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## 1 Basic

### 1.1 .vimrc

```
syn on
se ai nu ru cul mouse=a
se cin et ts=2 sw=2 sts=2
so $VIMRUNTIME/mswin.vim
colo desert
se gfn=Monospace\ 14
```

### 1.2 Big Primes

```
179424691, 179424697, 179424719, 179424731, 179424743
1000049341, 1000049371, 1000049383, 1000049419
```

### 1.3 Misc

```
#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }

#define SECS (clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;
```

### 1.4 python-related

```
from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision
```

```
itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cost):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cost = ((cost + 1) / two) ** itwo
        sinT = (1 - cost * cost) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))
```

## 2 flow

### 2.1 ISAP

```
#define SZ(c) ((int)(c).size())
struct Maxflow { // 0(V^2 * E)
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
```

```

    G[i].clear();
    iter[i] = d[i] = gap[i] = 0;
}
}
void addEdge(int u, int v, int c) {
    G[u].push_back(Edge(v, c, SZ(G[v])));
    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
}
int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
        Edge &e = G[p][i];
        if(e.c > 0 && d[p] == d[e.v]+1) {
            int f = dfs(e.v, min(flow, e.c));
            if(f) {
                e.c -= f;
                G[e.v][e.r].c += f;
                return f;
            }
        }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

## 2.2 MinCostFlow

```

struct MinCostMaxFlow{ // O(V^2 * F)
typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFf = 1000000;
    static const Tcost INFc = 1e9;
    struct Edge{
        int v, cap;
        Tcost w;
        int rev;
        Edge(){}
        Edge(int t2, int t3, Tcost t4, int t5)
        : v(t2), cap(t3), w(t4), rev(t5) {}
    };
    int V, s, t;
    vector<Edge> g[MAXV];
    void init(int n){
        V = n+2;
        s = n+1, t = n+2;
        for(int i = 0; i <= V; i++) g[i].clear();
    }
    void addEdge(int a, int b, int cap, Tcost w){
        g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
        g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
    }
    Tcost d[MAXV];
    int id[MAXV], mom[MAXV];
    bool inqu[MAXV];
    queue<int> q;
    Tcost solve(){
        int mxf = 0; Tcost mnc = 0;
        while(1){
            fill(d, d+1+V, INFc);
            fill(inqu, inqu+1+V, 0);
            fill(mom, mom+1+V, -1);
            mom[s] = s;
            d[s] = 0;
            q.push(s); inqu[s] = 1;
            while(q.size()){
                int u = q.front(); q.pop();
                inqu[u] = 0;
                for(int i = 0; i < (int) g[u].size(); i++){

```

```

                    Edge &e = g[u][i];
                    int v = e.v;
                    if(e.cap > 0 && d[v] > d[u]+e.w){
                        d[v] = d[u]+e.w;
                        mom[v] = u;
                        id[v] = i;
                        if(!inqu[v]) q.push(v), inqu[v] = 1;
                    }
                }
            }
            if(mom[t] == -1) break;
            int df = INFf;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mxf += df;
            mnc += df*d[t];
        }
        return mnc;
    }
} flow;

```

## 2.3 Dinic

```

struct Dinic{ // O(V^2 * E)
    static const int MXN = 10000;
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while ( BFS() )
            res += DFS(s,2147483647);
        return res;
    }
} flow;

```

## 2.4 KM

```
const int INF=1016; /// $\text{max}(a[i][j])$ 
const int MAXN=650;
int a[MAXN][MAXN]; // weight  $[x][y]$ , two set of vertex
int N; // two set: each set have exactly N vertex
int match[MAXN*2], weight[MAXN*2];
bool vis[MAXN*2];

bool DFS(int x) {
    vis[x]=1;
    for(int i=0;i<N;i++) {
        if(weight[x]+weight[N+i]!=a[x][i]) continue;
        vis[N+i]=1;
        if(match[N+i]==-1 || (!vis[match[N+i]]&&DFS(match[N+i]))) {
            match[N+i]=x;
            match[x]=N+i;
            return 1;
        }
    }
    return 0;
}

int KM() { //  $O(V^3)$ 
    fill(weight, weight+N*N, 0);
    for(int i=0;i<N;i++) {
        for(int j=0;j<N;j++)
            weight[i]=max(weight[i], a[i][j]);
    }
    fill(match, match+N*N, -1);
    for(int u=0;u<N;u++) {
        fill(vis, vis+N*N, 0);
        while(!DFS(u)) {
            int d=INF;
            for(int i=0;i<N;i++) {
                if(!vis[i]) continue;
                for(int j=0;j<N;j++)
                    if(!vis[N+j])
                        d=min(d, weight[i]+weight[N+j]-a[i][j]);
            }
            for(int i=0;i<N;i++)
                if(vis[i])
                    weight[i]-=d;
            for(int i=N;i<N+N;i++)
                if(vis[i])
                    weight[i]+=d;
            fill(vis, vis+N*N, 0);
        }
    }
    int ans=0;
    for(int i=0;i<N+N;i++) ans+=weight[i];
    return ans;
}
```

## 2.5 DMST

```
/*
 * Edmond's algoirthm for Directed MST
 * runs in  $O(VE)$ 
 */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
    int u, v, c;
    Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
    fill(con, con+V+1, 0);
    int r1 = 0, r2 = 0;
    while(1){
```

```
        fill(mnInW, mnInW+V+1, INF);
        fill(prv, prv+V+1, -1);
        REP(i, 1, E){
            int u=edges[i].u, v=edges[i].v, c=edges[i].c;
            if(u != v && v != root && c < mnInW[v])
                mnInW[v] = c, prv[v] = u;
        }
        fill(vis, vis+V+1, -1);
        fill(cyc, cyc+V+1, -1);
        r1 = 0;
        bool jf = 0;
        REP(i, 1, V){
            if(con[i]) continue;
            if(prv[i] == -1 && i != root) return -1;
            if(prv[i] > 0) r1 += mnInW[i];
            int s;
            for(s = i; s != -1 && vis[s] == -1; s = prv[s])
                vis[s] = i;
            if(s > 0 && vis[s] == i){
                // get a cycle
                jf = 1; int v = s;
                do{
                    cyc[v] = s, con[v] = 1;
                    r2 += mnInW[v]; v = prv[v];
                }while(v != s);
                con[s] = 0;
            }
        }
        if(!jf) break;
        REP(i, 1, E){
            int &u = edges[i].u;
            int &v = edges[i].v;
            if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
            if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
            if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
            if(u == v) edges[i--] = edges[E--];
        }
        return r1+r2;
    }
}
```

## 2.6 SW min-cut

```
// global min cut
struct SW{ //  $O(V^3)$ 
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n; FZ(edge); FZ(del);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++)
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for (int i=0; i<n; i++)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0,x,y; i<n-1; i++){
            search(x,y);
            res = min(res,wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++)
                edge[x][j] = (edge[j][x] += edge[y][j]);
        }
        return res;
    }
}
```

```
}graph;
```

## 2.7 Max Cost Circulation

```
struct MaxCostCirc {
    static const int MAXN = 33;
    int n, m;
    struct Edge { int v, w, c, r; };
    vector<Edge> g[ MAXN ];
    int dis[ MAXN ], prv[ MAXN ], prve[ MAXN ];
    bool vis[ MAXN ];
    int ans;
    void init( int _n, int _m ) : n(_n), m(_m) {}
    void adde( int u, int v, int w, int c ) {
        g[ u ].push_back( { v, w, c, SZ( g[ v ] ) } );
        g[ v ].push_back( { u, -w, 0, SZ( g[ u ] )-1 } );
    };
}
bool poscyc() {
    fill( dis, dis+n+1, 0 );
    fill( prv, prv+n+1, 0 );
    fill( vis, vis+n+1, 0 );
    int tmp = -1;
    FOR( t, n+1 ) {
        REP( i, 1, n ) {
            FOR( j, SZ( g[ i ] ) ) {
                Edge& e = g[ i ][ j ];
                if( e.c && dis[ e.v ] < dis[ i ]+e.w ) {
                    dis[ e.v ] = dis[ i ]+e.w;
                    prv[ e.v ] = i;
                    prve[ e.v ] = j;
                    if( t == n ) {
                        tmp = i;
                        break;
                    }
                }
            }
        }
    }
    if( tmp == -1 ) return 0;
    int cur = tmp;
    while( !vis[ cur ] ) {
        vis[ cur ] = 1;
        cur = prv[ cur ];
    }
    int now = cur, cost = 0, df = 100000;
    do{
        Edge &e = g[ prv[ now ] ][ prve[ now ] ];
        df = min( df, e.c );
        cost += e.w;
        now = prv[ now ];
    }while( now != cur );
    ans += df*cost; now = cur;
    do{
        Edge &e = g[ prv[ now ] ][ prve[ now ] ];
        Edge &re = g[ now ][ e.r ];
        e.c -= df;
        re.c += df;
        now = prv[ now ];
    }while( now != cur );
    return 1;
}
} circ;
```

## 2.8 Max flow with lower/upper bound

```
// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ];
int solve(){
    flow.init( n );
    for( int i = 0; i < m; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1; i <= n; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] )
            flow.addEdge( flow.s, i, in[ i ] - out[ i ] );
    }
    // original sink to source
    flow.addEdge( n, 1, INF );
    if( flow.maxflow() != nd )
        // no solution
        return -1;
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0; i < flow.G[ flow.s ].size(); i ++ ){
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0; i < flow.G[ flow.t ].size(); i ++ ){
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s, 1, INF );
    flow.addEdge( n, flow.t, INF );
    flow.reset();
    return ans + flow.maxflow();
}
```

## 2.9 Relabel to Front

