**Signals and Systems Final**

**10:20a.m. ~ 12:20p.m., June 20, Fri., 2008**

- Closed book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120, time allocation: 1 point/min
- Total 4 pages in one B4 sheet

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1. **[4]** What is the group delay $\tau(\omega)$ for a system with frequency response $H(j\omega)$? Explain what that means.

2. **[4]** A linear, time-invariant continuous-time system $H(j\omega)$ is distortionless within a signal band, $|\omega| < \omega_c$, if for any input signal $x(t)$ with $X(j\omega) = 0$, $|\omega| \geq \omega_c$, the output is of the form $y(t) = kx(t - t_0)$ for some fixed $k$ and $t_0$. What is the condition for the frequency response of this system, $H(j\omega)$, to be distortionless within a signal band, $|\omega| < \omega_c$?

3. **[8]** $x_c(t)$ is a continuous-time signal, $x_d[n] = x_c(nT)$, and

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT).$$

The discrete-time Fourier transform of $x_d[n]$ is $X_d(e^{j\Omega})$, while the continuous-time Fourier transform of $x_p(t)$ is $X_p(j\omega)$. Find the relationship between $X_d(e^{j\Omega})$ and $X_p(j\omega)$.

4. **[12]** A signal $x_c(t)$ is echoed, so the actually received signal is

$$y_c(t) = x_c(t) + ax_c(t - T_0)$$

An echo canceller is shown in Figure 4, with a goal to have $z_c(t) = x_c(t)$.

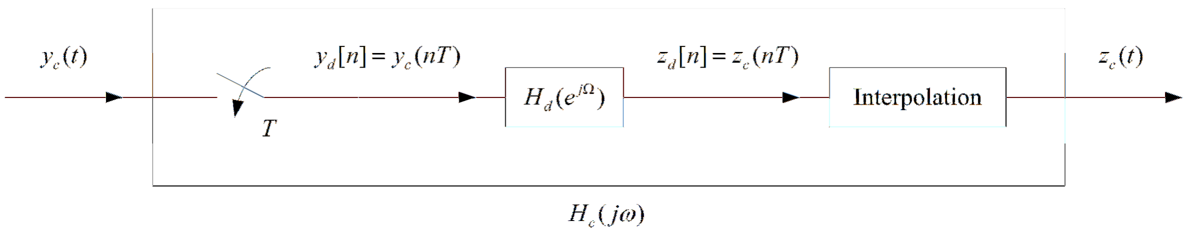



Figure 4

(a) **[8]** Assume the sampling theorem is satisfied, the interpolation is perfect so $z_c(nT) = z_d[n]$, and $T_0 = n_0 T$, $n_0$ is an integer. Find the difference equation relating $y_d[n]$ and $z_d[n]$ and the frequency response $H_d(e^{j\Omega})$.

(b) **[4]** What is the continuous-time frequency response $H_c(j\omega)$ between $y_c(t)$ and $z_c(t)$?

5. **[14]** Let $x[n]$ be a discrete-time signal with

$$\left|X(e^{j\omega})\right| = 0, \ \tfrac{2}{9}\pi \le |\omega| \le \pi$$

So the sampling frequency seems to be too high. The spectrum is shown in Figure 5.

