Electromagnetics II Midterm Exam (10:10am-11:50am) 11/14/2005

註: 每題 20 分，只計得分較高之 5 題，合計滿分 100 分。

1. **Finding fields and power flow for a parallel-plate line for specified voltage along the line.** A parallel-plate transmission line is made up of perfect conductors of width $w = 0.1$m and lying in the planes $x = 0$ and $x = 0.01$ m. The medium between the conductors is a nonmagnetic ($\mu = \mu_0$), perfect dielectric. For a uniform plane wave propagating along the line, the voltage along the line is given by
$$V(z,t) = 10\cos(3\pi\times10^8 t - 2\pi z)\ \mathrm{V}$$
Neglecting fringing of fields, find: (a) the electric field intensity $E_x(z,t)$ of the wave; (b) the magnetic field intensity $H_y(z,t)$ of the wave; (c) the current $I(z,t)$ along the line ; and (d) the power flow $P(z,t)$ down the line.

2. **Time-domain analysis of a transmission-line system using the bounce-diagram technique.** In the system shown in Fig.P2, the switch S is closed at $t = 0$. Assume $V_g(t)$ to be a direct voltage of 90 V and draw the voltage and current bounce diagrams. From these bounce diagrams, sketch: (a) the line voltage and line current versus $t$ (up to $t = 7.25\ \mu$s) at $z = 0$, $z = l$, and $z = l/2$; and (b) the line voltage and line current versus $z$ for $t = 1.2\ \mu$s and $t = 3.5\ \mu$s.

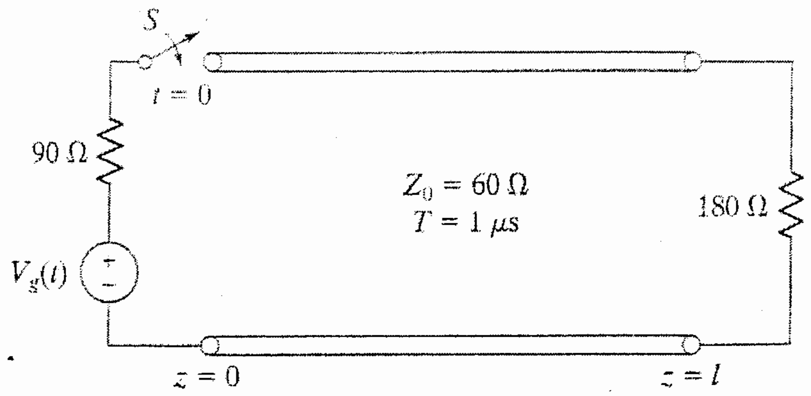



Figure P2

3. **A transmission-line system with inductive discontinuity.** In the system shown in Fig.P3, the switch $S$ is closed at $t = 0$, with the lines uncharged and with zero current in the inductor. Obtain the solution for the line voltage versus time at $z = l+$.

