(2012) Microelectronics III midterm exam

date: 2012/11/06

1. (25%) The circuit shown in Fig. 1(a) can be used to approximate an inductor; assuming the opamp is ideal.
   (1) For Fig. 1(a), derive an expression for the input impedance, Z_{in}. (10%)
   (2) Z_{in} can be modeled as that shown in Fig. 1(b). Please find expressions for L, R_S, and R_P, in terms of C, R_1, and R_2. (10%)
   (3) To realize a high-Q inductor, how would you choose R_1 and R_2. You must explain your reasons. (5%)

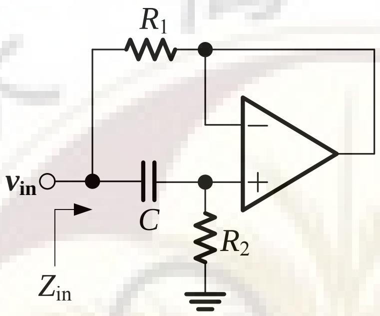


Fig. 1(a)

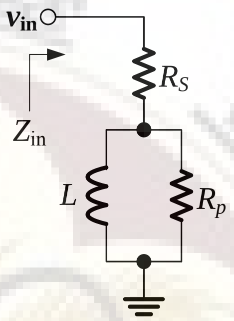
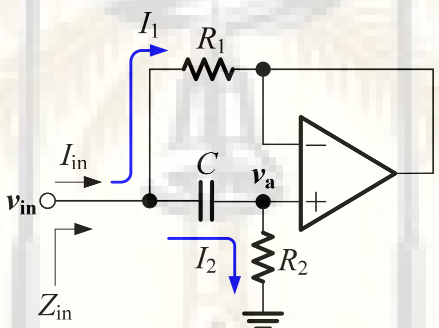


Fig. 1(b)

(Solution)

(1) For Fig. 1(a), derive an expression for the input impedance, Z_{in}. (10%)

Answer:



$$I_{in} = I_1 + I_2 = (v_{in} - v_a)\frac{1}{R_1} + v_a\frac{1}{R_2} = v_{in}\left(1 - \frac{sCR_2}{1+sCR_2}\right)\frac{1}{R_1} + v_{in}\frac{sC}{1+sCR_2}$$

$$= v_{in}\left[\left(\frac{1+sCR_2}{1+sCR_2} - \frac{sCR_2}{1+sCR_2}\right)\frac{1}{R_1} + \frac{sC}{1+sCR_2}\right] = v_{in}\left[\left(\frac{1}{1+sCR_2}\right)\frac{1}{R_1} + \frac{sC}{1+sCR_2}\right]$$

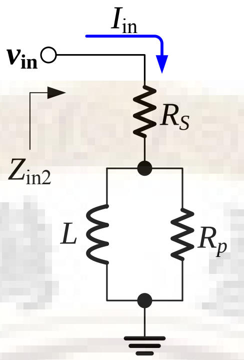
$$= v_{in}\left[\frac{1}{(1+sCR_2)R_1} + \frac{sC}{1+sCR_2}\left(\frac{R_1}{R_1}\right)\right] = v_{in}\left[\frac{1+sCR_1}{(1+sCR_2)R_1}\right]$$

$$Z_{in} = \frac{v_{in}}{I_{in}} = \frac{(1+sCR_2)R_1}{1+sCR_1}$$

$$where: v_a = v_{in} \times \frac{R_2}{R_2 + \frac{1}{sC}} = v_{in} \times \frac{sCR_2}{1+sCR_2}$$

(2) Z_{in} can be modeled as that shown in Fig. 1(b). Please find expressions for L, R_S, and R_P, in terms of C, R_1, and R_2. (10%)

