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Electromagnetics I

Final Exam (10:20am–12:00noon) 6/26/06

1. (5%) What is the definition of the polarization vector in a dielectric material in terms of dipole moments?

2. (5%) Find the unit vector normal to the surface $y = x^2$ at the point (2, 4, 1).

3. (5%) For a material medium with permittivity ϵ, permeability μ, and conductivity σ, what is its complex intrinsic impedance?

4. (20%) Consider a source consisting of an infinite sheet lying in the xy-plane in free space. On this infinite plane sheet, a uniformly distributed current flows in the negative x-direction, as given by $\mathbf{J}_S(t) = -J_S(t)\mathbf{a}_x$ for $z = 0$, where $J_S(t)$ is given in Fig. P4. Please find and sketch (a) H_y versus z for $t = 1\,\mu$s and (b) E_x versus z for $t = 1.5\,\mu$s.

5. (25%) Consider a similar source as in Problem 4 but with a different time-varying form of $\mathbf{J}_S(t)$. (a) (20%) The source generates sinusoidally time-varying uniform plane waves propagating in the $\pm z$-direction in free space. Consider the $+z$ propagetion having the following characteristics: $f = 200$ MHz and polarization is left circular with the electric field at $z = 0^+$ and $t = 0$ having an x-component equal to $-2E_0$ and a y-component equal to $1.5E_0$. Please write the expressions for the electric- and magnetic-field intensities for this wave. (b) (5%) What is the expression for $\mathbf{J}_S(t)$.

6. (20%) Consider a plane interface between air and a half-space good conductor having permittivity ϵ, permeability μ, and conductivity σ, as shown in Fig. P6. Uniform plane waves at angular frequency ω are propagating along the x-direction and that in the good conductor propagating in the $+x$-direction with the propagation constant $\alpha + j\beta$ (you do not need to derive specific expressions for α and β). At the interface, $x = 0$, the magnetic field intensity is found to be $\mathbf{H}(0, t) = H_0 \cos \omega t \; \mathbf{a}_y$ (A/m).
   (a) What will be the electric field intensity $\mathbf{E}(0, t)$ right at the interface? Please give the expression. (6%)
   (b) Give the expression for $\mathbf{H}(x, t)$ in the good conductor. (4%)
   (c) Give the expression for $\mathbf{E}(x, t)$ in the good conductor. (4%)
   (d) What is the time-average power flow per unit area along the x-direction in the good conductor? (6%)

7. (20%) Consider a capacitor consisting of two concentric spheres with the inner sphere having radius a and the outer sphere having radius b. Between the spheres is filled a lossless dielectric with permittivity ϵ. Assume that the electric potential at the inner sphere is V_0 and that at the outer sphere is zero.
   (a) Give the expression for the electric potential distribution between the spheres. (6%)
   (b) Give the expression for the electric field between the spheres. (4%)
   (c) What are the surface charge densities on the two spheres. (6%)

(d) What is the capacitance? (4%)
(Note: The two independent solutions for the (θ, ϕ)-independent Laplace equation in spherical coordinate systems are $1/r$ and a constant. The r-component of the gradient is simply the derivative with respect to r.)

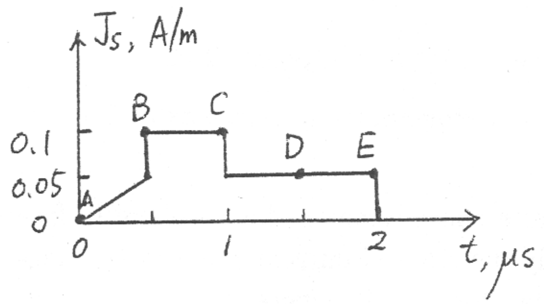

Fig. P4

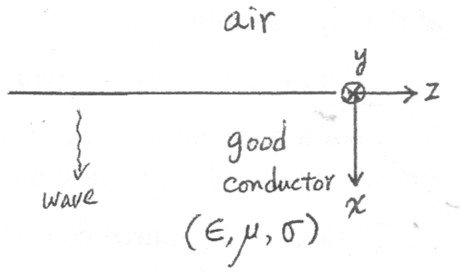

Fig. P6