

Complex Analysis: Quiz 2
10:20 AM - 11:20 PM, May 5, 2009.

[1] (20 %) Prove that

$$\int_0^\infty \frac{x^a}{(1+x^2)^2} dx = \frac{\pi(1-a)}{4 \cos(\pi a/2)}, \quad -1 < a < 3$$

[2] (20 %) Prove that

$$\int_0^\infty \frac{x^{-a}}{1+2x \cos \lambda + x^2} dx = \frac{\pi}{\sin a\pi} \cdot \frac{\sin a\lambda}{\sin \lambda}, \quad |a| < 1, |\lambda| < \pi$$

[3] (15 %) Prove that

$$\text{PV} \int_0^\infty \frac{x^{a-1}}{1-x} dx = \pi \cot a\pi, \quad 0 < a < 1$$

[4] (15 %) Evaluate

$$\int_0^{2\pi} \frac{1}{(2 + \cos \theta)^2} d\theta$$

[5] (15 %) Evaluate

$$\text{PV} \int_{-\infty}^\infty \frac{1}{(x^2+1)(x^2+9)} dx$$

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[6] (15 %) Evaluate

$$\text{PV} \int_0^\infty \frac{x \sin x}{x^2+9} dx$$