

Topic 2: Differential Equations

□ Concept Overview

Differential equations describe how quantities change with respect to one another. They are essential in modeling dynamic systems in physics, biology, and engineering, and increasingly important in deep learning (e.g., neural ODEs).

□ First-order Ordinary Differential Equations (ODEs)

- General form: $\frac{dy}{dx} = f(x, y)$
 - Solution via separation of variables or numerical methods (Euler, RK4).
 - Used in modeling simple growth/decay, neuron models (e.g., LIF model).
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```
import numpy as np
import matplotlib.pyplot as plt

# Euler's Method to solve dy/dx = -2y, y(0) = 1
def dydx(y, x):
    return -2 * y

x = np.linspace(0, 5, 100)
y = np.zeros(len(x))
y[0] = 1
h = x[1] - x[0]

for i in range(1, len(x)):
    y[i] = y[i-1] + h * dydx(y[i-1], x[i-1])

# True solution: y = exp(-2x)
plt.plot(x, y, label='Euler Approximation')
plt.plot(x, np.exp(-2*x), '--', label='Exact Solution')
plt.title("Solving dy/dx = -2y using Euler's Method")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)
plt.show()
```

□ Flowchart – Euler's Method

1. Start

2. Define differential equation $dy/dx = f(x, y)$
 3. Input initial conditions (x_0, y_0) , step size h , and x range
 4. For each step:
 - Compute $y_{n+1} = y_n + h \cdot f(x_n, y_n)$
 - Update $x_n = x_n + h$
 5. Repeat until final x
 6. Output (x, y) values
 7. End
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