

Department of Electrical Engineering, National Taiwan University
Engineering Mathematics-Differential Engineering, 2011, Fall
 Final Examination

2012/01/11 Wednesday, 10:20-12:10

1. (6 scores) Solve the following differential equation using Laplace transform

$$x'' - 3x' + 2x = 100\delta(t-3); \quad x(0) = 4, \quad x'(0) = 0$$

2. (6 scores) Find the inverse Laplace transform for the following equation

$$\frac{1}{s} \tanh s$$

$$(\text{Hint: recall that } \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \text{ and } \frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots)$$

3. (6 scores) Solve $x(t)$ from the following differential equation with the input $f(t)$ is a square wave

$$x'' + x = f(t); \quad x(0) = x'(0) = 1; \quad f(t) = \begin{cases} 1, & 2n < t < 2n+1, \quad n = 0, 1, 2, 3, \dots \\ 0, & 2n+1 < t < 2n+2, \quad n = 0, 1, 2, 3, \dots \end{cases}$$

4. (6 scores) Find the Laplace transform of $x(t)$ from the following integral equation

$$x(t) - 1 - 3t + 25 \int_0^t \int_0^\tau x(h) dh d\tau = \int_0^t \int_0^\tau e^{4h} \sin(5h) dh d\tau; \quad 0 \leq t \leq \infty$$

5. (3 scores) Find an example that $f(t)$ is of exponential order but df/dt is not.

6. (3 scores) Find the Laplace transform of

$$\int_0^t \cosh 3(t-\tau) d\tau$$

7. (9 scores) Use the Fourier transform (or the Fourier cosine transform, Fourier sine transform) to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \quad x > 0, \quad 0 < y < \pi,$$

$$u(0, y) = 1, \quad u(x, 0) = 1, \quad u(x, \pi) = e^{-x}.$$

8. (26 scores) Solve the following PDEs.

(a) $y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} = 0$

(8 scores)

(b) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$

$$u(0, t) = f(t), \quad u(2, t) = 0, \quad u(x, 0) = \sin\left(\frac{\pi}{2}x\right), \quad \left.\frac{\partial}{\partial t}u(x, t)\right|_{t=0} = 1.$$

(9 scores)

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$$u(0, y) = f(y), \quad u(2, y) = h(y), \quad u(x, 0) = 0, \quad u(x, 3) = 0$$

(Express the solution in terms of $f(y)$ and $h(y)$) (9 scores)

9. (10 scores) Using Fourier cosine series to solve the following DE,

$$\frac{1}{4}x'' + 12x = f(t), \quad x(0) = 1, \quad x'(0) = 0$$

$$f(t) = 2\pi t - t^2, \quad 0 < t < 2\pi; \quad f(t + 2\pi) = f(t)$$

10. (10 scores) Using Fourier series of $f(t)$ to show that

$$f(t) = \begin{cases} 0, & -\pi < t < 0 \\ \sin t, & 0 \leq t < \pi \end{cases}$$

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \frac{1}{7 \cdot 9} + \dots$$

11. (10 scores) Using $y = x^{1/2}w(t)$ and $t = \frac{2}{3}\alpha x^{3/2}$ to show the general

solution of Airy's DE $y'' + \alpha^2 xy = 0, x > 0$ can be expressed in terms of Bessel functions.

12. (5 scores) Use the method of Frobenius to obtain the general solution of DE $xy'' - xy' + 7 = 0$ on $(0, \infty)$.

- 期末考成績預計公告時間：1/16（一）中午 12:00 公告於二館助教公布欄
- 期末考預計看考卷時間：1/16（一）下午 13:00-15:00 在博理 113 教室
- 如有更改，將另行公告於二館助教公布欄與 ptt 電機系功課板，請各位同學密切注意。
- 祝各位學弟妹期末考順利，新年快樂！

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