4+4+8 16

Electromagnetics I

Midterm Exam (10:20am–12:10noon) $dv = r^2 \sin\theta\, dr\, d\theta\, d\phi$ 4+4 4/25/14

1. (a) (4%) What are the spherical coordinates of the point: $x = -2$, $y = 2$, $z = -2\sqrt{2}$?

   (b) (5%) Consider two points $P(r,\ \theta,\ \phi)$ and $Q(r + dr,\ \theta + d\theta,\ \phi + d\phi)$ at differential distance in the spherical coordinate system. Please write the expressions for the three side lengths that would define the differential volume formed by incrementing the coordinates from point P to point Q. $d\mathbf{l} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$

   (c) (6%) Consider the curve $x = y = z^3$. The expression for the differential length vector $d\mathbf{l}$ along the curve at the point (1, 1, 1) can be written as $d\mathbf{l} = \mathbf{A}dz$. What is the expression for the vector $\mathbf{A}$? $d\mathbf{l}_1 \times d\mathbf{l}_2$

2. (10%) Find the unit vector normal to the surface $2x^2 + y^2 = 6$ at the point ($\sqrt{2}$, $\sqrt{2}$, 1).

   $d\mathbf{l}_1 = dx\,\mathbf{a}_x + dy\,\mathbf{a}_y$

3. (a) (6%) In the cylindrical coordinate system, a linear velocity vector field is given by $\mathbf{v} = \omega r \mathbf{a}_\phi$. If this field is expressed in the Cartesian coordinate system as $\mathbf{v} = v_x(x, y)\mathbf{a}_x + v_y(x, y)\mathbf{a}_y$, what are $v_x(x, y)$ and $v_y(x, y)$? $\frac{-2\sqrt{2}}{\sqrt{2}}$

   (b) (4%) Obtain the equation for the direction lines in the Cartesian coordiante system. (0, 0, 1) (0, 2, 5)

   0, 2, 4

4. (a) (5%) Consider an infinitesimal current element $Idl\mathbf{a}_z$ located at the origin. Write the equation for the direction line of the magnetic flux density due to this current element passing through the point ($x = 0$, $y = 2$, $z = 5$).

   (b) (10%) For the current element $I\,dx\,(\mathbf{a}_x + \mathbf{a}_y)$ (A-m) situated at the point (1, −2, 2), find the magnetic flux density at the point (2, −3, 4). $\frac{dx}{0} = \frac{dy}{2} = \frac{dz}{4}$

5. (5%) Write the integral form for the law of convervation of charge.

$2dz = 4dy$

$-1\ -1$

$B = \frac{\mu_0}{4\pi} \frac{I\,d\mathbf{l} \times \mathbf{R}}{R^3}$ $\frac{dx}{0} = \frac{dy}{0} = \frac{dz}{I\,dl}$ $dz = 2dy$

$z = 2y + c$

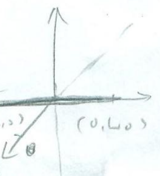
$R = (1, -1, 2)$ $Il + c = z$

$R = \sqrt{4+1+1} = \sqrt{6}$

$\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$ $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$

H_x H_y H_z H_x H_y H_z

$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}, \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}, \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$



$\nabla \times E = -\frac{\partial B}{\partial t}$ $dl =$ $\sqrt{2}B_0 L\, x$

$\sqrt{2}B_0 \quad 0 \quad B_0 \quad \sqrt{2}B_0 \quad 0 \quad B_0$
$0 \quad y \quad 0 \quad 0 \quad y \quad 0$
$-B_0 y \quad 0 \quad \sqrt{2}B_0 y$

6. (10%) At time $t = 0$ a rigid straight line section of metallic wire of length $2L$ (m) is situated on the y-axis with its tips located at (0, L, 0) and (0, $-L$, 0), respectively, and in the whole free space there exists a uniform magnetic field, $\mathbf{B} = \sqrt{2}B_0\mathbf{a}_x + B_0\mathbf{a}_z$ Wb/m^2, where B_0 is a positive constant. Now the line section is moved with constant velocity, v (m/s), along the z-direction. What is the open-circuit voltage induced between the tips of the line section? Also explain which tip has higher potential.