<u>*Answer*:</u>



$$Z_{\text{in2}} = R_S + (sL // R_P) = R_S + \frac{1}{\dfrac{1}{sL} + \dfrac{1}{R_P}} = R_S + \frac{(sL)R_P}{R_P + sL} = R_S + \frac{sL}{1 + s\dfrac{L}{R_P}} \;.....(1)$$

where :

$$Z_{\text{in}} = \frac{(1+sCR_2)R_1}{1+sCR_1} = \frac{R_1 + sCR_2R_1}{1+sCR_1} = \frac{R_1 + sCR_1R_2 + sCR_1^2 - sCR_1^2}{1+sCR_1} = \frac{R_1(1+sCR_1) + sCR_1(R_2 - R_1)}{1+sCR_1}$$

$$= R_1 + \frac{sCR_1(R_2 - R_1)}{1+sCR_1} \;.....(2)$$

Comparison (1) with (2)

$$\begin{cases} Z_{\text{in2}} = R_S + \dfrac{sL}{1 + s\dfrac{L}{R_P}} \\[2ex] Z_{\text{in}} = R_1 + \dfrac{sCR_1(R_2 - R_1)}{1+sCR_1} = R_1 + \dfrac{sCR_1(R_2 - R_1)}{1 + s\dfrac{CR_1(R_2 - R_1)}{(R_2 - R_1)}} \end{cases} \rightarrow \begin{cases} R_S = R_1 \\ L = CR_1(R_2 - R_1) \\ R_P = R_2 - R_1 \end{cases}$$

(3) To realize a high-Q inductor, how would you choose R_1 and R_2. You must explain your reasons. (5%)

<u>*Answer*:</u>

For a high-Q inductor, R_P should be larger, and R_S should be very small. According to the statement, we should choose large R_2 and small R_1.

Note: The following derivations are only for reference. Those who take the exam do not need to provide them on their answer sheets.

$$P_{\text{in2}}(s) = \frac{1}{2} Z_{\text{in2}} |I|^2 = \frac{1}{2}|I|^2 \left(R_S + \frac{sL}{1 + s\dfrac{L}{R_P}} \right) = \frac{1}{2}|I|^2 R_S + \frac{1}{2}|I|^2 \left(\frac{sLR_P}{R_P + sL} \right)$$

$$\rightarrow P_{\text{in2}}(s)=\frac{1}{2}|I|^2 R_S+\frac{1}{2}|I|^2\frac{sLR_P(R_P-sL)}{(R_P+sL)(R_P-sL)}$$

$$P_{\text{in2}}(j\omega)=\frac{1}{2}|I|^2 R_S+\frac{1}{2}|I|^2\frac{j\omega LR_P(R_P-j\omega L)}{(R_P+j\omega L)(R_P-j\omega L)}=\frac{1}{2}|I|^2 R_S+\frac{1}{2}|I|^2\frac{j\omega LR_P^2+\omega^2L^2R_P}{R_P^2+\omega^2L^2}$$

$$=\frac{1}{2}|I|^2\left(R_S+\frac{\omega^2L^2R_P}{R_P^2+\omega^2L^2}\right)+j\omega\frac{1}{2}|I|^2\left(\frac{LR_P^2}{R_P^2+\omega^2L^2}\right)=P_{\text{loss}}+j\omega\cdot W$$

$$Q=\omega\frac{\text{average energy stored}}{\text{energy loss/second}}=\omega\frac{\frac{1}{2}|I|^2\left(\frac{LR_P^2}{R_P^2+\omega^2L^2}\right)}{\frac{1}{2}|I|^2\left(R_S+\frac{\omega^2L^2R_P}{R_P^2+\omega^2L^2}\right)}=\omega\frac{\left(\frac{LR_P^2}{R_P^2+\omega^2L^2}\right)}{\left(R_S+\frac{\omega^2L^2R_P}{R_P^2+\omega^2L^2}\right)}$$

$$=\omega\left.\frac{LR_P^2}{R_S\left(R_P^2+\omega^2L^2\right)+\omega^2L^2R_P}\right|_{R_S<<1}\approx\frac{\omega LR_P^2}{\omega^2L^2R_P}=\frac{R_P}{\omega L},\quad \text{for } R_S<<1$$

2. (25%) Plot the transfer characteristic of the circuit in Fig. 2.