```
// O(N^3), 0-base
struct Edge{
    int from, to, cap, flow;
    Edge(int _from, int _to, int _cap, int _flow = 0):
        from(_from), to(_to), cap(_cap), flow(_flow) {}
};
struct PushRelabel{
    int n;
    vector<Edge> edges;
    vector<int> count, h, inQ, excess;
    vector<vector<int>> > G;
    queue<int> Q;
    PushRelabel(int _n):
        n(_n), count(_n<<1), G(_n), h(_n), inQ(_n), excess(_n) {}
    void addEdge(int from, int to, int cap) {
        G[from].push_back(edges.size());
        edges.push_back(Edge(from, to, cap));
        G[to].push_back(edges.size());
        edges.push_back(Edge(to, from, 0));
    }
    void enqueue(int u) {
        if(!inQ[u] && excess[u] > 0) Q.push(u), inQ[u] = true;
    }
    void Push(int EdgeIdx) {
        Edge &e = edges[EdgeIdx];
        int toPush = min<int>(e.cap - e.flow, excess[e.from]);
        if(toPush > 0 && h[e.from] > h[e.to]) {
            e.flow += toPush;
            excess[e.to] += toPush;
            excess[e.from] -= toPush;
            edges[EdgeIdx+1].flow -= toPush;
            enqueue(e.to);
        }
    }
    void Relabel(int u) {
        count[h[u]] --; h[u] = 2*n-2;
        for( size_t i = 0; i < G[u].size(); ++i ) {
            Edge &e = edges[G[u][i]];
            if(e.cap > e.flow) h[u] = min(h[u], h[e.to]);
        }
        count[++h[u]] += 1;
    }
    void gapRelabel(int height) {

```

```

    for (int u = 0; u < n; ++u) if(h[u] >= height && h[u] < n) {
        count[h[u]] -= 1;
        count[h[u] = n] += 1;
        enqueue(u);
    }
}
void Discharge(int u) {
    for (size_t i = 0; excess[u] > 0 && i < G[u].size(); ++i)
        Push(G[u][i]);
    if(excess[u] > 0) {
        if(h[u] < n && count[h[u]] < 2) gapRelabel(h[u]);
        else Relabel(u);
    }
    else if(!Q.empty()) { // dequeue
        Q.pop();
        inQ[u] = false;
    }
}
int solve(int src, int snk) {
    h[src] = n; inQ[src] = inQ[snk] = true;
    count[0] = n - (count[n] = 1);
    for (size_t i = 0; i < G[src].size(); ++i) {
        excess[src] += edges[G[src][i]].cap;
        Push(G[src][i]);
    }
    while (!Q.empty())
        Discharge(Q.front());
    return excess[snk];
}
};

```

## 2.10 Flow Method

Maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ ;  
 with the corresponding symmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y \geq c, y \geq 0$ .

Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
 with the corresponding asymmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y = c, y \geq 0$ .

Minimum vertex cover on bipartite graph =  
 Maximum matching on bipartite graph =  
 Max flow with source to one side, other side to sink

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side **and** with unused edges only. Equivalently, dfs from source with unused edges only **and** without visiting sink. Then, a vertex is chosen

iff. it is on the left side **and** without visited **or** on the right side **and** visited through dfs.

Maximum density subgraph  $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:

For a fixed D, construct a Max flow model as follow:

Let S be Sum of all weight( **or** inf)

1. from source to each node with cap = S
2. For each  $(u,v,w)$  in E,  $(u \rightarrow v, \text{cap}=w), (v \rightarrow u, \text{cap}=w)$
3. For each node v, from v to sink with cap =  $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$

where  $\text{deg}[v] = \sum \text{weight of edge associated with } v$   
 If  $\text{maxflow} < S * |V|$ , D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

## 3 Math

### 3.1 FFT

```

// const int MAXN = 262144; (must be 2^k)
// O(n*log(n))
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}

```

### 3.2 NTT

```

typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
   n    2^n    p    a    root
   16   65536   65537   1    3
   20   1048576  7340033  7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >= 1, bs = (bs * bs) % P)
            if(b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
        }
    }
}

```

```

    }
    }
    theta = (theta * 2) % MAXN;
}
int i = 0;
for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^ k); k >= 1);
    if (j < i) swap(a[i], a[j]);
}
if (inv_ntt) {
    LL ni = inv(n,P);
    reverse(a+1, a+n);
    for (i = 0; i < n; i++)
        a[i] = (a[i] * ni) % P;
}
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

### 3.3 Fast Walsh Transform

```

/* xor convolution:
* x = (x0,x1), y = (y0,y1)
* z = (x0y0 + x1y1, x0y1 + x1y0)
* =>
* x' = (x0+x1, x0-x1), y' = (y0+y1, y0-y1)
* z' = ((x0+x1)(y0+y1), (x0-x1)(y0-y1))
* z = (1/2) * z'
* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL pw(LL x, LL k) {
    LL res = 1;
    for (LL bs = x; k; k >= 1, bs = (bs * bs) % MOD)
        if (k & 1) res = (res * bs) % MOD;
    return res;
}
inline LL inv(LL x) {
    return pw(x, MOD-2);
}
inline void fwt(LL x[ MAXN ], int N, bool inv=0) {
    for (int d = 1; d < N; d <= 1) {
        int d2 = d << 1;
        for (int s = 0; s < N; s += d2)
            for (int i = s, j = s+d; i < s+d; i++, j++) {
                LL ta = x[i], tb = x[j];
                x[i] = ta+tb;
                x[j] = ta-tb;
                if (x[i] >= MOD) x[i] -= MOD;
                if (x[j] < 0) x[j] += MOD;
            }
    }
    if (inv)
        for (int i = 0; i < N; i++) {
            x[i] *= inv(N);
            x[i] %= MOD;
        }
}

```

### 3.4 Poly operator

```

struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
    NTT<P, root, MAXN> ntt;
    static int nxt2k(int x) {
        int i = 1; for (; i < x; i <= 1); return i;
    }
    void Mul(int n, LL a[], int m, LL b[], LL c[]) {
        static LL aa[MAXN], bb[MAXN];
        int N = nxt2k(n+m);
        copy(a, a+n, aa); fill(aa+n, aa+N, 0);
        copy(b, b+m, bb); fill(bb+m, bb+N, 0);

```

```

        ntt(N, aa); ntt(N, bb);
        FOR(i, N) c[i] = aa[i] * bb[i] % P;
        ntt(N, c, 1);
    }
    void Inv(int n, LL a[], LL b[]) {
        // ab = aa^-1 = 1 mod x^(n/2)
        // (b - a^-1)^2 = 0 mod x^n
        // bb - a^-2 + 2 ba^-1 = 0
        // bba - a^-1 + 2b = 0
        // bba + 2b = a^-1
        static LL tmp[MAXN];
        if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
        Inv((n+1)/2, a, b);
        int N = nxt2k(n*2);
        copy(a, a+n, tmp);
        fill(tmp+n, tmp+N, 0);
        fill(b+n, b+N, 0);
        ntt(N, tmp); ntt(N, b);
        FOR(i, N) {
            LL t1 = (2 - b[i] * tmp[i]) % P;
            if (t1 < 0) t1 += P;
            b[i] = b[i] * t1 % P;
        }
        ntt(N, b, 1);
        fill(b+n, b+N, 0);
    }
    void Div(int n, LL a[], int m, LL b[], LL d[], LL r[]) {
        // Ra = Rb * Rd mod x^(n-m+1)
        // Rd = Ra * Rb^-1 mod
        static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
        if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0); return;}
        // d: n-1 - (m-1) = n-m (n-m+1 terms)
        copy(a, a+n, aa); copy(b, b+m, bb);
        reverse(aa, aa+n); reverse(bb, bb+m);
        Inv(n-m+1, bb, tb);
        Mul(n-m+1, ta, n-m+1, tb, d);
        fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
        // r: m-1 - 1 = m-2 (m-1 terms)
        Mul(m, b, n-m+1, d, ta);
        FOR(i, n) {r[i] = a[i] - ta[i]; if (r[i] < 0) r[i] += P;}
    }
    void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i] = i * a[i] % P; }
    void Sx(int n, LL a[], LL b[]) {
        b[0] = 0;
        FOR(i, n) b[i+1] = a[i] * ntt.iv[i+1] % P;
    }
    void Ln(int n, LL a[], LL b[]) {
        // Integral a' a^-1 dx
        static LL a1[MAXN], a2[MAXN], b1[MAXN];
        int N = nxt2k(n*2);
        dx(n, a, a1); Inv(n, a, a2);
        Mul(n-1, a1, n, a2, b1);
        Sx(n+n-1-1, b1, b);
        fill(b+n, b+N, 0);
    }
    void Exp(int n, LL a[], LL b[]) {
        // Newton method to solve g(a(x)) = ln b(x) - a(x)
        // = 0
        // b' = b - g(b(x)) / g'(b(x))
        // b' = b (1 - lnb + a)
        static LL lnb[MAXN], c[MAXN], tmp[MAXN];
        assert(a[0] == 0); // dont know exp(a[0]) mod P
        if (n == 1) {b[0] = 1; return;}
        Exp((n+1)/2, a, b);
        fill(b+(n+1)/2, b+n, 0);
        Ln(n, b, lnb);
        fill(c, c+n, 0); c[0] = 1;
        FOR(i, n) {
            c[i] += a[i] - lnb[i];
            if (c[i] < 0) c[i] += P;
            if (c[i] >= P) c[i] -= P;
        }
        Mul(n, b, n, c, tmp);
        copy(tmp, tmp+n, b);
    }
} polyop;

