Signals and Systems Midterm

9:10a.m. ~ 11:10p.m., May 3, Mon., 2010

- Closed book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120
- Total 4 pages in one B4 sheet

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1. Find the even and odd components of the following signals.

   (a) [3] $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

   (b) [3] $x(t) = 5\cos(3t) + \sin(3t - \frac{\pi}{2})$

2. For each of the following signals, determine whether it is periodic, and if it is, find the fundamental period.

   (a) [2] $x(t) = \sin^3(2t)$

   (b) [2] $x[n] = \cos(2n)$

   (c) [2] $x(t) = te^{\sin(t)}$

   (d) [2] $x(t) = e^{-j10t} + e^{j15t}$

   (e) [2] $x[n] = \cos(\frac{\pi}{8}n^2)$

3. [5] Assume that an real-valued continuous-time signal is expressed as

$$x(t) = x_e(t) + x_o(t),$$

where $x_e(t)$ and $x_o(t)$ are, respectively, the even and odd components of $x(t)$.
Show that the energy of the signal $x(t)$ is equal to the sum of the energy of the even component $x_e(t)$, and the energy of the odd component $x_o(t)$. That is, show that

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^{\,2}(t)dt + \int_{-\infty}^{\infty} x_o^{\,2}(t)dt$$

4. [8] Suppose the input $x(t)$ and impulse response $h(t)$ of an LTI system are, respectively, given by

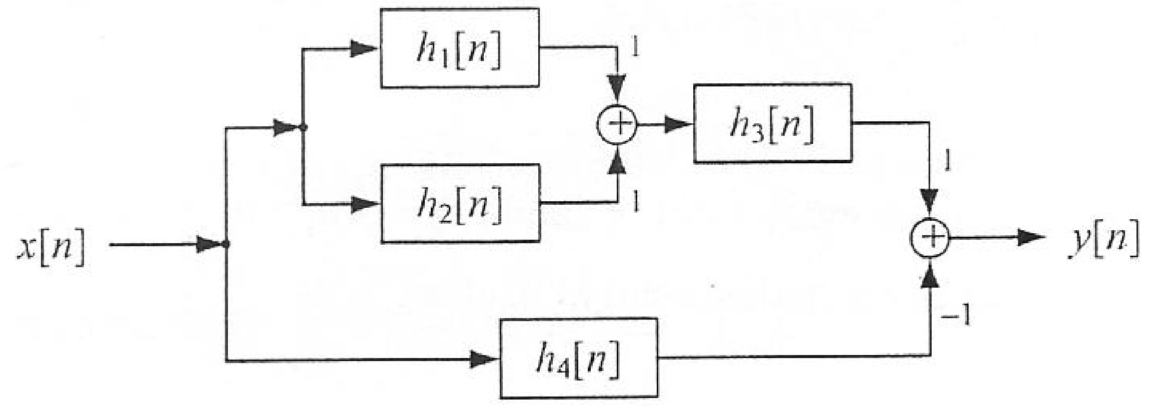
$$x(t) = (t-1)[u(t-1) - u(t-3)] \quad \text{and} \quad h(t) = u(t+1) - 2u(t-2).$$

Find the output of the system.

5. [6] Consider the interconnection of four LTI systems, as depicted in the following figure. The impulse response of these systems are

$$h_1[n] = u[n]$$
$$h_2[n] = u[n+2] - u[n]$$
$$h_3[n] = \delta[n-2]$$
$$h_4[n] = a^n u[n]$$

Find the impulse response h[n] of the overall system.

