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## 1 Basic

### 1.1 .vimrc

```
| syn on
| se ai nu ru cul mouse=a
| se cin et ts=2 sw=2 sts=2
| so $VIMRUNTIME/mswin.vim
| colo desert
| se gfn=Monospace\ 14
```

#### 1.2 Increase Stack Size

```
//stack resize
asm( "mov %0, %%esp\n" :: "g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
   const rlim_t ks = 64*1024*1024;
   struct rlimit rl;
   int res=getrlimit(RLIMIT_STACK, &rl);
   if(res==0){
     if(rl.rlim_cur<ks){
        rl.rlim_cur=ks;
        res=setrlimit(RLIMIT_STACK, &rl);
   }
   }
}</pre>
```

#### 1.3 Misc

```
#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }
#define SECs (clock() / CLOCKS_PER_SEC)
struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;
```

## 2 flow

## 2.1 ISAP

```
#define SZ(c) ((int)(c).size())
struct Maxflow {
  static const int MAXV = 20010;
  static const int INF = 1000000;
  struct Edge {
    int v, c, r;
Edge(int _v, int _c, int _r):
       v(_v), c(_c), r(_r) {}
  int s, t;
  vector<Edge> G[MAXV*2];
  int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
  void flowinit(int x) {
     tot = x+2;
    s = x+1, t = x+2;
for(int i = 0; i <= tot; i++) {
       G[i].clear()
       iter[i] = d[i] = gap[i] = 0;
  void addEdge(int u, int v, int c) {
  G[u].push_back(Edge(v, c, SZ(G[v]) ));
  G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
  int dfs(int p, int flow) {
     if(p == t) return flow;
```

```
for(int &i = iter[p]; i < SZ(G[p]); i++) {</pre>
      Edge &e = G[p][i]
      if(e.c > 0 \& d[p] == d[e.v]+1) {
        int f = dfs(e.v, min(flow, e.c));
        if(f) {
          e.c -= f;
          G[e.v][e.r].c += f;
          return f;
      }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
      d[p]++;
      iter[p] = 0;
      ++gap[d[p]];
    return 0;
  int maxflow() {
    //puts("MF");
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res:
} flow;
Maxflow::Edge e(1, 1, 1);
```

## 2.2 MinCostFlow

```
A template for Min Cost Max Flow
  tested with TIOJ 1724
struct MinCostMaxFlow{
  static const int MAXV = 20010;
  static const int INF = 10000000000:
  struct Edge{
    int v, cap, w, rev;
    Edge(){}
    Edge(int t2, int t3, int t4, int t5)
    : v(t2), cap(t3), w(t4), rev(t5) {}
  int V, s, t;
  vector<Edge> g[MAXV];
  void init(int n){
    V = n+2;
    s = n+1, t = n+2;
    for(int i = 1; i <= V; i++) g[i].clear();</pre>
  void addEdge(int a, int b, int cap, int w){
    g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
    g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
 int d[MAXV], id[MAXV], mom[MAXV];
bool inqu[MAXV];
  int qu[2000000], ql, qr;
  //the size of qu should be much large than MAXV
  int mncmxf(){
    int mxf = 0, mnc = 0;
    while(1){
      fill(d+1, d+1+V, INF);
      fill(inqu+1, inqu+1+V, 0);
      fill(mom+1, mom+1+V, -1);
      mom[s] = s;
      d[s] = 0;
      q\bar{l} = 1, qr = 0;

qu[++qr] = s;
      inqu[s] = 1;
      while(ql <= qr){</pre>
        int u = qu[ql++];
        inqu[u] = 0;
        for(int i = 0; i < (int) g[u].size(); i++){</pre>
          Edge &e = g[u][i];
          int v = e.v;
          if(e.cap > 0 \& d[v] > d[u]+e.w){
            d[v] = d[u] + e.w;
            mom[v] = u;
            id[v] = i;
            if(!inqu[v]) qu[++qr] = v, inqu[v] = 1;
```

```
}
       if(mom[t] == -1) break ;
       int df = INF;
       for(int u = t; u != s; u = mom[u])
       df = min(df, g[mom[u]][id[u]].cap);
for(int u = t; u != s; u = mom[u]){
         Edge &e = g[mom[u]][id[u]];
          e.cap
         g[e.v][e.rev].cap += df;
       mxf += df;
       mnc += df*d[t];
     return mnc;
} flow;
```

## 2.3 Hungarian

```
#define NIL -1
#define INF 100000000
int n,matched:
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN],ylabel[MAXN];
int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-
     cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
  int i,delta=INF;
  for(i=0;i<n;i++) if(!sett[i]) delta=min(slack[i],</pre>
       delta);
  for(i=0;i<n;i++) if(sets[i]) xlabel[i]-=delta;</pre>
  for(i=0;i<n;i++) {</pre>
    if(sett[i]) ylabel[i]+=delta;
    else slack[i]-=delta;
inline void add_sets(int x) {
  int i;
  sets[x]=1;
  for(i=0;i<n;i++) {</pre>
     if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {</pre>
       slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
       prev[i]=x;
  }
inline void augment(int final) {
  int x=prev[final],y=final,tmp;
  matched++
  while(1) {
    tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
     if(y==NIL) return;
    x=prev[y];
inline void phase() {
  int i,y,root;
  for(i=0;i<n;i++) { sets[i]=sett[i]=0; slack[i]=INF; }</pre>
  for(root=0;root<n&xy[root]!=NIL;root++);</pre>
  add_sets(root);
  while(1) {
    relabel();
     for(y=0;y<n;y++) if(!sett[y]&&slack[y]==0) break;</pre>
    if(yx[y]==NIL) { augment(y); return; }
    else { add_sets(yx[y]); sett[y]=1; }
  }
inline int hungarian() {
  int i,j,c=0;
  for(i=0;i<n;i++)</pre>
    xy[i]=yx[i]=NIL
    xlabel[i]=ylabel[i]=0;
     for(j=0;j<n;j++) xlabel[i]=max(cost[i][j],xlabel[i</pre>
```

```
for(i=0;i<n;i++) phase();
for(i=0;i<n;i++) c+=cost[i][xy[i]];
return c;
}</pre>
```

## 2.4 Hungarian Unbalanced

```
const int nil = -1;
const int inf = 10000000000:
int xn,yn,matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN], ylabel[MAXN];
int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-
   cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
  int i,delta=inf;
  for(i=0;i<yn;i++) if(!sett[i]) delta=min(slack[i],</pre>
       delta);
  for(i=0;i<xn;i++) if(sets[i]) xlabel[i]-=delta;</pre>
  for(i=0;i<yn;i++) {</pre>
     if(sett[i]) ylabel[i]+=delta;
     else slack[i]-=delta;
  }
inline void add_sets(int x) {
  int i:
  sets[x]=1;
  for(i=0;i<yn;i++) {</pre>
    if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {</pre>
       slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
       prev[i]=x;
  }
inline void augment(int final) {
  int x=prev[final],y=final,tmp;
  matched++;
  while(1) {
     tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
     if(y==nil) return;
     x=prev[y];
  }
inline void phase() {
  int i,y,root;
  for(i=0;i<xn;i++) sets[i]=0;</pre>
  for(i=0;i<yn;i++) { sett[i]=0; slack[i]=inf; }</pre>
  for(root=0;root<xn&xy[root]!=nil;root++);</pre>
  add_sets(root);
  while(1) -
     relabel():
     for(y=0;y<yn;y++) if(!sett[y]&&slack[y]==0) break;</pre>
     if(yx[y]==nil) { augment(y); return; }
     else { add_sets(yx[y]); sett[y]=1; }
  }
inline int hungarian() {
  int i,j,c=0;
  matched=0;
  // we must have "xn<yn"
  bool swapxy=0;
  if(xn>yn) {
     swapxy=1;
     int mn=max(xn,yn);
     swap(xn,yn);
     for(int i=0;i<mn;i++)</pre>
       for(int j=0;j<i;j++)</pre>
         swap(cost[i][j],cost[j][i]);
  for(i=0;i<xn;i++) {</pre>
     xy[i]=nil;
     xlabel[i]=0;
     for(j=0;j<yn;j++) xlabel[i]=max(cost[i][j],xlabel[i</pre>
          1);
  for(i=0;i<yn;i++) {</pre>
    yx[i]=nil;
```

```
ylabel[i]=0;
}
for(i=0;i<xn;i++) phase();
for(i=0;i<xn;i++) c+=cost[i][xy[i]];
// recover cost matrix (if necessary)
if(swapxy) {
   int mn=max(xn,yn);
   swap(xn,yn);
   for(int i=0;i<mn;i++)
      for(int j=0;j<i;j++)
      swap(cost[i][j],cost[j][i]);
}
// need special recovery if we want more info than
      matching value
return c;
}</pre>
```

#### 2.5 DMST

```
* Edmond's algoirthm for Directed MST
 * runs in O(VE)
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
  int u, v, c;
  Edge(){}
  Edge(int x, int y, int z) :
    u(x), v(y), c(z){}
int V, E, root
Edge edges[MAXE]
inline int newV(){
  ٧++;
  return V;
inline void addEdge(int u, int v, int c){
  E++:
  edges[E] = Edge(u, v, c);
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
  fill(con, con+V+1, 0);
  int r1 = 0, r2 = 0;
  while(1){
    fill(mnInW, mnInW+V+1, INF);
    fill(prv, prv+V+1, -1);
    REP(i, 1, E){
      int u=edges[i].u, v=edges[i].v, c=edges[i].c;
      if(u != v && v != root && c < mnInW[v])
        mnInW[v] = c, prv[v] = u;
    fill(vis, vis+V+1, -1);
    fill(cyc, cyc+V+1, -1);
    r1 = 0;
    bool jf = 0;
    REP(i, 1, V){
      if(con[i]) continue ;
if(prv[i] == -1 && i != root) return -1;
      if(prv[i] > 0) r1 += mnInW[i];
      int s;
      for(s = i; s != -1 && vis[s] == -1; s = prv[s])
        vis[s] = i;
      if(s > 0 \&\& vis[s] == i){
         // get a cycle
        if = 1;
        int v = s;
        do{
          cyc[v] = s, con[v] = 1;
          r2 += mnInW[v];
          v = prv[v];
        }while(v != s);
        con[s] = 0;
      }
    if(!jf) break ;
    REP(i, 1, E){
      int &u = edges[i].u;
```

```
int &v = edges[i].v;
if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
if(u == v) edges[i--] = edges[E--];
}
return r1+r2;
}
```

#### 2.6 SW min-cut

```
// global min cut
struct SW{ // O(V^3)
  static const int MXN = 514;
  int n,vst[MXN],del[MXN];
  int edge[MXN][MXN],wei[MXN];
  void init(int _n){
    n = _n;
    FZ(edge);
    FZ(del);
  void add_edge(int u, int v, int w){
    edge[u][v] += w;
    edge[v][u] += w;
  void search(int &s, int &t){
    FZ(vst); FZ(wei);
s = t = -1;
    while (true){
       int mx=-1, cur=0;
       for (int i=0; i<n; i++)
         if (!del[i] && !vst[i] && mx<wei[i])</pre>
      cur = i, mx = wei[i];
if (mx == -1) break;
       vst[cur] = 1;
       s = t:
       t = cur;
       for (int i=0; i<n; i++)</pre>
         if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){</pre>
       search(x,y);
       res = min(res,wei[y]);
       del[y] = 1;
       for (int j=0; j<n; j++)
  edge[x][j] = (edge[j][x] += edge[y][j]);</pre>
    return res;
  }
}graph;
```

### 2.7 Max Cost Circulation

```
Edge& e = g[ i ][ j ];
if( e.c && dis[ e.v ] < dis[ i ]+e.w ) {
              dis[e.v] = dis[i] + e.w;
             prv[_e.v] = i;
              prve[ e.v ] = j;
              if( t == n ) {
                tmp = i;
                break;
           }
        }
      }
     if( tmp == -1 ) return 0;
    int cur = tmp;
    while( !vis[ cur ] ) {
       vis[ cur ] = 1;
       cur = prv[ cur ];
    int now = cur;
    int cost = 0 , df = 100000;
    do{
      Edge &e = g[ prv[ now ] ][ prve[ now ] ];
df = min( df , e.c );
       cost += e.w;
    now = prv[ now ];
}while( now != cur );
    ans += df*cost;
    now = cur;
    do{
       Edge &e = g[prv[now]][prve[now]];
       Edge &re = g[now][e.r];
      e.c -= df;
re.c += df;
       now = prv[now];
    }while( now != cur');
    return 1;
} circ;
```

## 2.8 Max flow with lower/upper bound

```
// Max flow with lower/upper bound on edges
// source = 1 , sink = n
int in[ N ] , out[ N ];
int l[ M ] , r[ M ] , a[ M ] , b[ M ];
int solve(){
  flow.init( n );
for( int i = 0 ; i < m ; i ++ ){
     in[r[i]] += a[i];
     out[ l[ i ] ] += a[ i ];
flow.addEdge( l[ i ] , r[ i ] , b[ i ] - a[ i ] );
// flow on edge from l[ i ] to r[ i ] should
     // be in [a[ i ], b[ i ]].
  int nd = 0;
  for( int i = 1 ; i <= n ; i ++ ){
  if( in[ i ] < out[ i ] ){</pre>
        flow.addEdge( i , flow.t , out[ i ] - in[ i ] );
nd += out[ i ] - in[ i ];
     if( out[ i ] < in[ i ] )
        flow.addEdge( flow.s , i , in[ i ] - out[ i ] );
  // original sink to source
  flow.addEdge( n , 1 , INF );
if( flow.maxflow() != nd )
  // no solution
     return -1;
   int ans = flow.G[ 1 ].back().c; // source to sink
  flow.G[1].back().c = flow.G[n].back().c = 0;
  // take out super source and super sink
  for( size_t i = 0 ; i < flow.G[ flow.s ].size() ; i</pre>
     flow.G[ flow.s ][ i ].c = 0;
Edge &e = flow.G[ flow.s ][ i ];
     flow.G[ e.v ][ e.r ].c = 0;
  for( size_t i = 0 ; i < flow.G[ flow.t ].size() ; i</pre>
     ++ ){
flow.G[ flow.t ][ i ].c = 0;
```

```
Edge &e = flow.G[ flow.t ][ i ];
  flow.G[ e.v ][ e.r ].c = 0;
}
flow.addEdge( flow.s , 1 , INF );
flow.addEdge( n , flow.t , INF );
flow.reset();
return ans + flow.maxflow();
}
```

