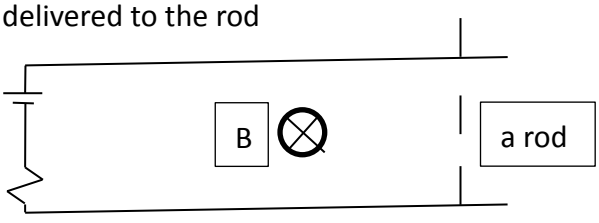
2007 Electromagnetics one Midterm

1. Find the algebraic equation for the direction lines for the vector field given in spherical coordinates by (5cosθ$\boldsymbol{a_r}$+sinθ$\boldsymbol{a_\theta}$)
2. (a) What is the Biot-Savart law? State its mathematical expression.
   (b) Is it exactly valid for computing the magnetic field due to a time-varying current distribution?
3. (a) What is Gauss' law for the electric field? Describe it both in integral form and in differential form.
   (b) Is it valid for the case of time-varying source and field distribution?
4. In a Cartesian coordinate system, constant current I flows along a straight wire from a point charge Q1(t) located at the origin to a point charge $Q_2(t)$ located at $(0,0,5)$
   (a) Find the line integral of magnetic field H along the square closed path having the vertices at $(5,5,0)$, $(-5,5,0)$, $(-5,-5,0)$ and $(5,-5,0)$and traversed in that order. (Express the answer in terms of current I)
   (b) If the current I is time-varying, i.e.,I=I(t),(0<z<5),does the answer in (a) need to be modified? Explain why.
   (c) If $Q_2(t)$ is slowly moved from $(0,0,5)$to the origin (keeping I constant), how does the answer of (a) gradually change? Explain briefly.
5. For the electric field $\vec{\mathbf{E}}(\mathrm{y},\mathrm{z},\mathrm{t}) = e^{-\alpha y}\cos(\omega t - \beta z)\mathbf{a}_{\boldsymbol{x}}$ in free space $(\mu_{0,}\varepsilon_0)$, find the necessary condition relating ω,α,β, $\mu_{0,}$ and ε for the field to satisfy both of Maxwell's curl equations.
6. A rectangular loop of conducting wire with three sides foxed and the fourth side movable is situated in a plane perpendicular to a uniform magnetic flux density B=0.5wb/m^2, as illustrated in Fig.P.6.(圖如下)
   A battery V_B=12V and a resistor R=4Ω are connected in series to the loop. The movable side consists of a conducting rod of length l=1m and of mass 0.2kg. Assume that the resistance of the conducting wire and the movable rod is 0.
   (a) With no friction and no load on the rod, calculate its initial acceleration and final velocity (in vector form)
   (b) If the rod must pull a load requiring a force of 1 newton in the x-direction, what will be its final velocity (in vector form)
   (c) If the system is to act as a generator to produce a 1A charging current through the battery, in what direction and how fast must the rod be pulled?
   (d) How much force (in vector form) is required to maintain this motion?
   (e) Calculate the electric power delivered by the rod and compute this with the mechanical power delivered to the rod



7.Consider a vector field $\vec{\mathbf{A}} = -mysinx\mathbf{a}_{\boldsymbol{x}}$+cosx$\mathbf{a}_{\boldsymbol{y}}$
   (a) Evaluate the line integral $\int \vec{\mathbf{A}} \times d\vec{\mathbf{l}}$ from point (x_1, y_1, z_1) to(x_2, y_2, z_2)along the straight line connecting them.
   (b) Find $\boldsymbol{\nabla} \times \vec{\boldsymbol{A}}$
   (c) Use the results of (a) and (b) to verify Stoke's theorem.