微分方程期末考

2001/1/11

1. (a) Specify the regular and irregular singular points of the given
(35%) differential equation *(8%)*

$$x^2(x^2-9)^2y'' - (x^2-9)y' + xy = 0$$

(b) Find the two power series solutions for the given equation about x = 0 *(12%)*

$$y'' - xy' - y = 0$$

(c) Find two linearly independent solutions of the given equation about x = 0 *(15%)*

$$2x^2y'' + xy' - (x+1)y = 0$$

2. (a) Please solve the following system of differential equations *(5%)*
(20%)

$$x' = -4x + y + z$$
$$y' = x + 5y - z$$
$$z' = y - 3z$$

(b) Please find the general solution of the following system of differential equations *(8%)*

$$x' = 2x + y + 6z$$
$$y' = 2y + 5z$$
$$z' = 2z$$

(c) Please find the general solution of the following system of differential equations *(7%)*

$$x' = -3x + y + 3t$$
$$y' = 2x - 4y + e^{-t}$$

3. Suppose an n x n matrix A has distinct eigenvalues $\lambda 1, \lambda 2, \ldots, \lambda n$
(10%) corresponding to eigenvectors $K1, K2, \ldots, Kn$. If P is the matrix whose columns are eigenvectors $K1, K2, \ldots, Kn$, then it can be shown that $A = PDP^{-1}$, where D is defined by

$$D = \begin{pmatrix} \lambda 1 & 0 & \cdots & 0 \\ 0 & \lambda 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda n \end{pmatrix}$$

(a) Show that $e^{At} = Pe^{Dt}P^{-1}$ *(5%)*

(b) If $A = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix}$, please compute e^{At} and explicitly express it in the matrix form. *(5%)*

4. (a) What is an even function? What is an odd function? *(5%)*
(15%)
(b) Show that the product of an even function and an odd function is odd. *(5%)*

(c) If $f(x)$ is even, then the Fourier series coefficients of sine terms (i.e. b_n's) are always zero. Why? *(5%)*

5. Solve $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$ for $0 < x < 1$ and $0 < y < 1$ subject to

(10%) $u(x,0) = 0$, $u(x,1) = 0$, $u(0,y) = 0$, $u(1,y) = 1$

6. Find the Fourier transform of the function $f(x) = e^{-|x|}$ for x in $(-\infty, \infty)$.

(10%)