Microelectronics II midterm exam

date: 2013/04/24

姓名：________________
學號：________________

注意事項：1. 題目卷、答案卷，皆要填寫考生姓名與學號；
2. 考試完畢，請將題目卷、答案卷一併繳回，未繳回者，不予計分；

1. (22%) Fig. 1 is a MOS cascode amplifier. Assume that two transistors, Q_1 and Q_2, have the same g_m and r_o.
   (1) Find A_{v1}, A_{v2}, R_{in2} and R_o, where $A_{v1} = v_{o1}/v_i$ and $A_{v2} = v_o/v_{o1}$. (12%)
   (2) If $R_L = 2r_o$, find A_{v1} and R_{in2}. (6%)
   (3) Explain how to select R_L so that A_{v1} can be comparable with the intrinsic gain of Q_1 (in the same order of magnitude). (4%)

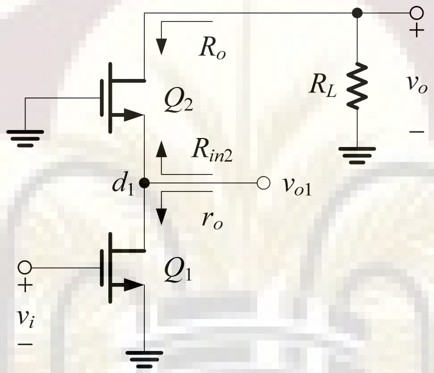


Fig. 1

(Solution)

(1) Find A_{v1}, A_{v2}, R_{in2} and R_o, where $A_{v1} = v_{o1}/v_i$ and $A_{v2} = v_o/v_{o1}$. (12%)

Answer:

$$A_v = A_{v1}A_{v2} = -g_{m1}\left(g_{m2}r_{o2}r_{o1} // R_L\right) = -g_{m1}\left(g_{m2}r_{o2}r_o // R_L\right), where: r_o = r_{o1}$$

$$A_{v1} = \frac{v_{o1}}{v_i} = -g_{m1}\left(r_{o1} // R_{in2}\right) = -g_{m1}\left(r_o // R_{in2}\right)$$

$$A_{v2} = \frac{A_v}{A_{v1}} = \frac{g_{m2}r_{o2}r_{o1} // R_L}{r_{o1} // R_{in2}} = \frac{g_{m2}r_{o2}r_{o1} // R_L}{r_{o1} // \left(\frac{R_L + r_{o2}}{1 + g_{m2}r_{o2}}\right)} = \frac{\frac{1}{r_{o1}} + \frac{1 + g_{m2}r_{o2}}{R_L + r_{o2}}}{\frac{1}{g_{m2}r_{o2}r_{o1}} + \frac{1}{R_L}} = \frac{\frac{\left(R_L + r_{o2}\right) + \left(1 + g_{m2}r_{o2}\right)r_{o1}}{r_{o1}\left(R_L + r_{o2}\right)}}{\frac{R_L + g_{m2}r_{o2}r_{o1}}{g_{m2}r_{o2}r_{o1}R_L}}$$

$$\to A_{v2} = \frac{\left(R_L + r_{o2}\right) + \left(1 + g_{m2}r_{o2}\right)r_{o1}}{r_{o1}\left(R_L + r_{o2}\right)} \frac{g_{m2}r_{o2}r_{o1}R_L}{R_L + g_{m2}r_{o2}r_{o1}} = \left.\frac{\left(R_L + r_{o2}\right) + \left(1 + g_{m2}r_{o2}\right)r_{o1}}{R_L + r_{o2}} \frac{g_{m2}r_{o2}R_L}{R_L + g_{m2}r_{o2}r_{o1}}\right|_{\because g_{m2}r_{o2}r_{o1} >> r_{o1}}$$

$$\to A_{v2} \approx \left.\frac{R_L + r_{o2} + g_{m2}r_{o2}r_{o1}}{R_L + r_{o2}} \frac{g_{m2}r_{o2}R_L}{R_L + g_{m2}r_{o2}r_{o1}}\right|_{g_{m2}r_{o2}r_{o1} >> r_{o2}} \approx \frac{R_L + g_{m2}r_{o2}r_{o1}}{R_L + r_{o2}} \frac{g_{m2}r_{o2}R_L}{R_L + g_{m2}r_{o2}r_{o1}} = \frac{g_{m2}r_{o2}R_L}{R_L + r_{o2}}$$

$$\to A_{v2} = g_{m2}\left(r_{o2} // R_L\right)$$

$$R_{in2} = \frac{R_L + r_{o2}}{1 + g_{m2}r_{o2}}$$

$$R_o = r_{o2} + \left(1 + g_{m2}r_{o2}\right)r_{o1} \approx g_{m2}r_{o2}r_{o1} = g_{m2}r_{o2}r_o$$

(2) If $R_L = 2r_o$, find A_{v1} and R_{in2}. (6%)

