電機系大一機率期末考 88.06.07 A.M.10:10~11:50(共一頁)

(20%) 1. Let X be a geometric random variable with parameter p.

(a) Show that the moment generating function of X is given by

$$M_X(t) = pe^t/(1 - qe^t), \quad q = 1 - p, \quad t < -\ln q. \quad (10\%)$$

(b) Use $M_X(t)$ to find $E(X)$ and $Var(X)$. (10%)

(15%) 2. A point is selected at random from the bounded region between two curves $y = x^2-1$ and $y = 1-x^2$. Let X be the x-coordinate and let Y be the y-coordinate of the point selected.

(a) Find the joint probability density function of X and Y. (4%)

(b) Find the marginal probability density function of X and of Y, $f_X(x)$ and $f_Y(y)$. (8%)

(c) Are X and Y independent? (3%)

(10%) 3. Prove the weak law of large numbers.

(10%) 4. State the central limit theorem.

(10%) 5. Prove that if $Cov(X,Y) = 0$, then

$$\rho(X+Y, X-Y) = \frac{Var(X) - Var(Y)}{Var(X) + Var(Y)}$$

(10%) 6. Let X be a random variable with the density function

$$f(x) = \frac{1}{\pi(1+x^2)}, \qquad -\infty < x < \infty$$

(X is called a Cauchy random variable.) Find the density function of Z=arctan X.

(25%) 7. In this problem, we consider a production line manufactures 1000-Ω resistors that must satisfy a 10% tolerance.

(a) If resistance is adequately descried by a Gaussian random variable X for which expectation of X is $\mu = 1000\Omega$ and standard deviation $\sigma = 40\Omega$. What fraction of the resistors is expected to be rejected?(15%)

(b) If a machine is not properly adjusted, the product resistance changes to the case where $\mu = 1050\Omega$. What fraction is now rejected?(10%)