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1 Basic

1.1 .vimrc

```
syn on
se ai nu ru cul mouse=a
se cin et ts=2 sw=2 sts=2
so $VIMRUNTIME/mswin.vim
colo desert
se gfn=Monospace\ 14
```

1.2 Misc

```
#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }

#define SECS (clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key, int, KeyHasher> map_t;
```

2 flow

2.1 ISAP

```
#define SZ(c) ((int)(c).size())
struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
        G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
    }
    int dfs(int p, int flow) {
        if(p == t) return flow;
        for(int &i = iter[p]; i < SZ(G[p]); i++) {
            Edge &e = G[p][i];
            if(e.c > 0 && d[p] == d[e.v]+1) {
                int f = dfs(e.v, min(flow, e.c));
                if(f) {
                    e.c -= f;
                    G[e.v][e.r].c += f;
                    return f;
                }
            }
        }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
```

```

}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

2.2 MinCostFlow

```

/*
    A template for Min Cost Max Flow
    tested with TIOJ 1724
*/
struct MinCostMaxFlow{
    typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFf = 1000000;
    static const Tcost INFc = 1e9;
    struct Edge{
        int v, cap;
        Tcost w;
        int rev;
        Edge(){
            Edge(int t2, int t3, Tcost t4, int t5)
                : v(t2), cap(t3), w(t4), rev(t5) {}
    };
    int V, s, t;
    vector<Edge> g[MAXV];
    void init(int n){
        V = n+2;
        s = n+1, t = n+2;
        for(int i = 0; i <= V; i++) g[i].clear();
    }
    void addEdge(int a, int b, int cap, Tcost w){
        g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
        g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
    }
    Tcost d[MAXV];
    int id[MAXV], mom[MAXV];
    bool inqu[MAXV];
    queue<int> q;
    Tcost solve(){
        int mxf = 0; Tcost mnc = 0;
        while(1){
            fill(d, d+1+V, INFc);
            fill(inqu, inqu+1+V, 0);
            fill(mom, mom+1+V, -1);
            mom[s] = s;
            d[s] = 0;
            q.push(s); inqu[s] = 1;
            while(q.size()){
                int u = q.front(); q.pop();
                inqu[u] = 0;
                for(int i = 0; i < (int) g[u].size(); i++){
                    Edge &e = g[u][i];
                    int v = e.v;
                    if(e.cap > 0 && d[v] > d[u]+e.w){
                        d[v] = d[u]+e.w;
                        mom[v] = u;
                        id[v] = i;
                        if(!inqu[v]) q.push(v), inqu[v] = 1;
                    }
                }
            }
            if(mom[t] == -1) break;
            int df = INFf;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mxf += df;
            mnc += df*d[t];
        }
        return mnc;
    }
}

```

```

} flow;

```

2.3 Hungarian

```

#define NIL -1
#define INF 100000000
int n,matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN],ylabel[MAXN];
int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
    int i,delta=INF;
    for(i=0;i<n;i++) if(!sett[i]) delta=min(slack[i],delta);
    for(i=0;i<n;i++) if(sets[i]) xlabel[i]-=delta;
    for(i=0;i<n;i++) {
        if(sett[i]) ylabel[i]+=delta;
        else slack[i]-=delta;
    }
}
inline void add_sets(int x) {
    int i;
    sets[x]=1;
    for(i=0;i<n;i++) {
        if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {
            slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
            prev[i]=x;
        }
    }
}
inline void augment(int final) {
    int x=prev[final],y=final,tmp;
    matched++;
    while(1) {
        tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
        if(y==NIL) return;
        x=prev[y];
    }
}
inline void phase() {
    int i,y,root;
    for(i=0;i<n;i++) { sets[i]=sett[i]=0; slack[i]=INF; }
    for(root=0;root<n&&xy[root]!=NIL;root++);
    add_sets(root);
    while(1) {
        relabel();
        for(y=0;y<n;y++) if(!sett[y]&&slack[y]==0) break;
        if(yx[y]==NIL) { augment(y); return; }
        else { add_sets(yx[y]); sett[y]=1; }
    }
}
inline int hungarian() {
    int i,j,c=0;
    for(i=0;i<n;i++) {
        xy[i]=yx[i]=NIL;
        xlabel[i]=ylabel[i]=0;
        for(j=0;j<n;j++) xlabel[i]=max(cost[i][j],xlabel[i]);
    }
    for(i=0;i<n;i++) phase();
    for(i=0;i<n;i++) c+=cost[i][xy[i]];
    return c;
}

```

2.4 Hungarian Unbalanced

```

const int nil = -1;
const int inf = 1000000000;
int xn,yn,matched;
int cost[MAXN][MAXN];
bool sets[MAXN]; // whether x is in set S
bool sett[MAXN]; // whether y is in set T
int xlabel[MAXN],ylabel[MAXN];

```

```

int xy[MAXN],yx[MAXN]; // matched with whom
int slack[MAXN]; // given y: min{xlabel[x]+ylabel[y]-
cost[x][y]} | x not in S
int prev[MAXN]; // for augmenting matching
inline void relabel() {
    int i,delta=inf;
    for(i=0;i<yn;i++) if(!sett[i]) delta=min(slack[i],
        delta);
    for(i=0;i<xn;i++) if(sett[i]) xlabel[i]-=delta;
    for(i=0;i<yn;i++) {
        if(sett[i]) ylabel[i]+=delta;
        else slack[i]-=delta;
    }
}
inline void add_sets(int x) {
    int i;
    sets[x]=1;
    for(i=0;i<yn;i++) {
        if(xlabel[x]+ylabel[i]-cost[x][i]<slack[i]) {
            slack[i]=xlabel[x]+ylabel[i]-cost[x][i];
            prev[i]=x;
        }
    }
}
inline void augment(int final) {
    int x=prev[final],y=final,tmp;
    matched++;
    while(1) {
        tmp=xy[x]; xy[x]=y; yx[y]=x; y=tmp;
        if(y==nil) return;
        x=prev[y];
    }
}
inline void phase() {
    int i,y,root;
    for(i=0;i<xn;i++) sett[i]=0;
    for(i=0;i<yn;i++) { sett[i]=0; slack[i]=inf; }
    for(root=0;root<xn&&xy[root]!=nil;root++);
    add_sets(root);
    while(1) {
        relabel();
        for(y=0;y<yn;y++) if(!sett[y]&&slack[y]==0) break;
        if(yx[y]==nil) { augment(y); return; }
        else { add_sets(yx[y]); sett[y]=1; }
    }
}
inline int hungarian() {
    int i,j,c=0;
    matched=0;
    // we must have "xn<yn"
    bool swapxy=0;
    if(xn>yn) {
        swapxy=1;
        int mn=max(xn,yn);
        swap(xn,yn);
        for(int i=0;i<mn;i++)
            for(int j=0;j<i;j++)
                swap(cost[i][j],cost[j][i]);
    }
    for(i=0;i<xn;i++) {
        xy[i]=nil;
        xlabel[i]=0;
        for(j=0;j<yn;j++) xlabel[i]=max(cost[i][j],xlabel[i]);
    }
    for(i=0;i<yn;i++) {
        yx[i]=nil;
        ylabel[i]=0;
    }
    for(i=0;i<xn;i++) phase();
    for(i=0;i<xn;i++) c+=cost[i][xy[i]];
    // recover cost matrix (if necessary)
    if(swapxy) {
        int mn=max(xn,yn);
        swap(xn,yn);
        for(int i=0;i<mn;i++)
            for(int j=0;j<i;j++)
                swap(cost[i][j],cost[j][i]);
    }
    // need special recovery if we want more info than
    // matching value
    return c;
}

```

}

2.5 DMST

```

/*
 * Edmond's algoirthm for Directed MST
 * runs in O(VE)
 */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
    int u, v, c;
    Edge(){}
    Edge(int x, int y, int z) :
        u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){
    V++;
    return V;
}
inline void addEdge(int u, int v, int c){
    E++;
    edges[E] = Edge(u, v, c);
}
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
    fill(con, con+V+1, 0);
    int r1 = 0, r2 = 0;
    while(1){
        fill(mnInW, mnInW+V+1, INF);
        fill(prv, prv+V+1, -1);
        REP(i, 1, E){
            int u=edges[i].u, v=edges[i].v, c=edges[i].c;
            if(u != v && v != root && c < mnInW[v])
                mnInW[v] = c, prv[v] = u;
        }
        fill(vis, vis+V+1, -1);
        fill(cyc, cyc+V+1, -1);
        r1 = 0;
        bool jf = 0;
        REP(i, 1, V){
            if(con[i]) continue;
            if(prv[i] == -1 && i != root) return -1;
            if(prv[i] > 0) r1 += mnInW[i];
            int s;
            for(s = i; s != -1 && vis[s] == -1; s = prv[s])
                vis[s] = i;
            if(s > 0 && vis[s] == i){
                // get a cycle
                jf = 1;
                int v = s;
                do{
                    cyc[v] = s, con[v] = 1;
                    r2 += mnInW[v];
                    v = prv[v];
                }while(v != s);
                con[s] = 0;
            }
        }
        if(!jf) break;
        REP(i, 1, E){
            int &u = edges[i].u;
            int &v = edges[i].v;
            if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
            if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
            if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
            if(u == v) edges[i--] = edges[E--];
        }
    }
    return r1+r2;
}

```

2.6 SW min-cut

```
// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n,vst[MXN],del[MXN];
    int edge[MXN][MXN],wei[MXN];
    void init(int _n){
        n = _n;
        FZ(edge);
        FZ(del);
    }
    void add_edge(int u, int v, int w){
        edge[u][v] += w;
        edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++){
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            }
            if (mx == -1) break;
            vst[cur] = 1;
            s = t;
            t = cur;
            for (int i=0; i<n; i++){
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
            }
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0,x,y; i<n-1; i++){
            search(x,y);
            res = min(res,wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++){
                edge[x][j] = (edge[j][x] += edge[y][j]);
            }
        }
        return res;
    }
}graph;
```

2.7 Max Cost Circulation

```
struct MaxCostCirc {
    static const int MAXN = 33;
    int n, m;
    struct Edge {
        int v, w, c, r;
    };
    vector<Edge> g[ MAXN ];
    int dis[ MAXN ], prv[ MAXN ], prve[ MAXN ];
    bool vis[ MAXN ];
    int ans;
    void init( int _n, int _m ) : n(_n), m(_m) {}
    void adde( int u, int v, int w, int c ) {
        g[ u ].push_back( { v, w, c, SZ( g[ v ] ) } );
        g[ v ].push_back( { u, -w, 0, SZ( g[ u ] )-1 } );
    }
    bool poscyc() {
        fill( dis, dis+n+1, 0 );
        fill( prv, prv+n+1, 0 );
        fill( vis, vis+n+1, 0 );
        int tmp = -1;
        FOR( t, n+1 ) {
            REP( i, 1, n ) {
                FOR( j, SZ( g[ i ] ) ) {
                    Edge& e = g[ i ][ j ];
                    if( e.c && dis[ e.v ] < dis[ i ]+e.w ) {
                        dis[ e.v ] = dis[ i ]+e.w;
                        prv[ e.v ] = i;
                        prve[ e.v ] = j;
                        if( t == n ) {
                            tmp = i;
                            break;
                        }
                    }
                }
            }
        }
    }
};
```

