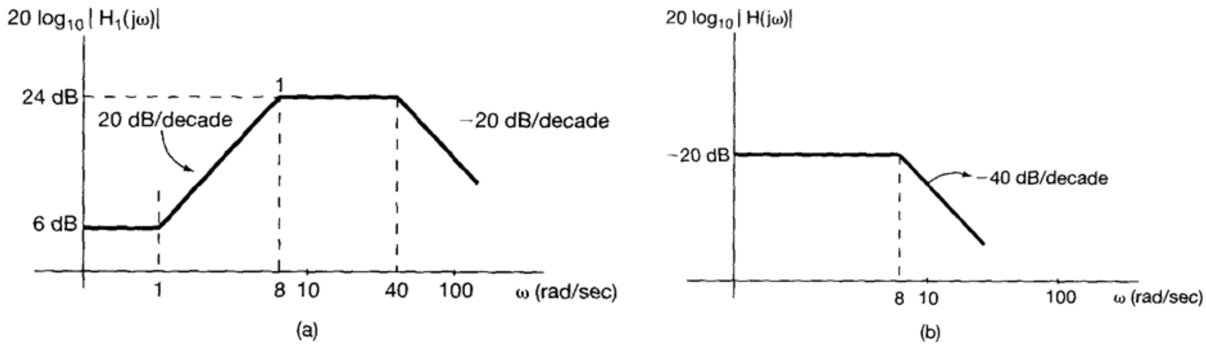
Signals and Systems Final Exam

9:20a.m. ~ 11:20a.m., June 22, Fri., 2007

- Closed book, but open 1 sheet (both sides, 2 pages) of personal notes of A4 size
- Total score: 120
- Total 3 pages in one B4 sheet

---

1. **[12]** A continuous-time LTI system with frequency response $H(j\omega)$ is constructed from two continuous-time LTI systems with frequency responses $H_1(j\omega)$ and $H_2(j\omega)$, respectively. The straight-line approximations of the Bode magnitude plots of $H_1(j\omega)$ and $H(j\omega)$ are shown in the following figure. All of the poles and zeros of $H_1(s)$ and $H(s)$ are on the real axis.

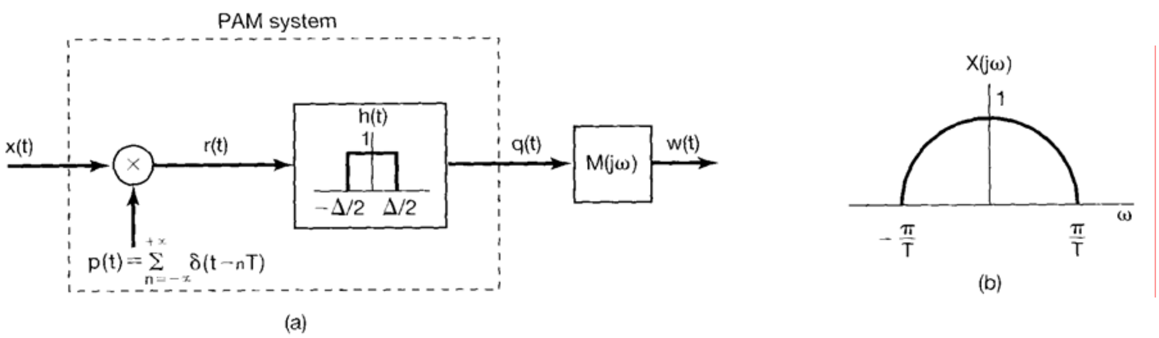


a) **[4]** Specify $H_2(j\omega)$ if $H_1(j\omega)$ and $H_2(j\omega)$ are connected in cascade form.

b) **[4]** Specify $H_2(j\omega)$ if $H_1(j\omega)$ and $H_2(j\omega)$ are connected in parallel form.

c) **[4]** Specify $H_2(j\omega)$ if $H_1(j\omega)$ and $H_2(j\omega)$ are connected in negative feedback form with $H_2(j\omega)$ in the feedback loop.

2. **[12]** Figure (a) below shows a model of amplitude modulation system using a pulse-train carrier.



a) **[4]** Let $x(t)$ be a band-limited signal with $X(j\omega)=0$ for $|\omega| \ge \pi/T$, as shown above. Determine and sketch $R(j\omega)$ and $Q(j\omega)$.

b) **[4]** Find the maximum value of Δ such that $w(t)=x(t)$ with an appropriate filter $M(j\omega)$.

c) **[4]** Determine and sketch the compensating filter $M(j\omega)$ such that $w(t)=x(t)$.

