Department of Electrical Engineering, National Taiwan University

**Engineering Mathematics-Differential Equation, 2010, Fall**

# Midterm Examination

2010/11/10 Wednesday, 10:20-12:10

**1. Find the general solutions of the following DEs. (26 scores)**

(a)

$$y''(x) = x\cos(4x) + \cos(2x)$$

(6 scores)

(b)

$$y^{(5)}(x) - 2y^{(4)}(x) + 4y'''(x) - 8y''(x) + 4y'(x) - 8y = 0$$

(7 scores)

(c)

$$x^2y''(x) + 2x^2y'(x) + (2x - 2)y(x) = 0$$

(Hint: $y(x) = \frac{1}{x}$ is one of the solutions)

(7 scores)

(d)

$$y'''(x) + y''(x) = 1$$

(6 scores)

**2. Suppose that a place can accommodate at most 10000 people. (9 scores)**

(a) Use the logistic equation to express the change of the population of the place.

$$\frac{dP(x)}{dt} = ?$$

(3 scores)

(b) Furthermore, suppose that 1% of the people move out of the place every year. How do we modify the logistic equation?

(3 scores)

(c) Among the following methods (linear equations, separable variables, homogeneous equations, Bernoulli's equations, exact equations), which ones can be used for solving the logistic equations in (a) and (b)?

(3 scores)

**3. Please find the general solution of the given differential equation. Give the largest interval I over which the general solution is defined.**

$$x^3y''' - 2x^2y'' + 2xy' = 8 + 2\sin(\ln x)$$

**(10 scores)**

4. **Please solve the given system of differential equations. Give the largest interval I over which the general solution is defined.**

$$x = \frac{dy}{dt}$$

$$y = -\frac{2}{3}\frac{d}{dt}(xy), \qquad y(t=0) = 0$$

**(10 scores)**

5. **Please find the general solution of the given differential equation in form of an infinite series. Give the largest interval I over which the general solution is defined.**

$$y'' - 4y = 4x^2 - 3 + \frac{e^{2x}}{x}$$

**(10 scores)**

6. **Solve**

$$x^2y'(x) + 2xy(x) - x + 1 = 0$$

**with**

$$y(1) = 0$$

**(15 scores)**

7. **(20 scores)**

(a) The differential equation

$$\frac{dy(x)}{dx} = p(x) + q(x)y(x) + r(x)y^2(x)$$

is known as Riccati's equation.

Suppose $y_1(x)$ is a given particular solution. Verify that the substitution

$$y(x) = y_1(x) + \frac{1}{v(x)}$$

transforms the Riccati's equation into the linear equation

$$\frac{dv(x)}{dx} + \big(q(x) + 2r(x)y_1(x)\big)v(x) = -r(x)$$

(10 scores)

(b) Solve

$$\frac{dy(x)}{dx} + 2xy(x) = 1 + x^2 + y^2(x)$$

Note

$$y_1(x) = x$$

is one particular solution.

(10 scores)