

Problem 1. Edge Detection

a. Motivation and approach

根據題目要分別做出一階微分、二階微分與 canny 三種方法

一階微分法是先算出各 pixel 的 gradient 值，再將大於等於某閾值者標示為邊界

一階微分算法有四種:

3points: $\sqrt{(F - A_7)^2 + (F - A_5)^2}$

A_0	A_1	A_2
A_7	$F(j, k)$	A_3
A_6	A_5	A_4

roberts: $\sqrt{(F - A_4)^2 + (A_3 - A_5)^2}$

A_0	A_1	A_2
A_7	$F(j, k)$	A_3
A_6	A_5	A_4

$$\frac{1}{(K + 2)^2} \sqrt{(A_2 + KA_3 + A_4 - A_0 - KA_7 - A_6)^2 + (A_0 + KA_1 + A_2 - A_6 - KA_5 - A_4)^2}$$

Prewitt: $K = 1$

Sobel: $K = 2$

A_0	A_1	A_2
A_7	$F(j, k)$	A_3
A_6	A_5	A_4

二階微分法是先選擇性使用 Gaussian Low-pass Filter 降噪

其後算出各 pixel 的二階微分值，對每一 pixel 判斷其 4 個方向上是否兩邊各為一正一負，

若是且此兩二階微分相差在一閾值外則標示為邊界點

Finally, we perform zero crossing process.

$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

Gaussian Low-pass Filter

Let $I = \begin{bmatrix} P_{(i-1,j-1)} & P_{(i-1,j)} & P_{(i-1,j+1)} \\ P_{(i,j-1)} & P_{(i,j)} & P_{(i,j+1)} \\ P_{(i+1,j-1)} & P_{(i+1,j)} & P_{(i+1,j+1)} \end{bmatrix}$ become the partition matrix of image, if

one of the following conditions is true, then the pixel is the edge.

(1) $P_{(i-1,j-1)}P_{(i+1,j+1)} < 0$ and $|P_{(i-1,j-1)} - P_{(i+1,j+1)}| > T$

(2) $P_{(i-1,j+1)}P_{(i+1,j-1)} < 0$ and $|P_{(i-1,j+1)} - P_{(i+1,j-1)}| > T$

(3) $P_{(i-1,j)}P_{(i+1,j)} < 0$ and $|P_{(i-1,j)} - P_{(i+1,j)}| > T$

(4) $P_{(i,j-1)}P_{(i,j+1)} < 0$ and $|P_{(i,j-1)} - P_{(i,j+1)}| > T$

where T is the threshold, the negative product means they have different sign.

zero-crossing 判斷

二次微分算法有三種:

4-neighbor

$$\frac{1}{4} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

8-neighbor non separable

$$\frac{1}{8} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

8-neighbor separable

$$\frac{1}{8} \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

Canny 法將照片以某 Gaussian Low-pass Filter 降噪，再用 Sobel 算出各 pixel 的一階微分值與方向。為了只保留二階微分為 zero-crossing 的 pixel，判斷所有 pixel 其一階微分是否為 gradient 方向上的最大一階微分，若不是則將一階微分值設為零。接下來由兩個閾值將 pixel 分類為確定邊界、可能為邊界、還是不為邊界。最後再用 connected component labeling 判斷可能為邊界的點是否與已確定為邊界的點相鄰(8-connected)，相鄰者則標示為邊界。

Gaussian Low-pass Filter

$$\frac{1}{159} \begin{bmatrix} 2 & 4 & 5 & 4 & 2 \\ 4 & 9 & 12 & 9 & 4 \\ 5 & 12 & 15 & 12 & 5 \\ 4 & 9 & 12 & 9 & 4 \\ 2 & 4 & 5 & 4 & 2 \end{bmatrix}$$

雙閾值分類

Label each pixels according to two threshold: T_H, T_L

- $G_N(x, y) \geq T_H$ Edge Pixel
- $T_H > G_N(x, y) \geq T_L$ Candidate Pixel
- $G_N(x, y) < T_L$ Non-edge Pixel