```
    }
}
if( tmp == -1 ) return 0;
int cur = tmp;
while( !vis[ cur ] ) {
    vis[ cur ] = 1;
    cur = prv[ cur ];
}
int now = cur;
int cost = 0, df = 100000;
do{
    Edge &e = g[ prv[ now ] ][ prve[ now ] ];
    df = min( df, e.c );
    cost += e.w;
    now = prv[ now ];
}while( now != cur );
ans += df*cost;
now = cur;
do{
    Edge &e = g[ prv[ now ] ][ prve[ now ] ];
    Edge &re = g[ now ][ e.r ];
    e.c -= df;
    re.c += df;
    now = prv[ now ];
}while( now != cur );
return 1;
}
} circ;
```

2.8 Max flow with lower/upper bound

```
// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ];
int solve(){
    flow.init( n );
    for( int i = 0; i < m; i ++ ){
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
        // flow on edge from l[ i ] to r[ i ] should
        // be in [a[ i ], b[ i ]].
    }
    int nd = 0;
    for( int i = 1; i <= n; i ++ ){
        if( in[ i ] < out[ i ] ){
            flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] ){
            flow.addEdge( flow.s, i, in[ i ] - out[ i ] );
        }
    }
    // original sink to source
    flow.addEdge( n, 1, INF );
    if( flow.maxflow() != nd ){
        // no solution
        return -1;
    }
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0; i < flow.G[ flow.s ].size(); i ++ ){
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0; i < flow.G[ flow.t ].size(); i ++ ){
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s, 1, INF );
    flow.addEdge( n, flow.t, INF );
    flow.reset();
    return ans + flow.maxflow();
}
```

2.9 Flow Method

Maximize $c^T x$ subject to $Ax \leq b, x \geq 0$;
with the corresponding symmetric dual problem,
Minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$.

Maximize $c^T x$ subject to $Ax \leq b$;
with the corresponding asymmetric dual problem,
Minimize $b^T y$ subject to $A^T y = c, y \geq 0$.

Minimum vertex cover on bipartite graph =
Maximum matching on bipartite graph =
Max flow with source to one side, other side to sink

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side **and** with unused edges only. Equivalently, dfs from source with unused edges only **and** without visiting sink. Then, a vertex is chosen
iff. it is on the left side **and** without visited **or** on the right side **and** visited through dfs.

Maximum density subgraph $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:

For a fixed D , construct a Max flow model as follow:
Let S be Sum of all weight(**or** inf)

1. from source to each node with cap = S
2. For each (u,v,w) in E , $(u \rightarrow v, \text{cap}=w)$, $(v \rightarrow u, \text{cap}=w)$
3. For each node v , from v to sink with cap = $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$

where $\text{deg}[v] = \sum \text{weight of edge associated with } v$
If $\text{maxflow} < S * |V|$, D is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0 .

有源匯，有下界，最大流，無費用。

先從 t 連向 s ，容量設為無限大。這樣就變成了無源匯的情況。將每條有下界的邊先滿上下界的流量，然後更新盈餘量（入的流量-出的流量）。新建超級源 ss 和超級匯 tt ，若某個點 u 的盈餘量 >0 則 $ss \rightarrow u$ ，容量為 u 的盈餘量。否則 $u \rightarrow tt$ ，容量為 u 的盈餘量的相反數。如果一個點的盈餘量 >0 ，則它是一定要流出去的，所以要從 ss 連向它，使它去找這些流量的出路。建完了圖以後求一遍最大流，如果從 ss 連出的所有邊都滿流，則有解。在得到的殘留網路（原圖）上再求一次最大流即可。

3 Math

3.1 FFT

```
// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first
//
// To implement poly. multiply:
//
// fft( n , a );
// fft( n , b );
// for( int i = 0 ; i < n ; i++ )
//   c[ i ] = a[ i ] * b[ i ];
// fft( n , c , 1 );
//
// then you have the result in c :: [cplx]
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
```

```
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv)
        for (i = 0; i < n; i++)
            a[i] /= n;
}
```

3.2 NTT

```
typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
n    2^n    p    a    root
5    32     97    3    5
6    64    193    3    5
7    128    257    2    3
8    256    257    1    3
9    512    7681   15   17
10   1024   12289   12   11
11   2048   12289   6    11
12   4096   12289   3    11
13   8192   40961   5    3
14   16384  65537   4    3
15   32768  65537   2    3
16   65536  65537   1    3
17   131072 786433   6   10
18   262144 786433   3   10 (605028353,
    2308, 3)
19   524288 5767169  11   3
20   1048576 7340033   7   3
21   2097152 23068673  11   3
22   4194304 104857601 25   3
23   8388608 167772161 20   3
24   16777216 167772161 10   3
25   33554432 167772161 5    3 (1107296257, 33,
    10)
26   67108864 469762049 7    3
27   134217728 2013265921 15   31 */
// (must be 2^k)
// To implement poly. multiply:
// NTT<P, root, MAXN> ntt;
// ntt( n , a ); // or ntt.tran( n , a );
// ntt( n , b );
// for( int i = 0 ; i < n ; i++ )
//   c[ i ] = a[ i ] * b[ i ];
// ntt( n , c , 1 );
//
// then you have the result in c :: [LL]

template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b >= 1; bs = (bs * bs) % P) {
            if(b&1) res=(res*bs)%P;
        }
        return res;
    }
};
```

```

}
static LL inv(LL a, LL b) {
    if(a==1) return 1;
    return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
}
LL omega[MAXN+1];
NTT() {
    omega[0] = 1;
    LL r = bigmod(root, (P-1)/MAXN);
    for (int i=1; i<=MAXN; i++)
        omega[i] = (omega[i-1]*r)%P;
}
// n must be 2^k
void tran(int n, LL a[], bool inv_ntt=false) {
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            LL w = omega[i*theta*MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = a[j] - a[k];
                if (x < 0) x += P;
                a[j] += a[k];
                if (a[j] > P) a[j] -= P;
                a[k] = (w * x) % P;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
        LL ni = inv(n,P);
        reverse(a+1, a+n);
        for (i = 0; i < n; i++)
            a[i] = (a[i] * ni) % P;
    }
}
void operator()(int n, LL a[], bool inv_ntt=false) {
    tran(n, a, inv_ntt);
}
};

const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

3.3 Fast Walsh Transform

```

/* xor convolution:
* x = (x0,x1) , y = (y0,y1)
* z = ( x0y0 + x1y1 , x0y1 + x1y0 )
* =>
* x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
* z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
* z = (1/2) * z'
* or convolution:
* x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
* and convolution:
* x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL pw( LL x , LL k ) {
    LL res = 1;
    for( LL bs = x ; k ; k >>= 1, bs = (bs * bs)%MOD ){
        if( k&1 ) res = ( res * bs ) % MOD;
    }
    return res;
}
inline LL inv( LL x ) {
    return pw( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {

```

```

        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 ) {
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
        }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N );
            x[ i ] %= MOD;
        }
}

```

3.4 BigInt

```

struct BigInt {
    static const int LEN = 60;
    static const int BIGMOD = 10000;
    int s, vl, v[LEN];
    // vector<int> v;
    BigInt() : s(1) { vl = 0; }
    BigInt(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while(a)
            push_back(a % BIGMOD),
            a /= BIGMOD;
    }
    BigInt(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-')
            stPos = 1, s = -1;
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD)
                push_back(num), num = 0, q = 1;
        }
        if (num) push_back(num);
        n();
    }
    int len() const { return vl; } // return SZ(v);
    bool empty() const { return len() == 0; }
    void push_back(int x) { v[vl++] = x; } // v.PB(x);
    void pop_back() { vl--; } // v.pop_back();
    int back() const { return v[vl-1]; } // return v.back()
    void n(){ while (!empty() && !back()) pop_back(); }
    void resize(int nl) {
        vl = nl; fill(v, v+vl, 0);
        // v.resize(nl); fill(ALL(v), 0);
    }
    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for(int i=len()-2; i>=0; i--) printf("%.4d", v[i]);
    }
    int cp3(const BigInt &b) const {
        if (s != b.s) return s - b.s;
        if (s == -1) return -(*this).cp3(-b);
        if (len() != b.len()) return len()-b.len(); //int
        for (int i=len()-1; i>=0; i--)
            if (v[i]!=b.v[i]) return v[i]-b.v[i];
        return 0;
    }
    bool operator<(const BigInt &b) const {
        return cp3(b)<0;
    }
    bool operator<=(const BigInt &b) const {
        return cp3(b)<=0;
    }
    bool operator==(const BigInt &b) const {
        return cp3(b)==0;
    }
    bool operator!=(const BigInt &b) const {
        return cp3(b)!=0;
    }
    bool operator>(const BigInt &b) const

```



```

    { return cp3(b)>0; }
    bool operator>=(const Bigint &b) const
    { return cp3(b)>=0; }
    Bigint operator - () const {
        Bigint r = (*this); r.s = -r.s; return r;
    }
    Bigint operator + (const Bigint &b) const {
        if (s == -1) return -(*this)+(-b);
        if (b.s == -1) return (*this)-(-b);
        Bigint r;
        int nl = max(len(), b.len());
        r.resize(nl + 1);
        // directly add TODO
        r.n(); return r;
    }
    Bigint operator - (const Bigint &b) const {
        if (s == -1) return -(*this)-(-b);
        if (b.s == -1) return (*this)+(-b);
        if ((*this) < b) return -(b-(*this));
        Bigint r;
        r.resize(len());
        // directly sub TODO
        r.n(); return r;
    }
    Bigint operator * (const Bigint &b) {
        Bigint r;
        r.resize(len() + b.len() + 1);
        r.s = s * b.s;
        // directly mul TODO
        r.n(); return r;
    }
    Bigint operator / (const Bigint &b) {
        Bigint r;
        r.resize(max(1, len()-b.len()+1));
        int oriS = s;
        Bigint b2 = b; // b2 = abs(b)
        s = b2.s = r.s = 1;
        for (int i=r.len()-1; i>=0; i--) {
            int d=0, u=BIGMOD-1;
            while(d<u) {
                int m = (d+u+1)>>1;
                r.v[i] = m;
                if((r*b2) > (*this)) u = m-1;
                else d = m;
            }
            r.v[i] = d;
        }
        s = oriS; r.s = s * b.s;
        r.n(); return r;
    }
    Bigint operator % (const Bigint &b)
    { return (*this)-(*this)/b*b; }
};

```

3.5 Linear Recurrence

```

LL n, m;
LL dp[ N + N ];
void pre_dp(){
    dp[ 0 ] = 1;
    LL bdr = min( m + m , n );
    for( LL i = 1 ; i <= bdr ; i ++ )
        for( LL j = i - 1 ; j >= max( 0LL , i - m ) ; j -- )
            dp[ i ] = add( dp[ i ] , dp[ j ] );
}
vector<LL> Mul( vector<LL>& v1, vector<LL>& v2 ){
    int _sz1 = (int)v1.size();
    int _sz2 = (int)v2.size();
    assert( _sz1 == m );
    assert( _sz2 == m );
    vector<LL> _v( m + m );
    for( int i = 0 ; i < m + m ; i ++ ) _v[ i ] = 0;
    // expand
    for( int i = 0 ; i < _sz1 ; i ++ )
        for( int j = 0 ; j < _sz2 ; j ++ )
            _v[ i + j + 1 ] = add( _v[ i + j + 1 ] ,
                                mul( v1[ i ] , v2[ j ] ) );
    // shrink
    for( int i = 0 ; i < m ; i ++ )
        for( int j = 1 ; j <= m ; j ++ )

