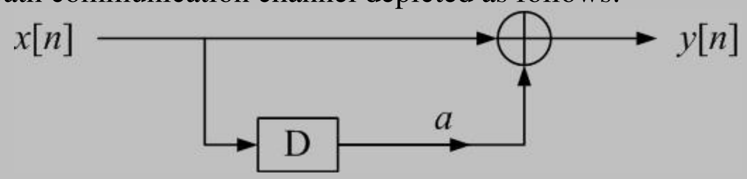
Signals and Systems, Midterm Exam Solutions (Draft)

Spring 2007, Edited by bypeng

1. **[10]** Consider designing a discrete-time inverse system to eliminate the distortion associated with multipath propagation in a data communication problem, where the propagation channel consists of a direct path and a number of reflected paths. For simplicity, let us consider the discrete-time model of a two-path communication channel depicted as follows:



   a) **[2]** Write down the difference equation describing the two-path communication channel system.
   b) **[6]** Find the impulse response of a causal inverse system that will recover $x[n]$ from $y[n]$.
   c) **[2]** Check if the inverse system is stable and explain the physical meaning of the condition you have derived.

Solution:

a) $y[n] = x[n] + ax[n-1]$

b) Since the inverse system is causal, the impulse response $g[n]$ can be written

$$g[n] = \sum_{l=0}^{\infty} g_l \delta[n-l]$$

We also observe that the system in LTI, and then by the definition of the inverse system we consider the convolution of $y[n]$ and $g[n]$ which is given by

$$\begin{aligned}
y[n]*g[n] &= \sum_{k=-\infty}^{\infty} y[n-k]g[k] = \sum_{k=-\infty}^{\infty} y[n-k]\sum_{l=0}^{\infty} g_l\delta[k-l] = \sum_{k=-\infty}^{\infty}\sum_{l=0}^{\infty} y[n-k]g_l\delta[k-l] \\
&= \sum_{k=-\infty}^{\infty} g_k y[n-k] = \sum_{k=-\infty}^{\infty} g_k (x[n-k] + ax[n-k-1]) \\
&= g_0(x[n] + ax[n-1]) + g_1(x[n-1] + ax[n-2]) + \ldots + g_k(x[n-k] + ax[n-k-1]) + \ldots \\
&= g_0 x[n] + (g_0 a + g_1)x[n-1] + (g_1 a + g_2)x[n-2] + \ldots + (g_k a + g_{k+1})x[n-k] + \ldots \\
&= x[n]
\end{aligned}$$

Now we find that $g_0 = 1$, $g_1 = -a$, $g_2 = a^2$, …, and in general $g_k = (-a)^k$ for each integer k.

So $g[n] = \sum_{l=0}^{\infty} (-a)^l \delta[n-l] = (-a)^n u[n]$.

c) The inverse system is stable if $|a| < 1$. This means if the reflected path needs to be weaker or the signals cannot be recovered in practical approaches.

2. **[12]** Consider the signal

$$x[n] = \cos\left(\frac{7\pi}{16}n\right) + \cos\left(\frac{9\pi}{16}n\right)$$

a) **[4]** Compute the discrete-time Fourier transform (DTFT) of the signal.

b) **[8]** Now compute the DTFT of only a portion of the signal by multiplying $x[n]$ with a windowing function $w[n]$,

$$w[n] = \begin{cases} 1, & |n| \le M \\ 0, & |n| > M \end{cases}$$

Plot the DTFT of the truncated signal with $M = 8$ and $M = 40$ to evaluate the effect of truncating a signal on the DTFT.

Solution:

a) We have

$$X(e^{j\omega}) = \mathcal{F}\left\{\cos\left(\frac{7\pi}{16}n\right) + \cos\left(\frac{9\pi}{16}n\right)\right\} = \mathcal{F}\left\{\cos\left(\frac{7\pi}{16}n\right)\right\} + \mathcal{F}\left\{\cos\left(\frac{9\pi}{16}n\right)\right\}$$

$$= \pi\left(\sum_{l=-\infty}^{\infty} \delta\left(\omega - \frac{7\pi}{16} - 2\pi l\right) + \delta\left(\omega + \frac{7\pi}{16} - 2\pi l\right) + \delta\left(\omega - \frac{9\pi}{16} - 2\pi l\right) + \delta\left(\omega + \frac{9\pi}{16} - 2\pi l\right)\right)$$