b. Original Images

sample1.raw



sample2.raw

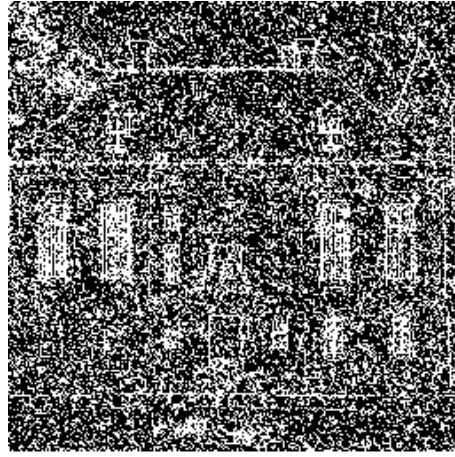


sample3.raw

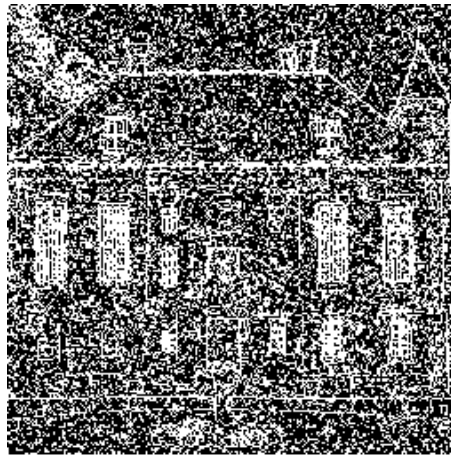


c. Output Images

first-order derivative 3points on three inputs (threshold=40)



first-order derivative roberts on three inputs (threshold=40)



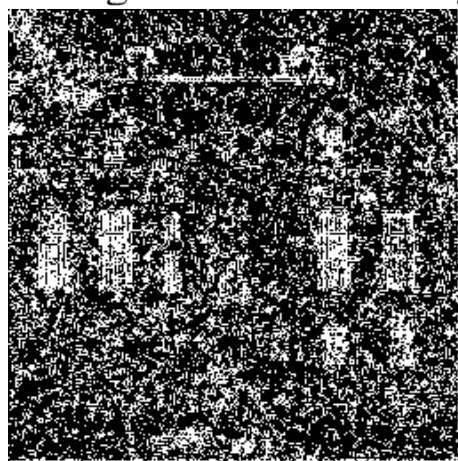
first-order derivative prewitt on three inputs (threshold=35)



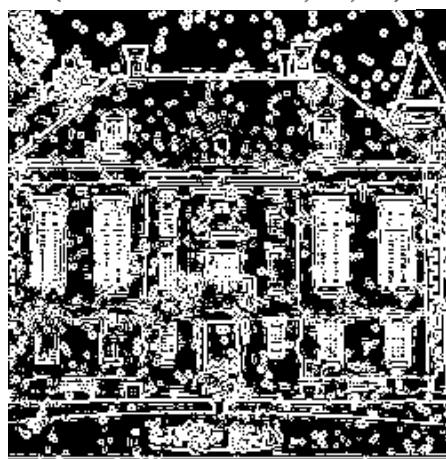
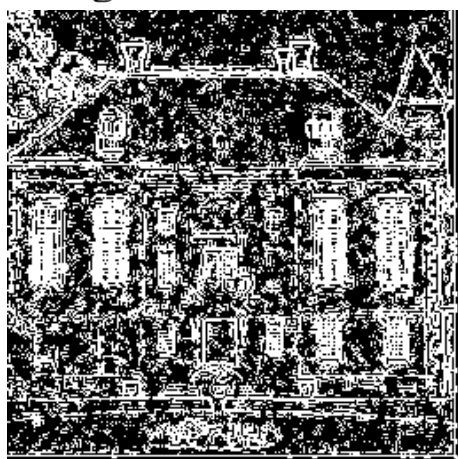
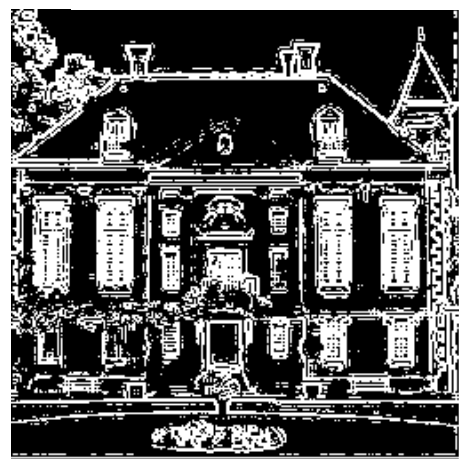
first-order derivative sobel on three inputs (threshold=40)