```

```

        _v[ i + j ] = add( _v[ i + j ] , _v[ i ] );
    for( int i = 0 ; i < m ; i ++ )
        _v[ i ] = _v[ i + m ];
    _v.resize( m );
    return _v;
}
vector<LL> I, A;
void solve(){
    pre_dp();
    if( n <= m + m )
        { printf( "%lld\n" , dp[ n ] ); exit( 0 ); }
    I.resize( m ); A.resize( m );
    for( int i = 0 ; i < m ; i ++ ) I[ i ] = A[ i ] = 1;
    // dp[ n ] = /Sum_{i=0}^{m-1} A_i * dp[ n - i - 1 ]
    LL dlt = ( n - m ) / m, rdlt = dlt * m;
    while( dlt ){
        if( dlt & 1LL ) I = Mul( I , A );
        A = Mul( A , A );
        dlt >>= 1;
    }
    LL ans = 0;
    for( int i = 0 ; i < m ; i ++ )
        ans = add(ans, mul(I[i], dp[n - i - 1 - rdlt]));
    printf( "%lld\n" , ans );
}

```

3.6 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(LL a, LL n, LL u, int t){
    LL x = mypow(a, u, n);
    for(int i=0; i<t; i++){
        LL nx = mul(x, x, n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x = nx;
    }
    return x!=1;
}
bool miller_rabin(LL n, int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    LL u=n-1; int t=0;
    // n-1 = u*2^t
    while(!(u&1)) u>>=1, t++;
    while(s--){
        LL a=randll()%(n-1)+1;
        if(witness(a, n, u, t)) return 0;
    }
    return 1;
}

```

3.7 Simplex

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
               double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
}

```

```

for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m - 1; ++j) d[i][j] = -a[i][j];
    d[i][m - 1] = 1;
    d[i][m] = b[i];
    if (d[r][m] > d[i][m]) r = i;
}
for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
d[n + 1][m - 1] = -1;
for (double dd;; ) {
    if (r < n) {
        int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
        d[r][s] = 1.0 / d[r][s];
        for (int j = 0; j <= m; ++j)
            if (j != s) d[r][j] *= -d[r][s];
        for (int i = 0; i <= n + 1; ++i) if (i != r) {
            for (int j = 0; j <= m; ++j) if (j != s)
                d[i][j] += d[r][j] * d[i][s];
            d[i][s] *= d[r][s];
        }
    }
    r = -1; s = -1;
    for (int j = 0; j < m; ++j)
        if (s < 0 || ix[s] > ix[j]) {
            if (d[n + 1][j] > eps ||
                (d[n + 1][j] > -eps && d[n][j] > eps))
                s = j;
        }
    if (s < 0) break;
    for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
        if (r < 0 ||
            (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
            (dd < eps && ix[r + m] > ix[i + m]))
            r = i;
    }
    if (r < 0) return -1; // not bounded
}
if (d[n + 1][m] < -eps) return -1; // not executable
double ans = 0;
for (int i = 0; i < m; i++) x[i] = 0;
for (int i = m; i < n + m; ++i) { // the missing
    enumerated x[i] = 0
    if (ix[i] < m - 1) {
        ans += d[i - m][m] * c[ix[i]];
        x[ix[i]] = d[i - m][m];
    }
}
return ans;
}

```

3.8 Faulhaber

```

/* faulhaber 's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK];
// bernoulli number
int inv[MAXK+1];
// inverse
int cm[MAXK+1][MAXK+1]; // combinatorics
int co[MAXK][MAXK+2];
// coefficient of x^j when p=i
inline int getinv(int x) {
    int a=x, b=mod, a0=1, a1=0, b0=0, b1=1;
    while(b) {
        int q, t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for (int i=0; i<=MAXK; i++) {
        cm[i][0]=cm[i][i]=1;
        for (int j=1; j<i; j++)
            cm[i][j]=add(cm[i-1][j-1], cm[i-1][j]);
    }
}

```

```

/* inverse */
for (int i=1; i<=MAXK; i++) inv[i]=getinv(i);
/* bernoulli */
b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
for (int i=2; i<=MAXK; i++) {
    if (i&1) { b[i]=0; continue; }
    b[i]=1;
    for (int j=0; j<i; j++)
        b[i]=sub(b[i], mul(cm[i][j], mul(b[j], inv[i-j+1])));
}
/* faulhaber */
// sigma_x=1~n {x^p} = 1/(p+1) * sigma_j=0~p { C(p+1, j) * B_j * n^(p-j+1) }
for (int i=1; i<=MAXK; i++) {
    co[i][0]=0;
    for (int j=0; j<=i; j++)
        co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
}
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n, int p) {
    int sol=0, m=n;
    for (int i=1; i<=p+1; i++) {
        sol=add(sol, mul(co[p][i], m));
        m = mul(m, n);
    }
    return sol;
}

```

3.9 Chinese Remainder

```

int pfn;
// number of distinct prime factors
int pf[MAXN]; // prime factor powers
int rem[MAXN]; // corresponding remainder
int pm[MAXN];
inline void generate_primes() {
    int i, j;
    pnum=1;
    prime[0]=2;
    for (i=3; i<=MAXVAL; i+=2) {
        if (!nprime[i]) continue;
        prime[pnum++]=i;
        for (j=i*i; j<=MAXVAL; j+=i) nprime[j]=1;
    }
}
inline int inverse(int x, int p) {
    int q, tmp, a=x, b=p;
    int a0=1, a1=0, b0=0, b1=1;
    while(b) {
        q=a/b; tmp=b; b=a-b*q; a=tmp;
        tmp=b0; b0=a0-b0*q; a0=tmp;
        tmp=b1; b1=a1-b1*q; a1=tmp;
    }
    return a0;
}
inline void decompose_mod() {
    int i, p, t=mod;
    pfn=0;
    for (i=0; i<pnum && prime[i]<=t; i++) {
        p=prime[i];
        if (t%p==0) {
            pf[pfn]=1;
            while (t%p==0) {
                t/=p;
                pf[pfn]*=p;
            }
            pfn++;
        }
    }
    if (t>1) pf[pfn++]=t;
}
inline int chinese_remainder() {
    int i, m, s=0;
    for (i=0; i<pfn; i++) {
        m=mod/pf[i];
        pm[i]=(LL)m*inverse(m, pf[i])%mod;
        s=(s+(LL)pm[i]*rem[i])%mod;
    }
}

```



```

}
return s;
}

```

3.10 Pollard Rho

```

// does not work when n is prime
LL f(LL x, LL mod){
    return add(mul(x,x,mod),1,mod);
}
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

3.11 Poly Generator

```

struct PolyGen{
    /* for a nth-order polynomial f(x), *
    * given f(0), f(1), ..., f(n) *
    * express f(x) as sigma_i{c_i*C(x,i)} */
    int n;
    vector<LL> coef;
    // initialize and calculate f(x), vector _fx should
    // be filled with f(0) to f(n)
    PolyGen(int _n,vector<LL> _fx):n(_n),coef(_fx){
        for(int i=0;i<n;i++){
            for(int j=n;j>i;j--){
                coef[j]-=coef[j-1];
            }
        }
        // evaluate f(x), runs in O(n)
        LL eval(int x){
            LL m=1, ret=0;
            for(int i=0;i<=n;i++){
                ret+=coef[i]*m;
                m=m*(x-i)/(i+1);
            }
            return ret;
        }
    }
};

```

3.12 Matrix Pseudo Inverse

```

Mat pinv( Mat m ){
    Mat res = I;
    FZ( used );
    for( int i = 0 ; i < W ; i ++ ){
        int piv = -1;
        for( int j = 0 ; j < W ; j ++ ){
            if( used[ j ] ) continue;
            if( abs( m.v[ j ][ i ] ) > EPS ){
                piv = j;
                break;
            }
        }
        if( piv == -1 ) continue;
        used[ i ] = true;
        swap( m.v[ piv ], m.v[ i ] );
        swap( res.v[ piv ], res.v[ i ] );
        LD rat = m.v[ i ][ i ];
        for( int j = 0 ; j < W ; j ++ ){
            m.v[ i ][ j ] /= rat;
            res.v[ i ][ j ] /= rat;
        }
        for( int j = 0 ; j < W ; j ++ ){
            if( j == i ) continue;

```

```

            rat = m.v[ j ][ i ];
            for( int k = 0 ; k < W ; k ++ ){
                m.v[ j ][ k ] -= rat * m.v[ i ][ k ];
                res.v[ j ][ k ] -= rat * res.v[ i ][ k ];
            }
        }
    }
    for( int i = 0 ; i < W ; i ++ ){
        if( used[ i ] ) continue;
        for( int j = 0 ; j < W ; j ++ ){
            res.v[ i ][ j ] = 0;
        }
    }
    return res;
}

```

3.13 ax+by=gcd

```

PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

3.14 Discrete sqrt

```

void calCH(int &t, int &h, const int p) {
    int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(int a, int p, int &x, int &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        int t, h, b, pb; calCH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
            } while (mypow(b, p / 2, p) != p - 1);
            pb = mypow(b, h, p);
        } int s = mypow(a, h / 2, p);
        for (int step = 2; step <= t; step++) {
            int ss = (((LL)(s * s) % p) * a) % p;
            for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
            if (ss + 1 == p) s = (s * pb) % p;
            pb = ((LL)pb * pb) % p;
        } x = ((LL)s * a) % p; y = p - x;
    } return true;
}

```

3.15 SchreierSims

```

// time: O(n^2 lg^3 |G| + t n lg |G|)
// mem : O(n^2 lg |G| + tn)
// t : number of generator
namespace SchreierSimsAlgorithm{
    typedef vector<int> Permu;
    Permu inv( const Permu& p ){
        Permu ret( p.size() );
        for( int i = 0 ; i < int(p.size()); i ++ )
            ret[ p[ i ] ] = i;
        return ret;
    }
    Permu operator*( const Permu& a, const Permu& b ){
        Permu ret( a.size() );
        for( int i = 0 ; i < (int)a.size(); i ++ )
            ret[ i ] = b[ a[ i ] ];
        return ret;
    }
    typedef vector<Permu> Bucket;
    typedef vector<int> Table;
    typedef pair<int,int> pii;
    int n, m;
    vector<Bucket> bkts, bktsInv;
    vector<Table> lookup;
}

