

工程數學-複變  
期中考  
共 100 分  
考試時間 100 分鐘

4/22/03

1. (10%) Suppose  $u$  and  $v$  are the harmonic generation function forming the real and imaginary parts of an analytic function. Show that the level curves  $u(x,y)=c_1$  and  $v(x,y)=c_2$  are orthogonal.
2. (10%) Calculate  $\tan^{-1} 3i$
3. (10%) Compute all values of  $3^i$ .
4. (10%) Find the fourth root of  $2+i$ .

5. Evaluate  $\int_{1-i}^{1+\sqrt{3}i} \left( \frac{1}{z} + \frac{1}{z^2} \right) dz$  for

(a) (10%)  $C$  is any contour in the right half plane  $\text{Re}(z) > 0$ .

(b) (10%)  $C$  is the composed by two straight line segments between  $(1-i, -1)$  and  $(-1, 1+\sqrt{3}i)$ .

6. (15%) Evaluate  $\oint_C \left( \frac{\cosh z}{(z-\pi)^3} - \frac{\sin^2 z}{(2z-\pi)^3} \right) dz$ , where  $C$  is a simple closed contour

with positive direction along the circle  $|z|=3$ .

7. (10%) Find the circle and radius of convergence of the power series of

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k 2^k} (z-1-i)^k.$$

8. Suppose the principle branch of the logarithm  $f(z) = \text{Ln}(z) = \log_e |z| + i \text{Arg}(z)$  is expanded in a Taylor Series with center  $z_0 = -1+i$ .

(a) (5%) What is the radius of the largest circle centered at  $z_0 = -1+i$  within which  $f(z) = \text{Ln}(z) = \log_e |z| + i \text{Arg}(z)$  is analytic?

(b) (5%) Show that within the circle, the Taylor series for  $f(z) = \text{Ln}(z) = \log_e |z| + i \text{Arg}(z)$  is

$$\text{Ln}(z) = \frac{1}{2} \log_e 2 + \frac{3\pi}{4} i - \sum_{k=1}^{\infty} \frac{1}{k} \left( \frac{1+i}{2} \right)^k (z+1-i)^k.$$

(c) (5%) What is the radius of convergence for the power series in part (b)?