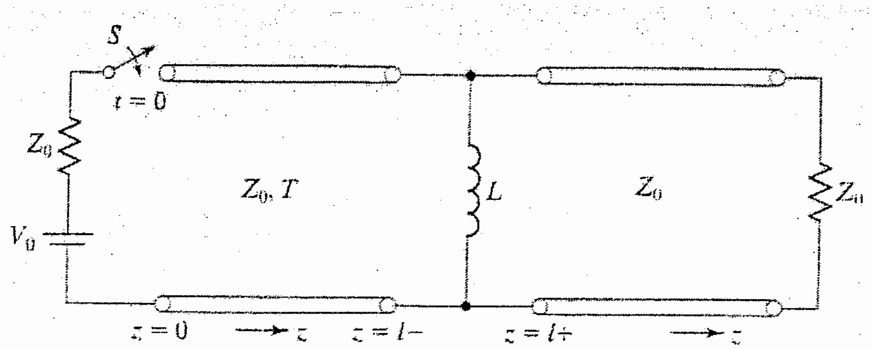



Figure P3

4. Fig. P4 shows a transmission line system driven by a switching gate. Transmission line 3 (Tx 3) is a short stub between transmission line 1 (Tx1) and transmission line 2 (Tx2). A capacitive load $C_L$ is terminated at the end of Tx2, and the end of Tx3 is open circuit. The per-unit-length coupling inductance and capacitance between Tx1 (or Tx2) and transmission line 4 (Tx4) are *Lm* and *Cm*, respectively. As shown in Fig. P4, the characteristic impedances of Tx1, Tx2 and Tx4 are all $Z_0$. The characteristic impedance of Tx3 is $Z_1$. The delay of Tx1, Tx2, Tx3, and Tx4 are *T/2*, *T/2*, *T/4*, and *T*, respectively. The propagation velocity of all transmission lines is $V_p$.
   a. Assume the initial voltage and current on the transmission line system is both zero, and switch $S_1$ and $S_2$ are opened. At t = 0, $S_1$ is closed. Please derive and sketch the voltage waveform *Vc(t)* on the load $C_L$ (point A) for $0 \le t \le 1.75T$. (The mutual coupling with Tx4 can be ignored here.)
   b. The timing requirement for the voltage waveform *Vc(t)* is $V_c(t) \ge V_{th}$ at $t \ge T + C_L Z_0$. Please design the minimum characteristic impedance of the short stub $Z_1$ to satisfy this timing requirement. It is assumed $C_L Z_0 << T/4$.

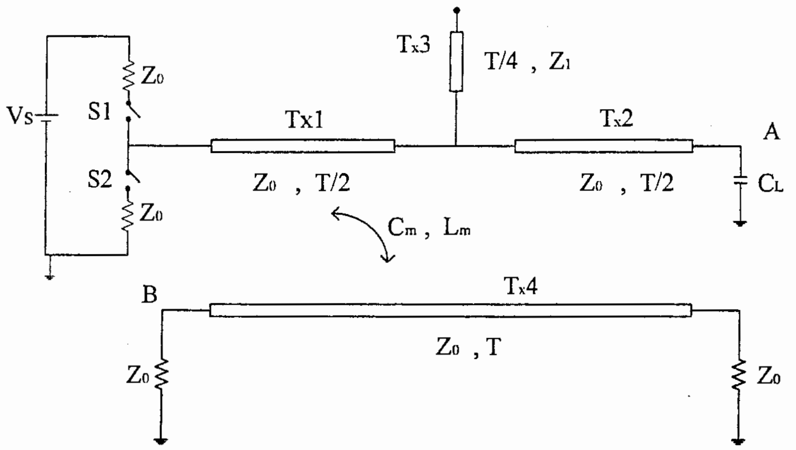



Figure P4

5. In the system shown in Fig.P4, after the transmission lines system enters the steady state, the switch $S_1$ is opened and $S_2$ is closed simultaneously at time *Ts*. Please derive and sketch the coupling waveform at the near end of Tx4 (point B) for $T_s \le t \le T_s + T$. It is assumed that the system is weakly coupling.

6. In the system shown in Fig. P6, steady-state conditions are established with the switch *S* closed. At the switch *S* is opened.
   a. Find the energy stored in the system at t = $0^-$.
   b. Obtain the solutions for the voltages across $R_{L1}$ and $R_{L2}$ for t > 0.
   c. Show that the total energy dissipated in $R_{L1}$ and $R_{L2}$ for t > 0 is equal to the energy stored in the system at t = $0^-$.

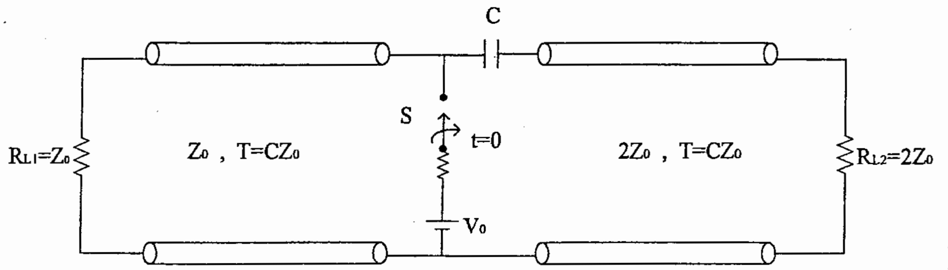



Figure P6

1. $V(z,t) = 10\cos(3\pi\times10^8 t - 2\pi z)$ V

$w = 0.1$ m

$d = 0.01$ m

(a) $E_x = \frac{V}{d} = 1000\cos(3\pi\times10^8 t - 2\pi z)$ V/m

(b) $H_y = \frac{E_x}{\eta}$

$v_p = \frac{3\pi\times10^8}{2\pi} = 1.5\times10^8$ m/s $= \frac{c}{2} = \frac{1}{\sqrt{\mu_0\cdot4\epsilon_0}}$