```

```

int fastFilter( const Permu &g, bool addToG = 1 ){
    n = bkts.size();
    Permu p;
    for( int i = 0 ; i < n ; i ++ ){
        int res = lookup[ i ][ p[ i ] ];
        if( res == -1 ){
            if( addToG ){
                bkts[ i ].push_back( p );
                bktsInv[ i ].push_back( inv( p ) );
                lookup[ i ][ p[ i ] ] = (int)bkts[i].size()-1;
            }
            return i;
        }
        p = p * bktsInv[i][res];
    }
    return -1;
}

long long calcTotalSize(){
    long long ret = 1;
    for( int i = 0 ; i < n ; i ++ )
        ret *= bkts[i].size();
    return ret;
}

bool inGroup( const Permu &g ){
    return fastFilter( g, false ) == -1;
}

void solve( const Bucket &gen, int _n ){
    n = _n, m = gen.size(); // m perm[0..n-1]s
    //clear all
    bkts.clear();
    bktsInv.clear();
    lookup.clear();
}

for(int i = 0 ; i < n ; i ++ ){
    lookup[i].resize(n);
    fill(lookup[i].begin(), lookup[i].end(), -1);
}

Permu id( n );
for(int i = 0 ; i < n ; i ++ ) id[i] = i;
for(int i = 0 ; i < n ; i ++ ){
    bkts[i].push_back(id);
    bktsInv[i].push_back(id);
    lookup[i][i] = 0;
}

for(int i = 0 ; i < m ; i ++ )
    fastFilter( gen[i] );
queue< pair<pii,pii> > toUpd;
for(int i = 0; i < n; i ++ )
    for(int j = i; j < n; j ++ )
        for(int k = 0; k < (int)bkts[i].size(); k ++ )
            for(int l = 0; l < (int)bkts[j].size(); l ++ )
                toUpd.push( {pii(i,k), pii(j,l)} );
while( !toUpd.empty() ){
    pii a = toUpd.front().first;
    pii b = toUpd.front().second;
    toUpd.pop();
    int res = fastFilter(bkts[a.first][a.second] *
                        bkts[b.first][b.second]);
    if(res == -1) continue;
    pii newPair(res, (int)bkts[res].size() - 1);
    for(int i = 0; i < n; i ++ )
        for(int j = 0; j < (int)bkts[i].size(); ++j){
            if(i <= res)
                toUpd.push(make_pair(pii(i, j), newPair));
            if(res <= i)
                toUpd.push(make_pair(newPair, pii(i, j)));
        }
}
}
}

```

3.16 Discrete K-th sqrt

```

// x^K mod P = A
const int LimitSave = 100000;
LL _mod( LL a , LL mo ){return ( a % mo + mo ) % mo;}
bool ext_gcd(LL A, LL B, LL C, LL &x, LL &y, LL &gn){
    LL t;
    if( A == 0 ){
        gn = B;

```

```

        if( _mod(C, B) == 0 )
            { x = 0; y = C / B; return true; }
        return false;
    }
    if( ext_gcd( _mod(B, A), A, C, y, t, gn ) )
        { x = t - LL(B / A) * y; return true; }
    return false;
}

LL Division( LL A, LL B, LL modular ){
    LL gcdnum, K, Y;
    ext_gcd(modular, B, A, K, Y, gcdnum);
    Y = _mod(Y, modular);
    return Y < 0 ? Y + modular : Y;
}

struct tp{
    LL expo, res;
}data[ LimitSave + 100 ];
bool compareab( const tp &a, const tp &b )
{ return a.res < b.res; }
bool Binary_Search( LL key, LL &pos ){
    LL start, stop;
    start=1; stop=LimitSave;
    while( start <= stop ){
        pos = (start + stop)/2;
        if( data[pos].res == key ) return true;
        if( data[pos].res < key ) start = pos + 1;
        else stop = pos - 1;
    }
    return false;
}

LL get_log( LL root , LL A , LL mod ){
    LL i, j, times, XD, XT, position;
    if( mod - 1 < LimitSave ){
        LL now = 1;
        for( i = 0 ; i < mod ; i ++ ){
            if( now == A ) return i;
            now = _mod( now * root , mod );
        }
    }
    data[1].expo = 0; data[1].res = 1;
    for( i = 1 ; i < LimitSave ; i ++ ){
        data[i+1].expo=i;
        data[i+1].res=_mod(data[i].res*root,mod);
    }
    sort(data+1,data+LimitSave+1,compareab);
    times=myspow(root,LimitSave,mod);
    j=0; XD=1;
    while( 1 ){
        XT = Division(A, XD, mod);
        if( Binary_Search( XT, position ) )
            return j + data[position].expo;
        j = j + LimitSave;
        XD = _mod(XD * times, mod);
    }
}

LL P, K, A;
vector<LL> ans;
LL get_ordinroot( LL p ){
    LL primes[ 100 ];
    LL tot = 0, tp = P - 1;
    for( LL i = 2 ; i * i <= P - 1 ; i ++ )
        if( _mod( tp , i ) == 0 ){
            primes[ ++ tot ]=i;
            while( _mod(tp,i) == 0 ) tp /= i;
        }
    if( tp != 1 ) primes[ ++ tot ] = tp;
    for( LL i = 2 ; i ++ ){
        bool ok = true;
        for( LL j = 1 ; j <= tot ; j ++ )
            if( mypow(i, (P-1)/primes[j], P ) == 1 )
                { ok = false; break; }
        if( ok ) return i;
    }
}

//x^K mod P = A
void work_ans() {
    cin>>P>>K>>A;
    A = A % P;
    ans.clear(); // roots in ans
    if( A == 0 )
        { ans.push_back( 0 ); return; }
    LL root, logs, delta, deltapower, now, gcdnum, x, y;

```

```

root=get_originroot(P);
logs=get_log(root,A,P);
if( ext_gcd(K, P-1, logs, x, y, gcdnum) ){
    delta=(P-1) / gcdnum;
    x = _mod(x, delta);
    if(x < 0) x += delta;
    now = mypow(root, x, P);
    deltapower = mypow(root, delta, P);
    while(x < P-1 ){
        ans.push_back(now);
        now=_mod(now * deltapower, P);
        x=x+delta;
    }
}
}
}

```

3.17 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

3.18 Roots of Polynomial

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int n;
int sign( double x ){
    return (x < -eps)?(-1):(x>eps);
}
double f(double a[], int n, double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++){
        sum=sum+a[i]*tmp;
        tmp=tmp*x;
    }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l;
    if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){
        x[1]=-a[0]/a[1];
        nx=1;
        return;
    }
    double da[10], dx[10];
    int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);

```

```

    if(tmp<inf) x[++nx]=tmp;
}
int main() {
    scanf("%d",&n);
    for(int i=n;i>=0;i--) scanf("%lf",&a[i]);
    int nx;
    solve(n,a,x,nx);
    for(int i=1;i<=nx;i++) printf("%.6f\n",x[i]);
}

```

3.19 Mod

```

/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.
int _fd(int a,int b){ return a<0?((-a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r){
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r){
    m=abs(m);
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r){
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}

```

3.20 Primes and μ function

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
 * 999983, 1097774749, 1076767633, 100102021, 999997771
 * 1001010013, 1000512343, 987654361, 999991231
 * 999888733, 98789101, 987777733, 999991921, 1010101333
 * 1010102101, 1000000000039, 100000000000037
 * 2305843009213693951, 4611686018427387847
 * 9223372036854775783, 18446744073709551557 */
int mu[ N ], p_tbl[ N ];
vector<int> primes;
void sieve() {
    mu[ 1 ] = p_tbl[ 1 ] = 1;
    for( int i = 2 ; i < N ; i ++ ){
        if( !p_tbl[ i ] ){
            p_tbl[ i ] = i;
            primes.push_back( i );
            mu[ i ] = -1;
        }
        for( int p : primes ){
            int x = i * p;
            if( x >= M ) break;
            p_tbl[ x ] = p;
            mu[ x ] = -mu[ i ];
            if( i % p == 0 ){
                mu[ x ] = 0;
                break;
            }
        }
    }
}
vector<int> factor( int x ){
    vector<int> fac{ 1 };
    while( x > 1 ){
        int fn = SZ(fac), p = p_tbl[ x ], pos = 0;
        while( x % p == 0 ){
            x /= p;
            for( int i = 0 ; i < fn ; i ++ )
                fac.PB( fac[ pos ++ ] * p );
        }
    }
    return fac;
}

```

3.21 Result

```

/*
Lucas' Theorem:
For non-negative integer n,m and prime P,
 $C(m,n) \bmod P = C(m/M,n/M) * C(m\%M,n\%M) \bmod P$ 
= mult_i ( C(m_i,n_i) )
where m_i is the i-th digit of m in base P.

Pick's Theorem
 $A = i + b/2 - 1$ 

Kirchhoff's theorem
 $A_{\{ii\}} = \deg(i)$ ,  $A_{\{ij\}} = (i,j) \in E ? -1 : 0$ 
Deleting any one row, one column, and cal the det(A)
*/

```

4 Geometry

4.1 halfPlaneIntersection

4.2 Intersection of 2 lines

```

Pt interPnt( Line l1, Line l2, bool &res ){
    Pt p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
    double f1 = (p2 - p1) ^ (q1 - p1);
    double f2 = (p2 - p1) ^ (p1 - q2);
    double f = (f1 + f2);
    if( fabs(f) < eps)
    { res = false; return {0, 0}; }
    res = true;
    return q1 * (f2 / f) + q2 * (f1 / f);
}

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    bool res;
    Pt p = interPnt(l1, l2, res);
    return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}

/* If no solution, check: 1. ret.size() < 3
* Or more precisely, 2. interPnt(ret[0], ret[1])
* in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
*/

/* --- Line.FI --- Line.SE --- */
vector<Line> halfPlaneInter( vector<Line> lines ){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for( int i=0; i<sz; i++) {
        ord[i] = i;
        Pt d = lines[i].SE - lines[i].FI;
        ata[i] = atan2(d.Y, d.X);
    }
    sort( ord.begin(), ord.end(), [&](int i, int j) {
        if( fabs(ata[i] - ata[j]) < eps )
            return ( (lines[i].SE - lines[i].FI) ^
                    (lines[j].SE - lines[j].FI) ) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for( int i=0; i<sz; i++)
        if ( !i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps )
            fin.PB(lines[ord[i]]);
    deque<Line> dq;
    for( int i=0; i<(int)(fin.size()); i++) {
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[(int)(dq.size()-2)],
                    dq[(int)(dq.size()-1)]))
            dq.pop_back();
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while( (int)(dq.size()) >= 3 and
        not isin(dq[0], dq[(int)(dq.size()-2)],
            dq[(int)(dq.size()-1)]))

```

```

        dq.pop_back();
        while( (int)(dq.size()) >= 3 and
            not isin(dq[(int)(dq.size()-1)], dq[0], dq[1]))
            dq.pop_front();
        vector<Line> res(dq.begin(), dq.end());
        return res;
    }
}

```

4.3 Intersection of 2 segments

```

int ori( const PLL& o, const PLL& a, const PLL& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return ret / max( 1ll, abs( ret ) );
}

// p1 == p2 || q1 == q2 need to be handled
bool banana( const PLL& p1, const PLL& p2,
              const PLL& q1, const PLL& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1, p2, q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
            ( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
            ( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
            ( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
    }
    return (ori( p1, p2, q1 ) * ori( p1, p2, q2 ) <= 0) &&
        (ori( q1, q2, p1 ) * ori( q1, q2, p2 ) <= 0);
}

```

4.4 Intersection of circle and segment

```

bool Inter( const Pt& p1, const Pt& p2, Circle& cc ){
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * ( dp * ( p1 - cc.O ) );
    double c = cc.O * cc.O + p1 * p1 - 2 * ( cc.O * p1 )
        - cc.R * cc.R;
    double bb4ac = b * b - 4 * a * c;
    return !( fabs( a ) < eps or bb4ac < 0 );
}