```



### 3.5 Linear Recurrence

```
LL n, m, dp[ N + N ];
// dp[ n ] = \sum_{i=0}^{m-1} A_i * dp[ n - i - 1 ]
void pre_dp( const vector<LL>& A ){
    dp[ 0 ] = 1;
    LL bdr = min( m + m , n );
    for( LL i = 1 ; i <= bdr ; i ++ ){
        dp[ i ] = 0;
        for( LL j = i - 1 ; j >= max( 0LL , i - m ) ; j -- )
            dp[ i ] = add( dp[ i ] , mul( dp[ j ] , A[ i - j - 1 ] ) );
    }
}
vector<LL> Mul( vector<LL>& v1, vector<LL>& v2 ){
    int _sz1 = (int)v1.size(), _sz2 = (int)v2.size();
    assert( _sz1 == m ); assert( _sz2 == m );
    vector<LL> _v( m + m );
    for( int i = 0 ; i < m + m ; i ++ ) _v[ i ] = 0;
    // expand
    for( int i = 0 ; i < _sz1 ; i ++ )
        for( int j = 0 ; j < _sz2 ; j ++ )
            _v[ i + j + 1 ] = add( _v[ i + j + 1 ] , mul( v1[ i ] , v2[ j ] ) );
    // shrink
    for( int i = 0 ; i < m ; i ++ )
        for( int j = 1 ; j <= m ; j ++ )
            _v[ i + j ] = add( _v[ i + j ] , _v[ i ] );
    for( int i = 0 ; i < m ; i ++ ) _v[ i ] = _v[ i + m ];
    _v.resize( m );
    return _v;
}
vector<LL> I, A;
LL solve(){ // A should be filled
    pre_dp( A );
    if( n <= m + m ) return dp[ n ];
    I.resize( m );
    for( int i = 0 ; i < m ; i ++ ) I[ i ] = 1;
    LL dlt = ( n - m ) / m, rdlt = dlt * m;
    while( dlt ){
        if( dlt & 1LL ) I = Mul( I , A );
        A = Mul( A , A ); dlt >>= 1;
    }
    LL ans = 0;
    for( int i = 0 ; i < m ; i ++ )
        ans = add( ans, mul( I[ i ], dp[ n - i - 1 - rdlt ] ) );
    return ans;
}
```

### 3.6 Miller Rabin

```
// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(LL a, LL n, LL u, int t){
    LL x = mypow(a, u, n);
    for( int i = 0; i < t; i ++ ) {
        LL nx = mul(x, x, n);
        if( nx == 1 && x != 1 && x != n - 1 ) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(LL n, int s = 100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if( n < 2 ) return 0;
    if( !(n & 1) ) return n == 2;
    LL u = n - 1; int t = 0;
    // n - 1 = u * 2^t
    while( !(u & 1) ) u >>= 1, t ++;
    while( s -- ){
        LL a = randll() % (n - 1) + 1;
        if( witness(a, n, u, t) ) return 0;
    }
}
```

```
return 1;
}
```

### 3.7 Simplex

```
const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax <= b, x >= 0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
               double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for( int i = 0; i < n + m; ++i ) ix[i] = i;
    for( int i = 0; i < n; ++i ) {
        for( int j = 0; j < m - 1; ++j ) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if( d[r][m] > d[i][m] ) r = i;
    }
    for( int j = 0; j < m - 1; ++j ) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for( double dd;; ) {
        if( r < n ) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for( int j = 0; j <= m; ++j )
                if( j != s ) d[r][j] *= -d[r][s];
            for( int i = 0; i <= n + 1; ++i ) if( i != r ) {
                for( int j = 0; j <= m; ++j ) if( j != s )
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for( int j = 0; j < m; ++j )
            if( s < 0 || ix[s] > ix[j] ) {
                if( d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps) )
                    s = j;
            }
        if( s < 0 ) break;
        for( int i = 0; i < n; ++i ) if( d[i][s] < -eps ) {
            if( r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]) )
                r = i;
        }
        if( r < 0 ) return -1; // not bounded
    }
    if( d[n + 1][m] < -eps ) return -1; // not executable
    double ans = 0;
    for( int i = 0; i < m; i ++ ) x[i] = 0;
    for( int i = m; i < n + m; ++i ) { // the missing
        enumerated x[i] = 0
        if( ix[i] < m - 1 ) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}
```

### 3.8 Faulhaber

```
/* faulhaber's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
```

```

int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinactories
int co[MAXK][MAXK+2]; // coeeficient of  $x^j$  when  $p=i$ 
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
    /* inverse */
    for(int i=1; i<=MAXK; i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2; i<MAXK; i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0; j<i; j++)
            b[i]=sub(b[i],
                mul(cm[i][j], mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    //  $\sigma_x = 1 \sim n \{x^p\} =$ 
    //  $1/(p+1) * \sigma_j = 0 \sim p \{C(p+1, j) * B_j * n^{p-j+1}\}$ 
    for(int i=1; i<MAXK; i++) {
        co[i][0]=0;
        for(int j=0; j<=i; j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
    /* sample usage: return f(n,p) =  $\sigma_x = 1 \sim n (x^p)$  */
    inline int solve(int n, int p) {
        int sol=0, m=n;
        for(int i=1; i<=p+1; i++) {
            sol=add(sol, mul(co[p][i], m));
            m = mul(m, n);
        }
        return sol;
    }
}

```

### 3.9 Chinese Remainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL, LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

### 3.10 Pollard Rho

```

// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
    }
}

```

```

    if (res!=0 && res!=n) return res;
}
}

```

### 3.11 ax+by=gcd

```

PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

### 3.12 Discrete sqrt

```

void calch(int &t, int &h, const int p) {
    int tmp=p-1; for(t=0; (tmp&1)==0; tmp/=2) t++; h=tmp;
}
// solve equation  $x^2 \bmod p = a$ 
bool solve(int a, int p, int &x, int &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a, (p+1)/4, p); y=p-x; return true;
    } else {
        int t, h, b, pb; calch(t, h, p);
        if (t >= 2) {
            do { b = rand() % (p - 2) + 2;
                } while (mypow(b, p / 2, p) != p - 1);
            pb = mypow(b, h, p);
            int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((LL)(s * s) % p) * a) % p;
                for(int i=0; i<t-step; i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((LL)pb * pb) % p;
                x = ((LL)s * a) % p; y = p - x;
            } return true;
        }
    }
}

```

### 3.13 Romberg

```

// Estimates the definite integral of
//  $\int_a^b f(x) dx$ 
template<class T>
double romberg(T& f, double a, double b, double eps=1e-8){
    vector<double> t; double h=b-a, last, curr; int k=1, i=1;
    t.push_back(h*(f(a)+f(b))/2);
    do{ last=t.back(); curr=0; double x=a+h/2;
        for(int j=0; j<k; j++) curr+=f(x), x+=h;
        curr=(t[0] + h*curr)/2; double k1=4.0/3.0, k2=1.0/3.0;
        for(int j=0; j<i; j++){ double temp=k1*curr-k2*t[j];
            t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
        } t.push_back(curr); k*=2; h/=2; i++;
    }while( fabs(last-curr) > eps);
    return t.back();
}

```

### 3.14 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```



### 3.15 Roots of Polynomial

```
const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int n;
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++)
        { sum=sum+a[i]*tmp; tmp=tmp*x; }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
}
int main() {
    scanf("%d",&n);
    for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
    int nx;
    solve(n,a,x,nx);
    for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);
}
```

### 3.16 Result

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^+$  and prime  $P$ ,  $C(m, n) \bmod P = \prod C(m_i, n_i)$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem :  $A = i + b/2 - 1$
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? -1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$

## 4 Geometry

### 4.1 Points class

```
typedef double type;
typedef pair<type,type> Pt;
typedef pair<Pt,Pt> Line;
typedef pair<Pt,type> Circle;
#define X first
#define Y second
#define O first
#define R second
Pt operator+( const Pt& p1 , const Pt& p2 ){
    return { p1.X + p2.X , p1.Y + p2.Y };
}
Pt operator-( const Pt& p1 , const Pt& p2 ){
    return { p1.X - p2.X , p1.Y - p2.Y };
}
Pt operator*( const Pt& tp , const type& tk ){
    return { tp.X * tk , tp.Y * tk };
}
Pt operator/( const Pt& tp , const type& tk ){
    return { tp.X / tk , tp.Y / tk };
}
type operator*( const Pt& p1 , const Pt& p2 ){
    return p1.X * p2.X + p1.Y * p2.Y;
}
type operator^( const Pt& p1 , const Pt& p2 ){
    return p1.X * p2.Y - p1.Y * p2.X;
}
type norm2( const Pt& tp ){
    return tp * tp;
}
double norm( const Pt& tp ){
    return sqrt( norm2( tp ) );
}
Pt perp( const Pt& tp ){
    return { tp.Y , -tp.X };
}
```

### 4.2 halfPlaneIntersection

### 4.3 Intersection of 2 lines

```
Pt interPnt( Line l1, Line l2, bool &res ){
    Pt p1, p2, q1, q2;
    tie(p1, p2) = l1; tie(q1, q2) = l2;
    double f1 = (p2 - p1) ^ (q1 - p1);
    double f2 = (p2 - p1) ^ (p1 - q2);
    double f = (f1 + f2);
    if( fabs(f) < eps){ res=0; return {0, 0}; }
    res = true;
    return q1 * (f2 / f) + q2 * (f1 / f);
}
bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    bool res; Pt p = interPnt(l1, l2, res);
    return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}
/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
 */
/* --^-- Line.FI --^-- Line.SE --^-- */
vector<Line> halfPlaneInter( vector<Line> lines ){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for( int i=0; i<sz; i++) {
        ord[i] = i;
        Pt d = lines[i].SE - lines[i].FI;
        ata[i] = atan2(d.Y, d.X);
    }
    sort( ord.begin(), ord.end(), [&](int i, int j) {
        if( fabs(ata[i] - ata[j]) < eps )
            return ( (lines[i].SE - lines[i].FI) ^
                    (lines[j].SE - lines[i].FI) ) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for( int i=0; i<sz; i++)
        if ( !i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps )
            fin.PB(lines[ord[i]]);
    deque<Line> dq;
    for( int i=0; i<(int)(fin.size()); i++) {
```

