DIFFERENTIAL EQUATION FINAL

Dept. of Elec. Eng., National Taiwan University

Total=105 pts

14:10-15:50, Jan 16, 2003

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1. (20%)Consider the following system of the differential equations,

$$\begin{cases}(y''-5y'+4y)-(2z'-10z)=0\\ 2y-(z'-5z)=0\end{cases}$$

(a) (5%)Transform the equations into a system of linear first-order differential equations, X' = AX.

(b) (15%)Find the general solution with X' = AX.

2. (15%)Find the general solution of

$$X'=\begin{pmatrix}1 & 2\\ -\frac{1}{2} & 1\end{pmatrix}X+\begin{pmatrix}\csc t\\ \sec t\end{pmatrix}\cdot e^{t}$$

3. (a) (5%) Please find the Fourier transform of the function $f(x)=\delta(x)$ ($\delta(x)$ is Dirac delta function)

(b) (8%) Please solve the boundary-value problem

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\quad -\infty<x<\infty,\ t>0$$

$$u(x,0)=\delta(x),\quad -\infty<x<\infty$$

(c) (7%) Please solve the boundary-value problem

$$k\frac{\partial^2 u}{\partial x^2}-v\frac{\partial u}{\partial x}=\frac{\partial u}{\partial t},\quad -\infty<x<\infty,\ t>0$$

$$u(x,0)=\delta(x),\quad -\infty<x<\infty$$

(Hint: use the substitutions $x'=x-vt$ and $t'=t$)

4. (8%)For the wave equation $c^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$, subject to the initial condition

$$u(x,0)=\sin x,\quad \left.\frac{\partial u}{\partial t}\right|_{t=0}=\cos x$$

the solution can be obtained without using separating variables. Please find the solution.

(Hint: use the substitutions $\xi=x+ct$ and $\eta=x-ct$)

5. (7%) Use separation of variables to find product solutions for the following partial differential equation.

$$x\frac{\partial u}{\partial x} = y\frac{\partial u}{\partial y}$$

6. (5%) Find the general solution of the given differential equation on (0, ∞) in terms of sine/cosine functions

$$4x^2y''+4xy'+(36x^2-1)y=0$$

7. (a) (10%) Find the Fourier series of f(x) on the given interval

$$f(x)=\begin{cases}0 & .....-\pi<x<0\\ \cos 2x & .....0\le x<\pi\end{cases}$$

(b) (5%) Use the result of part (a) to find the sum of the following series

$$\frac{1}{3}+\frac{3}{1*5}-\frac{5}{3*7}+\frac{7}{5*9}-\frac{9}{7*11}+.....$$

8. (15%) The differential equation

$$x^2y''+xy'+(x^2-\frac{9}{25})y=0$$

(a) (3%) Show that x=0 is a regular singular point of the given differential equation.

(b) (6%) Show that the indicial root of the singularity x=0 do not differ by an integer.

(c) (6%) Use the method of Frobenius to obtain two linearly independent series solutions about x=0. Form the general solution on (0, ∞) (Write explicitly the first three nonzero terms for each solution).