<u>*Answer*:</u>

$$A_{v1} = \frac{v_{o1}}{v_i} = -g_{m1}\left(r_o \,//\, R_{in2}\right) = -g_{m1}\frac{r_o R_{in2}}{r_o + R_{in2}} = -g_{m1}\frac{r_o \dfrac{3r_o}{1+g_{m2}r_o}}{r_o + \dfrac{3r_o}{1+g_{m2}r_o}} = -g_{m1}\frac{\dfrac{3r_o}{1+g_{m2}r_o}}{1+\dfrac{3}{1+g_{m2}r_o}}$$

$$\rightarrow A_{v1} = -g_{m1}\left(\frac{3r_o}{4+g_{m2}r_o}\right)$$

$$R_{in2} = \left.\frac{R_L + r_{o2}}{1+g_{m2}r_{o2}}\right|_{R_L=2r_o} = \frac{3r_o}{1+g_{m2}r_o}$$

(3) Explain how to select R_L so that A_{v1} can be comparable with the intrinsic gain of Q_1 (in the same order of magnitude). (4%)

<u>*Answer*:</u>

$$A_{v1} = \frac{v_{o1}}{v_i} = -g_{m1}\left(r_{o1} \,//\, R_{in2}\right) = -g_{m1}\left(r_o \,//\, R_{in2}\right) = -g_{m1}\left(r_o \,//\, \frac{R_L + r_{o2}}{1+g_{m2}r_{o2}}\right) \text{....}(1)$$

Intrinsic gain of Q_1,

$$A_{v1}' = -g_{m1}\left(r_o \,//\, R_L\right) \text{....}(2)$$

Let $(1) = (2)$,

$$r_o \,//\, \frac{R_L + r_{o2}}{1+g_{m2}r_{o2}} = r_o \,//\, R_L \rightarrow \frac{1}{r_o} + \frac{1+g_{m2}r_{o2}}{R_L + r_{o2}} = \frac{1}{r_o} + \frac{1}{R_L} \rightarrow \frac{1+g_{m2}r_{o2}}{R_L + r_{o2}} = \frac{1}{R_L}$$

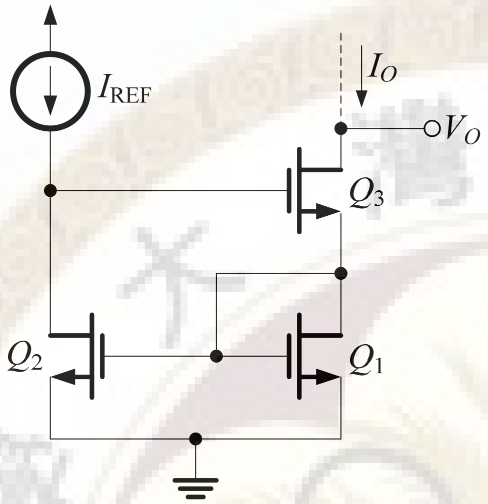
$$\rightarrow R_L\left(1+g_{m2}r_{o2}\right) = R_L + r_{o2} \rightarrow g_{m2}r_{o2}R_L = r_{o2} \rightarrow R_L = \frac{1}{g_{m2}}$$

2. Fig. 2 shows a MOS Wilson current source. Assume the transistors have identical parameters.

(1) Find the output resistance in terms of g_m and r_o of the transistors. (10%)

(2) The V_{DS} of Q_2 is higher than that of Q_1, which results in difference between I_{REF} and I_O. If $V_{GS3} = V_{GS2} = V_{GS1}$ and $V_A = 20\ V_{GS1}$, find I_O/I_{REF}. (8%)

(Hint: $I_D = (1/2)k_n'(W/L)(V_{GS}-V_t)^2(1+V_{DS}/V_A)$)



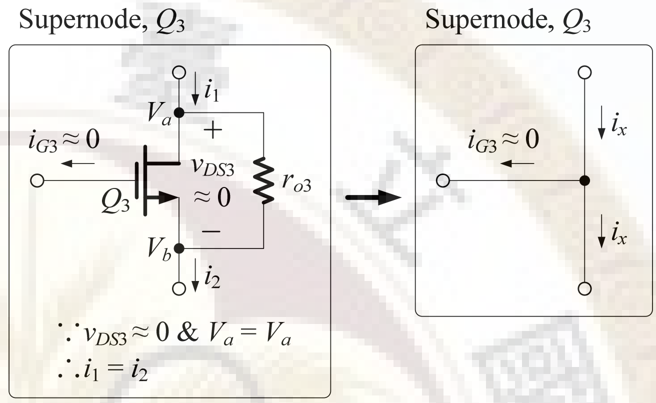
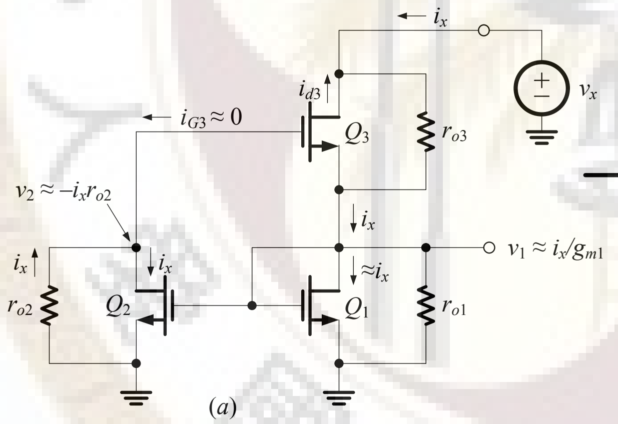


