# Complex Analysis

Quiz 1 (陳士元老師班)

1. If $u(x,y) = x^3 + 3x^2y + \boldsymbol{a}xy^2 + \boldsymbol{b}y^3$ is a harmonic function, find *a* and *b* and a function $v(x,y)$ such that $f(x,y) = u(x,y) + v(x,y)$ is analytic. **(10%)**

2. For each complex function given below, point out **where** the function is (or is not) differentiable and/or analytic and give the reason. **(15%)**

(a) $f(z) = e^{-y}\sin x - ie^{-y}\cos x$ (b) $f(z) = (2x - x^3 - xy^2) + i(x^2y + y^3 - 2y)$

(c) $f(z) = |z|^2$

3. Please find all values of the given quantity. **(5%, 10%, 10%)**

(a) $\ln(-e^3)$ (b) $(1+i)^{(1+i)^2}$

(c) $tan^{-1}(2i)$

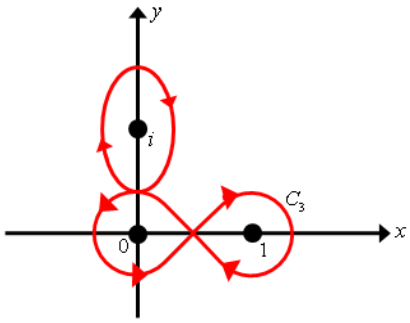
4. Evaluate the following **(40%)**.

(a) $\int_c \sinh(z)\,dz$, where *C* is an arbitrary path connecting from 1 to $1 + \frac{\pi}{2}i$.

(b) $\oint_c (2z^2 + 7z + 3)^{-1}\,dz$, where *C* is the ellipse $x^2/4 + y^2 = 1$

(c) $\oint_c \left(\frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}\right) dz$ , *C* is $|z| = 3$

(d) $\int_{C_3} \frac{1}{z^2 - z} + \frac{1}{z - i}\,dz$ , $C_3$ is shown in the right figure.




5. Without solving the contour integral, show that **(10%)**

$$\left|\oint_C \frac{2z^2 - 1}{z^4 + 5z^2 + 4}\,dz\right| \le \frac{2\pi R(2R^2 + 1)}{(R^2 - 1)(R^2 - 4)}\ ,\ C: |z| = R\ (R > 2)$$