#### 2.9 Flow Method

```
Maximize c^T x subject to Ax \leq b, x \geq 0;
with the corresponding symmetric dual problem,
Minimize b^T y subject to A^T y \geq c, y \geq 0.
Maximize c^T x subject to Ax \le b;
with the corresponding asymmetric dual problem,
Minimize b^T y subject to A^T y = c, y \ge 0.
Minimum vertex cover on bipartite graph =
Maximum matching on bipartite graph =
Max flow with source to one side, other side to sink
To reconstruct the minimum vertex cover, dfs from each
unmatched vertex on the left side and with unused edges
only. Equivalently, dfs from source with unused edges
only and without visiting sink. Then, a vertex is
    chosen
iff. it is on the left side and without visited or on
the right side and visited through dfs.
Maximum density subgraph ( \sum W_e + \sum W_v ) / |V|
Binary search on answer:
For a fixed D, construct a Max flow model as follow:
Let S be Sum of all weight( or inf)
1. from source to each node with cap = S

    For each (u,v,w) in E, (u->v,cap=w), (v->u,cap=w)
    For each node v, from v to sink with cap = S + 2 * D - deg[v] - 2 * (W of v)

where deg[v] = \sum weight of edge associated with v
If maxflow < S * IVI, D is an answer.
Requiring subgraph: all vertex can be reached from
```

#### 

## 3 Math

source with

#### 3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
//
// To implement poly. multiply:
//
// fft( n , a );
// fft( n , b );
// for( int i = 0 ; i < n ; i++ )
// c[ i ] = a[ i ] * b[ i ];
// fft( n , c , 1 );
//
// then you have the result in c :: [cplx]
typedef long double ld;</pre>
```

```
typedef complex<ld> cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
  for(int i=0; i<=MAXN; i++)
  omega[i] = exp(i * 2 * PI / MAXN * I);</pre>
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {
  cplx w = omega[inv ? MAXN-(i*theta%MAXN)</pre>
                              : i*theta%MAXN];
       for (int j = i; j < n; j += m) {
         int k = j + mh;
         cplx x = a[j] - a[k];
         a[j] += a[k];
         a[k] = w * x;
      }
    theta = (theta * 2) % MAXN;
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
     for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);</pre>
  if (inv)
    for (i = 0; i < n; i++)
a[i] /= n;
```

### 3.2 NTT

```
typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
         2<sup>n</sup>
   n
                                        root
         32
                      97
                                   3
                                        5
   6
         64
                      193
                                   3
                                        5
         128
                      257
                                        3
                                        3
   8
         256
                      257
                                   1
         512
                      7681
                                   15
                                        17
   10
         1024
                      12289
                                  12
                                        11
         2048
                                   6
   11
                      12289
                                        11
         4096
                      12289
                                   3
                                        11
   13
         8192
                      40961
                                   5
                                        3
   14
         16384
                      65537
                                        3
   15
         32768
                      65537
                      65537
   16
         65536
                                  1
                                        3
   17
         131072
                      786433
                                   6
                                        10
         262144
                      786433
                                  3
                                        10 (605028353,
   18
        2308, 3)
         524288
                      5767169
                                   11
                                        3
   20
         1048576
                      7340033
                                        3
   21
         2097152
                      23068673
                                   11
                                        3
         4194304
                                   25
                                        3
                      104857601
   23
         8388608
                                  20
                                        3
                      167772161
   24
         16777216
                      167772161
                                  10
   25
         33554432
                      167772161
                                        3 (1107296257, 33,
        10)
         67108864
                      469762049 7
         134217728
   27
                      2013265921 15
                                        31 */
// (must be 2^k)
// To implement poly. multiply:
// NTT<P, root, MAXN> ntt;
// ntt( n , a ); // or ntt.tran( n , a );
// ntt( n , b );
// for( int i = 0 ; i < n ; i++ )
// c[i] = a[i] * b[i];
// ntt( n , c , 1 );
// then you have the result in c :: [LL]
template<LL P, LL root, int MAXN>
struct NTT{
 static LL bigmod(LL a, LL b) {
```

```
LL res = 1:
     for (LL bs = a; b; b >>= 1, bs = (bs * bs) % P) {
       if(b&1) res=(res*bs)%P;
     return res;
  static LL inv(LL a, LL b) {
     if(a==1)return 1;
     return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
  LL omega[MAXN+1];
  NTT() {
     omega[0] = 1;
     LL r = bigmod(root, (P-1)/MAXN);
     for (int i=1; i<=MAXN; i++)
  omega[i] = (omega[i-1]*r)%P;</pre>
  // n must be 2^k
  void tran(int n, LL a[], bool inv_ntt=false){
     int basic = MAXN / n;
     int theta = basic;
     for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
for (int i = 0; i < mh; i++) {
    LL w = omega[i*theta%MAXN];</pre>
          for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
             LL x = a[j] - a[k];
             if (x < 0) x += P;
             a[j] += a[k];
if (a[j] > P) a[j] -= P;
a[k] = (w * x) % P;
        theta = (theta * 2) % MAXN;
     int i = 0;
     for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
  if (j < i) swap(a[i], a[j]);</pre>
     if (inv_ntt) {
        LL ni = inv(n,P);
       reverse( a+1 , a+n );

for (i = 0; i < n; i++)

a[i] = (a[i] * ni) % P;
  }
  void operator()(int n, LL a[], bool inv_ntt=false) {
     tran(n, a, inv_ntt);
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;
```

### 3.3 Fast Walsh Transform

```
/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
* z = (x0y0 + x1y1 , x0y1 + x1y0 )
 * x' = (x0+x1, x0-x1), y' = (y0+y1, y0-y1)
* z' = ((x0+x1)(y0+y1), (x0-x1)(y0-y1))
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1 < 20) + 10;
const LL MOD = 1e9+7;
inline LL pw( LL x , LL k ) {
  LL res = 1;
  for( LL bs = x ; k ; k >>= 1, bs = (bs * bs)%MOD ){
  if( k&1 ) res = ( res * bs ) % MOD;
  return res;
inline LL inv( LL x ) {
```

```
return pw( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <<= 1 ) {
        int d2 = d <<1;
        for( int s = 0 ; s < N ; s += d2 ) {
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ) {
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
                }
        }
}
if( inv )
        for( int i = 0 ; i < N ; i++ ) {
                x[ i ] *= inv( N );
                x[ i ] *= MOD;
        }
}
3.4 BigInt</pre>
```

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int s, vl, v[LEN];
       vector<int> v
  Bigint() : s(1) \{ vl = 0; \}
  Bigint(long long a) {
     s = 1; vl = 0;
if (a < 0) { s = -1; a = -a; }
     while(a)
       push_back(a % BIGMOD),
       a \neq BIGMOD;
  Bigint(string str) {
     s = 1; vl = 0;
     int stPos = 0, num = 0;
     if (!str.empty() && str[0] == '-')
       stPos = 1, s = -1;
     for (int i=$Z(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
  if ((q *= 10) >= BIGMOD)
         push_back(num), num = 0, q = 1;
     if (num) push_back(num);
    n();
  int len() const { return vl; } // return SZ(v);
  bool empty() const { return len() == 0; }
  void push_back(int x) { v[vl++] = x; } // v.PB(x);
void pop_back() { vl--; } // v.pop_back();
int back() const { return v[vl-1]; } // return v.back
       ()
  void n(){ while (!empty() && !back()) pop_back(); }
  void resize(int nl) {
    vl = nl; fill(v, v+vl, 0);
// v.resize(nl); fill(ALL(v), 0);
  void print() const {
     if (empty()) { putchar('0'); return; }
     if (s == -1) putchar('-');
printf("%d", back());
     for(int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  int cp3(const Bigint &b)const {
     if (s != b.s) return s - b.s;
     if (s == -1) return -(-*this).cp3(-b);
     if (len() != b.len()) return len()-b.len();//int
     for (int i=len()-1; i>=0; i--)
       if (v[i]!=b.v[i]) return v[i]-b.v[i];
     return 0;
  bool operator<(const Bigint &b)const
     { return cp3(b)<0; }
  bool operator<=(const Bigint &b)const</pre>
  { return cp3(b)<=0; }
bool operator==(const Bigint &b)const
     { return cp3(b)==0; }
```

```
bool operator!=(const Bigint &b)const
  { return cp3(b)!=0; }
bool operator>(const Bigint &b)const
  { return cp3(b)>0; }
bool operator>=(const Bigint &b)const
  { return cp3(\dot{b})>=0; }
Bigint operator - () const {
  Bigint r = (*this); r.s = -r.s; return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  // directly add TODO
  r.n(); return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b)
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  // directly sub TODO
  r.n(); return r;
Bigint operator * (const Bigint &b) {
  Bigint r;
  r.resize(len() + b.len() + 1);
r.s = s * b.s;
  // directly mul TODO
  r.n(); return r;
Bigint operator / (const Bigint &b) {
  Bigint r;
  r.resize(max(1, len()-b.len()+1));
  int oriS = s;
  Bigint b2 = b; // b2 = abs(b)
  s = b2.s = r.s = 1;
  for (int i=r.len()-1; i>=0; i--) {
    int d=0, u=BIGMOD-1;
    while(d<u) {</pre>
       int m = (d+u+1)>>1;
       r.v[i] = m;
       if((r*b2) > (*this)) u = m-1;
       else d = m;
    r.v[i] = d;
  s = oriS; r.s = s * b.s;
  r.n(); return r;
Bigint operator % (const Bigint &b)
{ return (*this)-(*this)/b*b; }
```

### 3.5 Linear Recurrence

```
LL n, m;
LL dp[N+N];
void_pre_dp(){
  dp[0] = 1;
  LL bdr = min(m + m, n);
  for( LL i = 1 ; i <= bdr ; i ++ )
for( LL j = i - 1 ; j >= max(0LL , i - m) ; j -- )
     dp[i] = add(dp[i], dp[j]);
vector<LL> Mul( vector<LL>& v1, vector<LL>& v2 ){
  int _sz1 = (int)v1.size();
  int _sz2 = (int)v2.size();
 assert( _sz1 == m );
 assert( _sz2 == m );
 vector<LL> _v( m + m );
  for( int i = 0; i < m + m; i ++) _v[ i ] = 0;
// expand
 // shrink
```

```
for( int i = 0 ; i < m ; i ++ )
  for( int j = 1 ; j <= m ; j ++ )
    _v[ i + j ] = add( _v[ i + j ] , _v[ i ] );
for( int i = 0 ; i < m ; i ++ )
    _v[ i ] = _v[ i + m ];</pre>
   _v.resize( m );
   return _v;
}
vector<LL> I, A;
void solve(){
   pre_dp();
   if(n \le m + m)
   { printf( "%lld\n" , dp[ n ] ); exit( 0 ); }
   I.resize( m ); A.resize( m );
for( int i = 0 ; i < m ; i ++ ) I[ i ] = A[ i ] = 1;</pre>
// dp[ n ] = Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
   LL dlt = ( n - m ) / m, rdlt = dlt *
while( dlt ){
      if( dlt & 1LL ) I = Mul( I , A );
      A = Mul(A, A);
      dlt >>= 1;
   \tilde{L}L ans = 0;
   for( int i = 0 ; i < m ; i ++ )</pre>
   ans = add(ans, mul(I[i], dp[n - i - 1 - rdlt]));
printf( "%lld\n" , ans );
```

### 3.6 Miller Rabin

```
// n < 4,759,123,141
                              3: 2, 7, 61
// n < 1,122,004,669,633
                                   2, 13, 23, 1662803
// n < 3,474,749,660,383
                                    6
                                      : pirmes <= 13
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, -n2] if
// you want to use magic.
bool witness(LL a,LL n,LL u,int t){
  LL x=mypow(a,u,n);
  for(int i=0;i<t;i++) {</pre>
    LL nx=mul(x,x,n);
     if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
  return x!=1;
bool miller_rabin(LL n,int s=100) {
  // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;
  if(!(n\&1)) return n == 2;
  LL u=n-1; int t=0;
// n-1 = u*2^t
  while(!(u&1)) u>>=1, t++;
  while(s--){
    LL a=randll()%(n-1)+1;
     if(witness(a,n,u,t)) return 0;
  return 1;
}
```

