E

Complex Analysis (Mid-term Exam.)

April 18, 2006 (10:20am – 12:10am)

1. True or false (If it is false, please explain the reasons briefly) (20%)
   (a) $\operatorname{Ln} z$ is analytic for $|z|>0$ and its derivative is $1/z$.
   (b) A function f is analytic in a simple connected domain D and C is any contour in D. Then $\int_C f(z)dz$ is independent of the path C.
   (c) A function f is analytic at point z_0 if and only if f is differentiable at z_0 and every point in every neighborhood of z_0.
   (d) The only bounded entire function is zero.
   (e) If f is analytic in a simply connected domain D. Then f possesses derivatives of all orders at every point z in D, and they are all analytic in D.
2. Suppose the function $f(z)=u(r,\theta)+iv(r,\theta)$ is analytic at point z whose polar coordinates are (r,θ). Please prove (15%)
   (a) the Cauchy-Riemann equations in the polar coordinate is $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$, and
   (b) the derivative of f at (r,θ) is $f'(z)=e^{-i\theta}(\frac{\partial u}{\partial r}+i\frac{\partial v}{\partial r})$.
3. Suppose the function $f(z)=u(x,y)+iv(x,y)$ is analytic in domain D. Then the real and imaginary parts of f can be used to define two families of curves, $u(x,y)=c_1$ and $v(x,y)=c_2$, in D, where c_1 and c_2 are arbitrary real constants. Please prove that these two families of curves are orthogonal. (20%)
4. Please find all values of the given quantity: (20%)
   (a) $\sinh^{-1} i$
   (b) $\cosh(1+\frac{\pi}{6}i)$
   (c) $\ln(-2+2i)$
   (d) $(1+i)^{1-i}$
5. Please find the values (15%)
   (a) $\oint_C (\frac{e^z}{z+3}-3\bar{z})dz$, where $|z|=1$
   (b) $\int_\pi^i e^z \cos z dz$
   (c) $\oint_C Ln(z+10)dz$, where $|z|=2$
6. Please find the values (10%)
   (a) $\oint_C \frac{z^2}{z^2+4}dz$, where $|z-i|=2$
   (b) $\oint_C \frac{1}{z^2(z-1)^2}dz$, where $|z-2|=5$