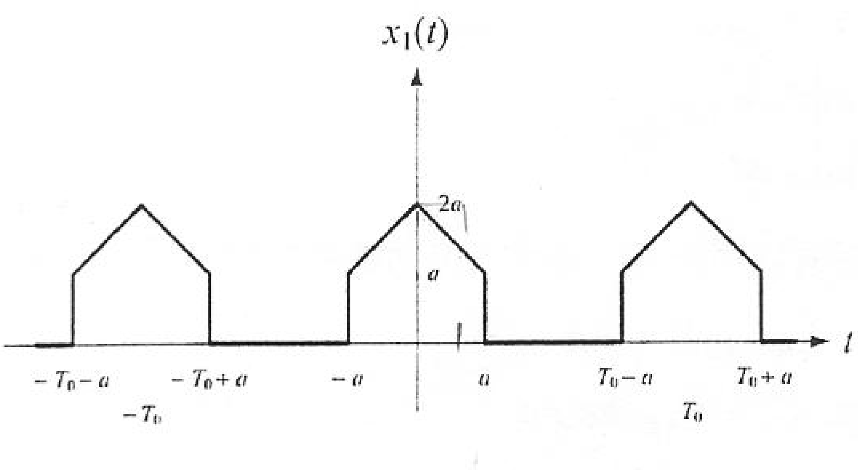


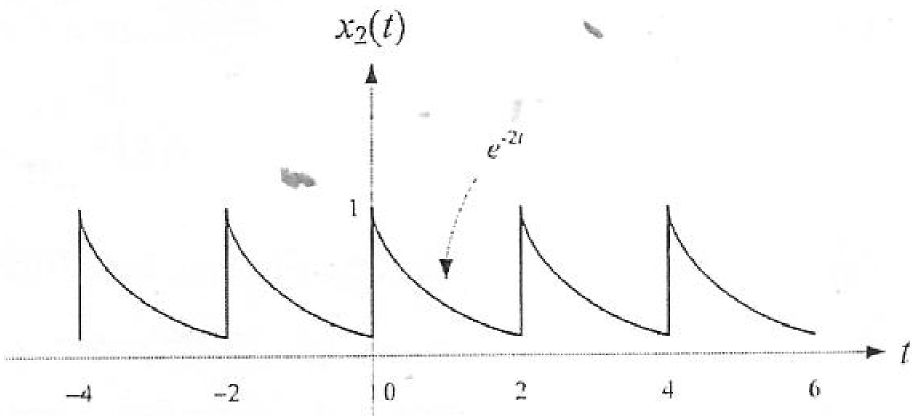
6. [6] Consider an LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation

$$\frac{dy(t)}{dt} + 4y(t) = x(t).$$

If $x(t) = e^{-t}[\cos(3t) + j\sin(3t)]u(t)$, find the output $y(t)$.

7. [8] Determine the Fourier series coefficients of the following periodic signals.





8. [8] Find the time-domain signal $x(t)$ corresponding to the following Fourier series coefficient

$$a_k = \left(\frac{1}{2}\right)^{|k|} e^{\frac{jk\pi}{20}}$$

in the following form

$$x(t) = \frac{F + G\sin(Ht + I)}{A + B\cos(Dt + E)}$$

where $A, B, \ldots, I$ are some constants and assume that the fundamental period is $T = 2$.

NOTE: There are problems in the back.

9. [6] Define the trigonometric Fourier series pair

$$x(t) = B_0 + \sum_{k=1}^{\infty}\left[B_k \cos(k\omega_0 t) + A_k \sin(k\omega_0 t)\right]$$

$$B_0 = \frac{1}{T}\int_0^T x(t)dt, \quad B_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt \text{ for } k > 0$$

$$A_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt$$

Show that

$$B_k = a_k + a_{-k} \text{ for every valid } k\text{, and } A_k = j(a_k - a_{-k}) \text{ for } k \neq 0$$

where a_k are Fourier series coefficients taught in class.

10. Solve the following problems.

(a) [4] Use the Fourier transform analysis equation to calculate the Fourier transform of

$$x(t) = e^{-2|t-1|}$$

(a) [4] Use the Fourier transform synthesis equation to calculate the inverse Fourier transform of

$$X(j\omega) = \begin{cases} 2 & 0 \le \omega \le 2 \\ -2 & -2 \le \omega < 0 \\ 0 & |\omega| > 2 \end{cases}$$

11. Let

$$x[n] = \begin{cases} 1 & |n| \le M \\ 0 & |n| > M \end{cases}$$

(a) [2] Find the discrete-time Fourier transform $X(e^{j\omega})$ of $x[n]$.

(b) [4] Calculate the values of $X(e^{j\omega})$ for $\omega = 2\pi$ and -4π.

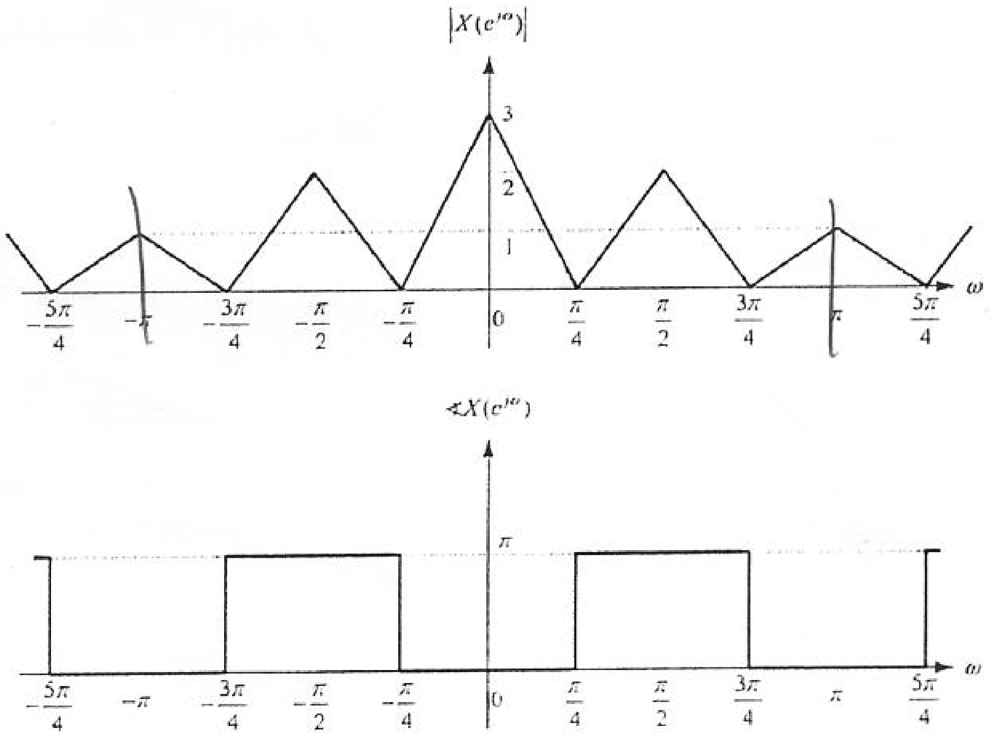
(c) [4] Calculate all the possible values ω such that $X(e^{j\omega}) = 0$.