Fig. 2

(Solution)

(1) Find the output resistance in terms of g_m and r_o of the transistors. (10%)

<u>*Answer*:</u>



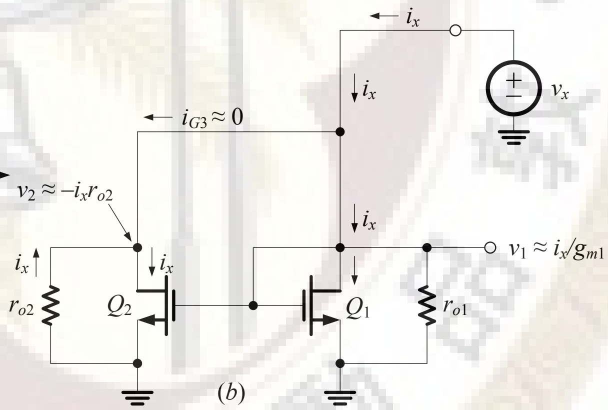


Fig. 2(1.1) Fig. 2(1.2)

where: the small-signal equivalent circuit of Q_3 is shown in Fig. 2(1.2).

Note: The test takers do not need to plot the circuit shown in Fig. 2(1.1) and Fig. 2(1.2).They are illustrated to help understand the process of finding the answers.

$$i_{d3} = g_{m3}v_{gs3} = g_{m3}\left(v_{g3} - v_{s3}\right) = g_{m3}\left(-i_x r_{o2} - \frac{i_x}{g_{m1}}\right) = -i_x g_{m3}\left(r_{o2} + \frac{1}{g_{m1}}\right) \approx -\left(g_{m3}r_{o2}\right)i_x$$

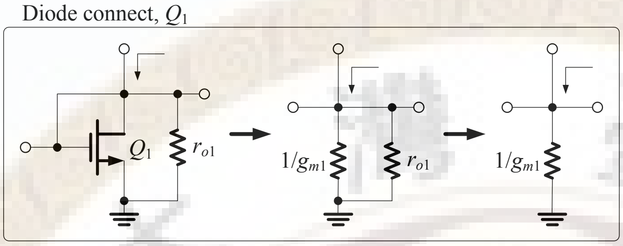
$$v_x = -i_{d3}r_{o3} + v_1 = g_{m3}r_{o2}r_{o3}i_x + \frac{i_x}{g_{m1}} = \left(g_{m3}r_{o2}r_{o3} + \frac{1}{g_{m1}}\right)i_x \approx g_{m3}r_{o2}r_{o3}i_x$$

$$R_o = \frac{v_x}{i_x} = \left(g_{m3}r_{o3}\right)r_{o2}$$

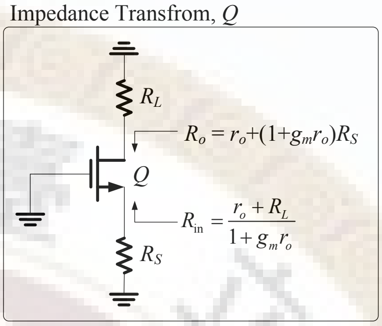
直觀解法：因為 Q_1 為 diode connect 電路，因此，在求該電路之小訊號輸出阻抗為目的時，Q_1 並不會參與阻抗轉換之過程，亦即，從 drain 端看入之阻抗是 source 端看到阻抗之$(1+g_m r_o)$倍。

原因是 Q_1 為 diode connect 電路，在此電路中不具電晶體放大之功能。而真是會參與這樣轉換的電晶體，即只剩(Q_2, Q_3)。

Diode connect, Q_1

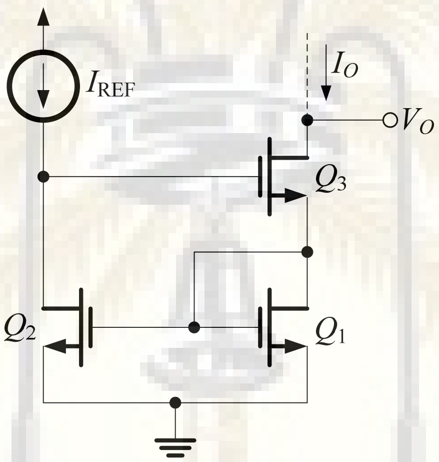


Impedance Transfrom, Q



(2) The V_{DS} of Q_2 is higher than that of Q_1, which results in difference between I_{REF} and I_O. If $V_{GS3} = V_{GS2} = V_{GS1}$ and $V_A = 20\ V_{GS1}$, find I_O/I_{REF}. (8%) (Hint: $I_D = (1/2)k_n'(W/L)(V_{GS}-V_t)^2(1+V_{DS}/V_A)$)

Answer:



3. (27%) Fig. 3 shows a differential cascode amplifier with an active load formed by a Wilson current mirror. Assume all the transistors are identical.