```

while((int)(dq.size()) >= 2 and
not isin(fin[i], dq[(int)(dq.size()-2),
dq[(int)(dq.size()-1)]))
dq.pop_back();
while((int)(dq.size()) >= 2 and
not isin(fin[i], dq[0], dq[1]))
dq.pop_front();
dq.push_back(fin[i]);
}
while( (int)(dq.size()) >= 3 and
not isin(dq[0], dq[(int)(dq.size()-2),
dq[(int)(dq.size()-1)]))
dq.pop_back();
while( (int)(dq.size()) >= 3 and
not isin(dq[(int)(dq.size()-1], dq[0], dq[1]))
dq.pop_front();
vector<Line> res(dq.begin(),dq.end());
return res;
}

```

#### 4.4 Intersection of 2 segments

```

int ori( const Pt& o , const Pt& a , const Pt& b ){
LL ret = ( a - o ) ^ ( b - o );
return (ret > 0) - (ret < 0);
}
// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
const Pt& q1 , const Pt& q2 ){
if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
if( ori( p1 , p2 , q1 ) ) return false;
return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
}
return (ori( p1, p2, q1 ) * ori( p1, p2, q2 ) <= 0) &&
(ori( q1, q2, p1 ) * ori( q1, q2, p2 ) <= 0);
}

```

#### 4.5 Intersection of circle and segment

```

bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
Pt dp = p2 - p1;
double a = dp * dp;
double b = 2 * ( dp * ( p1 - cc.o ) );
double c = cc.o * cc.o + p1 * p1 - 2 * ( cc.o * p1 )
- cc.R * cc.R;
double bb4ac = b * b - 4 * a * c;
return !( fabs( a ) < eps or bb4ac < 0 );
}

```

#### 4.6 Intersection of 2 polygons

```

double area(Pt* ps,int n){
ps[n]=ps[0];
double res=0;
for(int i=0;i<n;i++){
res+=ps[i].x*ps[i+1].y-ps[i].y*ps[i+1].x;
}
return res/2.0;
}
int lineCross(Pt a,Pt b,Pt c,Pt d,Pt&p){
double s1,s2;
s1=cross(a,b,c);
s2=cross(a,b,d);
if(sig(s1)==0&&sig(s2)==0) return 2;
if(sig(s2-s1)==0) return 0;
p.x=(c.x*s2-d.x*s1)/(s2-s1);
p.y=(c.y*s2-d.y*s1)/(s2-s1);
return 1;
}
void polygon_cut(Pt*p,int&n,Pt a,Pt b){
static Pt pp[maxn];
int m=0;p[n]=p[0];
for(int i=0;i<n;i++){

```

```

if(sig(cross(a,b,p[i]))>0) pp[m++]=p[i];
if(sig(cross(a,b,p[i]))!=sig(cross(a,b,p[i+1])))
)
lineCross(a,b,p[i],p[i+1],pp[m++]);
}
n=0;
for(int i=0;i<m;i++)
if(!i||!(pp[i]==pp[i-1]))
p[n++]=pp[i];
while(n>1&&p[n-1]==p[0])n--;
}
double intersectArea(Pt a,Pt b,Pt c,Pt d){
Pt o(0,0);
int s1=sig(cross(o,a,b));
int s2=sig(cross(o,c,d));
if(s1==0||s2==0)return 0.0;
if(s1==1) swap(a,b);
if(s2==1) swap(c,d);
Pt p[10]={o,a,b};
int n=3;
polygon_cut(p,n,o,c);
polygon_cut(p,n,c,d);
polygon_cut(p,n,d,o);
double res=fabs(area(p,n));
if(s1*s2==1) res=-res;return res;
}

```

#### 4.7 Circle cover

```

#define N 1021
struct CircleCover{
int C; Circ c[ N ];
bool g[ N ][ N ], overlap[ N ][ N ];
// Area[i] : area covered by at least i circles
D Area[ N ];
void init( int _C ){ C = _C; }
bool CCinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
Pt o1 = a.o , o2 = b.o;
D r1 = a.R , r2 = b.R;
if( norm( o1 - o2 ) > r1 + r2 ) return false;
if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
return true;
D d2 = ( o1 - o2 ) * ( o1 - o2 );
D d = sqrt(d2);
if( d > r1 + r2 ) return false;
Pt u=(o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
D A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
Pt v=Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
p1 = u + v; p2 = u - v;
return true;
}
struct Teve {
Pt p; D ang; int add;
Teve() {}
Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
bool operator<(const Teve &a)const
{return ang < a.ang;}
}eve[ N * 2 ];
// strict: x = 0, otherwise x = -1
bool disjunct( Circ& a, Circ& b, int x )
{return sign( norm( a.o - b.o ) - a.R - b.R ) > x;}
bool contain( Circ& a, Circ& b, int x )
{return sign( a.R - b.R - norm( a.o - b.o ) ) > x;}
bool contain(int i, int j){
/* c[j] is non-strictly in c[i]. */
return (sign(c[i].R - c[j].R) > 0 ||
(sign(c[i].R - c[j].R) == 0 && i < j) ) &&
contain(c[i], c[j], -1);
}
void solve(){
for( int i = 0 ; i <= C + 1 ; i ++ )
Area[ i ] = 0;
for( int i = 0 ; i < C ; i ++ )
for( int j = 0 ; j < C ; j ++ )
overlap[i][j] = contain(i, j);
for( int i = 0 ; i < C ; i ++ )
for( int j = 0 ; j < C ; j ++ )
g[i][j] = !(overlap[i][j] || overlap[j][i] ||
disjunct(c[i], c[j], -1));
for( int i = 0 ; i < C ; i ++ ){

```

```

int E = 0, cnt = 1;
for( int j = 0 ; j < C ; j ++ )
    if( j != i && overlap[j][i] )
        cnt ++;
for( int j = 0 ; j < C ; j ++ )
    if( i != j && g[i][j] ){
        Pt aa, bb;
        CCinter(c[i], c[j], aa, bb);
        D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
        D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
        eve[E ++] = Teve(bb, B, 1);
        eve[E ++] = Teve(aa, A, -1);
        if(B > A) cnt ++;
    }
if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
else{
    sort( eve , eve + E );
    eve[E] = eve[0];
    for( int j = 0 ; j < E ; j ++ ){
        cnt += eve[j].add;
        Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
        D theta = eve[j + 1].ang - eve[j].ang;
        if (theta < 0) theta += 2. * pi;
        Area[cnt] +=
            (theta - sin(theta)) * c[i].R*c[i].R * .5;
    }
}
}
}
};

```

## 4.8 Convex Hull trick

```

/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));

```

```

        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt
            (p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points
    // return false if inside CH
    bool get_tang(Pt p, int &i0, int &i1) {
        if (contain(p)) return false;
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p)
            - lower.begin();
        bi_search(0, id, p, i0, i1);
        bi_search(id, (int)lower.size(), p, i0, i1);
        id = lower_bound(upper.begin(), upper.end(), p,
            greater<Pt>()) - upper.begin();
        bi_search((int)lower.size() - 1, (int)lower.size()
            - 1 + id, p, i0, i1);
        bi_search((int)lower.size() - 1 + id, (int)lower.
            size() - 1 + (int)upper.size(), p, i0, i1);
        return true;
    }
    // 3. Find tangent points of a given vector
    // ret the idx of vertex has max cross value with vec
    int get_tang(Pt vec){
        pair<LL, int> ret = get_tang(upper, vec);
        ret.second = (ret.second+(int)lower.size()-1)%n;
        ret = max(ret, get_tang(lower, vec));
        return ret.second;
    }
    // 4. Find intersection point of a given line
    // return 1 and intersection is on edge (i, next(i))
    // return 0 if no strictly intersection
    bool get_intersection(Pt u, Pt v, int &i0, int &i1){
        int p0 = get_tang(u - v), p1 = get_tang(v - u);
        if (sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u)<0){
            if (p0 > p1) swap(p0, p1);
            i0 = bi_search(u, v, p0, p1);
            i1 = bi_search(u, v, p1, p0 + n);
            return 1;
        }
        return 0;
    }
};

```

## 4.9 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,

```

```

        v.Y * c + sign2 * h * v.X };
Pt p1 = c1.0 + n * c1.R;
Pt p2 = c2.0 + n * ( c2.R * sign1 );
if( fabs( p1.X - p2.X ) < eps and
    fabs( p1.Y - p2.Y ) < eps )
    p2 = p1 + perp( c2.0 - c1.0 );
ret.push_back( { p1 , p2 } );
}
return ret;
}

```

#### 4.10 Lower Concave Hull

```

/****
maintain a "concave hull" that support the following
1. insertion of a line
2. query of height(y) on specific x on the hull
****/
/* set as needed */
typedef long double LD;
const LD eps=1e-9;
const LD inf=1e19;
class Seg {
public:
    LD m,c,x1,x2; // y=mx+c
    bool flag;
    Seg(
        LD _m,LD _c,LD _x1=-inf,LD _x2=inf,bool _flag=0)
        :m(_m),c(_c),x1(_x1),x2(_x2),flag(_flag) {}
    LD evaly(LD x) const { return m*x+c; }
    const bool operator<(LD x) const{return x2-eps<x;}
    const bool operator<(const Seg &b) const {
        if(flag||b.flag) return *this<b.x1;
        return m+eps<b.m;
    }
};
class LowerConcaveHull { // maintain a hull like: \_/_/
public:
    set<Seg> hull;
    /* functions */
    LD xintersection(Seg a,Seg b)
    { return (a.c-b.c)/(b.m-a.m); }
    inline set<Seg>::iterator replace(set<Seg> &
        hull,set<Seg>::iterator it,Seg s) {
        hull.erase(it);
        return hull.insert(s).first;
    }
    void insert(Seg s) {
        // insert a line and update hull
        set<Seg>::iterator it=hull.find(s);
        // check for same slope
        if(it!=hull.end()) {
            if(it->c+eps>=s.c) return;
            hull.erase(it);
        }
        // check if below whole hull
        it=hull.lower_bound(s);
        if(it!=hull.end())&&
            s.evaly(it->x1)<=it->evaly(it->x1)+eps) return;
        // update right hull
        while(it!=hull.end()) {
            LD x=xintersection(s,*it);
            if(x>=it->x2-eps) hull.erase(it++);
            else {
                s.x2=x;
                it=replace(hull,it,Seg(it->m,it->c,x,it->x2));
                break;
            }
        }
        // update left hull
        while(it!=hull.begin()) {
            LD x=xintersection(s,*(--it));
            if(x<=it->x1+eps) hull.erase(it++);
            else {
                s.x1=x;
                it=replace(hull,it,Seg(it->m,it->c,it->x1,x));
                break;
            }
        }
        // insert s

