Electromagnetics (II): Quiz 2

10:20 AM - 12:20 PM, December 17, 2012.

[1] (20 %) Figure 1 shows a lossy transmission line with characteristic impedance Z_0. Assume the per-meter inductance, capacitance, resistance and conductance are L, C, R and G, respectively.

(a) (4 %) Draw an equivalent circuit of a small segment of this transmission line with length Δz, and derive the telegrapher's equations for the voltage wave $V(z)$ and the current wave $I(z)$.

(b) (3 %) Solve the telegrapher's equations for the voltage wave $V(z)$.

(c) (2 %) Derive the current wave $I(z)$ by substituting the voltage wave in (b) into one of the telegrapher's equations.

(d) (2 %) Derive the formulas of characteristic impedance Z_0 and propagation constant γ in terms of L, C, R and G.

(e) (3 %) Define a reflection coefficient $\Gamma(z)$ as the ratio of the voltage wave propagating in the $-z$ direction to that propagating in the z direction. A complex load Z_L is connected at $z = 0$, solve $\Gamma(0)$ in terms of Z_0 and Z_L.

(f) (3 %) Define impedance $Z(z)$ as the ratio between the total voltage to the total current at z. Derive $Z(z)$ as a function of Z_L and Z_0.

(g) (3 %) Derive the time-averaged power $\langle P(z)\rangle = \frac{1}{2}\mathrm{Re}\{V(z)I^*(z)\}$.

[2] (20 %) Figure 2 shows a parallel-plate waveguide (PPWG) made of perfect electric conductor (PEC). In order to demonstrate that the physical nature is independent of the coordiante system, the coordinate system in this figure is deliberately made different from those used in the lecture and the textbook. Assume the fields are independent of x. Consider a TM wave guided along the z direction with phase constant β_z, and the amplitude of the magnetic field is H_0.

(a) (3 %) Write down the expression of the magnetic field.

(b) (3 %) Derive the electric field using the Ampere's law $\nabla \times \bar{H} = j\omega\epsilon\bar{E}$.

(c) (3 %) Impose proper boundary conditions to determine all possible modes, and label them as TM_n modes.

(d) (3 %) Derive the formulas of cutoff wavenumber k_c, cutoff frequency ω_c, cutoff wavlength λ_c, phase constant β_z, and guided wavelength λ_g of the TM_n mode.

(e) (3 %) Write down all the field components of the lowest mode.

(f) (5 %) If the plates are made of good conductor with finite σ, calculate the ohmic power loss using the Poynting vector. Assume the magnetic and the electric fields are approximately the same as those derived in (a) and (b) with PEC plates.

[3] (20 %) Figure 3 shows a uniform plane wave of TE polarization incident upon a dielectric interfce at $x = 0$. Assume the fields are independent of y, and the propagation vectors of the incident, reflected and transmitted waves are

$$\bar{\beta}_i = -\hat{x}\beta_{ix} + \hat{z}\beta_{iz}$$
$$\bar{\beta}_r = \hat{x}\beta_{rx} + \hat{z}\beta_{rz}$$
$$\bar{\beta}_t = -\hat{x}\beta_{tx} + \hat{z}\beta_{tz}$$

respectively.

(a) (4 %) Write the expressions of the incident, reflected and transmitted electric fields. Assume the magnitude of the incident electric field is E_0, express the reflected and transmitted electric fields in term of E_0, reflection coefficient R and transmission coefficient T.

(b) (3 %) Derive the expressions of the incident, reflected and transmitted magnetic fields using the Faraday's law $\nabla \times \bar{E} = -j\omega\mu\bar{H}$.

(c) (3 %) Impose proper boundary conditions at $x = 0$ to derive two equations for the reflection coefficient R and the transmission coefficient T.

(d) (3 %) Explain why the phase-matching condition must be satisfied from either one of the two equations in (c), then derive the law of reflection and the law of refraction from this phase-matching condition.

(e) (4 %) Solve the two equations in (c) for the reflection coefficient R and the transmission coefficient T. The explicit expressions of R and T should include terms of θ_i, θ_t, $\eta_1 = \sqrt{\mu_1/\epsilon_1}$ and $\eta_2 = \sqrt{\mu_2/\epsilon_2}$ only.

(f) (3 %) Derive the formula of critical angle at which total reflection takes place.

[4] (20 %) Figure 4 shows a dielectric slab guide which is symmtric with respect to z. Note that the coordinate system in this figure is deliberately made different from those used in the lecture and the textbook. Assume the fields are independent of y.

(a) (4 %) Consider odd TE modes with E_y an odd function of z. Write down the expression of the electric fields.

(b) (4 %) Derive the magnetic fields using the Faraday's law.

(c) (4 %) Impose proper boundary conditions to derive an equation from which the propagation constant can be determined.

