微分方程期中考

Dept. of Electrical Engineering

Total = 100 points (2 pages) 11/12/2003

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1. (15 %) Solve the differential equations

(a) $\dfrac{dy}{dx}=\dfrac{xy+3y-x-3}{xy-2y+x-2}$ (5%)

(b) $\dfrac{dy}{dx}=\tan^2(x+y)$ (5%)

(c) $(y+xy^2)dx+(5y-x+y^2\sin y)dy=0$ (5%)

2. (10%)(a) Please show that the Ricatti's equation

$$\frac{dy}{dx}+P(x)y+Q(x)y^2=R(x)$$

can be reduced to a linear second-order equation.

(Hint: use the substitution, $y=\dfrac{du/dx}{Qu}$.) (5%)

(b) If y_1 is a solution of the above linear second-order equation, please show that the second solution can be written as $y_2 = y_1 + 1/v$, where v satisfies a linear first-order equation. Write down this linear first-order equation. (5%)

3. (5%) Write down the form of a particular solution of

$y^{(4)}-2y''+y=\cosh x$ in terms of hyperbolic functions.

4. (10%) Consider the Differential Equation

$$x^2y''-(x^2+2x)y'+(x+2)y=x^3$$

(a) Verify that $y_1 = x$ is one solution of the associated homogeneous equation.(2%)

(b) Using the method of reduction of order to find the general solution of the Differential Equation on the interval $(0, \infty)$. (8%)

5. (10%) Solve the initial-value problems.

(a) $y^{(5)} = y'$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$, $y'''(0) = 1$, $y^{(4)}(0) = 2$. (5%)

(b) y'' + 4y = g(x), y(0) = 1, y'(0) = 2

$$\text{where } g(x) = \begin{cases} \cos x & 0 \le x \le \frac{\pi}{2} \\ 0 & x > \frac{\pi}{2} \end{cases}$$ (5%)

6. (15%) Solve the given system of equations with the initial value x(0) = 1 and y(0) = 1.

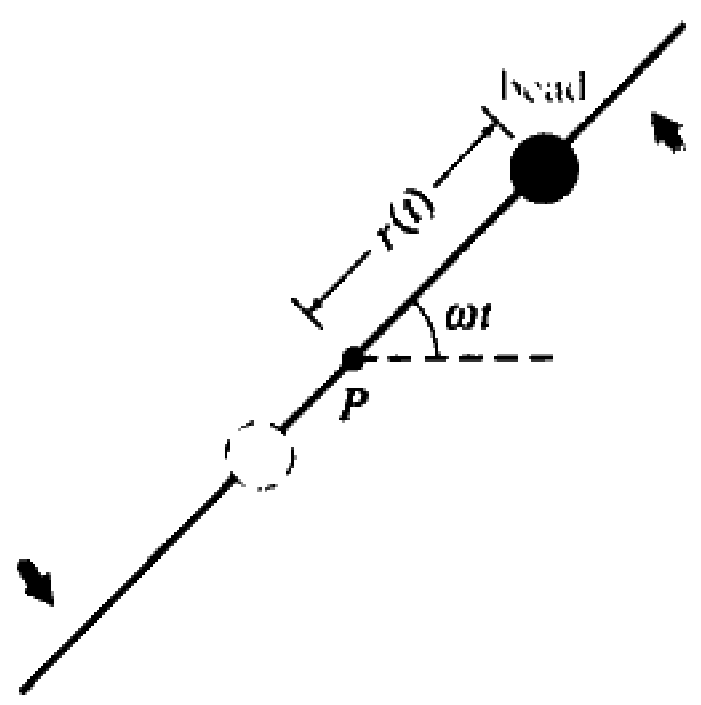
$$x(t) + \frac{dy(t)}{dt} + 2y(t) = \cos 2t + 3e^t$$

$$\frac{dx(t)}{dt} + 2x(t) + y(t) = 2\cos 2t - 2\sin 2t + e^t$$

7. (10%) Find the general solution of

$$x^2 \frac{d^2 y(x)}{dx^2} + 3x\frac{dy(x)}{dx} + y(x) = \frac{\ln x}{x}, \text{ where } x > 0$$

8. (25%) A bead with mass m is constrained to slide along a frictionless rod of length L. The rod is rotating in a vertical plane with a constant angular velocity ω about a pivot P fixed at the midpoint of the rod, but the design of the pivot allows the bead to move along the entire length of the rod. Let r(t) denote the position of the bead relative to this rotation coordinate system as shown in the following figure.



(a) Suppose g is the gravitational acceleration. Use the Newton's law to find the differential equation for r(t) and find the solution subject to the initial conditions r(0) = r_0 and r'(0) = v_0. (15%)

(b) Please determine the values of r_0 and v_0 for which a simple harmonic oscillation of the bead is observed in the rotation coordinate system. What is the minimum length of the rod for which it can accommodate the simple harmonic oscillation of the bead? (10%)