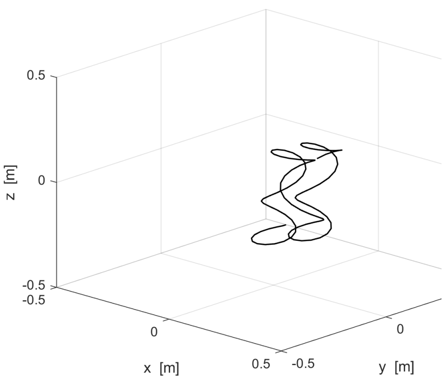
Animation of charged particle motion in a Penning trap

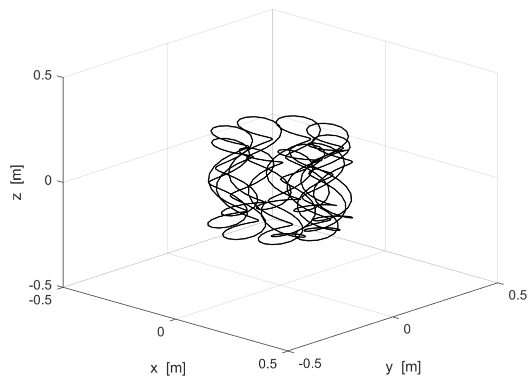
Penning trap simulation code and animation screenshots

```
clear all
close all

timesteps = 1000; % sets the number of time steps for the simulation
q = 1.602E-19; % charge of particle in trap
m = 1.67E-27; % mass of particle in trap
rho0 = 1; % minimum radial distance to trap electrodes
z0 = 1; % minimum axial distance to trap electrodes
d = sqrt(0.5*(z0^2 + (rho0)^2/2)); %characteristic trap dimension

V0 = 1E6; % potential constant V0
B0 = 0.5; % magnitude of uniform magnetic field

% Initialise vectors of displacement of particle in x, y, z directions
x = zeros(1,timesteps);
y = zeros(1,timesteps);
z = zeros(1,timesteps);

h = 1E-8; % set length of discrete time steps

x(1) = 0.1; y(1) = 0.1; z(1) = 0.1;

vx = 1E6; vy= 1E6; vz = 1E6;

x(2) = x(1) + vx*2*h;
y(2) = y(1) +vy*2*h;
z(2) = z(1) +vz*2*h;

%repeatedly used constants
kE = q*(h^2)*V0/(2*m*d^2); kB = q*h*B0/(m*2);
kz = h^2*q*V0/(m*d^2);

%displacement of particle
for t = 2:timesteps-1
    x(t+1) = (1/(1+kB^2))*(2*x(t)-x(t-1)+kE*x(t)-kB*y(t-1)+kB*(2*y(t) - y(t-1) + kE*y(t) + kB*(x(t-1))));
    y(t+1) = 2*y(t) - y(t-1) + kE*y(t) - kB*(x(t+1)-x(t-1)) ;
    z(t+1) = 2*z(t) - z(t-1) - kz*z(t);
end
```

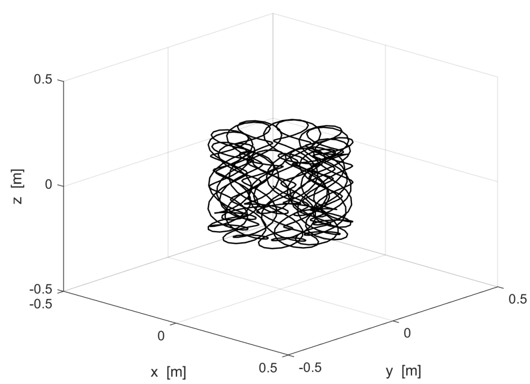
```
%final plot of particle trajectory
figure (1) %------------------------------------------------------------
set(gcf,'units','normalized','position',[0.670,0.6,0.3,0.3]);
plot3(x,y,z,'b','LineWidth',1);
xlabel('x [m]'); ylabel('y [m]'); zlabel('z [m]');
grid on
set(gca,'fontsize',14);

%animation of particle trajectory
figure(2) %-------------------------------------------------------------
curve = animatedline('linewidth',1);
 set(gca,'XLim',[-0.5 0.5],'YLim',[-0.5 0.5],'ZLim',[-0.5 0.5])
 view(43,24);
 xlabel('x [m]'); ylabel('y [m]'); zlabel('z [m]');
for i = 1:timesteps
    addpoints(curve,x(i),y(i),z(i))
    grid 'on'
    drawnow
end
```



