

National Taiwan University
Department of Electrical Engineering

工程數學-複變

Final Exam

6/13/00

1. (20%) Expand $f(z) = \frac{7z-3}{z(z-1)}$ in a Laurent series valid for
 - (a) $0 < |z| < 1$
 - (b) $0 < |z-1| < 1$
2. (10%) Determine the order of the pole at 0 for $f(z) = \frac{1}{1-e^z}$.
3. (10%) Evaluate $\oint_C \frac{\tan z}{z} dz$ along the contour $C: |z-1| = 2$.
4. (10%) Use an indented contour and residues to establish the result of

$$P.V. \int_{-\infty}^{\infty} dx \frac{\sin x}{x(x^2+1)} = \pi(1-e^{-1}).$$
5. (10%) Using residues, find the inverse Laplace transform of $\frac{z}{z^2+9}$.
6. (10%) Sum the series $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n^2+1}$.
7. (10%) Find the image of the region defined by $-\pi/2 \leq x \leq \pi/2$, $y \geq 0$, under the complex mapping $w = (\sin z)^{1/4}$. What is the image of the line segment $[-\pi/2, \pi/2]$ on the x-axis?
8. (10%) Find a transformation that will map the domain $0 < \arg z < \pi/2$ from the z-plane onto $|w| < 1$ in the w-plane.
9. (10%) Use the Schwarz-Christoffel formula to construct a conformal mapping $w=f(z)$ from the upper half plane to the given square region using $x_1=-1$, $x_2=0$, $x_3=1$, $x_4=2$. Find $f'(z)$. Do not attempt to obtain $f(z)$.

