Signals and Systems, Final Exam

9:10-11:10, 6/23/06

- Close book but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120 points. Time allocation: 1 point/minute

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1 (10). Consider the linear constant-coefficient second-order differential equation:

$$\frac{d^2}{dt^2}y(t) + 2\zeta w_n \frac{d}{dt}y(t) + w_n^2 y(t) = w_n^2 x(t).$$

(a) Find the frequency response $H(jw)$ of the system.
(b) For $0 < \zeta < \sqrt{2}/2$, what is the frequency w_{m} where $|H(jw_{\mathrm{m}})|$ has a maximum value?
(c) What is the maximum value of $|H(jw_{\mathrm{m}})|$ at the frequency in (b)?

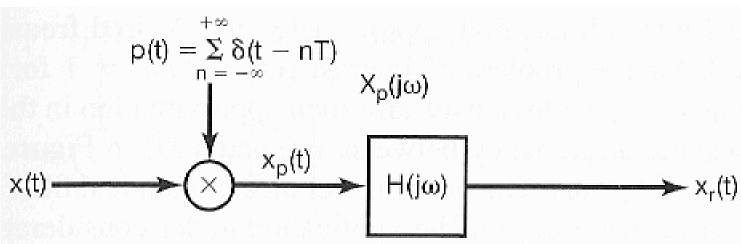
2 (10). Consider a continuous-time LTI system with frequency response $H(jw) = |H(jw)|e^{j\sphericalangle H(jw)}$ and real impulse $h(t)$. Suppose that we apply an input $x(t) = \sin(w_0 t + \phi_0)$ to the system. The resulting output can be shown to be of the form $y(t) = Ax(t - t_0)$, where A is a nonnegative real number representing an amplitude-scaling factor and t_0 is a time delay.
(a) Express A in terms of $|H(jw_0)|$
(b) Express t_0 in terms of $\sphericalangle H(jw_0)$

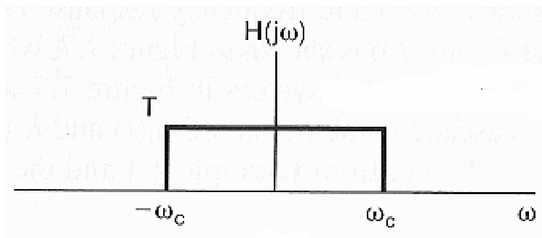
3 (6). Compute the group delay of the following frequency response.

$$H(jw) = \frac{1}{(jw+1)(jw+3)}.$$

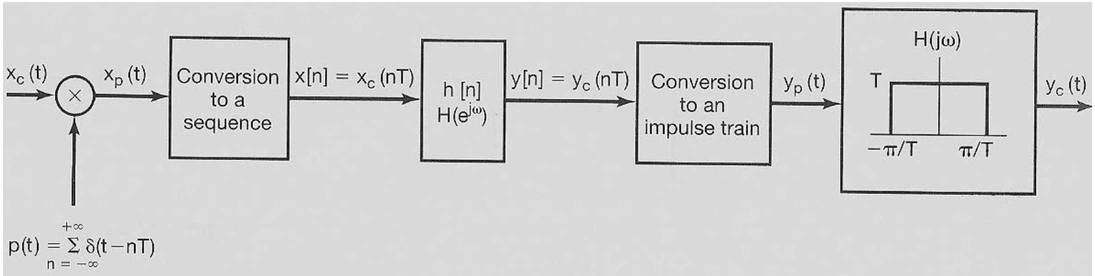
Note that $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$.

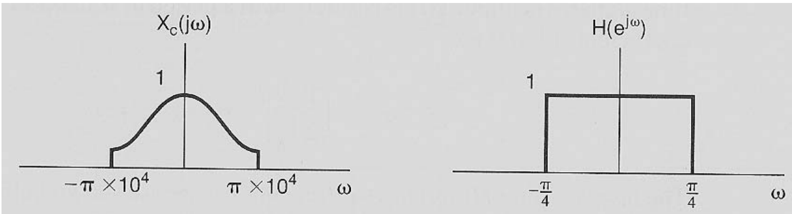
4 (12). Let $x(t) = \cos(w_0 t)$ and $x(t)$ is sampled and filtered by the following system with a sampling time T, $w_s = 2\pi/T$, and $w_c = w_s/2$.
(a) If $w_0 = 2w_s/6$, identify the signal $x_r(t)$.
(b) If $w_0 = 5w_s/6$, identify the signal $x_r(t)$.
(c) Does any aliasing occur in (a) and/or (b)? Justify your answer.





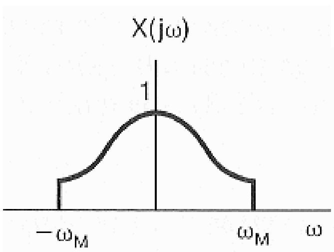
5 (20). The following figure shows the overall system for filtering a contiuous-time signal using a discrete-time filter. If $X_c(jw)$ and $H(e^{jw})$ are as shown in the following figure, with $1/T =$ 10kHz, sketch $X_p(jw), X(e^{jw}), Y(e^{jw}), Y_p(jw)$, and $Y_c(jw)$. Note that you need to specify all the critical values on the horizontal and vertical axes of the above plots.

