Electromagnetics (II) Final Exam. **June 12, 2000**

1. A d-c voltage V_0 is applied at $t = 0$ directly to the input terminals of an open-circuited lossless transmission line of characteristic impedance R_0 and length ℓ as shown in Fig. 1. Sketch the voltage and current waves on the line (i.e., $V(z)$ and $I(z)$ versus z, using the arrow symbol to indicate the directions of waves) for the following time intervals: (1) $0 < t < T(= \ell / u)$, (2) $T < t < 2T$, (3) $2T < t < 3T$, (4) $3T < t < 4T$. What happens after $t = 4T$? (18%)

Fig. 1

2. A load impedance $25 + j20(\Omega)$ is connected to a lossless transmission line of length 0.202λ and characteristic impedance $50\,(\Omega)$. Use a Smith chart to find (1) the standing-wave ratio, (2) the voltage reflection coefficient at the load end, (3) the input impedance, (4) the input admittance, and (5) the location of the voltage maximum on the line. (15%)

3. The TM_n mode field in a parallel-plate waveguide is expressed as

$$\overline{E} = A_n \left\{ \hat{z} \sin\left(\frac{n\pi y}{b}\right) - \hat{y}\frac{\gamma}{h}\cos\left(\frac{n\pi y}{b}\right) \right\} e^{-\gamma z}$$

$$\overline{H} = A_n \hat{x} \frac{j\omega\varepsilon}{h}\cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

where $h = \dfrac{n\pi}{b}$ and $\gamma = \sqrt{h^2 - \omega^2 \mu_0 \varepsilon_0}$.

(a) Find the cutoff frequency of the TM_n mode. (4%)

(b) Describe the behavior of the TM_n mode when the operating frequency is below the cutoff frequency. (4%)

(c) Compute the time-averaged power carried by this TM_n mode per unit width when the operating frequency is higher than the cutoff frequency. (5%)

4. For a rectangular waveguide of dimensions $a = 3.75cm$ and $b = 1.25cm$, and having a dielectric of $\varepsilon = 6.25\varepsilon_0$ and $\mu = \mu_0$, find all propagating modes for $f = 5GHz$ and for each mode, find the values of phase constant β, guide wavelength λ_g, phase velocity u_p, and wave impedance Z. (20%)

5. Consider TM waves with angular frequency ω along a dielectric slab, which is situated in free space (ε_0, μ_0), as shown in Fig. 2, where ε_d and μ_d are the permittivity and permeability, respectively, of the dielectric slab. Let

$E_z(y,z) = A\sin k_y y e^{-j\beta z}$ in the dielectric region, $|y| \le d/2$, and $E_z(y,z) = Be^{-\alpha(y-d/2)}$ in the upper free-space region, $y \ge d/2$.

(a) Give an expression relating k_y and β. (4%)

(b) Give an expression relating α and β. (4%)

(c) What is the value of β when the wave is at cutoff? (2%)

(d) Derive expressions for the nonzero field components in the dielectric region. (8%)

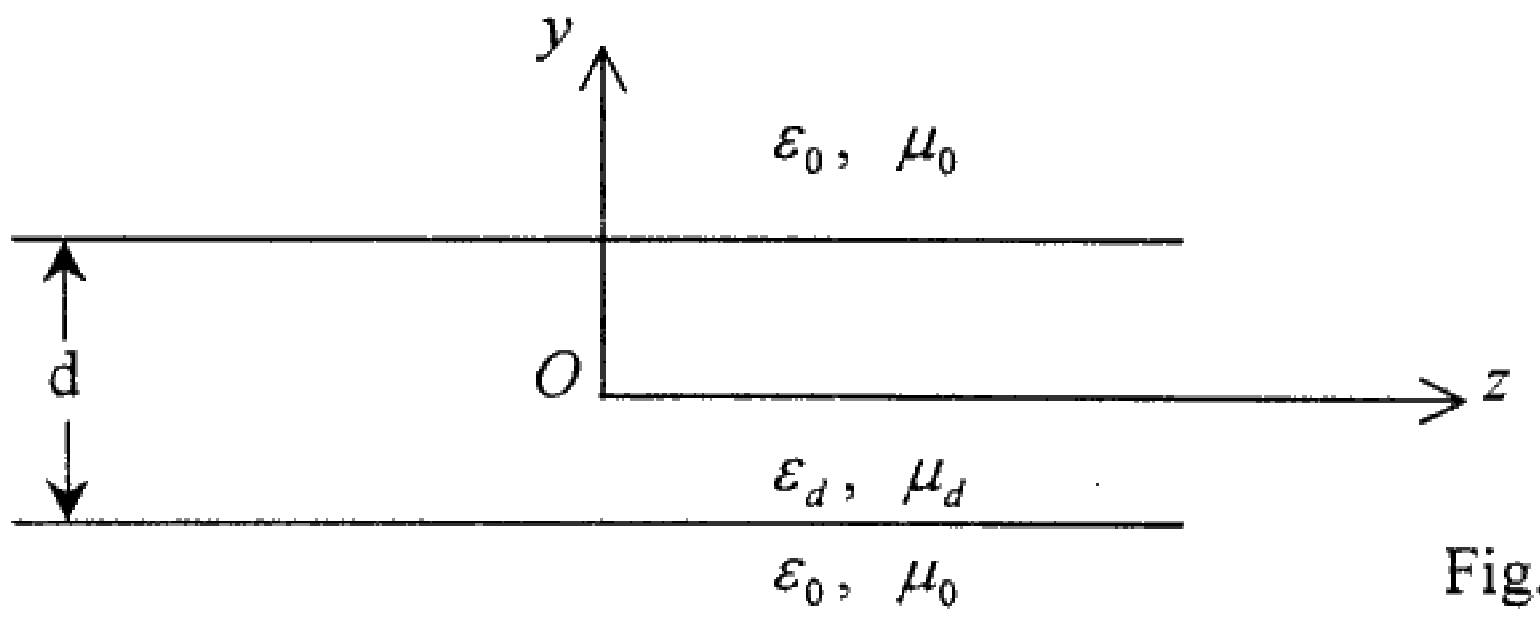

Fig. 2

6. The electromagnetic field of a Hertzian dipole situated at the origin of a spherical coordinate system (R,θ,ϕ) in free space, of length $d\ell$ and with current $i(t) = I\cos\omega t$ on it, is

$$H_\phi = -\frac{Id\ell}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$E_R = -\frac{Id\ell}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$E_\theta = -\frac{Id\ell}{4\pi}\eta_0\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

(a) What is the definition of the far zone? (2%)

(b) Write expressions for the far-zone leading fields for the given Hertzian dipole. (5%)

(c) Give an expression for the time-average Poynting vector (including the direction) at a point (R,θ,ϕ) in the far zone. (5%)

(d) Plot the radiation pattern in the $\phi = 0^\circ$ plane. (2%)

(e) Give an expression for the radiation intensity U. (2%)