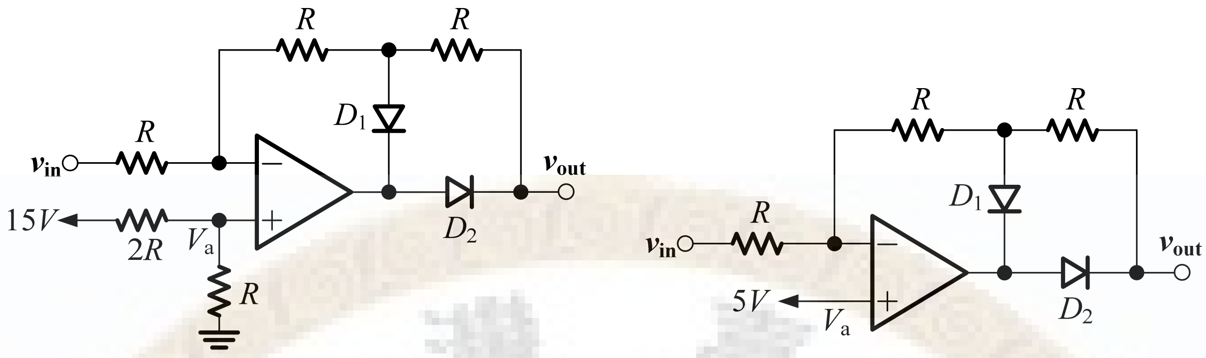
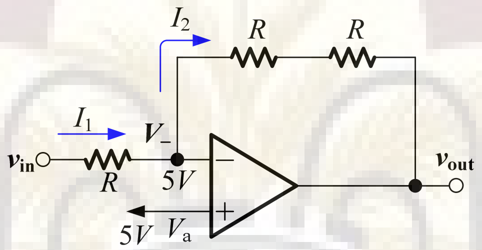


Fig. 2

<u>*Answer*</u>: $V_a = 15V \times \dfrac{R}{2R+R} = 5V$

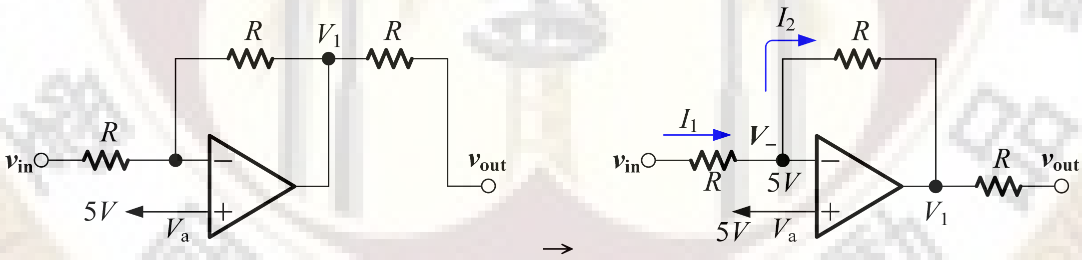
(1) If $v_{in} < V_a = 5\text{V}$ (i.e., $V_+ > V_-$), $v_{out} = L_+$. and D_1 OFF, D_2 ON.



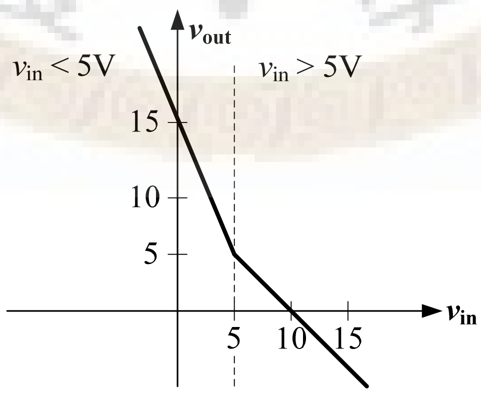
$$\frac{v_{in}-V_-}{R}=\frac{V_- - v_{out}}{2R} \to \frac{2R}{R}(v_{in}-V_-)=V_- - v_{out} \to 2(v_{in}-V_-)=V_- - v_{out}$$

$$\to v_{out}=V_- - 2(v_{in}-V_-)=5-(2v_{in}-10)=-2v_{in}+15$$

(2) If $v_{in} > V_a = 5\text{V}$ (i.e., $V_+ < V_-$), $v_{out} = L_-$. and D_1 ON, D_2 OFF.