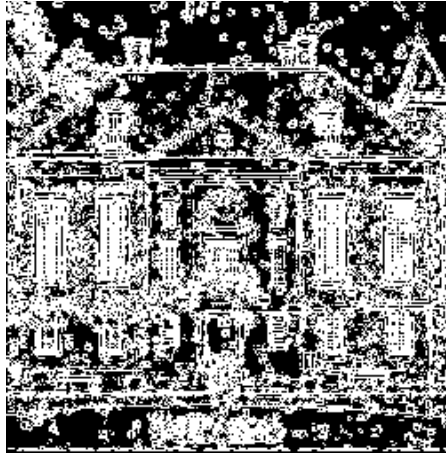
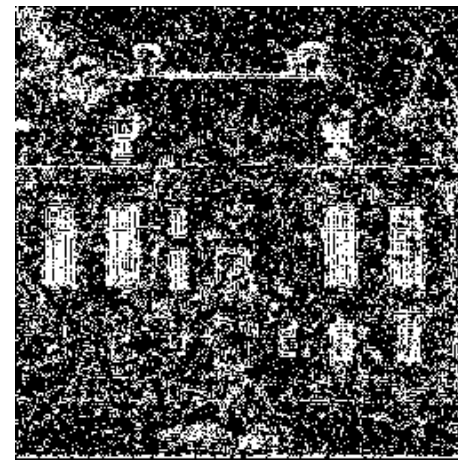
second-order derivative 4-neighbor on three images (threshold = 25,50,25)



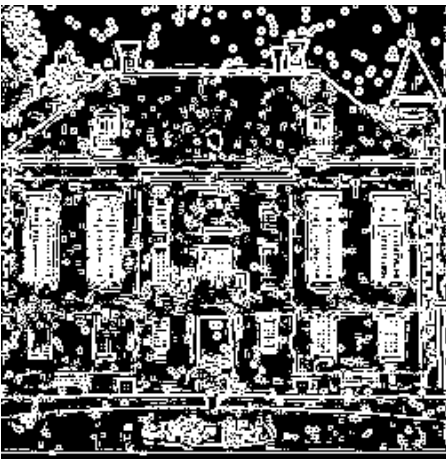
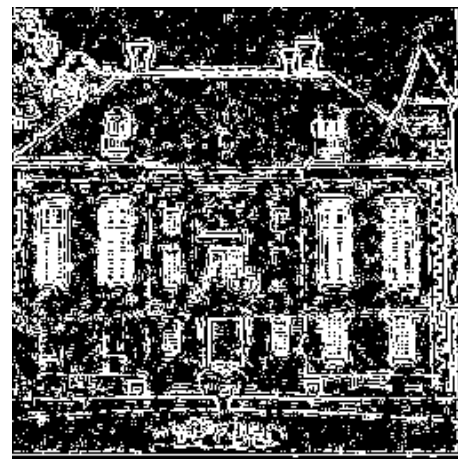
second-order derivative 4-neighbor with Gaussian blur (threshold = 3, 3, 3)



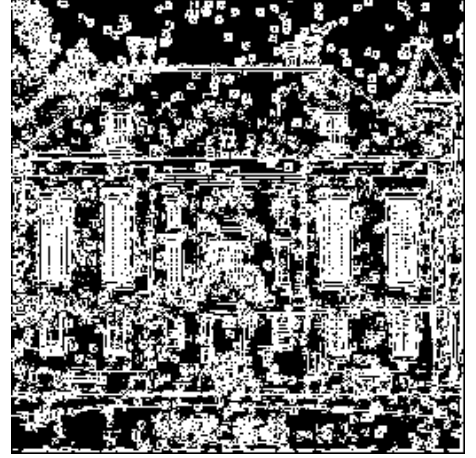
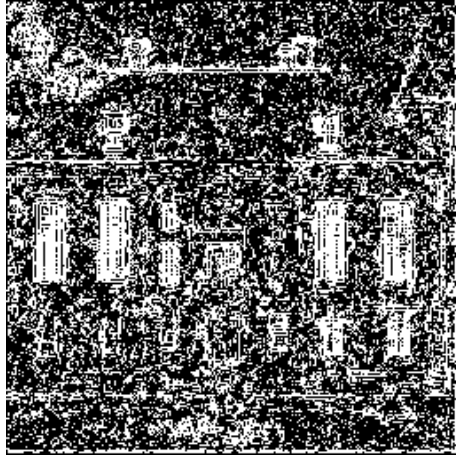
second-order derivative 8-neighbor non separable on three inputs (threshold = 20,50,20)



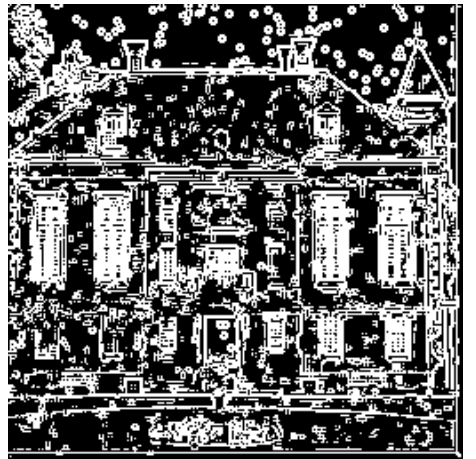
second-order derivative 8-neighbor non separable with Gaussian blur (threshold = 3,5,5)



