Electromagnetics II final exam

date: 2010/01/11

姓名：______________

學號：______________

注意事項：1. 題目卷(本頁)與答案卷(第一頁)，請皆確實填寫考生姓名與學號；
2. 考試完畢，請將題目卷、答案卷一併繳回，未繳回者，則不予計分；
3. 題目一共六題，每題 20 分，採計得分最高之五題。

1. (20%) Figure 1(a) shows an infinitesimal segment of transmission line with inductance $L\triangle z$ henry, capacitance $C\triangle z$ farad, and negative conductance $-G\triangle z$ siemens.
   (1) (3%) Apply Kirchhoff's current law (KCL) and voltage law (KVL) to derive two differential equations between $V(z)$ and $I(z)$.
   (2) (2%) Derive a differential equation for $V(z)$ by substituting one of the two differential equations into the other.
   (3) (2%) Find the propagation constant $\gamma = \alpha + j\beta$ of the traveling-wave solutions of $V(z)$.
   (4) (2%) Express the general solution of $V(z)$ in terms of the two traveling-wave solutions.
   (5) (2%) Express the general solution of $I(z)$ in terms of the same traveling-wave solutions as in $V(z)$.
   (6) (2%) Derive the formula of characteristic impedance Z_0.
   (7) (2%) If $G << \omega C$, find the approximate formulas of α and β in terms of ω, L, C and G.
   (8) (3%) If a load Z_L is connected to the transmission line as shown in Figure 1(b), find the voltage reflection coefficient Γ_L at $z = 0$ in terms of Z_L and Z_0.
   (9) (2%) Define the impedance as $Z(z) = V(z)/I(z)$. Express $Z(z)$ in terms of Γ_L, Z_0 and γ.

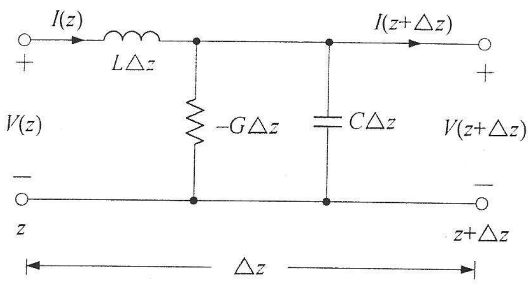

Figure 1(a)

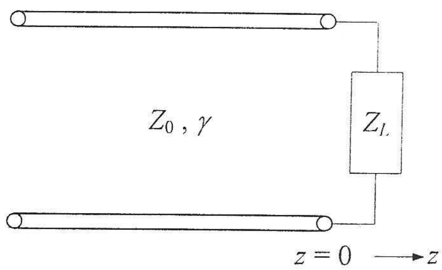

Figure 1(b)

2. (20%) Consider a plane wave propagating along the z direction in a homogeneous medium, its propagation constant is $\gamma = \alpha + j\beta$. The polarization of the magnetic field is $\hat{y}$, and the amplitude of the magnetic field at $z = 0$ is H_0.

The Ampere's law: $\nabla \times \bar{H} = j\omega\varepsilon\bar{E} + \bar{J}$
The Faraday's law: $\nabla \times \bar{E} = -j\omega\mu\bar{H}$

(1) (2%) Write down the functional form of the magnetic field.

(2) (3%) If the medium has permittivity ε_0, permeability μ_0 and conductivity of σ, apply the Ampere's law to find the functional form of the electric field.

(3) (3%) Apply the Faraday's law to the electric field in (2) to derive the magnetic field, then compare this magnetic field with that in (1) to find the functional form of α and β in terms of ω, μ_0, ε_0 and σ, assuming $\sigma >> \omega\varepsilon_0$.

(4) (2%) Based on α derived in (3) to find the propagation distance over which the field magnitude decays by $1/e$.

(5) (3%) If the medium has permittivity $-\varepsilon_0$ and permeability $-\mu_0$, apply the Ampere's law to the magnetic field in (1) to find the functional form of the electric field.

(6) (2%) Apply the Faraday's law to the electric field in (5) to derive the magnetic field, then compare this magnetic field with that in (1) to find the functional form of α and β in terms of ω, μ_0, ε_0.