$$\frac{v_{in}-V_-}{R}=\frac{V_- - V_1}{R} \to v_{in}-V_- = V_- - V_1 \to V_1 = -v_{in}+2V_- \to v_{out} = -v_{in}+10$$



3. (25%)

(1) For the circuit in Fig. 3, find the transfer function T(s) = V_o/V_i. (10%)

(2) Choose C = 2 nF. If the circuit is used to reject a frequency component at 10 kHz, find the value of R. (10%)

(3) For a 5-kHz input sine wave with amplitude of 2 V, find the voltage amplitude at the output. (5%)

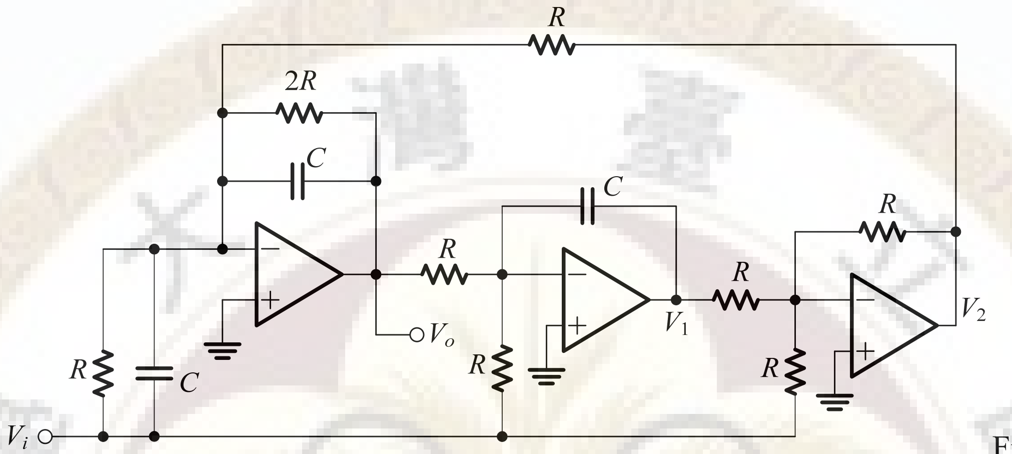


Fig. 3

(1) For the circuit in Fig. 3, find the transfer function T(s) = V_o/V_i. (10%)

$$V_o = -\frac{2R//\frac{1}{sC}}{R//\frac{1}{sC}}V_i - \frac{2R//\frac{1}{sC}}{R}V_2 = -\frac{\frac{1}{\frac{1}{2R}+sC}}{\frac{1}{\frac{1}{R}+sC}}V_i - \frac{\frac{1}{\frac{1}{2R}+sC}}{R}\left(\frac{1}{sCR}V_i - V_i + \frac{1}{sCR}V_o\right)$$

$$= -\frac{\frac{1}{R}+sC}{\frac{1}{2R}+sC}V_i - \frac{1}{\frac{1}{2}+sCR}\left(\frac{1}{sCR}V_i - V_i + \frac{1}{sCR}V_o\right) = -\frac{2+s2RC}{1+s2RC}V_i - \frac{2}{1+s2RC}\left(\frac{1}{sCR}V_i - V_i + \frac{1}{sCR}V_o\right)$$

$$= -\frac{2+s2RC}{1+s2RC}V_i - \frac{1}{sCR}\frac{2}{1+s2RC}V_i + \frac{2}{1+s2RC}V_i - \frac{1}{sCR}\frac{2}{1+s2RC}V_o$$

$$\to V_o\left(1+\frac{2}{sCR}\frac{1}{1+s2RC}\right) = \left(\frac{2}{1+s2RC} - \frac{2+s2RC}{1+s2RC} - \frac{1}{sCR}\frac{2}{1+s2RC}\right)V_i$$