$$= \pi\left(\sum_{l=-\infty}^{\infty} \delta\left(\omega + \frac{9\pi}{16} - 2\pi l\right) + \delta\left(\omega + \frac{7\pi}{16} - 2\pi l\right) + \delta\left(\omega - \frac{7\pi}{16} - 2\pi l\right) + \delta\left(\omega - \frac{9\pi}{16} - 2\pi l\right)\right)$$

b) We know that $W(e^{j\omega}) = \dfrac{\sin[\omega(M + \frac{1}{2})]}{\sin\frac{\omega}{2}}$, and by $x[n]y[n] \xleftrightarrow{DTFT} \dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$,

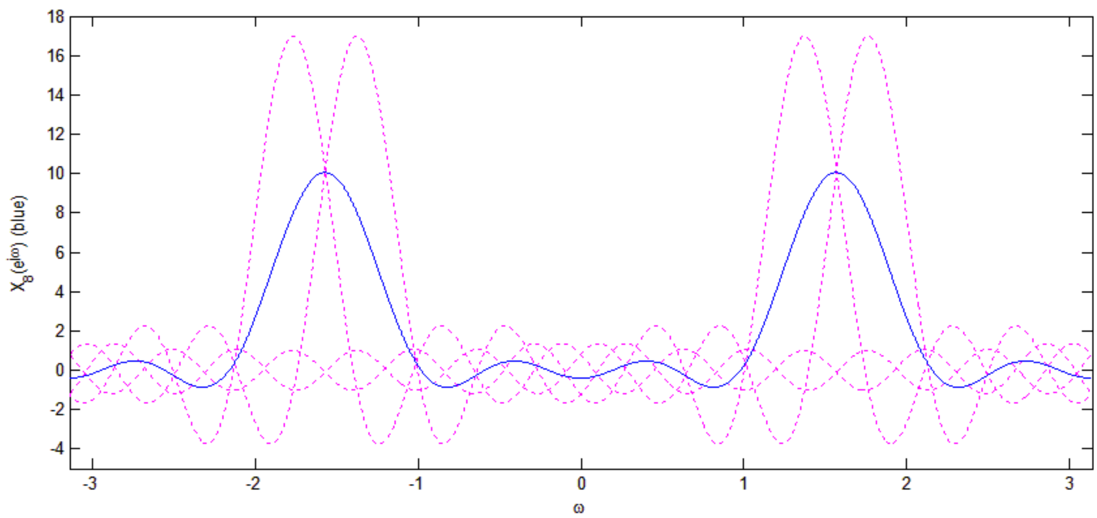
we have that

$$\mathcal{F}\{x[n]w[n]\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \pi\left[\delta\left(\theta + \frac{9\pi}{16}\right) + \delta\left(\theta + \frac{7\pi}{16}\right) + \delta\left(\theta - \frac{7\pi}{16}\right) + \delta\left(\theta - \frac{9\pi}{16}\right)\right]\frac{\sin[(\omega-\theta)(M + \frac{1}{2})]}{\sin\frac{\omega-\theta}{2}}d\theta$$

