2008 微分方程期中考

1. (5%) Without solving, match a solution curve of $y''+y=f(x)$ shown in the Fig. 1 with one of the following functions: (Briefly discuss your reasoning)

   (a) $f(x)=1$, (b) $f(x)=e^{-x}$,

   (c) $f(x)=e^{x}$, (d) $f(x)=\sin 2x$,

   (e) $f(x)=e^{x}\sin x$,

   (f) $f(x)=\sin x$

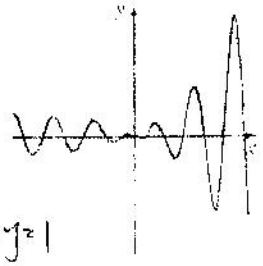

Fig. 1

2. (7%) Solve $y'=y^2-9$.

3. (10%) Solve $y'=\dfrac{1}{(x+y^2)}$.

4. (8%) Solve $x(x+y)^2dx+(2x^2y+x^3-x)dy=0, y(1)=1$.

5. (10%) Solve $2(3y^2-t^2)dy+tydt=0$.

6. (10%) Solve $y''-3y'+2y=3e^{-x}, y(0)=1, y'(0)=0$.

7. (10%) Solve $x^2y''-5xy'+10y=0$ for $x<0$.

8. (15%) Solve $\begin{aligned}\frac{dx}{dt}&=4x-3y\\ \frac{dy}{dt}&=6x-7y\end{aligned}$ with $x(0)=2, y(0)=-1$.

9. (10%) Consider the boundary-value problem $y''+\lambda y=0, y(0)=0, y(\pi/2)=0$
   Discuss: Is it possible to determine values of λ so that the problem possesses
   (a) trivial solutions? (b) nontrivial solutions?

10. (20%) Solve the given initial-value problem and give the largest internal I on which the solution is defined.

    (a) (5%) $y(\ln x-\ln y)dx=(x\ln x-x\ln y-y)dy, y(1)=1$

    (b) (5%) $xy'=y\ln(xy), y(1)=1$

    (c) (5%) $xy(y')+y^2=32x, y(1)=1$

    (d) (5%) $y''-y=\cosh(x), y(0)=2, y'(0)=12$