Department of Electrical Engineering, National Taiwan University

Engineering Mathematics-Differential Engineering, 2012, Fall

Final Examination

2013/01/09 Wednesday, 10:20-12:10

1. (10 scores) Suppose that the weight function $w(x) = |x|$

(a) (5 scores) Determine the value of c_1 such that e^x and $e^x + c_1 e^{-x}$ are orthogonal with respect to $w(x)$ on the interval $[-1, 1]$

(b) (5 scores) Determine the value of c_2 such that $\exp(x^3)$ and $\exp(x^4) + c_2 \exp(x^3)$ are orthogonal with respect to $w(x)$ on the interval $[-1, 1]$

2. (10 scores)

(a) (5 scores) Find the Fourier series of $f(t)$:

$$f(t) = \begin{cases} t, & 0 < t < 1/2 \\ -1+t, & 1/2 < t < 1 \end{cases}, \qquad f(t+1) = f(t)$$

(b) (5 scores) Solve the following DE:

$$\frac{d}{dt}x(t) + 4x(t) = f(t)$$

3. (10 scores) <u>Use the Fourier transform, the Fourier sine transform, or the Fourier cosine transform</u> to solve the following PDE

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 2, \quad y > 0,$$

$$u(x, 0) = 0, \text{ for } 0 < x < 2, \quad u(0, y) = f(y), \quad u(2, y) = 0 \quad \text{for } y > 0.$$

4. (20 scores) Solve the following partial differential equations <u>by the methods of separation of variables</u>:

(a) (10 scores) $\frac{\partial}{\partial x}u = 2\frac{\partial}{\partial y}u$.

(b) (10 scores) $16\frac{\partial^2}{\partial x^2}u = \frac{\partial^2}{\partial y^2}u \quad 0 < x < 2, y > 0$

subject to $\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=2} = 0, \quad y > 0,$

$u(x,0) = x, \quad \left.\frac{\partial u}{\partial y}\right|_{y=0}, \quad 0 < x < 2,$

5. (25 scores) Please solve the given differential equation.

$xy'' + 2y' + \lambda xy = 0$

(a) (7 scores) Using the method of Frobenius to find two series solutions about the regular singular point, $x = 0$.

(b) (3 scores) Express the series solutions of (a) in terms of elementary functions. (Elementary functions are finite combinations of integer powers of x, roots, exponential and logarithmic functions, and trigonometric and inverse trigonometric functions.)

(c) (5 scores) Find two solutions in terms of Bessel functions.

($a = -\frac{1}{2}, p = \frac{1}{2}, b = \sqrt{\lambda}$) (Hint: $Y_\nu(x) = \dfrac{\cos(\nu\pi)J_\nu(x) - J_{-\nu}(x)}{\sin(\nu\pi)}$)

(d) (7 scores) Using the Laplace transform to solve above DE with initial conditions of $y(0) = 1$, and $y'(0) = 0$. ($\lambda = 1$)

(e) (3 scores) Express the series solutions of (a) in terms of real-valued elementary functions ($\lambda = -1$)

6. (15 scores) Please evaluate the following inverse Laplace transform

(a) (5 scores) $L^{-1}\left\{\dfrac{3s+1}{s^2(s+1)^3}\right\}$ (Do not evaluate the integral)

(b) (2 scores) $L^{-1}\left\{\ln\dfrac{s-3}{s+1}\right\}$

(c) (3 scores) $L^{-1}\left\{\dfrac{8k^3 s}{(s^2+k^2)^3}\right\}$

(d) (5 scores) $L^{-1}\left\{\dfrac{F(s-a)}{s-a}\right\}$ ($L^{-1}\{F(s)\} = f(t)$, Express the solution in terms of $f(t)$)

7. (5 scores) Using the method of Frobenius to find two series solutions about x = 0.

$xy'' + (1-x)y' - y = 0$

8. (5 scores) Is the function $f(t)$ of exponential order? Can you show that the Laplace transform of $f(t)$ exists?

$f(t) = 2te^{t^2}\cos(e^{t^2})$

- **期末考成績預計公告時間：1/15（二）中午 12: 00 公告於二館助教公布欄**
- **期末考預計看考卷時間：1/15（二）下午 16: 00-17: 30 在博理 113 教室**
- **如有更改，將另行公告於二館助教公布欄與 ptt 電機系功課板。**