# 3.7 Simplex

```
const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXM], d[MAXN][MAXM];
double x[MAXM]
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// \max{cx}  subject to \{Ax <= b, x >= 0\}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
                 double c[MAXM], int n, int m){
  ++m;
  int r = n, s = m - 1;
  memset(d, 0, sizeof(d));
  for (int i = 0; i < n + m; ++i) ix[i] = i; for (int i = 0; i < n; ++i) {
```

```
for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
d[i][m - 1] = 1;
d[i][m] = b[i];</pre>
   if (d[r][m] > d[i][m]) r = i;
for (int j = 0; j < m - 1; ++j) d[n][j] = c[j]; d[n + 1][m - 1] = -1;
for (double dd;; ) {
   if (r < n) {
     int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
d[r][s] = 1.0 / d[r][s];
for (int j = 0; j <= m; ++j)
    if (j != s) d[r][j] *= -d[r][s];</pre>
     for (int i = 0; i <= n + 1; ++i) if (i != r) {
  for (int j = 0; j <= m; ++j) if (j != s)
    d[i][j] += d[r][j] * d[i][s];
        d[i][s] *= d[r][s];
  }
   r = -1; s = -1;
  for (int j = 0; j < m; ++j)
if (s < 0 || ix[s] > ix[j]) {
        if (d[n + 1][j] > eps | I
              (d[n + 1][j] > -eps && d[n][j] > eps))
   if (s < 0) break;
   for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
     if (r < 0 ||
           (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s])
                 < -eps ||
           (dd < eps \&\& ix[r + m] > ix[i + m]))
        r = i;
   if (r < 0) return -1; // not bounded
if (d[n + 1][m] < -eps) return -1; // not executable</pre>
double ans = 0;
for(int i=0; i<m; i++) x[i] = 0;</pre>
for (int i = m; i < n + \overline{m}; ++i) \{ // \text{ the missing } \}
      enumerated x[i] = 0
  if (ix[i] < m - 1){
  ans += d[i - m][m] * c[ix[i]];</pre>
     x[ix[i]] = d[i-m][m];
  }
return ans;
```

## 3.8 Faulhaber

```
/* 'faulhabers formula -
 * cal power sum formula of all p=1\sim k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK];
// bernoulli number
int inv[MAXK+1];
// inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2];
// coeeficient of x^j when p=i
inline int getinv(int x) {
  int a=x,b=mod,a0=1,a1=0,b0=0,b1=1;
  while(b) {
    int q,t;
    q=a/b; t=b; b=a-b*q; a=t;
    t=b0; b0=a0-b0*q; a0=t;
    t=b1; b1=a1-b1*q; a1=t;
  return a0<0?a0+mod:a0;</pre>
inline void pre() {
  /* combinational */
  for(int i=0;i<=MAXK;i++) {</pre>
     cm[i][0]=cm[i][i]=1;
    for(int j=1;j<i;j++)
  cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);</pre>
   /* inverse */
  for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);</pre>
```

```
/* bernoulli */
  b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
  for(int i=2;i<MAXK;i++) {</pre>
    if(i&1) { b[i]=0; continue; }
    b[i]=1;
    for(int j=0; j<i; j++)</pre>
      b[i]=sub(b[i], mul(cm[i][j],mul(b[j], inv[i-j+1])
  }
/* faulhaber */
  // sigma_x=1~n \{x^p\} = 1/(p+1) * sigma_j=0~p \{ C(p+1, p+1) \}
       i) * Bj * n^{(p-j+1)}
  for(int i=1;i<MAXK;i++) {</pre>
    co[i][0]=0;
    for(int j=0;j<=i;j++)</pre>
      co[i][i-j+1]= mul(inv[i+1], mul(cm[i+1][j], b[j])
}
/* sample usage: return f(n,p) = sigma_x=1\sim (x^p) */
inline int solve(int n,int p) {
  int sol=0, m=n;
  for(int i=1;i<=p+1;i++) {</pre>
    sol=add(sol,mul(co[p][i],m));
    m = mul(m, n);
  return sol;
```

## 3.9 Chinese Remainder

```
int pfn;
// number of distinct prime factors
int pf[MAXN]; // prime factor powers
int rem[MAXN]; // corresponding remainder
int pm[MAXN];
inline void generate_primes() {
  int i,j;
  pnum=1
  prime[0]=2;
  for(i=3;i<MAXVAL;i+=2) {</pre>
     if(nprime[i]) continue;
     prime[pnum++]=i;
     for(j=i*i;j<MAXVAL;j+=i) nprime[j]=1;</pre>
  }
inline int inverse(int x,int p) {
  int q, tmp, a=x, b=p;
  int a0=1,a1=0,b0=0,b1=1;
  while(b) {
    q=a/b; tmp=b; b=a-b*q; a=tmp;
tmp=b0; b0=a0-b0*q; a0=tmp;
     tmp=b1; b1=a1-b1*q; a1=tmp;
  return a0;
inline void decompose_mod() {
  int i,p,t=mod;
  pfn=0;
  for(i=0;i<pnum&&prime[i]<=t;i++) {</pre>
     p=prime[i];
     if(t\%p==0)^{-1}
       pf[pfn]=1;
       while(t%p==0) {
        t/=p
         pf[pfn]*=p;
       pfn++;
  if(t>1) pf[pfn++]=t;
}
inline int chinese_remainder() {
  int i,m,s=0;
  for(i=0;i<pfn;i++) {</pre>
    m=mod/pf[i];
    pm[i]=(LL)m*inverse(m,pf[i])%mod;
    s=(s+(LL)pm[i]*rem[i])%mod;
  return s;
}
```

### 3.10 Pollard Rho

```
// does not work when n is prime
LL f(LL x, LL mod){
   return add(mul(x,x,mod),1,mod);
}
LL pollard_rho(LL n) {
   if(!(n&1)) return 2;
   while(true){
      LL y=2, x=rand()%(n-1)+1, res=1;
      for(int sz=2; res==1; sz*=2) {
        for(int i=0; i<sz && res<=1; i++) {
            x = f(x, n);
            res = __gcd(abs(x-y), n);
      }
      y = x;
   }
   if (res!=0 && res!=n) return res;
}</pre>
```

## 3.11 Poly Generator

```
struct PolyGen{
     ' for a nth-order polynomial f(x), *
   * given f(0), f(1), ..., f(n) *
* express f(x) as sigma_i{c_i*C(x,i)} */
  vector<LL> coef;
  // initialize and calculate f(x), vector _fx should
  // be filled with f(0) to f(n)
  PolyGen(int _n,vector<LL> _fx):n(_n),coef(_fx){
     for(int i=0;i<n;i++)</pre>
       for(int j=n; j>i; j--)
   coef[j]-=coef[j-1];
  \frac{1}{1} evaluate f(x), runs in O(n)
  LL eval(int x){
     LL m=1, ret=0;
     for(int i=0;i<=n;i++){
  ret+=coef[i]*m;</pre>
       m=m*(x-i)/(i+1);
     return ret;
};
```

#### 3.12 Matrix Pseudo Inverse

```
Mat pinv( Mat m ){
   Mat res = I;
   FZ( used );
   for( int i = 0 ; i < W ; i ++ ){</pre>
       int piv = -1;
       for( int j = 0 ; j < W ; j ++ ){
  if( used[ j ] ) continue;
  if( abs( m.v[ j ][ i ] ) > EPS ){
              piv = j;
              break;
          }
       if( piv == -1 ) continue;
      used[ i ] = true;
swap( m.v[ piv ], m.v[ i ] );
swap( res.v[ piv ], res.v[ i ] );
       LD rat = m.v[i][i];
for( int j = 0 ; j < W ; j ++ ){
    m.v[i][j] /= rat;
    res.v[i][j] /= rat;</pre>
       for( int j = 0; j < W; j ++){
          if( j == i ) continue;
          rat = m.v[ j ][ i ];

for( int k = 0 ; k < W ; k ++ ){

    m.v[ j ][ k ] -= rat * m.v[ i ][ k ];

    res.v[ j ][ k ] -= rat * res.v[ i ][ k ];
      }
   for( int i = 0 ; i < W ; i ++ ){</pre>
```

```
if( used[ i ] ) continue;
for( int j = 0 ; j < W ; j ++ )
  res.v[ i ][ j ] = 0;</pre>
   return res;
}
3.13 ax+by=gcd
PII gcd(int a, int b){
   if(b == 0) return {1, 0};
   PII q = gcd(b, a \% b);
   return {q.second, q.first - q.second * (a / b)};
3.14 Discrete sqrt
void calcH(int &t, int &h, const int p) {
   int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
// solve equation x^2 \mod p = a
bool solve(int a, int p, int &x, int &y) {
   if(p == 2) { x = y = 1; return true; }
int p2 = p / 2, tmp = mypow(a, p2, p);
if (tmp == p - 1) return false;
if ((p + 1) % 4 == 0) {
      x=mypow(a,(p+1)/4,p); y=p-x; return true;
      int t, h, b, pb; calcH(t, h, p);
      if (t >= 2) {
         do {b = rand() % (p - 2) + 2;
} while (mypow(b, p / 2, p) != p - 1);
      pb = mypow(b, h, p);
} int s = mypow(a, h / 2, p);
for (int step = 2; step <= t; step++) {
  int ss = (((LL)(s * s) % p) * a) % p;</pre>
         for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
if (ss + 1 == p) s = (s * pb) % p;</pre>
         pb = ((LL)pb * pb) % p;
      x = ((LL)s * a) % p; y = p - x;
   } return true;
3.15 SchreierSims
// time: O(n^2 \lg^3 \lg + t n \lg \lg)
// mem : 0(n^2 \lg |G| + tn)
```

```
// t : number of generator
namespace SchreierSimsAlgorithm{
  typedef vector<int> Permu;
  Permu inv( const Permu& p ){
     Permu ret( p.size() );
for( int i = 0; i < int(p.size()); i ++ )</pre>
       ret[ p[ i ] ] = i;
     return ret:
  Permu operator*( const Permu& a, const Permu& b ){
     Permu ret( a.size() );
     for( int i = 0 ; i < (int)a.size(); i ++ )
  ret[ i ] = b[ a[ i ] ];</pre>
     return ret:
  typedef vector<Permu> Bucket;
typedef vector<int> Table;
  typedef pair<int,int> pii;
  int n, m;
  vector<Bucket> bkts, bktsInv;
  vector<Table> lookup;
  int fastFilter( const Permu &g, bool addToG = 1 ){
     n = bkts.size();
     Permu p;
     for( int i = 0 ; i < n ; i ++ ){
  int res = lookup[ i ][ p[ i ] ];</pre>
       if( res == -1 ){
          if( addToG ){
            bkts[ i ].push_back( p );
bktsInv[ i ].push_back( inv( p ) );
            lookup[ i ][ p[i] ] = (int)bkts[i].size()-1;
          return i;
```

```
p = p * bktsInv[i][res];
      return -1:
   long long calcTotalSize(){
      long long ret = 1;
for( int i = 0 ; i < n ; i ++ )</pre>
        ret *= bkts[i].size();
      return ret;
   bool inGroup( const Permu &g ){
      return fastFilter( g, false ) == -1;
   void solve( const Bucket &gen, int _n ){
     n = _n, m = gen.size(); // m perm[0..n-1]s
      {//clear all
        bkts.clear();
         bktsInv.clear();
        lookup.clear();
      for(int i = 0; i < n; i ++){
        lookup[i].resize(n);
         fill(lookup[i].begin(), lookup[i].end(), -1);
      Permu id( n );
      for(int i = 0'; i < n; i ++ ) id[i] = i;
for(int i = 0; i < n; i ++ ){</pre>
        bkts[i].push_back(id)
        bktsInv[i].push_back(id);
        lookup[i][i] = \vec{0};
      for(int i = 0 ; i < m ; i ++)
  fastFilter( gen[i] );</pre>
      queue< pair<pii,pii> > toUpd;
     for(int i = 0; i < n; i ++)
  for(int j = i; j < n; j ++)
    for(int k = 0; k < (int)bkts[i].size(); k ++)
      for(int l = 0; l < (int)bkts[j].size(); l ++)
            toUpd.push( {pii(i,k), pii(j,l)} );
while( !toUpd.empty() ){
    pii a = toUpd.front().first;
    pii b = toUpd.front().second:</pre>
        pii b = toUpd.front().second;
        toUpd.pop();
        int res = fastFilter(bkts[a.first][a.second] *
                                      bkts[b.first][b.second]);
        if(res == -1) continue:
        pii newPair(res, (int)bkts[res].size() - 1);
         for(int i = 0; i < n; i ++)
  for(int j = 0; j < (int)bkts[i].size(); ++j){</pre>
              if(i <= res)
                 toUpd.push(make_pair(pii(i , j), newPair));
                 toUpd.push(make_pair(newPair, pii(i, j)));
   }
}
```

