2012 Fall Electromagnetics II – Quiz I

Date: 22/10/2012

Problem 1

In the system shown in Fig. 1, assume that V_g is a constant voltage source of 80 V and the switch S is closed at t = 0. Find and sketch: (a) the line voltage versus z for t = 0.2 μs; and (b) the line current versus z for t = 0.4 μs; (c) the line voltage versus t for z = 30 m; and (d) the line current versus t for z = -40 m. $R_S = 60\Omega$.

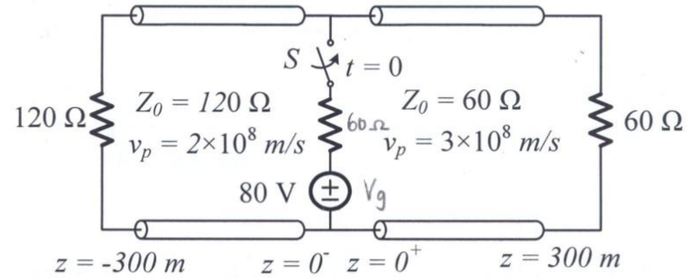
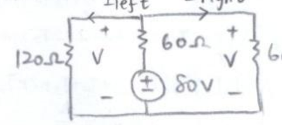


Fig. 1

at t=0



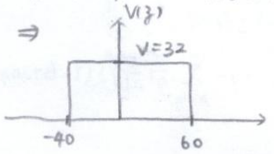
$V = 80\times\frac{40}{60+40} = 32$ (V)

$60//120 = \frac{60\times120}{60+120} = 40$

$T_{d,\,left} = \frac{300}{2\times10^8} = \frac{3}{2}\times10^{-6} = 1.5\mu$. $T_{d,\,right} = \frac{300}{3\times10^8} = 1\mu$.

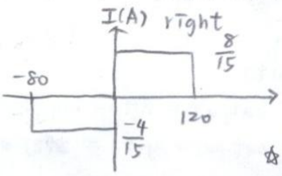
(a) $t = 0.2\mu s \rightarrow$ left wave 走了 $300\times\frac{0.2\mu}{1.5\mu} = 40$ m , right wave 走了 $300\times\frac{0.2\mu}{1\mu} = 60$ m

$\Rightarrow$



(b) $t = 0.4 \rightarrow$ left wave 走了 $300\times\frac{0.4\mu}{1.5\mu} = 80$ (m)

$300\times\frac{0.4}{1\mu} = 120$ (m)



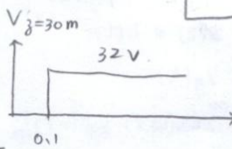
$I_{Left} = \frac{V}{120\Omega} = \frac{4}{15}$ (A)

$I_{right} = \frac{V}{60\Omega} = \frac{8}{15}$ A

☆ 定義向+z方向的电流為+

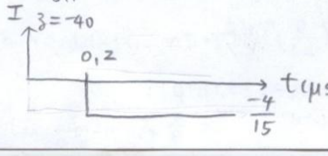
(c) $z = 30$ m.

$\frac{30}{3\times10^8} = 10^{-7} = 0.1\mu$



(d). $z = -40$

$\frac{40}{2\times10^8} = 0.2\mu$.



Problem 2

In the system shown in Fig. 2: **(a)** find the output voltage V_o across the 150 Ω resistor for $V_g(t) = \delta(t)$; and **(b)** find [redacted] the amplitude of $V_o(t)$ versus ω for $V_g(t) = \cos\omega t$.