$\therefore \epsilon_r = 4$

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2} = 60\pi$

$\therefore H_y = \frac{50}{3\pi}\cos(3\pi\times10^8 t - 2\pi z)$ A/m

(c) $I = H_y w$

$= \frac{5}{3\pi}\cos(3\pi\times10^8 t - 2\pi z)$ A

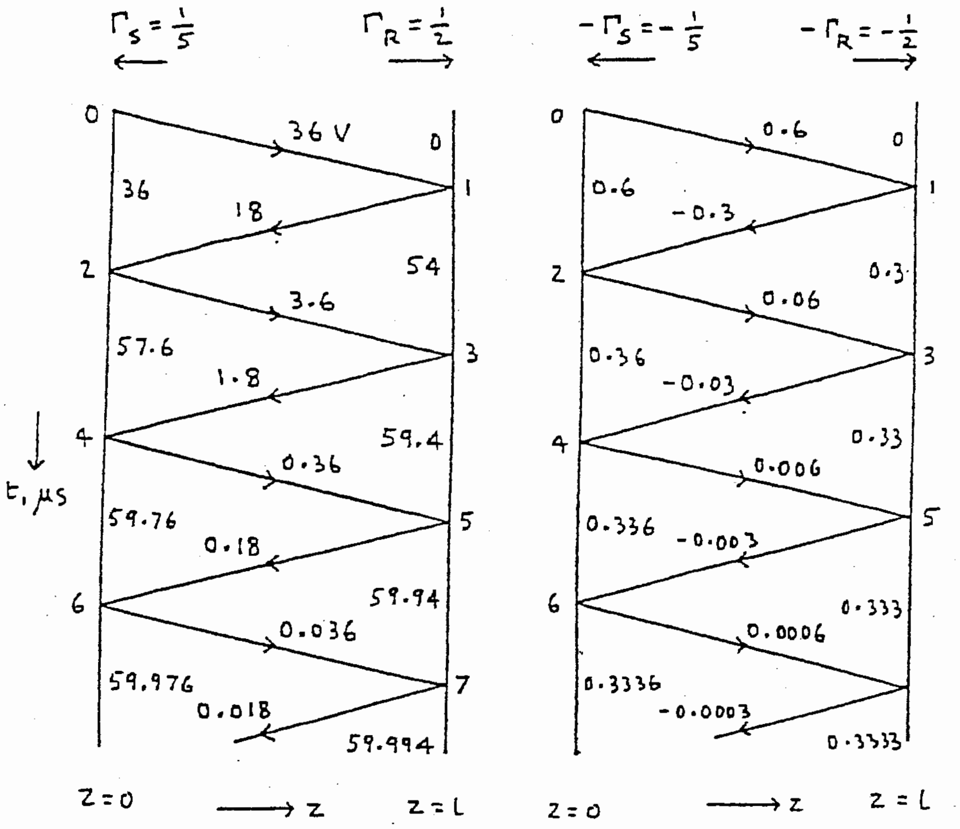
(d) $P = VI$

$= \frac{50}{3\pi}\cos^2(3\pi\times10^8 t - 2\pi z)$ W

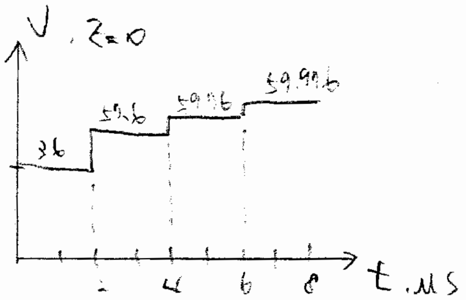
2.

$$V^+ = 90 \times \frac{60}{90+60} = 36\,V, \quad \Gamma_R = \frac{180-60}{180+60} = \frac{1}{2}$$