(1) Plot the small-signal model of the Wilson current mirror (3%)

(2) Find the output resistance of the Wilson current mirror (8%)

(3) Plot the small-signal model of the bipolar cascode composed of Q_2 and Q_4 (3%)

(4) Find the output resistance of the bipolar cascode composed of Q_2 and Q_4 (8%)

(5) Prove that the differential gain of this differential cascode amplifier is given approximately by $A_d = (\beta g_m r_o)/3$ (5%)

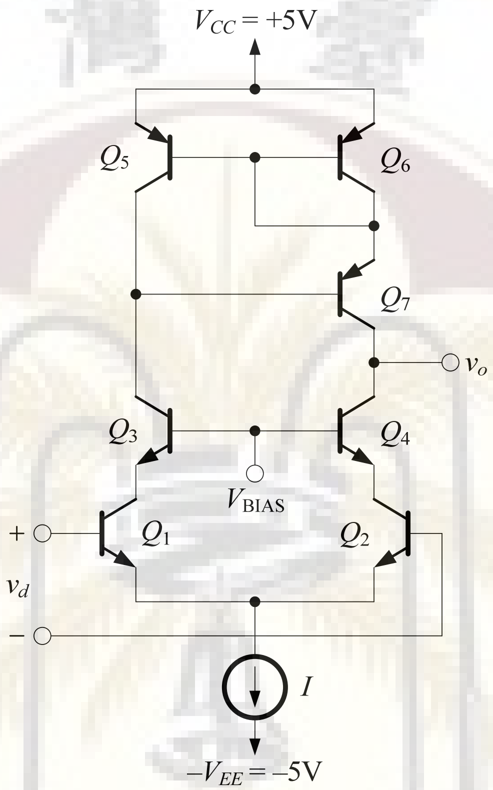


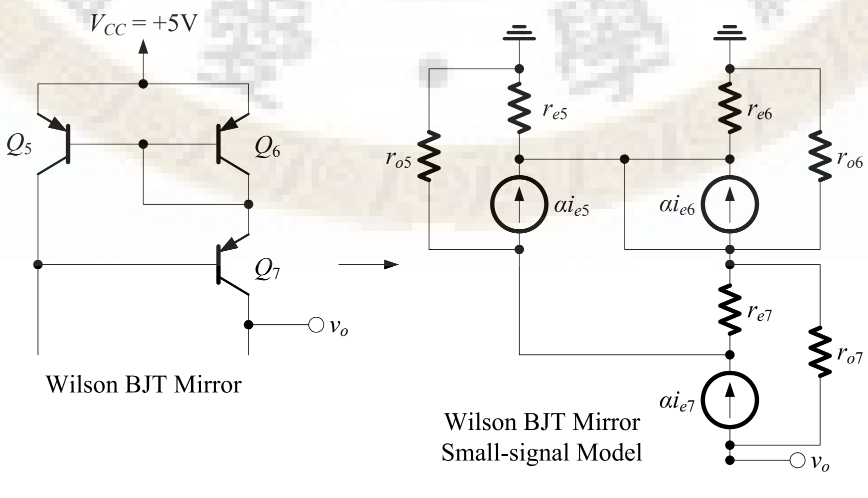
Fig. 3

(Solution)

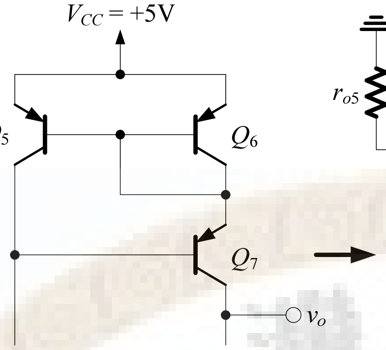
(1) Plot the small-signal model of the Wilson current mirror (3%)

Answer:

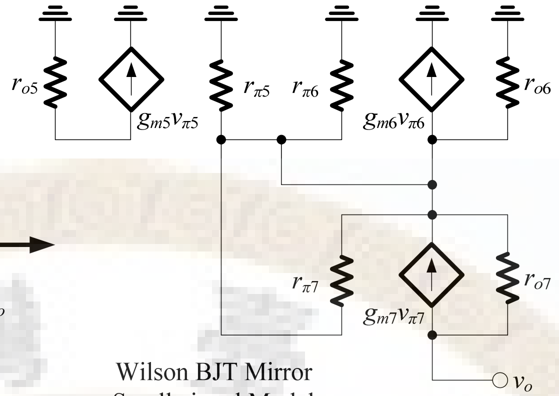
(Case 1) T model small-signal equivalent circuits.



(Case 2) π model small-signal equivalent circuits.