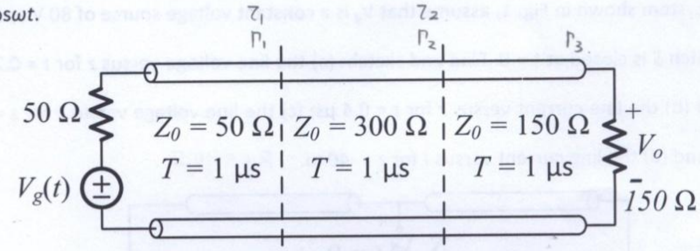


Fig. 2

(a). $V_g(t) = \delta(t)$

$$\Gamma_1 = \frac{300-50}{300+50} = \frac{250}{350} = \frac{5}{7}, \quad \Gamma_2 = \frac{150-300}{150+300} = \frac{-150}{450} = -\frac{1}{3}$$

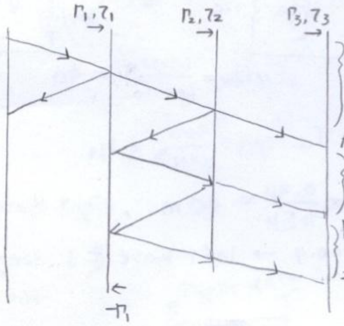
$$\Gamma_3 = \frac{150-150}{150+150} = 0.$$

$$\rightarrow \tau_1 = 1+\Gamma_1 = \frac{12}{7}, \quad \tau_2 = 1+\Gamma_2 = 1-\frac{1}{3} = \frac{2}{3}. \quad \tau_3 = 1+\Gamma_3 = 1.$$

$$V^+ = V_g \times \frac{50}{50+50} = \frac{1}{2} V_g. \Rightarrow V_g = A\delta(t-(T_1+T_2+T_3)) + (-\Gamma_1\Gamma_2)A\,\delta(t-(T_1+T_2+T_3+2T$$

$$+ A(-\Gamma_1\Gamma_2)^2\delta(t-(T_1+T_2+T_3+2\times 2T_2)$$

$$= \sum_{n=0}^{\infty} A(-\Gamma_1\Gamma_2)^n \delta(t-(T_1+T_2+T_3+2n$$



$\} T_1+T_2+T_3$

$A = V^+ \times \tau_1 \times \tau_2 \times \tau_3$

$\} 2T_2$

$B = V^+ \times \tau_1 \times \Gamma_2 \times (-\Gamma_1) \times \tau_2 \times \tau_3$

$= A \times (-\Gamma_1\Gamma_2)$

$\} 2T_2$

$C = A \times (-\Gamma_1\Gamma_2)^2$

$A = \frac{1}{2} \times \frac{12}{7} \times \frac{2}{3} \times 1 = \frac{4}{7}$

$-\Gamma_1\Gamma_2 = -\frac{5}{7} \times -\frac{1}{3} = \frac{5}{21}$

$T_1+T_2+T_3 = 3\,\mu s$

$2T_2 = 2\,\mu s$

$$\Downarrow$$

$$V_g = \sum_{n=0}^{\infty} \frac{4}{7}\left(\frac{5}{21}\right)^n \delta(t-(3+2n$$

(b) $V_g(t) = \cos(\omega t)$

If input $= \delta(t) \longrightarrow$ output $= h(t) =$ impulse response.

then input $= x(t) \longrightarrow$ output $= y(t) = x(t) * h(t)$

Now $x(t) = \cos(\omega t)$, $h(t) =$ (a) 的 $V_g(t)$

$$\Rightarrow y(t) = \cos\omega(t) * V_g(t)|_{(a)} = \int_{-\infty}^{\infty} \cos(\omega\tau)\, V_g(t-\tau)|_{(a)}\, d\tau.$$

$$= \int_{-\infty}^{\infty} \cos(\omega\tau) \sum_{n=0}^{\infty} \frac{4}{7}\left(\frac{5}{21}\right)^n \delta(t-\tau-(3+2n)\mu s)\, d\tau.$$

$$= \sum_{n=0}^{\infty} \frac{4}{7}\left(\frac{5}{21}\right)^n \cos[\omega(t-(3+2n)\mu)]$$

$$\Rightarrow \text{phasor } \overline{Y}(\omega) = \sum_{n=0}^{\infty} \frac{4}{7}\left(\frac{5}{21}\right)^n e^{-j\omega(3+2n)\mu} = \frac{4}{7} e^{-j3\omega\mu} \sum_{n=0}^{\infty} \left(\frac{5}{21}\right)^n \left(e^{-j\omega 2\mu}\right)^n$$

$$\frac{4}{7} e^{-j3\omega\mu} \frac{1}{1-\frac{5}{21}\cdots}$$

$$\Uparrow$$

$$\frac{\frac{4}{7} e^{-j3\omega\mu}}{1-\frac{5}{21} e^{-j2\omega\mu}}$$

小考

3.

