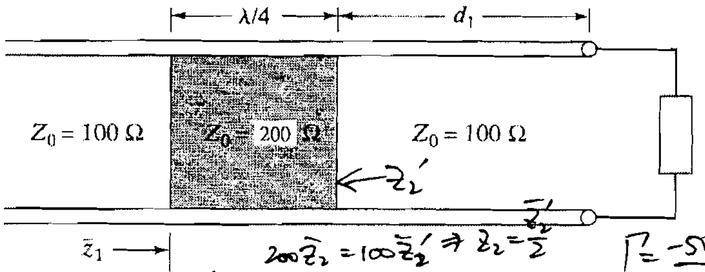
Electromagnetics Ⅱ Final Exam (10:20-12:00) 01/16/2006

註:每題 20 分，只計得分較高之 5 題，合計滿分 100 分。

1. A $\lambda/4$ section of characteristic impedance ~~50~~ 100 Ω is used to minimize the SWR to the left of the section. Please **(a)** plot the locus of $\bar{z}_1$, in Smith chart versus d_1 and using the Smith chart, **(b)**find the minimum value of d_1 that minimized the SWR and **(c)** the minimum value of the SWR.



$\leftarrow \bar{z}_2'$ $\quad \bar{z}_2'$

$200\bar{z}_2 = 100\bar{z}_2' \Rightarrow \bar{z}_2 = \frac{\bar{z}_2'}{2}$

$100\bar{z}_1 = \bar{z}_1' 200 \rightarrow \bar{z}_1' = \frac{1}{\bar{z}_2}$

$\Gamma = \frac{-50+j50}{150+j50} = \frac{-1+j}{3+j} = \frac{(-1+j)(3-j)}{10} = \frac{-2}{10} + \frac{4}{10}j$

$\frac{1}{\sqrt{3}\tan x}$ $\quad \tan^{-1}\left(\frac{1}{\sqrt{3}\tan x}\right)$

2. A resonator system is made up of two transmission line sections of characteristic impedances Z_{01} and Z_{02}, lengths ℓ_1 and ℓ_2; and same phase velocity v_p. The resonator is short-circuited at one end while open-circuited at the other end. Let the wavelength corresponding to the smallest resonant frequency f be $\lambda = v_p/f$.

(a) Find the characteristic equation for the resonant frequency f.

(b) In case of same characteristic impedance, i.e., $Z_{01} = Z_{02}$, find the total length $\ell = \ell_1 + \ell_2$ in terms of the wavelength λ.

(c) If we hope to reduce the total length required to achieve the same smallest resonant frequency, should we choose Z_{01} larger or smaller than Z_{02}?

(d) Following (c), determine the best $\boxed{\text{ratio between } \ell_1 \text{ and } \ell_2}$ such that the total length ℓ can be minimized while keeping the smallest resonant frequency unchanged.

(e) If $Z_{01}/Z_{02} = \sqrt{3}$, find the minimal length ℓ in terms of λ by (d). Compare the result with that in (b). How much is the length reduction?

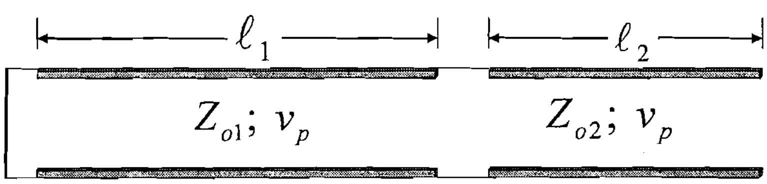
$\tan a \cdot \tan b = x$

$\tan a + \frac{x}{\tan a}$

$a + \tan^{-1}\left(\frac{x}{\tan a}\right)$

$\ln \tan a + \ln \tan b = x$

$x \cdot \tan\beta\ell_1 = \frac{1}{\tan\beta\ell_2}$



$\Gamma = -1$ $\quad \Gamma = e^{-2j\beta\ell_2}$ $\quad \Gamma = 1$

$\bar{z} = z_{02}\frac{1+e^{-2j\beta\ell_2}}{1-e^{-2j\beta\ell_2}} = z_{02}\cdot\frac{e^{j\beta\ell_2}+e^{-j\beta\ell_2}}{e^{j\beta\ell_2}-e^{-j\beta\ell_2}} = z_{02}\frac{\cos\beta\ell_2}{j\tan\beta\ell_2}$

$\lambda = \frac{v_p}{f}$

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p}\cdot\ell_1$

3. The electric field of a right-hand circularly polarized uniform plane wave propagating in free space is given by $\tilde{E} = 10(A\hat{x} + j0.4\hat{y} + B\hat{z})e^{j(0.6y-0.8z)}$ where A and B are constants.

(a) Determine the frequency of the wave.

(b) What is the direction of propagation?

(c) Find the values of A and B.

(d) Obtain the associated magnetic field in phasor form.

(e) Find the time-average power flow per unit area normal to the direction of propagation.

4. A uniform plane wave having the electric field

$\tan a \cdot \tan b = $ 定值 $= k$

$$\mathbf{E} = E_0(\mathbf{a}_y)\cdot\cos\left[6\pi\times10^8 t - \sqrt{2}\pi(x+z)\right]$$

$\sin a \cdot \sin b = k\cos a \cdot \sin b$

is incident on the interface between free space and a dielectric medium of $\varepsilon = 1.5\varepsilon_0$ and $\mu = \mu_0$. The angle of incidence is 45° as shown in Fig.P4. Obtain the expressions for the electric fields of the reflected and transmitted waves.

