Study of Dynamic Biped Locomotion on Rugged Terrain
— Theory and Basic Experiment —

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***Abstract* — This paper introduces a new control method for biped locomotion on rugged terrain. We assume the ideal robot model which has massless legs. When a particular constraint control is applied to the ideal biped, the dynamics of the robot becomes completely linear. We call such motion the Linear Inverted Pendulum Mode and use it to develop the control scheme of the biped walking on rugged terrain. In the linear inverted pendulum mode, walking on a particular rugged ground is shown to be equivalent to walking on a level ground. It is also shown that the additional use of the ankle torque makes our control scheme robust and applicable to a real biped robot with mass legs. To ascertain our theory, we built an experimental biped robot. As the basic experiment, our robot achieved stable walking on a level ground.**

I. INTRODUCTION

A lot of studies of a biped locomotion have been conducted worldwide[1]-[14]. One of the important approaches is to extract a dominant feature of its dynamical behavior. Because biped dynamics is high-order and nonlinear, it is difficult to understand unless we make some kind of simplification.

A good analyzing technique helps us understand the dynamics of biped locomotion. Sometimes, it also helps us establish the control law. Golliday and Hemami decoupled the high-order system dynamics of biped into independent low-order subsystems using state feedback [2]. Miyazaki and Arimoto used a singular perturbation technique, and showed that biped locomotion can be divided into two modes: a fast mode and a slow mode [8]. Furusho and Masubuchi derived a reduced order model as a dominant subsystem which approximates the original high-order model very well by the application of local feedback at each articular joint of the biped robot [1]. Raibert used symmetry to analyze his one, two, and four legged robots which are controlled by the algorithm named three parts control [10].

In this paper, we introduce a new technique for analyzing the dynamic walk of a biped. First, we consider applying constraint control to an ideal robot so that the body of the robot moves on a particular straight line and rotates at a constant angular velocity. As a result of such control, we find that the dynamics of the center of mass of the body becomes completely linear. In addition, the dynamics of the constrained system has many advantages in designing biped walking motion. We call such motion of the ideal model the *Linear Inverted Pendulum Mode.*

Based on the dynamics of the linear inverted pendulum mode, we develop the control scheme of the biped walk on rugged terrain. Additional use of the ankle torque is considered to make the control scheme robust and applicable to a real biped robot with mass legs.

To ascertain our theory, we built an experimental biped robot. As the basic experiment, our robot achieved stable walking on a level ground.

II. LINEAR INVERTED PENDULUM MODE

A. Motion Equation of Support Phase

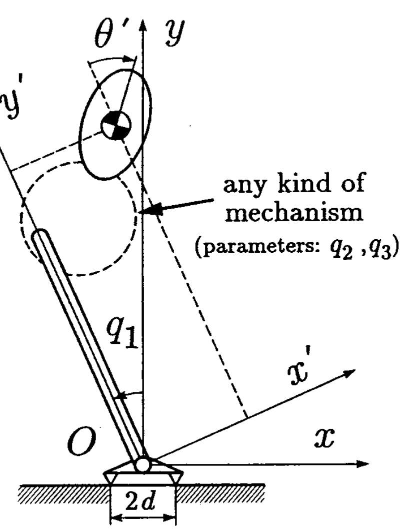

Fig.1 Generalized biped robot

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We assume a biped robot which has a mass body and two massless legs and is restricted to move in a sagittal plane (that is defined by a vertical axis and the direction of locomotion). We also consider the state in which only one leg supports the body. Figure 1 shows such robot model. As a generalized structure of the leg, we assume the first link is connected to the ankle joint and any kind of two D.O.F. mechanism drives the body with respect to the first link.

The angle of the first link, q_1 is measured from the vertical axis. q_2 and q_3 are the joint angles of the mechanism that drives the body. With respect to the base coordinate frame $O-xy$, the position and the attitude of the body are represented as follows.

$$x = x'(q_2,q_3)\cos q_1 + y'(q_2,q_3)\sin q_1 \tag{1}$$

$$y = -x'(q_2,q_3)\sin q_1 + y'(q_2,q_3)\cos q_1 \tag{2}$$

$$\theta = q_1 + \theta'(q_2,q_3) \tag{3}$$

where $x'(q_2,q_3)$ and $y'(q_2,q_3)$ denote the body position and $\theta'(q_2,q_3)$ denotes the attitude with respect to the coordinate frame that is embedded in the first link, $O-x'y'$. From these equations, Jacobian matrix J is determined as follows.