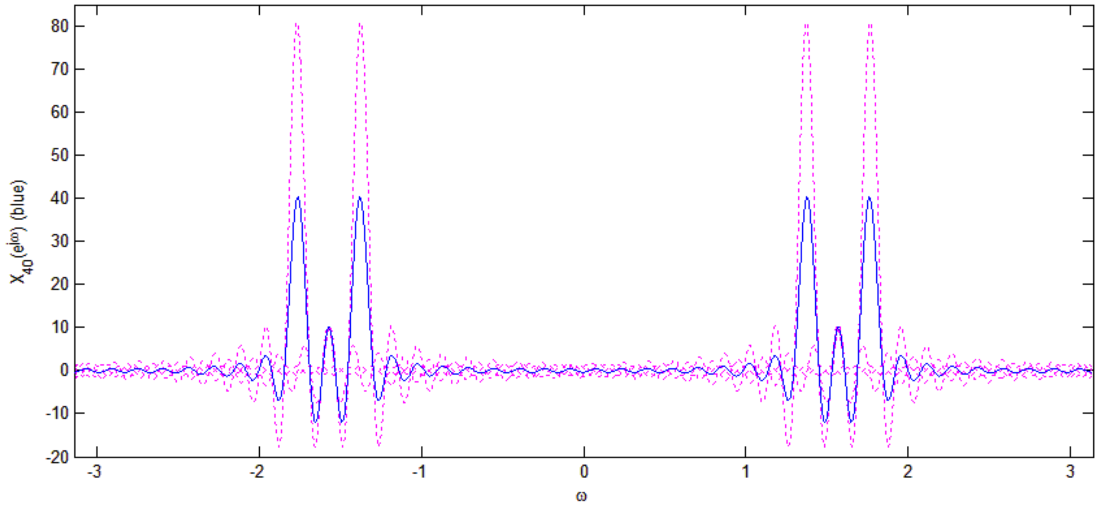
$$= \frac{1}{2}\left[\frac{\sin[(\omega + \frac{9\pi}{16})(M + \frac{1}{2})]}{\sin\frac{\omega + \frac{9\pi}{16}}{2}} + \frac{\sin[(\omega + \frac{7\pi}{16})(M + \frac{1}{2})]}{\sin\frac{\omega + \frac{7\pi}{16}}{2}} + \frac{\sin[(\omega - \frac{7\pi}{16})(M + \frac{1}{2})]}{\sin\frac{\omega - \frac{7\pi}{16}}{2}} + \right.$$

$$\left.\frac{\sin[(\omega - \frac{9\pi}{16})(M + \frac{1}{2})]}{\sin\frac{\omega - \frac{9\pi}{16}}{2}}\right]$$

For $M = 8$, the plot of the DTFT is as the following graph:



And for $M = 40$, the plot of the DTFT is as the following graph:



We can see that if the window is not "wide" enough, there will be aliasing on the spectrum of the sampled signal.

3. **[12]** Given the Fourier transform pair $x[n] \xleftrightarrow{F} X(e^{j\omega})$, determine if each of the following statements is true or false. Justify your answer.

a) **[3]** If $X(e^{j\omega}) = X(e^{j(\omega-1)})$, then $x[n] = 0$ for $|n| > 0$.

b) **[3]** If $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, then $x[n] = 0$ for $|n| > 0$.

c) **[3]** If $X(e^{j\omega}) = X(e^{j\omega/2})$, then $x[n] = 0$ for $|n| > 0$.

d) **[3]** If $X(e^{j\omega}) = X(e^{j2\omega})$, then $x[n] = 0$ for $|n| > 0$.

Solution:

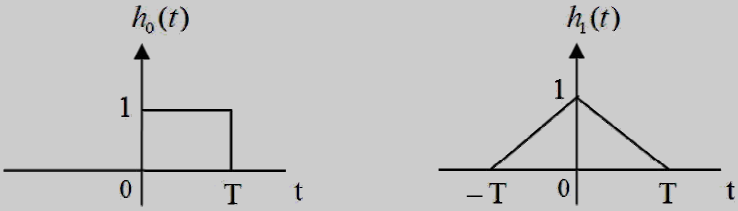
a) True. $X(e^{j\omega}) = X(e^{j(\omega-1)})$, $x[n] = e^{jn}x[n]$, $x[n]\cdot(1-e^{jn}) = 0$. Observing that $1-e^{jn} = 0$ only if $n = 0$, we conclude that $x[n] = 0$ for $|n| > 0$.

b) False. $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, $x[n] = e^{j\pi n}x[n]$, $x[n]\cdot(1-e^{j\pi n}) = x[n]\cdot(1-(-1)^n) = 0$. We conclude that $x[n] = 0$ only for n being odd.

c) False. $X(e^{j\omega}) = X(e^{j\omega/2})$, $x[n] = \dfrac{x[n]+e^{j\pi n}x[n]}{2}$, $x[n] = e^{j\pi n}x[n] = (-1)^n x[n]$. We conclude that $x[n] = 0$ only for n being odd.

d) True. $X(e^{j\omega}) = X(e^{j2\omega})$, $x[n] = x_{(2)}[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$. By induction, we conclude that $x[n] = 0$ for $|n| > 0$.

