Department of Electrical Engineering, National Taiwan University

Engineering Mathematics-Differential Engineering, 2011, Fall

Midterm Examination

2010/11/09 Wednesday, 10:20-12:10

1. **(11 scores) Given a differential equation**

$$y' + 4y^2 = x, \quad y(0) = 1$$

(a) Show that when $x \to \infty, y \approx ax^n, n > 0$; find *a* and *n*.
(Hint: Substitute $y = ax^n, n > 0$, into the equation and show that y' is negligible compared to x and y^2 when $x \to \infty$) (6 scores)

(b) If the initial condition changes to $y(0) = 2$, will the *a* and *n* change? Why?
(Hint: follow the same step as in (a)) (5 scores)

2. **(8 scores) Solve the given differential equation**

$$xy' - (1 + x)y = x^2y^2$$

3. **(8 scores) Solve the given differential equation**

$$(x + 2)^2y' - 4xy - 8y = x + 5$$

4. **(8 scores) Solve the given differential equation**

$$\frac{dx}{dt} = -\frac{1}{10}\sqrt{2gx}, \qquad x(0) = 100$$

5. **(28 scores) Find the solutions of the following differential equations:**

(a) $y^{(3)}(x) + y''(x) + 3y'(x) + 3y(x) = x$ (7 scores)

(b) $y''(x) + y'(x) + 2y(x) = \cosh x$ (7 scores)

(c) $y''(x) + 2y'(x) + y(x) = e^{-x} + \cos x$ (7 scores)

(d) $\frac{4}{3}y''(x) + 2y'(x) + \left(x^{-1} - x^{-2}\right)y(x) = 0$

(Hint: $y(x) = x^{-1/2}$ is one of the solution) (7 scores)

6. **(7 scores) Suppose that the mass of an object is 5kg and the force acting on the object is *F* = 12 newtons. Also suppose that the initial location is *x* = 0 and the initial velocity is 4 m/sec. Use the differential equation to determine the location *x*(*t*) for *t* > 0.**

7. (30 scores) Please solve the given differential equations. <u>Give the largest interval I over which the general solution is defined.</u>

(a)

$$2xyy' = 4x^2 + 3y^2$$

(5 scores)

(b)

$$\frac{d^4y}{dt^4} - 6\frac{d^3y}{dt^3} + 11\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + e^{4t}y = 0$$

(Hint: $x = e^t$)

(5 scores)

(c)

$$2xe^{2y}\frac{dy}{dx} = 3x^4e^{2y/3} + e^{2y}$$

(5 scores)

(d)

$$x^4y^{(4)} - 6x^3y^{(3)} + 33x^2y'' - 105xy' + 169y = 0$$

(Hint: $y_1 = x^3\,cos(2\,ln\,x)$)

(5 scores)

(e)

$$\frac{dy}{dx} = \left[\left(\frac{d^2x}{dy^2}\right)^{\frac{2}{3}} - 1\right]^{-0.5}$$

(10 scores)