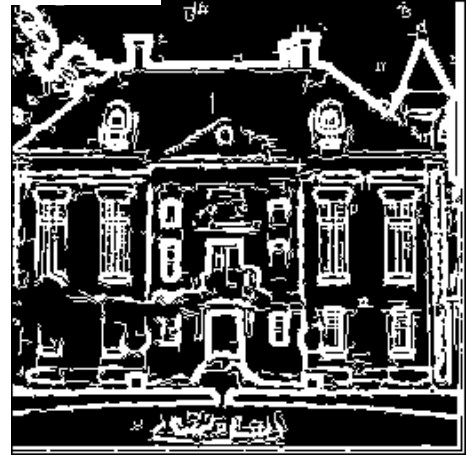
second-order derivative 8-neighbor separable on three inputs (threshold = 20,30,20)



second-order derivative 8-neighbor separable with Gaussian blur (threshold = 3,5,5)



canny on three inputs (threshold = 10,23,10)



d. Discussion of Results

可以看得出來 second-order 的方法對於 noise 最敏感，去噪後雖然有改善但仍不及 first-order 與 canny。另外因為去噪是用 Gaussian Filter，所以對 salt & pepper noise 效果較不顯著。

First-order 中的 prewitt 和 sobel 明顯在 uniform noise 表現得比 3points 和 roberts 好，這是因為 prewitt 和 sobel 在計算一階微分時考慮了更多鄰近 pixel 的值，因此減低了 noise 造成的影響。最後，canny 的 edge 看起來比較寬一點，可能跟 threshold 的選擇有關。

Problem 2. Geometrical Modification

a. Motivation and approach

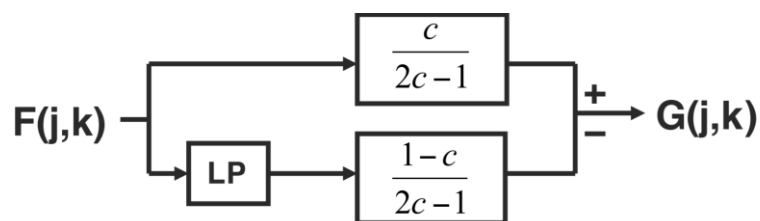
edge crispening 有兩種做法：

1. 直接用 high-pass filter 對整張圖做 filtering

high-pass filters

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

2. 使用 unsharp crispening



$$G(j,k) = \frac{c}{2c-1} F(j,k) - \frac{1-c}{2c-1} F_L(j,k), \text{ where } \frac{3}{5} \leq c \leq \frac{5}{6}$$

warping 先判斷原圖座標點(x,y)與原點(中心點)距離是否超過 127.5

是則設為 255，否則利用右方公式將正方形圖片轉為圓形

因為漩渦是越接近中心旋轉越明顯，

因此最後用公式 $\theta \propto \frac{1}{\sqrt{u^2+v^2}}$ 旋轉造成漩渦效果

$$u = \frac{x}{\sqrt{1+x^2y^2}}, \quad v = \frac{y}{\sqrt{1+x^2y^2}}$$
$$\begin{bmatrix} \cos\left(\frac{\text{para}}{\sqrt{u^2+v^2}}\right) & -\sin\left(\frac{\text{para}}{\sqrt{u^2+v^2}}\right) \\ \sin\left(\frac{\text{para}}{\sqrt{u^2+v^2}}\right) & \cos\left(\frac{\text{para}}{\sqrt{u^2+v^2}}\right) \end{bmatrix}$$

b. Original Images



c. Output Images

high-pass filtering -1 5 -1



high-pass filtering -1 9 -1



unsharp with $c = \frac{3}{5}$



unsharp with $c = \frac{5}{6}$



swirled disk of parameter = 0.4



swirled disk of parameter = 0.6



d. Discussion of Results

直接使用 high-pass filter 做出來的效果明顯較粗糙，而-1 9 -1 的又比-1 5 -1 的還嚴重。

unsharp 方法可以得到比較接近原圖的結果，而參數 c 越小 low-pass filtering 的結果會負的越多

因此銳利化效果 $c=3/5$ 的會比 $c=5/6$ 的明顯。

漩渦效果的部分可以明顯看出來當旋轉參數越大，旋轉效果就越明顯。