```

4.5 Intersection of polygon and circle

```

Pt ORI, info[ N ];
D r; int n;
// Divides into multiple triangle, and sum up
// oriented area
D area2(Pt pa, Pt pb){
    if( norm(pa) < norm(pb) ) swap(pa, pb);
    if( norm(pb) < eps ) return 0;
    D S, h, theta;
    D a = norm( pb ), b = norm( pa ), c = norm( pb - pa );
    D cosB = (pb * (pb - pa)) / a / c, B = acos(cosB);
    D cosC = (pa * pb) / a / b, C = acos(cosC);
    if(a > r){
        S = (C/2)*r*r;
        h = a*b*sin(C)/c;
        if ( h < r && B < PI/2 ) S -= (acos(h/r)*r*r - h*sqrt(r*r-h*h));
    } else if(b > r){
        theta = PI - B - asin(sin(B)/r*a);
        S = .5*a*r*sin(theta) + (C-theta)/2*r*r;
    } else S = .5*sin(C)*a*b;
    return S;
}

D area() {
    D S = 0;
    for(int i = 0; i < n; ++i)
        S += abs( area2(info[i], info[i + 1]) ) * sign( det(
            info[i], info[i + 1]));
    return fabs(S);
}

```

4.6 Intersection of 2 circles

4.7 Circle cover

```
#define N 1021
struct CircleCover{
    int C; Circle c[ N ];
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool CCinter( Circle& a , Circle& b , Pt& p1 , Pt& p2 )
    {
        Pt o1 = a.O , o2 = b.O;
        D r1 = a.R , r2 = b.R;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2))
            ;
        D A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d)
            );
        Pt v = Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Tevent {
        Pt p; D ang; int add;
        Tevent() {}
        Tevent(Pt _a, D _b, int _c): p(_a), ang(_b), add(_c)
        {}
        bool operator<(const Tevent &a)const
        {return ang < a.ang;}
    }eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjunct( Circle& a, Circle& b, int x ){
        return sign( norm( a.O - b.O ) - a.R - b.R ) > x;
    }
    bool contain( Circle& a, Circle& b, int x ){
        return sign( a.R - b.R - norm( a.O - b.O ) ) > x;
    }
    bool contain(int i, int j){ /* c[j] is non-strictly
        in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjunct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
                if( j != i && overlap[j][i] )
                    cnt ++;
            for( int j = 0 ; j < C ; j ++ )
                if( i != j && g[i][j] ){
                    Pt aa, bb;
                    CCinter(c[i], c[j], aa, bb);
                    D A = atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X)
                        ;
                    D B = atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X)
                        ;
                    eve[E ++] = Tevent(bb, B, 1);
                    eve[E ++] = Tevent(aa, A, -1);
                    if(B > A) cnt ++;
                }
            if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
            else{
                sort( eve , eve + E );
                eve[E] = eve[0];
                for( int j = 0 ; j < E ; j ++ ){
                    cnt += eve[j].add;
                    Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
                }
            }
        }
    }
};
```

```
D theta = eve[j + 1].ang - eve[j].ang;
if (theta < 0) theta += 2. * pi;
Area[cnt] += ( theta - sin(theta) ) * c[i].R
            * c[i].R * .5;
    }
    }
    }
};
```

4.8 Convex Hull trick

```
/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0 }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt
            (p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }
    }
};
```



```

    }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
    return 1;
}
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0, i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p, int &i0, int &i1) {
    if (contain(p)) return false;
    i0 = i1 = 0;
    int id = lower_bound(lower.begin(), lower.end(), p)
        - lower.begin();
    bi_search(0, id, p, i0, i1);
    bi_search(id, (int)lower.size(), p, i0, i1);
    id = lower_bound(upper.begin(), upper.end(), p,
        greater<Pt>()) - upper.begin();
    bi_search((int)lower.size() - 1, (int)lower.size()
        - 1 + id, p, i0, i1);
    bi_search((int)lower.size() - 1 + id, (int)lower.
        size() - 1 + (int)upper.size(), p, i0, i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<LL, int> ret = get_tang(upper, vec);
    ret.second = (ret.second+(int)lower.size()-1)%n;
    ret = max(ret, get_tang(lower, vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i, next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u, Pt v, int &i0, int &i1){
    int p0 = get_tang(u - v), p1 = get_tang(v - u);
    if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
        if (p0 > p1) swap(p0, p1);
        i0 = bi_search(u, v, p0, p1);
        i1 = bi_search(u, v, p1, p0 + n);
        return 1;
    }
    return 0;
}
}
};

```

4.9 Tangent line of two circles

```

vector<Line> go( const Circle& c1 , const Circle& c2 ){
    vector<Line> ret;
    double d_sq = norm2( c1.0 - c2.0 );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.0 - c1.0 ) / d;
    for( int sign1 = 1 ; sign1 >= -1 ; sign1 -= 2 ){
        double c = ( c1.R - sign1 * c2.R ) / d;
        if( c * c > 1 ) continue;
        double h = sqrt( max( 0.0 , 1.0 - c * c ) );
        for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
            Pt n;
            n.X = v.X * c - sign2 * h * v.Y;
            n.Y = v.Y * c + sign2 * h * v.X;
            Pt p1 = c1.0 + n * c1.R;
            Pt p2 = c2.0 + n * ( c2.R * sign1 );
            if( fabs( p1.X - p2.X ) < eps and
                fabs( p1.Y - p2.Y ) < eps )
                p2 = p1 + perp( c2.0 - c1.0 );
            ret.push_back( { p1 , p2 } );
        }
    }
    return ret;
}

```

4.10 KD Tree

```

const int MXN = 100005;

struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;

```

```

        int id,f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;

    long long dis2(int x1, int y1, int x2, int y2) {
        long long dx = x1-x2;
        long long dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }

        tree[M].R = build_tree(M+1, R, dep+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }

        return tree+M;
    }
    int touch(Node* r, int x, int y, long long d2){
        long long dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
            r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Node* r, int x, int y, int &mID, long
        long &md2) {
        if (!r || !touch(r, x, y, md2)) return;
        long long d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id;
            md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {
            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
    int query(int x, int y) {
        int id = 1029384756;
        long long d2 = 102938475612345678LL;
        nearest(root, x, y, id, d2);
        return id;
    }
}tree;

```


4.11 Lower Concave Hull

```

****
maintain a "concave hull" that support the following
1. insertion of a line
2. query of height(y) on specific x on the hull
****/
/* set as needed */
typedef long double LD;
const LD eps=1e-9;
const LD inf=1e19;
class Seg {
public:
    LD m,c,x1,x2; // y=mx+c
    bool flag;
    Seg(
        LD _m,LD _c,LD _x1=-inf,LD _x2=inf,bool _flag=0)
        :m(_m),c(_c),x1(_x1),x2(_x2),flag(_flag) {}
    LD evaly(LD x) const {
        return m*x+c;
    }
    const bool operator<(LD x) const {
        return x2-eps<x;
    }
    const bool operator<(const Seg &b) const {
        if(flag!=b.flag) return *this<b.x1;
        return m+eps<b.m;
    }
};

class LowerConcaveHull { // maintain a hull like: \_/_/
public:
    set<Seg> hull;
    /* functions */
    LD xintersection(Seg a,Seg b) {
        return (a.c-b.c)/(b.m-a.m);
    }
    inline set<Seg>::iterator replace(set<Seg> &
        hull,set<Seg>::iterator it,Seg s) {
        hull.erase(it);
        return hull.insert(s).first;
    }
    void insert(Seg s) {
        // insert a line and update hull
        set<Seg>::iterator it=hull.find(s);
        // check for same slope
        if(it!=hull.end()) {
            if(it->c+eps>=s.c) return;
            hull.erase(it);
        }
        // check if below whole hull
        it=hull.lower_bound(s);
        if(it!=hull.end()&&
            s.evaly(it->x1)<=it->evaly(it->x1)+eps) return;
        // update right hull
        while(it!=hull.end()) {
            LD x=xintersection(s,*it);
            if(x>=it->x2-eps) hull.erase(it++);
            else {
                s.x2=x;
                it=replace(hull,it,Seg(it->m,it->c,x,it->x2));
                break;
            }
        }
        // update left hull
        while(it!=hull.begin()) {
            LD x=xintersection(s,*(--it));
            if(x<=it->x1+eps) hull.erase(it++);
            else {
                s.x1=x;
                it=replace(hull,it,Seg(it->m,it->c,it->x1,x));
                break;
            }
        }
        // insert s
        hull.insert(s);
    }
    void insert(LD m,LD c) { insert(Seg(m,c)); }
    LD query(LD x) { // return y @ given x
        set<Seg>::iterator it =
            hull.lower_bound(Seg(0.0,0.0,x,x,1));
        return it->evaly(x);
    }
};

```

```

}
};

```

4.12 Delaunay Triangulation

```

/*
Delaunay Triangulation:
Given a sets of points on 2D plane, find a
triangulation
such that no points will strictly inside circumcircle
of any triangle.

find : return a triangle contain given point
add_point : add a point into triangulation

A Triangle is in triangulation iff. its has_chd is 0.
Region of triangle u: iterate each u.edge[i].tri,
each points are u.p[(i+1)%3], u.p[(i+2)%3]
*/
const int N = 100000 + 5;
const int MAX_TRIS = N * 6;
double eps = 1e-6;
double sqr(double x) { return x*x; }
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
    const Pt& p4){
// return p4 is in circumcircle of tri(p1,p2,p3)
    double u11 = p1.X - p4.X;
    double u21 = p2.X - p4.X;
    double u31 = p3.X - p4.X;
    double u12 = p1.Y - p4.Y;
    double u22 = p2.Y - p4.Y;
    double u32 = p3.Y - p4.Y;
    double u13 = sqr(p1.X)-sqr(p4.X)+sqr(p1.Y)-sqr(p4.Y);
    double u23 = sqr(p2.X)-sqr(p4.X)+sqr(p2.Y)-sqr(p4.Y);
    double u33 = sqr(p3.X)-sqr(p4.X)+sqr(p3.Y)-sqr(p4.Y);
    double det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32
        - u11*u23*u32 - u12*u21*u33 + u11*u22*u33;
    return det > eps;
}
double side(const Pt& a, const Pt& b, const Pt& p)
{ return (b - a) ^ (p - a); }
typedef int SdRef;
struct Tri;
typedef Tri* TriRef;
struct Edge {
    TriRef tri;
    SdRef side;
    Edge() : tri(0), side(0) {}
    Edge(TriRef _tri, SdRef _side) : tri(_tri), side(
        _side) {}
};

struct Tri {
    Pt p[3];
    Edge edge[3];
    TriRef chd[3];
    Tri() {}
    Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const
    { return chd[0] != 0; }
    int num_chd() const {
        return chd[0] == 0 ? 0
            : chd[1] == 0 ? 1
            : chd[2] == 0 ? 2 : 3;
    }
    bool contains(Pt const& q) const {
        double a = side(p[0],p[1],q);
        double b = side(p[1],p[2],q);
        double c = side(p[2],p[0],q);
        return a >= -eps && b >= -eps && c >= -eps;
    }
} triange_pool[MAX_TRIS], *tot_tris;
void edge(Edge a, Edge b) {
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}
struct Trig { // Triangulation
    Trig() {

