Electromagnetics I
Final Exam (2 pages)

6/24/2011 10:20-12:00

Problem 1 (16%)

(a) Write down the general boundary conditions of electromagnetic fields across an interface of two different media in terms of electric and magnetic fields and the surface sources with four equations. (You need to draw a figure and indicate the media 1 and 2 with a unit vector normal to the surface.)

(b) Repeat (a), but replace one of the media with a perfect electric conductor.

Problem 2 (24%)

In the below figure, the region $z < 0$ is a perfect dielectric, whereas the region $z > 0$ is a perfect conductor. For an incident uniform plane wave having the electric field $\bar{E}_i = E_0[\cos(\omega t - kz)\hat{x} - \sin(\omega t - kz)\hat{y}]$ where $k = \omega\sqrt{\mu\varepsilon}$,

(a) Apply the proper boundary condition at z=0 and find the electric field of the reflected waves.

(b) Determine the polarization sense of the incident and reflected waves (linear/circular? right/left handed?), respectively.

(c) Express the total (incident + reflected) electric fields. Are they standing or traveling waves?

(d) Find the induced current density on the surface of the perfect conductor.

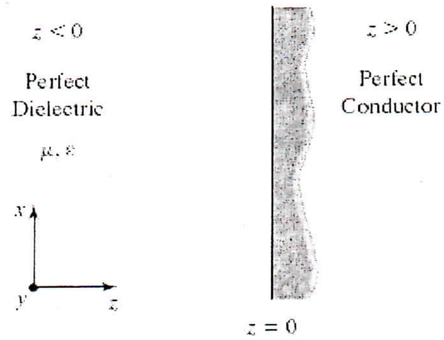

Figure of Problem-2

Problem 3 (20%)

A dielectric material of permittivity ε sliding freely in a cylindrical capacitor experiences a mechanical force $\mathbf{F}_e$ of electric origin in the axial direction, as shown below (neglecting the fringing effects at the edges).

(a) Find the expression of electric field in the region of $a < r < b$.

(b) Calculate the capacitance of such a configuration in terms of the intrusion x and total length L.

(c) Find the expression for $\mathbf{F}_e$.

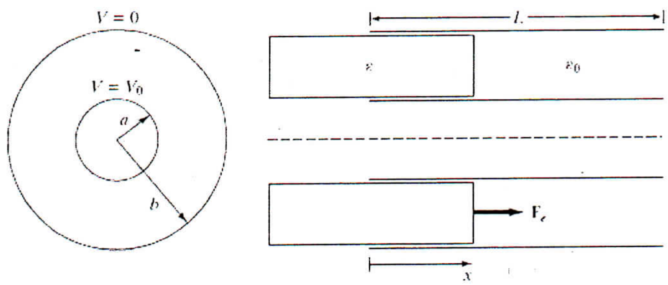

Figure of Problem-3

Problem 4 (15%)

For a coaxial cable structure, we calculate the internal inductance is based on the magnetic energy stored in the inner solid cylindrical conductor.

(a) Obtain the expression for the energy per unit length stored in the magnetic field internal to the current distribution.

(b) Denoting the energy from (a) equal to $\frac{1}{2}L_i I^2$, where I is the total current, find the internal inductance L_i per unit length.

(c) Now, similar to the approach of (a) and (b), find the external inductance of this structure based on the magnetic energy stored in the region between the inner and outer conductor.

Problem 5 (25%)

Consider two infinite plane parallel current sheets, spaced by $\lambda/4$ apart and carrying surface current density expressed by

$$\bar{J}_{s1} = \hat{x} J_{s0} \cos \omega t \quad \text{at } z = 0$$
$$\bar{J}_{s2} = \hat{y} J_{s0} \cos \omega t \quad \text{at } z = \lambda / 4$$

(a) Find the magnetic field $\bar{H}_1$ generated from the source-1 at $z = 0$.

(b) Find the magnetic field $\bar{H}_2$ generated from the source-2 at $z = \lambda / 4$.

(c) Find the total electric field $\bar{E}$ $(= \bar{E}_1 + \bar{E}_2)$ and its polarization sense (linear/circular? right/left handed?) in the regions of $z < 0$ and $z > \lambda / 4$, respectively.

(d) If you are free to change the current sources, how do you set up the sources such that the radiated E-fields are left-handed circularly polarized in the $z > \lambda / 4$ region, while there are no fields in the $z < 0$ region?