PH2102 Bonus Homework Notes

The code that is submitted uses Python 3.6.

First of all, I start the code by setting up all the important parameters:

1. Electron charge (q)
2. Penning trap magnetic field (B)
3. Penning trap potential (V)
4. Electron mass (m)
5. Step size for iteration (h)
6. Minimum axial distances from the trap electrodes (z_0)
7. Minimum radial distances from the trap electrodes (s_0)
8. Characteristic trap dimension (d). $d^2 = ½(z_0^2 + \rho_0^2/2)$
9. k1, k2, k3 are some constant containing q, B, m, V, h, and d. These constants were separated to make the code more well written.

Next, setting up all the initial values of the particle, labeled as x_0, v_0, and a_0, for position, velocity and acceleration respectively.

The main part of the code is about the iteration formula. From finite difference method, we know:

$$\mathrm{a}_j[\mathrm{i}] = \frac{\mathrm{x}_j[\mathrm{i+1}] - 2\mathrm{x}_j[\mathrm{i}] + \mathrm{x}_j[\mathrm{i-1}]}{h^2} \quad (1)$$

$$\mathrm{v}_j[\mathrm{i}] = \frac{\mathrm{x}_j[\mathrm{i+1}] - \mathrm{x}_j[\mathrm{i-1}]}{2h} \quad (2)$$

From Physical law (Lorentz force, Coulomb's force). We found that Penning trap acceleration have this relation:

$$\mathrm{a}_x[\mathrm{i}] = \frac{qV}{2md^2}x[i] + \frac{qB}{m}v_y[i] \quad (3)$$

$$\mathrm{a}_y[\mathrm{i}] = \frac{qV}{2md^2}y[i] - \frac{qB}{m}v_x[i] \quad (4)$$

$$\mathrm{a}_z[\mathrm{i}] = -\frac{qV}{md^2}z[i] \quad (5)$$

Let,

$$\mathrm{k}_1 = qBh/2m$$

$$\mathrm{k}_2 = qVh^2/2md^2$$

$$\mathrm{k}_3 = \frac{1}{(\mathrm{k}_1{}^2+1)}$$

Plugging in all the equations above and rearranging all the variable yields

$$x[i+1] = \mathrm{k}_3[(k_2+2)x[i] + (\mathrm{k}_1{}^2 - 1)\,\mathrm{x}[\mathrm{i}-1] + (\mathrm{k}_2+2)\mathrm{k}_1 y[i] - 2\mathrm{k}_1 y[i-1] \quad (6)$$

$$y[i+1] = \mathrm{k}_3[(k_2+2)y[i] + (\mathrm{k}_1{}^2 - 1)\,\mathrm{y}[\mathrm{i}-1] - (\mathrm{k}_2+2)\mathrm{k}_1 x[i] + 2\mathrm{k}_1 x[i-1] \quad (7)$$

$$z[i+1] = (2 - 2\mathrm{k}_2)z[i] - \mathrm{z}[\mathrm{i}-1] \quad (8)$$

From Eq. (6) to (8), now we can construct the iteration formula to produce the animation.

PARAMETER SETUP:

The value for the parameters need to be adjusted carefully, since putting values randomly may not produce the expected motion. For example, If we put B with a small value and a very high initial velocity The radius of the motion may be just too large to be observed, and the reverse yields the same result.

There is also another condition in which doesn't allow us to put the value of the step size (h) too high, because since we are using iteration method, it's simply just an approximation, if the approximation runs through thousands of loops then the error will propagate and become more significant. In this code I put the $\mathrm{h} = 0.01\,{}^{\mathrm{m}}/_{\mathrm{qB}}$. This value is set to be this way because, the increment ${}^{\mathrm{qBh}}/_{\mathrm{m}}$ must be significantly small. Here I set the constant to be 0.01, which is relatively small enough to get the desired result. We are free to change the constant, the lower it is, the result will be better, but as for the cost, the generated animation will get slower.

Here the motion observed in the animation is a particle's motion trapped inside Penning trap. It's a combination of magnetron, cyclotron, and axial rotation motion, in which it's rotation axis have its own angular frequency (ω).

SCREENSHOTS:

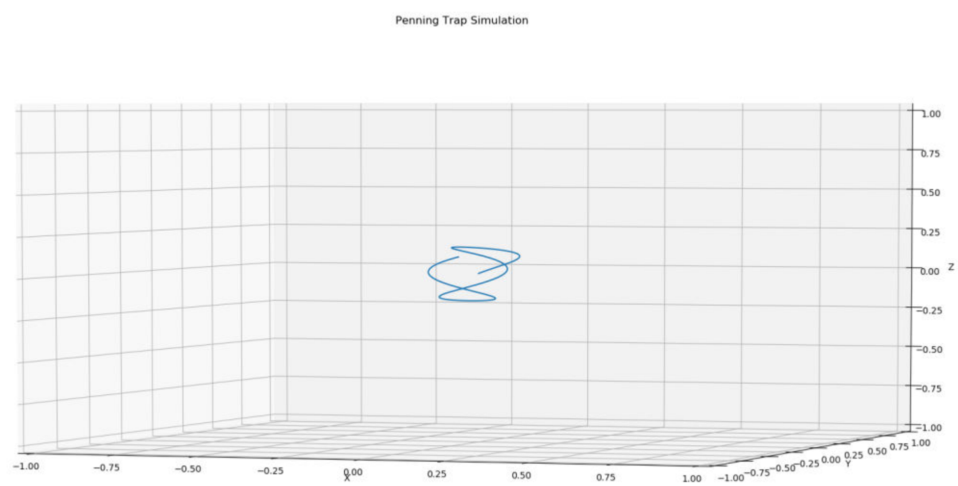

Figure 1

Figure 1 is a screenshot taken during the beginning of the animation. At first, the pattern seems to be quite chaotic. In the next figure, we will see how the pattern progresses.

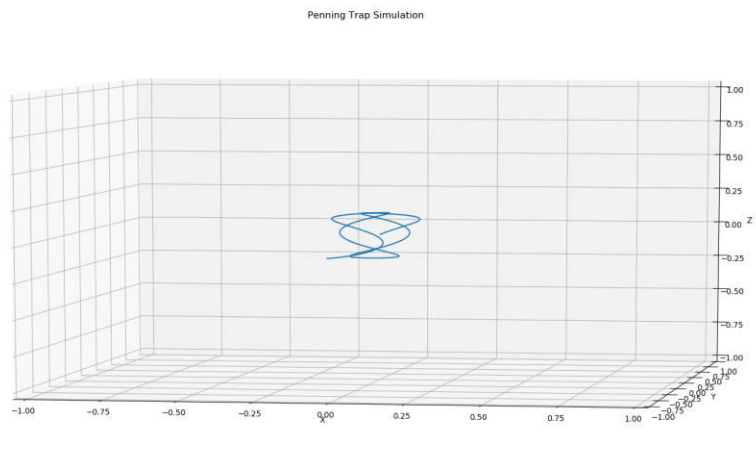

Figure 2

In this figure 2, I let the animation to continue a few more steps. Here is what I got, we can see that the particle is trapped in z direction, in the sense that the particle is oscillating back and forth in z-axis with amplitude of around 0.2. This oscillating motion in z-axis is called known as cyclotron oscillation/motion. But the overall pattern is still very abstract for us to understand at this point.

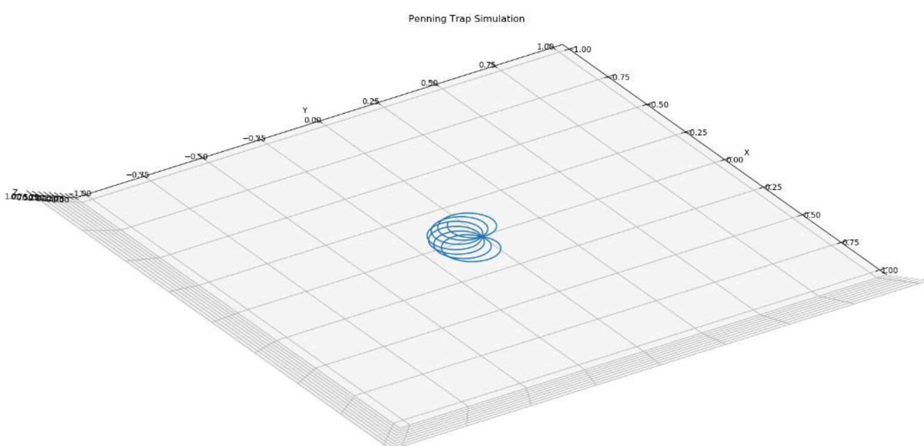

Figure 3

Figure 3 is taken almost at the same time with Figure 2 but from the different angle (viewed from z-axis). The pattern is clear if viewed from this particular angle. We can see that the particle spiral around a bigger circle centered at (0, 0). This motion is described as the magnetron and axial oscillatory motion. Axial oscillatory motion is the circular motion with the smaller radius, which is depicted in figure 3 as the small rings that goes around the bigger circle. The bigger circle in which the smaller ring rotates about is called the magnetron motion.