```

```

        hull.insert(s);
    }
    void insert(LD m,LD c) { insert(Seg(m,c)); }
    LD query(LD x) { // return y @ given x
        set<Seg>::iterator it =
            hull.lower_bound(Seg(0.0,0.0,x,x,1));
        return it->evaly(x);
    }
};

```

#### 4.11 Delaunay Triangulation

```

/* Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation such that no points will strictly
inside circumcircle of any triangle.

find : return a triangle contain given point
add_point : add a point into triangulation

A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]

calculation involves O(IV!^6) */
const int N = 100000 + 5;
const type inf = 2e3;
type eps = 1e-6; // 0 when integer
type sqr(type x) { return x*x; }
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
    const Pt& p4){
    type u11 = p1.X - p4.X; type u12 = p1.Y - p4.Y;
    type u21 = p2.X - p4.X; type u22 = p2.Y - p4.Y;
    type u31 = p3.X - p4.X; type u32 = p3.Y - p4.Y;
    type u13 = sqr(p1.X)-sqr(p4.X)+sqr(p1.Y)-sqr(p4.Y);
    type u23 = sqr(p2.X)-sqr(p4.X)+sqr(p2.Y)-sqr(p4.Y);
    type u33 = sqr(p3.X)-sqr(p4.X)+sqr(p3.Y)-sqr(p4.Y);
    type det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32
        -u11*u23*u32 - u12*u21*u33 + u11*u22*u33;
    return det > eps;
}
type side(const Pt& a, const Pt& b, const Pt& p)
{ return (b - a) ^ (p - a); }
typedef int SdRef;
struct Tri;
typedef Tri* TriRef;
struct Edge {
    TriRef tri; SdRef side;
    Edge():tri(0), side(0){}
    Edge(TriRef _tri, SdRef _side):tri(_tri), side(_side)
    {}
};
struct Tri {
    Pt p[3];
    Edge edge[3];
    TriRef chd[3];
    Tri() {}
    Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return chd[0] == 0 ? 0
            : chd[1] == 0 ? 1
            : chd[2] == 0 ? 2 : 3;
    }
    bool contains(Pt const& q) const {
        for( int i = 0 ; i < 3 ; i ++ )
            if( side(p[i], p[(i + 1) % 3] , q) < -eps )
                return false;
        return true;
    }
} pool[ N * 10 ], *tris;
void edge( Edge a, Edge b ){
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation

```

```

Trig(){
    the_root = // Tri should at least contain all
                points
                new(tris++)Tri(Pt(-inf,-inf),Pt(+inf+inf,-inf),Pt
                (-inf,+inf+inf));
}
TriRef find(Pt p){const{ return find(the_root,p); }
void add_point(const Pt& p){ add_point(find(the_root,
p),p); }
TriRef the_root;
static TriRef find(TriRef root, const Pt& p) {
    while( true ){
        if( !root->has_chd() )
            return root;
        for( int i = 0; i < 3 && root->chd[i] ; ++i )
            if (root->chd[i]->contains(p)) {
                root = root->chd[i];
                break;
            }
    }
    assert( false ); // "point not found"
}
void add_point(TriRef root, Pt const& p) {
    TriRef tab,tbc,tca;
    /* split it into three triangles */
    tab=new(tris++) Tri(root->p[0],root->p[1],p);
    tbc=new(tris++) Tri(root->p[1],root->p[2],p);
    tca=new(tris++) Tri(root->p[2],root->p[0],p);
    edge(Edge(tab,0), Edge(tbc,1));
    edge(Edge(tbc,0), Edge(tca,1));
    edge(Edge(tca,0), Edge(tab,1));
    edge(Edge(tab,2), root->edge[2]);
    edge(Edge(tbc,2), root->edge[0]);
    edge(Edge(tca,2), root->edge[1]);
    root->chd[0] = tab;
    root->chd[1] = tbc;
    root->chd[2] = tca;
    flip(tab,2);
    flip(tbc,2);
    flip(tca,2);
}
void flip(TriRef tri, SdRef pi) {
    TriRef trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0],tri->p[1],tri->p[2],trj->p[pj
    ])) return;
    /* flip edge between tri,trj */
    TriRef trk = new(tris++) Tri(tri->p[(pi+1)%3], trj
    ->p[pj], tri->p[pi]);
    TriRef trl = new(tris++) Tri(trj->p[(pj+1)%3], tri
    ->p[pi], trj->p[pj]);
    edge(Edge(trk,0), Edge(trl,0));
    edge(Edge(trk,1), tri->edge[(pi+2)%3]);
    edge(Edge(trk,2), trj->edge[(pj+1)%3]);
    edge(Edge(trl,1), trj->edge[(pj+2)%3]);
    edge(Edge(trl,2), tri->edge[(pi+1)%3]);
    tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
    trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
    flip(trk,1); flip(trk,2);
    flip(trl,1); flip(trl,2);
}
};
vector<TriRef> triang;
set<TriRef> vst;
void go( TriRef now ){
    if( vst.find( now ) != vst.end() )
        return;
    vst.insert( now );
    if( !now->has_chd() ){
        triang.push_back( now );
        return;
    }
    for( int i = 0 ; i < now->num_chd() ; i ++ )
        go( now->chd[ i ] );
}
void build( int n , Pt* ps ){
    tris = pool;
    random_shuffle(ps, ps + n);
    Trig tri;
    for(int i = 0; i < n; ++ i)
        tri.add_point(ps[i]);
}

```

```

    go( tri.the_root );
}

```

## 4.12 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for( int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for( int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for( int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
        }
        return {cen,sqrt(r2)};
    }
} mec;

```

## 4.13 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
    }else swap(x1,z1), swap(x2,z2), swap(L,H);
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```



## 5 Graph

### 5.1 HeavyLightDecomp

```
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of
    // node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfsz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfsz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]),
            [&](int a, int b){return sz[a] > sz[b];});
        bool flag = 1;
        for(int& v:g[u]) if(v != prt[u][0]){
            if(flag) head[v] = head[u], flag = 0;
            dfshl(v);
            tr[u] = tr[v];
        }
    }
    inline int lca(int a, int b){ // O(log(V))
        if(dep[a] > dep[b]) swap(a, b);
        int diff = dep[b] - dep[a];
        REPD(k, LOG-1, 0) if(diff & (1<<k)){
            b = prt[b][k];
        }
        if(a == b) return a;
        REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
            a = prt[a][k]; b = prt[b][k];
        }
        return prt[a][0];
    }
    void init(int _n){
        n = _n; REP(i, 1, n) g[i].clear();
    }
    void addEdge(int u, int v){
        g[u].push_back(v);
        g[v].push_back(u);
    }
    void yutruli(){ // O(Vlog(V))
        dfsz(1, 0);
        ts = 0;
        dfshl(1);
        REP(k, 1, LOG-1) REP(i, 1, n)
            prt[i][k] = prt[prt[i][k-1]][k-1];
    }
    vector<PII> getPath(int u, int v){ // O((log(V))^2)
        vector<PII> res;
        while(tid[u] < tid[head[v]]){
            res.push_back(PII(tid[head[v]], tid[v]));
            v = prt[head[v]][0];
        }
        res.push_back(PII(tid[u], tid[v]));
        reverse(ALL(res));
        return res;
    }
    /* res : list of intervals from u to v
```

```

    * u must be ancestor of v
    * usage :
    * vector< PII >& path = tree.getPath( u , v )
    * for( PII tp : path ) {
    *     int l , r; tie( l , r ) = tp;
    *     upd( l , r );
    *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
    *     uu ~> vv is a heavy path on tree
    * }
    */
} tree;

```

### 5.2 DominatorTree

```
const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , m , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ];
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ]; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if( cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ) )
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n , int _m , int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i , 1 , n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){
                eval( w );
                if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
                    idom[ w ] = mn[ w ];
                else idom[ w ] = par[ u ];
            }
            cov[ par[ u ] ].clear();
        }
        REP( i , 2 , n ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
```



```

    if( idom[ u ] != sdom[ u ] )
        idom[ u ] = idom[ idom[ u ] ];
}
}
} domT;

```

### 5.3 MaxClique

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset< N > Int;
    Int linkto[ N ] , v[ N ];
    int n;
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            linkto[ i ].reset();
            v[ i ].reset();
        }
    }
    void addEdge( int a , int b ){
        v[ a ][ b ] = v[ b ][ a ] = 1;
    }
    int popcount(const Int& val)
    { return val.count(); }
    int lowbit(const Int& val)
    { return val._Find_first(); }
    int ans , stk[ N ];
    int id[ N ] , di[ N ] , deg[ N ];
    Int cans;
    void maxclique(int elem_num, Int candi){
        if(elem_num > ans){
            ans = elem_num;
            cans.reset();
            for( int i = 0 ; i < elem_num ; i ++ )
                cans[ id[ stk[ i ] ] ] = 1;
        }
        int potential = elem_num + popcount(candi);
        if(potential <= ans) return;
        int pivot = lowbit(candi);
        Int smaller_candi = candi & (~linkto[pivot]);
        while(smaller_candi.count() && potential > ans){
            int next = lowbit(smaller_candi);
            candi[next] = !candi[next];
            smaller_candi[ next ] = !smaller_candi[ next ];
            potential --;
            if(next == pivot || (smaller_candi & linkto[next]
                ).count() ){
                stk[elem_num] = next;
                maxclique(elem_num + 1, candi & linkto[next]);
            }
        }
    }
    int solve(){
        for( int i = 0 ; i < n ; i ++ ){
            id[ i ] = i;
            deg[ i ] = v[ i ].count();
        }
        sort( id , id + n , [&](int id1, int id2){
            return deg[id1] > deg[id2]; } );
        for( int i = 0 ; i < n ; i ++ )
            di[ id[ i ] ] = i;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                if( v[ i ][ j ] )
                    linkto[ di[ i ] ][ di[ j ] ] = 1;
        Int cand; cand.reset();
        for( int i = 0 ; i < n ; i ++ )
            cand[ i ] = 1;
        ans = 1;
        cans.reset(); cans[ 0 ] = 1;
        maxclique(0, cand);
        return ans;
    }
} solver;