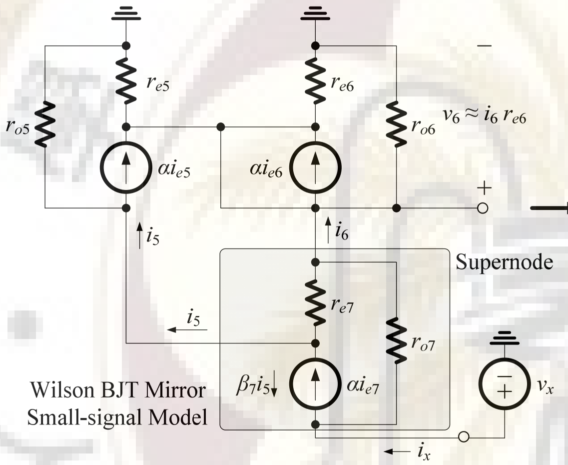
Wilson BJT Mirror



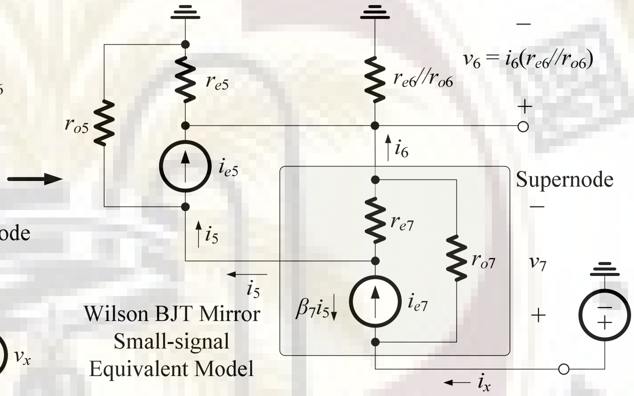
Wilson BJT Mirror
Small-signal Model

(2) Find the output resistance of the Wilson current mirror (8%)

Answer:



Wilson BJT Mirror
Small-signal Model



Wilson BJT Mirror
Small-signal
Equivalent Model

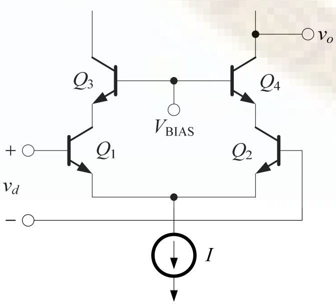
$$v_7 = \left(\beta_7 i_5 + i_x\right) r_{o7},\ v_6 = i_6\left(r_{e6} \,//\, r_{o6}\right) \approx i_6 r_{e6}$$

$$v_x = v_6 + v_7 = i_6 r_{e6} + \left(\beta_7 i_5 + i_x\right) r_{o7} = \frac{i_x}{2} r_{e6} + \left(\beta_7 \frac{i_x}{2} + i_x\right) r_{o7} = i_x \left[\frac{r_{e6}}{2} + \left(\frac{\beta_7}{2} + 1\right) r_{o7}\right] \approx i_x \frac{\beta_7}{2} r_{o7}$$

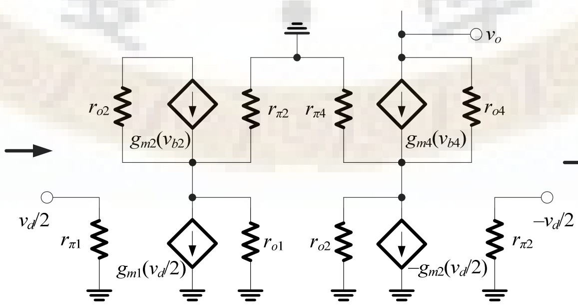
$$R_o = R_{o.upper} = \frac{v_x}{i_x} = \frac{\beta_7}{2} r_{o7},\ \ where: i_6 \approx i_5 = \frac{i_x}{2}$$

(3) Plot the small-signal model of the bipolar cascode composed of Q_2 and Q_4 (3%)

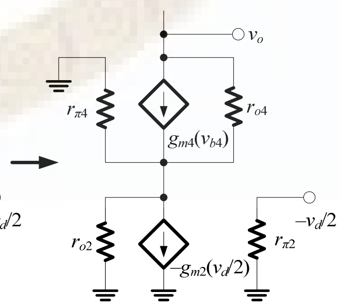
Answer:



Cascode BJT Differential Pair



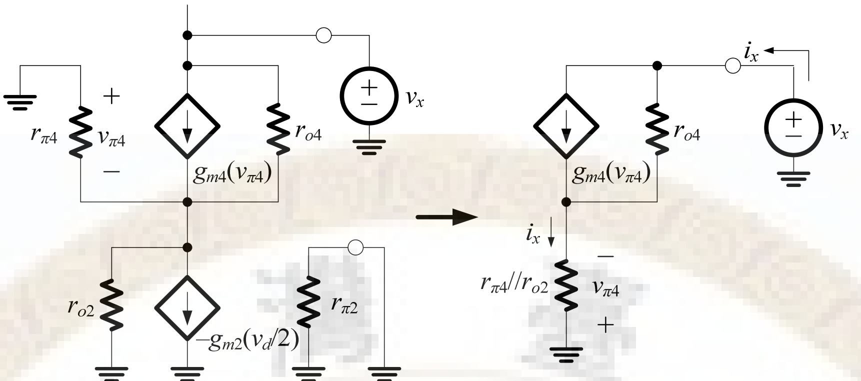
Small-signal Model of
Cascode BJT Differential Pair