## 3.16 Discrete K-th sqrt

```
// x^K \mod P = A
const int LimitSave = 100000;
LL _mod( LL a , LL mo ){return ( a % mo + mo ) % mo;}
bool ext_gcd(LL A, LL B, LL C, LL &x, LL &y, LL &gn){
  LL t;
  if( A == 0 ){
     gn = B;
     if( _{mod(C, B)} == 0 )
     { x = 0; y = C / B; return true; }
     return false;
  if( ext_gcd( _mod(B , A) , A , C , y , t , gn ) )
{ x = t - LL(B / A) * y; return true; }
  return false;
LL Division( LL A, LL B, LL modular ){
  LL gcdnum, K, Y;
  ext_gcd(modular, B, A, K, Y, gcdnum);
  Y = _{mod}(Y, modular);
  return Y < 0 ? Y + modular : Y;</pre>
```

```
struct tp{
  LL expo, res;
}data[ LimitSave + 100 ];
bool compareab( const tp &a, const tp &b )
{ return a.res < b.res; }
bool Binary_Search( LL key, LL &pos ){
  LL start, stop
  start=1; stop=LimitSave;
  while( start <= stop ){</pre>
     pos = (start + stop)/2;
     if( data[pos].res == key ) return true;
     if( data[pos].res < key ) start = pos + 1;</pre>
     else stop = pos - 1;
  return false;
LL get_log( LL root , LL A , LL mod ){
  LL i, j, times, XD, XT, position;
  if( mod - 1 < LimitSave ){</pre>
     LL now = 1;
     for( i = 0 ; i < mod ; i ++ ){
       if( now == A ) return i;
now = _mod( now * root , mod );
  data[1].expo = 0; data[1].res = 1;
  for( i = 1; i < LimitSave; i ++'){</pre>
     data[i+1].expo=i;
     data[i+1].res=_mod(data[i].res*root,mod);
  sort(data+1,data+LimitSave+1,compareab);
  times=mypow(root,LimitSave,mod);
  i=0; XD=1;
  while( 1 ){
     XT = Division(A, XD, mod);
     if( Binary_Search( XT, position ) )
       return j + data[position].expo;
     j = j + LimitSave;
     XD = _mod(XD * times, mod);
  }
LL P, K, A;
vector<LL> ans;
LL get_originroot( LL p ){
  LL primes[ 100 ];
LL tot = 0, tp = P - 1;
  for( LL i = 2 ; i * i <= P - 1 ; i ++ )
     if( _mod( tp , i ) == 0 ){
   primes[ ++ tot ]=i;
       while( _{mod(tp,i)} == 0 ) tp /= i;
  if( tp != 1 ) primes[ ++ tot] = tp;
  for( LL i = 2;; i ++){
    bool ok = true;
for( LL j = 1 ; j <= tot ; j ++ )</pre>
       if(mypow(i, (P-1)/primes[j], P) == 1)
        { ok = false; break; }
     if( ok ) return i;
  }
//x^K \mod P = A
void work_ans() {
  cin>>P>>K>>A;
  A = A \% P
  ans.clear(); // roots in ans
   if( A == 0 )
  { ans.push_back( 0 ); return; }
LL root,logs,delta,deltapower,now,gcdnum,x,y;
   root=get_originroot(P);
  logs=get_log(root,A,P);
if( ext_gcd(K, P-1, logs, x, y, gcdnum) ){
  delta=(P-1) / gcdnum;
     x = -mod(x, delta)
     if(x < 0) x += delta;
     now = mypow(root, x, P);
     deltapower = mypow(root, delta, P);
     while(x < P-1){
       ans.push_back(now);
       now=_mod(now * deltapower, P);
       x=x+delta;
```

```
}
3.17 Prefix Inverse
                                                                       r=(r\%m+m)\%m;
void solve( int m ){
                                                                       return _fd(a-r,m)*m+r;
  inv[ 1 ] = 1;
for( int i = 2 ; i < m ; i ++ )
  inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;</pre>
                                                                       m=abs(m);
                                                                       r=(r\%m+m)\%m;
3.18 Roots of Polynomial
                                                                       m=abs(m);
const double eps = 1e-12;
                                                                       a=_nt(a,m,r);
const double inf = 1e+12;
                                                                       b=pv(b,m,r)
double a[ 10 ], x[ 10 ];
int sign( double x ){
  return (x < -eps)?(-1):(x>eps);
double f(double a[], int n, double x){
  double tmp=1,sum=0;
  for(int i=0;i<=n;i++){</pre>
     sum=sum+a[i]*tmp;
     tmp=tmp*x;
  return sum;
double binary(double l,double r,double a[],int n){
                                                                  int mu[N], p_tbl[N];
  int sl=sign(f(a,n,l)), sr=sign(f(a,n,r));
                                                                  vector<int> primes;
  if(sl==0) return l;
  if(sr==0) return r;
  if(sl*sr>0) return inf;
  while(r-l>eps){
                                                                       if( !p_tbl[_i ] ){
     double mid=(l+r)/2;
                                                                         p_tbl[ i ] = i;
     int ss=sign(f(a,n,mid));
                                                                         primes.push_back( i );
     if(ss==0) return mid;
                                                                         mu[i] = -1;
     if(ss*sl>0) l=mid; else r=mid;
                                                                       for( int p : primes ){
  int x = i * p;
  return 1;
                                                                         if( x >= M ) break;
void solve(int n,double a[],double x[],int &nx){
                                                                         p_{tbl[x]} = p;
  if(n==1){
                                                                         mu[x] = -mu[i]
    x[1]=-a[0]/a[1];
                                                                         if( i % p == 0 ){
    nx=1;
                                                                           mu[x] = 0;
    return;
                                                                           break;
                                                                         }
  double da[10], dx[10];
                                                                       }
  int ndx;
for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
                                                                    }
                                                                  }
  solve(n-1,da,dx,ndx);
                                                                  vector<int> factor( int x ){
  nx=0;
                                                                     vector<int> fac{ 1 };
  if(ndx==0){
                                                                     while( x > 1 ){
  int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
     double tmp=binary(-inf,inf,a,n);
     if (tmp<inf) x[++nx]=tmp;</pre>
                                                                       while( x \% p == 0 ){
     return;
                                                                         x /= p;
for( int i = 0 ; i < fn ; i ++ )
  fac.PB( fac[ pos ++ ] * p );</pre>
  double tmp;
  tmp=binary(-inf,dx[1],a,n);
  if(tmp<inf) x[++nx]=tmp;</pre>
  for(int i=1;i<=ndx-1;i++){</pre>
                                                                     return fac;
     tmp=binary(dx[i],dx[i+1],a,n);
                                                                  }
     if(tmp<inf) x[++nx]=tmp;</pre>
                                                                  3.21
                                                                           Result
  tmp=binary(dx[ndx],inf,a,n);
  if(tmp<inf) x[++nx]=tmp;</pre>
int main() {
   scanf("%d",&n);
                                                                  Lucas Theorem:
  for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
                                                                     = mult_i ( C(m_i,n_i) )
  solve(n,a,x,nx);
  for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);</pre>
                                                                  Picks Theorem
                                                                     A = i + b/2 - 1
3.19 Mod
                                                                  Kirchhoff's theorem
/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
```

```
11
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.
int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r){
int _nt(int a,int m,int r){
      return _fd(a-r-1,m)*m+r+m;
int _ct(int a,int b,int m,int r){
       return (a>b)?0:((b-a+m)/m);
3.20 Primes and \mu function
/* 12721, 13331, 14341, 75577, 123457, 222557, 556679 * 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231

* 999888733, 98789101, 987777733, 999991921, 10

* 1010102101, 10000000000039, 1000000000000037
                                                                        1010101333
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
void sieve() {
  mu[ 1 ] = p_tbl[ 1 ] = 1;
   for( int i = 2 ; i < N ; i ++ ){</pre>
```

```
For non-negative integer n,m and prime P,
C(m,n) \mod P = C(m/M,n/M) * C(m%M,n%M) \mod P
where m_i is the i-th digit of m in base P.'
A_{ii} = deg(i), A_{ij} = (i,j) \in ? -1 : 0
Deleting any one row, one column, and cal the det(A)
```

# 4 Geometry

#### 4.1 halfPlaneIntersection

#### 4.2 Intersection of 2 lines

```
Pt interPnt( Line 11, Line 12, bool &res ){
  Pt p1, p2, q1, q2;
tie(p1, p2) = l1;
tie(q1, q2) = l2;
  double f1 = (p2 - p1) ^ (q1 - p1);
double f2 = (p2 - p1) ^ (p1 - q2);
double f = (f1 + f2);
  if( fabs(f) < eps)</pre>
  { res = false; return {0, 0}; }
  res = true;
return q1 * (f2 / f) + q2 * (f1 / f);
bool isin( Line 10, Line 11, Line 12 ){
  // Check inter(l1, l2) in l0
  bool res;
  Pt p = interPnt(l1, l2, res);
  return ( (10.SE - 10.FI) ^ (p - 10.FI) ) > eps;
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])</pre>
 * in all the lines. (use (l.S - l.F) \wedge (p - l.F) > 0
/* --^- Line.FI --^- Line.SE --^- */
vector<Line> halfPlaneInter( vector<Line> lines ){
  int sz = lines.size();
  vector<double> ata(sz), ord(sz);
  for( int i=0; i<sz; i++) {</pre>
     ord[i] = i;
    Pt d = lines[i].SE - lines[i].FI;
    ata[i] = atan2(d.Y, d.X);
  return ata[i] < ata[j];</pre>
  });
  vector<Line> fin;
for (int i=0; i<sz; i++)
   if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
       fin.PB(lines[ord[i]]);
  deque<Line> dq;
  for (int i=0; i<(int)(fin.size()); i++) {
  while((int)(dq.size()) >= 2 and
         not isin(fin[i], dq[(int)(dq.size())-2]
                              dq[(int)(dq.size())-1]))
       dq.pop_back();
     while((int)(dq.size()) >= 2 and
         not isin(fin[i], dq[0], dq[1]))
       dq.pop_front();
    dq.push_back(fin[i]);
  while( (int)(dq.size()) >= 3 and
    not isin(dq[0], dq[(int)(dq.size())-2]
                           dq[(int)(dq.size())-1]))
     dq.pop_back();
  while( (int)(dq.size()) >= 3 and
    not isin(dq[(int)(dq.size())-1], dq[0], dq[1]))
     dq.pop_front()
  vector<Line> res(dq.begin(),dq.end());
  return res;
```

### 4.3 Intersection of 2 segments

## 4.4 Intersection of polygon and circle

```
|Pt ORI , info[ N ];
Dr; int n;
// Divides into multiple triangle, and sum up
// oriented area
D area2(Pt pa, Pt pb){
   if( norm(pa) < norm(pb) ) swap(pa, pb);</pre>
   if( norm(pb) < eps ) return 0;</pre>
  D S, h, theta;
  D a = norm( pb ), b = norm( pa ), c = norm(pb - pa);
D cosB = (pb * (pb - pa)) / a / c, B = acos(cosB);
D cosC = (pa * pb) / a / b, C = acos(cosC);
   if(a > r){
     S = (C/2)*r*r
     h = a*b*sin(C)/c;
     if (h < r \&\& B < PI/2) S = (acos(h/r)*r*r - h*sqrt
          (r*r-h*h));
   else if(b > r){
     theta = PI - B - asin(sin(B)/r*a);
     S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
   else S = .5*sin(C)*a*b;
   return S;
D area() {
  DS = 0;
   for(int i = 0; i < n; ++i)
     S += abs( area2(info[i], info[i + 1]) * sign( det(
    info[i], info[i + 1]));
   return fabs(S);
```

### 4.5 Intersection of 2 circles

#### 4.6 Circle cover

```
#define N 1021
struct CircleCover{
  int C; Circle c[ N ];
  bool g[N][N], overlap[N][N];
  // Area[i]
               : area covered by at least i circles
  D Area[ N ];
  void init( int _C ){ C = _C; }
bool CCinter( Circle& a , Circle& b , Pt& p1 , Pt& p2
    Pt o1 = a.0 , o2 = b.0;
    D r1 = a.R , r2 = b.R;
D d2 = (o1 - o2) * (o1 - o2);
    D d = sqrt(d2);
if( d > r1 + r2 ) return false;
    Pt u = (01+02)*0.5 + (01-02)*((r2*r2-r1*r1)/(2*d2))
    D A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d)
    Pt v = Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
    p1 = u + v; p2 = u - v;
    return true;
  struct Tevent {
    Pt p; D ang; int add;
    Tevent() {}
    Tevent(Pt _a, D _b, int _c): p(_a), ang(_b), add(_c
         ) {}
    bool operator<(const Tevent &a)const
  {return ang < a.ang;}
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1</pre>
  bool disjuct( Circle& a, Circle &b, int x ){
    return sign( norm( a.0 - b.0 ) - a.R - b.R ) > x;
```

```
bool contain( Circle& a, Circle &b, int x ){
  return sign( a.R - b.R - norm( a.O - b.O ) ) > x;
  bool contain(int i, int j){ /* c[j] is non-strictly
        in c[i]. */
     return (sign(c[i].R - c[j].R) > 0 ||
(sign(c[i].R - c[j].R) == 0 && i < j) ) &&
                      contain(c[i], c[j], -1);
   void solve(){
     for( int i = 0 ; i <= C + 1 ; i ++ )
        Area[ i ] = 0;
     for( int i = 0 ; i < C ; i ++ )
  for( int j = 0 ; j < C ; j ++ )
    overlap[i][j] = contain(i, j);</pre>
     for( int i = 0 ; i < C ; i ++ )
  for( int j = 0 ; j < C ; j ++ )
    g[i][j] = !(overlap[i][j] || overlap[j][i] ||</pre>
                          disjuct(c[i], c[j], -1));
     for( int i = 0 ; i < C ; i ++ ){
        int E = 0, cnt = 1;
        for( int j = 0 ; j < C ; j ++ )
  if( j != i && overlap[j][i] )</pre>
             cnt ++;
        for( int j = 0 ; j < C ; j ++ )
  if( i != j && g[i][j] ){</pre>
             Pt aa, bb;
             CCinter(c[i], c[j], aa, bb);
             D A = atan2(aa.Y - c[i].0.Y, aa.X - c[i].0.X)
             D B = atan2(bb.Y - c[i].0.Y, bb.X - c[i].0.X)
             eve[É ++] = Tevent(bb, B, 1);
eve[E ++] = Tevent(aa, A, -1);
             if(B > A) cnt ++;
        if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
        else{
          sort( eve , eve + E );
          eve[E] = eve[0];
          for( int j = 0; j < E; j ++){
             cnt += eve[j].add;
             Area[cnt] \stackrel{--}{+}= (eve[j].p ^ eve[j + 1].p) * .5;
             }
     }
  }
};
```