(a)

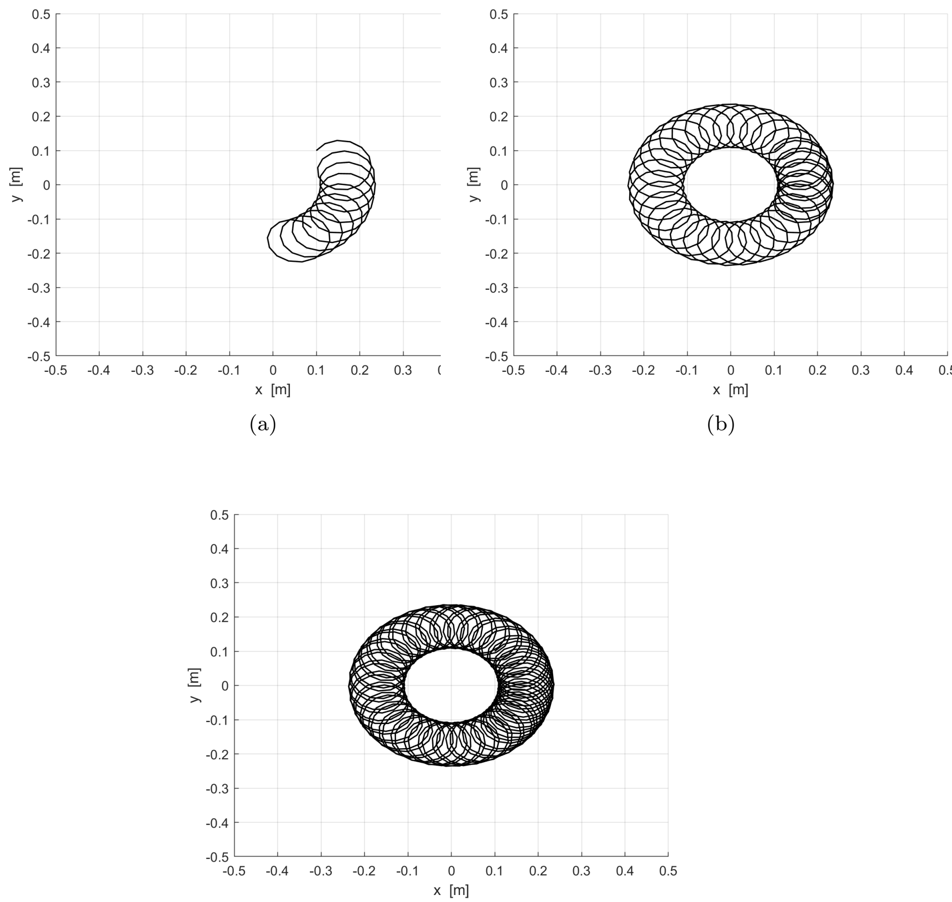


(b)



(c)

Figure 1: 'Front' view of Penning trap animation. The sub-figures (a), (b) and (c) show (in that order) the time evolution of the particle's trajectory in the presence of electric and magnetic fields in the Penning trap.



(a) (b) (c)

Figure 2: 'Top' view of Penning trap animation. The sub-figures (a), (b) and (c) show (in that order) the time evolution of the particle's trajectory in the presence of electric and magnetic fields in the Penning trap.

Animation of Charged Particle Motion in Penning Trap

Accompanying note for code

The purpose of a Penning trap is to contain charged particles within a spatial boundary using a combination of electric and magnetic fields. The details of the magnetic field strength $\vec{B}$ and the electric potential $V(\vec{r})$ are as follows:

$$\vec{B} = B_0\hat{z} \tag{1}$$

$$V(\vec{r}) = \frac{V_0}{2}\frac{z^2 - s^2/2}{d^2} \tag{2}$$

Rewriting Eqn (2) in Cartesian coordinates, we obtain

$$V(\vec{r}) = \frac{V_0}{2}\frac{z^2 - (x^2+y^2)/2}{d^2} \tag{3}$$

$$\begin{aligned} \vec{E}(\vec{r}) &= -\nabla V(\vec{r}) \\ &= -\frac{V_0}{2d^2}\left(-x\cdot\hat{x} - y\cdot\hat{y} + 2z\cdot\hat{z}\right) \end{aligned} \tag{4}$$

