Electromagnetics (II)

Final Examination

6/11/01

1. The electric field of a uniform plane wave propagating in free space is given in phasor form by $\bar{E} = 10(\hat{a}_x + j0.4\hat{a}_y + j0.3\hat{a}_z)e^{j(0.6y-0.8z)}$
(a) Determine the frequency of the wave. (b) What is the direction of propagation? (c) Obtain the associated magnetic field in phasor form. (d) Discuss the polarization of the wave. (e) Find the time-average power flow per unit area normal to the direction of propagation. (5% each)

2. The ω-β curve for a dispersive channel can be approximated by

$$\frac{1}{\omega^2} = \frac{1}{\omega_0^2} + \frac{k^2}{\beta^2}$$

in the vicinity of $\omega = 0.5\omega_0$, where k is a constant. Find the following: (a) the phase velocity for a signal of $\omega = 0.5\omega_0$; (b) the group velocity for a signal composed of two frequencies $0.4\omega_0$ and $0.6\omega_0$; and (c) the group velocity for a narrow-band signal having the center frequency $0.5\omega_0$. (5% each)

3. Referring to Fig. 9.23 for a dielectric slab waveguide, use Wave-Bounce Approach and the self-consistency condition, show that for TE modes

$$\tan\left(\frac{\pi d\sqrt{\varepsilon_{r1}}}{\lambda_0}\cos\theta_i - \frac{m\pi}{2}\right) = \frac{\sqrt{\sin^2\theta_i - (\varepsilon_2/\varepsilon_1)}}{\cos\theta_i}, \quad m = 0, 1, 2, \cdots (15\%)$$

then write the expression for the cutoff condition and find the cutoff frequency (5%)

Note: $\Gamma_\perp = \dfrac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$

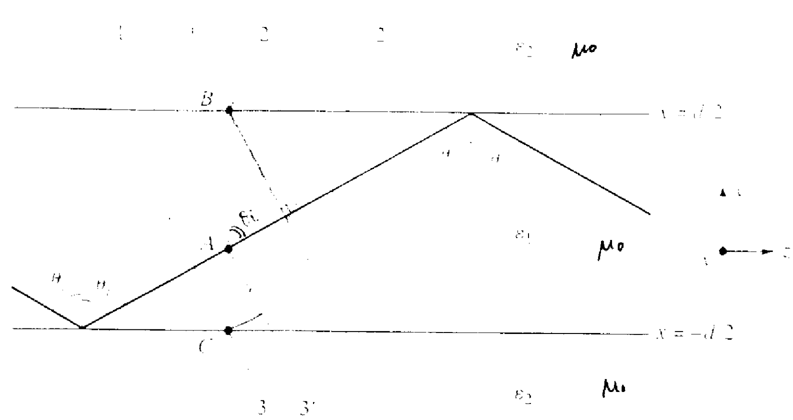

FIGURE 9.23 For explaining the self-consistency condition for waveguiding in a

4. Find the following field expressions and associated parameters for TM Modes in a rectangular waveguide. (Assuming the four conducting plates are situated at $x=0$, $x=a$, $y=0$, $y=b$).
(a) $\overline{E}_z$, (b) $\overline{E}_x$, (c) $\overline{E}_y$, (d) f_c, (e) λ_c, (f) λ_g, (g) v_{pz}, and (h) η_g.
[(b): 4%, (c): 4%, all others : 2% each]

5. For a rectangular waveguide of dimensions $a = 3.75$ cm and $b = 1.25$ cm, and having a dielectric of $\varepsilon = 6.25\,\varepsilon_0$ and $\mu = \mu_0$, find all propagating modes for $f = 5000$ MHz. (10%)

6. The power density pattern for an antenna located at the origin is given by

$$f(\theta,\phi) = \begin{cases} 1 & \text{for } 0 \le \theta \le \pi/6 \\ 0.25 & \text{for } \pi/3 \le \theta \le 2\pi/3 \\ 0.5 & \text{for } 5\pi/6 \le \theta \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

Find the directivity of the antenna. (10%).