Signals and Systems Final

10:10 a.m. ~ 12:10 p.m., Jan. 17, Fri., 2003

- Closed book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score : 120, time allocation : 1 point/minute

1.(6) Consider a discrete-time system $H(e^{j\omega})$ giving an output signal y[n] for an input signal x[n].

(a.)(3) Explain what the group delay $T(\omega)$ is for a given frequency ω.

(b.)(3) The system is called distortionless if $y[n] = ax[n-n_0]$. Describe the conditions on $H(e^{j\omega})$ for the system to be distortionless.

2.(10) Given a discrete-time signal x[n] with discrete-time Fourier transform $X(e^{j\omega})$, the sampled sequence $x_p[n]$ for a sampling period N and the decimated sequence $x_b[n]$ are respectively

$$x_p[n] = \sum_{k=-\infty}^{\infty} x[kN]\delta[n-kN],\ x_b[n] = x_p[nN].$$

(a.)(4) Derive the discrete-time Fourier transform $X_p(e^{j\omega})$ for $x_p[n]$ in terms of $X(e^{j\omega})$.

(b.)(6) Derive the relationship between $X_b(e^{j\omega})$ and $X_p(e^{j\omega})$.

3.(8) Consider the system below. The relationships between x[n], z[n] and between w[n], y[n] are respectively

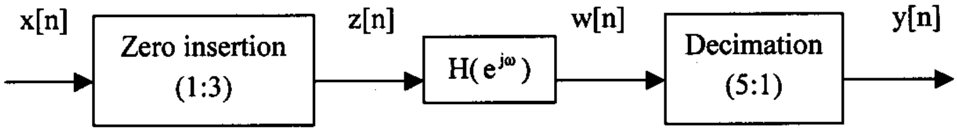
$$z[n] = x_{(3)}[n] = \begin{cases} x[n/3], \text{if } n \text{ is a multiple of } 3 \\ 0, \text{else.} \end{cases},\ y[n] = w[5n],$$

and $H(e^{j\omega})$ has the response

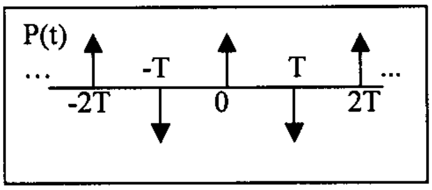
$$H(e^{j\omega}) = \begin{cases} 1, |\omega| \le \frac{\pi}{5} \\ 0, \frac{\pi}{5} < |\omega| < \pi \end{cases}$$

Now assume the input x[n] is $x[n] = \dfrac{\sin\omega_1 n}{\pi n},\ \ \omega_1 < \dfrac{3}{5}\pi$.

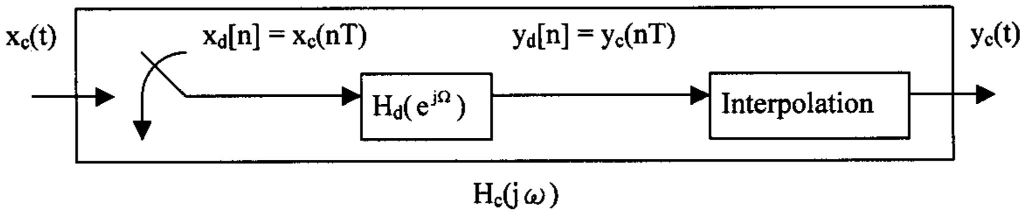
Find the output y[n].



4.(6) Assume a continuous-time signal x(t) with Fourier transform $X(j\omega)$ is sampled by an alternating impulse train p(t) as shown below (i.e., $+\delta(t-nT)$ for n even and $-\delta(t-nT)$ for n odd) to produce $x_p(t) = x(t)p(t)$. Find the Fourier transform $X_p(j\omega)$ for x_p~~[n]~~ in terms of $X(j\omega)$.
(t)



5.(8) Consider the following system which processes the continuous-time signal with discrete-time approaches. The discrete-time system $H_d(e^{j\Omega})$ is described by the difference equation below,