$$J = \frac{\partial \boldsymbol{p}}{\partial \boldsymbol{q}}$$

$$\boldsymbol{p} = (x,\, y,\, \theta)^{\mathrm{T}}$$

$$\boldsymbol{q} = (q_1,q_2,q_3)^{\mathrm{T}}$$

We assume there exist no singular points in the work space of the leg. Then the motion equation of Fig. 1 is represented as follows.

$$\boldsymbol{M}\ddot{\boldsymbol{p}} = (\boldsymbol{J}^{\mathrm{T}})^{-1}\boldsymbol{u} + \boldsymbol{M}\boldsymbol{g} \tag{4}$$

where

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\boldsymbol{u} = (u_1,u_2,u_3)^{\mathrm{T}}$$

$$\boldsymbol{g} = (0,-g,0)^{\mathrm{T}}$$

m : mass of the body
I : moment of inertia of the body
g : gravity acceleration

u_i is the generalized force that corresponds to q_i, and u_1 is the ankle torque of the support leg. Now, we premultiply eq. (4) by $\boldsymbol{J}^{\mathrm{T}}$ to obtain

$$\boldsymbol{J}^{\mathrm{T}}\boldsymbol{M}\ddot{\boldsymbol{p}} = \boldsymbol{u} + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{g} \ . \tag{5}$$

The first row of eq. (5) is as follows.

$$m(\frac{\partial x}{\partial q_1}\ddot{x} + \frac{\partial y}{\partial q_1}\ddot{y}) + I\frac{\partial \theta}{\partial q_1}\ddot{\theta} = u_1 - \frac{\partial y}{\partial q_1}mg \tag{6}$$

From eqs. (1) , (2) and (3), we can derive following equations.

$$\frac{\partial x}{\partial q_1} = -x'(q_2,q_3)\sin q_1 + y'(q_2,q_3)\cos q_1 = y$$

$$\frac{\partial y}{\partial q_1} = -x'(q_2,q_3)\cos q_1 - y'(q_2,q_3)\sin q_1 = -x$$

$$\frac{\partial \theta}{\partial q_1} = 1$$

By substituting these equations into eq. (6) , we obtain

$$m(y\ddot{x} - x\ddot{y}) + I\ddot{\theta} = u_1 + mgx \ . \tag{7}$$

Note that this equation does not depend on the structure of the mechanism that drives the body.

B. Derivation of the Linear Inverted Pendulum

Now, let us assume the motion of the body is controlled to be constrained by the following equations.

$$y = kx + y_H \quad (y_H \neq 0) \tag{8}$$

$$\dot{\theta} = \omega_c \tag{9}$$

Equation (8) is a constraint that keeps the center of the gravity of the body to move on a straight line that is defined by its intersection with y-axis, y_H and the slope, k. Equation (9) is a constraint that keeps the rotation rate of the body constant, ω_c. By differentiating eqs. (8) and (9), we obtain the constraints in terms of acceleration.

$$\ddot{y} = k\ddot{x} \tag{8'}$$

$$\ddot{\theta} = 0 \tag{9'}$$

By substituting eqs. (8)', (9)' and (8) into eq. (7), we obtain

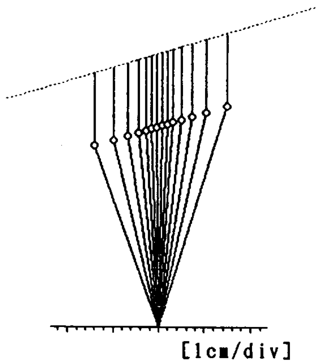
$$\ddot{x} = \frac{g}{y_H}x + \frac{1}{my_H}u_1 \tag{10}$$

Equation (10) is a motion equation of one D.O.F., because we impose two constraints upon the three D.O.F. mechanism. The forces of constraints are calculated from the second and the third raws of eq. (5).

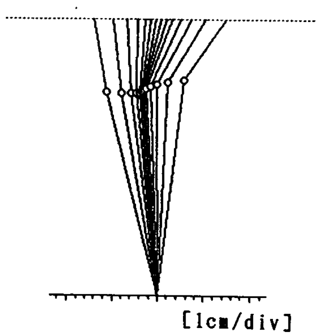
$$u_2 = m(\frac{\partial x}{\partial q_2} + k\frac{\partial y}{\partial q_2})\ddot{x} + \frac{\partial y}{\partial q_2}mg \tag{11}$$

$$u_3 = m(\frac{\partial x}{\partial q_3} + k\frac{\partial y}{\partial q_3})\ddot{x} + \frac{\partial y}{\partial q_3}mg \tag{12}$$

Figure 2 (a) and (b) show examples of constrained motions of the model which contains a variable length leg, a rigid body and a hip joint. The constraint line is sloped in Fig. 2(a) and the body rotates at a constant rate in Fig. 2(b). Both motions do without the ankle torque (u_1=0). Note that the horizontal components of the both motions of the center of mass of the body are the same, because both the examples have same y_H and the same initial condition of horizontal components of the motion.