```

### 5.4 Strongly Connected Component

```

struct Scc{ // 0(V+E)
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
};

```

### 5.5 Dynamic MST

```

/* Dynamic MST O( Q lg^2 Q )
(qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value
*/
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[
            ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){

```

```

    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
        a[ri]=rj; ans += z[id[i]];
        kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
    }
}
for(int i=1;i<=n;i++) a[i]=0;
for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
int n2=0;
for(int i=1;i<=n;i++) if(a[i]==0)
vd[i]=++n2;
for(int i=1;i<=n;i++) if(a[i])
vd[i]=vd[find(i)];
int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
for(int i=0;i<m1;i++) app[i]=-1;
for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
    Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
    Nz[m2]=z[ qx[i] ];
    app[qx[i]]=m2; m2++;
}
for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
for(int i=1;i<=n2;i++) a[i]=0;
for(int i=0;i<tm;i++){
    ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
        a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
        Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
    }
}
int mid=Q/2;
solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

## 5.6 Maximum General graph Matching

```

const int N = 514, E = (2e5) * 2;
struct Graph{ // O(V^0.5 * E) ...?
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;

```

```

        for(int i=1;i<=n;i++){
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        }
        return ans;
    }
} graph;

```

## 5.7 Minimum General Weighted Matching

```

struct Graph { // O(V*E*log(V))
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            for( int i = 0 ; i < n ; i ++ )
                onstk[ i ] = dis[ i ] = 0;
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for (int i=0; i<n; i++){
            ret += edge[i][match[i]];
        }
        ret /= 2;
        return ret;
    }
} graph;

```

## 5.8 Maximum General Weighted Matching

```

struct WeightGraph { // O(V*E*log(V))
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){
            edge(int ui,int vi,int wi)
                :u(ui),v(vi),w(wi){}
        };
        int n,n_x;
        edge g[N*2][N*2];
        int lab[N*2];
        int match[N*2],slack[N*2],st[N*2],pa[N*2];
        int flo_from[N*2][N+1],S[N*2],vis[N*2];
        vector<int> flo[N*2];
        queue<int> q;
        int e_delta(const edge &e){
            return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
        }
        void update_slack(int u,int x){
            if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]]
                [x]))slack[x]=u;
        }
        void set_slack(int x){
            slack[x]=0;
            for(int u=1;u<=n;++u)
                if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                    update_slack(u,x);
        }
        void q_push(int x){
            if(x<=n)q.push(x);
            else for(size_t i=0;i<flo[x].size();i++)
                q_push(flo[x][i]);
        }
        void set_st(int x,int b){
            st[x]=b;
            if(x>n)for(size_t i=0;i<flo[x].size();++i)
                set_st(flo[x][i],b);
        }
        int get_pr(int b,int xr){
            int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].
                begin();
            if(pr%2==1){
                reverse(flo[b].begin()+1,flo[b].end());
                return (int)flo[b].size()-pr;
            }else return pr;
        }
        void set_match(int u,int v){
            match[u]=g[u][v].v;
            if(u<=n) return;
            edge e=g[u][v];
            int xr=flo_from[u][e.u],pr=get_pr(u,xr);
            for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i
                ^1]);
            set_match(xr,v);
            rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end
                ());
        }
        void augment(int u,int v){
            for(;;){
                int xnv=st[match[u]];
                set_match(u,v);
                if(!xnv)return;
                set_match(xnv,st[pa[xnv]]);
                u=st[pa[xnv]],v=xnv;
            }
        }
        int get_lca(int u,int v){
            static int t=0;
            for(++t;u!=v;swap(u,v)){
                if(u==0)continue;
                if(vis[u]==t)return u;
                vis[u]=t;
                u=st[match[u]];
                if(u)u=st[pa[u]];
            }
            return 0;
        }
        void add_blossom(int u,int lca,int v){
            int b=n+1;
            while(b<=n_x&&st[b])++b;
            if(b>n_x)++n_x;
            lab[b]=0,S[b]=0;

```

```

            match[b]=match[lca];
            flo[b].clear();
            flo[b].push_back(lca);
            for(int x=u,y; x!=lca;x=st[pa[y]])
                flo[b].push_back(x),flo[b].push_back(y=st[match[x
                    ]]),q_push(y);
            reverse(flo[b].begin()+1,flo[b].end());
            for(int x=v,y; x!=lca;x=st[pa[y]])
                flo[b].push_back(x),flo[b].push_back(y=st[match[x
                    ]]),q_push(y);
            set_st(b,b);
            for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
            for(int x=1;x<=n_x;++x)flo_from[b][x]=0;
            for(size_t i=0;i<flo[b].size();++i){
                int xs=flo[b][i];
                for(int x=1;x<=n_x;++x)
                    if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b
                        ][x]))
                        g[b][x]=g[xs][x],g[x][b]=g[x][xs];
                for(int x=1;x<=n_x;++x)
                    if(flo_from[xs][x])flo_from[b][x]=xs;
            }
            set_slack(b);
        }
        void expand_blossom(int b){
            for(size_t i=0;i<flo[b].size();++i)
                set_st(flo[b][i],flo[b][i]);
            int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
            for(int i=0;i<pr;i+=2){
                int xs=flo[b][i],xns=flo[b][i+1];
                pa[xs]=g[xns][xs].u;
                S[xs]=1,S[xns]=0;
                slack[xs]=0,set_slack(xns);
                q_push(xns);
            }
            S[xr]=1,pa[xr]=pa[b];
            for(size_t i=pr+1;i<flo[b].size();++i){
                int xs=flo[b][i];
                S[xs]=-1,set_slack(xs);
            }
            st[b]=0;
        }
        bool on_found_edge(const edge &e){
            int u=st[e.u],v=st[e.v];
            if(S[v]==-1){
                pa[v]=e.u,S[v]=1;
                int nu=st[match[v]];
                slack[v]=slack[nu]=0;
                S[nu]=0,q_push(nu);
            }else if(S[v]==0){
                int lca=get_lca(u,v);
                if(!lca)return augment(u,v),augment(v,u),true;
                else add_blossom(u,lca,v);
            }
            return false;
        }
        bool matching(){
            memset(S+1,-1,sizeof(int)*n_x);
            memset(slack+1,0,sizeof(int)*n_x);
            q=queue<int>();
            for(int x=1;x<=n_x;++x)
                if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
            if(q.empty())return false;
            for(;;){
                while(q.size()){
                    int u=q.front();q.pop();
                    if(S[st[u]]==1)continue;
                    for(int v=1;v<=n_x;++v)
                        if(g[u][v].w>0&&st[u]!=st[v]){
                            if(e_delta(g[u][v])==0){
                                if(on_found_edge(g[u][v]))return true;
                            }else update_slack(u,st[v]);
                        }
                }
                int d=INF;
                for(int b=n+1;b<=n_x;++b)
                    if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
                for(int x=1;x<=n_x;++x)
                    if(st[x]==x&&slack[x]){
                        if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
                        else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x
                            ])/2);
                    }
            }

```

```

    }
    for(int u=1;u<=n;++u){
        if(S[st[u]]==0){
            if(lab[u]<=d)return 0;
            lab[u]-=d;
        }else if(S[st[u]]==1)lab[u]+=d;
    }
    for(int b=n+1;b<=n_x;++b)
        if(st[b]==b){
            if(S[st[b]]==0)lab[b]+=d*2;
            else if(S[st[b]]==1)lab[b]-=d*2;
        }
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
        if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta
            (g[slack[x]][x])==0)
            if(on_found_edge(g[slack[x]][x]))return true;
    for(int b=n+1;b<=n_x;++b)
        if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(
            b);
    }
    return false;
}
pair<long long,int> solve(){
    memset(match+1,0,sizeof(int)*n);
    n_x=n;
    int n_matches=0;
    long long tot_weight=0;
    for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
    int w_max=0;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v){
            flo_from[u][v]=(u==v?u:0);
            w_max=max(w_max,g[u][v].w);
        }
    for(int u=1;u<=n;++u)lab[u]=w_max;
    while(matching())n_matches++;
    for(int u=1;u<=n;++u)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;++u)
        for(int v=1;v<=n;++v)
            g[u][v]=edge(u,v,0);
}
} graph;

```

## 5.9 Minimum Steiner Tree

```

// Minimum Steiner Tree
// O(V^3AT + V^2 2AT)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1<<T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ] ,
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
}

```

