**Giải**

Xét các từ $w_i$ là độc lập.
Theo phân phối categorical:
$P(count(w_i, c_j)|\hat{P}(w_i|c_j)) = \prod_{w_i \in V} \hat{P}(w_i|c_j)^{count(w_i,c_j)}$
Đánh giá $\hat{P}(w_i|c_j)$ dựa trên Maximum log-likelihood:

$$\mathscr{L}\hat{P}(w_i|c_j) = \arg\max[P(count(w_i, c_j)|\hat{P}(w_i|c_j))] = \arg\max[\prod_{w_i \in V} \hat{P}(w_i|c_j)^{count(w_i,c_j)}]$$

$$= \arg\max \sum_{w_i \in V} count(w_i, c_j) log(\hat{P}(w_i|c_j))$$

Điều kiện chuẩn : $\sum_{W_i \in V} \hat{P}(w_i|c_j) = 1$
$\Rightarrow$ Áp dụng phương pháp nhân tử Lagrange:

$$\mathscr{L}(\hat{P}(w_i|c_j), \lambda) = \sum_{w_i \in V} count(w_i, c_j) \log(\hat{P}(w_i|c_j)) + \lambda[1 - \sum_{w_i \in V} \hat{P}(w_i|c_j)]$$

Lấy đạo hàm riêng của L :

$$\frac{\partial \mathscr{L}(\hat{P}(w_i|c_j), \lambda)}{\partial \hat{P}(w_i|c_j)} = \frac{count(w_i, c_j)}{\hat{P}(w_i|c_j)} - \lambda = 0$$

$$\Rightarrow \hat{P}(w_i|c_j) = \frac{count(w_i, c_j)}{\lambda} \quad (1)$$

$$\frac{\partial \mathscr{L}(\hat{P}(w_i|c_j), \lambda)}{\partial \lambda} = 1 - \sum_{w_i \in V} \hat{P}(w_i|c_j) = 0$$

$$\Rightarrow \sum_{w_i \in V} \hat{P}(w_i|c_j) = 1 \quad \mathbf{(2)}$$