Signals and Systems, Final

10:10-12:10, Jan 12, Fri, 2001

- Close book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120, Time allocation: 1 point/minute

1.(4) A linear, time-invariant system $H(e^{jw})$ is distortionless if for any input signal $x[n]$, the output signal is of the form $y[n]=Ax[n-n_0]$ for some certain constant A and constant integer n_0.

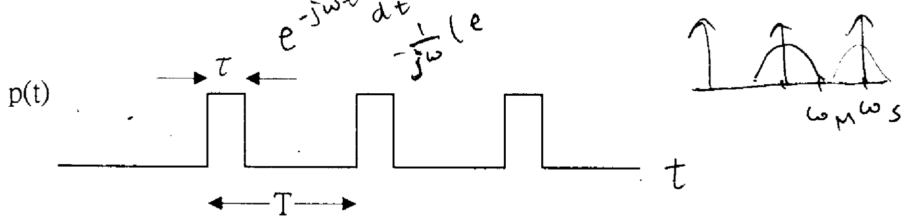
$Y(e^{j\omega}) = A e^{-j\omega n_0} X(\dots)$

What are the conditions on $H(e^{jw})$ for the system to be distortionless?

2.(4) What is the "group delay" for a system with a transfer function $H(j\omega)$? Explain what that means.

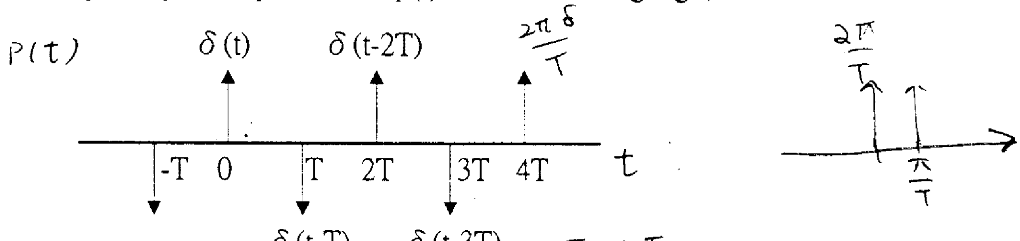
$H(j\omega)$

3.(8) A signal $x(t)$ is sampled by a rectangular pulse train $p(t)$ with pulse width τ and sampling period T, $\tau \ll T$,



so the sampled signal is $x_p(t)=x(t)p(t)$. Show if $x(t)$ can be recovered from $x_p(t)$, and if the sampling theorem applies here.

4.(8) A signal $x(t)$ is sampled by an impulse train $p(t)$ with alternating sign,



$T = 2T$. $\omega = \frac{2\pi}{2T} = \frac{\pi}{T}$

so that the sampled signal is $x_p(t)=x(t)p(t)$. Find $X_p(j\omega)$ in terms of $X(j\omega)$, the Fourier transform of $x(t)$.

5.(10) Two operators A and B are applied on a discrete-time signal $x[n]$,

$$x[n] \longrightarrow \boxed{A} \xrightarrow{y[n]} \boxed{B} \longrightarrow z[n]$$

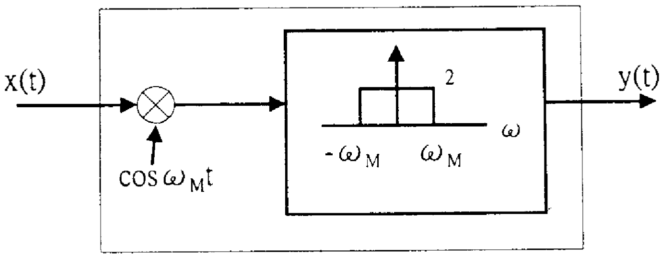
$$y[n]=\begin{cases} x(n/k), & \text{if n is a multiple of k} \\ 0, & \text{else} \end{cases}$$

$z[n]=y[nk]$

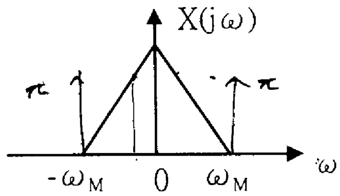
Find $Y(e^{j\Omega})$ and $Z(e^{j\Omega})$ in terms of $X(e^{j\Omega})$, where $X(e^{j\Omega})$, $Y(e^{j\Omega})$ and $Z(e^{j\Omega})$ are respectively the Fourier transforms of x[n], y[n] and z[n], and explain what the operators A and B really mean in both the time and frequency domains.

