Midterm

2000/4/25

1. (24%)

Determine the following statement is correct of not. If not, give the reason.

a) If the Cauchy-Riemann equations are satisfied at a point, then the function is analytic there.

b) A function f is analytic at a point z_0 if f can be expanded in a convergent power series centered at z_0.

c) A power series represents a continuous function at every point within and on its circle of convergence.

2. (16%)

Expand the given function $f(z)=\dfrac{z-1}{3-z}$ in a Taylor series centered at $z_0=1$.

3. (20%)

(i) Evaluate $(1-i)^{2i}$.

(ii) Give $z^4+z^3+z^2+z^1=0$. Find z.

4. (16%)

Evaluate $\oint_C \dfrac{2z+1}{z(z-1)^2}dz$, where C is the curve in Figure 1.

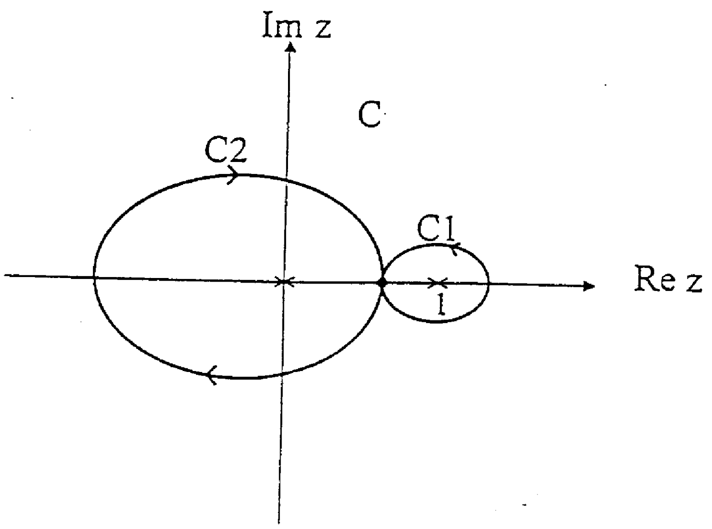

Figure 1

5. (24%)

(i) Let a function $f(z)$ be analytic through a simply connected domain D and let z_0 be the only zero (with order m) of $f(z)$ in D. Show that if C is a positively oriented (counterclockwize) simple closed countour in D that enclosed z_0, then $\frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)}dz = m$, where $f'(z) = \frac{df(z)}{dz}$.

Hint: $f(z)$ can be expressed as $(z-z_0)^m g(z)$, where $g(z) \neq 0$ and $g(z)$ is analytic in D.

(ii) Use the result of (i) to prove the following property. Let D be simply connected domain throughout which a function $f(z)$ is analytic and $f'(z) \neq 0$. Let C denote a simple closed contour in D, described in the positive sense, such that $f(z) \neq 0$ at any point on C. Then, if $f(z)$ has N distinct zeros interior to C, that number is given by $N = \frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)}dz$.