A. Definitions: write down the definitions or the theorems and answer the questions. (10%)

1. State the definition of a residue.
2. What is line-circle-preserving property in linear fractional transformation?

B. True or false (If it is false, explain briefly why it isn't true). (15%)

1. The only possible singularities of a rational function are poles.
2. The function $f(z)=(z+5)/(z\sin 3z)$ has a pole of order 4 at $z=0$.
3. If $f'(z)=z^{-1/2}(z+1)^{-1/2}(z-1)^{-1/2}$, then $w=f(z)$ maps the upper half-plane $y>0$ onto the interior of a rectangle.

$z(z-1)$ $\frac{z-5}{z(z-2+\frac{5}{z})}$

C. Find the Laurent series of $f(z)=\dfrac{z-5}{z(z^2-2z+5)}$ centered at $z=1$, in the following domains: (10%)

1. $|z-1|<1$
2. $1<|z-1|<2$
3. $|z-\cancel{2}|>2$ 1

$z=1\pm 2i$

$\frac{z-5}{z(z-1+2i)(z-1-2i)}$

D. Evaluate the following: (30%)

1. $\displaystyle\oint_C \frac{1}{(z-\pi i)(e^z-1)}dz$, C: $|z|=4$
2. $\displaystyle\oint_C \cot z\,dz$, C is the circle $|z-3|=2$
3. $\displaystyle\oint_C \frac{z+2}{\sinh z}dz$, C is the rectangle defined by $x=-1$, $x=1$, $y=4$, $y=-1$.
4. $\displaystyle\int_0^{2\pi}\frac{\sin^2\theta}{2+\cos\theta}d\theta$
5. Evaluate P.V. $\displaystyle\int_{-\infty}^{\infty}\frac{x^2}{(x^2-2x+2)(x^2+2x+2)}dx$

$= \frac{-4+2i}{(1+2i)(4i)}$ $= \frac{-4+2i}{4i-8}$ $\frac{1-2i-5}{(1-2i)(-4i)}$ $= \frac{-4+2i}{-4i+8}$ $= \frac{(2+i)(4-2i)}{4+2i}$ $= \frac{8-4i+4i+2}{20}$ $= \frac{1}{2}$

E. Use residue theorem to solve $L^{-1}\left\{\dfrac{e^{-2s}}{(s-1)(s-3)}\right\}$ for $\mathrm{Re}\{s\}>3$ (10%)

F. Evaluate P.V. $\displaystyle\int_0^{\infty}\frac{1-\cos x}{x^2}dx$ (15%)

G. Use appropriate conformal mapping to solve the given Dirichlet problem in Fig. 1.

$$\begin{cases} \left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)\varphi(x,y)=0 \text{ in } D \\ \varphi(x,y)=1, \text{ on } B_1,\ \varphi(x,y)=0, \text{ on } B_2 \end{cases}$$

D is the interior shaded area, and the boundary circles are $B_1=\{z \,\|z+1|=1\}$ and $B_2=\left\{z\,\left\|z+\dfrac{1}{2}\right|=\dfrac{1}{4}\right\}$

(10%)

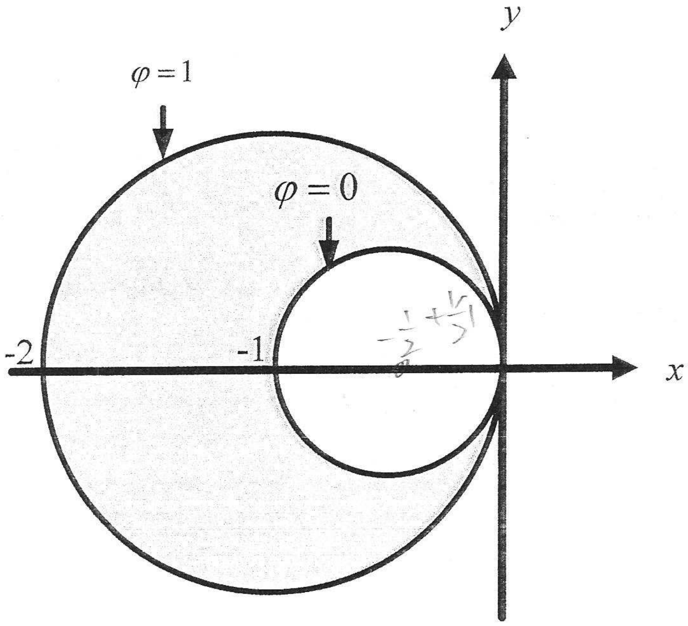

Fig. 1