線性代數期中考

89/04/22

1.(16points)True or False(2 points each)

(a) If $T:V\to W$ is linear and invertible, then $V=W$.

(b) If $T:V\to W$ is linear and $\dim(V)\geqq\dim(W)$, then $\text{nullity}(T)\geqq\dim(V)-\dim(W)$.

(c) Let $V=\{(a_1,a_2,a_3,a_4,a_5)\in R^5 \mid a_1-a_2=0, a_2-a_3=0, a_1-a_3=0\}$, then $\dim(V)=2$.

(d) $V=\{f\in P_{10}(R) \mid f(0)=0, f'(0)=0, f''(0)=0\}$ is a subspace of $P_{10}(R)$.

(e) $V=\{A\in M_{m\times n}(R) \mid A_{ij}$ are integers$\}$ is a subspace of $M_{m\times n}(R)$.

(f) $V=\{A\in M_{m\times n}(R) \mid A_{ij}$ are rational numbers$\}$ is a subspace of $M_{m\times n}(R)$.

(g) If $A\in M_{m\times n}(R)$ and $A^2=0$, then $A=0$.

(h) Let $I_V:V\to V$, let β and β' be two ordered bases of V, then $([I_V]_{\beta}^{\beta'})^{-1}=[I_V]_{\beta'}^{\beta}$.

2.(10points) Give the definition of the following(2 points each):

(a) Linearly independent set of vectors

(b) Inverse of a matrix

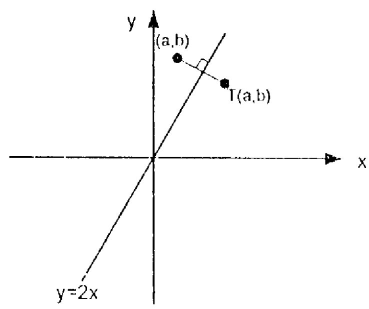
(c) Span(G), where $G\triangleq\{v_1,v_2,\ldots,v_n\}$ is a subset of a vector space V over a field F

For (d),(e), assume that $T: V\to W$ is a linear transformation, where V and W are vector spaces.

(d) Null space of T

(e) Rank of T

3.(10points) Let $T:R^2\to R^2$ denote reflection about the line $y=2x$ as shown in this figure. Find $T(a,b)$ for any $(a,b)\in R^2$



4.(10points) Let V and W be vector spaces and $T: V\to W$ be a linear transformation. Prove that T is one to one if and only if T carries linearly independent subsets of V onto linearly independent subsets of W.

5.(10points) Let $\beta\triangleq\{1-x, 1+2x\}$ be an ordered basis for $P_1(R)$. Furthermore, let $T:P_1(R)\to R^2$ be a linear transformation such that $T(1-x)=\begin{bmatrix}1\\-\frac{1}{2}\end{bmatrix}$ and $T(1+2x)=\begin{bmatrix}1\\1\end{bmatrix}$

(a) For any $f(x)\triangleq a_0+a_1x\in P_1(R)$, find $T(a_0+a_1x)$ in terms of a_0 and a_1.(5points)

(b) Let $\beta'\triangleq\{1,x\}$ and $\gamma'\triangleq\{(1,0),(0,1)\}$ be ordered bases for $P_1(R)$ and R^2, respectively. Find $[T]_{\beta'}^{\gamma'}$.(5points)

6.(10points) Let $f(x)\triangleq a_0+a_1x+a_2x^2+\ldots+a_{n-1}x^{n-1}+x^n$, where $a_0,a_1,\ldots,a_{n-1}\in R$. Prove that $\{f(x),f'(x),\ldots,f^{(n)}(x)\}$ is a basis for $P_n(R)$.

7.(15points) Let T and U be linear transformations from $M_{n\times n}(F)$ to $M_{n\times n}(F)$ defined by $T(A)=\dfrac{A+A^t}{2}$ and $U(A)=\dfrac{A-A^t}{2}$, respectively. Find the rank of T, U and T+U, respectively.

8.(10points)Let $W=\{f(x); f(x)=g(x)(x^2+1), g(x)\in P_{n-2}(F)\}$ where $\mathbb{C}$ is the set of all the complex numbers.

(a) Show that W is a subspace of $P_n(F)$.(5points)

(b) Find the dimension of W.(5points)

9.(10points)Let A and B be $n\times n$ matrices such that AB is invertible. Prove that A and B are invertible.