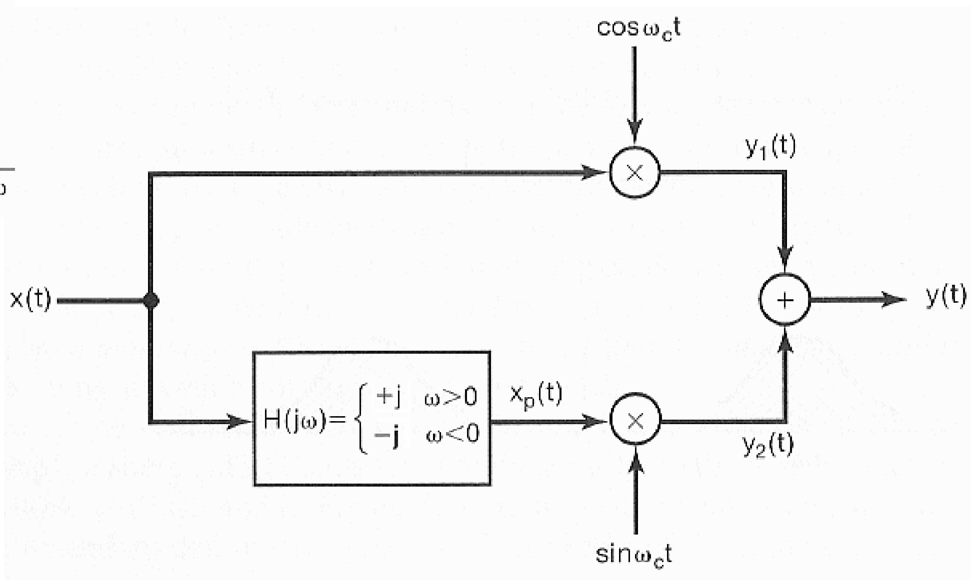




6 (4). What are amplitude modulation, phase modulation, and frequency modulation? Please decribe the them in terms of a modulating signal $x(t)$ and a carrier signal $c(t) = A\cos(w_c t + \theta_c)$.

7 (8). Consider the system for single-sideband amplitude modulation. If $X(jw)$ is shown as follows and $w_c > 2w_M$, please sketch the spectra of $Y_1(jw), X_p(jw), Y_2(jw)$, and $Y(jw)$.



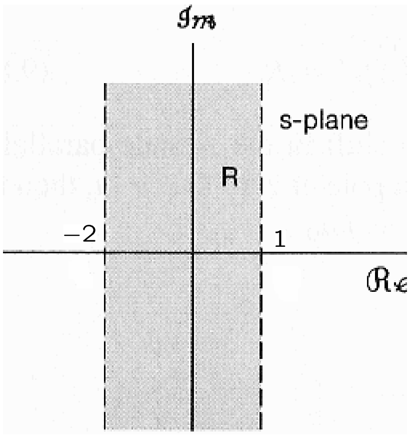


8 (8). If $X(s)$ is the Laplace transform of $x(t)$, that is,

$$x(t) \stackrel{\mathcal{L}}{\longleftrightarrow} X(s), \quad ROC = R,$$

where R is as shown in the following figure. Show the following time scaling property and plot the new ROC, R_a.

$$x(-0.3t) \stackrel{\mathcal{L}}{\longleftrightarrow} \frac{10}{3}X(-\frac{10}{3}s), \quad ROC = R_a$$



9 (10). Given a system function $H(s)$ as follows:

$$H(s) = \frac{-s}{s^2+3s+2}, \quad ROC = \{-2 < \mathcal{R}e\{s\} < -1\}$$

(a) Is the system stable? Justify your answer.
(b) Is the system causal, anticausal, or neither? Justify your answer.
(c) If the input to the system is an impulse funcion, that is, $x(t) = \delta(t)$, find the output $y(t)$.

10 (6). A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (a+1)\frac{d^2y(t)}{dt^2} + a(a+1)\frac{dy(t)}{dt} + a^2y(t) = \frac{dx(t)}{dt} - 2x(t)$$

Please use adder, amplifier, and integrator only and as few components as possible to Plot the block diagram representation of the system S.

11 (10). Consider a discrete-time signal:

$$x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[-n-1].$$

Find the z-transform.

12 (8). If

$$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z), \quad ROC = R_1,$$

and

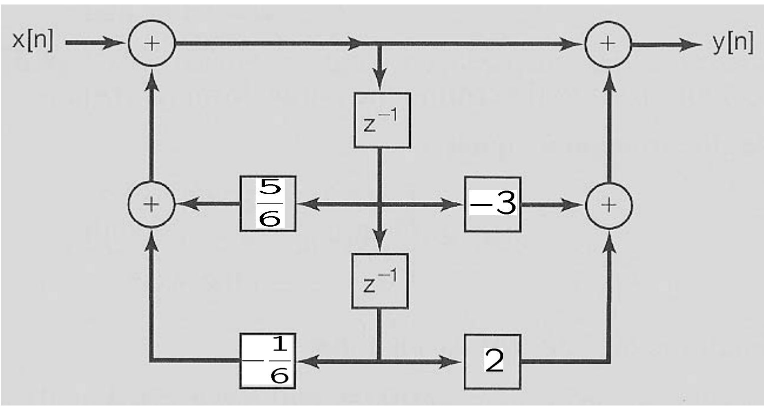
$$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z), \quad ROC = R_2,$$

show that

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z),$$

with ROC containing $R_1 \cap R_2$

13 (8). Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related through the block diagram representation shown in the figure.



(a) Determine a differential equation relating $y[n]$ and $x[n]$.
(b) Is the system stable?