(7) (2%) Calculate the Poynting vector $\bar{P} = \bar{E} \times \bar{H}^*$ of the plane wave using the magnetic field in (1) and the electric field in (5).

(8) (3%) Is the propagation direction of the plane wave the same as that of the Poynting vector? Briefly explain why.

3. (20%) Figure 3 shows a parallel-plate waveguide (PPWG) made of perfect electric conductor (PEC). Assume the medium between the two plates has permittivity ε_0 and permeability μ_0, and the fields are independent of y. Consider a TE wave guided along the z direction with phase constant β_z, and the amplitude of the electric field is E_0.

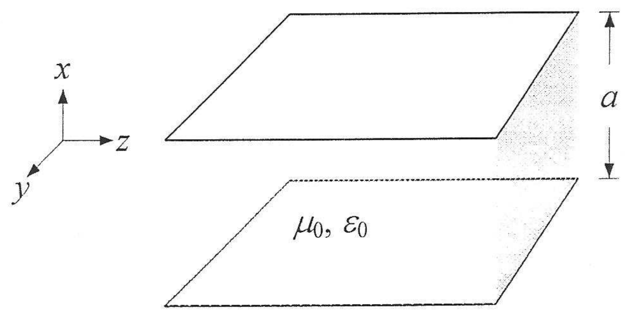

Figure 3

(1) (3%) Write down the expression of the electric field.

(2) (3%) Impose the boundary condition that the tangential electric field should vanish on PEC surface to determine all possible modes, and label them as TE_n modes.

(3) (3%) Derive the magnetic field of the TE_n mode using the Faraday's law.

(4) (3%) Derive the formulas of cutoff wavenumber k_c, cutoff frequency ω_c, cutoff wavelength λ_c, phase constant β_z, and guided wavelength λ_g for the TE_n mode.

(5) (4%) If the plates are made of good conductor with finite σ, calculate the ohmic power loss of the TE_n mode over a segment of length Δz in the z direction and one meter in the y direction. Assume the magnetic and the electric fields between the two plates are approximately the same as those with PEC plates.

(6) (4%) Estimate the attenuation constant of the TE_n mode.

4. (20%) Design a symmetric dielectric slab waveguide, with $\varepsilon_{r1} = 2.25$ and $\varepsilon_{r2} = 2.13$, by finding the value of d/λ_0 such that the TE_1 mode operates at 20% above its cutoff frequency.

5. (20%) A dielectric slab of thickness 4 cm and permittivity $2.25\varepsilon_0$ exists in an air-dielectric rectangular waveguide of dimensions a = 3 cm and b = 1.5 cm, as shown in Figure 5. Find the lowest frequency for which the dielectric slab is transparent (i.e., allows complete transmission) for $\mathrm{TE}_{1,0}$ mode propagation in the waveguide.

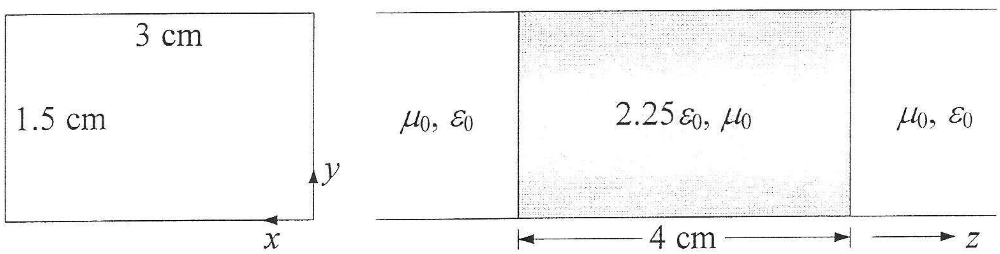

Figure 5

6. (20%) Compute the (peak) Power Gain of an antenna which has a power efficiency of 95% and the following radiation **E-field** pattern (independent of Φ):

$$E(\theta) = \begin{cases} 1, & 0^\circ \le \theta \le 30^\circ \\ 0.5, & 60^\circ \le \theta \le 120^\circ \\ 0.707, & 150^\circ \le \theta \le 180^\circ \\ 0, & 30^\circ < \theta < 60^\circ \text{ and } 120^\circ < \theta < 150^\circ \end{cases}$$

