機率期中考

4/19/99

1. Suppose that n random integers are selected from {1, 2, ..., N} with replacement.
   (a) What is the expected value of the largest number selected? (10%)
   (b) Show that for large N the answer is approximately nN/(n+1). (5%)

2. A random variable X is called symmetric if for all $x \in R$, $P(X \geq x) = P(X \leq -x)$. Prove that if X is symmetric, then for all $t > 0$, its distribution function F satisfies the following relations:
   (a) $P(|X| \leq t) = 2F(t) - 1$ (5%)
   (b) $P(|X| > t) = 2[1 - F(t)]$ (5%)
   (c) ~~$P(X \geq t)$~~ $= F(t) + F(-t) - 1$ (5%)

3. In this problem, we consider that a number is selected randomly from the set {0000, 0001, 0002, 0003, ..., 9998, 9999}. Find the probability that the sum of the first two digits of the number selected is equal to the sum of its last two digits. (15%)

4. In this problem, we consider that the probability of a bit error in a communication line is 10^{-3}. Each bit transmission corresponds to a Bernoulli trial with a "success" corresponding to a bit error in transmission. Let X be the number of bit errors in N transmissions.
   (a) Is X a binomial random variable? You must justify your answer. (3%)
   (b) Find the probability function P(X=x) of X. (3%)
   (c) Let N = 1000 which is assumed to be large enough. So X can be viewed as a Poisson random variable. Find the corresponding probability function P(X=i). (8%)
   (d) From (c), find the probability that a block of 1000 bits has five or more bit errors. (6%)

5. The coefficient of the quadratic equation $x^2 + bx + c = 0$ are determined by tossing a fair dice twice (the first outcome is b, the second outcome is c). Find the probability that the equation has real roots. (15%)

6. Throw a dice twice. Let A be the event that the first throw is 1, 2, or 3; B be the event that the first throw is 2, 4, or 6; and C be the event that the sum of the two throws is 5.
   (a) Find P(A|B), P(B|C), and P(C|A). (9%)
   (b) Find P(A|BC), P(B|CA), and P(C|AB). (9%)
   (c) Are the three events mutually independent. (2%)