(a) Slope trajectory
k=0.3, y_H=30.0[cm], ω_c=0.0[rad/sec],
$x(0)$=−7.0[cm], $\dot{x}(0)$=41.0[cm]



(b) Horizontal trajectory, rotating body
k=0.0, y_H=30.0[cm], ω_c=1.0[rad/sec],
$x(0)$=−7.0[cm], $\dot{x}(0)$=41.0[cm]

Fig.2 Linear inverted pendulum mode
m=2.0[kg], I=1.7×10^4[g·cm^2]
distance between hip joint and center of mass: 8.0[cm]

Equation (10) represents a class of the solution of the nonlinear equation (4). The features of eq. (10) are:

1. Linear differential equation in the Cartesian coordinate frame.
2. Does not depend on the structure of the leg.
3. Does not depend on the constraint parameters except y_H.

We call such motion the *Linear Inverted Pendulum Mode* and propose it for the design and the control of dynamic walking motion of a biped.

Equation (10) may look like the linealized motion equation of a simple inverted pendulum. Note, however, that eq. (10) can be used in the whole state space, while the linearized equation can be used only in the neighborhood of the equilibrium point. Therefore, the linear inverted pendulum mode is useful to design the biped locomotion of wide stride.

III. DESIGN OF WALKING MOTION

A. Change of the Slope of the Constraint Line

Equation (10) does not include the parameter k, so it is possible to change the slope of the constraint line without affecting the horizontal component of the body motion. To keep the parameter y_H constant, the slope is changed at the point of x=0. When the body passes x=0, the body must be driven by an impulse along a vertical axis.

$$f_y \Delta t = m(k_2 - k_1)\dot{x} \quad . \tag{13}$$

where k_1 and k_2 are the slopes of the constraint line before and after the change. If the constraint is realized by using local feedback control, the impulse is generated by the servo system, and we do not have to take the impulse into account in the design of walking motion.

B. Walking Motion and Support Leg Exchange

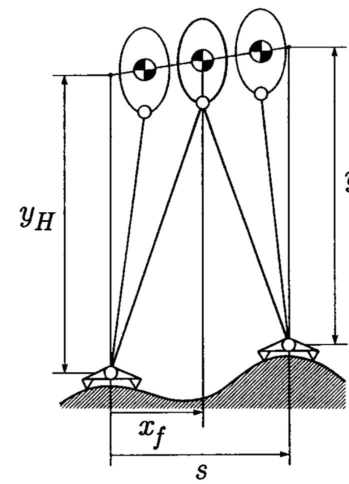


Fig.3 Support leg exchange

To design the nominal walking motion, we consider the motion that does not use the ankle torque ($u_1=0$), because the ankle torque is limited in magnitude and we want to reserve it in order to cope with the disturbances as shown in Section 4.

Figure 3 shows a support leg exchange. We specify that the body is constrained to the same line (same k, same y_H) over the leg exchange, and the support leg is assumed to be exchanged instantaneously. We assume that the horizontal component of the body velocity changes from v_- to v_+ at the instant of exchange as follows.

$$v_+ = ev_- \quad (0<e\leq 1) \tag{14}$$

where e is the parameter which depends on the velocity of the foot of the swing leg at the exchange, the compliance of the ground and so on.

When the ankle torque u_1 is zero, the first order integral of eq. (10) is

$$E = -\frac{g}{2y_H}x^2+\frac{1}{2}\dot{x}^2 \ . \tag{15}$$

In eq. (15) E is conserved during each step and characterizes the particular motion of the step. We call this the *orbital energy* [4]. Considering Fig. 5, we get equations of E_1 and E_2, the orbital energy before and after the support leg exchange.

$$E_1 = -\frac{g}{2y_H}x_f^2+\frac{1}{2}v_-^2 \tag{16}$$

$$E_2 = -\frac{g}{2y_H}(x_f-s)^2+\frac{1}{2}v_+^2 \tag{17}$$

where x_f is the horizontal position of the body at the support leg exchange and s the stride. When we specify the nominal motion by E_2, the leg exchange condition can be calculated from eqs. (14), (16) and (17).

