

1. (20%) Please prove the following statements:

- (a) The probability of selecting  $1/2$  in a random selection of a point from  $(0,1)$  is 0. (10%)
- (b) A point is selected at random from interval  $(0,1)$ . The probability that it is irrational is 1. (10%)

2. (10%) If two fair dice are rolled 10 times, what is the probability of at least one 6 (on either die) in exactly five of these 10 rolls.

3. (10%) From families with three children, a family is selected at random and found to have a boy. What is the probability that the boy has a brother and a sister? Assume that in a three-child family all gender distributions have equal probabilities.

4. (10%) An urn contains **five red** and **three blue** chips. Suppose that four of these chips are selected at random and transferred to a second urn, which was originally empty. If a random chip selected from this second urn is **red**, what is the probability that **two red** and **two blue** chips were transferred from the first urn to the second urn?

5. (10%) Let  $S$  be the sample space of a repeatable experiment. Let  $A$  and  $B$  be the mutually exclusive events of  $S$  with a probability of  $P(A)$  and  $P(B)$ , respectively. In **independent trials** of this experiment, what is the probability that the event  $A$  occurs before the event  $B$ ?

6. (8%) In this Problem, we consider that a random variable  $X$  has the distribution function  $F(t)$  given by

$$F(t) = \begin{cases} 0 & t < -1 \\ (1/4)t + 1/4 & -1 \leq t < 0 \\ 1/2 & 0 \leq t < 1 \\ (1/12)t + 7/12 & 1 \leq t < 2 \\ 1 & 2 \leq t \end{cases}$$

(a) Plot the graph of  $F(t)$ . (3%)

(b) Calculate the following probabilities: (5%)

$$P(X < 1), P(X=1), P(1 \leq X < 2), P(X > 1/2), \text{ and } P(1 < X \leq 6)$$

- 7.(12%)In this problem, we consider that three calls are observed at a telephone switch, where voice calls (V) and data calls (D) are equally likely. Assume that  $X$  denotes the number of voice calls,  $Y$  the number of data calls. Now let  $R = XY$ .
- (a) Find the corresponding values of the random variables  $X$ ,  $Y$ , and  $R$ . (3)
  - (b) Find the probability mass function (PMF) of  $R$ . (3)
  - (c) Find the expectation of  $R$ . (3)
  - (d) Find the variance of  $R$ . (3)

8.(10%)In this problem, we proceed with **Problem 7**.

- (a) Find the expectation of the random variable  $Q = f(R) = 2R - 3/2$ . (2)
- (b) Find the second moment of the random variable  $Q$  of (a). (2)
- (c) Find the variance of the random variable  $Q$  of (a). (2)
- (d) Find the standard deviation of the random variable  $Q$  of (a). (2)
- (e) Which one of  $R$  and  $Q$  is more concentrated about their expectations and why ? (2)

9.(10%)Suppose that, on the Richter scale, earthquakes of magnitude 5.5 or higher have probability 0.015 of damaging class-A bridges. Suppose that such intense earthquake occur following a Poisson random variable in average 1.5 times per year. If a class-A bridge is constructed to last at least 60 years, what is the probability that it will be undamaged by earthquakes for 60 years.