PH2102 - Electromagnetism Bonus Homework

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1 Objective

The goal is to make an animation of a charged particle in a Penning trap.

2 Crisis

As I have started this homework the night before it is due, the work is quite rush so some parts are omitted. As such, I am sorry. I just hope to earn some bonus mark by attempting the homework.

3 Research

The Penning trap has a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ which is in the z direction. Any charged particle will be able to remain in cyclotron motion with frequency:

$$\omega_c = \frac{eB}{m}$$

The particle motion is bound by superimposing an electric quadrupole potential:

$$V(\hat{\mathbf{r}}) = \frac{V_0}{2}\frac{z^2 - s^2/2}{d^2}$$

Where z and s are cylindrical coordinates, d is the trap dimension.

The electric field due to this potential is the negative gradient in cylindrical coordinates:

$$\mathbf{E} = -\nabla\mathbf{V} = \frac{V_0}{2d^2}s\hat{\mathbf{s}} - \frac{V_0}{d^2}z\hat{\mathbf{z}}$$

The motion of the particle is due to the Lorentzian Force:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

The acceleration is then:

$$a = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

With the few above relations at hand, we can obtain the equations of motion for the particle. [1]

$$\ddot{x} - \omega_c \dot{y} - \frac{1}{2}\omega_a^2 x = 0$$

$$\ddot{y} - \omega_c \dot{x} - \frac{1}{2}\omega_a^2 y = 0$$

$$\ddot{z} + \omega_a^2 z = 0$$

The first 2 equations gives rise to a set of 'modulated' circulating motion equations:

$$x(t) = A\cos(\bar{\omega_c} t) + B\cos(\bar{\omega_m} t)$$

$$y(t) = A\sin(\bar{\omega_c} t) + B\sin(\bar{\omega_m} t)$$

Where the $\bar{\omega_c}$ term is much higher than the $\bar{\omega_m}$ term.

The third equation gives a sinusoidal oscillation in the z axis:

$$z(t) = C\cos(\omega_a t)$$

4 Quick Implementation

Now that we have obtained the equation of motions of all 3 axes in Cartesian form, plugging these into a plot should be quite straightforward.

I have set the constant terms to arbitrary values tuned to fit the Penning trap.

$$x(t) = 4\cos(0.25t) + 0.3\cos(33t)$$

$$y(t) = 4\sin(0.25t) + 0.3\sin(33t)$$

$$z(t) = 3\cos(8t)$$

The plot obtained somewhat resembles the trajectory that was found from various sources. [1, 2]

Plot is shown in figure 1.

References

[1] Joon S. H. *Particle Motion in a Penning Trap.* `http://demonstrations.wolfram.com/ParticleMotionInAPenningTrap/` Wolfram Demonstrations Project, 2016.

[2] Gabrielse, G. *Chapter 2: Particle Motions in a Penning Trap* `http://gabrielse.physics.harvard.edu/gabrielse/papers/1990/1990_tjoelker/chapter_2.pdf` Harvard University, 1990.

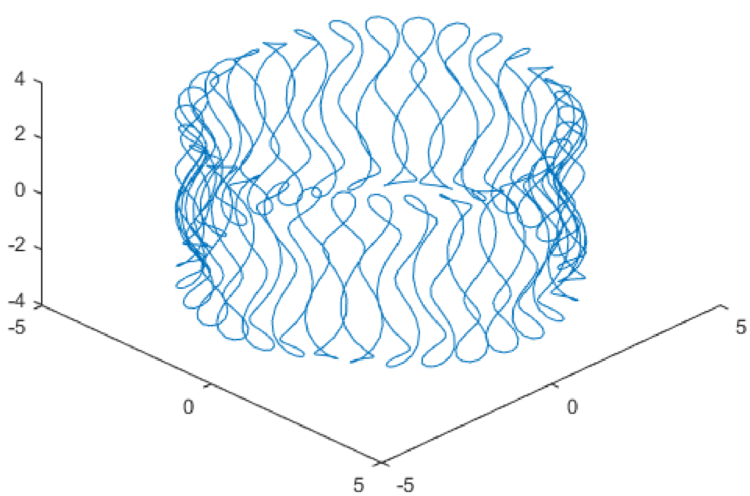

Figure 1: Trajectory of particle in Penning trap.