Electromagnetics (I) Final Examination June 27, 2005

1. The word "polarization" has two meanings in electromagnetics. One is related to the wave field and the other is related to the material property. Please explain both of them briefly. (10%)

2. (1) In discussing the time-varying electromagnetic field, why do we usually consider sinusoidally time-varying sources and wave fields? Explain briefly.
(2) In discussing the electromagnetic wave, why do we usually consider a uniform plane wave? Explain briefly.
(3) What is a phasor? Discuss its usefulness in electromagnetic theory briefly.
(4) To compute the time-average Poynting vector, we usually introduce a complex Poynting vector. Please write down the expression of that complex Poynting vector. Is it also a phasor? Explain briefly. (16%)
$E \times H^*$

3. An infinite plane sheet lying in the $z = 0$ plane carries a surface current of density $\vec{J}_S = 0.1\cos(8\pi\times10^8 t)\hat{x} A/m$. The region $z > 0$ is a perfect dielectric of $\varepsilon = 2.25\varepsilon_0$ and $\mu = \mu_0$, whereas the region $z < 0$ is a perfect dielectric of $\varepsilon = 9\varepsilon_0$ and $\mu = \mu_0$. Find $\vec{E}(z,t)$ and $\vec{H}(z,t)$ on both sides of the sheet. (20%)

4. Region 1 ($z < 0$) is free space whereas region 2 ($z > 0$) is a perfect conductor. A uniform plane wave with electric field $\vec{E}_i(z,t) = \hat{y}E_0 \sin\omega(t - \sqrt{\mu_0\varepsilon_0}\, z)$ is incident on the interface $z = 0$ from region 1. Find: (1) the reflected electric and magnetic fields in region 1; (2) the total electric and magnetic fields in region 1; (3) the surface current density on $z = 0$; (4) the instantaneous Poynting vector at $z < 0$; and (5) the time-average Poynting vector at $z < 0$. (6) If region 2 ($z > 0$) is a good (but not perfect) conductor, how would the result of (5) be changed? Discuss it qualitatively. (24%)
$\sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

5. Consider a uniform plane wave with electric field $\vec{E}(z,t) = \hat{y}E_0 \sin\omega(t - \sqrt{\mu_0\varepsilon_0}\, z)$ propagating in the free space.
(1) Find the instantaneous energy (electric + magnetic) density $u(z, t)$ associated with this uniform plane wave.
(2) Show that $\vec{E}\times\vec{H} = \frac{u}{\sqrt{\mu_0\varepsilon_0}}\hat{z}$, and explain its physical meaning. $\nabla\cdot(\phi a_x) = \frac{\partial}{\partial x}$
(3) Show that u satisfies a continuity equation $\nabla\cdot(\frac{u}{\sqrt{\mu_0\varepsilon_0}}\hat{z}) + \frac{\partial u}{\partial t} = 0$. Discuss the physical meaning briefly. (15%) $\boxed{\int \nabla\cdot A \, dv = \int A\cdot dS}$ $\int \frac{\partial}{\partial x}\phi \, dv = \int \phi a_x \, d\vec{S}$

6. Let ϕ be a scalar function and V be a volume bounded by a closed surface S with unit outer normal $\hat{n}$. (1) Show that $\int_V \nabla\phi dv = \oint_S \phi\hat{n} dS$. (Hint: Consider a vector function $\phi\vec{c}$ and use divergence theorem appropriately.) (2) Based on (1), give an alternative definition of the gradient operation by taking the limit of a surface integral. (3) Consider an infinitesimal cube of side δ around a point (x, y, z). Show that the definition in (2) can yield the differential expression of the gradient operation in the Cartesian coordinate system. (15%)