```

}
int solve( const vector<int>& ter ){
    int t = (int)ter.size();
    for( int i = 0 ; i < ( 1<<t ) ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            dp[ i ][ j ] = INF;
    for( int i = 0 ; i < n ; i ++ )
        dp[ 0 ][ i ] = 0;
    for( int msk = 1 ; msk < ( 1<<t ) ; msk ++ ){
        if( msk == ( msk & (-msk) ) ){
            int who = __lg( msk );
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
            continue;
        }
        for( int i = 0 ; i < n ; i ++ )
            for( int submsk = ( msk - 1 ) & msk ; submsk ;
                submsk = ( submsk - 1 ) & msk )
                dp[ msk ][ i ] = min( dp[ msk ][ i ] ,
                    dp[ submsk ][ i ] +
                    dp[ msk ^ submsk ][ i ] );
        for( int i = 0 ; i < n ; i ++ ){
            tdst[ i ] = INF;
            for( int j = 0 ; j < n ; j ++ )
                tdst[ i ] = min( tdst[ i ] ,
                    dp[ msk ][ j ] + dst[ j ][ i ] );
        }
        for( int i = 0 ; i < n ; i ++ )
            dp[ msk ][ i ] = tdst[ i ];
    }
    int ans = INF;
    for( int i = 0 ; i < n ; i ++ )
        ans = min( ans , dp[ ( 1<<t ) - 1 ][ i ] );
    return ans;
}
} solver;

```

## 5.10 BCC based on vertex

```

struct BccVertex { // O(V+E)
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            }else
                low[u] = min(low[u],dfn[v]);
        }
    }
}
vector<vector<int>>> solve() {
    vector<vector<int>>> res;
    for (int i=0; i<n; i++)
        dfn[i] = low[i] = -1;
    for (int i=0; i<n; i++)
        if (dfn[i] == -1) {
            top = 0;
            DFS(i,i);
        }
    REP(i,nScc) res.PB(sccv[i]);
}

```

```

    return res;
}
}graph;

```

## 5.11 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
                    prv[i+1][u] = v;
                    prve[i+1][u] = j;
                }
            }
        }
    }
    double solve(){
        // returns inf if no cycle, mmc otherwise
        double mmc=inf;
        int st = -1;
        bellman_ford();
        for(int i=0; i<n; i++) {
            double avg=-inf;
            for(int k=0; k<n; k++) {
                if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
                else avg=max(avg,inf);
            }
            if (avg < mmc) tie(mmc, st) = tie(avg, i);
        }
        FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
        for (int i=n; !vst[st]; st=prv[i--][st]) {
            vst[st]++;
            edgeID.PB(prve[i][st]);
            rho.PB(st);
        }
        while (vst[st] != 2) {
            int v = rho.back(); rho.pop_back();
            cycle.PB(v);
            vst[v]++;
        }
        reverse(ALL(edgeID));
        edgeID.resize(SZ(cycle));
        return mmc;
    }
} mmc;

```

## 5.12 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};

```

```

struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i+1], dp[i+1]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] =min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
        }
        mu=INF; LL bunbo=1;
        for(int i=1; i<=n; i++) if(dp[n][i] < INF){
            LL a=-INF, b=1;
            for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
                if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
                    a = dp[n][i]-dp[j][i];
                    b = n-j;
                }
            }
            if(mu*b > bunbo*a)
                mu = a, bunbo = b;
        }
        if(mu < 0) return -1; // negative cycle
        if(mu == INF) return INF; // no cycle
        if(mu == 0) return 0;
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w *= bunbo;
            }
            memset(p, 0, sizeof(p));
            queue<int> q;
            for(int i=1; i<=n; i++){
                q.push(i);
                inq[i] = true;
            }
            while(!q.empty()){
                int i=q.front(); q.pop(); inq[i]=false;
                for(int j=0; j<(int)g[i].size(); j++){
                    if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                        p[g[i][j].to] = p[i]+g[i][j].w-mu;
                        if(!inq[g[i][j].to]){
                            q.push(g[i][j].to);
                            inq[g[i][j].to] = true;
                        }
                    }
                }
            }
        }
        for(int i=1; i<=n; i++) grev[i].clear();
        for(int i=1; i<=n; i++){
            for(int j=0; j<(int)g[i].size(); j++){
                g[i][j].w += p[i]-p[g[i][j].to];
                grev[g[i][j].to].push_back(edge(i, g[i][j].w));
            }
        }
        LL mlcd = n*mu;
        for(int i=1; i<=n; i++){
            bn=mlcd/mu, bsz=0;
            memset(hd, 0, sizeof(hd));
            fill(d+i+1, d+n+1, INF);
            b_insert(d[i]=0, i);
            for(int j=0; j<bn-1; j++) for(int k=hd[j]; k; k=
                b[k].next){

```

```

    int u = b[k].u;
    LL du = b[k].d;
    if(du > d[u]) continue;
    for(int l=0; l<(int)g[u].size(); l++) if(g[u][l].to > i){
        if(d[g[u][l].to] > du + g[u][l].w){
            d[g[u][l].to] = du + g[u][l].w;
            b_insert(d[g[u][l].to], g[u][l].to);
        }
    }
    for(int j=0; j<(int)grev[i].size(); j++) if(grev[i][j].to > i)
        mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
    return mldc / bunbo;
}
} graph;

```

### 5.13 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v, d;
        nd(int ui = 0, int vi = 0, int di = INF)
        { u = ui; v = vi; d = di; }
    };
    struct heap{
        nd* edge; int dep; heap* chd[4];
    };
    static int cmp(heap* a, heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; LL d; heap* H; nd* E;
        node(){
            node(LL _d, int _v, nd* _E)
            { d = _d; v = _v; E = _E; }
            node(heap* _H, LL _d)
            { H = _H; d = _d; }
            friend bool operator<(node a, node b)
            { return a.d > b.d; }
        };
        int n, k, s, t, dst[ N ];
        nd *nxt[ N ];
        vector<nd*> g[ N ], rg[ N ];
        heap *nullNd, *head[ N ];
        void init( int _n, int _k, int _s, int _t ){
            n = _n; k = _k; s = _s; t = _t;
            for( int i = 1; i <= n; i ++ ){
                g[ i ].clear(); rg[ i ].clear();
                nxt[ i ] = head[ i ] = NULL;
                dst[ i ] = -1;
            }
        }
        void addEdge( int ui, int vi, int di ){
            nd* e = new nd(ui, vi, di);
            g[ ui ].push_back( e );
            rg[ vi ].push_back( e );
        }
        queue<int> dfsQ;
        void dijkstra(){
            while(dfsQ.size()) dfsQ.pop();
            priority_queue<node> Q;
            Q.push(node(0, t, NULL));
            while (!Q.empty()){
                node p = Q.top(); Q.pop();
                if(dst[p.v] != -1) continue;
                dst[ p.v ] = p.d;
                nxt[ p.v ] = p.E;
                dfsQ.push( p.v );
                for(auto e: rg[ p.v ])
                    Q.push(node(p.d + e->d, e->u, e));
            }
        }
        heap* merge(heap* curNd, heap* newNd){
            if(curNd == nullNd) return newNd;
            heap* root = new heap;
            memcpy(root, curNd, sizeof(heap));

```

```

            if(newNd->edge->d < curNd->edge->d){
                root->edge = newNd->edge;
                root->chd[2] = newNd->chd[2];
                root->chd[3] = newNd->chd[3];
                newNd->edge = curNd->edge;
                newNd->chd[2] = curNd->chd[2];
                newNd->chd[3] = curNd->chd[3];
            }
            if(root->chd[0]->dep < root->chd[1]->dep)
                root->chd[0] = merge(root->chd[0], newNd);
            else
                root->chd[1] = merge(root->chd[1], newNd);
            root->dep = max(root->chd[0]->dep, root->chd[1]->dep) + 1;
            return root;
        }
    }
    vector<heap*> V;
    void build(){
        nullNd = new heap;
        nullNd->dep = 0;
        nullNd->edge = new nd;
        fill(nullNd->chd, nullNd->chd+4, nullNd);
        while(not dfsQ.empty()){
            int u = dfsQ.front(); dfsQ.pop();
            if(!nxt[ u ]) head[ u ] = nullNd;
            else head[ u ] = head[nxt[ u ]->v];
            V.clear();
            for( auto&& e : g[ u ] ){
                int v = e->v;
                if( dst[ v ] == -1 ) continue;
                e->d += dst[ v ] - dst[ u ];
                if( nxt[ u ] != e ){
                    heap* p = new heap;
                    fill(p->chd, p->chd+4, nullNd);
                    p->dep = 1;
                    p->edge = e;
                    V.push_back(p);
                }
            }
            if(V.empty()) continue;
            make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
            for( size_t i = 0; i < V.size(); i ++ ){
                if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
                else V[i]->chd[2]=nullNd;
                if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
                else V[i]->chd[3]=nullNd;
            }
            head[u] = merge(head[u], V.front());
        }
    }
    vector<LL> ans;
    void first_K(){
        ans.clear();
        priority_queue<node> Q;
        if( dst[ s ] == -1 ) return;
        ans.push_back( dst[ s ] );
        if( head[s] != nullNd )
            Q.push(node(head[s], dst[s]+head[s]->edge->d));
        for( int _ = 1; _ < k and not Q.empty(); _ ++ ){
            node p = Q.top(); Q.pop();
            ans.push_back( p.d );
            if(head[ p.H->edge->v ] != nullNd){
                q.H = head[ p.H->edge->v ];
                q.d = p.d + q.H->edge->d;
                Q.push(q);
            }
        }
        for( int i = 0; i < 4; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->edge->d;
                Q.push( q );
            }
    }
    void solve(){
        dijkstra();
        build();
        first_K();
    }
}