(Q_2, Q_4) Small-signal Model of
Cascode BJT Differential Pair

(4) Find the output resistance of the bipolar cascode composed of Q_2 and Q_4 (8%)

Answer:



Small-signal Output Resistance Model of (Q_2, Q_4)

Small-signal Output Resistance Model of (Q_2, Q_4)

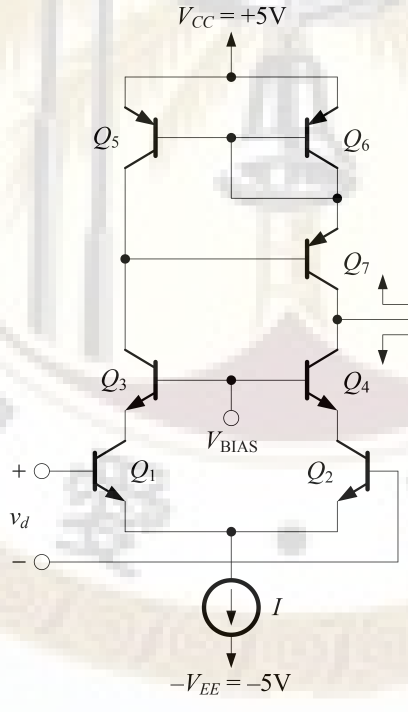
$$v_x = (i_x - g_{m4}v_{\pi 4})r_{o4} + i_x(r_{\pi 4} // r_{o2}) = i_x\left[1 + g_{m4}(r_{\pi 4} // r_{o2})\right]r_{o4} + i_x(r_{\pi 4} // r_{o2})$$

$$where: -v_{\pi 4} = i_x(r_{\pi 4} // r_{o2})$$

$$R_o = R_{o.lower} = \frac{v_x}{i_x} = \left[1 + g_{m4}(r_{\pi 4} // r_{o2})\right]r_{o4} + (r_{\pi 4} // r_{o2}) \approx g_{m4}(r_{\pi 4} // r_{o2})r_{o4} \approx g_{m4}r_{\pi 4}r_{o4} = \beta_4 r_{o4}$$

(5) Prove that the differential gain of this differential cascode amplifier is given approximately by $A_d = (\beta g_m r_o)/3$ (5%)

Answer:



$$R_{o.upper} = \frac{\beta_7}{2}r_{o7},\ R_{o.lower} = \beta_4 r_{o4} \rightarrow R_o = R_{o.upper} // R_{o.lower} = \left(\frac{1}{2}\beta r_o\right) // \beta r_o = \frac{1}{3}\beta r_o$$

$$A_d = G_m R_o = \frac{1}{3}(\beta g_m r_o)$$

4. (13%) Fig. 4 shows a MOS differential amplifier. Assume Q_1 and Q_2 are in saturation.

(1) Find the currents i_{D1} and i_{D2} in terms of $v_d \equiv v_{G1} - v_{G2}$ and other semiconductor parameters (8%)

(2) When the tail current I is steered entirely into Q_1. Find v_d in this case. (5%)

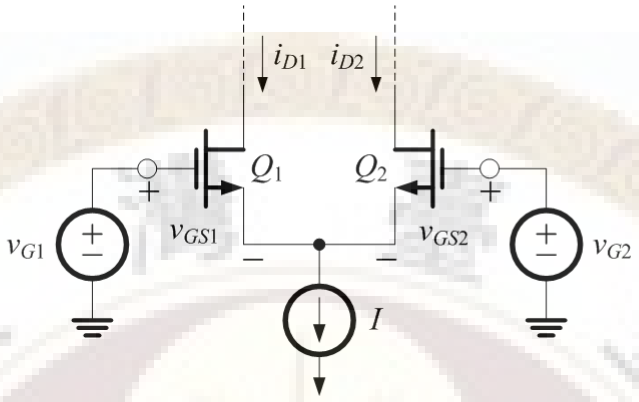


Fig. 4

(Solution)

(1) Find the currents i_{D1} and i_{D2} in terms of $v_d \equiv v_{G1} - v_{G2}$ and other semiconductor parameters (8%)