$$\to V_o\left(1+s2RC+\frac{2}{sCR}\right) = \left[2-(2+s2RC)-\frac{2}{sCR}\right]V_i = \left[-s2RC-\frac{2}{sCR}\right]V_i$$

$$\frac{V_o}{V_i} = \frac{-s2RC-\frac{2}{sCR}}{1+s2RC+\frac{2}{sCR}} = \frac{-s2RC(sCR)-2}{(sCR)+s2RC(sCR)+2} = \frac{-s^2 2R^2C^2-2}{s^2 2R^2C^2+sRC+2} = -\frac{s^2+\left(\frac{1}{RC}\right)^2}{s^2+s\frac{1}{2RC}+\left(\frac{1}{RC}\right)^2}$$

$$where:\ V_1 = \frac{-\frac{1}{sC}}{R}(V_i+V_o) = \frac{-1}{sCR}(V_i+V_o),$$

$$V_2 = \frac{-R}{R}(V_i+V_1) = -V_i - V_1 = -V_i - \frac{-1}{sCR}(V_i+V_o) = -V_i + \frac{1}{sCR}(V_i+V_o) = \frac{1}{sCR}V_i - V_i + \frac{1}{sCR}V_o$$

(2) Choose C = 2 nF. If the circuit is used to reject a frequency component at 10 kHz, find the value of R. (10%)

Answer:

$$\frac{V_o}{V_i}=-\frac{s^2+\left(\frac{1}{RC}\right)^2}{s^2+s\frac{1}{2RC}+\left(\frac{1}{RC}\right)^2}=-\frac{s^2+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2},$$

$$f_0=\frac{1}{2\pi\cdot RC}=10kHz,\rightarrow R=\frac{1}{2\pi\cdot 10kHz\cdot C}=\frac{1}{2\pi\cdot 10\times 10^3\cdot 2\times 10^{-9}}=\frac{1}{4\pi\times 10^{-5}}=7.96K\Omega$$

(3) For a 5-kHz input sine wave with amplitude of 2 V, find the voltage amplitude at the output. (5%)

Answer:

$$T(s)=\frac{V_o(s)}{V_i(s)}=-\frac{s^2+\left(\frac{1}{RC}\right)^2}{s^2+s\frac{1}{2RC}+\left(\frac{1}{RC}\right)^2},\rightarrow T(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}=-\frac{\left(\frac{1}{RC}\right)^2-\omega^2}{\left[\left(\frac{1}{RC}\right)^2-\omega^2\right]+j\omega\frac{1}{2RC}}$$

$$\frac{V_o(f)}{V_i(f)}=-\frac{\left(\frac{1}{RC}\right)^2-(2\pi f)^2}{\left[\left(\frac{1}{RC}\right)^2-(2\pi f)^2\right]+j(2\pi f)\frac{1}{2RC}}=-\frac{1-(2\pi f\cdot RC)^2}{1-(2\pi f\cdot RC)^2+j(\pi f\cdot RC)}$$

$$\left.\frac{V_o(f)}{V_i(f)}\right|_{f=5kHz}=-\frac{1-\left(\frac{1}{2}\right)^2}{1-\left(\frac{1}{2}\right)^2+j\left(\frac{1}{4}\right)}=-\frac{\frac{3}{4}}{\frac{3}{4}+j\left(\frac{1}{4}\right)}=-\frac{3}{3+j}=-\frac{3}{\sqrt{3^2+1}\angle\tan^{-1}\left(\frac{1}{3}\right)}=-0.95\angle-18.43^{o}$$

$$\left.\frac{V_o(f)}{V_i(f)}\right|_{f=5kHz}=-0.95,\rightarrow \left|V_o(f)\right|_{f=5kHz}=-0.95\times\left|V_i(f)\right|_{f=5kHz}=-0.95\times 2=-1.9V$$

$$where:\ f_0=\frac{1}{2\pi\cdot RC}=10kHz,\rightarrow RC=\frac{1}{2\pi\cdot 10kHz}$$

$$(a)\left.(2\pi f\cdot RC)\right|_{f=5kHz}=\frac{2\pi\cdot 5kHz}{2\pi\cdot 10kHz}=\frac{1}{2},(b)\left.(\pi f\cdot RC)\right|_{f=5kHz}=\frac{\pi\cdot 5kHz}{2\pi\cdot 10kHz}=\frac{1}{4}$$

4. (16%)

(1) Symbolically derive the loop gain of the Wien-bridge oscillator (4%) shown in Fig. 4.