## 4.7 Convex Hull trick

```
/* Given a convexhull, answer querys in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
  int n;
  vector<Pt> a;
   vector<Pt> upper, lower;
  Conv(vector < Pt > \_a) : a(\_a){}
     n = a.size();
     int ptr = 0;
for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
for(int i=ptr; i<n; ++i) upper.push_back(a[i]);</pre>
     upper.push_back(a[0]);
  int sign( LL x ){ // fixed when changed to double
  return x < 0 ? -1 : x > 0 }
  pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
     int l = 0, r = (int)conv.size() - 2;
     for( ; l + 1 < r; ){
  int mid = (l + r) / 2;</pre>
        if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
        else \bar{l} = mid;
```

```
void upd_tang(const Pt &p, int id, int &i0, int &i1){
  if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
  if(det(a[i1] - p, a[id] - p) < 0) i1 = id;</pre>
void bi_search(int l, int r, Pt p, int &i0, int &i1){
  if(l == r) return;
  upd_tang(p, 1 % n, i0, i1);
  int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
  for(; l + 1 < r; ) {
  int mid = (l + r) / 2;
    int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
    if (smid == sl) l = mid;
    else r = mid;
  upd_tang(p, r % n, i0, i1);
int bi_search(Pt u, Pt v, int l, int r) {
  int sl = sign(det(v - u, a[l % n] - u));
  for(; l + 1 < r; ) {
    int mid = (l + r) / 2;
int smid = sign(det(v - u, a[mid % n] - u));
    if (smid == sl) l = mid;
    else r = mid;
  return 1 % n;
// 1. whether a given point is inside the CH
bool contain(Pt p) {
  if (p.X < lower[0].X | | p.X > lower.back().X)
       return 0;
  int id = lower_bound(lower.begin(), lower.end(), Pt
       (p.X, -INF)) - lower.begin();
  if (lower[id].X == p.X) {
  if (lower[id].Y > p.Y) return 0;
}else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;</pre>
  id = lower_bound(upper.begin(), upper.end(), Pt(p.X
        INF), greater<Pt>()) - upper.begin();
  if (upper[id].X == p.X) {
  if (upper[id].Y < p.Y) return 0;</pre>
  }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;</pre>
  return 1;
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
  if (contain(p)) return false;
  i0 = i1 = 0;
  int id = lower_bound(lower.begin(), lower.end(), p)
  - lower.begin();
bi_search(0, id, p, i0, i1);
bi_search(id, (int)lower.size(), p, i0, i1);
  id = lower_bound(upper.begin(), upper.end(), p,
       greater<Pt>()) - upper.begin();
  bi_search((int)lower.size() - 1, (int)lower.size()
       -1 + id, p, i0, i1);
  bi_search((int)lower.size() - 1 + id, (int)lower.
       size() - 1 + (int)upper.size(), p, i0, i1);
  return true:
\frac{1}{2} 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
  pair<LL, int> ret = get_tang(upper, vec);
  ret.second = (ret.second+(int)lower.size()-1)%n;
  ret = max(ret, get_tang(lower, vec));
  return ret.second;
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
 int p0 = get_tang(u - v), p1 = get_tang(v - u);
if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){</pre>
   if (p0 > p1) swap(p0, p1);
   i0 = bi_search(u, v, p0, p1);
   i1 = bi_search(u, v, p1, p0 + n);
   return 1;
 }
```

```
4.8 KD Tree
```

return 0:

**}**;

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int x,,,
int id,f;
    ' *! . *R;
  }tree[MXN];
  int n;
  Node *root;
  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
long long dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x;
static bool cmpy(Node& a, Node& b){ return a.y<b.y;</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
for (int i=0; i<n; i++) {</pre>
       tree[i].id = i;
       tree[i].x = ip[i].first;
       tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
         cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
    tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
       tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
       tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
       tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
       tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M;
  int touch(Node* r, int x, int y, long long d2){
  long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
         r->y2+dis)
       return 0;
    return 1;
  void nearest(Node* r, int x, int y, int &mID, long
       long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 | I (d2 == md2 && mID < r->id)) {
       mID = r->id;
       md2 = d2;
     // search order depends on split dim
    if ((r->f == 0 \&\& x < r->x) | |
         (r->f == 1 && y < r->y)) {
       nearest(r->L, x, y, mID, md2);
       nearest(r->R, x, y, mID, md2);
    } else {
```

```
nearest(r->R, x, y, mID, md2);
    nearest(r->L, x, y, mID, md2);
}
int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
}tree;
```

#### 4.9 Lower Concave Hull

```
maintain a "concave hull" that support the following
  1. insertion of a line
  2. query of height(y) on specific x on the hull
 ****/
/* set as needed */
typedef long double LD;
const LD eps=1e-9;
const LD inf=1e19;
class Seg {
 public:
  LD m,c,x1,x2; // y=mx+c
  bool flag;
  Seg(
    LD _m,LD _c,LD _x1=-inf,LD _x2=inf,bool _flag=0)
:m(_m),c(_c),x1(_x1),x2(_x2),flag(_flag) {}
  LD evaly(LD x) const {
    return m*x+c;
  const bool operator<(LD x) const {</pre>
    return x2-eps<x;</pre>
  const bool operator<(const Seg &b) const {</pre>
    if(flag||b.flag) return *this<b.x1;</pre>
    return m+eps<b.m;</pre>
class LowerConcaveHull { // maintain a hull like: \__/
 public:
  set<Seg> hull;
  /* functions
  LD xintersection(Seg a,Seg b) {
    return (a.c-b.c)/(b.m-a.m);
  inline set<Seg>::iterator replace(set<Seg> &
      hull, set<Seg>::iterator it, Seg s) {
    hull.erase(it);
    return hull.insert(s).first;
  void insert(Seg s) {
    // insert a line and update hull
    set<Seg>::iterator it=hull.find(s);
    // check for same slope
    if(it!=hull.end()) {
      if(it->c+eps>=s.c) return;
      hull.erase(it);
    // check if below whole hull
    it=hull.lower_bound(s);
    if(it!=hull.end()&&
       s.evaly(it->x1)<=it->evaly(it->x1)+eps) return;
    // update right hull
    while(it!=hull.end()) {
  LD x=xintersection(s,*it);
      if(x>=it->x2-eps) hull.erase(it++);
      else {
         s.x2=x;
         it=replace(hull,it,Seg(it->m,it->c,x,it->x2));
        break;
    // update left hull
    while(it!=hull.begin()) {
      LD x=xintersection(s,*(--it)):
      if(x<=it->x1+eps) hull.erase(it++);
      else {
        s.x1=x;
```

```
it=replace(hull,it,Seg(it->m,it->c,it->x1,x));
    break;
}

// insert s
hull.insert(s);
}

void insert(LD m,LD c) { insert(Seg(m,c)); }
LD query(LD x) { // return y @ given x
    set<Seg>::iterator it =
        hull.lower_bound(Seg(0.0,0.0,x,x,1));
    return it->evaly(x);
}
};
```

## 4.10 Delaunay Triangulation

```
Delaunay Triangulation:
Given a sets of points on 2D plane, find a
    triangulation
such that no points will strictly inside circumcircle
of any triangle.
find : return a triangle contain given point
add_point : add a point into triangulation
A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)\%3], u.p[(i+2)\%3]
const int N = 100000 + 5
const int MAX_TRIS = N * 6;
double eps = 1e-6;
double sqr(double x) { return x*x; }
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
    const Pt& p4){
// return p4 is in circumcircle of tri(p1,p2,p3)
  double u11 = p1.X - p4.X;
double u21 = p2.X - p4.X;
  double u31 = p3.X - p4.X;
  double u12 = p1.Y - p4.Y;
  double u22 = p2.Y - p4.Y;
  double u32 = p3.Y - p4.Y;
  double u13 = sqr(p1.X)-sqr(p4.X)+sqr(p1.Y)-sqr(p4.Y);
  double u23 = sqr(p2.X)-sqr(p4.X)+sqr(p2.Y)-sqr(p4.Y);
  double u33 = sqr(p3.X)-sqr(p4.X)+sqr(p3.Y)-sqr(p4.Y);
  double det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32
        - u11*u23*u32 - u12*u21*u33 + u11*u22*u33;
  return det > eps;
double side(const Pt& a, const Pt& b, const Pt& p)
{ return (b - a) ^ (p - a); }
typedef int SdRef;
struct Tri;
typedef Tri* TriRef;
struct Edge {
  TriRef tri;
             side;
  SdRef
  Edge() : tri(0), side(0) {}
  Edge(TriRef _tri, SdRef _side) : tri(_tri), side(
       _side) {}
struct Tri {
  Pt p[3];
  Edge edge[3];
  TriRef chd[3];
  Tri() {}
  Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
  p[0] = p0; p[1] = p1; p[2] = p2;
    chd[0] = chd[1] = chd[2] = 0;
  bool has_chd()_const
  { return chd[0] != 0; }
  int num_chd() const {
    return chd[0] == 0 ? 0
: chd[1] == 0 ? 1
      : chd[2] == 0 ? 2 : 3;
  bool contains(Pt const& q) const {
    double a = side(p[0], p[1], q);
```

```
double b = side(p[1],p[2],q);
double c = side(p[2],p[0],q);
     return a >= -eps && b >= -eps && c >= -eps;
} triange_pool[MAX_TRIS], *tot_tris;
void edge(Edge a, Edge b) {
  if(a.tri) a.tri->edge[a.side] = b;
  if(b.tri) b.tri->edge[b.side] = a;
struct Trig { // Triangulation
  Trig() {
     const double LOTS = 1e6;
     the_root = new(tot_tris++) Tri(Pt(-LOTS,-LOTS),Pt(+
          LOTS, -LOTS), Pt(0,+LOTS));
//
   tools
  TriRef find(Pt p) const
{ return find(the_root,p); }
  void add_point(const Pt& p)
  { add_point(find(the_root,p),p); }
   tools
  TriRef the_root;
  static TriRef find(TriRef root, const Pt& p) {
     for( ; ; ) {
   if (!root->has_chd())
          return root;
       for (int i = 0; i < 3 && root->chd[i] ; ++i)
          if (root->chd[i]->contains(p)) {
            root = root->chd[i];
            break:
           "point not found"
     }
  }
  void add_point(TriRef root, Pt const& p) {
     TriRef tab, tbc, tca;
     /* split it into three triangles */
     tab=new(tot_tris++) Tri(root->p[0],root->p[1],p);
tbc=new(tot_tris++) Tri(root->p[1],root->p[2],p);
tca=new(tot_tris++) Tri(root->p[2],root->p[0],p);
     edge(Edge(tab,0), Edge(tbc,1));
     edge(Edge(tbc,0), Edge(tca,1));
edge(Edge(tca,0), Edge(tab,1));
     edge(Edge(tab,2), root->edge[2]);
     edge(Edge(tbc,2), root->edge[0]);
edge(Edge(tca,2), root->edge[1]);
     root->chd[0] = tab;
     root->chd[1] = tbc;
     root->chd[2] = tca;
     flip(tab,\bar{2});
     flip(tbc,2);
     flip(tca,2);
  void flip(TriRef tri, SdRef pi) {
     TriRef trj = tri->edge[pi].tri;
     int pj = tri->edge[pi].side;
     if (!trj) return;
     if (!in_cc(tri->p[0],tri->p[1],tri->p[2],trj->p[pj
          ])) return;
     /* flip edge between tri,trj */
     TriRef trk = new(tot_tris++) Tri(tri->p[(pi+1)%3],
     trj->p[pj], tri->p[pi]);
TriRef trl = new(tot_tris++) Tri(trj->p[(pj+1)%3],
          tri->p[pi], trj->p[pj])
     edge(Edge(trk,0), Edge(trl,0));
     edge(Edge(trk,1), tri->edge[(pi+2)%3]);
     edge(Edge(trk,2), trj->edge[(pj+1)%3]);
     edge(Edge(trl,1), trj->edge[(pj+2)%3]);
edge(Edge(trl,2), tri->edge[(pi+1)%3]);
     tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
     flip(trk,1); flip(trk,2);
     flip(trl,1); flip(trl,2);
  }
};
int n;
Pt ps[N];
void build(){
  tot_tris = triange_pool;
  scanf( "%d"
                 , &n );
  for(int i = 0; i < n; ++ i) {</pre>
     int x, y;
```

```
scanf("%d%d", &x, &y);
  ps[ i ] = { x , y };
}
random_shuffle(ps, ps + n);
Trig tri;
for(int i = 0; i < n; ++ i)
  tri.add_point(ps[i]);
}</pre>
```

## 4.11 Min Enclosing Circle

```
struct Mec{
  // return pair of center and r
  static const int N = 101010;
  Pt p[ N ], cen;
  double r2
  void init( int _n , Pt _p[] ){
    n = _n;
     memcpy( p , _p , sizeof(Pt) * n );
  double sqr(double a){ return a*a; }
  Pt center(Pt p0, Pt p1, Pt p2) {
     Pt a = p1-p0;
     Pt b = p2-p0;
     double c1=norm2( a ) * 0.5;
double c2=norm2( b ) * 0.5;
     double d = a \wedge b;
     double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
     double y = p0.Y + (a.X * c2 - b.X * c1) / d;
return Pt(x,y);
  pair<Pt,double> solve(){
     random_shuffle(p,p+n);
     r2=0:
     for (int i=0; i<n; i++){
       if (norm2(cen-p[i]) <= r2) continue;</pre>
       cen = p[i];
       r2 = 0;
       for (int j=0; j<i; j++){</pre>
          if (norm2(cen-p[j]) <= r2) continue;</pre>
         cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
r2 = norm2(cen-p[j]);
         for (int k=0; k<j; k++){
  if (norm2(cen-p[k]) <= r2) continue;
  cen = center(p[i],p[k]);</pre>
            r2 = norm2(cen-p[k]);
       }
     return {cen,sqrt(r2)};
} mec;
```

### 4.12 Min Enclosing Ball

```
// Pt : { x
                    , z }
#define N 202020
int n, nouter; Pt pt[ N ], outer[4], res;
double radius,tmp;
void ball() {
  Pt q[3]; double m[3][3], sol[3], L[3], det;
  int i,j; res.x = res.y = res.z = radius = 0;
switch ( nouter ) {
     case 1: res=outer[0]; break;
     case 2: res=(outer[0]+outer[1])/2; radius=norm2(res
             outer[0]); break;
     case 3:
       for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q
[i] * q[j])**;
        for (i=0; i<2; ++i) sol[i]=(q[i] * q[i])
        if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps</pre>
       L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
res=outer[0]+q[0]*L[0]+q[1]*L[1];
       radius=norm2(res, outer[0]);
        break;
     case 4:
```

