Electromagnetics I　　　Midterm Examination　　　4/25/2005

Duration: 100 minutes

Close Book

Dictionary and Calculator Allowed

$\nabla\times H = J + \frac{\partial D}{\partial t}$

If it appears that necessary information is missing in a problem, make a reasonable assumption and state the assumption.

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2\times\frac{\sqrt{3}}{2}\right)$　$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \sqrt{3}\right)$

Honor code: I neither receive nor give any assistance during the exam. (Write the code on your solution sheets and sign your name next to it.)

$r\sin\theta\cos\phi,\ r\sin\theta\sin\phi,\ r\cos\theta$

BEHD

$2\times\frac{1}{2}\times\frac{\sqrt{2}}{2},\ 2\times\frac{1}{2}\times\frac{\sqrt{2}}{2}$　$\left(1, \frac{\pi}{4}, \sqrt{3}\right)$

Problem 1. (15 %)

In spherical coordinates, a vector is expressed as $\bar{A} = 2\hat{a}_\theta$ at the position $r = 2, \theta = \pi/6, \phi = \pi/4$. Convert the vector and position to one in Cartesian coordinates and one in Cylindrical coordinates.

$\hat{a}_r + \cot\theta\,\hat{a}_\theta$　$\nabla\times B = J + \frac{\partial D}{\partial t}$

Problem 2. (15%)

$\nabla\cdot B = 0$

Given a vector field $\bar{A} = (1/r)\hat{a}_r + \cot\theta\,\hat{a}_\theta$ in spherical coordinates,

$\nabla\times E = -\frac{\partial B}{\partial t}$

(a) Find the condition that satisfies $\nabla\cdot\bar{A} = 0$. (see the useful formula in backside)

(b) Can it be realized as a magnetic field ? Why ?

$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & -2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$　$\hat{a}_r + \cot\theta\,\hat{a}_\theta$　$\frac{\cos\theta}{\sin\theta}$

$\nabla\cdot D = \rho$

$\nabla\cdot J = -\frac{\partial\rho}{\partial t}$

Problem 3. (25%)

A spaceship, with charge Q and constant velocity $\bar{V}$, is passing through a region of uniform electric and magnetic fields given by $\bar{E} = E_0(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$ and $\bar{B} = B_0(\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z)$ in free space.

$F = q\vec{E} + q\vec{V}\times\vec{B}$　$\frac{\sqrt{3}}{2}\times\frac{\sqrt{2}}{2}$

(a) Show that the electric and magnetic fields are perpendicular to each other.

(b) Is it possible for the spaceship to pass the field region with constant velocity without detouring(偏離軌道)? (Assuming steering power off during the passing)

$-\frac{4}{3}+1$

(c) If your answer in (b) is no, state the reasons and go to the next problem. If yes, how do you control the velocity $\bar{V}$ of the spaceship before entering the field region.

$\frac{4}{3}-2$

(d) Find the minimum velocity $(\bar{V})_{\min}$ in (c). (including the speed and direction)

(see the backside)

$\frac{\frac{4}{3}-2}{-2} =$　$1-r=0$　$\frac{4E_0}{3B_0} - \frac{2E_0}{B_0}$

$\oint \vec{E}\cdot d\vec{l} =$

Problem 4. (25%)

(a) A rigid rectangular loop of wire is hung by pivoting one side along the x-axis, as shown below. The loop is free to swing without friction under the influence of gravity and in the presence of a uniform magnetic field $\overline{B} = B_0\hat{z}$. The induced electromotive force (emf) around the close loop is behaving a function of time as the loop swings. Find the **emf at α=0** in terms of B_0, a, b, ω, where ω is the angular ~~velocity~~ while the loop swinging toward the vertical. Does the loop swing **faster** or **slower** than in the absence of the magnetic field? **Why** ?

slower

(b) Repeat (a), but change the magnetic field as $\overline{B} = B_0\hat{x}$.

(c) Repeat (a), but change the magnetic field as $\overline{B} = B_0\hat{y}$.

$\frac{d\alpha}{dt} = -\omega$

$\vec{F} = i\vec{l}\times\vec{B}$

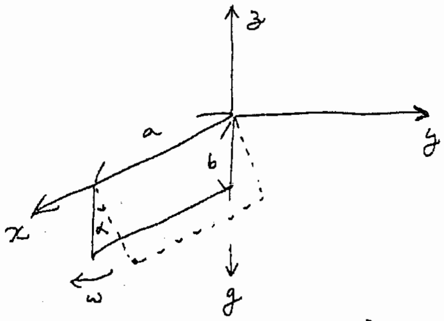

Figure of Problem 4

$\nabla\times\vec{E} = -\frac{\partial B}{\partial t}$

$\nabla\times\vec{H} = 0\,\frac{\partial D}{\partial t}$

Problem 5. (20%)

The rectangular cavity resonator is a box comprising the region of free space within $0 < x < a$, $0 < y < b$, and $0 < z < d$, and bounded by metallic walls on all of its six sides. The time-varying electric field inside the resonator is given by

$$\overline{E} = E_0 \sin\frac{\pi x}{a}\sin\frac{\pi z}{d}\cos\omega t\,\hat{a}_y$$

where E_0 is a constant and ω is the radian frequency of oscillation. Find the value of ω that satisfies both of Maxwell's curl equations (Ampere's and Faraday's Laws). What do you find the physical meaning of ω in this problem ?

$\frac{1}{r^2} - \frac{1}{r} = 0$

$1 - r = 0$

$D = \varepsilon_0 E$

Useful formula:

$$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

in spherical coordinates.

$\frac{1-r}{r^2} = 0$

$r = 1$

$$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\cos\theta)$$