# Complex Analysis: Quiz 3

10:20 AM - 11:20 AM, June 9, 2009.

[1] (20 %) Apply the residue theorem to find the inverse Laplace transform of

$$\frac{e^{-3s}}{(s^2+1)(s^2+4)}$$

[2] (20 %) Let $f$ be an analytic function that maps a domain $D$ onto a domain $D'$. If $U$ is harmonic in $D'$, then the real-valued function $u(x,y) = U(f(z))$ is harmonic in $D$. Prove it.

[3] (15 %) Find the image curve in the $w$-plane of curve $e^x \cos y = 1$ under $w = e^z$.

[4] (15 %) Determine where the given complex mapping

$$f(z) = \pi i - \frac{1}{2}[\mathrm{Ln}(z+1) + \mathrm{Ln}(z-1)]$$

is conformal.

[5] (15 %) Find the image of disk $|z| \leq 1$ mapped by the bilinear transformation

$$T(z) = \frac{z+1}{z-1}$$

[6] (15 %) Find $f'(z)$ that maps $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$ to the four corners of a square in the $w$-plane at $w_1 = 0$, $w_2 = 1$, $w_3 = 1+i$, and $w_4 = i$, respectively.