```
for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
    [i]=(q[i] * q[i]);</pre>
             (i=0;i<3;++i) for(j=0;j<3;++j) m[i][j]=(q[i]
                q[j])*2;
        det = m[0][0]*m[1][1]*m[2][2]
          + m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[2][1]*m[1][0]
- m[0][2]*m[1][1]*m[2][0]
           - m[0][1]*m[1][0]*m[2][2]
            - m[0][0]*m[1][2]*m[2][1];
        if ( fabs(det)<eps ) return;</pre>
        for (j=0; j<3; ++j) {
          for (i=0; i<3; ++i) m[i][j]=sol[i];
L[j]=( m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
                    + m[0][2]*m[2][1]*m[1][0]
                    - m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
                    - m[0][0]*m[1][2]*m[2][1]
                 ) / det;
           for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;</pre>
        } res=outer[0];
        for (i=0; i<3; ++i ) res = res + q[i] * L[i];
        radius=norm2(res, outer[0]);
void minball(int n){ ball();
  if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
    if( norm2(res, pt[i]) - radius > eps ){
        outer[ nouter ++ ] = pt[ i ]; minball(i); --
              nouter:
        if(i>0){ Pt Tt = pt[i]
          memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
}}}
void solve{
  // n points in pt
  random_shuffle(pt, pt+n); radius=-1;
  for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
  nouter=1, outer[0]=pt[i], minball(i);
printf("%.5f\n",sqrt(radius));
```

## 4.13 Minkowski sum

```
/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
int pos( const Pt& tp ){
  if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
  return tp.Y > 0 ? 0 : 1;
#define N_300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
  int pa=pos( a ),pb=pos( b );
  if(pa==pb) return (a^b)>0;
  return pa<pb;</pre>
int minkowskiSum(int n,int m){
  int i,j,r,p,q,fi,fj;
for(i=1,p=0;i<n;i++){</pre>
    if( pt[i].Y<pt[p].Y ||</pre>
         (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
  for(i=1,q=0;i<m;i++)
    if( qt[i].Y<qt[q].Y ||</pre>
         (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
  rt[0]=pt[p]+qt[q];
  r=1; i=p; j=q; fi=fj=0;
while(1){
    if((fj&&j==q) ||
        ( (!filli!=p) &&
          cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
      rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
      p=(p+1)%n;
      fi=1;
    }else{
      rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
      q=(q+1)\%m;
      fj=1;
```

```
if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
    else rt[r-1]=rt[r];
    if(i==p && j==q) break;
  return r-1;
void initInConvex(int n){
  int i,p,q;
  LL Ly,Ry;
Lx=INF; Rx=-INF
  for(i=0;i< n;i++){
    if(pt[i].X<Lx) Lx=pt[i].X;</pre>
    if(pt[i].X>Rx) Rx=pt[i].X;
  Ly=Ry=INF;
  for(i=0;i<n;i++){
  if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i;</pre>
    if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }</pre>
  for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
  qt[dn]=pt[q]; Ly=Ry=-INF;
  for(i=0;i<n;i++){
  if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i;
    if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
  for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
  rt[un]=pt[q];
inline int inConvex(Pt p){
  int L,R,M;
  if(p.X<Lx || p.X>Rx) return 0;
  L=0; R=dn;
  while(L<R-1){ M=(L+R)/2;
    if(p.X<qt[M].X) R=M; else L=M; }</pre>
    if(tri(qt[L],qt[R],p)<0) return 0;</pre>
    L=0;R=un
    while(L<R-1){ M=(L+R)/2;
       if(p.X<rt[M].X) R=M; else L=M; }</pre>
       if(tri(rt[L],rt[R],p)>0) return 0;
int main(){
  int n,m,i;
  Pt p;
  scanf("%d",&n);
  for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);</pre>
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);</pre>
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
  scanf("%d",&m);
  for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);</pre>
  n=minkowskiSum(n,m);
  for(i=0;i<n;i++) pt[i]=rt[i];</pre>
  initInConvex(n);
  scanf("%d",&m);
  for(i=0;i<m;i++){
    scanf("%1ld %1ld",&p.X,&p.Y);</pre>
    p.X*=3; p.Y*=3;
    puts(inConvex(p)?"YES":"NO");
}
```

#### 4.14 Heart of Triangle

```
Pt inCenter( Pt &A, Pt &B, Pt &C) { // III double a = norm(B-C), b = norm(C-A), c = norm(A-B); return (A * a + B * b + C * c) / (a + b + c); }

Pt circumCenter( Pt &a, Pt &b, Pt &c) { // III Pt bb = b - a, cc = c - a; double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc); return a+Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d; }

Pt othroCenter( Pt &a, Pt &b, Pt &c) { // III Pt ba = b - a, ca = c - a, bc = b - c; double Y = ba.Y * ca.Y * bc.Y, A = ca.X * ba.Y - ba.X * ca.Y, x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A, y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
```

```
return Pt(x0, y0);
```

Graph

5

## 5.1 HeavyLightDecomp

```
#define SZ(c) (int)(c).size()
#define ALL(c) (c).begin(), (c).end()
#define REP(i, s, e) for(int i = (s); i <= (e); i++)</pre>
#define REPD(i, s, e) for(int i = (s); i \ge (e); i \ge (e)
typedef tuple< int , int > tii;
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
  int n;
  vector<int> g[MAXN];
  int sz[MAXN], dep[MAXN];
  int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
  // ts : timestamp , useless after yutruli
      tid[ u ] : pos. of node u in the seq.
tdi[ i ] : node at pos i of the seq.
  //
       tl , tr[ u ] : subtree interval in the seq. of
  //
       node u
  int mom[MAXN][LOG], head[MAXN];
  // head[ u ] : head of the chain contains u
  void dfssz(int u, int p){
     dep[u] = dep[p] + 1;
    mom[u][0] = p;
     sz[u] = 1;
     head[u] = u;
     for(int& v:g[u]) if(v != p){
       dep[v] = \overline{dep[u]} + 1;
       dfssz(v, u)
       sz[u] += sz[v];
    }
  }
  void dfshl(int u){
    //printf("dfshl %d\n", u);
     ts++
     tid[u] = tl[u] = tr[u] = ts;
     tdi[tid[u]] = u;
     sort(ALL(g[u]),
           [&](int a, int b){return sz[a] > sz[b];});
     bool flag = 1;
     for(int& v:g[u]) if(v != mom[u][0]){
       if(flag) head[v] = head[u], flag = 0;
       dfshl(v);
       tr[u] = tr[v];
    }
  inline int lca(int a, int b){
  if(dep[a] > dep[b]) swap(a, b);
  //printf("lca %d %d\n", a, b);
  int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
   //printf("b %d\n", mom[b][k]);</pre>
       b = mom[b][k];
     if(a == b) return a;
     REPD(k, LOG-1, 0) if(mom[a][k] != mom[b][k]){
       a = mom[a][k];
       b = mom[b][k];
    return mom[a][0];
  void init( int _n ){
    REP( i , 1 , n ) g[ i ].clear();
  void addEdge( int u , int v ){
    g[ u ].push_back( v );
g[ v ].push_back( u );
  void yutruli(){
    dfssz(1, 0);
     ts = 0:
    REP(k, 1, LOG-1) REP(i, 1, n)
  mom[i][k] = mom[mom[i][k-1]][k-1];
```

```
vector< tii > getPath( int u , int v ){
    vector< tii > res;
while( tid[ u ] < tid[ head[ v ] ] ){</pre>
      res.push_back( tii(tid[ head[ v ] ] , tid[ v ]) )
      v = mom[head[v]][0];
    }
    res.push_back( tii( tid[ u ] , tid[ v ] ) );
    reverse( ALL( res ) );
    return res:
     u must be ancestor of v
     * usage :
     * vector< tii >& path = tree.getPath( u , v )
       for( tii tp : path ) {
  int l , r;tie( l , r ) = tp;
  upd( l , r );
         uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
         uu ~> vv is a heavy path on tree
  }
} tree;
```

### 5.2 DominatorTree

```
const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)</pre>
#define REPD(i, s, e) for(int i=(s); i>=(e); i--)
  vector< int > g[ MAXN ] , pred[ MAXN ];
vector< int > cov[ MAXN ];
int dfn[ MAXN ] , nfd[ MAXN ] , ts;
int par[ MAXN ];
  int sdom[ MAXN ] , idom[ MAXN ];
int mom[ MAXN ] , mn[ MAXN ];
inline bool cmp( int u , int v )
{ return dfn[ u ] < dfn[ v ]; }</pre>
  int eval( int u ){
  if( mom[ u ] == u ) return u;
     int res = eval( mom[ u ] );
if(cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ))
    mn[ u ] = mn[ mom[ u ] ];
     return mom[ u ] = res;
  void init( int _n , int _m , int _s ){
     ts = 0; n = _n; m = _m; s = _s;
     REP( i, 1, n ) g[ i ].clear(), pred[ i ].clear();
  void addEdge( int u , int v ){
  g[ u ].push_back( v );
     pred[ v ].push_back( u );
  void dfs( int u ){
     ts++;
dfn[ u ] = ts;
     nfd[ts] = u;
     for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
  par[ v ] = u;
        dfs(v);
     }
  }
  void build(){
     REP( i , 1 , n ){
  dfn[ i ] = nfd[ i ] = 0;
        cov[ i ].clear();
mom[ i ] = mn[ i ] = sdom[ i ] = i;
     dfs( s );
     REPD( i , n , 2 ){
  int u = nfd[ i ];
        if( u == 0 ) continue ;
        for( int v : pred[ u ] ) if( dfn[ v ] ){
           eval(v);
           if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
              sdom[u] = sdom[mn[v]];
        cov[ sdom[ u ] ].push_back( u );
```

```
mom[ u ] = par[ u ];
for( int w : cov[ par[ u ] ) {
    eval( w );
    if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
        idom[ w ] = mn[ w ];
    else idom[ w ] = par[ u ];
}
cov[ par[ u ] ].clear();
}
REP( i , 2 , n ) {
    int u = nfd[ i ];
    if( u == 0 ) continue;
    if( idom[ u ] != sdom[ u ] )
        idom[ u ] = idom[ idom[ u ] ];
}
}
domT;
```

## 5.3 MaxClique

```
struct MaxClique {
  static const int MV = 210;
  int V , ans , dp[MV];
  int el[MV][MV/30+1], s[MV][MV/30+1];
  vector<int> sol;
  void init(int v) {
     V = v; ans = 0;
FZ(el); FZ(dp);
  }
/* Zero Base */
  void addEdge(int u, int v) {
     if(u > v) swap(u, v);
if(u == v) return;
     el[u][v/32] = (1 << (v%32));
  bool dfs(int v, int k) {
     int c = 0, d = 0;
     for(int i=0; i<(V+31)/32; i++) {
       s[k][i] = el[v][i];
       if(k != 1) s[k][i] &= s[k-1][i];
       c += __builtin_popcount(s[k][i]);
     if(c == 0) {
       if(k > ans) {
         ans = k;
         sol.clear();
         sol.push_back(v);
         return 1;
       }
       return 0;
     for(int i=0; i<(V+31)/32; i++) {</pre>
       for(int a = s[k][i]; a; d++) {
  if(k + (c-d) <= ans) return 0;</pre>
         int lb = a&(-a), lg = 0;
         a \sim 1b;
         while(lb!=1) {
            lb = (unsigned int)(lb) >> 1;
            lg ++;
         int u = i*32 + lq;
         if(k + dp[u] \ll ans) return 0;
         if(dfs(u, k+1)) {
            sol.push_back(v);
            return 1;
         }
       }
     }
    return 0;
  int solve() {
     for(int i=V-1; i>=0; i--) {
       dfs(<u>i</u>, 1);
       dp[i] = ans;
     return ans;
};
```

### 5.4 Number of Maximal Clique

```
// bool g[][] : adjacent array indexed from 1 to n
void dfs(int sz){
  int i, j, k, t, cnt, best = 0;
if(ne[sz]==ce[sz]){ if (ce[sz]==0) ++ans; return; }
  for(t=0, i=1; i<=ne[sz]; ++i){
  for (cnt=0, j=ne[sz]+1; j<=ce[sz]; ++j)</pre>
    if (!g[lst[sz][i]][lst[sz][j]]) ++cnt;
     if (t==0 || cnt<best) t=i, best=cnt;</pre>
  } if (t && best<=0) return;</pre>
  swap(lst[sz][k], lst[sz][i]);
    } i=lst[sz][k]; ne[sz+1]=ce[sz+1]=0;
    for (j=1; j<k; ++j)if (g[i][lst[sz][j]])
    lst[sz+1][++ne[sz+1]]=lst[sz][j];</pre>
    for (ce[sz+1]=ne[sz+1], j=k+1; j<=ce[sz]; ++j)
if (g[i][lst[sz][j]]) lst[sz+1][++ce[sz+1]]=lst[sz</pre>
          ][j];
    dfs(sz+1); ++ne[sz]; --best;
    for (j=k+1, cnt=0; j<=ce[sz]; ++j) if (!g[i][lst[sz</pre>
          ][j]]) ++cnt;
     if (t==0 || cnt<best) t=k, best=cnt;
     if (t && best<=0) break;</pre>
void work(){
 ne[0]=0; ce[0]=0;
for(int i=1; i<=n; ++i) lst[0][++ce[0]]=i;
  ans=0; dfs(0);
```

## 5.5 Strongly Connected Component

