DIFFERENTIAL EQUATION MIDTERM

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Total=105 pts Nov 21, 2002

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1. (27 %) Solve the differential equations

   (a) $e^x y^2 \frac{dy}{dx} = e^{-y} + e^{-2x-y}$ (5%)

   (b) $2xy\frac{dy}{dx} + 3x + 2y^2 = 0$ (8%)

   (c) $\frac{dy}{dx} = \sin(2x+y)$ (7%)

   (d) $xy' = y\ln(xy)$ (7%)

2. (8%) If $y_1 = \frac{2}{x}$ is a known solution of the differential equation $\frac{dy}{dx} = -\frac{4}{x^2} - \frac{y}{x} + y^2$
Find a one-parameter family of solutions for the differential equation.

3. (12%) Find the general solution for the following differential equation.

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \frac{\ln(x^3)}{x}$$

4. (7%) An LR series circuit has a variable inductor with the inductance defined by

$$L = \begin{cases} 1 - \frac{t}{10} & \ldots\ldots\ldots 0 \le t < 10 \\ 0 & \ldots\ldots\ldots\ldots\ldots t \ge 10 \end{cases}$$

Find the current i(t) if the resistance is 0.2 ohm, the impressed voltage is E(t)=4 and i(0)=0.

5. (23%) Suppose $f_1(x) = x^2$ and $f_2(x) = x - 1$.
   (a) Please find an operator L_1 which can annihilate $f_1(x)$ and $f_2(x)$ simultaneously and find the general solution of $L_1 y(x) = 0$. (4%)
   (b) If $f_1(x)$ and $f_2(x)$ are two independent solutions of some differential equation and their linear combination is also a solution, please find the associated differential equation $L_2 y(x) = 0$. (7%)
   (c) Please find a finite range where the solution subject to two initial conditions of $L_2 y(x) = 0$ is unique. (5%)
   (d) If the coefficient of the highest derivative in $L_2 y(x)$ is 1, please find a particular solution of $L_2 y(x) = x(x-2)\sin x$. (7%)

6. (10%) The potential V(r) in the coaxial cable showing in Figure1 is determined from the boundary-value problem:

$$r\frac{d^2V}{dr^2}+\frac{dV}{dr}=0, \qquad a<r<b,\ V(a)=V_1,\ V(b)=V_2.$$

Find the solution of $V(r)$

7. (18%)

(a) Show that the solution of the initial-value problem

$$\frac{d^2x}{dt^2}+\omega^2x=2\cos^2(\gamma t)-1, \qquad x(0)=0,\ x'(0)=0$$

is $x(t)=\frac{-2}{\omega^2-4\gamma^2}\sin\left[(\gamma-\frac{\omega}{2})t\right]\sin\left[(\gamma+\frac{\omega}{2})t\right]$

(Hint: $\sin u*\sin v=\frac{1}{2}[\ \cos(u-v)-\cos(u+v)\ ]$) (8%)

(b) If we define $\varepsilon=\gamma-\frac{\omega}{2}$, show that when ε is small an approximation solution is $X(t)=\frac{1}{4\varepsilon\gamma}\sin(\varepsilon t)\sin(2\gamma t)$ (7%)

(c) Find the dash curve (or envelope) E(t) of the graph of x(t) in Figure 2 (3%)

