Final Examination for Electromagnetics (I) Jan. 17, 2000

1. A circular disk of radius b and thickness t carries a current in a circular direction about the center. A cylindrical coordinate system has the origin at the center of the disk and has the z axis coinciding with the axis of symmetry of the disk. The current density in the disk is $\overline{J} = k\rho\hat{\varphi}$ (A/m^2), where k is a positive real constant. Find the magnetic intensity $\overline{H}$ at the center of the disk, assuming that the thickness t is very small compared with b. (12%)

2. A ferromagnetic sphere of radius b is magnetized uniformly with a magnetization $\overline{M} = M\hat{z}$.
   (1) Determine the equivalent magnetization current density $\overline{J}_m$ and surface current density $\overline{J}_{ms}$.
   (2) Determine the magnetic flux density at the center of the sphere. (16%)

3. Consider an infinitely long solenoid with air core having a radius r_o and n closely wound turns per unit length and carrying a current I. The magnetic flux density inside the coil can be expressed by $\overline{B} = B_z\hat{z}$ approximately.
   (1) Find B_z inside the coil.
   (2) If the core has a relative permeability μ_r, find the inductances of the coils per unit length with the following parameters. (12%)

	r_o (cm)	n (turns/cm)	μ_r
Coil A	1	10	1
Coil B	0.5	50	3000

4. Consider a long solenoid of length l and of radius a, ($a << l$). The permeability of its iron core is μ and a current I flows in this solenoid with n closely wound coil-turns per unit length. Assume that the iron core is partially withdrawn so that only a length $x_0 = l/2$ is left within the solenoid, as shown in Fig. P4.
   (1) Find the force (in a vector form) acting on the core.
   (2) If $x_0 \cong 0$ or $x_0 \cong l$, does the result of (1) have to be modified? Discuss it briefly. (20%)

5. An open-circuited rectangular conducting loop is situated near a very long wire carrying a low-frequency alternating current $I(t) = I_0 \sin \omega t$, as shown in Fig. P5.
   (1) Find the open-circuit voltage $V(t)$ induced across terminals a and b.
   (2) If a resistor R is connected across terminals a and b, is the amplitude of the voltage across R greater than, less than or equal to the open-circuit voltage mentioned in (1)? Explain why briefly. (Assume that the resistance of the rectangular loop can be neglected.) (20%)

6. (1) State Ampere's circuital law and Biot-Savart law and describe the physical meanings in words.
   (2) Compare the usefulness of these two laws in determining the static magnetic field generated by a steady-current carrying circuit. (10%)

7. Write the differential form of Maxwell's equations, and identify each equation with the proper experimental law. (10%)

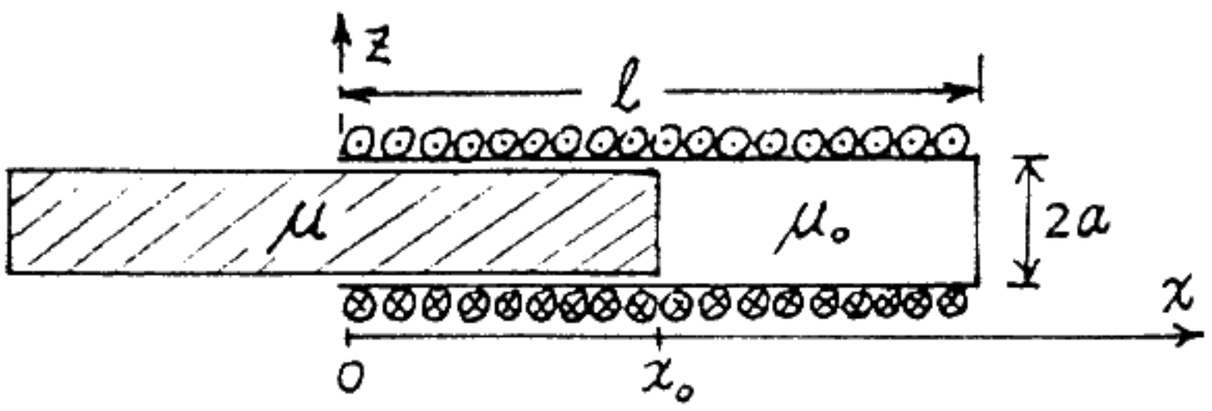

Fig. P4

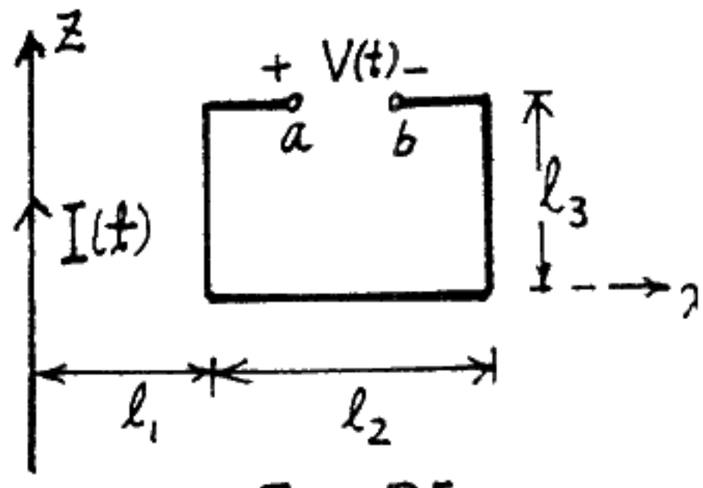

Fig. P5