Probability Midterm Examination

1. The coefficients of the quadratic equation $x^2 + bx + c = 0$ are determined by tossing a fair die twice (the first outcome is b, the second one is c).
   (i) Define the probability space of the die tossing experiment. (10%)
   (ii) Express the event that the quadratic equation has real roots in terms of the probability space. (5%)
   (iii) Find the probability that the equation has real roots. (5%)

2. Among 85 students in a class, 43 of them earned A's on the midterm exam, 38 earned A's on the final exam, and 25 did not earn A's on either exam. What is the probability that a randomly selected student from this class earned A on both exams? (10%)

3. Urn I contains 3 black balls and 3 white balls. While Urn II contains 4 black balls and 2 white balls. Suppose one Urn is randomly selected and then two balls are drawn from the selected Urn.
   (i) If the first ball is white, what is the probability that the second ball is white ? (6%)
   (ii) If the second ball is white, what is the probability that the first ball is white ?(6%)
   (iii) If both balls are white, what is the probability that the selected Urn is Urn I ?(6%)

4. In the experiment of rolling two fair dice successively, what is the probability that a sum of 4 appears before a sum of 7 ? (12%)

5. In this problem, consider the experiment of rolling two fair dice. Let X be the sum of two numbers that appear.
   (i) Find the probability function of X. (5%)
   (ii) Find the cumulative distribution function (CDF) of X. Please sketch the CDF. (5%)
   (iii) Find the probability of $4 \leq X \leq 9$. (5%)

6. In this problem, consider that X is a Poisson random variable with parameter λ. Show that the maximum of $P(X=i)$ occurs at $[\lambda]$, where $[\lambda]$ is the greatest integer less than or equal to λ. (10%)

7. In this problem, consider that a Bernoulli trial with probability of success P is performed n times independently. Let X be the number of experiments until the first success occurs.
   (i) Find the probability function of X. (5%)
   (ii) Find the expectation of X. (5%)
   (iii) Find the variance of X. (5%)