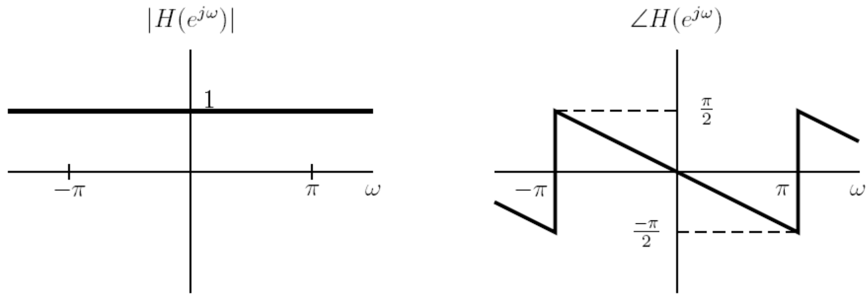
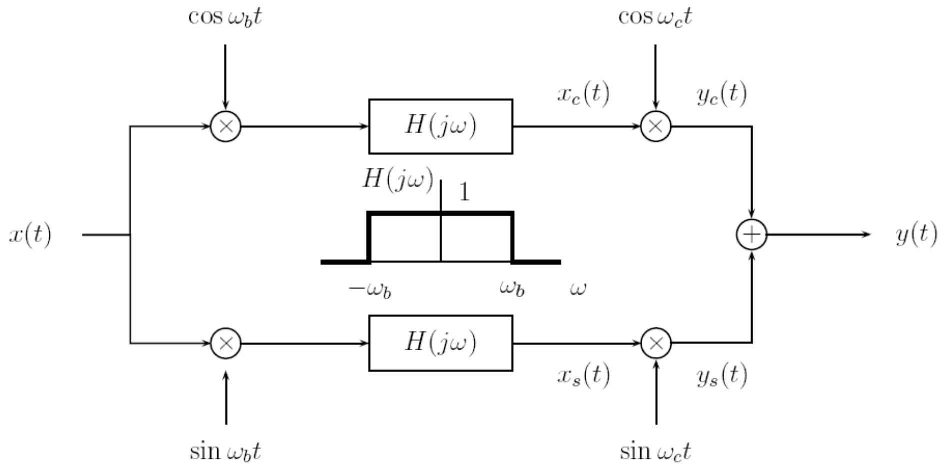
1. [6] The magnitude and phase of the frequency response of a DT LTI system are given below:

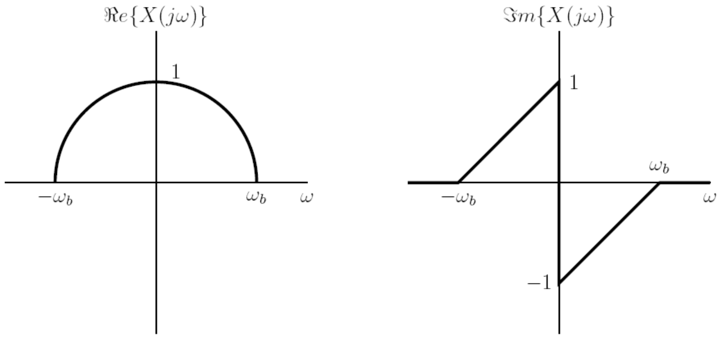


Suppose the input sequence to the system is $x[n] = \cos(\frac{5\pi}{2}n - \frac{\pi}{4})$. Determine the output.

2. [12] Suppose the Fourier transform $X(j\omega)$ of the input $x(t)$ to the following system



has real and imaginary parts given below:



(a) Provide a sketch of the real and imaginary parts of $X_s(j\omega)$. [3]
(b) Provide a sketch of the real and imaginary parts of $Y_s(j\omega)$. [3]
(c) Provide a sketch of the real and imaginary parts of $Y(j\omega)$. [3]
(d) What small change would you make in the system to create a lower sideband modulation? [3]

3. [18] Suppose the system function of a system is

$$H(s) = \frac{10(1-s)}{(1+s)(10+s)}$$

(a) Draw the block diagram of the system in direct, cascade, and parallel forms. [6]
(b) Sketch the Bode plot for $H(j\omega)$. [6]
(c) Use pole-zero plot to determine the magnitude and phase of $H(j\omega)$ graphically. [6]

4. [9] Given the following conditions of a CT LTI system:

   A. The system function is rational and has only two poles at $s=-1$ and $s=3$.
   B. If $x(t)$=1, then $y(t)$=0.

   (a) Determine the region of convergence of the system. [5]
   (b) Is the system causal? Justify your answer. [2]
   (c) Is the system stable? Justify your answer. [2]

5. [5] Determine if the following statement is correct or not:

   "If the Laplace transform $X(s)$ of $x(t)$ is rational, then $x(t)$ is right sided if and only if $x(t)$ is causal."

