Signals and Systems Final

9:10a.m. ~ 11:10a.m., June 21, Mon., 2010

- Closed book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120
- Total 3 pages in one B4 sheet

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1. Consider a system $H(j\omega)$ giving an output signal $y(t)$ for an input signal $x(t)$.

   (a) [3] Explain what the group delay $T(\omega)$ is for a frequency ω.

   (b) [4] The system is called distortionless if $y(t) = Kx(t-t_0)$. Describe the conditions for $H(j\omega)$ to be distortionless.

2. [8] $x(t)$ is a continuous-time signal, $p(t)$ is the shape of a narrow pulse with duration $\tau < T$, and $y(t)$ is a pulse train modulated by the samples of $x(t)$,

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)p(t-nT)$$

Explain why and how $x(t)$ can be recovered from $y(t)$ under what kind of conditions.

3. $x[n]$ is a discrete-time signal, $x_p[n]$ is the sampled version of $x[n]$ with sampling period N,

$$x_p[n] = \begin{cases} x[n] & \text{, if } n \text{ is an integer multiple of } N \\ 0 & \text{, else} \end{cases}$$

while $x_b[n]$ is a decimated version of $x[n]$,

$$x_b[n] = x_p[nN]$$

   (a) [8] Derive the relationship between $X(e^{j\omega})$, $X_p(e^{j\omega})$ and $X_b(e^{j\omega})$, the discrete-time Fourier transform of $x[n]$, $x_p[n]$ and $x_b[n]$, and explain what these relationships mean in frequency domain.

   (b) [8] Explain why and how it is possible to recover $x[n]$ from $x_b[n]$ under what kind of conditions, how this can be done in the time-domain and what that means in frequency domain.

4. $x_1(t)$, $x_2(t)$ are two signals with $|X_1(j\omega)| = 0$, $|\omega| > \omega_1$ and $|X_2(j\omega)| = 0$, $|\omega| > \omega_2$. Find the conditions for the sampling frequency $\omega_s = \frac{2\pi}{T}$ to avoid aliasing when the signal $y(t)$ below is sampled:

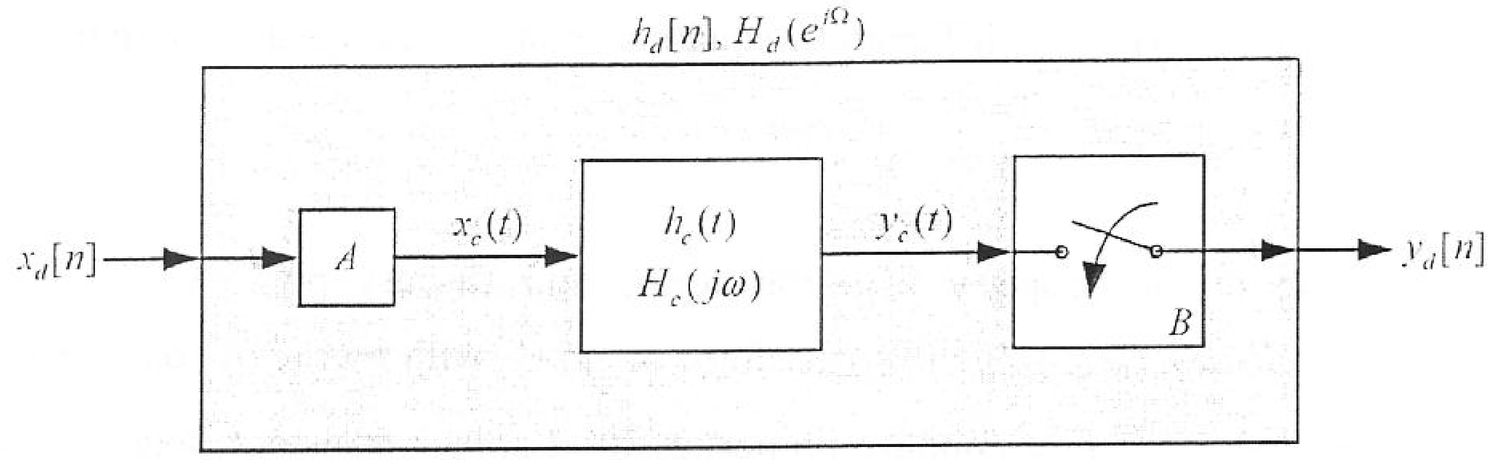
   (a) [2] $y(t) = x_1(t) - bx_2(t-t_0)$

   (b) [2] $y(t) = x_1(t)x_2(t)$

   (c) [2] $y(t) = \frac{d}{dt}x_1(t)$

   (d) [2] $y(t) = x_2(t)\cos(\omega_2 t)$

5. [10] An engineer has a continuous-time system $H_c(j\omega)$ or $h_c(t)$, and he wishes to design a discrete-time version $H_d(e^{j\Omega})$ or $h_d[n]$ of it using the existing system $H_c(j\omega)$ or $h_c(t)$, with input $x_d[n]$ and output $y_d[n]$ both at sampling period T. He comes up with the following design:



