

機率期中考

4/19/99

1. Suppose that n random integers are selected from $\{1, 2, \dots, N\}$ with replacement.
 - (a) What is the expected value of the largest number selected? (10%)
 - (b) Show that for large N the answer is approximately $nN/(n+1)$. (5%)
2. A random variable X is called symmetric if for all $x \in \mathbb{R}$, $P(X \geq x) = P(X \leq -x)$.
Prove that if X is symmetric, then for all $t > 0$, its distribution function F satisfies the following relations:
 - (a) $P(|X| \leq t) = 2F(t) - 1$ (5%)
 - (b) $P(|X| > t) = 2[1 - F(t)]$ (5%)
 - (c) $P(X \geq t) = F(t) + F(-t) - 1$ (5%)
3. In this problem, we consider that a number is selected randomly from the set $\{0000, 0001, 0002, 0003, \dots, 9998, 9999\}$. Find the probability that the sum of the first two digits of the number selected is equal to the sum of its last two digits. (15%)
4. In this problem, we consider that the probability of a bit error in a communication line is 10^{-3} . Each bit transmission corresponds to a Bernoulli trial with a "success" corresponding to a bit error in transmission. Let X be the number of bit errors in N transmissions.
 - (a) Is X a binomial random variable? You must justify your answer. (3%)
 - (b) Find the probability function $P(X=x)$ of X . (3%)
 - (c) Let $N = 1000$ which is assumed to be large enough. So X can be viewed as a Poisson random variable. Find the corresponding probability function $P(X=i)$. (8%)
 - (d) From (c), find the probability that a block of 1000 bits has five or more bit errors. (6%)
5. The coefficient of the quadratic equation $x^2 + bx + c = 0$ are determined by tossing a fair dice twice (the first outcome is b , the second outcome is c). Find the probability that the equation has real roots. (15%)
6. Throw a dice twice. Let A be the event that the first throw is 1, 2, or 3; B be the event that the first throw is 2, 4, or 6; and C be the event that the sum of the two throws is 5.
 - (a) Find $P(A|B)$, $P(B|C)$, and $P(C|A)$. (9%)
 - (b) Find $P(A|BC)$, $P(B|CA)$, and $P(C|AB)$. (9%)
 - (c) Are the three events mutually independent. (2%)