   Justify your answer with a proof if it is yes or with a counterexample if it is no.

6. [18] Consider that a causal linear time-invariant (LTI) system has its input $x[n]$ and output $y[n]$ given by the following difference equation:

$$y[n] - y[n-1] - y[n-2] = x[n-1] \qquad \text{(I)}$$

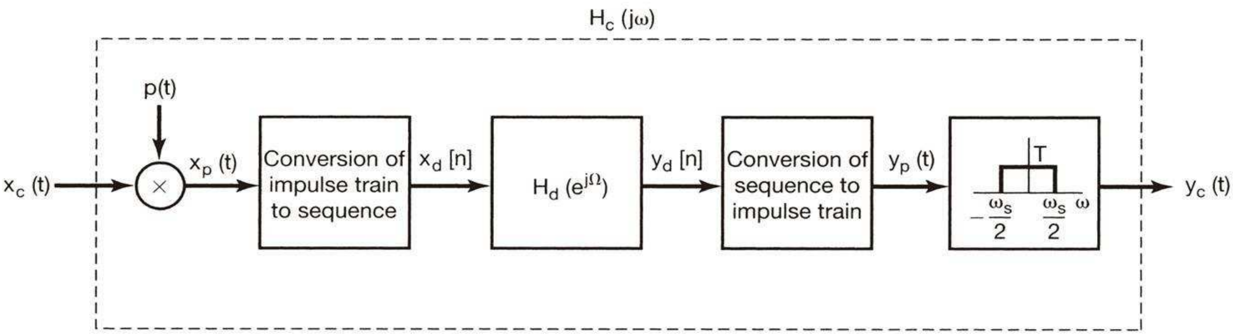
   (a) Find the system function $H(z)$. Justify your answer. [3]
   (b) Sketch the poles and zeros of $H(z)$. [3]
   (c) Determine the region of convergence of $H(z)$. [3]
   (d) Find the unit impulse response of the LTI system. Justify your answer. [3]
   (e) Is the system stable? Using Part (c) to justify your answer. [3]
   (f) Find a stable but noncausal LTI system whose unit impulse response satisfies the same difference equation given by (I). Justify your answer. [3]

7. [9] Consider a causal linear time-invariant (LTI) system with unit impulse response $h(t)$ satisfies the following properties:

   (1) $dh(t)/dt + 2h(t) = (K+\exp\{-4t\})u(t)$, where K is an unknown constant and $u(t)$ is the unit step function.
   (2) Let the input to the LTI system be $x(t) = \exp\{2t\}$ for all t, then the output of the LTI system is $y(t) = (1/6)\exp\{2t\}$ for all t.

   (a) Find the constant K. [3]
   (b) Determine the system function $H(s)$ of the LTI system. [3]
   (c) Determine the region of convergence for $H(s)$. [3]

8. [15] Consider the following system for filtering a continuous-time signal using a discrete-time filter, where $p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$ denotes the sampling function with sampling period T and sampling frequency $\omega_s = 2\pi/T$. Let the input signal $x_c(t) = \sin(\pi t/T)/\pi t$ and $y_c(t) = dx_c(t)/dt$

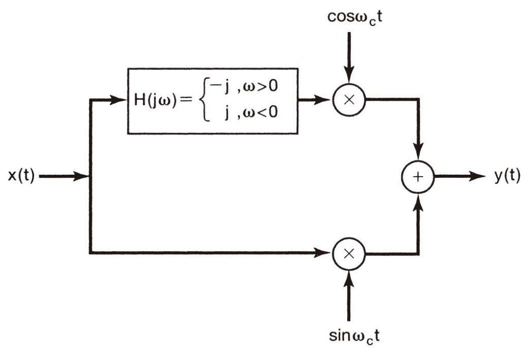


(a) Find $x_d[n]$. [5]

(b) Find $y_d[n]$. [5]

(c) Find the impulse response $h_d[n]$ of the filter $H_d(e^{j\Omega})$. [5]

9. [8] Consider a system for generating a modulated signal $y(t)$ from the message signal $x(t)$ as follows:



Assume that the message signal $x(t)$ has its spectrum $X(j\omega) = 0$ for $|\omega| > \omega_M$ and $\omega_c > \omega_M$.

(a) Is $y(t)$ a real modulated signal if $x(t)$ is real ? Justify your answer. [4]

(b) How do you recover the message signal $x(t)$ from the modulated signal $y(t)$? Justify your answer. [4]