Linear Algebra Quiz 1

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USE OF ALL AUTOMATIC COMPUTING MACHINES IS PROHIBITED

1. Judge if the following statements are true or false. Give concise proofs to the true statements, and counterexamples to the false statements.

   (a) Let S be a nonempty set of vectors in $\mathcal{R}^n$, and let $\mathbf{v}$ be in $\mathcal{R}^n$. The spans of S and $S \cup \{\mathbf{v}\}$ are equal if and only if $\mathbf{v}$ is in S. (20%)

   (b) For any matrix A with the reduced row echelon form R there is a unique invertible matrix P such that $A = PR$. (20%)

2. Suppose A is an $m \times n$ matrix and B is an $n \times p$ matrix. Prove that rank of the matrix product AB is smaller than or equal to rank of A. (30%)

3. For the set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$, find all subsets of S that have the same span as S. (30%)