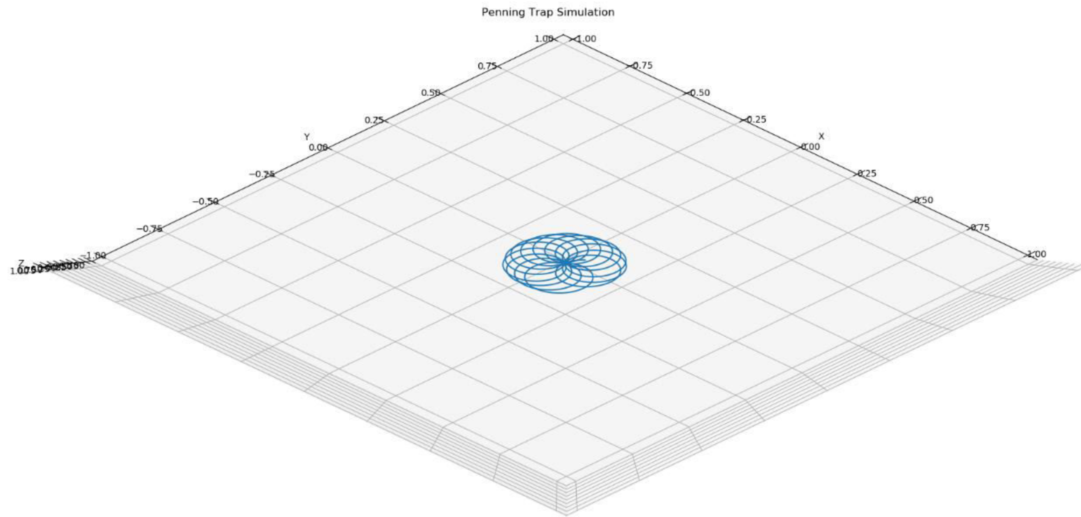

Figure 4

Figure 4 is the trace of the particle after a few minutes and almost complete a full loop of its magnetron cycle, viewed from z-axis

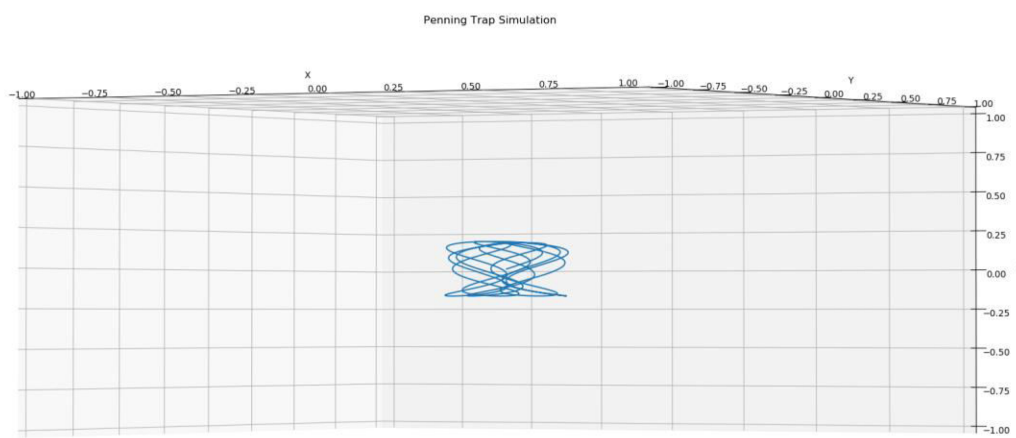

Figure 5

Figure 5 is also the trace of the particle after a few minutes and almost complete a full loop of its magnetron cycle but, viewed from x and y-axis.

Figure 4 and 5 confirm that the particle is indeed trapped in all axis and agrees with 3 circular motion that have been described through the 3 previous figures (Figure 1,2, and 3)

Additional screenshots:

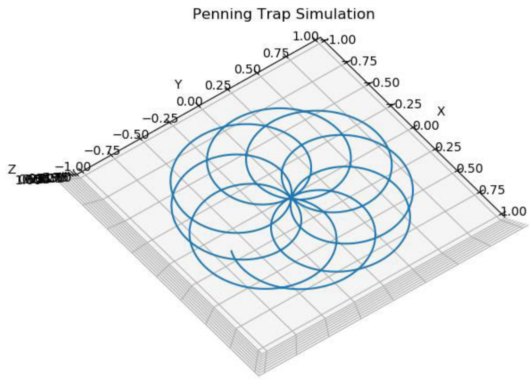

Figure 6

Figure 6 is the motion of the particle viewed from z-axis, when I changed the magnetic field (B) to be smaller than the previous screenshots. It makes sense that the radius will be larger since the Lorentz force acting on the particle is smaller. The B-field also affects the frequency of the cyclotron motion. You can see that the ring is now more loose compared to the previous figures.

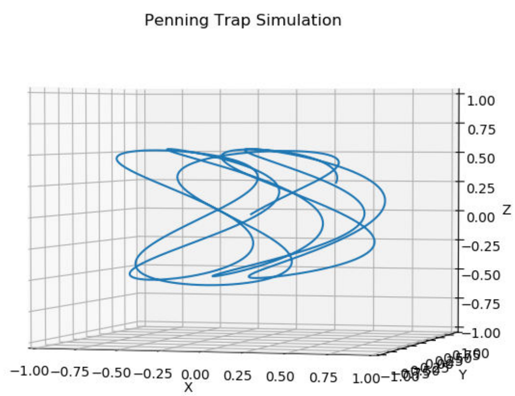

Figure 7

Figure 7 is essentially the same as figure 6 but, viewed from x and y-axis. The particle is still trapped in all axes but it only changes in radius, and frequency of the oscillatory motion.

In conclusion, Penning trap trapped a particle in all directions by making it oscillate in various axes, in this case is z-axis and radial axis. There are 3 oscillatory motion encountered in Penning trap, which is magnetron motion, cyclotron motion, and axial oscillation. These 3 motions are described in the explanation of the figures above.