```



```
} solver;
```

## 6 String

### 6.1 PalTree

```
/*
 * sfail: compressed fail links with same diff
 * 0(lgn): length of sfail link path
 */
const int MAXN = 1e6+10;
struct PalT{
    int tot,lst;
    int nxt[MAXN][26], len[MAXN];
    int fail[MAXN], diff[MAXN], sfail[MAXN];
    char* s;
    int newNode(int l, int _fail) {
        int res = ++tot;
        fill(nxt[res], nxt[res]+26, 0);
        len[res] = l, fail[res] = _fail;
        diff[res] = l - len[_fail];
        if (diff[res] == diff[_fail])
            sfail[res] = sfail[_fail];
        else
            sfail[res] = _fail;
        return res;
    }
    void push(int p) {
        int np = lst;
        int c = s[p] - 'a';
        while (p-len[np]-1 < 0 || s[p] != s[p-len[np]-1])
            np = fail[np];
        if ((lst=nxt[np][c])) return;
        int nq_f = 0;
        if (len[np]+2 == 1) nq_f = 2;
        else {
            int tf = fail[np];
            while (p-len[tf]-1 < 0 || s[p] != s[p-len[tf]-1])
                tf = fail[tf];
            nq_f = nxt[tf][c];
        }
        int nq = newNode(len[np]+2, nq_f);
        nxt[np][c] = nq;
        lst=nq;
    }
    void init(char* _s){
        s = _s;
        tot = 0;
        newNode(-1, 1);
        newNode(0, 1);
        diff[2] = 0;
        lst = 2;
    }
} palt;
```

### 6.2 SAIS

```
const int N = 300010;
struct SA{ // O(N)
#define REP(i,n) for (int i=0; i<int(n); i++)
#define REP1(i,a,b) for (int i=(a); i<=int(b); i++)
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int* s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
        }
    }
};
```

```
        hei[r[i]] = ans;
    }
}
void sais(int* s, int* sa, int* p, int* q, bool* t,
    int* c, int n, int z){
    bool uniq = t[n-1] = true, neq;
    int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
        lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
    memcpy(x, c, sizeof(int) * z); \
    XD; \
    memcpy(x + 1, c, sizeof(int) * (z - 1)); \
    REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
        ]-1]]++ = sa[i]-1; \
    memcpy(x, c, sizeof(int) * z); \
    for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
        ]-1]) sa[-x[s[sa[i]-1]]] = sa[i]-1;
    MS0(c, z);
    REP(i,n) uniq &= ++c[s[i]] < 2;
    REP(i,z-1) c[i+1] += c[i];
    if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[-x[s[i
        ]]] = p[q[i]=nn++] = i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[-x[s[p[
        nsa[i]]]] = p[nsa[i]]);
}
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}
```

### 6.3 SuffixAutomata

```
const int MAXM = 1000010;
struct SAM{ // O(N)
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
                while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            }
        }
    }
};
```

```

    int nq = newNode();
    mx[nq] = mx[p]+1;
    for(int i = 0; i < 33; i++)
        nxt[nq][i] = nxt[q][i];
    mom[nq] = mom[q];
    mom[q] = nq;
    mom[np] = nq;
    for(; p && nxt[p][c] == q; p = mom[p])
        nxt[p][c] = nq;
    }
    }
    lst = np;
}
void push(char *str){
    for(int i = 0; str[i]; i++)
        push(str[i]-'a'+1);
    }
}
sam;

```

## 6.4 Aho-Corasick

```

int counts[105]; // added strings
int indexCounter;
struct Node{
    int cnt,dp;
    vector<int> indices;
    Node *go[26], *fail;
    Node (){
        cnt = 0; dp = -1; fail = 0;
        memset(go,0,sizeof(go));
    }
};
Node pool[1048576];

struct AAutomata{ // O(N)
    Node *root;
    int nMem;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init()
    { nMem = 0; root = new_Node(); }
    void add(const string &str)
    { insert(root,str,0); }
    void insert(Node *cur, const string &str, int pos){
        if (pos >= (int)str.size()) {
            cur->cnt++;
            cur->indices.push_back(indexCounter++);
            return;
        }
        int c = str[pos]-'a';
        if (cur->go[c] == 0)
            cur->go[c] = new_Node();
        insert(cur->go[c],str,pos+1);
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front();
            que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    if (!ptr) fr->go[i]->fail = root;
                    else fr->go[i]->fail = ptr->go[i];
                    que.push(fr->go[i]);
                }
            }
        }
    }
    void query(const string& str) {
        int ans=0,k,len=str.size();
        Node *p=root;
        for(int i=0; i<len; i++){
            k=str[i]-'a';
            while(!p->go[k]&&p!=root)
                p=p->fail;

```

```

        p=p->go[k];
        if(!p)p=root;
        Node *temp=p;
        while(temp!=root){
            ans+=temp->cnt;
            for (int k=0; k<temp->indices.size(); ++k)
                counts[temp->indices[k]]++;
            temp=temp->fail;
        }
    }
};

```

## 6.5 Z Value

```

char s[MAXN];
int len,z[MAXN];
void Z_value() { // O(N)
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++) {
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 6.6 BWT

```

struct BurrowsWheeler{ // O(N)
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
        for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
            res[ i ] = ori[ a[ ptr ] ];
            ptr = a[ ptr ];
        }
        res[ len ] = 0;
    }
} bwt;

```

## 6.7 ZValue Palindrome

```

int len, zv[MAX*2];
char ip[MAX], op[MAX*2];
int main(){ // O(N)
    cin >> ip; len = strlen(ip);
    int l2 = len*2 - 1;
    for(int i=0; i<l2; i++){
        if(i&1) op[i] = '@';
        else op[i] = ip[i/2];
    }
    int l=0, r=0; zv[0] = 1;
    for(int i=1; i<l2; i++){
        if (i > r ){
            l = r = i;
            while( l>0 && r<l2-1 && op[l-1] == op[r+1] )

```

```

    l --, r ++;
    zv[i] = (r-l+1);
} else {
    int md = (l+r)/2, j = md + md - i;
    zv[i] = zv[j];
    int q = zv[i] / 2, nr = i + q;
    if( nr == r ) {
        l = i + i - r;
        while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
            l --, r ++;
        zv[i] = r - l + 1;
    } else if( nr > r )
        zv[i] = (r - i) * 2 + 1;
    }
}
}

```

## 6.8 Smallest Rotation

```

string mcp(string s){ // O(N)
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
// O(K*log(N)+R), K: # of matches, R: # of elements
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2],b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];

```

```

    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<=2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 7 Data Structure

### 7.1 Treap

```

struct Treap{ // Each op: Av: O(log(n)), W: O(n)
    int sz, val, pri, tag;
    Treap *l, *r;
    Treap( int _val ){
        val = _val; sz = 1;
        pri = rand(); l = r = NULL; tag = 0;
    }
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;
        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a, Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r, b );
        pull( a );
        return a;
    } else {
        push( b );
        b->l = merge( a, b->l );
        pull( b );
        return b;
    }
}
void split( Treap *t, int k, Treap*&a, Treap*&b ){
    if( !t ) { a = b = NULL; return; }

```

```

push( t );
if( Size( t->l ) + 1 <= k ){
    a = t;
    split( t->r , k - Size( t->l ) - 1 , a->r , b );
    pull( a );
}else{
    b = t;
    split( t->l , k , a , b->l );
    pull( b );
}
}
}

```

## 7.2 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay { // Each operation: O(log(n))
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay (int _val=-1) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;

Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}

int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}

```

```

}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}

Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}

bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}

Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}

```

## 7.3 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t; //
    less_equal
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

    rope<char> r[ 2 ];
    r[ 1 ] = r[ 0 ]; // persistenet
    string t = "abc";
    r[ 1 ].insert( 0 , t.c_str() );
    r[ 1 ].erase( 1 , 1 );
    cout << r[ 1 ].substr( 0 , 2 );
}

```

```
}
}
```

## 7.4 Seg Tree

```
#define maxn 200005
typedef long long ll;
struct seg {
    int lson, rson;
    int smallCount;
};

seg tree[maxn*10];

void build(ll root, ll l, ll r) {
    tree[root].smallCount = 0;
    if (l == r) return;
    tree[root].lson = (root << 1);
    tree[root].rson = (root << 1) + 1;
    build(tree[root].lson, l, (l+r)/2);
    build(tree[root].rson, (l+r)/2+1, r);
}

bool update(ll root, ll l, ll r, ll ind) {
    ll lid = tree[root].lson, rid = tree[root].rson;
    if (l == r) {
        tree[root].smallCount++;
        return true;
    } else if ((ind <= (l+r) / 2)) {
        if (update(lid, l, (l+r)/2, ind) == false)
            return false;
    } else {
        if (update(rid, (l+r)/2+1, r, ind) == false)
            return false;
    }
    tree[root].smallCount = min(tree[lid].smallCount,
                                tree[rid].smallCount);
    return true;
}
```

## 7.5 Dynamic Seg Tree

```
#include <bits/stdc++.h>
using namespace std;

struct node;
node *newNode();

struct node {
    int lv, rv, sum;
    node *left, *right;
    node() : left(NULL), right(NULL), sum(0) {}
    inline void init(int l, int r) {
        lv = l; rv = r;
    }
    inline void extend() {
        if (!left) {
            int m = (lv + rv) / 2;
            left = newNode();
            right = newNode();
            left->init(lv, m);
            right->init(m + 1, rv);
        }
    }
    int getSum(int l, int r) {
        if (r < lv || rv < l) return 0;
        if (l <= lv && rv <= r) return sum;
        extend();
        return left->getSum(l, r) + right->getSum(l, r);
    }
    void update(int p, int newVal) {
        if (lv == rv) {
            sum = newVal;
            return;
        }
        extend();
        (p <= left->rv ? left : right)->update(p, newVal);
        sum = left->sum + right->sum;
    }
}
```

```
}
};
node *newNode() {
    static int bufSize = 1e7;
    static node buf[(int) 1e7];
    assert(bufSize);
    return &buf[--bufSize];
}

int main() {
    node *rmq = newNode();
    rmq->init(0, 1e9);
}
```

## 7.6 Fenwick Tree

```
typedef long long ll;
ll sum(ll x, ll c[]) {
    ll ans = 0;
    for (ll i = x; i > 0; i -= (i & (-i)))
        ans += c[i];
    return ans;
}

void modify(ll x, ll d, ll c[]) {
    for (ll i = x; i <= M; i += (i & (-i)))
        c[i] += d;
}
```