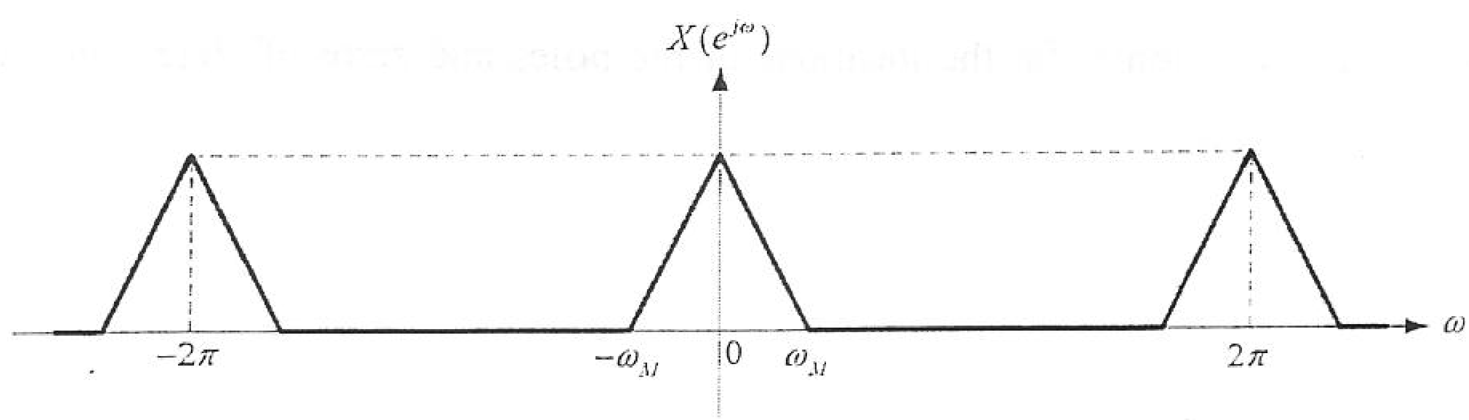
where $x_d[n] = x_c(nT)$ and $y_d[n] = y_c(nT)$. So the operator A includes conversion of $x_d[n]$ to an impulse train plus an ideal low-pass filter, and the operator B is sampling at a period of T. Now derive the relationship between $H_c(j\omega)$ and $H_d(e^{j\Omega})$.

6. [10] Explain what the frequency modulation (FM) and phase modulation (PM) are, what the instantaneous frequency is and how it can be used to analyze FM and PM, and what the relationship between FM and PM is in terms of differentiation.

7. [6] A discrete-time signal $x[n]$ is discrete-time modulated by a discrete-time carrier signal $c[n]$,

$$c[n] = A\cos(\omega_c nT) \quad w_cT$$

to produce a modulated signal $y[n] = x[n]c[n]$. Assume $X(e^{j\omega})$ looks like the following.



Plot the spectra $C(e^{j\omega})$ and $Y(e^{j\omega})$, and show the condition(s) for ω_c to avoid aliasing.

8. [5] Assume a system $h(t)$ has a Laplace transform $H(s) = \dfrac{s^2 - s + 1}{s^2 + s + 1}$, $\text{ROC} = \left\{ s \mid \mathcal{Re}\{s\} > -\dfrac{1}{2} \right\}$.

Use pole-zero plot and geometric evaluation to find out $|H(j\omega)|$ for all ω.

NOTE: There are problems in the back.

9. When an input signal $x(t) = e^{-3t}u(t)$ is applied to a linear time-invariant system, the output signal is $y(t) = (e^{-t} - e^{-2t})u(t)$.

   (a) [4] Find the system function $H(s)$ and its region of convergence.

   (b) [4] Determine the causality and stability of the system.

   (c) [3] Write down the differential equation characterizing the system (with initial rest condition).

10. Prove the following property of z-transform, and discuss possible changes of region of convergence, if any. $X(z)$ is the z-transform of $x[n]$ with region of convergence R. For each case, write down the corresponding properties for Laplace transform and discrete-time Fourier transform, if any.

   (a) [9] $z_0^n x[n] \xleftrightarrow{Z} X(z/z_0)$

   (b) [9] $x_{(k)}[n] \xleftrightarrow{Z} X(z^k)$, where $x_{(k)}[n] = \begin{cases} x[n/k] & \text{, if } n \text{ is a multiple of } k \\ 0 & \text{, else} \end{cases}$

11. [9] A system has a system function $H(z) = \dfrac{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$. Draw the block diagram of the system in (a) direct form, (b) parallel form and (c) cascade form.

12. [12] An engineer is designing a system $H(z)$ with the requirement that both $H(z)$ and $H^{-1}(z)$ have to be causal and stable, where $H^{-1}(z)$ is the inverse system of $H(z)$. Can you describe the condition(s) for the locations of the poles and zeros of $H(z)$ in order to meet the above requirement?