```
struct Scc{
  int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
  void init(int _n){
    n = _n;
for (int_i=0; i<MXN; i++){
       E[i].clear()
       rE[i].clear();
  }
  void add_edge(int u, int v){
    E[u].PB(v);
     rE[v].PB(u);
  void DFS(int u){
     vst[u]=1;
     for (auto v : E[u])
       if (!vst[v]) DFS(v);
     vec.PB(u);
  void rDFS(int u){
     vst[u] = 1;
     bln[u] = nScc;
     for (auto v : rE[u])
       if (!vst[v]) rDFS(v);
  void solve(){
    nScc = 0:
     vec.clear();
     FZ(vst);
     for (int i=0; i<n; i++)
  if (!vst[i]) DFS(i);</pre>
     reverse(vec.begin(), vec.end());
     FZ(vst);
     for (auto v : vec){
       if (!vst[v]){
         rDFS(v);
         nScc++;
  }
};
```

## 5.6 Dynamic MST

```
/* Dynamic MST O( Q lg^2 Q )
  (qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
  delete an edge: (i, \infty)
```

```
add an edge: change from \infty to specific value
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
  int root=xx; while(a[root]) root=a[root];
  int next; while((next=a[xx])){a[xx]=root; xx=next; }
  return root;
bool cmp(int_aa,int bb){ return tz[aa]<tz[bb]; }</pre>
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
     int *z,int m1,long long ans){
  if(Q==1){
    for(int i=1;i<=n;i++) a[i]=0;
    z[qx[0]]=qy[0]; tz = z;
for(int i=0;i<m1;i++) id[i]=i;
    sort(id,id+m1,cmp); int ri,rj;
     for(int i=0;i<m1;i++){</pre>
       ri=find(x[id[i]]); rj=find(y[id[i]]);
if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
    printf("%lld\n",ans);
    return;
  int ri,rj;
  //contract
  kt=0;
  for(int i=1;i<=n;i++) a[i]=0;</pre>
  for(int i=0;i<Q;i++){</pre>
    ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[
         ri]=rj;
  int tm=0;
  for(int i=0;i<m1;i++) extra[i]=true;</pre>
  for(int i=0;i<Q;i++) extra[ qx[i] ]=false;</pre>
  for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;</pre>
  tz=z; sort(id,id+tm,cmp);
  for(int i=0;i<tm;i++){</pre>
    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
       a[ri]=rj; ans += z[id[i]];
       kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
    }
  for(int i=1;i<=n;i++) a[i]=0;
  for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);</pre>
  int n2=0;
  for(int i=1;i<=n;i++) if(a[i]==0)</pre>
  vd[i]=++n2;
  for(int i=1;i<=n;i++) if(a[i])</pre>
  vd[i]=vd[find(i)];
  int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
  for(int i=0;i<m1;i++) app[i]=-1;
for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
    Nz[m2]=z[ qx[i] ];
app[qx[i]]=m2; m2++;
  for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[</pre>
  i]]; }
for(int i=1;i<=n2;i++) a[i]=0;
  for(int i=0;i<tm;i++){</pre>
    ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
       a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
       Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
  int mid=Q/2;
  solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
  solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
  scanf("%d%d",&n,&m);
  for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);</pre>
  scanf("%d",&0):
  for(int i=0; i<0; i++){ scanf("%d%d",qx+i,qy+i); qx[i
```

```
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }
```

## 5.7 Minimum General Weighted Matching

```
struct Graph {
  // Minimum General Weighted Matching (Perfect Match)
  static const int MXN = 105;
  int n, edge[MXN][MXN];
  int match[MXN],dis[MXN],onstk[MXN];
  vector<int> stk;
  void init(int _n) {
    n = _n;
for( int i = 0 ; i < n ; i ++ )
  for( int j = 0 ; j < n ; j ++ )</pre>
         edge[ i ][ j ] = 0;
  void add_edge(int u, int v, int w)
  \{ edge[u][v] = edge[v][u] = w; \}
  bool SPFA(int u){
    if (onstk[u]) return true;
    stk.PB(u);
    onstk[u] = 1;
    for (int v=0; v<n; v++){</pre>
       if (u != v && match[u] != v && !onstk[v]){
         int m = match[v]
         if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
           dis[m] = dis[u] - edge[v][m] + edge[u][v];
           onstk[v] = 1;
           stk.PB(v);
           if (SPFA(m)) return true;
           stk.pop_back();
           onstk[v] = 0;
        }
      }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  int solve() {
    // find a match
    for (int i=0; i<n; i+=2){</pre>
      match[i] = i+1;
      match[i+1] = i;
    while (true){
      int found = 0;
      for( int i = 0 ; i < n ; i ++ )
onstk[ i ] = dis[ i ] = 0;
for (int i=0; i<n; i++){
         stk.clear()
         if (!onstk[i] && SPFA(i)){
           found = 1;
           while (SZ(stk)>=2){
             int u = stk.back(); stk.pop_back();
             int v = stk.back(); stk.pop_back();
             match[u] = v;
             match[v] = u;
        }
      if (!found) break;
    int ret = 0;
    for (int i=0; i<n; i++)
      ret += edge[i][match[i]];
    ret /= 2;
    return ret;
}graph;
```

## 5.8 Minimum Steiner Tree

```
// Minimum Steiner Tree
// O(V 3^T + V^2 2^T)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
  int n , dst[V][V] , dp[1 << T][V] , tdst[V];</pre>
```

```
void init( int _n ){
      n = _n;
for( int i = 0 ; i < n ; i ++ ){</pre>
        for( int j = 0; j < n; j ++ )

dst[i][j] = INF;

dst[i][i] = 0;
      }
   }
   void add_edge( int ui , int vi , int wi ){
  dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
  dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
   void shortest_path(){
      for( int k = 0 ; k < n ; k ++ )
         for( int i = 0; i < n; i ++)
           int solve( const vector<int>& ter ){
      int t = (int)ter.size();
      for( int i = 0 ; i < ( 1 << t ) ; i ++ )
  for( int j = 0 ; j < n ; j ++ )
    dp[ i ][ j ] = INF;</pre>
      for( int i = 0 ; i < n ; i ++ )
         dp[0][i] = 0;
      for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
  if( msk == ( msk & (-msk) ) ){</pre>
            int who = __lg( msk );
            for( int i = 0 ; i < n ; i ++ )
  dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];</pre>
            continue;
         for( int i = 0 ; i < n ; i ++ )</pre>
            for( int submsk = ( msk - 1 ) & msk ; submsk ;
                        submsk = (submsk - 1) & msk)
                 dp[ msk ][ i ] = min( dp[ msk ][ i ],
                                       dp[ submsk ][ i ] +
                                      dp[ msk ^ submsk ][ i ] );
         for( int i = 0 ; i < n; i ++){
           tdst[ i ] = INF;
            for( int j = 0 ;
              or( int j = 0 ; j < n ; j ++ )
tdst[ i ] = min( tdst[ i ],
                             dp[ msk ][ j ] + dst[ j ][ i ] );
         for( int i = 0 ; i < n ; i ++
  dp[ msk ][ i ] = tdst[ i ];</pre>
      int ans = INF;
      for( int i = 0 ; i < n ; i ++ )
ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
      return ans;
} solver;
```

# 5.9 BCC based on vertex

```
struct BccVertex {
  int n,nScc,step,dfn[MXN],low[MXN];
  vector<int> E[MXN],sccv[MXN];
  int top,stk[MXN];
  void init(int _n) {
    n = _n;
nScc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    stk[top++] = u;
    for (auto v:E[u]) {
      if (v == f) continue;
if (dfn[v] == -1) {
         DFS(v,u);
low[u] = min(low[u], low[v]);
         if (low[v] >= dfn[u]) {
           int z;
           sccv[nScc].clear();
```

```
do {
            z = stk[--top]
            sccv[nScc].PB(z);
          } while (z != v);
          sccv[nScc].PB(u);
          nScc++;
      } else {
        low[u] = min(low[u],dfn[v]);
      }
 vector<vector<int>> solve() {
    vector<vector<int>> res;
    for (int i=0; i<n; i++) {</pre>
      dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++) {
      if (dfn[i] == -1) {
        top = 0;
        DFS(i,i);
    REP(i,nScc) res.PB(sccv[i]);
   return res:
}graph;
```

## 5.10 Graph Hash

```
$$F_t(i) =
   (F_{t-1}(i) \times A +
   \sum_{i\rightarrow j} F_{t-1}(j) \times B +
   \sum_{j\rightarrow i} F_{t-1}(j) \times C +
   D \times (i = a))\ mod\ P
$$
for each node i, iterate t times.
t, A, B, C, D, P are hash parameter
```

# 6 String

## 6.1 PalTree

```
const int MAXN = 200010;
struct PalT{
  struct Node{
    int nxt[ 33 ] , len , fail;
    ll cnt;
 int tot , lst;
Node nd[ MAXN * 2 ];
  char* s:
  int newNode( int l , int _fail ){
    int res = ++tot;
    memset( nd[ res ].nxt , 0 , sizeof nd[ res ].nxt );
    nd[ res ].len = l;
nd[ res ].cnt = 0;
    nd[ res ].fail = _fail;
    return res;
 void push( int p ){
    int np = lst;
int c = s[ p ] - 'a';
    while(p - nd[np].len - 1 < 0
      || s[p]!=s[p-nd[np].len - 1])
np = nd[np].fail;
    if( nd[ np ].nxt[ c ] ){
  nd[ nd[ np ].nxt[ c ] ].cnt++;
      lst = nd[ np ].nxt[ c ];
      return;
    int nq = newNode( nd[ np ].len + 2 , 0 );
    nd[ nq ].cnt++;
    nd[ np ].nxt[ c ] = nq;
    lst = nq;
    if( nd[ nq ].len == 1 ){
      nd[nq].fail = 2;
      return;
```

```
int tf = nd[ np ].fail;
   nd[ nq ].fail = nd[ tf ].nxt[ c ];
    return ;
  void init( char* _s ){
    tot = 0;
    newNode( -1 , 1 );
newNode( 0 , 1 );
    lst = 2
    for( int i = 0 ; s[ i ] ; i++ )
      push( i );
  void yutruli(){
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
     PD( i , tot , 1 )
nd[ nd[ i ].fail ].cnt += nd[ i ].cnt;
    REPD( i
    nd[ 1 ].cnt = nd[ 2 ].cnt = 0li;
} pA;
int main(){ pA.init( sa ); }
```

## 6.2 SuffixArray

```
const int MAX = 1020304;
 int ct[MAX], he[MAX], rk[MAX];
int sa[MAX], tsa[MAX], tp[MAX][2];
void suffix_array(char *ip){
   int len = strlen(ip);
   int alp = 256;
   memset(ct, 0, sizeof(ct));
for(int i=0;i<len;i++) ct[ip[i]+1]++;</pre>
   for(int i=1;i<alp;i++) ct[i]+=ct[i-1]</pre>
   for(int i=0;i<len;i++) rk[i]=ct[ip[i]];</pre>
   for(int i=1;i<len;i*=2){</pre>
      for(int j=0;j<len;j++)</pre>
         if(j+i>=len) tp[j][1]=0;
else tp[j][1]=rk[j+i]+1;
         tp[j][0]=rk[j];
      memset(ct, 0, sizeof(ct));
      for(int j=0;j<len;j++) ct[tp[j][1]+1]++;</pre>
      for(int j=1;j<len+2;j++) ct[j]+=ct[j-1];
for(int j=0;j<len;j++) tsa[ct[tp[j][1]]++]=j;</pre>
      memset(ct, 0, sizeof(ct));
for(int j=0;j<len;j++) ct[tp[j][0]+1]++;</pre>
      for(int j=1;j<len+1;j++) ct[j]+=ct[j-1];
for(int j=0;j<len;j++)</pre>
         sa[ct[tp[tsa[j]][0]]++]=tsa[j];
      rk[sa[0]]=0;
      for(int j=1;j<len;j++){</pre>
         if( tp[sa[j]][0] == tp[sa[j-1]][0] &&
    tp[sa[j]][1] == tp[sa[j-1]][1] )
            rk[sa[j]] = rk[sa[j-1]];
           rk[sa[j]] = j;
      }
   for(int i=0,h=0;i<len;i++){</pre>
      if(rk[i]==0) h=0;
      else{
         int j=sa[rk[i]-1];
         h=max(0,h-1);
         for(;ip[i+h]==ip[j+h];h++);
      he[rk[i]]=h;
}
 6.3 SAIS
```

```
const int N = 300010;
struct SA{
  #define REP(i,n) for ( int i=0; i<int(n); i++ )
  #define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
  bool _t[N*2];
```

