Electromagnetics II midterm exam

date: 2013/11/04

題目一共六題，每題 20 分，採計個人最高分五題。

1. (20%) Figure 1 depicts the cross section of a parallel-plate waveguide. The two plates are perfect conductor ($\sigma \to \infty$), separated by d (m), and each has a width of w (m). Two materials of ($\varepsilon_1 = 16\varepsilon_0$, $\mu_1 = \mu_0$) and ($\varepsilon_2 = 4\varepsilon_0$, $\mu_2 = 4\mu_0$), respectively, are inserted between these two plates. Assume a TEM wave propagates in the $\hat{z}$ direction, its electric field has only the $\hat{x}$ component, and its magnetic field has only the $\hat{y}$ component. The fields in the left-half of the cross section are $(\overline{E}_1, \overline{H}_1)$, and the fields in the right-half are $(\overline{E}_2, \overline{H}_2)$. The fringing effect is neglected.

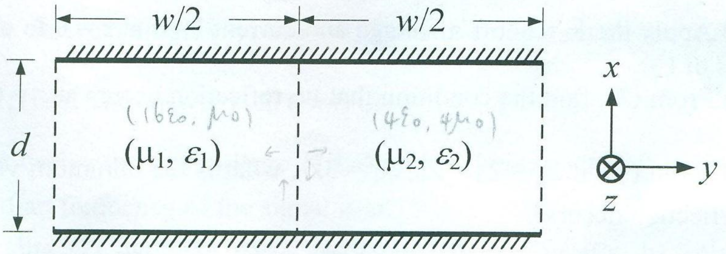


Figure 1

The following boundary conditions may be applicable:

(A) At the interface between two materials:

$$\hat{n}\times\overline{E}_1 = \hat{n}\times\overline{E}_2,\ \hat{n}\times\overline{H}_1 = \hat{n}\times\overline{H}_2,\ \hat{n}\cdot\overline{D}_1 = \hat{n}\cdot\overline{D}_2,\ \hat{n}\cdot\overline{B}_1 = \hat{n}\cdot\overline{B}_2.$$

(B) At the interface between the material and the plate: at the surface of perfect conductor.

$$\hat{n}\times\overline{E}_1 = 0,\ \hat{n}\times\overline{H}_1 = \overline{J}_s,\ \hat{n}\cdot\overline{D}_1 = \rho_s,\ \hat{n}\cdot\overline{B}_1 = 0.$$

(1) (2%) Specify the relation between $\overline{E}_1$ and $\overline{E}_2$.

(2) (2%) Specify the relation between $\overline{H}_1$ and $\overline{H}_2$.

(3) (3%) Derive the surface current density and the total surface current on the bottom plate, in terms of the fields.

(4) (3%) Derive the surface charge density and the total surface charge on the bottom plate, per meter in the $\hat{z}$ direction, in terms of the fields

(5) (3%) What is the capacitance of the waveguide per meter in the $\hat{z}$ direction?

(6) (3%) What is the inductance of the waveguide per meter in the $\hat{z}$ direction?

(7) (4%) If $\mu_1\varepsilon_1 \neq \mu_2\varepsilon_2$, what will happen to the wave propagation in the $\hat{z}$ direction?

2. (20%) Figure 2 depicts a cascade of transmission line segments. Assume a voltage wave, $V_{10}e^{-j\beta_1 z}$, is incident from segment 1.

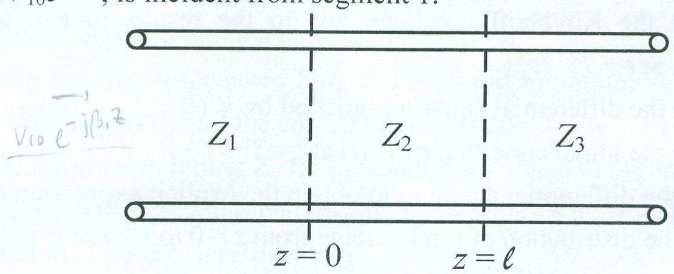


Figure 2

$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v_p}$

The voltage wave in each segment can be expressed as

$$V_1 = V_{10}e^{-j\beta_1 z} + \Gamma_{12}V_{10}e^{j\beta_1 z}$$
$$V_2 = V_{20}e^{-j\beta_2 z} + \Gamma_{23}V_{20}e^{j\beta_2 z}$$
$$V_3 = \tau_{23}V_{20}e^{-j\beta_3 z}$$

where $\beta_1 = \omega\sqrt{L_1C_1}$; $\beta_2 = \omega\sqrt{L_2C_2}$, and $\beta_3 = \omega\sqrt{L_3C_3}$.

(1) (5%) Derive the current waves, I_1, I_2 and I_3, using the Telegrapher's equation,
$$\frac{\partial V(z)}{\partial z} = -j\omega L I(z)$$

(2) (5%) Apply the Kirchhoff's voltage and current laws at $z = \ell$ to solve Γ_{23} and τ_{23}.

(3) (5%) Apply the Kirchhoff's voltage and current laws at $z = 0$ to express Γ_{12} in terms of Γ_{23}.