(d) (4 %) Solve the equation derived in (c) graphically, and explain how to use that graph to determine the cutoff frequency of the lowest TE mode.

(e) (4 %) If $\epsilon_2 < \epsilon_1$, will odd TE modes exist ? Briefly explain why.

[5] (25 %) Consider the rectangular waveguide shown in Figure 5.

(a) (5 %) Decompose the electric and the magnetic fields as $\bar{E} = \bar{E}_s + \bar{E}_z$ and $\bar{H} = \bar{H}_s + \bar{H}_z$, decompose the ∇ operator as $\nabla = \nabla_s + \hat{z}\frac{\partial}{\partial z}$. Apply these decompositions to the Faraday's law and the Ampere's law such that $\bar{E}_s$ and $\bar{H}_s$ can be expressed in terms of $\bar{E}_z$ and $\bar{E}_z$ only.

(b) (5 %) Consider TE modes with $E_z = 0$ and $H_z \neq 0$. Apply the separation-of-variables technique to solve $\left(\nabla^2 + k^2\right) H_z = 0$ for a general expression of H_z, where $k^2 = \omega^2\mu\epsilon$.

(c) (3 %) Write down the expresions of $\bar{E}_s$ and $\bar{H}_s$ using the formulas derived in (a).

(d) (2 %) List all the boundary conditions that must be satisfied by the fields.

(e) (5 %) Impose the boundary conditions in (d) to determine all possible modes, and label them as TE_{mn} modes.

(f) (3 %) Derive the formulas of cutoff wavenumber k_c, cutoff frequency ω_c, cutoff wavlength λ_c, phase constant β_z, and guided wavelength λ_g of the TE_{mn} mode.

(g) (2 %) What happen to the phase constant β_z if $\omega < \omega_c$?

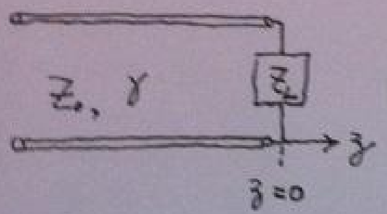

Figure 1

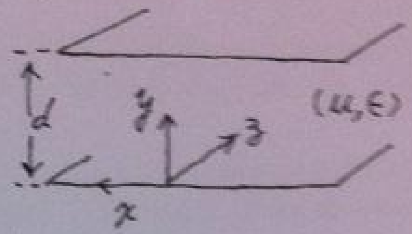

Figure 2

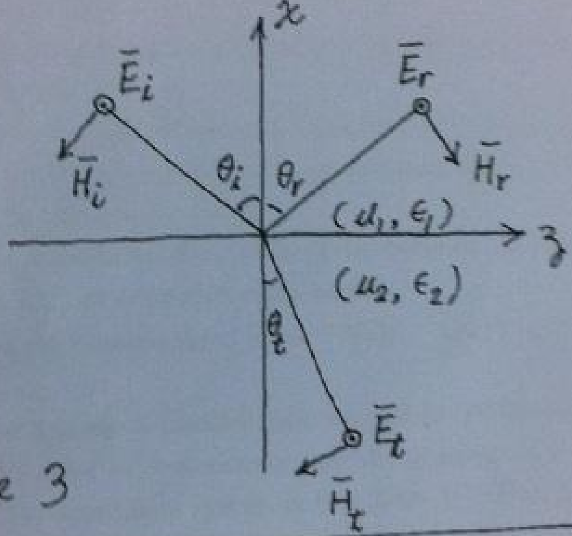

Figure 3

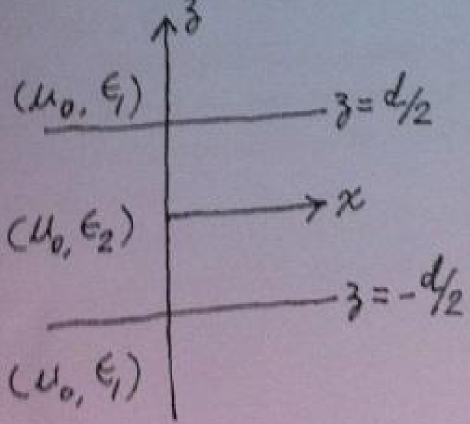

Figure 4

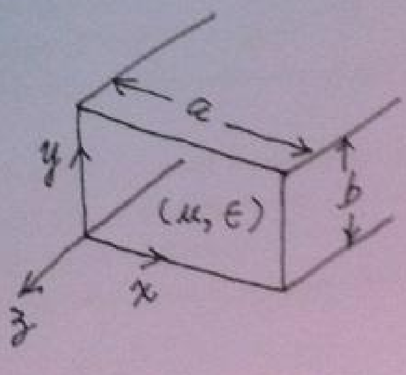

Figure 5