課程名稱：機率與統計 2008

1. Describe the Central Limit Theorem. (5%)

2. Suppose that X and Y are two independent zero mean Gaussian random varibles with variaces $(\sigma X)^2$ and $(\sigma Y)^2$.

   (a) What is the joint probability density function of (X,Y)? (5%)

3. Suppose that X and Y are two zero mean Gaussian random varibles with covariances $Cov[X,X]=(\sigma X)^2$, $Cov[X,Y]=\sigma XY$, and $Cov[Y,Y]=(\sigma Y)^2$. What is the probability density function of X+Y? (10%)

4. Consider two random variables X and Y.

   (a) Describe the condition that X and Y are uncorrelated implies that X and Y are independent. (5%)

   (b) Suppose Cov[X,Y]=0. Is it necessary that X and Y are independent? Please provide your reason. (5%)

5. Describe the condition that MAP test is equivalent to the maximum likelihood test. (5%)

6. Suppose that we know $E[X]=\mu X$ in advance. Is

$$(1/n) \sum_{i=1}^{n} (X_i-\mu X)^2$$

an unbiased estimate of Var[X]? Prove your answer. (5%)

7. X and Y are two random variables with a joint probability density function as

$$f_{XY}(x,y)=\begin{cases} cxy, & 1 \leqq x \leqq y \leqq 2; \\ 0, & \text{otherwise.} \end{cases}$$

(a) c=? (5%)

(b) Are X and Y independent? Explain why. (5%)

(c) Derive the conditional probability distribution function $F_{Y|X}(y|x)$. (5%)

(d) Var[E[Y|X]]=? (5%)

(e) Correlation coefficient $\rho(X,Y)$=? (5%)

(f) Let U=X+Y and W=3X+4Y. Find the covariance matrix of [U W]'. (5%)

8. You are counting the numbers of buses, cars and bicycles passing by where you stand. Let KB, KC and Kb be the numbers you get during time [0,T], which are independent Poisson random varibles with parameters λB, λC and λb.

(a) Prove that K=KB+KC+Kb is also a Poisson with parameter $\lambda=\lambda B+\lambda C+\lambda b$.

(Hint: Use moment generating function) (10%)

(b) Let event A be the event that "when a vehicle passes by, it is a bicycle". P[A]=? (5%)

9. Let X be binomial random varible with parameters (n,p) and n=3. For the following hypothesis test

$$Ho: p=1/2 \quad \text{vs} \quad Ha: p=2/3$$

we reject Ho as X=0 or 3. Please calculate

α = the probability of making a type I error

β = the probability of making a type II error. (10%)