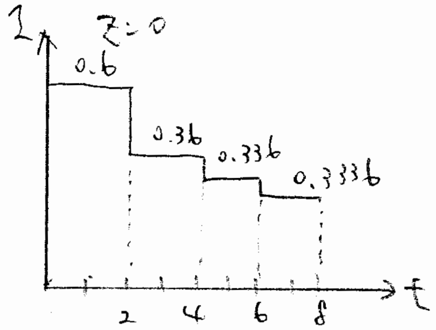
$$I^+ = \frac{60}{60} = 1\,A, \quad \Gamma_S = \frac{90-60}{90+60} = \frac{1}{5}$$



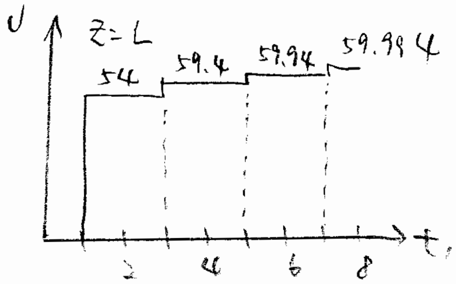
2 (continued)



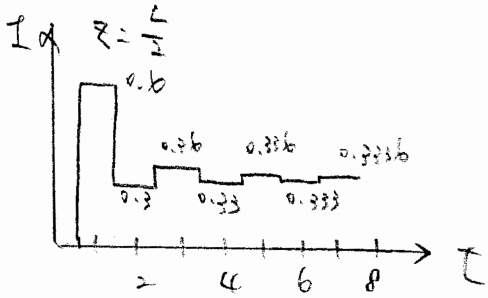
V, z=0
36
59.6
59.96
59.996
2
4
6
8
t, μs

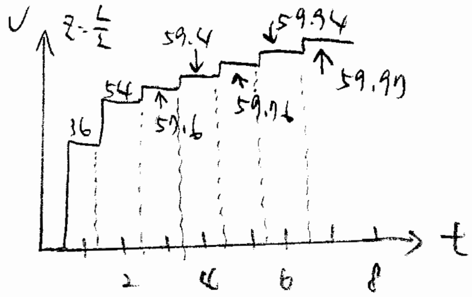
I
z=0
0.6
0.36
0.336
0.3336
2
4
6
8
t

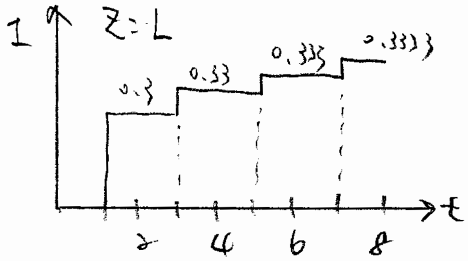
V
z=L
54
59.4
59.94
59.994
2
4
6
8
t

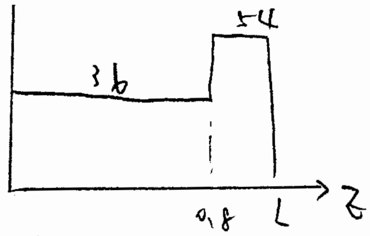
I
z=L/2
0.6
0.36
0.336
0.3336
0.3
0.33
0.333
2
4
6
8
t

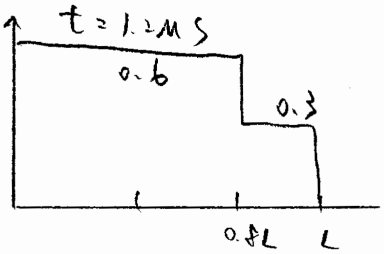
V
z=L/2
36
54
59.6
59.4
59.96
59.94
59.99
2
4
6
8
t

