Electromagnetics (I) **Final Examination** Jan. 14, 2002

1. The electric field intensity of a uniform plane wave is $\bar{E} = E_0 \sin(6\pi \times 10^9 t - 20\pi y)\hat{z}$. Ignoring relativistic effects, find the Doppler shift in frequency for each of the following cases: (a) observer moving in the positive y-direction with speed 10^3 m/s; and (b) observer moving along the line $x = -2y = 2z$ in the sense of decreasing x with speed 10^3 m/s. (10%)
2. An infinite plane sheet lying in the $z = 0$ plane carries a surface current of density $\bar{J}_S = 0.1\cos(6\pi \times 10^8 t)\hat{x} A/m$. The region $z > 0$ is a perfect dielectric of $\varepsilon = 2.25\varepsilon_0$ and $\mu = \mu_0$, whereas the region $z < 0$ is a perfect dielectric of $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$. Find $\bar{E}$ and $\bar{H}$ on both sides of the sheet. (20%)
3. The electric and magnetic field intensities in the radiation field of an antenna located at the origin are given in spherical coordinates by
$$\bar{E} = E_0 \frac{\sin\theta}{r} \sin\omega(t - r\sqrt{\mu_0\varepsilon_0})\hat{\theta} V/m, \quad \bar{H} = \frac{E_0}{\sqrt{\mu_0/\varepsilon_0}} \frac{\sin\theta}{r} \sin\omega(t - r\sqrt{\mu_0\varepsilon_0})\hat{\phi} A/m.$$
Find: (a) the instantaneous Poynting vector; (b) the instantaneous power radiated by the antenna by evaluating the surface integral of the instantaneous Poynting vector over a spherical surface of radius r centered at the antenna and enclosing the antenna; and (c) the time-average power radiated by the antenna. (12%)
4. Region 1 ($z < 0$) is free space, whereas region 2 ($z > 0$) is a perfect conductor. A uniform plane wave with electric field $\bar{E}_i = E_0 \cos\omega(t - \sqrt{\mu_0\varepsilon_0} z)\hat{x}$ is incident on the interface $z = 0$ from region 1. Find: (a) the reflected wave electric and magnetic fields in region 1; (b) the total electric and magnetic fields in region 1; (c) the surface current density on $z = 0$; (d) the instantaneous Poynting vector at $z < 0$; and (e) the time-average Poynting vector at $z < 0$. (f) If region 2 ($z > 0$) is a good (but not perfect) conductor, how would the result of (e) be changed? Discuss it qualitatively. (24%)
5. Shown in Fig. 1 is a solenoid with two coaxial windings. One coil is of N_1 turns and length l_1, and the other is of N_2 turns and length l_2 ($l_2 < l_1$). The solenoid has a core of radius a and permeability μ. Find the mutual inductance between the coils. (10%)
6. A battery of dc voltage V is applied across two parallel perfect conductor plates each having a large area S. The space between the plates is filled with two different lossy dielectrics of thickness d_1 and d_2 ($d_1 + d_2 << \sqrt{S}$), permittivities ε_1 and ε_2, and conductivities σ_1 and σ_2, respectively, as shown in Fig. 2. Find: (a) the electric field distribution between the plates; (b) the current density between the plates; (c) the total resistance between the plates; and (d) the surface charge densities on the plates and at the interface of two dielectrics. (e) Draw an equivalent circuit of this system. (f) If the battery is disconnected, what would happen to this system? Give a qualitative description. (24%)

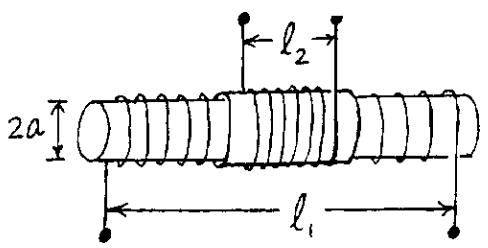

Fig. 1

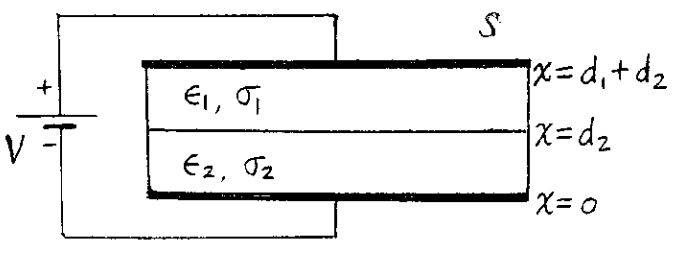

Fig. 2