, $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$ $\because \vec{B} = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix}$

$$= \frac{qV_0}{2d^2}\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} + q\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}\times\begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix}$$

$$\vec{F} = q\frac{V_0}{2d^2}\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} + \begin{pmatrix} qV_yB_0 \\ -qV_xB_0 \\ 0 \end{pmatrix} = \frac{qV_0}{2d^2}\begin{pmatrix} x \\ y \\ 2z \end{pmatrix} + qB_0\begin{pmatrix} V_y \\ -V_x \\ 0 \end{pmatrix}$$

acceleration, $\vec{a} = \frac{q}{m}\frac{V_0}{2d^2}\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} + \frac{qB_0}{m}\begin{pmatrix} V_y \\ -V_x \\ 0 \end{pmatrix}$

$$= \frac{q}{m}\begin{bmatrix} B_0V_y + \frac{V_0}{2d^2}x \\ \frac{V_0}{2d^2}y - B_0V_x \\ -\frac{V_0}{d^2}z \end{bmatrix}$$

$\therefore$ The above equation can be expressed as a system of 3 2^{nd} order ODE

$$\ddot{x} = \frac{q}{m}\left(B_0V_y + \frac{V_0}{2d^2}x\right)$$

$$\ddot{y} = \frac{q}{m}\frac{V_0}{2d^2}y - B_0V_x$$

$$\ddot{z} = \frac{q}{m}\left[-\frac{V_0}{d^2}z\right]$$

By converting, $\dot{x} = V_x$, $\dot{y} = V_y$, $\dot{z} = V_z$, we can convert the 3 2^{nd} order ODE into 6 1^{st} order ODE.

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$$\dot{x} = V_x$$
$$\dot{V}_x = \frac{q}{m}\left[B_0 V_y + \frac{V_0}{2d^2}x\right]$$

$$\dot{y} = V_y$$
$$\dot{V}_y = \frac{q}{m}\left[\frac{V_0}{2d^2}y - B_0 V_x\right]$$

$$\dot{z} = V_z$$
$$\dot{V}_z = -\frac{q}{m}\left[\frac{V_0}{d^2}\right]z$$

Plug ~~ing~~ these equations into the code f function handle
where $Y(1) \Rightarrow V_x$,
$Y(2) \Rightarrow x$
$Y(3) \Rightarrow V_y$
$Y(4) \Rightarrow y$
$Y(5) \Rightarrow V_z$
$Y(6) \Rightarrow z$

Solving the 6 1st order ODE using inbuilt ode45 function is a shorter and more efficient method to retrieve the trajectory solution. =)
The said method was taught by Dr Fedor Duzhin in the module Ordinary Differential Equations.