(a) $$\frac{V_0}{2}+V^- = L\frac{d\left(\frac{V_0}{2Z_0}-\frac{V^-}{Z_0}\right)}{dt} + \left(\frac{V_0}{2Z_0}-\frac{V^-}{Z_0}\right)Z_0$$

$$\cancel{\frac{V_0}{2}} + V^- = \frac{-L}{Z_0}\frac{dV^-}{dt} + \cancel{\frac{V_0}{2}} - V^-$$

$$\underline{\underline{V^- + \frac{L}{2Z_0}\frac{dV^-}{dt} = 0}}$$

(b) at $t=T$ L open $[I]_{t=T}=0$

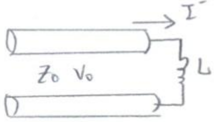
$$\frac{V_0}{2Z_0}-\frac{V^-}{Z_0}=0 \qquad \underline{\underline{[V^-]_{t=T}=\frac{V_0}{2}}}$$

(c) $$V^- = A\cdot e^{-\frac{2Z_0}{L}(t-T)}$$

又 $[V^-]_{t=T}=\frac{V_0}{2}$ $\Rightarrow$ $\underline{\underline{V^-=\frac{V_0}{2}e^{-\frac{2Z_0}{L}(t-T)}}}$ $t>T$

4

(a)

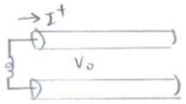


$\underline{\underline{V^-}}$

(b) $$V_0+V^- = L\cdot\frac{d(I^-)}{dt} = L\frac{d\left(-\frac{V^-}{Z_0}\right)}{dt} = -\frac{L}{Z_0}\frac{dV^-}{dt}$$

$$\therefore \underline{\underline{\frac{L}{Z_0}\frac{dV^-}{dt}+V^-=-V_0}}$$

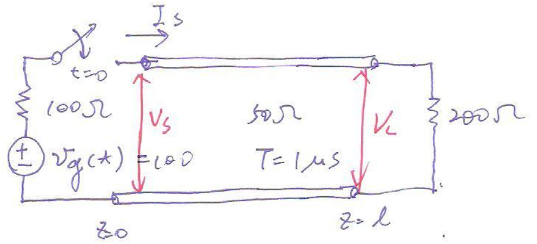
(c)



$\underline{\underline{V^+}}$

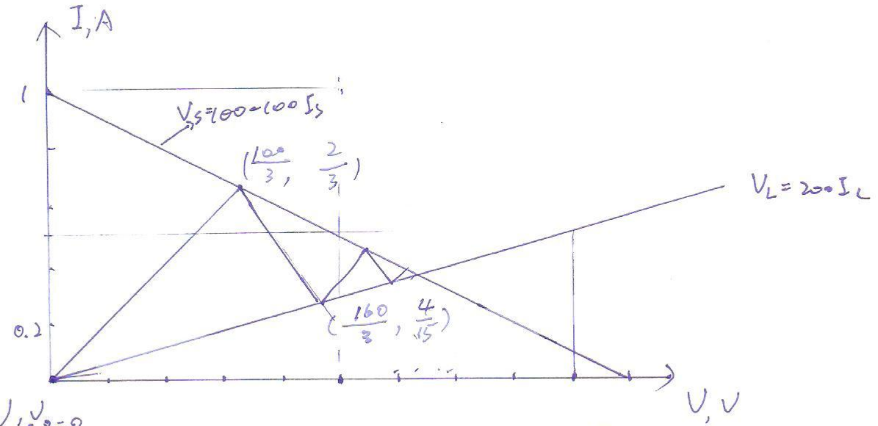
(d) $$V_0+V^+ = L\cdot\frac{d(-I^+)}{dt} = -\frac{L}{Z_0}\frac{dV^+}{dt} \qquad \therefore \underline{\underline{\frac{L}{Z_0}\frac{dV^+}{dt}+V^+=-V_0}}$$