Useful formulas: One could refer to the useful formulas as shown in the followings.

$$v_{pz} = \frac{\omega}{\beta_z};\ \frac{1}{v_g} = \frac{d\beta_z}{d\omega};\ v_g = \frac{\omega_B - \omega_A}{\beta_{zB} - \beta_{zA}}$$

$$\tan\left(\frac{\pi d\sqrt{\varepsilon_{r1}}}{\lambda_0}\cos\theta_i - \frac{m\pi}{2}\right) = \frac{\sqrt{\sin^2\theta_i - (\varepsilon_2/\varepsilon_1)}}{\cos\theta_i}, m = 0,1,2,...$$

$$\begin{cases}\beta_{x1}^2 + \beta_z^2 = \omega^2\mu_0\varepsilon_1 \\ -\alpha_{x2}^2 + \beta_z^2 = \omega^2\mu_0\varepsilon_2\end{cases}, \quad \rightarrow \frac{\alpha_{x2}}{\beta_{x1}} = \sqrt{\frac{\omega^2\mu_0(\varepsilon_1 - \varepsilon_2)}{\beta_{x1}^2} - 1}$$

$$\tan\left(\beta_{x1}\frac{d}{2}\right) = \sqrt{\frac{\omega^2\mu_0(\varepsilon_1 - \varepsilon_2)}{\beta_{x1}^2} - 1}, \quad \tan\left(\beta_{x1}\frac{d}{2}\cos\theta_i\right) = \sqrt{\frac{\omega^2\mu_0(\varepsilon_1 - \varepsilon_2)}{\omega^2\mu_0\varepsilon_1\cos^2\theta_i} - 1}$$

$$\tan\left(\frac{\pi d\sqrt{\varepsilon_{r1}}}{\lambda_0}\cos\theta_i\right) = \frac{\sqrt{\sin^2\theta_i - (\varepsilon_2/\varepsilon_1)}}{\cos\theta_i}, \quad \tan[f(\theta_i)] = \begin{cases} g(\theta_i), & m = 0,2,4,... \\ \dfrac{-1}{g(\theta_i)}, & m = 1,3,5,... \end{cases}$$

$$f_c = \frac{mc}{2d\sqrt{\varepsilon_{r1} - \varepsilon_{r2}}}, m = 0,1,2,...; \quad f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}, \quad v_{pz} = \frac{1}{\sqrt{\mu\varepsilon}\cdot\sqrt{1-(f_c/f)^2}}$$

Table 1: Field expresssions and associated parameters for TE and TM modes in a rectangular waveguide

Transverse electric (TE) waves

Field expressions:

$(m, n = 0, 1, 2, ...,$ but not both zero)

$$\bar{E}_z = 0$$

$$\bar{H}_Z = \bar{A}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{E}_x = j\frac{\lambda_c^2}{4\pi^2}\omega\mu\frac{n\pi}{b}\bar{A}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{E}_y = -j\frac{\lambda_c^2}{4\pi^2}\omega\mu\frac{m\pi}{a}\bar{A}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{H}_x = \mp\frac{\bar{E}_y}{\eta_g}, \quad \bar{H}_y = \pm\frac{\bar{E}_x}{\eta_g}$$

$$\eta_g = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1-(f_c/f)^2}} = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

Transverse magnetic (TM) waves

Field expressions:

$(m, n = 1, 2, 3, ...)$

$$\bar{H}_Z = 0$$

$$\bar{E}_z = \bar{A}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{E}_x = \mp j\frac{\lambda_c^2}{2\pi\lambda_g}\frac{m\pi}{a}\bar{A}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{E}_y = \mp j\frac{\lambda_c^2}{2\pi\lambda_g}\frac{n\pi}{b}\bar{A}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{\mp j\beta_z z}$$

$$\bar{H}_x = \mp\frac{\bar{E}_y}{\eta_g}, \quad \bar{H}_y = \pm\frac{\bar{E}_x}{\eta_g}$$

$$\eta_g = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}, \quad \lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}, \quad \lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

$$v_{pz} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(f_c/f)^2}} = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-(\lambda/\lambda_c)^2}},$$