

# Linear Algebra Quiz 2

Dept. of Elec. Eng., National Taiwan University

May 17, 2006

USE OF ALL AUTOMATIC COMPUTING MACHINES IS PROHIBITED

1. Judge if the following statements are true or false. Give a concise proof to each true statement, and a counterexample to each false statement.

(a) For any subspace  $W$  of  $\mathcal{R}^n$ , the orthogonal projection matrix  $P_W$  for  $W$  equals  $(P_W)^T$ . (20%)

(b) If two  $n \times n$  matrices have the same characteristic polynomial, then they have the same eigenvectors. (20%)

2. Given a linear operator  $T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_1 + 3x_2 \\ -4x_1 + 6x_2 \end{pmatrix}$  on  $\mathcal{R}^2$ , find a linear operator  $G$  on  $\mathcal{R}^2$  such that  $T = G^2$ .

Is the answer unique? (30%)

3. Given a vector  $v = \begin{pmatrix} 7 \\ 4 \\ 1 \\ 2 \end{pmatrix}$  and a subspace  $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 2 \\ 2 \end{pmatrix} \right\}$  of  $\mathcal{R}^4$ , find an orthonormal basis of  $W^\perp$ ,

and two vectors  $w \in W$  and  $z \in W^\perp$  such that  $v = w + z$ . (30%)