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Discrete Mathematics

Fall 2006, Final Exam

1. (10 pts) **True or False?** Score = max{0, right - $\frac{1}{2}$ wrong}. No explanations are needed.

   (a) Relation $R = \{(1,1),(1,2)\}$ (over the set $\{1,2\}$) is an antisymmetric relation.

   (b) A *partial order* relation is a relation which is reflexive, asymmetric and transitive.

   (c) If a *poset* has a least element, then the element is unique.

   (d) If (S,R) is a poset, then (S,R^{-1}) is also a poset. (Here R^{-1} denotes the inverse of R.)

   (e) There is a simple graph with 4 vertices each having degrees 2, 2, 3, 3.

   (f) Both K_9 and K_{10} have an Euler cycle.

   (g) Graph $K_{5,4}$ has 20 edges.

   (h) Let G be a simple graph with 5 vertices and 9 edges. Then G is always planar.

   (i) Every loop-free connected planar graph has a vertex v with degree(v) < 6.

   (j) Let $\mathcal{R}$ be the relation on the set of all ordered pairs of positive integers such that $(a,b)\mathcal{R}(c,d)$ if and only if $ad = bc$. The $\mathcal{R}$ is an equivalence relation.

2. (10 pts) Define *Ramsey*(g,y) to be the smallest number n such that any green/yellow coloring of the edges of an n-clique will contain a green g-clique or a yellow y-clique, where g, y are natural numbers. Use an example to show that *Ramsey*$(3,3) > 5$. That is, construct a graph with two colors (green and yellow) such that the graph neither has a green 3-clique nor has a yellow 3-clique.

3. (15 pts) If $A = \{1,2,3,4\}$ and $R = \{(1,2),(1,3),(2,4),(3,2)\}$ be a binary relation over A. Answer the following questions. No explanations are needed. (Recall that $R^2 = R \circ R$.)

   (a) R^2=?

   (b) R^3 =?

   (c) R^4 =?

   (d) What is the symmetric closure of R?

   (e) What is the transitive closure of R?

4. (10 pts) Let $A = \{1,2,3,4,5\} \times \{1,2,3,4,5\}$, and define relation $\mathcal{R}$ on A by $(x_1,y_1)\mathcal{R}(x_2,y_2)$ if $x_1 + y_1 = x_2 + y_2$.

   (a) Prove that $\mathcal{R}$ is an equivalence relation.

   (b) Determine the equivalence classes $[(1,3)]$ and $[(1,1)]$ (i.e., for each of the two equivalence classes, list all of its elements).

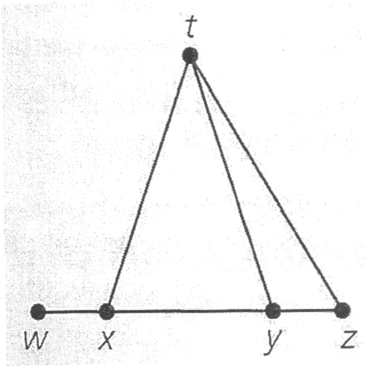
5. (10 pts) Solve the following recurrence relation exactly. Show your derivation in detail.

$$a_{n+2} - 4a_{n+1} + 3a_n = -200,\ n \geq 0,$$
$$a_0 = 3000,\ a_1 = 3300.$$

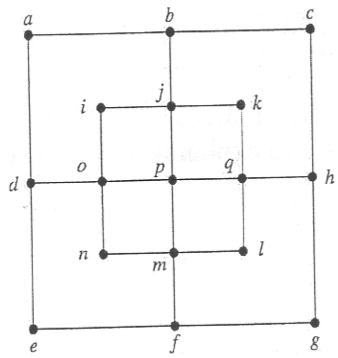
6. (15 pts) Consider the following graph G

   (a) (10 pts) Compute the the chromatic polynomial $P(G, \lambda)$. Show your derivation in detail.

   (b) (5 pts) Find the chromatic number $\chi(G)$.



7. (10 pts) Prove (in a rigorous way) that the following graph does not have a Hamilton circuit.



8. (20 pts) Answer the following questions:

   (a) Construct a graph G_1 which has a Hamilton circuit but not a Euler circuit

   (b) Construct a graph G_2 which has a Euler circuit but not a Hamilton circuit

   (c) Construct a graph G_3 which has both a Hamilton circuit and a Euler circuit

   (d) Construct a graph G_4 which is not planar but has a Hamilton circuit.

   (e) Construct a graph G_5 which is homeomorphic (but not isomorphic) to $K_{3,3}$.