Electromagnetic (I) Second Examination

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1. Given a scalar field Φ and a vector field $\vec{A}$, derive the expressions of

   $\nabla\Phi$

   $\nabla\cdot\vec{A}$

   $\nabla^2\Phi$

   in cylindrical coordinates. (Note: $\nabla\cdot\vec{A}=\lim_{\Delta v\to 0}\frac{\oint_s \vec{A}\cdot d\vec{s}}{\Delta v}$) (15 %)

2. Use the Maxwell's equations to show the electric field $\vec{E}$ and magnetic field $\vec{H}$ in the free space without any sources satisfy the following equations: (15 %)

$$\nabla^2\vec{E}-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{E}=0$$

$$\nabla^2\vec{H}-\mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{H}=0$$

3. An infinitely extended positive line charge ρ_l is distributed over the axis of an infinitely extended cylindrical dielectric shell of an inner radius R_i and outer radius R_0, as shown in Fig. 1. The dielectric constant of the shell is ε_r. Determine the electric field $\vec{E}$, electric flux density $\vec{D}$, polarization vector $\vec{P}$, and potential function V as functions of the radial distance r for $0<r<\infty$. Assume $V(R_i)=0$. (15 %)

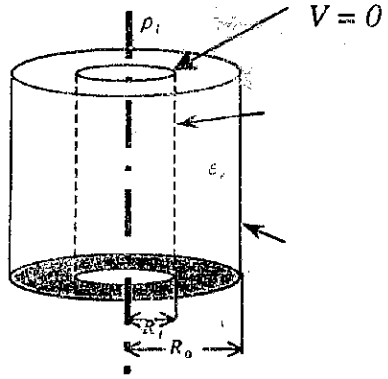

Fig. 1

4. A Current I flows along a straight wire from a point charge $Q_1(t)$ located at (0,0,0) to a point charge $Q_2(t)$ at $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ as shown in Fig. 2. Find the absolute value of the line integral of $\vec{H}$ around the closed loop in terms of I. (15 %)

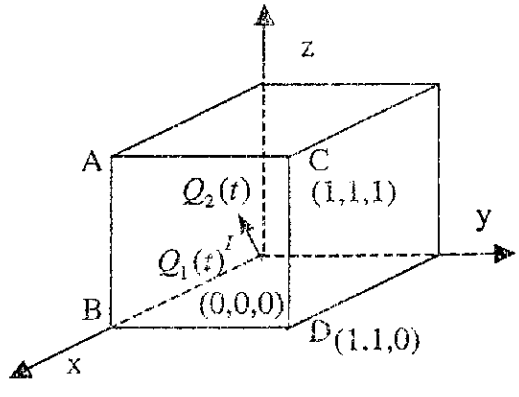

Fig. 2

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5. Two dielectric media with permittivities ε_1 and ε_2 are separated by a boundary with no free charge as shown in Fig. 3. The electric field intensity in medium 1 at the point P_1 has a magnitude E_1 and makes an angle α_1 with the normal. Determine the magnitude and direction of the electric field intensity E_2 at point P_2 in media 2. in terms of α_1 and E_1.

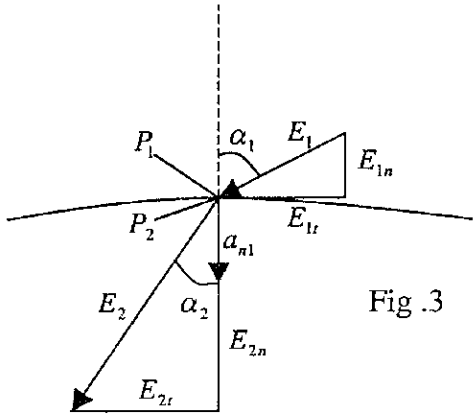

Fig .3

(15 %)

6. A volume charge is distributed throughout a sphere of radius a(m), and centered at the origin, with uniform density $\rho_0(C/m^3)$. Find the energy stored in the electric field of this charge distribution. (15 %)

7. What is the displacement current? Compare and contrast the displacement current with the current due to flow of charges. (10 %)