$dF = dl \times B$

7. (a) (10%) From Maxwell's curl equations, obtain the particular differential equations necessary for the analysis of the case with the source volume current density $\mathbf{J} = J_y(z, t)\mathbf{a}_y$.

$\nabla \times H = J + \frac{\partial D}{\partial t}$ $\nabla \cdot J = -\frac{\partial \rho}{\partial t}$ $2\sin \pi(t-1)$

(b) (15%) Now consider the case of an infinite plane sheet lying in the $z = 0$ plane in free space carrying a surface current of density $\mathbf{J_S} = f(t)\mathbf{a}_y$ A/m, where $f(t) = 0$ for $t < 0$; $f(t) = \sin\frac{2\pi}{T}t$ for $0 \leq t < 1\,\mu$s, where $T = 2\,\mu$s; $f(t) = 2\sin\frac{2\pi}{T}(t - \frac{T}{2})$ for $1\,\mu\text{s} \leq t < 2\,\mu$s; $f(t) = 0$ for $t \geq 2\,\mu$s. Let the generated electromagnetic waves be with $\mathbf{E} = E_x\mathbf{a}_x + E_y\mathbf{a}_y + E_z\mathbf{a}_z$ and $\mathbf{H} = H_x\mathbf{a}_x + H_y\mathbf{a}_y + H_z\mathbf{a}_z$. (b1) Find and sketch E_x versus t, E_y versus t, and E_z versus t in the $z = 0$ plane; (b2) H_x versus t, H_y versus t, and H_z versus t in the $z = 300$ m plane; (b3) E_x versus z, E_y versus z, and E_z versus z for $t = 1.5\,\mu$s. Please clearly specify the magnitudes and units for these field components in the plots.

$2\sin\pi(t-1)$ $\frac{4}{2}$ $\frac{6}{2}$ $\frac{5}{2}$ $\sin \pi t$ $\frac{3}{2}$ $\pi\ \frac{1}{2}$

$\frac{150}{3\times10^8}$ 0.5 $\frac{450}{3\times10^8} = 1.5$ 0.5 1

8. (10%) A point charge with Q (coulombs) is located at $(-1, -1, 2)$ in the Cartesian coordinate system. Please calculate the folowing surface integral in the $z = 0$ plane of the electric flux density due to this point charge: $\int_{-1}^{1}\int_{-1}^{1} \mathbf{D} \cdot \mathbf{a}_z\, dx\, dy$.

$\frac{300}{3\times10^8}$ $\frac{3}{2}$ $\pi(t-1)$

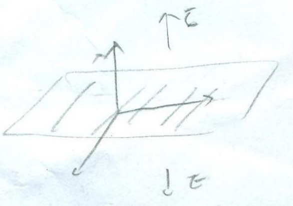
1.5×300 $E = \frac{\eta_0}{2} J_s (t \mp \frac{z}{v_p})$ $\nabla \times E = -\frac{\partial B}{\partial t}$ $\frac{300}{3\times10^8}$

$\frac{10^6}{3\times10^8}$ $H = \pm\frac{1}{2} J_s (t \mp \frac{z}{v_p})$

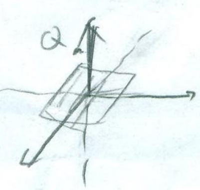
$\sqrt{2}B_0 \quad 0 \quad B_0 \quad \sqrt{2}B_0 \quad 0 \quad B_0$
$0 \quad 0 \quad v \quad 0 \quad 0 \quad v$
$0 \quad -\sqrt{2}B_0 v$

$2\sin\frac{2\pi}{T}(t - \frac{T}{2})$

$E =$



$\oint H \cdot dl = J + \frac{d}{dt}\int D\, ds$ $V =$ $F = qE$ $t-1$



~~$\sqrt{2}B_0 \quad 0 \quad B_0$~~ B_0

$\sin \pi t$ $\sin \pi t$