$$x_f = \frac{s-\sqrt{s^2-(1-e^2)(s^2+(2y_H/g)(E_2-e^2E_1))}}{1-e^2} \quad (0<e<1)$$

$$x_f = \frac{y_H}{gs}(E_2-E_1)+\frac{s}{2} \quad (e=1) \tag{18}$$

C. Design of Walking Motion on Rugged Terrain

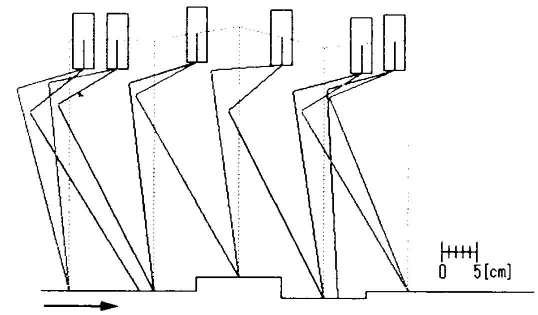
The algorithm for designing the walking motion on a rugged ground is as follows.

step 1. Decide the landing points on the ground.

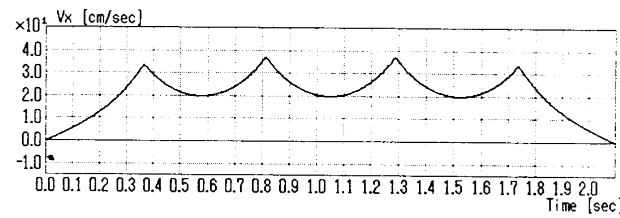
step 2. Define the constraint line by connecting the points that are at the height y_H from the landing points.

step 3. Calculate the orbital energy E for each support phase from the nominal motion and calculate the body position, x_f at the support leg exchange using eq. (18).

Figure 4 shows an example of the walking motion on a rugged ground.



(a) Stick diagram



(b) Horizontal component of body velocity

Fig.4 Designed walk on a rugged ground

IV. ROBUST WALK USING THE ANKLE TORQUE

The biped robot suffers many kinds of disturbances during walking, and the disturbances mainly affect the horizontal motion of the body.

$$\ddot{x} = \frac{g}{y_H}x + \frac{1}{my_H}u_1 + d \tag{19}$$

where d is the disturbance force.

To consider the control of the real biped robot, we assume d also includes the effect of the masses of the legs. Under the disturbances, the motion without control using u_1 never follows the trajectory that we designed in Section 3.

This problem is solved when we use the ankle torque to control the horizontal motion of the body. For example, we can use following simple control law.

$$u_1 = f_1(x-x^*)+f_2(\dot{x}-\dot{x}^*) \tag{20}$$

where x^* and $\dot{x}^*$ give the nominal trajectory of an ideal biped as shown in Section 3, and f_1 and f_2 are feedback gains.

In eq. (20) the ankle torque u_1 is used to cancel the disturbance force d. The magnitude of the ankle torque is limited to satisfy the condition that the foot is kept in secure contact with the ground. Therefore, it is necessary that the masses of the legs are small enough compared with the mass of the body.

V. EXPERIMENT

A. Hardware

The experimental walking robot was designed to move in the sagittal plane. The robot has legs of the parallel link structure which are driven by four DC servo motors contained in the body (Fig. 5). The ankle joint of each leg is driven by a small DC motor mounted on the leg. A potentiometer is mounted on each joint of the leg and a rotary encoder on each motor which drives the leg.

The aim in employing the parallel link structure is to approximate the massless leg model as closely as possible by placing the motors in the body and reducing the weight of the legs.

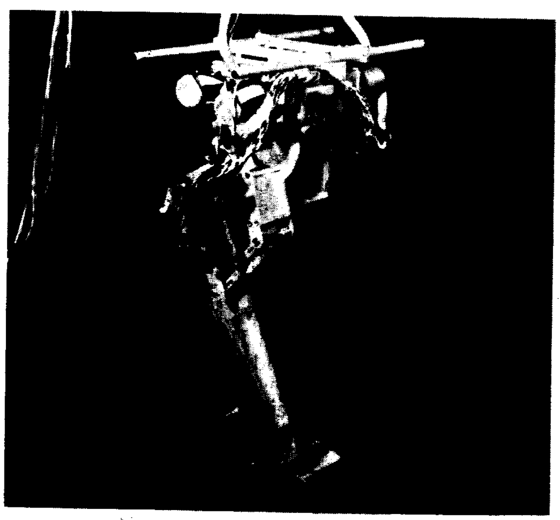


Fig.5 Biped walking robot

We use a digital signal processor (DSP, NEC μPD77230) and a microcomputer system (Intel 80386+80387) to control our biped. The DSP executes PID control for each joint, constraint control of support leg, trajectory generation for swing leg, and so on. These calculations are repeated every 1 millisecond. The microcomputer system executes more symbolic processes, such as support leg exchange and trajectory planning. The DSP and the microcomputer can communicate via a common memory.