12. For the right figure showing the frequency-domain representation $X(e^{j\omega})$, determine whether the time-domain signal $x[n]$ is

(a) [3] real or complex-valued,

(b) [3] even or odd, and

(c) [3] periodic or aperiodic.



13. Consider the following causal system

$$y[n] = x[n] + ax[n-1], \quad |a| < 1.$$

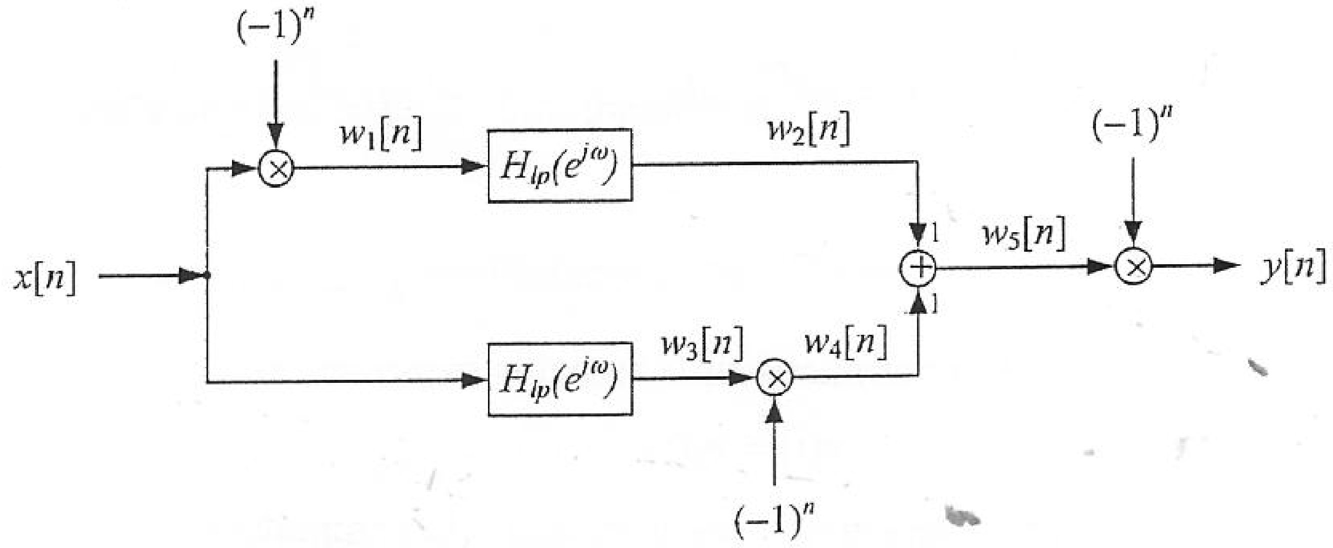
(a) [3] Find the impulse response $h[n]$ of the system.

(b) [3] Find the impulse response $h_i[n]$ of the inverse system of the system, where the inverse system will recover $x[n]$ from $y[n]$.

14. [12] Determine the Fourier transform $G(j\omega)$ of the Gaussian pulse function $g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$.

(Hint: $g(t)$ satisfies the facts $\frac{d}{dt} g(t) = -tg(t)$ and $\int_{-\infty}^{\infty} g(t)dt = 1$.)

15. Consider the system shown in the following figure with input $x[n]$ and output $y[n]$. The LTI systems with $H_{lp}(e^{j\omega})$ are ideal low-pass filters with cutoff frequency $\frac{\pi}{4}$ and unity gain in the passband.



(a) [6] Compute and plot the frequency response $H(e^{j\omega})$ of the overall system from $x[n]$ to $y[n]$.

(b) [6] If the frequency response $X(e^{j\omega})$ of the input $x[n]$ is shown in the following, please find the frequency response $Y(e^{j\omega})$ of the output $y[n]$.