Electromagnetics I (Final Examination)

2001.1.10

1. The electric field of a uniform plane wave in free space is given by
$$\overline{E} = \cos(\omega t - \beta z)\hat{x} + 2\sin(\omega t - \beta z)\hat{y}.$$
Find (a) the corresponding magnetic field $\overline{H}$, (b) the polarization, and (c) the instantaneous and time-average Poynting vectors of the wave. (20%)

2. Consider a uniform plane wave propagated in the seawater whose material parameters are given by σ=4 S/m, $\varepsilon = 80\,\varepsilon_0$, and $\mu = \mu_0$. The electric field of the uniform plane wave, at z=0, propagating in the +z-direction is given by
$$\overline{E}(z=0,t) = \hat{x}(\cos 2\pi \times 10^7 t).$$
Find (a) the propagation parameters α, β, λ, v_P, and intrinsic impedance η, (b) the electric and magnetic fields in z>0, and (c) the time-average Poynting vector in z>0. (20%)

3. Region 1 (z<0) is free space (σ_1=0, $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$) and region 2 (z>0) is the seawater (σ_2=4 S/m, $\varepsilon_2 = 80\,\varepsilon_0$, $\mu_2 = \mu_0$). Let a uniform plane wave having the electric field
$$\overline{E}^i = E_0 \cos(2\pi \times 10^7 t - \beta z)\hat{x}$$
be normally incident on the interface z=0 from region 1. Find (a) the reflection and transmission coefficients, (b) the reflected-wave electric and magnetic fields, and (c) the transmitted-wave electric and magnetic fields. (20%)

4. Let ($\overline{E}$, $\overline{H}$) be the fields of the uniform plane wave excited by an infinite plane current sheet
$$\overline{J}_s(z=0,t) = -\hat{y}J(t) \quad (A/m)$$
at z=0 which is introduced in a perfect dielectric region whose material parameters are (ε, μ). Find the expressions for the fields in z>0 and z<0. (20%)

5. (a) From Maxwell's equations in free space (ε_0, μ_0), show that the wave equation for electric field $\overline{E}$ may be written as
$$\nabla^2 \overline{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \overline{E}}{\partial t^2}.$$
(b) Consider the vector fields of the form:
$$\frac{\overline{E}}{\overline{H}}(\overline{r},t) = \frac{\overline{E}_0}{\overline{H}_0} \exp j(\omega t - k_x x - k_y y - k_z z) = \frac{\overline{E}_0}{\overline{H}_0} \exp j(\omega t - \overline{k}\cdot\overline{r})$$
where $\overline{r} = x\hat{x} + y\hat{y} + z\hat{z}$, $\overline{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$, and ($\overline{E}_0$, $\overline{H}_0$) are constant vectors. Find the relation among ω, k_x, k_y, k_z such that the vector $\overline{E}(\overline{r},t)$ will be a solution of the wave equation.
(c)Find the relation between $\overline{E}(\overline{r},t)$ and $\overline{H}(\overline{r},t)$. (20%)

Formulas:

$$\nabla \times \nabla \times \overline{A} = \nabla(\nabla \cdot \overline{A}) - \nabla^2 \overline{A}$$
$$\nabla \times (\phi \overline{A}) = (\nabla \phi) \times \overline{A} + \phi \nabla \times \overline{A}$$