Electromagnetics (II) Midterm Exam. May 3, 2000

1. In vacuum, a time-varying charge density $\rho(\vec{r},t)$ and a time-varying current density $\vec{J}(\vec{r},t)$ are distributed over a finite region of volume V'. Under the Lorentz condition, what are the scalar potential $V(\vec{r}_1,t_1)$ and vector potential $\vec{A}(\vec{r}_1,t_1)$ at an external point $\vec{r}_1$ on instant t_1 due to these source distributions? Write down the integral expressions of the results directly and interpret the physical meaning of time retardation briefly. (15%)

2. A time-varying electric field

$$\vec{E}(x,y,z,t) = \hat{y}\sin\left[2\pi\left(10^8 t + \frac{\sqrt{3}}{4}y - \frac{1}{4}z\right) + \frac{\pi}{3}\right] + \hat{z}\sqrt{3}\sin\left[2\pi\left(10^8 t + \frac{\sqrt{3}}{4}y - \frac{1}{4}z\right) + \frac{\pi}{3}\right]$$

exists in an infinitely extended dielectric medium of permittivity $\varepsilon_0\varepsilon_r$ and permeability μ_0. Here all the quantities are in SI units.
(a) Show that the $\vec{E}$-field propagates as a uniform plane wave.
(b) Find the phase velocity of this uniform plane wave.
(c) Find the dielectric constant ε_r.
(d) Find the unit vector indicating the direction of wave propagation.
(e) Describe the polarization of the wave.
(f) Find the corresponding $\vec{H}$-field. (18%)

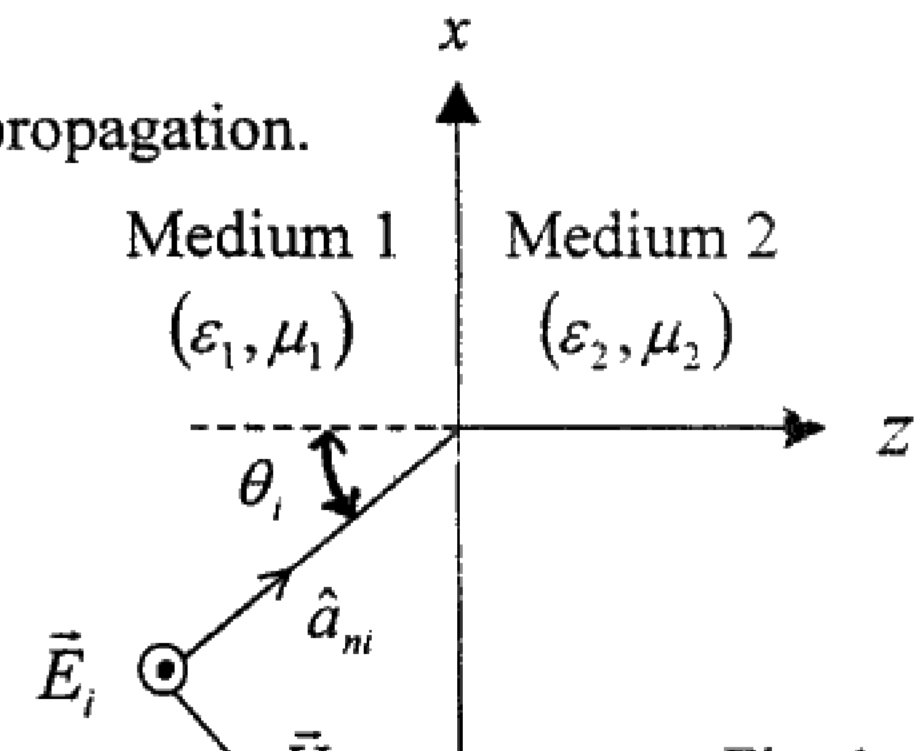

Fig. 1

3. Consider the x-y plane as the interface between two nonconducting media 1 and 2, as shown in Fig. 1. A plane wave with the electric field vector expressed as $\vec{E}_i(x,z) = \hat{y}E_{io}e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$ is incident upon the interface with an angle of incidence θ_i.
(a) Give an expression for its magnetic field intensity vector. (2%)
(b) Give expressions for the electric field intensity and the magnetic field intensity vectors of the reflected wave. (4%)
(c) Give expressions for the electric field intensity and the magnetic field intensity vectors of the transmitted wave. (4%)
(d) Derive the expressions for the reflection and transmission coefficients. (10%)
(e) If $\mu_1 = \mu_2$, please discuss if it is possible that the transmission coefficient is unity. (5%)
(f) Let $\mu_1 = \mu_2$, $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = \varepsilon_0$ and $\theta_i = 60°$, please give an expression for the magnitude of the transmitted electric field as a function of z. (5%)
(g) What is the phase velocity of the transmitted wave for the case in part (f)? (3%)

4. Four lossless transmission lines are connected as shown in Fig. 2 with a $100\,\Omega$ resistor termination. Find lengths L1, L2 and L3 for a matched condition at AA' points. (15%)

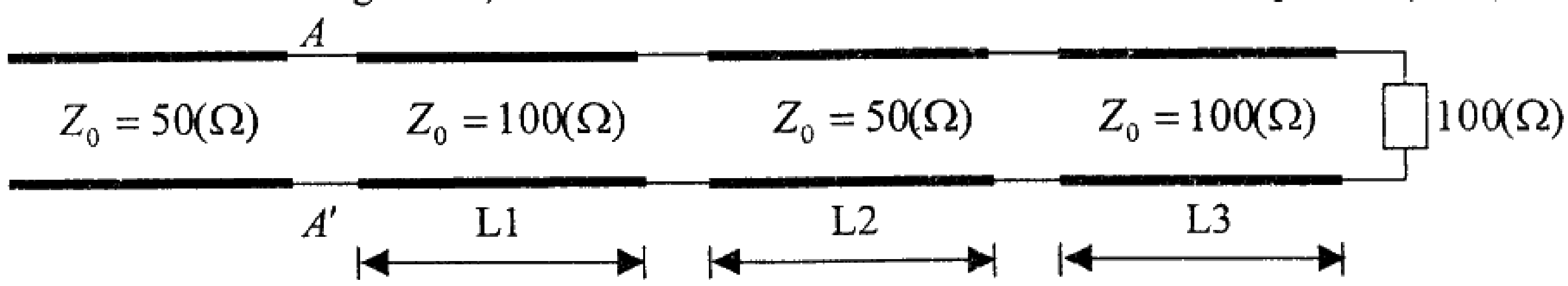

Fig. 2

5. The standing-wave ratio on a lossless $50(\Omega)$ transmission line terminated in an unknown load impedance is 2.0, and the nearest voltage minimum is at a distance 0.125λ from the load. Determine
(a) the reflection coefficient Γ of the load (9%)
(b) the unknown load impedance Z_L (10%)