```

```

const double LOTS = 1e6;
the_root = new(tot_tris++) Tri(Pt(-LOTS,-LOTS),Pt(+
LOTS,-LOTS),Pt(0,+LOTS));
}
// tools
TriRef find(Pt p) const
{ return find(the_root,p); }
void add_point(const Pt& p)
{ add_point(find(the_root,p),p); }
// tools
TriRef the_root;
static TriRef find(TriRef root, const Pt& p) {
    for(;;) {
        if (!root->has_chd())
            return root;
        for (int i = 0; i < 3 && root->chd[i] ; ++i)
            if (root->chd[i]->contains(p)) {
                root = root->chd[i];
                break;
            }
    }
    // "point not found"
}
void add_point(TriRef root, Pt const& p) {
    TriRef tab,tbc,tca;
    /* split it into three triangles */
    tab=new(tot_tris++) Tri(root->p[0],root->p[1],p);
    tbc=new(tot_tris++) Tri(root->p[1],root->p[2],p);
    tca=new(tot_tris++) Tri(root->p[2],root->p[0],p);
    edge(Edge(tab,0), Edge(tbc,1));
    edge(Edge(tbc,0), Edge(tca,1));
    edge(Edge(tca,0), Edge(tab,1));
    edge(Edge(tab,2), root->edge[2]);
    edge(Edge(tbc,2), root->edge[0]);
    edge(Edge(tca,2), root->edge[1]);
    root->chd[0] = tab;
    root->chd[1] = tbc;
    root->chd[2] = tca;
    flip(tab,2);
    flip(tbc,2);
    flip(tca,2);
}
void flip(TriRef tri, SdRef pi) {
    TriRef trj = tri->edge[pi].tri;
    int pj = tri->edge[pi].side;
    if (!trj) return;
    if (!in_cc(tri->p[0],tri->p[1],tri->p[2],trj->p[pj]
    )) return;
    /* flip edge between tri,trj */
    TriRef trk = new(tot_tris++) Tri(tri->p[(pi+1)%3],
    trj->p[pj], tri->p[pi]);
    TriRef trl = new(tot_tris++) Tri(trj->p[(pj+1)%3],
    tri->p[pi], trj->p[pj]);
    edge(Edge(trk,0), Edge(trl,0));
    edge(Edge(trk,1), tri->edge[(pi+2)%3]);
    edge(Edge(trk,2), trj->edge[(pj+1)%3]);
    edge(Edge(trl,1), trj->edge[(pj+2)%3]);
    edge(Edge(trl,2), tri->edge[(pi+1)%3]);
    tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
    trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
    flip(trk,1); flip(trl,2);
}
}
int n;
Pt ps[N];
void build(){
    tot_tris = triange_pool;
    scanf("%d", &n);
    for(int i = 0; i < n; ++ i) {
        int x, y;
        scanf("%d%d", &x, &y);
        ps[i] = { x , y };
    }
    random_shuffle(ps, ps + n);
    Trig tri;
    for(int i = 0; i < n; ++ i)
        tri.add_point(ps[i]);
}

```

4.13 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
            return {cen,sqrt(r2)};
        }
    }
} mec;

```

4.14 Min Enclosing Ball

```

// Pt : { x , y , z }
#define N 202020
int n, nouter; Pt pt[N ], outer[4], res;
double radius,tmp;
void ball() {
    Pt q[3]; double m[3][3], sol[3], L[3], det;
    int i,j; res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2: res=(outer[0]+outer[1])/2; radius=norm2(res
        , outer[0]); break;
        case 3:
            for (i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for (i=0; i<2; ++i) for(j=0; j<2; ++j) m[i][j]=(q[i]
            * q[j])*2;
            for (i=0; i<2; ++i) sol[i]=(q[i] * q[i]);
            if (fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps
            ) return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;
            res=outer[0]+q[0]*L[0]+q[1]*L[1];
            radius=norm2(res, outer[0]);
            break;
        case 4:
            for (i=0; i<3; ++i) q[i]=outer[i+1]-outer[0], sol
            [i]=(q[i] * q[i]);
            for (i=0; i<3; ++i) for(j=0; j<3; ++j) m[i][j]=(q[i]
            * q[j])*2;
            det= m[0][0]*m[1][1]*m[2][2]
            + m[0][1]*m[1][2]*m[2][0]
            + m[0][2]*m[1][0]*m[2][1]
            - m[0][2]*m[1][1]*m[2][0]

```

```

- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1];
if ( fabs(det)<eps ) return;
for (j=0; j<3; ++j) {
    for (i=0; i<3; ++i) m[i][j]=sol[i];
    L[j]=( m[0][0]*m[1][1]*m[2][2]
+ m[0][1]*m[1][2]*m[2][0]
+ m[0][2]*m[1][0]*m[2][1]
- m[0][2]*m[1][1]*m[2][0]
- m[0][1]*m[1][0]*m[2][2]
- m[0][0]*m[1][2]*m[2][1]
) / det;
    for (i=0; i<3; ++i) m[i][j]=(q[i] * q[j])*2;
} res=outer[0];
for (i=0; i<3; ++i) res = res + q[i] * L[i];
radius=norm2(res, outer[0]);
}
}
void minball(int n){ ball();
    if( nouter < 4 ) for( int i = 0 ; i < n ; i ++ )
        if( norm2(res, pt[i]) - radius > eps ){
            outer[ nouter ++ ] = pt[ i ]; minball(i); --
            nouter;
        }
        if(i>0){ Pt Tt = pt[i];
            memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt
            ;
        }
}
}
void solve{
    // n points in pt
    random_shuffle(pt, pt+n); radius=-1;
    for(int i=0;i<n;i++) if(norm2(res,pt[i])-radius>eps)
        nouter=1, outer[0]=pt[i], minball(i);
    printf("%.5f\n",sqrt(radius));
}
}

```

4.15 Minkowski sum

```

/* convex hull Minkowski Sum*/
#define INF 1000000000000000LL
int pos( const Pt& tp ){
    if( tp.Y == 0 ) return tp.X > 0 ? 0 : 1;
    return tp.Y > 0 ? 0 : 1;
}
#define N 300030
Pt pt[ N ], qt[ N ], rt[ N ];
LL Lx,Rx;
int dn,un;
inline bool cmp( Pt a, Pt b ){
    int pa=pos( a ),pb=pos( b );
    if(pa==pb) return (a^b)>0;
    return pa<pb;
}
int minkowskiSum(int n,int m){
    int i,j,r,p,q,fi,fj;
    for(i=1,p=0;i<n;i++){
        if( pt[i].Y<pt[p].Y ||
            (pt[i].Y==pt[p].Y && pt[i].X<pt[p].X) ) p=i; }
    for(i=1,q=0;i<m;i++){
        if( qt[i].Y<qt[q].Y ||
            (qt[i].Y==qt[q].Y && qt[i].X<qt[q].X) ) q=i; }
    rt[0]=pt[p]+qt[q];
    r=1; i=p; j=q; fi=fj=0;
    while(1){
        if((fj&&j==q) ||
            ( (!fj||i==p) &&
              cmp(pt[(p+1)%n]-pt[p],qt[(q+1)%m]-qt[q]) ) ){
            rt[r]=rt[r-1]+pt[(p+1)%n]-pt[p];
            p=(p+1)%n;
            fi=1;
        }else{
            rt[r]=rt[r-1]+qt[(q+1)%m]-qt[q];
            q=(q+1)%m;
            fj=1;
        }
        if(r<=1 || ((rt[r]-rt[r-1])^(rt[r-1]-rt[r-2]))!=0)
            r++;
        else rt[r-1]=rt[r];
        if(i==p && j==q) break;
    }
    return r-1;
}
}

```

```

void initInConvex(int n){
    int i,p,q;
    LL Ly,Ry;
    Lx=INF; Rx=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X<Lx) Lx=pt[i].X;
        if(pt[i].X>Rx) Rx=pt[i].X;
    }
    Ly=Ry=INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y<Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y>Ry){ Ry=pt[i].Y; q=i; }
    }
    for(dn=0,i=p;i!=q;i=(i+1)%n){ qt[dn++]=pt[i]; }
    qt[dn]=pt[q]; Ly=Ry=-INF;
    for(i=0;i<n;i++){
        if(pt[i].X==Lx && pt[i].Y>Ly){ Ly=pt[i].Y; p=i; }
        if(pt[i].X==Rx && pt[i].Y<Ry){ Ry=pt[i].Y; q=i; }
    }
    for(un=0,i=p;i!=q;i=(i+n-1)%n){ rt[un++]=pt[i]; }
    rt[un]=pt[q];
}
inline int inConvex(Pt p){
    int L,R,M;
    if(p.X<Lx || p.X>Rx) return 0;
    L=0;R=dn;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<qt[M].X) R=M; else L=M; }
    if(tri(qt[L],qt[R],p)<0) return 0;
    L=0;R=un;
    while(L<R-1){ M=(L+R)/2;
        if(p.X<rt[M].X) R=M; else L=M; }
    if(tri(rt[L],rt[R],p)>0) return 0;
    return 1;
}
int main(){
    int n,m,i;
    Pt p;
    scanf("%d",&n);
    for(i=0;i<n;i++) scanf("%lld%lld",&pt[i].X,&pt[i].Y);
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    scanf("%d",&m);
    for(i=0;i<m;i++) scanf("%lld%lld",&qt[i].X,&qt[i].Y);
    n=minkowskiSum(n,m);
    for(i=0;i<n;i++) pt[i]=rt[i];
    initInConvex(n);
    scanf("%d",&m);
    for(i=0;i<m;i++){
        scanf("%lld %lld",&p.X,&p.Y);
        p.X*=3; p.Y*=3;
        puts(inConvex(p)?"YES":"NO");
    }
}

```

4.16 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 内心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a+Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt orthoCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
        A = ca.X * ba.Y - ba.X * ca.Y,
        x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
        y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

5 Graph

5.1 HeavyLightDecomp

```
#define SZ(c) (int)(c).size()
#define ALL(c) (c).begin(), (c).end()
#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
typedef tuple< int , int > tii;
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruli
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of
    // node u
    int mom[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfsz(int u, int p){
        dep[u] = dep[p] + 1;
        mom[u][0] = p;
        sz[u] = 1;
        head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfsz(v, u);
            sz[u] += sz[v];
        }
    }
    void dfshl(int u){
        //printf("dfshl %d\n", u);
        ts++;
        tid[u] = tl[u] = tr[u] = ts;
        tdi[tid[u]] = u;
        sort(ALL(g[u]),
            [&](int a, int b){return sz[a] > sz[b];});
        bool flag = 1;
        for(int& v:g[u]) if(v != mom[u][0]){
            if(flag) head[v] = head[u], flag = 0;
            dfshl(v);
            tr[u] = tr[v];
        }
    }
    inline int lca(int a, int b){
        if(dep[a] > dep[b]) swap(a, b);
        //printf("lca %d %d\n", a, b);
        int diff = dep[b] - dep[a];
        REPD(k, LOG-1, 0) if(diff & (1<<k)){
            //printf("b %d\n", mom[b][k]);
            b = mom[b][k];
        }
        if(a == b) return a;
        REPD(k, LOG-1, 0) if(mom[a][k] != mom[b][k]){
            a = mom[a][k];
            b = mom[b][k];
        }
        return mom[a][0];
    }
    void init( int _n ){
        n = _n;
        REP( i , 1 , n ) g[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        g[ v ].push_back( u );
    }
    void yutruli(){
        dfsz(1, 0);
        ts = 0;
        dfshl(1);
        REP(k, 1, LOG-1) REP(i, 1, n)
            mom[i][k] = mom[mom[i][k-1]][k-1];
    }
    vector< tii > getPath( int u , int v ){
        vector< tii > res;
```

