$\mathscr{L}(\hat{P}(c_j), \Lambda) = \sum_{j=1}^{k} Count(c_j) \log(\hat{P}(c_j)) + \lambda(1 - \sum_{j=1}^{k} \hat{P}(c_j))$

Lấy đạo hàm riêng cho $\mathscr{L}$

$$\frac{\partial \mathscr{L}(\hat{P}(c_j), \lambda)}{\partial \hat{P}(c_j)} = \frac{Count(c_j)}{\hat{P}(c_j)} = 0$$

$$\Rightarrow \hat{P}(c_j) = \frac{Count(c_j)}{\lambda} (1)$$

$$\frac{\partial \mathscr{L}(\hat{P}(c_j), \lambda)}{\partial \lambda} = 1 - \sum_{j=1}^{k} \hat{P}(c_j) = 0$$

$$\Rightarrow \sum_{j=1}^{k} = 1 (2)$$

Thế (1) vào (2) :

$$\Leftrightarrow \sum_{j=1}^{k} \frac{Count(c_j)}{\lambda} = 1$$

$$\Leftrightarrow \frac{1}{\lambda} \sum_{j=1}^{k} Count(c_j) = 1$$

$$\Leftrightarrow \frac{N_{doc}}{\lambda} = 1$$

$$\Rightarrow \lambda = N_{doc}$$

$$\Rightarrow \hat{P}(c_j) = \frac{Count(c_j)}{\lambda} = \frac{Count(c_j)}{N_{doc}} (dpcm)$$