4. **[16]** Let the impulse train $g(t)=\sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$, where $x(t)$ is a continuous-time signal, be the input to two filters with impulse response $h_0(t)$ and $h_1(t)$ as depicted in the following figure:



Let $x_0(t)=g(t)*h_0(t)$ and $x_1(t)=g(t)*h_1(t)$.

a) **[6]** Suppose the Fourier transform (FT) of $x(t)$ is $X(j\omega)$. What is the FT of $g(t)$?

b) **[4]** In terms of filtering operation, what do these two filters do to the impulse train?

c) **[6]** Specify the frequency response of a filter that produces $x_1(t)$ as its output when $x_0(t)$ is the input.

Solution:

a) Observing that $g(t)=\sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)=\sum_{n=-\infty}^{\infty} x(t)\delta(t-nT)=x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$, and

$\mathcal{F}\left\{\sum_{n=-\infty}^{\infty}\delta(t-nT)\right\}=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{2\pi k}{T})$, by $x(t)y(t)\xleftrightarrow{CTFT}\frac{1}{2\pi}\left[X(j\omega)*Y(j\omega)\right]$, we have that

$$G(j\omega)=\frac{1}{2\pi}\left[X(j\omega)*\left(\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-\frac{2\pi k}{T})\right)\right]=\frac{1}{T}\sum_{k=-\infty}^{\infty}X(j(\omega-\frac{2\pi k}{T}))$$

b) h_0 is a zero-order hold filter, and h_1 is a linear interpolation filter.

c) Consider the convolution

$$h_0(t)*\left[Ah_0(t-t_0)\right]=\int_{-\infty}^{\infty}Ah_0(\tau)h_0(t-\tau-t_0)d\tau=\int_0^T Ah_0(t-\tau-t_0)d\tau\overset{\tau_0\triangleq t-\tau-t_0}{=}\int_{t-t_0-T}^{t-t_0}Ah_0(\tau_0)d\tau_0$$

$$=\begin{cases}0 & t-t_0<0\\ \int_0^{t-t_0}Ad\tau_0 & 0<t-t_0<T\\ \int_{t-t_0-T}^{T}Ad\tau_0 & T<t-t_0<2T\\ 0 & t-t_0>2T\end{cases}=\begin{cases}0 & t<t_0\\ A(t-t_0) & t_0<t<T+t_0\\ 2AT-A(t-t_0) & T+t_0<t<2T+t_0\\ 0 & t>2T+t_0\end{cases}$$

Compared with $h_1(t)=\begin{cases}0 & t<-T\\ \frac{t}{T}+1 & -T<t<0\\ -\frac{t}{T}+1 & 0<t<T\\ 0 & t>T\end{cases}$, we can find that the impulse response satisfies

$t_0=-T$ and $A=\frac{1}{T}$. So the impulse response is $\frac{1}{T}h_0(t+T)=\begin{cases}\frac{1}{T} & -T<t<0\\ 0 & \text{otherwise}\end{cases}$, and the corresponding frequency response is $e^{j\omega\frac{T}{2}}\frac{2\sin\frac{\omega T}{2}}{\omega T}$.

5. **[10]** Let T be the period of a continuous-time periodic signal. Prove

a) **[5]** $T\sum_{k=-\infty}^{\infty}\delta(t-kT)=\sum_{k=-\infty}^{\infty}e^{j2\pi kt/T}$

b) **[5]** $u[n]\xleftrightarrow{F}\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega-2\pi k)$

Solution:

a) Consider the Fourier series coefficient of the left part:

$$a_k=\frac{1}{T}\int_T T\sum_{k=-\infty}^{\infty}\delta(t-kT)e^{-jk\frac{2\pi}{T}t}dt=e^{-jk\frac{2\pi}{T}T}=e^{-jk2\pi}=1$$

Therefore the Fourier series representation is given by

$$T\sum_{k=-\infty}^{\infty}\delta(t-kT)=\sum_{k=-\infty}^{\infty}e^{j2\pi kt/T}$$

b) Observing that $u[n]=\sum_{k=-\infty}^{n}\delta[k]$, by the accumulation property

$$\sum_{k=-\infty}^{n}x[k]\xleftrightarrow{DTFT}\frac{1}{1-e^{-j\omega}}X(e^{j\omega})+\pi X(e^{j0})\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$$

We have that

$$u[n]\xleftrightarrow{DTFT}\frac{1}{1-e^{-j\omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$$

6. **[14]** Answer the following questions.