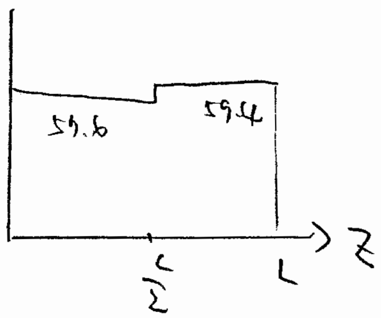
I
z=L
0.3
0.33
0.333
0.3333
2
4
6
8
t

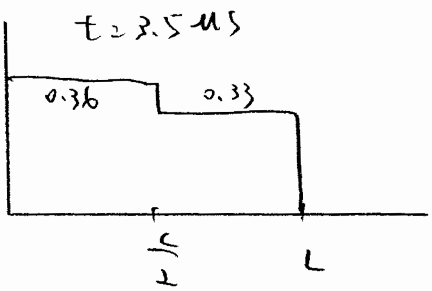
36
54
0.8
L
z

t=1.2μs
0.6
0.3
0.8L
L

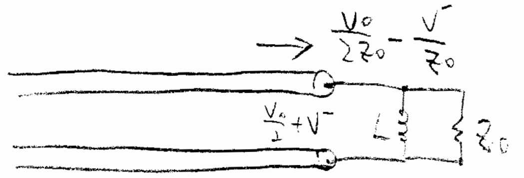



59.6
59.4
L/2
L
z

t=3.5μs
0.36
0.33
L/2
L

3.




$z = L^-$

$$\frac{V_0}{2Z_0} - \frac{V^-}{Z_0} = \frac{1}{L}\int\left(\frac{V_0}{2}+V^-\right)dt + \frac{1}{Z_0}\left(\frac{V_0}{2}+V^-\right)$$

$$-\frac{1}{Z_0}\frac{dV^-}{dt} = \frac{1}{L}\left(\frac{V_0}{2}+V^-\right) + \frac{1}{Z_0}\frac{dV^-}{dt}$$

$$\frac{2L}{Z_0}\frac{dV^-}{dt} + V^- = \frac{-V_0}{2}$$

$$V^- = -\frac{V_0}{2} + Ae^{-\left(\frac{Z_0}{2L}\right)t}$$

At $t = T^+$, current through $L$ is zero,

$$\therefore \left[\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right]_{t=T^+} = \frac{1}{Z_0}\left[\frac{V_0}{2}+V^-\right]_{t=T^+}$$

$$\left[V^-\right]_{t=T^+} = 0$$

$$0 = -\frac{V_0}{2} + Ae^{-(Z_0/2L)T}$$

$$A = \frac{V_0}{2}e^{(Z_0/2L)T}$$

$$V^-(t) = -\frac{V_0}{2} + \frac{V_0}{2}e^{-\left(\frac{Z_0}{2L}\right)(t-T)}$$

$$[V]_{\ell=\ell+} = [V]_{z=\ell-} = \frac{V_0}{2} + V^-$$

$$= \underline{\frac{V_0}{2}e^{-(Z_0/2L)(t-T)}} \quad \text{for } t \geq T$$

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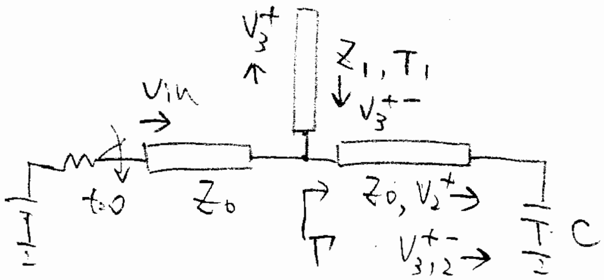
4.




(a) a, $V_{in} = \frac{Z_0}{Z_0+Z_0} V_s = \frac{1}{2} V_s$

b, $\Gamma_{stub} = \frac{(Z_1 // Z_0) - Z_0}{(Z_1 // Z_0) + Z_0} = -\frac{Z_0}{2Z_1+Z_0}$

$\tau_{stub} = 1 + \Gamma = \frac{2Z_1}{2Z_1+Z_0}$

c, $V_3^+ = \tau_{stub} V_{in} = V_2^+ = \frac{2Z_1}{2Z_1+Z_0} \cdot \left(\frac{1}{2} V_s\right) = \frac{Z_1 V_s}{2Z_1+Z_0}$

d, $V_3^{+-} = 2V_3^+$ (open) $= \frac{2Z_1 V_s}{2Z_1+Z_0}$

e, $\tau' = 1 + \frac{\frac{Z_0}{2} - Z_1}{\frac{Z_0}{2} + Z_1} = \frac{2Z_0}{2Z_1+Z_0}$ (from stub (TX3) to (TX2))