```
while( tid[ u ] < tid[ head[ v ] ] ){
    res.push_back( tii( tid[ head[ v ] ] , tid[ v ] ) );
    v = mom[ head[ v ] ][ 0 ];
}
res.push_back( tii( tid[ u ] , tid[ v ] ) );
reverse( ALL( res ) );
return res;
/*
 * res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< tii >& path = tree.getPath( u , v )
 * for( tii tp : path ) {
 *     int l , r; tie( l , r ) = tp;
 *     upd( l , r );
 *     uu = tree.tdi[ l ] , vv = tree.tdi[ r ];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;
```

5.2 DominatorTree

```
const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n , m , s;
    vector< int > g[ MAXN ] , pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ] , nfd[ MAXN ] , ts;
    int par[ MAXN ];
    int sdom[ MAXN ] , idom[ MAXN ];
    int mom[ MAXN ] , mn[ MAXN ];
    inline bool cmp( int u , int v )
    { return dfn[ u ] < dfn[ v ] ; }
    int eval( int u ){
        if( mom[ u ] == u ) return u;
        int res = eval( mom[ u ] );
        if( cmp( sdom[ mn[ mom[ u ] ] ] , sdom[ mn[ u ] ] ) )
            mn[ u ] = mn[ mom[ u ] ];
        return mom[ u ] = res;
    }
    void init( int _n , int _m , int _s ){
        ts = 0; n = _n; m = _m; s = _s;
        REP( i , 1 , n ) g[ i ].clear(), pred[ i ].clear();
    }
    void addEdge( int u , int v ){
        g[ u ].push_back( v );
        pred[ v ].push_back( u );
    }
    void dfs( int u ){
        ts++;
        dfn[ u ] = ts;
        nfd[ ts ] = u;
        for( int v : g[ u ] ) if( dfn[ v ] == 0 ){
            par[ v ] = u;
            dfs( v );
        }
    }
    void build(){
        REP( i , 1 , n ){
            dfn[ i ] = nfd[ i ] = 0;
            cov[ i ].clear();
            mom[ i ] = mn[ i ] = sdom[ i ] = i;
        }
        dfs( s );
        REPD( i , n , 2 ){
            int u = nfd[ i ];
            if( u == 0 ) continue;
            for( int v : pred[ u ] ) if( dfn[ v ] ){
                eval( v );
                if( cmp( sdom[ mn[ v ] ] , sdom[ u ] ) )
                    sdom[ u ] = sdom[ mn[ v ] ];
            }
            cov[ sdom[ u ] ].push_back( u );
            mom[ u ] = par[ u ];
            for( int w : cov[ par[ u ] ] ){

```

```

    eval( w );
    if( cmp( sdom[ mn[ w ] ] , par[ u ] ) )
        idom[ w ] = mn[ w ];
    else idom[ w ] = par[ u ];
}
cov[ par[ u ] ].clear();
}
REP( i , 2 , n ){
    int u = nfd[ i ];
    if( u == 0 ) continue ;
    if( idom[ u ] != sdom[ u ] )
        idom[ u ] = idom[ idom[ u ] ];
}
}
} domT;

```

5.3 MaxClique

```

struct MaxClique {
    static const int MV = 210;
    int V , ans , dp[MV];
    int el[MV][MV/30+1] , s[MV][MV/30+1];
    vector<int> sol;
    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }
    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }
    bool dfs(int v, int k) {
        int c = 0, d = 0;
        for(int i=0; i<(V+31)/32; i++) {
            s[k][i] = el[v][i];
            if(k != 1) s[k][i] &= s[k-1][i];
            c += __builtin_popcount(s[k][i]);
        }
        if(c == 0) {
            if(k > ans) {
                ans = k;
                sol.clear();
                sol.push_back(v);
                return 1;
            }
            return 0;
        }
        for(int i=0; i<(V+31)/32; i++) {
            for(int a = s[k][i]; a ; d++) {
                if(k + (c-d) <= ans) return 0;
                int lb = a&(-a), lg = 0;
                a ^= lb;
                while(lb!=1) {
                    lb = (unsigned int)(lb) >> 1;
                    lg ++;
                }
                int u = i*32 + lg;
                if(k + dp[u] <= ans) return 0;
                if(dfs(u, k+1)) {
                    sol.push_back(v);
                    return 1;
                }
            }
        }
        return 0;
    }
    int solve() {
        for(int i=V-1; i>=0; i--) {
            dfs(i, 1);
            dp[i] = ans;
        }
        return ans;
    }
};

```

5.4 Number of Maximal Clique

```

// bool g[][] : adjacent array indexed from 1 to n
void dfs(int sz){
    int i, j, k, t, cnt, best = 0;
    if(ne[sz]==ce[sz]){ if (ce[sz]==0) ++ans; return; }
    for(t=0, i=1; i<=ne[sz]; ++i){
        for(cnt=0, j=ne[sz]+1; j<=ce[sz]; ++j)
            if (!g[lst[sz][i]][lst[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=i, best=cnt;
    } if (t && best<=0) return;
    for (k=ne[sz]+1; k<=ce[sz]; ++k) {
        if (t>0){ for (i=k; i<=ce[sz]; ++i)
            if (!g[lst[sz][t]][lst[sz][i]]) break;
            swap(lst[sz][k], lst[sz][i]);
        } i=lst[sz][k]; ne[sz+1]=ce[sz+1]=0;
        for (j=1; j<k; ++j)if (g[i][lst[sz][j]])
            lst[sz+1][++ne[sz+1]]=lst[sz][j];
        for (ce[sz+1]=ne[sz+1], j=k+1; j<=ce[sz]; ++j)
            if (g[i][lst[sz][j]]) lst[sz+1][++ce[sz+1]]=lst[sz][j];
        dfs(sz+1); ++ne[sz]; --best;
        for (j=k+1, cnt=0; j<=ce[sz]; ++j) if (!g[i][lst[sz][j]]) ++cnt;
        if (t==0 || cnt<best) t=k, best=cnt;
        if (t && best<=0) break;
    }
}
void work(){
    ne[0]=0; ce[0]=0;
    for(int i=1; i<=n; ++i) lst[0][++ce[0]]=i;
    ans=0; dfs(0);
}

```

5.5 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        FZ(vst);
        for (auto v : vec){
            if (!vst[v]){
                rDFS(v);
                nScc++;
            }
        }
    }
};

```

5.6 Dynamic MST


```

/* Dynamic MST O( Q lg^2 Q )
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infy)
add an edge: change from \infy to specific value
*/
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next; }
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
            int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);

```

```

    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

5.7 Minimum General Weighted Matching

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            for( int i = 0 ; i < n ; i ++ )
                onstk[ i ] = dis[ i ] = 0;
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for (int i=0; i<n; i++)
            ret += edge[i][match[i]];
        ret /= 2;
        return ret;
    }
}graph;

```

5.8 Minimum Steiner Tree


```
// Minimum Steiner Tree
// O(V^3AT + V^2 2^AT)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n, dst[V][V], dp[1 << T][V], tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ )
                dst[ i ][ j ] = INF;
            dst[ i ][ i ] = 0;
        }
    }
    void add_edge( int ui , int vi , int wi ){
        dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
        dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
    }
    void shortest_path(){
        for( int k = 0 ; k < n ; k ++ )
            for( int i = 0 ; i < n ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dst[ i ][ j ] = min( dst[ i ][ j ],
                        dst[ i ][ k ] + dst[ k ][ j ] );
    }
    int solve( const vector<int>& ter ){
        int t = (int)ter.size();
        for( int i = 0 ; i < ( 1 << t ) ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                dp[ i ][ j ] = INF;
        for( int i = 0 ; i < n ; i ++ )
            dp[ 0 ][ i ] = 0;
        for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
            if( msk == ( msk & (-msk) ) ){
                int who = __lg( msk );
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                continue;
            }
            for( int i = 0 ; i < n ; i ++ )
                for( int submsk = ( msk - 1 ) & msk ; submsk ; submsk = ( submsk - 1 ) & msk )
                    dp[ msk ][ i ] = min( dp[ msk ][ i ],
                        dp[ submsk ][ i ] +
                        dp[ msk ^ submsk ][ i ] );
            for( int i = 0 ; i < n ; i ++ ){
                tdst[ i ] = INF;
                for( int j = 0 ; j < n ; j ++ )
                    tdst[ i ] = min( tdst[ i ],
                        dp[ msk ][ j ] + dst[ j ][ i ] );
            }
            for( int i = 0 ; i < n ; i ++ )
                dp[ msk ][ i ] = tdst[ i ];
        }
        int ans = INF;
        for( int i = 0 ; i < n ; i ++ )
            ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
        return ans;
    }
} solver;
```

5.9 BCC based on vertex

```
struct BccVertex {
    int n, nScc, step, dfn[MAXN], low[MAXN];
    vector<int> E[MAXN], sccv[MAXN];
    int top, stk[MAXN];
    void init(int _n) {
        n = _n;
        nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v) {
        E[u].PB(v);
        E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
```

```
for (auto v:E[u]) {
    if (v == f) continue;
    if (dfn[v] == -1) {
        DFS(v,u);
        low[u] = min(low[u], low[v]);
        if (low[v] >= dfn[u]) {
            int z;
            sccv[nScc].clear();
            do {
                z = stk[--top];
                sccv[nScc].PB(z);
            } while (z != v);
            sccv[nScc].PB(u);
            nScc++;
        }
    } else {
        low[u] = min(low[u], dfn[v]);
    }
}
}
vector<vector<int>>> solve() {
    vector<vector<int>>> res;
    for (int i=0; i<n; i++) {
        dfn[i] = low[i] = -1;
    }
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) {
            top = 0;
            DFS(i,i);
        }
    }
    REP(i,nScc) res.PB(sccv[i]);
    return res;
}
}graph;
```

5.10 Graph Hash

```
$$F_t(i) =
(F_{t-1}(i) \times A +
\sum_{j \rightarrow i} F_{t-1}(j) \times B +
\sum_{i \rightarrow j} F_{t-1}(j) \times C +
D \times (i = a)) \bmod P
$$
for each node i, iterate t times.
t, A, B, C, D, P are hash parameter
```

6 String

6.1 PalTree

```
const int MAXN = 200010;
struct PalT{
    struct Node{
        int nxt[ 33 ], len , fail;
        ll cnt;
    };
    int tot , lst;
    Node nd[ MAXN * 2 ];
    char* s;
    int newNode( int l , int _fail ){
        int res = ++tot;
        memset( nd[ res ].nxt , 0 , sizeof nd[ res ].nxt );
        nd[ res ].len = l;
        nd[ res ].cnt = 0;
        nd[ res ].fail = _fail;
        return res;
    }
    void push( int p ){
        int np = lst;
        int c = s[ p ] - 'a';
        while( p - nd[ np ].len - 1 < 0
            || s[ p ] != s[ p - nd[ np ].len - 1 ] )
            np = nd[ np ].fail;
        if( nd[ np ].nxt[ c ] ){
```

```

    nd[ nd[ np ].nxt[ c ] ].cnt++;
    lst = nd[ np ].nxt[ c ];
    return ;
}
int nq = newNode( nd[ np ].len + 2 , 0 );
nd[ nq ].cnt++;
nd[ np ].nxt[ c ] = nq;
lst = nq;
if( nd[ nq ].len == 1 ){
    nd[ nq ].fail = 2;
    return ;
}
int tf = nd[ np ].fail;
while( p - nd[ tf ].len - 1 < 0
    || s[ p ] != s[ p - nd[ tf ].len - 1 ] )
    tf = nd[ tf ].fail;

nd[ nq ].fail = nd[ tf ].nxt[ c ];
return ;
}
void init( char* _s ){
    s = _s;
    tot = 0;
    newNode( -1 , 1 );
    newNode( 0 , 1 );
    lst = 2;
    for( int i = 0 ; s[ i ] ; i++ )
        push( i );
}
void yutruli(){
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
    REPD( i , tot , 1 )
        nd[ nd[ i ].fail ].cnt += nd[ i ].cnt;
    nd[ 1 ].cnt = nd[ 2 ].cnt = 0ll;
}
} pA;
int main(){ pA.init( sa ); }