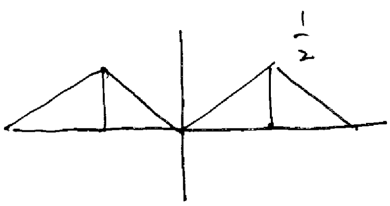
6.(6) Explain what phase modulation and frequency modulation are in terms of the "instantaneous frequency".

7.(8) The system below is called a "frequency inverter" in early days, where ω_M is the maximum



frequency for x(t), i.e. x(t) has a Fourier transform $X(j\omega)$ as below , $|X(j\omega)|=0$, $|\omega|>\omega_M$





Show why this system is called a "frequency inverter", and the system is its own inverse System .

8.(9)Let $X(s)=\dfrac{1}{(s+1)(s+2)}$ be the Laplace transform of x(t). Find x(t) if the Region of Convergence of X(s) is

(a)(3) Re{s}>-1

(b)(3) Re{s}<-2

(c)(3) -2<Re{s}<-1

e^{st}

9.(8) Find the inverse Laplace transform of $\dfrac{1}{(s+a)^n}$ with region of convergence Re{s}>-a .

10.(15) When an input signal $x(t)=e^{-3t}u(t)$ is applied to a linear, time-invariant system, the output signal is $y(t)=(e^{-t}-e^{-2t})u(t)$

(a)(6) Find the system function H(s) and its region of convergence

(b)(6) Determine the causality and stability of the system

(c)(3) Write down the differential equation characterizing the system(with initial rest condition)

11.(8) Let $x(n)=\left(\frac{1}{3}\right)^n \sin\left(\frac{\pi}{4}n\right)u[n]$ .Find the Z-transform of x[n], its region of convergence, and draw the pole-zero plot.

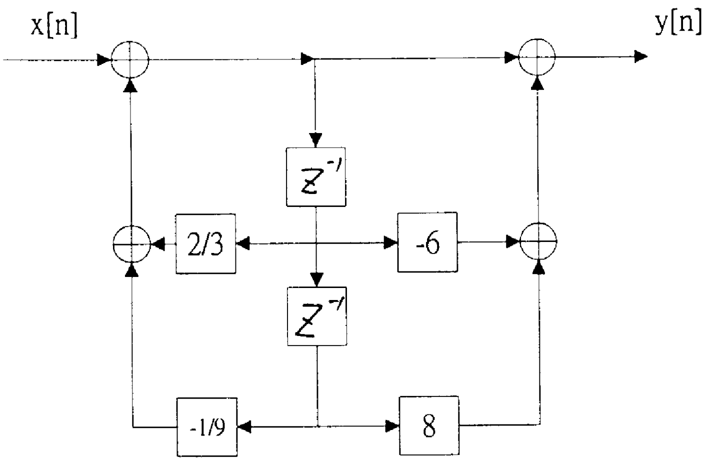
12.(16) Verify if the following properties of Z-transform is correct. If yes, prove it.If no, show it is not. R is the Region of Convergence(ROC) of X(z) , the Z-transform of x[n] .

(a)(8) $x_{(k)}[n] \overset{Z}{\leftrightarrow} X(z^k)$ with ROC=$R^{1/k}=\{z^{1/k} \mid z \in R\}$

where $x_{(k)}[n]=\begin{cases} x(n/k), & \text{if n is a multiple of k} \\ 0 & \text{, else} \end{cases}$ $\quad z^{-\frac{n}{k}} \quad z^{-nk}$

(b)(8) $\sum_{k=-\infty}^{n} x[k] \overset{Z}{\leftrightarrow} \frac{1}{1-z^{-1}}X(z)$ with ROC$\supset$[R$\cap$\{|z|>1\}]

13.(4) Write down the difference equation relating y[n] and x[n] of a causal, linear, time-invariant system below



14.(8) H(z) is the system function of a linear, time-invariant system, and H(z) is rational. Explain how and why the locations of poles of H(z) are related to the stability of the system.

15.(4)Show the time delay property of Unilateral Z-transform ,

$$x[n-1] \xleftrightarrow{Z_u} z^{-1}X(z)_u + x[-1]$$

where Z_u means unilateral Z-transform and $X(z)_u$ is the unilateral Z-transform of x[n] .