2010 Electromagnetics one Midterm

1. (6%) Consider two points $\mathrm{P}(r,\theta,\varphi)$ and $\mathrm{Q}(r+dr,\theta+d\theta,\varphi+d\varphi)$at differential distance in the spherical coordinate system. Please write the expressions for three side lengths that would define the differential volume formed by incrementing the coordinates from point P to point Q.

2. 8%) For the current element $\mathrm{Idx}(\boldsymbol{a_x}+\boldsymbol{a_y})$ (A-m) situated at the point $(1-2,2)$, find the magnetic flux density at the point$(2-1,3)$

3. (8%) Prove that for any vector field A the volume integral

$$\oint_V (\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\vec{\boldsymbol{A}})dv=0$$

   Please state clearly the theorems you use.

4. (8%) At time t=0 a rigid straight line section of metallic wire of length L(m) is situated on the y-axis with its tips located at $(0,\frac{L}{2},0)$and $(0,-\frac{L}{2},0)$ respectively, and in the whole free space there exists a uniform magnetic field, B = $\sqrt{3}B_0\boldsymbol{a_x}+B_0\boldsymbol{a_z}$ (Wb/m^2) , where Bo is a positive constant. Now the line section is moved with constant velocity, v(m/s), along the z-direction. What is the open-circuit voltage induced between the tips of the line section? Also explain which tip has higher potential.

5. (15%) Find the unit vector normal to the surface $2x^2+y^2=6$ at the point $(\sqrt{2},\sqrt{2},0)$

6. (20%) A circular ring of radius a lies in the xy-plane and has its center at the origin. The ring is coated with charge such that the charge density is given by$\rho_L=\rho_{L_0}sin\varphi$ (C/m), where φ is the angle variable as in the cylindrical or polar coordinate system. Find the electric field intensity vector at a point on the z-axis.

7. (20%) For the electric field $\vec{\mathbf{E}}=E_0e^{-kx}\cos(3\times10^8t-y)\boldsymbol{a_z}$ in free space (J=0), find the value(s) of k for which the field satisfies both of Maxwell's curl equations.

8. (15%) Current I flows along a straight wire from a point charge $Q_1(t)$ located at$(1,1,2)$ to a point charge$Q_2(t)$ located at $(0,0,1)$. Find the line integral of the magnetic field intensity vector $\vec{\boldsymbol{H}}$ along the square closed path having the vertices at$(1,1,0)$, $(-1,1,0)$,$(-1,-1,0)$, and $(1,-1,0)$ and traversed in that order.