a) **[6]** Consider a linear system with input $x(t)$ and output $y(t)$. Let $\Phi(t)$ be an eigenfunction of the system, i.e., if $x(t)=\Phi(t)$, then $y(t)=\lambda\Phi(t)$, where the complex number λ represents the eigenvalue associated with $\Phi(t)$. Assume that we input a signal $x(t)=\sum_{k=-\infty}^{\infty}a_k\Phi_k(t)$ to the system, where $\Phi_k(t)$ is an eigenfunction with a corresponding eigenvalue λ_k. Find the output $y(t)$ of the system in terms of $\{a_k\}$, $\{\Phi_k(t)\}$, and $\{\lambda_k\}$.

b) **[8]** Let the system be characterized by the differential equation

$$y(t)=t^2\frac{d^2x(t)}{dt^2}+t\frac{dx(t)}{dt}$$

Are $\Phi_k(t)=t^k$ the eigenfunctions of the system? You should justify your answer. If your answer is yes, then determine the corresponding eigenvalue λ_k.

Solution:

a) Let the system be S and we denote $y(t)=S[x(t)]$. Since S is linear, we have that

$$y(t)=S[x(t)]=S\left[\sum_{k=-\infty}^{\infty}a_k\Phi_k(t)\right]=\sum_{k=-\infty}^{\infty}a_kS\left[\Phi_k(t)\right]=\sum_{k=-\infty}^{\infty}a_k\lambda_k\Phi_k(t)$$

b) We have

$$y(t)=t^2\frac{d^2\Phi_k(t)}{dt^2}+t\frac{d\Phi_k(t)}{dt}=t^2\frac{d^2t^k}{dt^2}+t\frac{dt^k}{dt}=t^2k(k-1)t^{k-2}+tkt^{k-1}=(k^2-k+k)t^k=k^2t^k$$

So the $\Phi_k(t)=t^k$ are the eigenfunctions of the system and each of them is with eigenvalue k^2.

7. **[10]** Consider an important concept of the correlation between two signals. Let $x(t)$ and $y(t)$ be two signals; then the correlation function of $x(t)$ and $y(t)$ is defined as follows:

$$R_{xy}(t)=\int_{-\infty}^{\infty}x(\alpha+t)y(\alpha)d\alpha$$

a) **[5]** Prove that $R_{xy(t)}=x(t)*y(-t)$, where $*$ denotes the convolution integral.

b) **[5]** Find $R_{xy}(t)$ for $x(t)=2e^{-t}u(t)-3e^{-2t}u(t)$ and $y(t)=\cos(2\pi t)\cdot[u(t+2)-u(t-2)]$, where $u(t)$ denotes the unit step function.

Solution:

a) We have

$$x(t)*y(-t)=\int_{-\infty}^{\infty}x(\tau)y(-(t-\tau))d\tau=\int_{-\infty}^{\infty}x(\tau)y(-t+\tau)d\tau \overset{\tau'=-t+\tau}{=} \int_{-\infty}^{\infty}x(\tau'+t)y(\tau')d\tau'$$

$$=\int_{-\infty}^{\infty}x(\alpha+t)y(\alpha)d\alpha$$

b) We have

$$R_{xy}(t)=x(t)*y(-t)=\left[2e^{-t}u(t)-3e^{-2t}u(t)\right]*\left[\cos(-2\pi t)\cdot[u(-t+2)-u(-t-2)]\right]$$

$$=\left[2e^{-t}u(t)-3e^{-2t}u(t)\right]*\left[\cos(2\pi t)\cdot[u(2-t)-u(-2-t)]\right]$$

$$=\int_{-\infty}^{\infty}\left[2e^{-(t-\tau)}u(t-\tau)-3e^{-2(t-\tau)}u(t-\tau)\right]\left[\cos(2\pi\tau)\cdot[u(2-\tau)-u(-2-\tau)]\right]d\tau$$

$$=\int_{-2}^{2}\left[2e^{-(t-\tau)}u(t-\tau)-3e^{-2(t-\tau)}u(t-\tau)\right]\cos(2\pi\tau)d\tau$$

$$=\int_{t-2}^{t+2}u(\tau_0)\left(2e^{-\tau_0}-3e^{-2\tau_0}\right)\cos(2\pi(t-\tau_0))d\tau_0$$

When $t<-2$, we have that $R_{xy}(t)=0$.