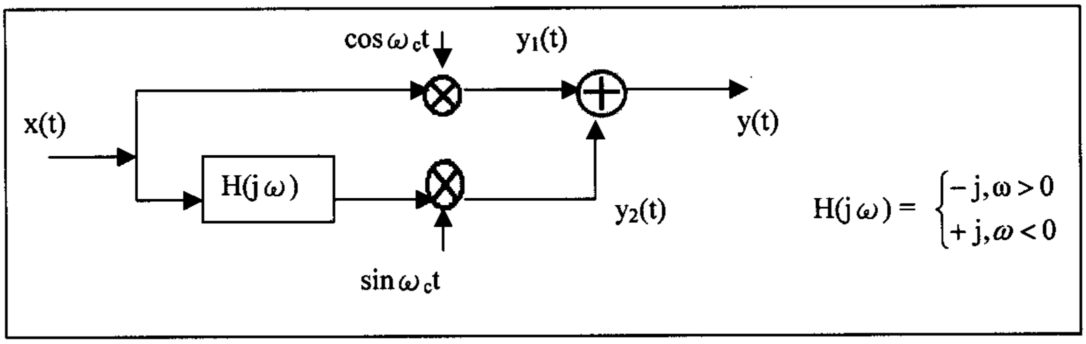
$$y_d[n] = \frac{1}{2} y_d[n-1] + x_d[n].$$

Assume the sampling theorem is satisfied. Find the frequency response $H_c(j\omega)$ for the continuous-time system relating input $x_c(t)$ and output $y_c(t)$.

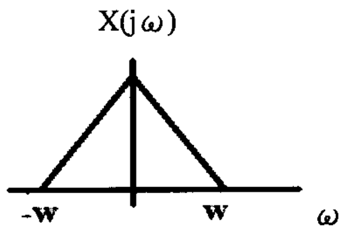
6.(9) Consider a continuous-time signal $x(t) = A\cos(\omega_0 t + \Phi)$ being sampled by a sampling frequency $\omega_s = 2\omega_0$, and the samples $x(nT)$ are used to obtain the reconstructed signal $x_r(t)$. Draw the time-domain samples and the corresponding $x_r(t)$ for three cases：

(a.)(3) $\Phi = 0$, **(b.)(3)** $\Phi = \frac{\pi}{4}$, **(c.)(3)** $0 < \Phi < \frac{\pi}{4}$.

7.(6) Consider the single-sideband modulation system below.



For a signal $x(t)$ with Fourier transform $X(j\omega)$ shown below. Draw the Fourier transform $Y_1(j\omega)$, $Y_2(j\omega)$ and $Y(j\omega)$ for $y_1(t)$, $y_2(t)$ and $y(t)$.



8.(6) For an angle modulated signal $y(t) = A\cos[\theta(t)]$, what is the instantaneous frequency $\omega_i(t)$? How is it related to the information-bearing signal $x(t)$ for phase modulation and frequency modulation respectively?

9.(6) Assume $x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$. Find the Laplace transform $X(s)$, draw the pole-zero plot, and identify the region of convergence for it.

10.(12) A linear time-invariant system produces an output $y(t) = [e^{-t} - e^{-2t}]u(t)$ for an input $x(t) = e^{-3t}u(t)$.

(a.)(3) Find the transfer function $H(s)$ of the system.

(b.)(6) Draw the pole-zero plot for $H(s)$. Is this system causal? stable?

(c.)(3) Write down a differential equation characterizing this system.

11.(18) Prove the following properties of z-transform, and discuss possible change of region of convergence, if any. $X(z)$ is the z-transform of $x[n]$ with region of convergence R. For each case, write down the corresponding properties for Laplace transform and discrete-time Fourier transform, if any.

(a.)(9) $z_0^n x[n] \overset{z}{\leftrightarrow} X(z/z_0)$

(b.)(9) $x_{(k)}[n] \overset{z}{\leftrightarrow} X(z^k)$, where $x_{(k)}[n] = \begin{cases} x[n/k], \text{if } n \text{ is a multiple of } k \\ 0, \text{else.} \end{cases}$

12.(9) Consider the z-transform $X(z) = \dfrac{3 - \frac{5}{6}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})}$, find out the corresponding time-domain signal $x[n]$ if the region of convergence is

(a.)(3) $|z| > \frac{1}{3}$, **(b.)(3)** $\frac{1}{3} > |z| > \frac{1}{4}$, and **(c.)(3)** $|z| < \frac{1}{4}$.

13.(6) Consider the three different forms of a filter $H(z)$ as below, determine whether each of them is approximately lowpass, bandpass or highpass, and explain why.

(a.)(2) $H(z) = \dfrac{z^{-1}}{1 + \frac{8}{9}z^{-1}}, |z| > \frac{8}{9}$. **(b.)(2)** $H(z) = \dfrac{1 + \frac{8}{9}z^{-1}}{1 - \frac{16}{9}z^{-1} + \frac{64}{81}z^{-2}}, |z| > \frac{8}{9}$.

(c.)(2) $H(z) = \dfrac{1}{1 + \frac{64}{81}z^{-2}}, |z| > \frac{8}{9}$.

14.(10) Consider the system as shown in the following.

(a.) Write down a difference equation relating y[n] and x[n].

(b.) Write down the transfer function H(z) for the system and draw the pole-zero plot.

(c.) Is this system stable?