09 Fall 微分方程期中考

1. Solve the following 1st order differential equations (show the explicit solutions).

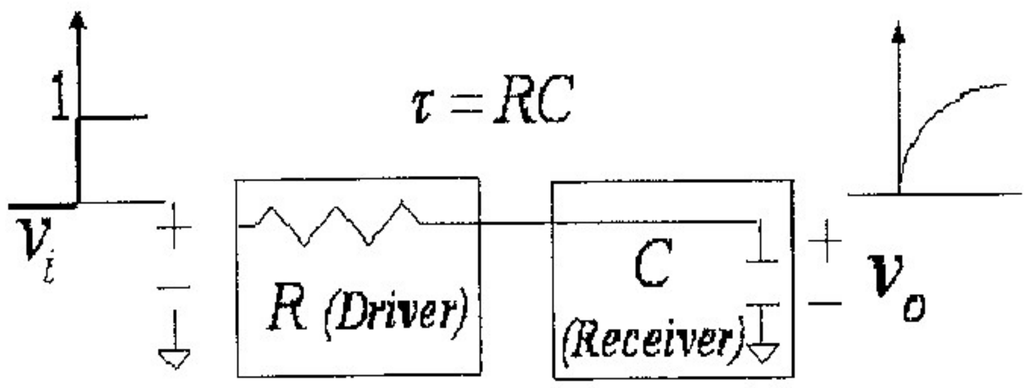
(a) (9%) $\frac{dy}{dx} = \frac{y^2 - 1}{2x}$

(b) (9%) $\frac{dy}{dx} = x + y$

(c) (8%) $\frac{dy}{dx} = y + 2y^3$

(d) (9%) $\frac{dy}{dx} = -\frac{x + 2y}{2x + 3}$

2. (7%) Please solve the differential equation of RC network and express the rise time, Tr, of the output voltage, Vo, in terms of the RC time constant, $\tau = RC$. (The rise time, Tr, is defined as the time differences between 10% to 90% of final voltage level.)



3. (3%) A cake's temperature is expressed in (1). Based on the value in Table (I), how long will it for this cake to cool off to a room temperature of T_{room}? (Within Thermometer's measurement accuracy 0.5° 2次, express the cooling time in terms of the time constant, τ.)

$$T(t) = T_{room} + 200e^{-t/\tau} \qquad (1)$$

Table (I)

	e^{-1}	e^{-2}	e^{-3}	e^{-4}	e^{-5}	e^{-6}	e^{-7}
Value	0.367879	0.135335	0.049787	0.018316	0.006738	0.002479	0.000912

4. (7%) Find solution of $xy'' = y' + (y')^3$.

5. (8%) Please solve the system of differential equations.

$$\begin{cases} \frac{dx}{dt} = 2x + 3y + 1 \\ \frac{dy}{dt} = -x - 2y + 4 \end{cases}$$

6. (7%) Solve the differential equation $x^2y'' + 4xy' + 2y = x$.

7. (8%) Find the complementary function y_c and the particular solution y_p of the differential equation $x^2y'' - 2xy' + 2y = x$.

8. $x''+\omega^2 x = \cos\gamma t, x(0) = x'(0) = 0$

(a) (12%)

Show that when the difference between ω and γ is small, an approximate solution, 'beats phenomenon', is

$$x(t) = \frac{1}{2}A(t)\frac{\sin(\gamma t)}{\gamma}$$

where $A(t)$ is function of t, ω and γ. Please also find $A(t)$.

(b) (3%)

When the $\omega = 2\pi \times 90.9MHz$ and $\gamma = 2\pi \times 90.924MHz$, find the 'envelop' frequency of the approximate solution in (a).

9. (10%) Please solve the differential equation $y'' - 2y' + y = e^t \arctan(t) + e^t$.