3. **[8]** Determine $X(s)$ and its region of convergence based on the following 5 descriptions about a signal $x(t)$, which is real, and its Laplace transform $X(s)$:

   i) $X(0) = 4$

   ii) $X(s)$ has exactly two poles

   iii) $X(s)$ has no zeros in the finite s-plane

   iv) $X(s)$ has a pole at $s = j - 2$

   v) $e^{3t}x(t)$ is absolutely integrable

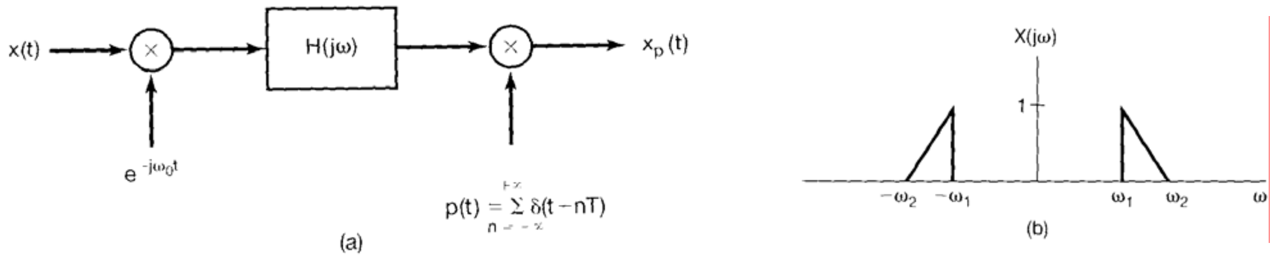
4. **[12]** Consider an LTI system with $H(z) = \dfrac{64}{49z^{-2} + 64}$, $|z| > \frac{7}{8}$.

   a) **[4]** Use geometric evaluation of the magnitude of the Fourier transform from the pole-zero plot to determine if the system is approximately lowpass, bandpass, or highpass.

   b) **[4]** Determine if the system is causal.

   c) **[4]** Determine $h[n]$.

5. **[16]** The sampling theorem states that a signal must be sampled at a rate larger than twice its highest frequency. For a bandpass signal with its energy concentrates in a narrow band, however, it is possible to have a sampling rate that is lower than twice the highest frequency of the signal to be sampled. To examine the possibility, let us apply the system shown in Figure (a) below to sample and reconstruct a bandpass signal $x(t)$, which is real, with $X(j\omega)$ nonzero only for $\omega_1 < |\omega| < \omega_2$, as shown in Figure (b) below. The system consists of multiplying the signal $x(t)$ by a complex-exponential $e^{-j\omega_0 t}$, sending the product to an ideal lowpass filter $H(j\omega)$, and then sampling the filtered signal.



(a) (b)

Suppose $\omega_1 > \omega_2 - \omega_1$ and the cutoff frequency of $H(j\omega)$ is $(\omega_2 - \omega_1)/2$.

   a) **[4]** Determine the frequency ω_0 for the complex exponential.

   b) **[4]** Determine the maximum sampling period T such that $x(t)$ is recoverable from $x_p(t)$.

   c) **[4]** Sketch $X_p(j\omega)$.

   d) **[4]** Determine a system to recover $x(t)$ from $x_p(t)$.

NOTE: There are problems in the back.

6. **[10]** Assume that the relationship of a continuous-time LTI system is given by

$$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+5y(t)=\frac{d^2x(t)}{dt^2}-2\frac{dx(t)}{dt}+x(t)$$

a) **[5]** Find the system function $H(s)$ of the continuous-time LTI system.

b) **[5]** Does this system have a stable and causal inverse? Why or why not?

7. **[14]** Assume that a discrete-time LTI system has the input and output related by the following difference equation: $y[n]-0.75y[n-1]+0.125y[n-2]=x[n]$.

a) **[8]** Find $y[n]$ by using z-transform when $x[n]=1$ for $n=0$ and $x[n]=0$ for $n\neq 0$.

b) **[6]** Verify the value of $y[0]$ in part a) by using the initial-value theorem.

8. **[16]** Consider a discrete-time LTI system with transfer function $H(z)=\frac{1-a^*z}{z-a}$, $|a|<1$, where a^* represents the complex conjugate of a.

a) **[4]** Sketch the pole-zero plot of $H(z)$ in the z-plane.

b) **[6]** Is $H(z)$ stable and causal? Why?

c) **[6]** Use the graphic method to show what the magnitude response of the system is.

9. **[10]** Consider a time-division multiplexing (TDM) system with a sampling rate of 3×10^6 Hz. The baseband signals $x_1(t)$, $x_2(t)$, …, $x_N(t)$ to be multiplexed are converted to pulse-amplitude modulation (PAM) signals, $y_1(t)$, $y_2(t)$, …, $y_N(t)$, respectively.

a) **[5]** Please state the principle of TDM.

b) **[5]** Let the pulses be 8×10^{-6} seconds in duration. Determine the maximum value of N to ensure an appropriate TDM operation.

10. **[10]** Consider a frequency-division multiplexing (FDM) system of discrete-time signals $x_i[n]$, $i=1, 2, 3, 4$. Let each $x_i[n]$ occupy the entire frequency band ($-\pi\le\omega\le\pi$).

a) **[5]** Please state the principle of FDM.

b) **[5]** Let each signal $x_i[n]$ be upsampled and then modulated with $\cos\left(\frac{i\pi n}{4}\right)$. Determine the minimum amount of upsampling that can be performed on each $x_i[n]$.