$V_{3,2}^{+-} = \tau' V_3^{+-} = \frac{2Z_0 Z_1}{(2Z_1+Z_0)^2} V_s$

f, $V_2^+$ arriving $C_L$ at T

$V_{3,2}^{+-}$ arriving $C_L$ at $T + \frac{T}{2}$

So, $V_2^+$ and $V_{3,2}^{+-}$ charging up $C_L$ with time constant $C_L Z_0$,

$V_c(t) = 0, \quad 0 \le t < T$

$$\begin{cases} V_c(t) = V_{c1}(t) = 2V_2^+\left(1 - e^{-(t-T)/C_L Z_0}\right), & T \le t < 1.5T \\ V_c(t) = V_{c1}(t) + V_{c2}(t) = V_{c1}(t) + 2V_{3,2}^{+-}\left(1 - e^{-(t-1.5T)/C_L Z_0}\right), & 1.5T \le t \le 1.75T \end{cases}$$

4 (b)

$$V_{c1}(t) = 2V_2^+\left(1 - e^{-(t-T)/(C_L Z_0)}\right)$$

$$t = T + C_L Z_0$$

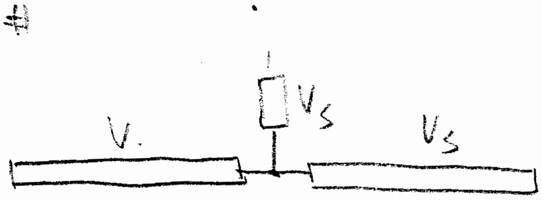
$$V_{c1} = 2V_2^+(1-e^{-1}) \geq V_{th}$$

$$\Rightarrow \frac{2Z_1 V_s}{2Z_1+Z_0}(1-e^{-1}) \geq V_{th}$$

$$\Rightarrow \frac{2Z_1}{2Z_1+Z_0} \geq \frac{V_{th}}{V_s(1-\frac{1}{e})}$$

$$\Rightarrow Z_1 \geq \frac{Z_0 V_{th}}{2V_s(1-e^{-1})-2V_{th}}$$ #

5.




a. At $t=T_s$

$$V^+ = -\frac{Z_0}{Z_0+Z_0}V_s = -\frac{V_s}{2}$$

$$I^+ = -V_0\frac{1}{R_L+Z_0}$$

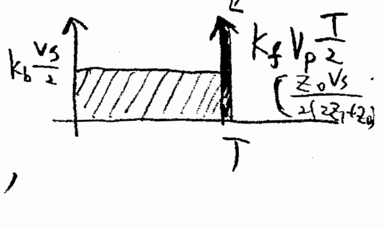
b. $$V^{+-} = \Gamma_{stub}\cdot V^+ = \frac{Z_0}{2Z_1+Z_0}\cdot\frac{V_s}{2} = \frac{Z_0 V_s}{2(2Z_1+Z_0)}$$

c. Coupling voltage at point B contributed from backward coupling from $V^+$ and forward coupling from $V^{+-}$

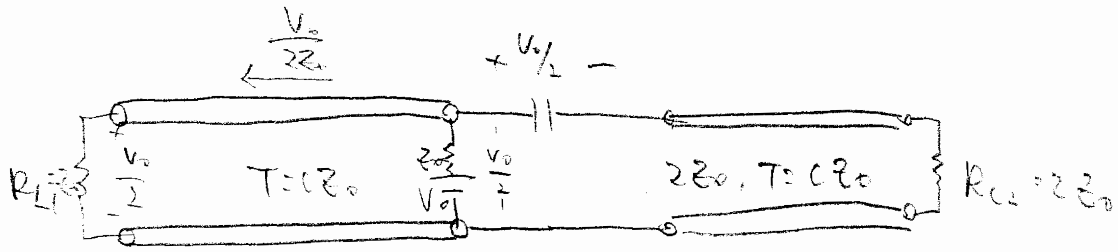
$$V_b(t) = K_b(V^+(t)-V^+(t-T)) = K_b\left[-\frac{V_s}{2}H(t)+\frac{V_s}{2}H(t-\tau)\right]$$

$$V_f(t) = \frac{V_p T}{2}K_f\,\delta(t-T)\times\frac{Z_0 V_s}{2(2Z_1+Z_0)}$$

$$K_b = \frac{1}{4}V_p\left(C_m Z_0+\frac{L_m}{Z_0}\right),\quad K_f = \frac{1}{2}\left(C_m Z_0-\frac{L_m}{Z_0}\right),$$



6.