When $-2<t<2$, we have that

$$R_{xy}(t)=\int_{0}^{t+2}\left(2e^{-\tau}-3e^{-2\tau}\right)\cos(2\pi(t-\tau))d\tau$$

$$=\int_{-2}^{t}\left(2e^{-(t-\tau)}-3e^{-2(t-\tau)}\right)\cos(2\pi\tau)d\tau$$

$$=2e^{-t}\int_{-2}^{t}e^{\tau}\cos(2\pi\tau)d\tau+3e^{-2t}\int_{-2}^{t}e^{2\tau}\cos(2\pi\tau)d\tau$$

$$=2e^{-t}\left(\tfrac{1}{4\pi^2+1}e^{\tau}\cos 2\pi\tau+\tfrac{2\pi}{4\pi^2+1}e^{\tau}\sin 2\pi\tau\right)_{\tau=-2}^{t}+3e^{-2t}\left(\tfrac{1}{2(\pi^2+1)}e^{2\tau}\cos 2\pi\tau+\tfrac{\pi}{2(\pi^2+1)}e^{2\tau}\sin 2\pi\tau\right)_{\tau=-2}^{t}$$

$$=2e^{-t}\left(\tfrac{1}{4\pi^2+1}e^{t}\cos 2\pi t+\tfrac{2\pi}{4\pi^2+1}e^{t}\sin 2\pi t-\tfrac{1}{4\pi^2+1}e^{-2}\right)+$$

$$3e^{-2t}\left(\tfrac{1}{2(\pi^2+1)}e^{2t}\cos 2\pi t+\tfrac{\pi}{2(\pi^2+1)}e^{2t}\sin 2\pi t-\tfrac{1}{2(\pi^2+1)}e^{-4}\right)$$

When $t>2$, we have that

$$R_{xy}(t)=\int_{t-2}^{t+2}\left(2e^{-\tau}-3e^{-2\tau}\right)\cos(2\pi(t-\tau))d\tau$$

$$=2e^{-t}\left(\tfrac{1}{4\pi^2+1}e^{\tau}\cos 2\pi\tau+\tfrac{2\pi}{4\pi^2+1}e^{\tau}\sin 2\pi\tau\right)_{\tau=-2}^{2}+3e^{-2t}\left(\tfrac{1}{2(\pi^2+1)}e^{2\tau}\cos 2\pi\tau+\tfrac{\pi}{2(\pi^2+1)}e^{2\tau}\sin 2\pi\tau\right)_{\tau=-2}^{2}$$

$$=2e^{-t}\left(\tfrac{1}{4\pi^2+1}e^{2}-\tfrac{1}{4\pi^2+1}e^{-2}\right)+3e^{-2t}\left(\tfrac{1}{2(\pi^2+1)}e^{4}-\tfrac{1}{2(\pi^2+1)}e^{-4}\right)$$

8. **[14]** Consider a system with the relationship of its input and output given by

$$y(t)=\int_0^t e^{-\tau}x(t-\tau)d\tau$$

a) **[5]** Find the system impulse response $h(t)$ of the system.

b) **[4]** Is the system causal ? You must justify your answer.

c) **[5]** Determine the output $y(t)$ of the system if the input $x(t)=u(t+1)$.

Solution:

a) The impulse response is given by

$$h(t)=\int_0^t e^{-\tau}\delta(t-\tau)d\tau=\int_0^t e^{-(t-\tau_0)}\delta(\tau_0)d\tau_0=e^{-t}$$

b) Consider the output when the input is $2\delta(t)$ but not $\delta(t)$.

$$h_2(t)=\int_0^t e^{-\tau}2\delta(t-\tau)d\tau=2\int_0^t e^{-(t-\tau_0)}\delta(\tau_0)d\tau_0=2e^{-t}$$

We find that even if $\delta(t)$ and $2\delta(t)$ are the same when $t<0$, the outputs when $t<0$ are not the same. Therefore the system is not causal.

c) We have that

$$y(t)=\int_0^t e^{-\tau}u(t-\tau+1)d\tau=\int_1^{t+1} e^{-(t-\tau_0+1)}u(\tau_0)d\tau_0$$

When $t<-1$,

$$y(t)=\int_1^0 e^{-(t-\tau_0+1)}d\tau_0=e^{-t-1}\int_1^0 e^{\tau_0}d\tau_0=e^{-t-1}(e^0-e)=e^{-(t+1)}-e^{-t}$$

When $t>-1$,

$$y(t)=\int_1^{t+1} e^{-(t-\tau_0+1)}d\tau_0=e^{-(t+1)}\int_1^{t+1} e^{\tau_0}d\tau_0=e^{-(t+1)}(e^{t+1}-e)=1-e^{-t}$$

(NOTE: Since the system is not time-invariant, the convolution sum property cannot be applied.)