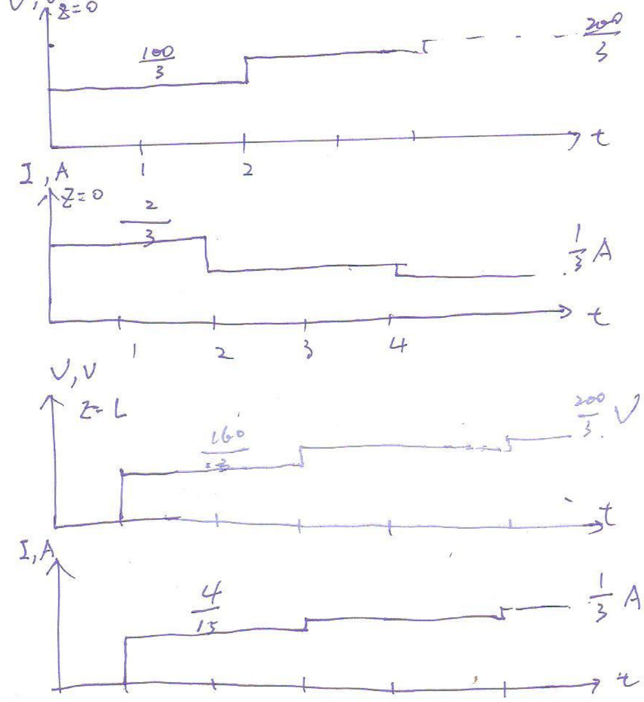
Problem 5.



Characteristic at $z=0$ is $V_s = 100 - 100 I_s$

Charateristic at $z=l$ is $V_L = 200 I_L$



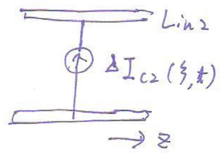


$V_{ss} = \frac{200}{3}$ V

$I_{ss} = \frac{1}{3}$ A.

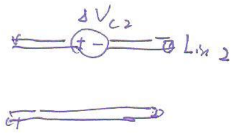
Problem 6.

(a)



$$\Delta I_{c2}(\xi,t) = C_m\,\Delta\xi\,\frac{\partial V_1(\xi,t)}{\partial t}$$

$$\Delta V_{c2}(\xi,t) = L_m\,\Delta\xi\,\frac{\partial I_1(\xi,t)}{\partial t}$$



$$\Delta V_2^+ = \frac{1}{2} Z_0\,\Delta I_{c2} - \frac{1}{2}\Delta V_{c2}$$

$$\Delta V_2^- = \frac{1}{2} Z_0\,\Delta I_{c2} + \frac{1}{2}\Delta V_{c2}$$

$$\Delta V_2^+(\xi,t) = \left\{\frac{1}{2} C_m Z_0 \frac{\partial V_1(\xi,t)}{\partial t} - \frac{1}{2} L_m \frac{\partial I_1(\xi,t)}{\partial t}\right]\Delta\xi$$

$$= \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)\frac{\partial V_1(\xi,t)}{\partial t}\,\Delta\xi$$

$$\Delta V_2^-(\xi,t) = \frac{1}{2}\left(C_m Z_0 + \frac{L_m}{Z_0}\right)\frac{\partial V_1(\xi,t)}{\partial t}\,\Delta\xi$$

$$V_2^+(z,t) = \int_0^z \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)\frac{\partial}{\partial t}\left\{V_1\left(t - \frac{\xi}{v_p} - \frac{z-\xi}{v_p}\right)\right]d\xi$$

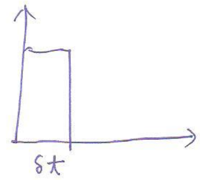
$$= \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)\int_0^z \frac{\partial V_1(t - z/v_p)}{\partial t}\,d\xi$$

$$\therefore\ V_2^+(z,t) = z\,k_f\,V_1'(t - z/v_p)$$

$$\text{for}\quad k_f = \frac{1}{2}\left(C_m Z_0 - \frac{L_m}{Z_0}\right)$$

(b)

Line 1



Line 2