工程數學-複變

期中考

共 100 分

考試時間 100 分鐘

4/23/02

1. (10%) Verify $Ln\dfrac{1+i}{1-i} = Ln(1+i) - Ln(1-i)$.

2. (10%) Evaluate $\sin\left(\dfrac{\pi}{2} + i\ln 2\right)$

3. (10%) Compute all values of 2^i.

4. (10%) Find the fourth root of $1+i$.

5. (10%) Evaluate $\oint_C \dfrac{e^{z^2}}{z-i}dz$, where C is a simple closed contour with positive direction along the circle $|z|$=2.

6. (10%) Evaluate $\oint_C \dfrac{e^{z^3}}{(z-i)^3}dz$, where C is a simple closed contour with positive direction along the circle $|z|$=3.

7. (10%) Evaluate $\oint_C \dfrac{z-4i}{z^3+4z}dz$, where C is a simple closed contour with positive direction along the circle $|z|$=3.

8. (15%) Evaluate $\int_{-1-\sqrt{3}i}^{-1+i}\left(\dfrac{1}{z}+\dfrac{1}{z^2}\right)dz$, the integration is along a contour C in the left half plane Re(z)<0.

9. (15%) Let $f(z) = z^n g(z)$, where n is a positive integer, $g(z)$ is an entire function, and $g(z) \neq 0$ for all z. Let C be a circle with center at the origin. Evaluate $\oint_C \dfrac{f'(z)}{f(z)}dz$.