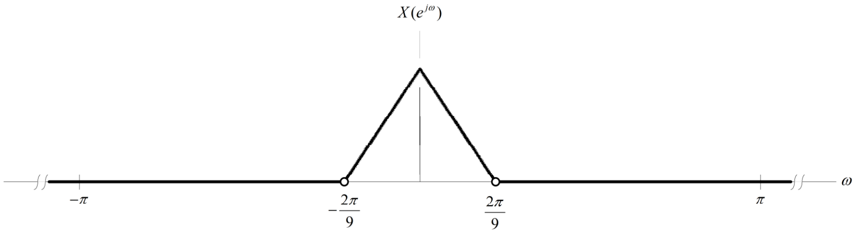



Figure 5

(a) **[4]** $x[n]$ can be decimated by a factor of integer $N$, i.e.,

$$x_b[n] = x[nN].$$

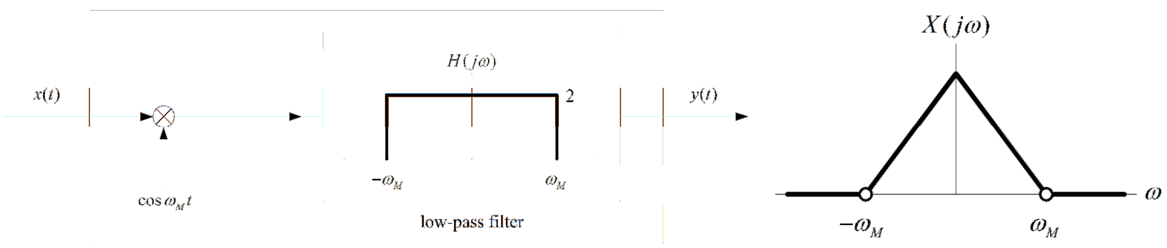
Find the maximum integer $N$ such that no aliasing will be introduced. Sketch $X_b(e^{j\omega})$ obtained in this case.

(b) **[10]** Is there any way to further reduce the sampling frequency without introducing aliasing effect? If yes, write it down, sketch the spectrum after each process, and explain what that means. If no, explain why.

6. **[8]** Explain why phase modulation with a signal $x(t)$ corresponds to frequency modulation with a signal $\frac{d}{dt}x(t)$.

7. **[8]** The system shown in Figure 7(a) is called a "frequency inverter" in early days, where $\omega_M$ is the frequency upper bound for the signal $x(t)$, i.e., $x(t)$ has a continuous-time Fourier transform $X(j\omega)$ shown in Figure 7(b), where $|X(j\omega)| = 0$ for $|\omega| \ge \omega_M$. $H(j\omega)$ is

$$H(j\omega) = \begin{cases} 2 & |\omega| \le \omega_M \\ 0 & else \end{cases}.$$




(a) (b)

Figure 7

Explain why this system is called a "frequency inverter", and discuss whether the system can be its own inverse system.

**NOTE: There are problems in the back.**

8. **[6]** Let $X(s)=\dfrac{1}{(s+1)(s+2)}$ be the Laplace transform of $x(t)$. Find $x(t)$ if the region of convergence is

(a) **[2]** $\Re e\{s\}>-1$

(b) **[2]** $\Re e\{s\}<-2$

(c) **[2]** $-2<\Re e\{s\}<-1$

9. **[8]** Find the inverse Laplace transform of $\dfrac{1}{(s+a)^n}$ with region of convergence $\Re e\{s\}>-a$.

10. **[6]** If $x(t)$ is right-sided, and if the line $\Re e\{s\}=\sigma_0$ is in the ROC of $X(s)$, are all values of $s$ for which $\Re e\{s\}>\sigma_0$ also in the ROC of $X(s)$? If yes, show it. If no, explain why.

11. **[9]** Use pole-zero plots to determine each of the following system functions to be lowpass, highpass, or bandpass:

(a) **[3]** $H(z)=\dfrac{z^{-1}}{1+\frac{8}{9}z^{-1}},\ |z|>\dfrac{8}{9}$

(b) **[3]** $H(z)=\dfrac{1+\frac{8}{9}z^{-1}}{1-\frac{16}{9}z^{-1}+\frac{64}{81}z^{-2}},\ |z|>\dfrac{8}{9}$

(c) **[3]** $H(z)=\dfrac{1}{1+\frac{64}{81}z^{-2}},\ |z|>\dfrac{8}{9}$

12. **[18]** $X(z)$ is the $z$-transform of $x[n]$ with region of convergence $\Re$. Prove the following properties of $z$-transform, and discuss the possible changes of region of convergence. For each case, write down the corresponding properties of Laplace transform and discrete-time Fourier transform, if any.

(a) **[9]** $x[n-n_0]\xleftrightarrow{Z}z^{-n_0}X(z)$

(b) **[9]** $x^*[n]\xleftrightarrow{Z}X^*(z^*)$

13. **[6]** Consider a system with $H(z) = \dfrac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$. Draw the block diagram of the system with minimum number of delays in

(a) **[2]** direct form.

(b) **[2]** cascade form.

(c) **[2]** parallel form.

14. **[9]** A causal, linear and time-invariant system is described by the following difference equation, where $x[n]$ is the input and $y[n]$ is the output,

$$y[n] - \frac{1}{2}y[n-1] = x[n] + \frac{1}{3}x[n-1].$$

Find the system function $H(z)$, its region of convergence, and discuss if it is stable.