微3.
100-1 丁班小考.

2011 年微分方程小考

1. Find power series solutions about $x = 0$ for the following DE. (20 %)

$$xy'' + (1-x)y' - y = 0$$

(註：這一題為 Sec. 6-2 的練習題)

2. Find the inverse Laplace transforms of (40 %)

(a) $\dfrac{1}{s^2+9}$

(b) $\dfrac{s+2}{s^2+4s+3}e^{-3s}$

(註：這一題為基本題，但第二小題要綜合多個觀念)

3. Use Fourier series to solve the differential equation (20 %)

$$x'' + 10x = f(t),$$

subject to the initial conditions $x(0) = 0$, $x'(0) = 0$, where

$$f(t) = \begin{cases} 5 & 0 < t < \pi \\ -5 & \pi < t < 2\pi \end{cases}, \qquad f(t) = f(t+2\pi)$$

(註：這一題為 2007 年的考古題)

4. Solve $u(x, t)$ (20 %)

$$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \qquad -\infty < x < \infty, \quad t > 0$$

$$u(x,0) = \begin{cases} 0 & x < -1 \\ -100 & -1 < x < 0 \\ 100 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$$

(註：這一題為 Sec. 14-4 的練習題)