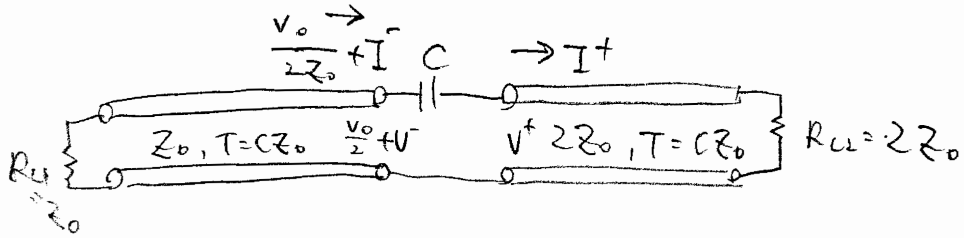



(a) Energy stored in the system

$$= \frac{1}{2}C\left(\frac{V_0}{2}\right)^2 + \frac{1}{2Z_0}\left(\frac{V_0}{2}\right)^2 CZ_0 + \frac{1}{2}\left(\frac{V_0}{2Z_0}\right)^2 Z_0(CZ_0)$$

$$= \frac{3}{8}CV_0^2$$

(b)




$$I^+ = \frac{V_0}{2Z_0} + I^-$$

$$\frac{V^+}{2Z_0} = \frac{V_0}{2Z_0} - \frac{V^-}{Z_0}$$

$$V^- = -\frac{V^+}{2} - \frac{V_0}{2}$$

$$C\frac{d}{dt}\left(\frac{V_0}{2} + V^- - V^+\right) = I^+$$

$$C\frac{d}{dt}\left(\frac{V_0}{2} - \frac{V^+}{2} - \frac{V_0}{2} - V^+\right) = \frac{V^+}{2Z_0}$$

$$3CZ_0\frac{dV^+}{dt} + V^+ = 0$$

$$V^+ = Ae^{-\frac{1}{3CZ_0}t}$$

$$\text{I.C.}\ \left[\frac{V_0}{2} + V^- - V^+\right]_{t=0} = \frac{V_0}{2}$$

6. (continued)

$$\left[-\frac{V^+}{2}-V^+\right]_{t=0}=\frac{V_0}{2}$$

$$\left[V^+\right]_{t=0}=-\frac{1}{3}V_0$$

$$\therefore\ V^+=-\frac{1}{3}V_0e^{-\frac{1}{3CZ_0}t}$$

$$V^-=-\frac{V_0}{2}+\frac{1}{6}V_0e^{-\frac{1}{3CZ_0}t}$$

$$V_{RL1}=\begin{cases}\frac{V_0}{2}, & t<T\\ \frac{1}{6}V_0e^{-\frac{1}{3CZ_0}(t-T)}, & t>T\end{cases}$$

$$V_{RL2}=\begin{cases}0, & t<T\\ -\frac{1}{3}V_0\,e^{-\frac{1}{3CZ_0}(t-T)}, & t>T\end{cases}$$

(c) RL1:

$$E_1\left(\frac{V_0}{2}\right)^2\frac{1}{Z_0}(CZ_0)+\frac{1}{Z_0}\int_T^\infty\frac{V_0^2}{36}e^{-\frac{2}{3CZ_0}(t-T)}dt$$

$$=\frac{1}{4}CV_0^2+\frac{V_0^2}{36Z_0}\left[\frac{e^{-\frac{2}{3CZ_0}(t-T)}}{-\frac{2}{3CZ_0}}\right]_{t=T}^{\infty}=\frac{7}{24}CV_0^2$$

RL2:

$$E_2=\frac{1}{2Z_0}\int_T^\infty\frac{V_0^2}{9}e^{-\frac{1}{3CZ_0}(t-T)}dt=\frac{1}{12}CV_0^2$$

Sum, $E_{total}=E_1+E_2=\left(\frac{7}{24}+\frac{1}{12}\right)CV_0^2=\underline{\frac{3}{8}CV_0^2}$

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