<u>*Answer*</u>:

$$(a)\begin{cases} i_{D1} = \dfrac{1}{2}k_n^{'}\dfrac{W}{L}\left(v_{GS1}-V_t\right)^2 \\ i_{D2} = \dfrac{1}{2}k_n^{'}\dfrac{W}{L}\left(v_{GS2}-V_t\right)^2 \end{cases} \rightarrow \begin{cases} \sqrt{i_{D1}} = \sqrt{\dfrac{1}{2}k_n^{'}\dfrac{W}{L}}\left(v_{GS1}-V_t\right)......(1) \\ \sqrt{i_{D2}} = \sqrt{\dfrac{1}{2}k_n^{'}\dfrac{W}{L}}\left(v_{GS2}-V_t\right)......(2) \end{cases}$$

$$v_d = v_{GS1} - v_{GS2} = v_{G1} - v_{G2}$$

$$(1)-(2),\rightarrow \sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}v_d......(3)$$

$$(3)^2 \rightarrow \left(\sqrt{i_{D1}} - \sqrt{i_{D2}}\right)^2 = \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2 \rightarrow i_{D1} + i_{D2} - 2\sqrt{i_{D1}}\sqrt{i_{D2}} = \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2$$

$$\rightarrow 2\sqrt{i_{D1}i_{D2}} = I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2......(4),\quad where: i_{D1}+i_{D2} = I......(5)$$

$$(b)\ \left(\sqrt{i_{D1}} + \sqrt{i_{D2}}\right)^2 = i_{D1} + i_{D2} + 2\sqrt{i_{D1}i_{D2}} = 2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2 \rightarrow \sqrt{i_{D1}} + \sqrt{i_{D2}} = \sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2}......(6)$$

$$(c)\ (3)+(6) \rightarrow 2\sqrt{i_{D1}} = \sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}v_d + \sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2} \rightarrow i_{D1} = \frac{1}{4}\left(\sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}v_d + \sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2}\right)^2$$

$$\rightarrow i_{D1} = \frac{1}{4}\left(\frac{1}{2}k_n^{'}\frac{W}{L}v_d^2 + 2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2 + 2\sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}v_d\sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2}\right)$$

$$= \frac{I}{2} + \sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}\left(\frac{v_d}{2}\right)\sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2} = \frac{I}{2} + \sqrt{\frac{1}{2}k_n^{'}\frac{W}{L}}\left(\frac{v_d}{2}\right)\sqrt{2I - \frac{1}{2}k_n^{'}\frac{W}{L}v_d^2}$$

$$= \frac{I}{2} + \sqrt{k_n^{'}\frac{W}{L}I}\left(\frac{v_d}{2}\right)\sqrt{1 - \frac{1}{I}k_n^{'}\frac{W}{L}\left(\frac{v_d}{2}\right)^2} = \frac{I}{2} + \sqrt{k_n^{'}\frac{W}{L}I}\left(\frac{v_d}{2}\right)\sqrt{1 - \frac{\left(v_d/2\right)^2}{I/\left(k_n^{'}\left(W/L\right)\right)}}$$

$$\rightarrow i_{D1} = \frac{I}{2} + \sqrt{k_n' \frac{W}{L} I}\left(\frac{v_d}{2}\right)\sqrt{1 - \frac{(v_d/2)^2}{I/\left(k_n'(W/L)\right)}}$$

(d) $\because i_{D1} + i_{D2} = I$

$$\rightarrow i_{D2} = I - i_{D1} = \frac{I}{2} - \sqrt{k_n' \frac{W}{L} I}\left(\frac{v_d}{2}\right)\sqrt{1 - \frac{(v_d/2)^2}{I/\left(k_n'(W/L)\right)}}$$

(e) At the bias point, $v_d = 0$, leading to

$$i_{D1} = i_{D2} = \frac{I}{2},\ v_{GS1} = v_{GS2} = V_{GS},$$

$$\rightarrow i_{D1} = i_{D2} = \frac{I}{2} = \frac{1}{2} k_n' \frac{W}{L}(v_{GS1} - V_t)^2 = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2 = \frac{1}{2} k_n' \frac{W}{L} V_{OV}^2$$

$$\rightarrow I = k_n' \frac{W}{L} V_{OV}^2 \rightarrow V_{OV} = \sqrt{\frac{I}{k_n'(W/L)}} \rightarrow \sqrt{k_n' \frac{W}{L}} = \frac{\sqrt{I}}{V_{OV}}$$

(f) $$i_{D1} = \frac{I}{2} + \sqrt{k_n' \frac{W}{L} I}\left(\frac{v_d}{2}\right)\sqrt{1 - \frac{(v_d/2)^2}{I/\left(k_n'(W/L)\right)}} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_d}{2}\right)\sqrt{1 - \left(\frac{v_d/2}{V_{OV}}\right)^2}$$

$$i_{D2} = \frac{I}{2} - \left(\frac{I}{V_{OV}}\right)\left(\frac{v_d}{2}\right)\sqrt{1 - \left(\frac{v_d/2}{V_{OV}}\right)^2}$$

(2) When the tail current I is steered entirely into Q_1. Find v_d in this case. (5%)