Local PID feedback control is applied to each joint except the ankle joint. The reference angles for the joints of the support leg are calculated from the measurement of the ankle joint angle to maintain the constraint condition. The torque reference for the ankle joint is calculated from eq. (20). The swing leg is controlled to reach the next landing point at the time of support leg exchange.

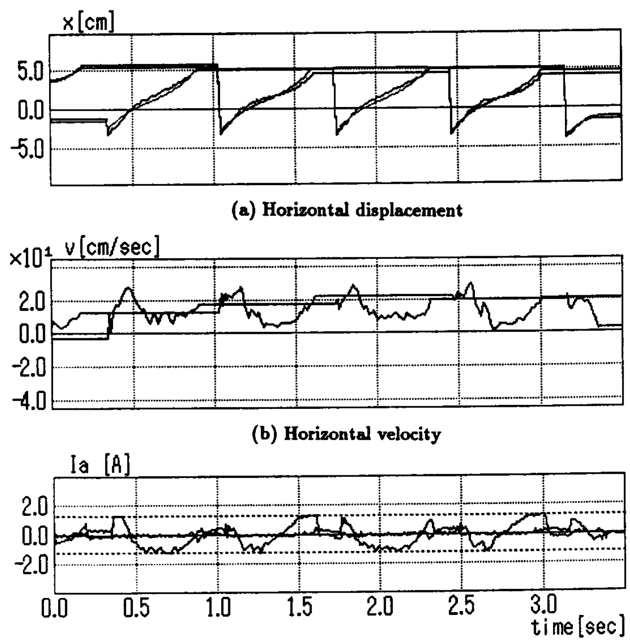
B. Experimental Result

Figure 6 shows the experimental data for the walking on the flat plane. The biped was controlled to walk five steps from the walking start to a standstill. Figure 6(a) shows the horizontal displacement of the body, and the thin line shows the nominal motion of the ideal model. The step width was specified to be 12.0 cm.

Figure 6(b) shows the horizontal velocity of the body. The speed at the moment when x=0 was specified to be 10.0 cm/sec.

Figure 6(c) shows the input current for the ankle actuator of the support leg. The current is limited to keep the foot in secure contact with the ground. The limits are shown by broken lines.

Figure 7 shows the photograph of the walking robot. The white line shows the trajectory of the LED mounted on the body.



(c) Current for ankle joint
Fig.6 Horizontal motion of the body

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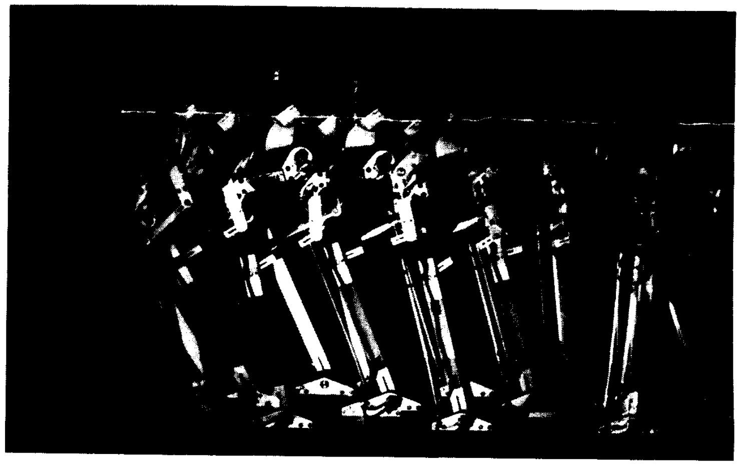

Fig.7 Walk on a level ground (8.3 msec intervals)

VI. CONCLUSION

From the ideal model of the biped, we derived a simple dynamics that is useful for designing and controlling biped locomotion. We name this the *Linear Inverted Pendulum Mode*, and presented the method to design the walking motion on rugged terrain.

Though the linear inverted pendulum mode is derived from the model whose legs have no masses, we show that, using the ankle torque, the robot with masses of legs can be controlled in the same sense.

To ascertain our theory, we built an experimental biped robot and realized the walk on a level ground as the basic experiment.

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