Department of Electrical Engineering

National Taiwan University

Probability and Statistics, Spring 2011

Final Examination

15:30-18:30, Thursday, June 23, 2011

（所有同學請先在答案卷上標註自己所屬的班級）

1. **(10 scores) Let X_1 and X_2 be independent and identical (iid) exponential random variables with $f(x) = \lambda e^{-\lambda x}, x \geq 0$. Find**
   (1) The MGF of $W = X_1 - X_2$ (7 scores) and
   (2) Variance of W (3 scores)

2. **(13 scores) Let $W_1 = X_1 - X_2$ and $W_2 = X_1 + X_2$ where X_1 and X_2 are iid uniform random variables between [-1, 1]. Find**
   (1) The joint pdf of W_1 and W_2. (5 scores)
   (2) Are W_1 and W_2 identical random variables? (5 scores)
   (3) Are they independent? (3 scores)

3. **(13 scores) Let the number of customers arriving at a bank follow a Poisson distribution with mean = λ customers per minute. The bank has two clerks serving the customers. Each customer gets a ticket (with a number on it) when entering the bank. If his/her number is even, he/she goes to clerk 1. Otherwise, he/she goes to clerk 2.**
   (1) If each ticket number is randomly and uniformly generated from [1, 2, 3, 4], what will be the distribution of the inter-arrival time of customers at clerk 2? (8 scores)
   (2) If the ticket numbers are sequentially incremented (as most of the banks do), what will be the distribution of the inter-arrival time of customers at clerk 1? (5 scores)

4. **(7 scores) Let the distance of a ball from the origin (X = 0) depend on how many students push it together. If the number of students that push the ball together is N, then the distance D follows an Erlang distribution $f_D(d) = \frac{\lambda^N d^{N-1} e^{-\lambda d}}{(N-1)!}, d \geq 0$. Now assume the number of students joining the game is geometrically distributed with a parameter p (i.e. $P_N(n) = (1-p)^{n-1}p, n = 1, 2, 3, \ldots,$). Please find E(D).**

5. **(7 scores) Let Z be the standard normal (Gaussian) distribution. Based on what you have learned from this course, find the tightest upper bound of P(|Z| > 3σ). Note that your answer must show the bound that is the tightest of all.**
6. **(11 scores) For n flips of a fair coin, where n is a positive integer, let *X* be the total number of heads and let *Y* be the number of head in the last flip.**
   (1) Find $E[X \mid Y > 0]$ and $Var[X \mid Y > 0]$ (7 scores)
   (2) Please find the correlation E[XY] of *X* and *Y* (4 scores)
7. **(27 scores) In a game show, the player must complete *n* independent random tests to complete the game, and the probability to pass a test is *p*. If the player passes the i^{th} test, there will be an award of X_i. Of course, she will get 0 award (i.e. not X_i) if she does not pass. Suppose that $X_1, \ldots X_n$, is a sequence of iid Poisson random variables, all with mean equal to *m*. If we let A denote the number of passed tests, and *Y* is the total award of the player at the end of this game.**
   (1) Please show that $E(Y) = mnp$ *(4 scores)*
   (2) Please derive the moment generating function of Y. (8 scores)
   (3) Please derive Var(*Y*) without using the results of 7(2). (5 scores)
   (4) Please use the Chebychev's inequality to obtain an upper bound for $P[Y > 200]$ if $m = 10$, $p = 0.5$, and $n = 20$ (5 scores)
   (5) Please use the central limit theorem approximation to estimate $P[Y > 200]$ if $m = 10$, $p = 0.5$, and $n = 20$. The answer can be expressed in Q or Φ function. (5 scores)
8. **(12 scores) Let $\mathbf{X} = (X_1, X_2)'$ be a Gaussian (0, Cx) vector, with the covariance matrix $Cx = \sigma^2 I$, where *I* is a 2 x 2 identity matrix. Now let Q be a rotation matrix such that y=Qx can rotate vector x by 90 degree.**
   (1) Please determine the joint PDF of Y_1 and Y_2, if **Y= QX** and $\mathbf{Y} = (Y_1, Y_2)'$. (8 scores)
   (2) Please determine whether Y_1 and Y_2 are orthogonal. (4 scores)

- 期末考成績預定公告時間：6/29（三）中午 12:00 公告於電機系助教公布欄
- 期末考預定看考卷時間：6/29（三）晚上 18:30-20:00 於電二 143
- 如有更改，將另行公告於電機系二館助教公布欄與 ptt 電機系功課板，請各位同學密切注意。
- 祝各位學弟妹們期末考順利，暑假愉快！

國立台灣大學電機工程學系
網路與多媒體實驗 兼任工程數學-微分方程/機率與統計 專任助教
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