$x^2 \cdot 0.4056 - x + 0.1736 = 0$

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$\dfrac{1 \pm \sqrt{1 - 4\times0.4056\times0.1736}}{2\times0.4056} =$

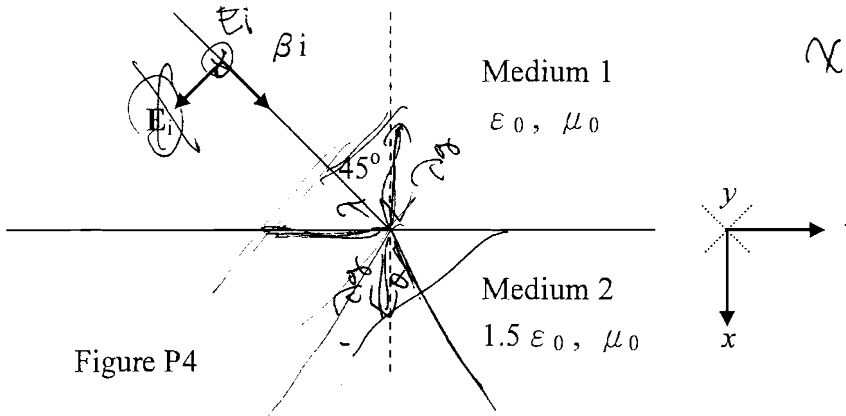

Figure P4

5. A rectangular waveguide of dimensions a = 4 cm and b = 2 cm has a dielectric discontinuity, as shown in Fig.P5. A $TM_{1,1}$ wave of frequency 10,000 MHz is incident on the discontinuity from the free-space side. **(a)** Find the SWR in the free space section. **(b)** Find the length and the permittivity of a quarter-wave section required to achieve a match between the two media. Assume $\mu = \mu_0$ for the quarter-wave section.

10^{10}

$\lambda = \dfrac{u_p}{f} = \dfrac{1/\sqrt{\mu\varepsilon}}{f\sqrt{\varepsilon_r}}$

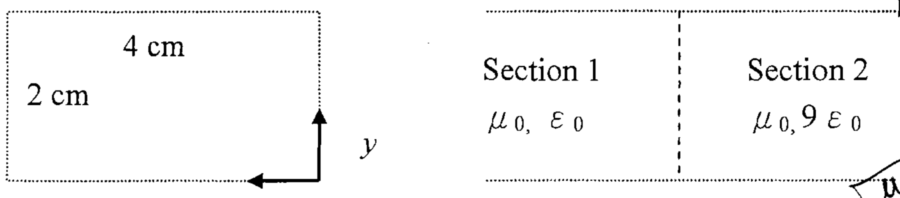

Figure P5

$\eta_{g3} = \sqrt{\dfrac{\mu_0}{\varepsilon_r\varepsilon_0}}\sqrt{1-\left(\dfrac{0.03/k}{0.05m}\right)^2}$

$\omega/\beta = v_p$

6 The power density pattern for an antenna located at the origin is given by

$$f(\theta,\phi)=\begin{cases}\csc^2\theta & \text{for } \pi/6 \le \theta \le \pi/2 \\ 0 & \text{Otherwise}\end{cases}$$

Find the directivity of the antenna.

Useful Formula

$$\Gamma_\perp = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$$

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$$

$$\tau_\parallel = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1}$$

TABLE 9.1 Field Expressions and Associated Parameters for TE and TM Modes in a Rectangular Waveguide

Transverse electric (TE) waves	Transverse magnetic (TM) waves
Field Expressions:	Field Expressions:
($m, n = 0, 1, 2, \ldots$, but not both zero)	($m, n = 1, 2, 3, \ldots$)
$\bar{E}_z = 0$	$\bar{H}_z = 0$
$\bar{H}_z = \bar{A}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$	$\bar{E}_z = \bar{A}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$
$\bar{E}_x = j\frac{\lambda_c^2}{4\pi^2}\omega\mu\frac{n\pi}{b}\bar{A}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$	$\bar{E}_x = \mp j\frac{\lambda_c^2}{2\pi\lambda_g}\frac{m\pi}{a}\bar{A}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$
$\bar{E}_y = -j\frac{\lambda_c^2}{4\pi^2}\omega\mu\frac{m\pi}{a}\bar{A}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$	$\bar{E}_y = \mp j\frac{\lambda_c^2}{2\pi\lambda_g}\frac{n\pi}{b}\bar{A}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$
$\bar{H}_x = \mp\frac{\bar{E}_y}{\eta_g}$	$\bar{H}_x = \mp\frac{\bar{E}_y}{\eta_g}$
$\bar{H}_y = \pm\frac{\bar{E}_x}{\eta_g}$	$\bar{H}_y = \pm\frac{\bar{E}_x}{\eta_g}$
$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$	$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$
$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$	$\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$
$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$	$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$
$v_{pz} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(\lambda/\lambda_c)^2}}$	$v_{pz} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(\lambda/\lambda_c)^2}}$
$\eta_g = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1-(f_c/f)^2}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1-(\lambda/\lambda_c)^2}}$	$\eta_g = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$