(4) (3%) From (3), find the condition that no reflection occurs at $z = 0$, that is, $\Gamma_{12} = 0$.

(5) (2%) From (4), if $Z_1 = Z_3 = Z_0$, $Z_2 = 3Z_0$, what is the minimum value of ℓ that no reflection occurs?

3. (20%) Figure 3 shows a transmission line loaded with a capacitor. At $t = 0 - \varepsilon$ (ε is an infinitesimal number), there is no residual charge in the capacitor, C.

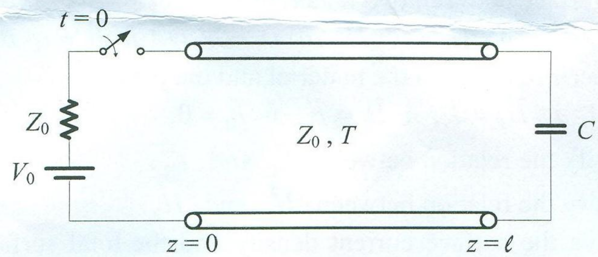


Figure 3

(1) (2%) At $t = 0 + \varepsilon$, what is the ratio between the voltage V_+ and the current I_+ at $z = 0 + \delta$ (δ is an infinitesimal number)?

(2) (3%) Apply the Kirchhoff's voltage law at $z = 0$ to find V_+.

(3) (2%) At $t = T + \varepsilon$, another voltage wave, V_-, is induced at the capacitor, C, and propagates in the $-\hat{z}$ direction. Write down the total current wave and the total voltage wave at $z = \ell$.

(4) (2%) Apply the Kirchhoff's voltage law to the results in (3), across the capacitor, C, at $t > T$.

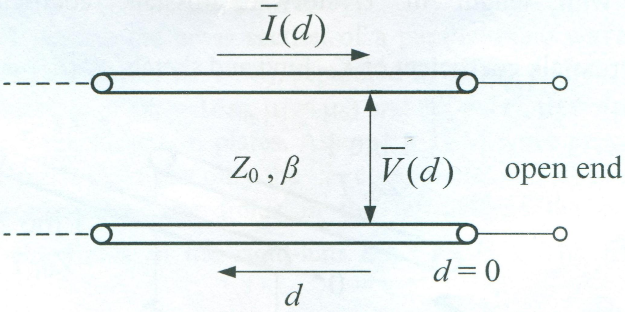
(5) (3%) Derive the differential equation satisfied by $V_-(t)$.

(6) (2%) Derive the initial condition of $V_-(t)$ at $t = T$.

(7) (3%) Solve the differential equation to obtain the explicit expression of $V_-(t)$.

(8) (3%) Draw the distribution of total voltage from $z = 0$ to $z = \ell$ at $t = 2T$.

4. (20%) Answer the following questions for the open-ended transmission line system.



(1) Please derive complex voltage $\bar{V}(d)$ and complex current $\bar{I}(d)$ in phasor form. (Hint: The answer is in terms of $\bar{V}^{+}$, Z_0, β and d.)

(2) Please derive the real voltage $V(d, t)$ and real current $I(d, t)$ in time domain if the radian frequency of the signal is ω.

(3) Following (2), will the wave propagate? If yes, what's the velocity? If no, please explain why.

(4) Calculate the time average power flow $\langle P \rangle$ on the transmission line.

(5) If the length of this line is ℓ, what's the input impedance of this line?

5. A quarter-wave transformer matching is used in a transmission line of characteristic impedance Z_0 with the load impedance Z_ℓ as shown in Figure 5. Please answer following questions if $Z_\ell = 30\ \Omega$:

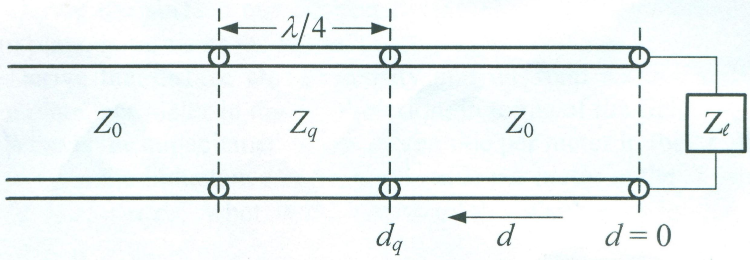


Figure 5

(1) When the line impedance $Z(d)$ is a real and maximum value, please find the shortest length d_q and the corresponding $Z(d_q)$.

(2) When the line impedance $Z(d)$ is a real and minimum value, please find the shortest length d_q and the corresponding $Z(d_q)$.

(3) Find the corresponding Z_q for the conditions stated in (1) and (2), respectively.

(4) Plot the standing wave patterns along the lines, i.e., $d > 0$, for the conditions stated in (1) and (2), respectively.

$\Gamma_R = \frac{Z_\ell - Z_0}{Z_\ell + Z_0}$

$z_0 \frac{Z_0}{Z_\ell}$

6. Please determine the induced wave voltages in the secondary line of a coupled pair of lines, with length of ℓ, forward-crosstalk coefficient of K_f, and backward-crosstalk coefficient of K_b. Find and sketch $V_2^+(\ell,t)$ and $V_2^-(0,t)$.