```

6.2 SAIS

```

const int N = 300010;
struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REP1(i,a,b) for ( int i=(a); i<=int(b); i++ )
    bool _t[N*2];
    int _s[N*2], _sa[N*2], _c[N*2], x[N], _p[N], _q[N*2],
        hei[N], r[N];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t,
        int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]
            ]-1]]++ = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]
            ]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];

```

```

    if(uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
    for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i
        +1] ? t[i+1] : s[i]<s[i+1]);
    MAGIC(REP1(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i
        ]]] = p[q[i]=nn++]=i);
    REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
        neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa
            [i])*sizeof(int));
        ns[q[lst=sa[i]]]=nmzx+=neq;
    }
    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx
        + 1);
    MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[
        nsa[i]]]] = p[nsa[i]]);
}
}sa;
int H[ N ], SA[ N ];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i=0; i<len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0,len)
}

```

6.3 SuffixAutomata

```

const int MAXM = 1000010;
struct SAM{
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode(){
        int res = ++tot;
        fill(nxt[res], nxt[res]+33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init(){
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c){
        int p = lst;
        int np = newNode();
        mx[np] = mx[p]+1;
        for(; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if(p == 0) mom[np] = root;
        else{
            int q = nxt[p][c];
            if(mx[p]+1 == mx[q]) mom[np] = q;
            else{
                int nq = newNode();
                mx[nq] = mx[p]+1;
                for(int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for(; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char *str){
        for(int i = 0; str[i]; i++)
            push(str[i]-'a'+1);
    }
} sam;

```

6.4 Aho-Corasick

```
struct AAutomata{
    struct Node{
        int cnt,dp;
        Node *go[26], *fail;
        Node (){
            cnt = 0; dp = -1; fail = 0;
            memset(go,0,sizeof(go));
        }
    };
    Node *root, pool[1048576];
    int nMem;
    Node* new_Node(){
        pool[nMem] = Node();
        return &pool[nMem++];
    }
    void init()
    { nMem = 0; root = new_Node(); }
    void add(const string &str)
    { insert(root,str,0); }
    void insert(Node *cur, const string &str, int pos){
        if (pos >= (int)str.size())
            { cur->cnt++; return; }
        int c = str[pos]-'a';
        if (cur->go[c] == 0)
            cur->go[c] = new_Node();
        insert(cur->go[c],str,pos+1);
    }
    void make_fail(){
        queue<Node*> que;
        que.push(root);
        while (!que.empty()){
            Node* fr=que.front();
            que.pop();
            for (int i=0; i<26; i++){
                if (fr->go[i]){
                    Node *ptr = fr->fail;
                    while (ptr && !ptr->go[i]) ptr = ptr->fail;
                    if (!ptr) fr->go[i]->fail = root;
                    else fr->go[i]->fail = ptr->go[i];
                    que.push(fr->go[i]);
                }
            }
        }
    }
};
```

6.5 Z Value

```
char s[MAXN];
int len,z[MAXN];
void Z_value() {
    int i,j,left,right;
    left=right=0; z[0]=len;
    for(i=1;i<len;i++){
        j=max(min(z[i-left],right-i),0);
        for(;i+j<len&&s[i+j]==s[j];j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}
```

6.6 BWT

```
struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
```

```
for( int i = 0 ; i < SIGMA ; i ++ )
    v[ i ].clear();
int len = strlen( ori );
for( int i = 0 ; i < len ; i ++ )
    v[ ori[i] - BASE ].push_back( i );
vector<int> a;
for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
    for( auto j : v[ i ] ){
        a.push_back( j );
        ori[ ptr ++ ] = BASE + i;
    }
for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
    res[ i ] = ori[ a[ ptr ] ];
    ptr = a[ ptr ];
}
res[ len ] = 0;
}
} bwt;
```

6.7 ZValue Palindrome

```
int len, zv[MAX*2];
char ip[MAX], op[MAX*2];
int main(){
    cin >> ip; len = strlen(ip);
    int l2 = len*2 - 1;
    for(int i=0; i<l2; i++){
        if(i&1) op[i] = '@';
        else op[i] = ip[i/2];
    }
    int l=0, r=0; zv[0] = 1;
    for(int i=1; i<l2; i++){
        if( i > r ){
            l = r = i;
            while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                l --, r ++;
            zv[i] = (r-l+1);
        }else{
            int md = (l+r)/2, j = md + md - i;
            zv[i] = zv[j];
            int q = zv[i] / 2, nr = i + q;
            if( nr == r ){
                l = i + i - r;
                while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                    l --, r ++;
                zv[i] = r - l + 1;
            }else if( nr > r )
                zv[i] = (r - i) * 2 + 1;
        }
    }
}
```

6.8 Smallest Rotation

```
string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}
```

7 Data Structure

7.1 Treap

```
struct Treap{
    int sz, val, pri, tag;
```

```

Treap *l , *r;
Treap( int _val ){
    val = _val; sz = 1;
    pri = rand(); l = r = NULL; tag = 0;
}
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;
        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r , b );
        pull( a );
        return a;
    }else{
        push( b );
        b->l = merge( a , b->l );
        pull( b );
        return b;
    }
}
void split( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( Size( t->l ) + 1 <= k ){
        a = t;
        split( t->r , k - Size( t->l ) - 1 , a->r , b );
        pull( a );
    }else{
        b = t;
        split( t->l , k , a , b->l );
        pull( b );
    }
}
}

```

7.2 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay () : val(-1), rev(0), size(0)
    { f = ch[0] = ch[1] = &nil; }
    Splay (int _val) : val(_val), rev(0), size(1)
    { f = ch[0] = ch[1] = &nil; }
    bool isr()
    { return f->ch[0] != this && f->ch[1] != this; }
    int dir()
    { return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
}

```

```

} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;
void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void evert(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    // evert(x);
    access(x);
    splay(x);
    evert(y);
    x->setCh(y, 1);
}
void cut(Splay *x, Splay *y){
    // evert(x);
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y){
    access(x);
    access(y);
    splay(x);
    int res = x->f->val;
    if (res == -1) res=x->val;
    return res;
}
int main(int argc, char** argv){
    scanf("%d%d", &N, &Q);
    for (int i=1; i<=N; i++){
        vt[i] = new (Splay::pmem++) Splay(i);
    }
    while (Q--) {
        char cmd[105];
        int u, v;
        scanf("%s", cmd);
        if (cmd[1] == 'i') {
            scanf("%d%d", &u, &v);
            link(vt[u], vt[v]);
        } else if (cmd[0] == 'c') {
            scanf("%d", &v);
            cut(vt[1], vt[v]);
        } else {

```

```

scanf("%d%d", &u, &v);
int res=ask(vt[u], vt[v]);
printf("%d\n", res);
}
}
}

```

7.3 Disjoint Set

```

struct DisjointSet{
    // save() is like recursive
    // undo() is like return
    int n, fa[ N ], sz[ N ];
    vector< pair<int*,int> > h;
    vector<int> sp;
    void init( int tn ){
        n=tn;
        for( int i = 0 ; i < n ; i ++ ){
            fa[ i ]=i;
            sz[ i ]=1;
        }
        sp.clear(); h.clear();
    }
    void assign( int *k, int v ){
        h.PB( {k, *k} );
        *k = v;
    }
    void save(){ sp.PB(SZ(h)); }
    void undo(){
        assert(!sp.empty());
        int last=sp.back(); sp.pop_back();
        while( SZ(h)!=last ){
            auto x=h.back(); h.pop_back();
            *x.first = x.second;
        }
    }
    int f( int x ){
        while( fa[ x ] != x ) x = fa[ x ];
        return x;
    }
    void uni( int x , int y ){
        x = f( x ); y = f( y );
        if( x == y ) return;
        if( sz[ x ] < sz[ y ] ) swap( x, y );
        assign( &sz[ x ], sz[ x ] + sz[ y ] );
        assign( &fa[ y ], x );
    }
}djs;

```

7.4 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int,int> umap_t;
typedef priority_queue<int> heap;
#include<ext/rope>
using namespace __gnu_cxx;
int main(){
    // Insert some entries into s.
    set_t s; s.insert(12); s.insert(505);
    // The order of the keys should be: 12, 505.
    assert(*s.find_by_order(0) == 12);
    assert(*s.find_by_order(3) == 505);
    // The order of the keys should be: 12, 505.
    assert(s.order_of_key(12) == 0);
    assert(s.order_of_key(505) == 1);
    // Erase an entry.
    s.erase(12);
    // The order of the keys should be: 505.
    assert(*s.find_by_order(0) == 505);
    // The order of the keys should be: 505.
    assert(s.order_of_key(505) == 0);

    heap h1 , h2; h1.join( h2 );

```

```

rope<char> r[ 2 ];
r[ 1 ] = r[ 0 ]; // persistenet
string t = "abc";
r[ 1 ].insert( 0 , t.c_str() );
r[ 1 ].erase( 1 , 1 );
cout << r[ 1 ].substr( 0 , 2 );
}

```

8 Others

8.1 Exact Cover Set

```

// given n*m 0-1 matrix
// find a set of rows s.t.
// for each column, there's exactly one 1
#include <stdio.h>
#include <string.h>
#define N 1024 //row
#define M 1024 //column
#define NM ((N+2)*(M+2))
char A[N][M]; //n*m 0-1 matrix
int used[N]; //answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=R[i]; j!=i; j=R[j] ){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
}
void resume(int c){
    for( int i=D[c]; i!=c; i=D[i] )
        for( int j=L[i]; j!=i; j=L[j] ){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    L[R[c]]=R[L[c]]=c;
}
int dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for( int i=R[0]; i!=0; i=R[i] )
        if(S[i]<md){ md=S[i]; c=i; }
    if(md==0) return 0;
    remove(c);
    for( int i=D[c]; i!=c; i=D[i] ){
        used[ROW[i]]=1;
        for( int j=R[i]; j!=i; j=R[j] ) remove(C[j]);
        if(dfs()) return 1;
        for( int j=L[i]; j!=i; j=L[j] ) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c);
    return 0;
}
int exact_cover(int n,int m){
    for( int i=0; i<=m; i++ ){
        R[i]=i+1; L[i]=i-1; U[i]=D[i]=i;
        S[i]=0; C[i]=i;
    }
    R[m]=0; L[0]=m;
    int t=m+1;
    for( int i=0; i<n; i++ ){
        int k=-1;
        for( int j=0; j<m; j++ ){
            if(!A[i][j]) continue;
            if(k==-1) L[t]=R[t]=t;
            else{ L[t]=k; R[t]=R[k]; }
            k=t; D[t]=j+1; U[t]=U[j+1];
            L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
            C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
        }
    }
    for( int i=0; i<n; i++ ) used[i]=0;
    return dfs();
}

```