Electromagnetics I **Midterm Examination** 2006.4.17

1. Let P and P' be the points with spherical coordinates $(1,\theta,\phi)$ and $(1,\theta',\phi')$, respectively. Let O be the origin of the coordinate system. Find

   1) the angle γ between the two vectors $\overline{OP}$ and $\overline{OP'}$,

   2) the distance between the points P and P'.

(14%)

2. In Cartesian coordinate system, let S be the triangular plane defined by $x+y+z=1$, $0<x,y,z<1$ and $\overline{A}$ be the vector given by $\overline{A}=3\hat{x}+2\hat{y}+\hat{z}$. Find

   1) the unit normal vector to the plane S,

   2) the surface integral $\int_S \overline{A}\cdot d\overline{S}$ over the plane S.

(14%)

3. Two equal and opposite point charges $+Q$ and $-Q$ are located at $\left(0,0,{}^{d}/_{2}\right)$ and $\left(0,0,-{}^{d}/_{2}\right)$, respectively. Such an arrangement is known as the electric dipole. Find the electric field intensity $\overline{E}\left(\bar{r}\right)$ at the point $\bar{r}$, due to the electric dipole, such that the spacing d is much smaller than the distance $r=\left|\bar{r}\right|$ from the origin.

(14%)

4. The Faraday disk generator consists of a circular metal disk rotating with a constant angular velocity ω in a uniform and constant magnetic field of flux density $\overline{B}=\hat{z}B_0$ that is parallel to the axis of rotation. Brush contacts are provided at the axis and on the rim of the disk, as shown in Fig.1. Determine the open-circuit voltage or emf of the generator if the disk radius is b.

(14%)

5. Let us consider the charge distribution given by

$$\rho = \begin{cases} \rho_0 \, x/a & , \quad -a < x < a \\ 0 & , \quad \text{otherwise} \end{cases}$$

where $\rho_0 \left(C/m^3 \right)$ is a constant. Find the displacement flux density vector $\overline{D}$ everywhere and plot D versus x.

(15%)

6. A coaxial cable consists of an inner conductor of radius a and an outer conductor (zero thickness) of radius b $(b > a)$. Assume that the cable is infinitely long and its axis is along the z-axis. Current I flows with uniform density in the z-direction in the inner conductor and returns with uniform density in the –z-direction in the outer conductor. Find the magnetic field intensity $\overline{H}$ everywhere and plot H versus r.

(15%)

7. Let an electric field in free space (μ_0, ε_0) be specified by

$$\overline{E}(z,t) = \hat{x} \, e^{j(\omega t - \beta z)} \ , \ j = \sqrt{-1} \, .$$

Find the condition on ω and β such that the field would satisfy both Faraday's and Ampere's circuital laws.

(14%)

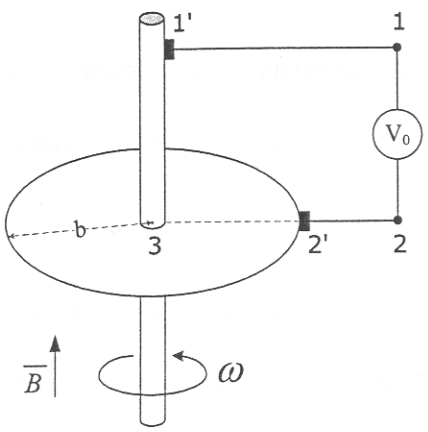

Fig.1.