Signals and Systems Midterm Exam 2011/5/2

1. [28] Suppose the output $y[n]$ of an LTI system is related to the input $x[n]$ by the following equation:

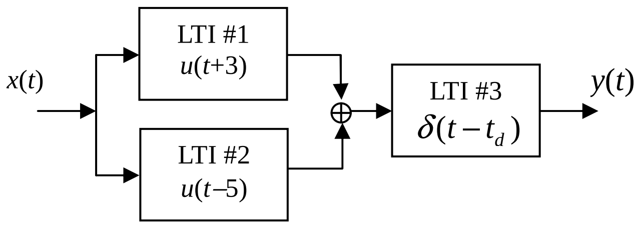
$$y[n] = \sum_{k=n-2}^{n+2} x[k].$$

(a) Is the system causal? Justify your answer. [4]
(b) Is this system time-invariant? Justify your answer. [4]
(c) Is this system linear? Justify your answer. [4]
(d) Determine the impulse response of the system. [4]
(e) Does the system introduce a delay to the input? Justify your answer. [4]
(f) Draw a diagram of the frequency response and show the frequencies at which the frequency response has value zero. [4]
(g) If the system equation becomes

$$y[n] = x[n-4] + x[n-2] + x[n] + x[n+2] + x[n+4],$$

determine the frequency response of this system by the property of time expansion. [4]

2. [12] The following block diagram depicts two LTI subsystems in parallel that are cascaded with a third LTI sub system. The impulse response of each subsystem is written within each block of the diagram.



(a) What is the impulse response of the overall system? [4]
(b) What should the time delay t_d be chosen so that the overall system is causal? [4]
(c) Which subsystems are stable? Is the overall system stable? [4]

3. [16] Consider the unit impulse response $h(t) = \cos(\pi t)\sin(10\pi t)$ of a linear-time-invariant system.

(a) Determine its fundamental period. [4]
(b) Determine the Fourier series coefficients of $h(t)$. [4]
(c) Determine the Fourier transform of the even part of $h(t)$. [4]
(d) Determine the response of the system to the input $x(t) = e^{-at}u(t),\ a > 0$.[4]

4. [12] An ideal ($-\pi/2$) phase shifter is defined by the frequency response

$$H(j\omega) = \begin{cases} e^{-j(\pi/2)}, & \omega > 0 \\ e^{j(\pi/2)}, & \omega < 0 \end{cases}.$$

(a) Find the impulse response of this phase shifter. [4]
(b) Find the output of this phase shifter due to an arbitrary input $x(t)$. [4]
(c) Find the output of this phase shifter when $x(t) = \cos\omega_0 t$.[4]

5. [10] Let an LTI system have its input $x[n]$ and output $y[n]$ characterized by the following difference equation

$$y[n] - y[n\text{-}1]/2 = x[n]$$

with the condition of initial rest.

(a) Find the unit impulse response of the system. [6]

(b) Is this system stable? Justify your answer. [4]

6. [10] Suppose that we have a linear system with the following differential equation

$$y(t) = t^2 \frac{d^2 x(t)}{dt^2} + t \frac{dx(t)}{dt}.$$

(a) Is the signal $s(t) = t^k$, where k is an integer, an eigenfunction of this system? Justify your answer.[5]

(b) Find the output of the system with the input signal given by

$$x(t) = 10t^{-10} + 3t + t^4/2 + 5. \text{ [5]}$$

7. [12] Let $X(j\omega)$ be the Fourier transform of a signal $x(t)$. Assume that another signal $g(t)$ has the same shape as that of $X(j\omega)$, i.e., $g(t) = X(jt)$.

(a) Show that the Fourier transform $G(j\omega)$ of $g(t)$ has the same shape as $2\pi x(-t)$. **[6]**

(b) Using the result of Part (a), find the Fourier transform of a signal $x(t) = e^{jQt}$, where Q is a real number. Justify your answer. **[6]**