<u>*Answer*</u>:

$$\rightarrow i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{v_d}{2}\right)\sqrt{1 - \left(\frac{v_d/2}{V_{OV}}\right)^2} = I$$

$$\rightarrow \left(\frac{I}{V_{OV}}\right)\left(\frac{v_d}{2}\right)\sqrt{1 - \left(\frac{v_d/2}{V_{OV}}\right)^2} = \frac{I}{2} \rightarrow \left(\frac{v_d}{V_{OV}}\right)\sqrt{1 - \left(\frac{v_d/2}{V_{OV}}\right)^2} = 1 \rightarrow \left(\frac{v_d}{V_{OV}}\right)^2\left[1 - \left(\frac{v_d/2}{V_{OV}}\right)^2\right] = 1$$

$$\rightarrow \left(\frac{v_d}{V_{OV}}\right)^2\left[1 - \frac{1}{4}\left(\frac{v_d}{V_{OV}}\right)^2\right] = 1 \rightarrow -\frac{1}{4}\left(\frac{v_d}{V_{OV}}\right)^4 + \left(\frac{v_d}{V_{OV}}\right)^2 - 1 = 0 \rightarrow \left(\frac{v_d}{V_{OV}}\right)^4 - 4\left(\frac{v_d}{V_{OV}}\right)^2 + 4 = 0$$

$$\rightarrow \left[\left(\frac{v_d}{V_{OV}}\right)^2 - 2\right]^2 = 0 \rightarrow \left(\frac{v_d}{V_{OV}}\right)^2 = 2 \rightarrow v_d = \sqrt{2} V_{OV} = \sqrt{\frac{2I}{k_n'(W/L)}}$$

5. (20%) The NMOS transistor in the discrete common-source amplifier of Fig. 5 is biased to have $g_m = 1\ mA/V$. Find
   (1) A_M (4%),
   (2) f_{p1} (4%), f_{p2} (4%),
   (3) f_{p3} (4%) and f_L (4%)

   in unit of Hz using the short circuit time-constant method (i.e., the low-frequency 3dB frequency is given by the summation of all poles). Do not forget the (2π).

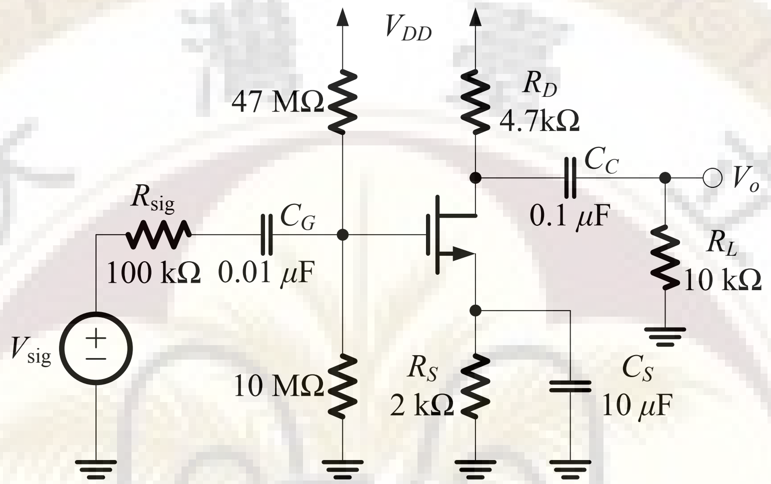


Fig. 5

(Solution)

(1) A_M (4%),

$$R_G = 47M\Omega // 10M\Omega = 8.25M\Omega \rightarrow (+1\%)$$

$$A_M = -\frac{R_G}{R_G + R_{\text{sig}}} g_m (R_D // R_L) \rightarrow (+2\%) = -\frac{8.25M\Omega}{8.25M\Omega + 0.1M\Omega}(1mA/V)(4.7k\Omega // 10k\Omega)$$

$$\rightarrow A_M = -3.16(V/V) \rightarrow (+1\%)$$

(2) f_{p1} (4%), f_{p2} (4%),

$$f_{p1}\Big|_{\text{@Gate Terminal}} = \frac{1}{2\pi C_G (R_G + R_{\text{sig}})} \rightarrow (+3\%) = \frac{1}{2\pi \times 0.01\mu F \times (8.25M\Omega + 0.1M\Omega)} = 1.9Hz \rightarrow (+1\%)$$

$$f_{p2}\Big|_{\text{@Source Terminal}} = \frac{1}{2\pi C_S \left(R_S // \frac{1}{g_m}\right)} \rightarrow (+3\%) = \frac{1}{2\pi \times 10\mu F \times (2k\Omega // 1k\Omega)} = 23.87Hz \rightarrow (+1\%)$$

(3) f_{p3} (4%) and f_L (4%)

$$f_{p3}\Big|_{\text{@Drain Terminal}} = \frac{1}{2\pi C_C (R_D + R_L)} \rightarrow (+3\%) = \frac{1}{2\pi \times 0.1\mu F \times (4.7k\Omega + 10k\Omega)} = 108.27Hz \rightarrow (+1\%)$$

$$f_L \approx f_{p3} \rightarrow (+3\%) = 108.27Hz \rightarrow (+1\%)$$