Complex Analysis: Quiz 1

10:20 AM - 11:20 PM, April 7, 2009.

[1] (15 %) Find the four fourth roots of $z = 1 + i$.

[2] (20 %) Verify that $u(x, y) = 4xy^3 - 4x^3y + x$ is harmonic. Find $v(x, y)$, the conjugate harmonic function of $u(x, y)$.

[3] (15 %) Find all values of $\sin^{-1}\sqrt{5}$.

[4] (15 %) Suppose z_0 is any constant complex number interior to any simple closed contour C. Show that

$$\oint_C \frac{dz}{(z - z_0)^n} = \begin{cases} 2\pi i, & n = 1 \\ 0, & n > 1 \end{cases}$$

[5] (20 %) Evaluate $\oint_C \frac{z+1}{z^4 + 4z^3} dz$, where C is the circle $|z| = 1$.

[6] (15 %) Expand $f(z) = \frac{1}{2+z}$ in the Taylor series centered at $z_0 = i$.