```
memcpy(_s, s, sizeof(int) * n);
           sais(_s, _sa, _p, _q, _t, _c, n, m);
           mkhei(n);
      void mkhei(int n){
           REP(i,n) r[\_sa[i]] = i;
            hei[0] = 0;
            REP(\bar{i},n) if(r[i]) {
                  int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
                  \label{eq:while} \begin{tabular}{ll} while(\_s[i+ans] == \_s[\_sa[r[i]-1]+ans]) & ans++; \\ \end{tabular}
                 hei[r[i]] = ans;
      void sais(int *s, int *sa, int *p, int *q, bool *t,
                  int *c, int n, int z){
            bool uniq = t[n-1] = true, neq;
            int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s + n,
                         lst = -1;
#define MSO(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
           memcpy(x + 1, c, sizeof(int) * (z - 1)); \
REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
                         ]-1]]++] = sa[i]-1; \setminus
           memcpy(x, c, sizeof(int) * z);
for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
                         ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
           MSO(c, z);
            REP(i,n) uniq \&= ++c[s[i]] < 2;
            REP(i,z-1) c[i+1] += c[i];
           if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i +1] ? t[i+1] : s[i] < s[i+1]);</pre>
           \label{eq:magic_replication} \text{MAGIC}(\bar{R}\text{EP1}(\bar{i},1,\bar{n}\text{-}1)\_\bar{i}\bar{f}(\bar{t}[\bar{i}] \& \bar{\&}^{\text{!}}\bar{t}[\bar{i}\text{-}1]) \ sa[\text{--x[s[i]}] \ sa[\text{--x[s[i]}]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]}]] \ sa[\text{--x[s[i]}]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]] \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]} \ sa[\text{--x[s[i]]]] \ sa[\text{--x[s[i]]]]} \ sa[\text{--x[s[i]]]
                         ]]]=p[q[i]=nn++]=i)
            REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
                 \label{eq:neq_lambda} \begin{subarray}{ll} neq=lst<0 | lmemcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa)) \\ \end{subarray}
                               [i])*sizeof(int));
                 ns[q[lst=sa[i]]]=nmxz+=neq;
           sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmxz
                            + 1);
            MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s[p[
                        nsa[i]]]] = p[nsa[i]];
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
      // should padding a zero in the back
     // ip is int array, len is array length
// ip[0..n-1] != 0, and ip[len] = 0
     ip[len++] = 0;
     sa.build(ip, len, 128);
for (int i=0; i<len; i++) {
    H[i] = sa.hei[i + 1];</pre>
            SA[i] = sa.\_sa[i + 1];
      // resulting height, sa array \in [0,len)
```

## SuffixAutomata

```
const int MAXM = 1000010;
struct SAM{
  int tot, root, lst, mom[MAXM], mx[MAXM];
int acc[MAXM], nxt[MAXM][33];
  int newNode(){
     int res = ++tot;
    fill(nxt[res], nxt[res]+33, 0);
mom[res] = mx[res] = acc[res] = 0;
     return res;
  void init(){
     tot = 0;
     root = newNode();
    mom[root] = 0, mx[root] = 0;
```

```
lst = root;
   void push(int c){
     int p = lst;
     int np = newNode();
     mx[np] = mx[p]+1;
     for(; p && nxt[p][c] == 0; p = mom[p])
       nxt[p][c] = np;
     if(p == 0) mom[np] = root;
     else{
       int q = nxt[p][c];
       if(mx[p]+1 == mx[q]) mom[np] = q;
         int nq = newNode();
         mx[nq] = mx[p]+1;
         for(int i = 0; i < 33; i++)
           nxt[nq][i] = nxt[q][i];
         mom[nq] = mom[q];
         mom[q] = nq;
         mom[np] = nq;
for(; p && nxt[p][c] == q; p = mom[p])
           nxt[p][c] = nq;
       }
     lst = np;
   void push(char *str){
     for(int i = 0; str[i]; i++)
       push(str[i]-'a'+1);
} sam;
```

#### 6.5 Aho-Corasick

```
struct ACautomata{
   struct Node{
     int cnt,dp
     Node *go[26], *fail;
     Node (){
       cnt = 0; dp = -1; fail = 0;
       memset(go,0,sizeof(go));
   Node *root, pool[1048576];
   int nMem;
   Node* new_Node(){
     pool[nMem] = Node();
     return &pool[nMem++];
   void init()
   { nMem = 0; root = new_Node(); }
   void add(const string &str)
   { insert(root, str,0); }
void insert(Node *cur, const string &str, int pos){
     if (pos >= (int)str.size())
     { cur->cnt++; return; }
     int c = str[pos]-'a';
     if (cur->go[c] == 0)
  cur->go[c] = new_Node();
     insert(cur->go[c],str,pos+1);
   void make_fail(){
     queue<Node*> que;
     que.push(root);
     while (!que.empty()){
  Node* fr=que.front();
       que.pop();
for (int i=0; i<26; i++){
         if (fr->go[i]){
            Node *ptr = fr->fail;
            while (ptr && !ptr->go[i]) ptr = ptr->fail;
            if (!ptr) fr->go[i]->fail = root;
            else fr->go[i]->fail = ptr->go[i];
            que.push(fr->go[i]);
       }
    }
  }
};
```

#### 6.6 Z Value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
   int i,j,left,right;
   left=right=0; z[0]=len;
   for(i=1;i<len;i++) {
      j=max(min(z[i-left],right-i),0);
      for(;i+j<len&&s[i+j]==s[j];j++);
      z[i]=j;
      if(i+z[i]>right) {
        right=i+z[i];
        left=i;
      }
   }
}
```

## 6.7 ZValue Palindrome

```
int len, zv[MAX*2];
char ip[MAX], op[MAX*2];
int main(){
  cin >> ip; len = strlen(ip);
int l2 = len*2 - 1;
  for(int i=0; i<l2; i++)
  if(i&1) op[i] = '@';</pre>
     else op[i] = ip[i/2];
  int l=0, r=0; zv[0] = 1;
for(int i=1; i<l2; i++){
  if( i > r ){
        while( l>0 && r<12-1 && op[l-1] == op[r+1] )
        l --, r ++;
zv[i] = (r-l+1);
     }else{
        int md = (1+r)/2, j = md + md - i;
        zv[i] = zv[j];
        int q = zv[i] / 2, nr = i + q;
        if( nr == r ){
           while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
          l --, r ++;
zv[i] = r - l + 1;
        }else if( nr > r )
           zv[i] = (r - i)^* 2 + 1;
  }
}
```

### 6.8 Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
  }
  int ans = i < n ? i : j;
  return s.substr(ans, n);
}</pre>
```

#### **7 Data Structure**

#### 7.1 Treap

```
struct Treap{
  int sz , val , pri , tag;
  Treap *l , *r;
  Treap( int _val ){
    val = _val; sz = 1;
    pri = rand(); l = r = NULL; tag = 0;
}
```

```
};
void push( Treap * a ){
  if( a->tag ){
    Treap *swp = a -> 1; a -> 1 = a -> r; a -> r = swp;
    if( a->l') a->l->tag ^= 1;
    if( a->r ) a->r->tag ^= 1;
    a \rightarrow tag = 0;
int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
  a\rightarrow sz = Size(a\rightarrow l) + Size(a\rightarrow r) + 1;
Treap* merge( Treap *a , Treap *b ){
  if( !a || !b ) return a ? a : b;
  if( a->pri > b->pri ){
    push( a );
    a \rightarrow r = merge(a \rightarrow r, b);
    pull( a );
     return a;
  }else{
    push( b );
    b->l = merge( a , b->l );
    pull( b );
    return b;
  }
}
void split( Treap *t , int k , Treap*&a , Treap*&b ){
  if( !t ){ a = b = NULL; return; }
  push( t):
  if( Size( t -> l ) + 1 <= k ){
    split( t->r , k - Size( t->l ) - 1 , a->r , b );
    pull( a );
  }else{
    split( t->l , k , a , b->l );
    pull( b );
}
```

## 7.2 Link-Cut Tree

```
const int MXN = 100005
const int MEM = 100005;
struct Splay {
  static Splay nil, mem[MEM], *pmem;
Splay *ch[2], *f;
  int val, rev, size;
  Splay () : val(-1), rev(0), size(0) 
{ f = ch[0] = ch[1] = &nil; }
                       : val(_val), rev(0), size(1)
  Splay (int _val)
  \{ f = ch[0] = ch[1] = &nil; \}
  bool isr()
  { return f->ch[0] != this && f->ch[1] != this; }
  int dir()
  { return f->ch[0] == this ? 0 : 1; }
  void setCh(Splay *c, int d){
     ch[d] = c
     if (c != &nil) c->f = this;
    pull();
  void push(){
  if( !rev ) return;
    swap(ch[0], ch[1]);
if (ch[0] != &nil) ch[0]->rev ^= 1;
if (ch[1] != &nil) ch[1]->rev ^= 1;
    rev=0;
  void pull(){
    size = ch[0] -> size + ch[1] -> size + 1;
     if (ch[0] != &nil) ch[0] -> f = this;
     if (ch[1] != &nil) ch[1]->f = this;
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
  Splay *p = x -> f
  int d = x->dir();
```

```
if (!p->isr()) p->f->setCh(x, p->dir());
   else x - > f = p - > f
   p->setCh(x->ch[!d], d);
   x \rightarrow setCh(p, !d);
   p->pull(); x->pull();
vector<Splay*> splayVec;
void splay(Splay *x){
   splayVec.clear();
   for (Splay *q=x;; q=q->f){
      splayVec.push_back(q);
      if (q->isr()) break;
   reverse(begin(splayVec), end(splayVec));
for (auto it : splayVec) it->push();
   while (!x->isr()) {
     if (x->f->isr()) rotate(x);
else if (x->dir()==x->f->dir())
        rotate(x->f),rotate(x);
      else rotate(x),rotate(x);
   }
Splay* access(Splay *x){
   Splay *q = nil;
for (;x!=nil;x=x->f){
     splay(x)
     x - setCh(q, 1);
     q = x;
   return q;
void evert(Splay *x){
   access(x);
   splay(x);
   x \rightarrow rev = 1;
   x->push(); x->pull();
void link(Splay *x, Splay *y){
// evert(x);
  access(x);
   splay(x);
   evert(y)
   x \rightarrow setCh(y, 1);
void cut(Splay *x, Splay *y){
     evert(x);
  access(y);
   splay(y)
   y->push();
   y->ch[0] = y->ch[0]->f = nil;
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y){
   access(x);
   access(y);
   splay(x);
   int res = x->f->val;
   if (res == -1) res=x->val;
   return res;
int main(int argc, char** argv){
   scanf("%d%d", &N, &Q);
   for (int i=1; i<=N; i++)</pre>
      vt[i] = new (Splay::pmem++) Splay(i);
   while (Q--) {
      char cmd[105];
     int u, v;
scanf("%s", cmd);
     scanf("%s", cma);
if (cmd[1] == 'i') {
    scanf("%d%d", &u, &v);
    link(vt[v], vt[u]);
    link(cond[0] == 'c') {
        scanf("%d", &v);
cut(vt[1], vt[v]);
     } else {
        scanf("%d%d", &u, &v);
int res=ask(vt[u], vt[v]);
        printf("%d\n", res);
  }
}
```

## 7.3 Disjoint Set

```
struct DisjointSet{
   // save() is like recursive
// undo() is like return
   int n, fa[ N ], sz[ N ];
vector< pair<int*,int> > h;
   vector<int> sp;
   void init( int tn ){
     n=tn;
for( int i = 0 ; i < n ; i ++ ){
  fa[ i ]=i;
        sz[ i ]=1;
     sp.clear(); h.clear();
   void assign( int *k, int v ){
    h.PB( {k, *k} );
      *k = v;
   void save(){ sp.PB(SZ(h)); }
   void undo(){
     assert(!sp.empty());
      int last=sp.back(); sp.pop_back();
     while( SZ(h)!=last ){
        auto x=h.back(); h.pop_back();
        *x.first = x.second;
   int f( int x ){
  while( fa[ x ] != x ) x = fa[ x ];
      return x;
   void uni( int x , int y ){
     x = f(x); y = f(y);
if(x == y) return;
if(sz[x] < sz[y]) swap(x, y)
     assign( &sz[ x ] , sz[ x ] + sz[ y ] );
assign( &fa[ y ] , x);
}djs;
```

## 7.4 Leftist Heap

```
const int MAXN = 10000;
struct Node{
  int num,lc,rc;
  Node() : num(0), lc(-1), rc(-1){}
  Node( int _v ) : num(_v), lc(-1), rc(-1){}
}tree[ MAXN ];
int merge( int x, int y ){
  if( x == -1 ) return y;
  if( y == -1 ) return x;
  if( tree[ x ].num < tree[ y ].num )
      swap(x, y);
  tree[ x ].rc = merge(tree[ x ].rc, y);
  swap(tree[ x ].lc, tree[ x ].rc);
  return x;
}
/* Usage
merge: root = merge(x, y)
delmin: root = merge(root.lc, root.rc)
*/</pre>
```

## 7.5 Black Magic

```
assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);
// Erase an entry.
s.erase(12);
// The order of the keys should be: 505.
assert(*s.find_by_order(0) == 505);
// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);
heap h1 , h2; h1.join( h2 );
}
```

## 8 Others

### 8.1 Exact Cover Set

```
// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#include <stdio.h>
#include <string.h>
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
  L[R[c]]=L[c]; R[L[c]]=R[c];
  for( int i=D[c]; i!=c; i=D[i] )
  for( int j=R[i]; j!=i; j=R[j] ){
       U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
void resume(int c){
  for( int i=D[c]; i!=c; i=D[i] )
  for( int j=L[i]; j!=i; j=L[j] ){
    U[D[j]]=D[U[j]]=j; S[C[j]]++;
  L[R[c]]=R[L[c]]=c;
int dfs(){
  if(R[0]==0) return 1;
  int md=100000000,c;
  for( int i=R[0]; i!=0; i=R[i] )
  if(S[i]<md){ md=S[i]; c=i; }
if(md==0) return 0;</pre>
  remove(c);
  for( int i=D[c]; i!=c; i=D[i] ){
    used[ROW[i]]=1;
     for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
     if(dfs()) return 1;
    for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
used[ROW[i]]=0;
  resume(c);
  return 0;
int exact_cover(int n,int m){
  for( int i=0; i<=m; i++ ){
    R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
     S[i]=0; C[i]=i;
  R[m]=0; L[0]=m;
  int t=m+1;
  for( int i=0; i<n; i++ ){</pre>
     int k=-1;
     for( int j=0; j<m; j++ ){</pre>
       if(!A[i][j]) continue;
       if(k==-1) L[t]=R[t]=t
       else{ L[t]=k; R[t]=R[k]; }
k=t; D[t]=j+1; U[t]=U[j+1];
L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
       C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
    }
  }
  for( int i=0; i<n; i++ ) used[i]=0;</pre>
  return dfs();
```