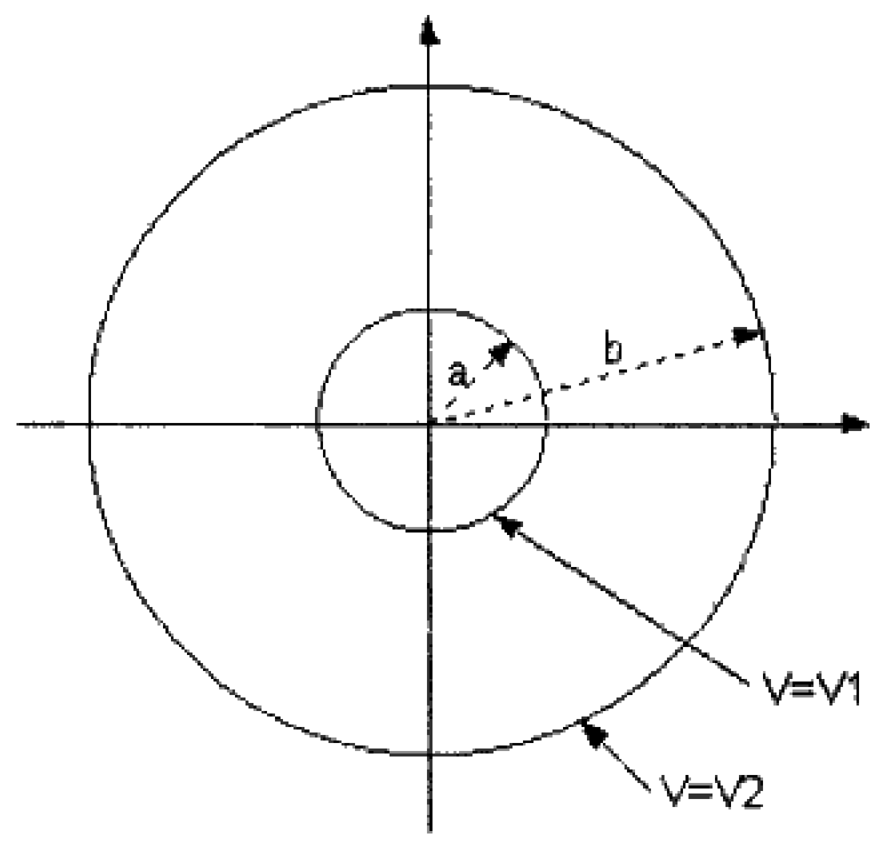

Figure 1

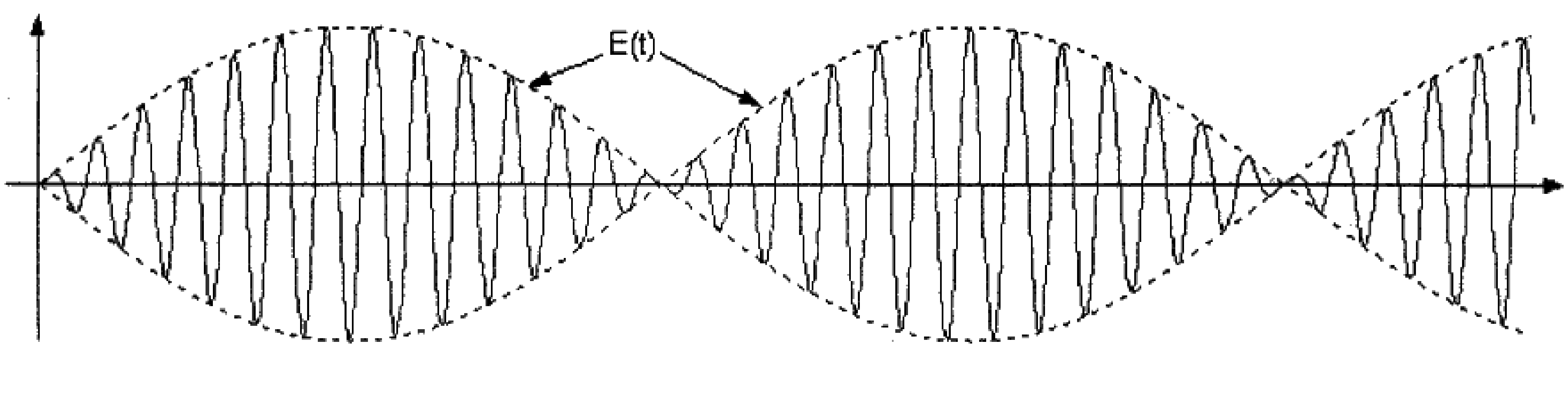

Figure 2