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Electromagnetics (I) Midterm Examination April 20, 2012

1. The forces experienced by a test charge q at a point in a region of electric and magnetic fields $\vec{E}$ and $\vec{B}$, respectively, are given as follows for three different velocities of the test charge, where v_0 and E_0 are constants. Please find $\vec{E}$ and $\vec{B}$ at that point. (10%)

$$\vec{F}_1 = qE_0(\hat{a}_x - \hat{a}_y - \hat{a}_z), \quad \text{for } \vec{v}_1 = v_0\hat{a}_x$$
$$\vec{F}_2 = qE_0(\hat{a}_x - \hat{a}_y + \hat{a}_z), \quad \text{for } \vec{v}_2 = v_0\hat{a}_y$$
$$\vec{F}_3 = \vec{0}, \quad \text{for } \vec{v}_3 = v_0\hat{a}_z$$

2. In Fig. 2, a cylindrical capacitor consists of an inner conductor of radius a and an outer conductor of radius b. The space between the conductors is free space and the length of the capacitor is L. Fringe effect can be neglected for $L >> a,b$.

(1) By applying a DC voltage V_0, there will be a charge $+Q$ on the inner cylinder and $-Q$ on the outer cylinder, please solve the $\vec{E}$ field in terms of Q for $a < r < b$. (5%)

(2) Following (1), please calculate $\nabla \bullet \vec{E}$ and $\nabla \times \vec{E}$ for $a < r < b$ and explain the results. (5%)

(3) Following (1), please calculate the voltage from the inner cylinder to the outer cylinder, and show that V_0 is independent of the path of integration. (5%)

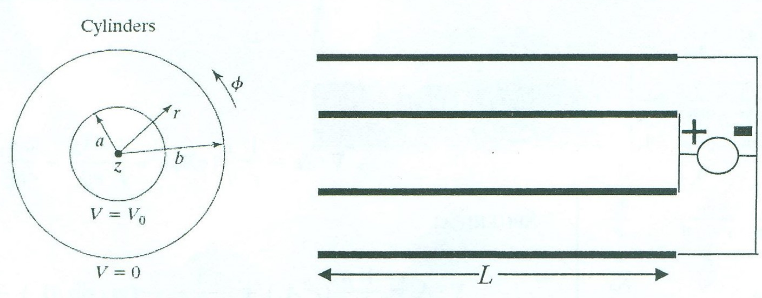
(4) Following (3), please calculate the capacitance by using $C=Q/V$. (5%)

(5) By applying a AC voltage $V_0(t) = \sin(2\pi \cdot 10^6 t)\sin(4\pi \cdot 10^6 t)$, there will be a charge $+Q(t)$ on the inner cylinder and $-Q(t)$ on the outer cylinder. Please follow the similar derivation of (1) and (3) for the relation between $Q(t)$ and $V_0(t)$ by assuming the voltage is independent of the path for integration of $\vec{E}$. (5%)

(6) Following (5), please calculate the capacitance by using $I = \dfrac{dQ}{dt} = C\dfrac{dV}{dt}$ and show that the result is the same as that in (4). (5%)

(7) Following (6), please calculate the root-mean-square value of current drawn from the voltage source. (5%)

(8) The solution in (5)-(6) is directly adopted from the electrostatic case in (1)-(4). Please explain what's questionable about the solution and under what condition it can be applied? (5%)



Top View Side View **Fig. 2 for Problem 2**

3. Current I flows along a straight wire from a point charge $Q_1(t)$ located at the origin to a point charge $Q_2(t)$ located at (0, 0, 2).

(1) Find <u>the line integral of $\vec{H}$ along the square closed path C</u> having the vertices at (2, 2, 0), (-2, 2, 0), (-2, -2, 0), and (2, -2, 0) and traversed in that order. Please solve it by using Ampere's law and considering the **plane** surface S bounded by C except for a slight **upward** bulge at the origin to avoid $Q_1(t)$ as shown in Fig. 3. (Express the answer in terms of I.) (5%)

(2) Redo (1) by considering the **plane** surface S bounded by C except for a slight **downward** bulge at the origin to avoid $Q_1(t)$. (5%)

(3) If $Q_2(t)$ is moved to infinity along z-axis, how does the answer in (1) change? (5%)

(4) If $Q_2(t)$ is slowly moved to the origin (keeping I constant) along z-axis, how does the answer in (1) gradually change? (5%)

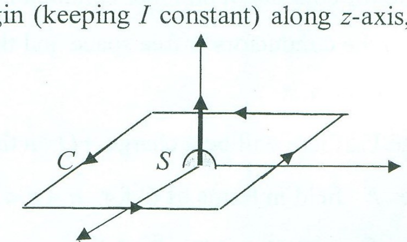


Fig. 3 for Problem 3

4. For the electric field $\vec{E} = E_0 e^{-kx}\cos(2\cdot 10^8 t - y)\hat{a}_z$ in free space ($\vec{J} = 0$), please find

(1) The value(s) of k for which the field satisfies both of Maxwell's curl equations. (10%)

(2) The magnetic field $\vec{H}$. (5%)

5. An infinite plane sheet lying in the $z = 0$ plane in free space carries a surface current of density $\vec{J}_s = -J_s(t)\hat{a}_y$ where $J_s(t)$ is as shown in Fig. 5. Find and sketch

(1) $E_y(t)$ for $z = 300\ m$ plnae. (5%)

(2) $E_y(z)$ for $t = 2\ \mu s$. (5%)

(3) $H_x(z)$ for $t = 4\ \mu s$. (5%)

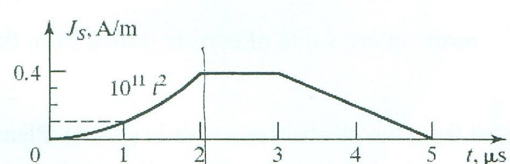


Fig. 5 for Problem 5

CYLINDRICAL

$$\nabla \times \mathbf{A} = \begin{vmatrix} \dfrac{\mathbf{a}_r}{r} & \mathbf{a}_\phi & \dfrac{\mathbf{a}_z}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

CYLINDRICAL

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

SPHERICAL

$$\nabla \times \mathbf{A} = \begin{vmatrix} \dfrac{\mathbf{a}_r}{r^2 \sin\theta} & \dfrac{\mathbf{a}_\theta}{r\sin\theta} & \dfrac{\mathbf{a}_\phi}{r} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

SPHERICAL

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$