Hence we obtain the force on the particle due to electric field, $\vec{F_E}(\vec{r})$, and the force on the particle due to magnetic field, $\vec{F_B}(\vec{r})$. q is the charge of the particle in the trap and $\vec{v}$ is the velocity vector of the particle. The total force on the particle F can then be obtained by summing $\vec{F_E}(\vec{r})$ and $\vec{F_B}(\vec{r})$.

$$\begin{aligned} \vec{F_E}(\vec{r}) &= q\cdot\vec{E}(\vec{r}) \\ &= \frac{qV_0}{d^2}\left(-\frac{x}{2}\cdot\hat{x} - \frac{y}{2}\cdot\hat{y} + z\cdot\hat{z}\right) \end{aligned} \tag{5}$$

$$\begin{aligned} \vec{F_B}(\vec{r}) &= \vec{B}q\vec{v} \\ &= qB_0\left(v_y\cdot\hat{x} - v_x\cdot\hat{y}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} \vec{F}(\vec{r}) &= \vec{F_E}(\vec{r}) + \vec{F_B}(\vec{r}) \\ &= q\left[\left(\frac{V_0x}{2d^2} + B_0v_y\right)\cdot\hat{x} + \left(\frac{V_0y}{2d^2} - B_0v_x\right)\cdot\hat{y} - \frac{V_0z}{d^2}\cdot\hat{z}\right] \end{aligned} \tag{7}$$

From Equation (7), we can compute the total acceleration, $\vec{a}$, of the particle due to the electric and magnetic fields. m is the mass of the particle in the trap.

$$\begin{aligned}\vec{a} &= \frac{1}{m}\cdot\vec{F}(\vec{r}) \\ &= \frac{q}{m}\left[\left(\frac{V_0 x}{2d^2} + B_0 v_y\right)\cdot\hat{x} + \left(\frac{V_0 y}{2d^2} - B_0 v_x\right)\cdot\hat{y} - \frac{V_0 z}{d^2}\cdot\hat{z}\right]\end{aligned} \tag{8}$$

The components of acceleration are as follows:

$$a_x = \frac{q}{m}\left(\frac{V_0 x}{2d^2} + B_0 v_y\right) \tag{9}$$

$$a_y = \frac{q}{m}\left(\frac{V_0 y}{2d^2} - B_0 v_x\right) \tag{10}$$

$$a_z = -\frac{q V_0 z}{m d^2} \tag{11}$$

Using the numerical analysis method of finite differences, we are able to approximate the components of acceleration and velocity in the forms shown below:

$$a_x(t) = \frac{x(t+1) - 2x(t) + x(t-1)}{h^2} \tag{12}$$

$$a_y(t) = \frac{y(t+1) - 2y(t) + y(t-1)}{h^2} \tag{13}$$

$$a_z(t) = \frac{z(t+1) - 2z(t) + z(t-1)}{h^2} \tag{14}$$

$$v_x(t) = \frac{x(t+1) - x(t-1)}{2h} \tag{15}$$

$$v_y(t) = \frac{y(t+1) - y(t-1)}{2h} \tag{16}$$

$$v_z(t) = \frac{z(t+1) - z(t-1)}{2h} \tag{17}$$

Manipulating Equations (12), (13), (14), (15), (16), (17) we get:

$$x(t+1) = 2x(t) - x(t-1) + \frac{qh^2}{m}\left(\frac{V_0 x(n)}{2d^2} + B_0\left(\frac{y(t+1) - y(t-1)}{2h}\right)\right) \tag{18}$$

$$y(t+1) = 2y(t) - y(t-1) + \frac{qh^2}{m}\left(\frac{V_0 y(n)}{2d^2} - B_0\left(\frac{x(t+1) - x(t-1)}{2h}\right)\right) \tag{19}$$

$$z(t+1) = 2z(t) - z(t-1) - \frac{h^2 q V_0 z}{m d^2}\cdot z(t) \tag{20}$$

$$\text{We define} \quad k_E = \frac{qh^2V_0}{2md^2}, \quad k_B = \frac{qhB_0}{2m}, \quad k_z = \frac{h^2qV_0}{md^2}, \quad A = y(t+1) + k_B \cdot x(t+1)$$

By substituting Equation (19) into Equation (18) and making $x(t+1)$ the subject we obtain:

$$x(t+1) = \frac{1}{1+k_B^2}\left[(2+k_E)\,x(t) - x(n-1) - k_B \cdot y(n-1) + k_B \cdot A\right] \tag{21}$$

Equations (21), (19) and (20) can then be used to simulate the motion of the particle in the Penning trap.