(2) Determine the frequency of oscillation (4%) and the condition to start oscillation (4%).

(3) Find the frequency of oscillation numerically in units of Hz when R = 10 kΩ and C = 16 nF (4%).

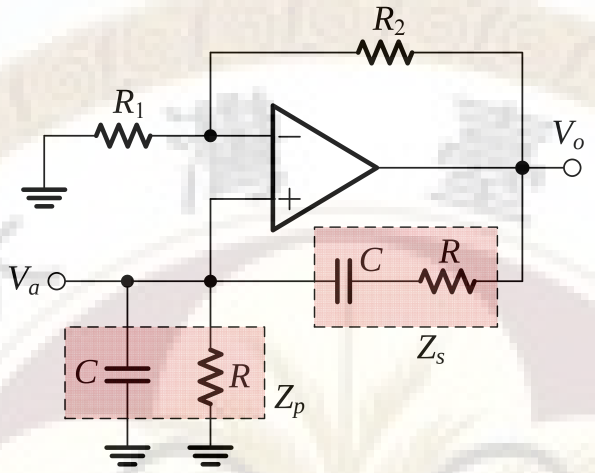


Fig. 4

(Solution)

(1) Symbolically derive the loop gain of the Wien-bridge oscillator (4%) shown in Fig. 4.

Answer:

$$L(s)=\left[1+\frac{R_2}{R_1}\right]\frac{Z_p}{Z_p+Z_s}=\frac{1+\frac{R_2}{R_1}}{1+Z_sY_p}$$

$$L(j\omega)=\frac{1+\frac{R_2}{R_1}}{1+\left(R+\frac{1}{j\omega C}\right)\left(\frac{1}{R}+j\omega C\right)}=\frac{1+\frac{R_2}{R_1}}{3+\left(\frac{1}{j\omega RC}+j\omega CR\right)}=\frac{1+\frac{R_2}{R_1}}{3+j\left(\omega CR-\frac{1}{\omega RC}\right)}$$

$$where: Z_s=R+\frac{1}{j\omega C}, Y_p=\left(\frac{1}{R}+j\omega C\right)$$

(2) Determine the frequency of oscillation (4%) and the condition to start oscillation (4%).

$$L(j\omega)=\frac{1+\frac{R_2}{R_1}}{3+j\left(\omega CR-\frac{1}{\omega RC}\right)}, \to \omega_0=\frac{1}{RC}\to(+4\%)$$

$$\to L(j\omega_0)=\frac{1+\frac{R_2}{R_1}}{3}=1, \to \frac{R_2}{R_1}=2(\text{Condition to start oscillation})\to(+4\%)$$

(3) Find the frequency of oscillation numerically in units of Hz when R = 10 kΩ and C = 16 nF (4%).

$$\omega_0=\frac{1}{RC}\to f_0=\frac{1}{2\pi RC}=\frac{1}{2\pi\times 10^4\times 16\times 10^{-9}}=994.72Hz\to(+4\%)$$

5. (9%) Consider a modification of the circuit of Fig. 5 in which R_1 is replaced by a pair of diodes connected in parallel in opposite directions. For $L_+ = -L_- = 12$ V, $R_2 = R = 10$ kΩ, $C = 0.1$ μF, and the diode voltage as a constant denoted V_D, find an expression for frequency as a function of V_D.