9. **[12]** Consider a real continuous-time (CT) signal $x(t)$. Assume that $x(t)$ is periodic with period being 6. Some Fourier coefficients of $x(t)$ are given as follows: $a_k=0$ for $k=0$ and $k>2$, and a_1 is positive real number. Moreover, $x(t)=-x(t-3)$ and the total average power of $x(t)$ equals $1/2$. Find the CT signal $x(t)$.

Solution:

The only unknown Fourier series coefficients are a_1, a_{-1}, a_2 and a_{-2} only. Since $x(t)$ is real and a_1 is real and positive, we know that $a_1=a_{-1}$ and $a_2=a_{-2}$, and $x(t)$ must be in the form

$$x(t)=2a_1\cos(\tfrac{\pi}{3}t)+A_2\cos(\tfrac{2\pi}{3}t+\theta)$$

Since $x(t)=-x(t-3)$, we have that

$$2a_1\cos(\tfrac{\pi}{3}t)+A_2\cos(\tfrac{2\pi}{3}t+\theta)=-2a_1\cos(\tfrac{\pi}{3}t-\pi)-A_2\cos(\tfrac{2\pi}{3}t-2\pi+\theta)$$

or

$$2a_1\left[\cos(\tfrac{\pi}{3}t)+\cos(\tfrac{\pi}{3}t-\pi)\right]+A_2\left[\cos(\tfrac{2\pi}{3}t+\theta)+\cos(\tfrac{2\pi}{3}t-2\pi+\theta)\right]=0$$

$$\Rightarrow\ A_2\left[\cos(\tfrac{2\pi}{3}t+\theta)+\cos(\tfrac{2\pi}{3}t+\theta)\right]=0\ \Rightarrow\ A_2=0\ \Rightarrow\ a_2=a_{-2}=0$$

Now by Parseval's relation, we get

$$|a_1|^2+|a_{-1}|^2=\tfrac{1}{2}$$

So $a_1=a_{-1}=\frac{1}{2}$ and then $x(t)=\cos\frac{\pi t}{3}$.

10. **[10]** Consider a system with the relationship of its input and output given by

$$y(t) = \int_t^{t+1} x(\tau - \alpha) d\tau$$

where α is a constant.

a) **[3]** Is the system invertible? You must justify your answer.

b) **[3]** Is the system stable? You must justify your answer.

c) **[4]** Find the values of α so that the system is causal.

Solution:

a) No, it is not invertible. Consider the cases $x_1(t) = \cos 2\pi t$ and $x_2(t) = \sin 2\pi t$:

$$y_1(t) = \int_t^{t+1} \cos[2\pi(\tau - \alpha)] d\tau = \left(\frac{\sin 2\pi(\tau - \alpha)}{2\pi} \right)_t^{t+1} = \frac{1}{2\pi} \{ \sin[2\pi(t - \alpha) + 2\pi] - \sin[2\pi(t - \alpha)] \} = 0$$

$$y_1(t) = \int_t^{t+1} \sin[2\pi(\tau - \alpha)] d\tau = \left(-\frac{\cos 2\pi(\tau - \alpha)}{2\pi} \right)_t^{t+1} = \frac{1}{2\pi} \{ \cos[2\pi(t - \alpha)] - \sin[2\pi(t - \alpha) + 2\pi] \} = 0$$

Clearly, we find a pair of inputs resulting in the same output. So the system is not invertible.

b) Yes. If $|x(t)| < B$, then $y(t) = \int_t^{t+1} x(\tau - \alpha) d\tau < \int_t^{t+1} B d\tau = B$.

c) The values of $x(t)$ in the interval $[t_0 - \alpha, t_0 + 1 - \alpha]$ influence the value of $y(t_0)$ for any t_0. To make the system causal, we need $t_0 + 1 - \alpha \le t_0$, or $\alpha \ge 1$.