微分方程期末考

考試時間：14:10-15:50 2002/1/17

總分：105

1. Find the general solution of the following differential equation by the power series method. (17%)

$$xy''+(1-x)y'-y=0$$

2. Use the change of variables $y=x^r u(x)$ (r is a constant) to make the equation

$$x^2y''+2xy'+\lambda^2x^2y=0, x>0$$

become an ordinary differential equation of u(x). What's the r value? Please solve the equation of u(x) and find the general solution of y(x). (18%)

3. Solve $\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ with $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. (15%)

4. Let $S=\{P_0(x),P_1(x),P_2(x)\}$ where $P_0(x)=1, P_1(x)=x$ and $P_2(x)=\frac{1}{2}(3x^2-1)$. Show that S is an orthogonal set on interval [-1,1]. (10%)

5. Let $f(x)=\begin{cases} 0, & -\pi<x<-\frac{\pi}{2} \\ 1, & -\frac{\pi}{2}\le x\le\frac{\pi}{2} \\ 0, & \frac{\pi}{2}<x<\pi \end{cases}$

(a) Find the Fourier series of $f(x)$. (5%)

(b) From (a) show that $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=\frac{\pi}{4}$. (5%)

6. Use separation of variables to find product solutions for the following partial differential equation. (7%)

$$y\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}=0$$

7. For the wave equation $c^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$, the solution can be obtained without using separating variables. Please find the general solution. (8%)
(Hint : use the substitutions $\xi = x+ct$ and $\eta = x\text{-}ct$)

8. (a) Please find the Fourier transform of the function (5%)

$$f(x)=\exp(-\frac{x^2}{4p^2}) \qquad \text{(Hint : } \int_{-\infty}^{\infty} e^{-t^2}\,dt=\sqrt{\pi}\text{)}$$

(b) Please solve the boundary-value problem (7%)

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}, \quad -\infty<x<\infty\,,\ t>0$$

$u(x,0)=\exp(-x^2)$, $-\infty<x<\infty$

(c) Please solve the boundary-value problem (8%)

$$k\frac{\partial^2 u}{\partial x^2}-v\frac{\partial u}{\partial x}=\frac{\partial u}{\partial t}, \quad -\infty<x<\infty\,,\ t>0$$

$u(x,0)=\exp(-x^2)$, $-\infty<x<\infty$

(Hint : use the substitutions $x' = x\text{-}vt$ and $t' = t$)