Department of Electrical Engineering, National Taiwan University

Engineering Mathematics-Differential Engineering, 2012, Fall

Midterm Examination

2010/11/07 Wednesday, 10:20-12:10

1. (40 scores) Find the solutions of the following DEs

(a) $y^{(3)}(x)+y'(x)=1+\cosh x$ (8 scores)

(b) $y^{(3)}(x)-y(x)=e^{x}$ (8 scores)

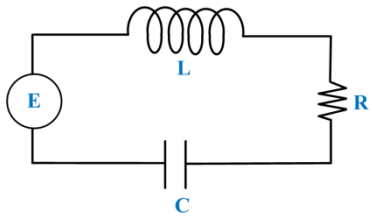
(c) $(x-1)y''(x)-y'(x)+2x^{-2}y(x)=0$

(Hint: $y = x^2$ is one of the solutions) (8 scores)

(d) $y^{(3)}(x)=1/x$ (8 scores)

(e) $y^{(4)}(x)-y''(x)=4x+2xe^{-x}$ (8 scores)

2. (10 scores) For the RLC circuit as follows:



(a) What is the condition where the charge $q(t)$ on the capacitor can be expressed as the form of $q(t)=A\cos(\omega t+\phi)+B$ where A, ω, ϕ, and B are some real constants? (5 scores)

(b) What is the condition where $q(t)$ can be expressed as the form of $q(t)=(At+B)e^{-\alpha t}+C$ where A, B, C, and α are some real constants?

(5 scores)

3. **(10 scores) Please find a one-parameter family of solutions for the differential equations.**

$$y'\left(\frac{x^2}{ln\,y}\right)+\frac{4y}{ln\,y}=x^2y(ln\,y)-xy$$

where $y_1 = exp\left(\frac{2}{x}\right)$ is a known solution of the equation.

(Hint: If $\frac{dy}{dx}=P(x)+Q(x)y+R(x)y^2$ and $y_0(x)$ is one of the solution, then the general solution of the DE can be solved by $y(x)=y_0(x)+u(x)$)

4. **(10 scores) A portion of a uniform chain of length 8 ft is loosely coiled around a peg at the edge of a high horizontal platform, and the remaining portion of the chain hangs at rest over the edge of the platform. Suppose the length of the overhanging chain is 3 ft, that the chain weights 2 lb/ft, and that the positive direction is downward. Starting at *t* = 0 seconds, the weight of the overhanging portion causes the chain on the table to uncoil smoothly and to fall to the floor. If x(t) denotes the length of the chain overhanging the table at time t > 0, then v = dx / dt is its velocity. <u>(Note: The gravity constant is 32 when using the imperial unit lb, ft)</u>**
   (a) When all resistive forces are ignored, please construct a mathematical model relating *v* to *x* in detailed steps. (4 scores)
   (b) Please solve (a) using the method of 'exact equations', and give the largest interval I over which the general solution is defined. (3 scores)
   (c) Please solve (a) using the method of 'substitutions', and give the largest interval I over which the general solution is defined. (3 scores)
5. **(30 scores) Please solve the given differential equations. <u>Give the largest interval I over which the general solution is defined.</u>**
   (a) Find an explicit solution of $(x^2-y^2)dx+xydy=0, y(1)=-\sqrt{2}$
   (5 scores)
   (b) $4x^2y''+y=0, y(-1)=2, y'(-1)=4$
   (5 scores)
   (c) $2y''=3y^2, y(0)=1, y'(0)=1$
   (5 scores)
   (d) $y'''+(6e^x-3)y"+(2-6e^x+e^{2x})y'-34ye^{3x}=0$
   (5 scores)
   (e) $\left(x-\sqrt{x^2+y^4}\right)dx+2y^3dy=0$
   (Hint: The substitution $z=x^2+u^2, dz=?\,dx+?\,du$ may be applied when solving this DE)
   (10 scores)