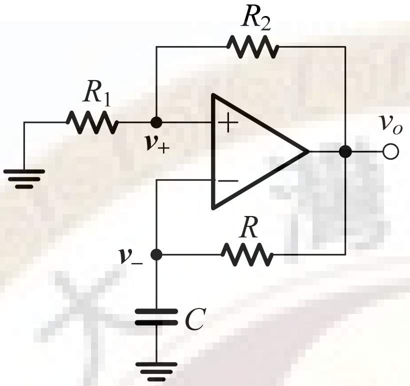


Fig. 5

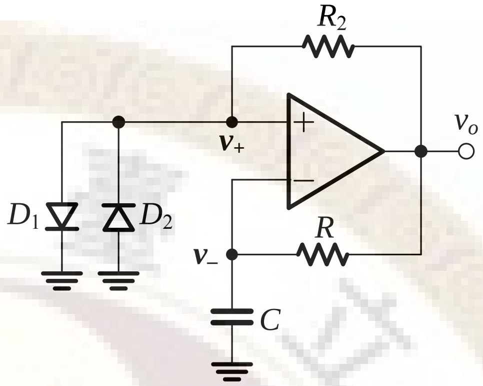
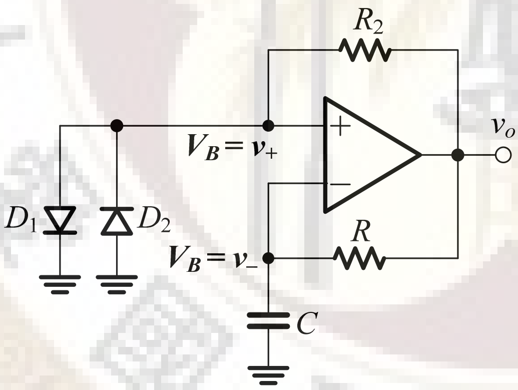
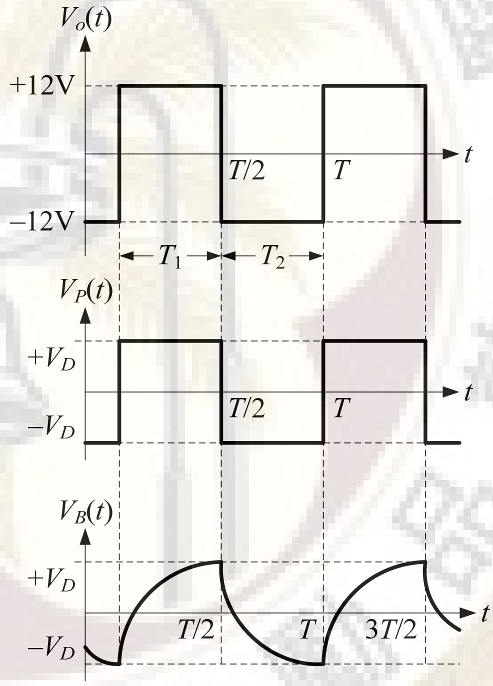


Fig. 5(a)

(Solution)





$$V_C(t)=V(\infty)+\left[V(0)-V(\infty)\right]e^{\frac{-1}{RC}t},V_B(\infty)=12V,V_B(0)=-V_D,V_B(T_1)=V_D,T_1=T_2=\frac{T}{2},$$

During $T_1: V_B(t)=12+(-V_D-12)e^{\frac{-1}{RC}t},\rightarrow V_B(T_1)=V_D=12-(V_D+12)e^{\frac{-1}{RC}T_1}$

$$\rightarrow T_1=\frac{T}{2}=-RC\times\ln\left(\frac{12-V_D}{12+V_D}\right)=RC\times\ln\left(\frac{12+V_D}{12-V_D}\right)=10\times10^3\times0.1\times10^{-6}\times\ln\left(\frac{12+V_D}{12-V_D}\right),$$

$$T=2\times10^{-3}\times\ln\left(\frac{12+V_D}{12-V_D}\right),\rightarrow f=\frac{1}{T}=\frac{1}{2\times10^{-3}\times\ln\left(\frac{12+V_D}{12-V_D}\right)}=\frac{500}{\ln\left(\frac{12+V_D}{12-V_D}\right)}$$