



**Faculty of Automation and  
Computer Science**

# **NONLINEAR ARX IDENTIFICATION**

System Identification semester project

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**2022 - 2023**

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**Datafile indexes: 16/19**



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# Problem Description

**ARX black-box model** for a **dynamic system** from adjustable model orders and degree

**Linear regression** to determine the parameters

One-step ahead **Prediction & Simulation** implementations

Tune model orders and degree to obtain **best performance**

# Approximator Structure

## ARX

Easily implementable method that produces parametric, polynomial models

Output  $y(k)$  computed based on previous inputs and outputs

Orders  $\underline{n_a}$  &  $\underline{n_b}$  are not needed to be higher than  $\underline{m}$

General ARX model structure

$$y(k) = -a_1y(k-1)-a_2y(k-2)- \dots - a_{n_a}y(k-n_a) + b_1u(k-1)+b_2u(k-2) + \dots + b_{n_b}u(k-n_b) + e(k)$$

## NARX

Nonlinear dependence between previous outputs and inputs

Combines multiple regressors of different powers

Vector of delayed inputs & outputs:  $d$

Polynomial of degree  $m$ :  $p$

Parameters: coefficients of the polynomial

# Our Solution

## 1. Preparation

Load the data – index: 19

Build 'Comb\_inator' function

## 2. Find MSE optim

## 3. Prediction and Simulation

## 4. Representative plots

# Prediction vs Simulation

**Prediction:** predict solution from the output and the calculated parameters

- Create vector of combinations for each row
- Create identification and validation matrices for combinations
- Calculate approximator,  $\Theta$  based on  $\phi_{id}$
- Calculate approximation  $\hat{y}$  based on validation matrix and  $\Theta$

**Simulation:** solution based on previously predicted outputs and parameters

- Initialise  $\hat{y}_{sim}$  for simulation
- Create combination vector for each row
- Calculate each  $\hat{y}_{sim}$  based on combinations
- Store all combinations in matrix

# Forming the Combinations

## Function 'Comb\_inator' – key features

Creates vector of delayed inputs/outputs

Makes all combinations based around the polynomial degree,  $m$ .

Adjusts vector  $d$ , so that we don't have null elements in between - first  $n_a$  rows

Creates an auxiliary vector to filter out the null elements – correct length

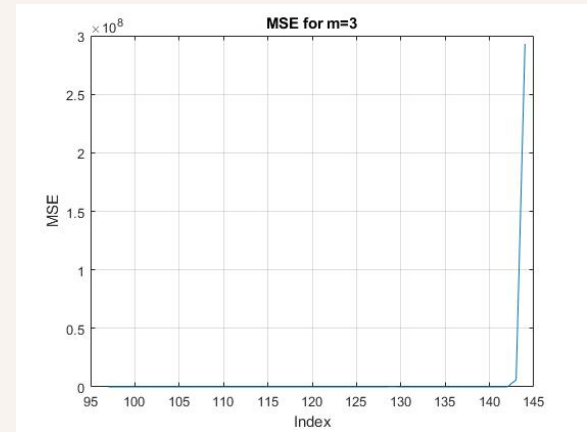
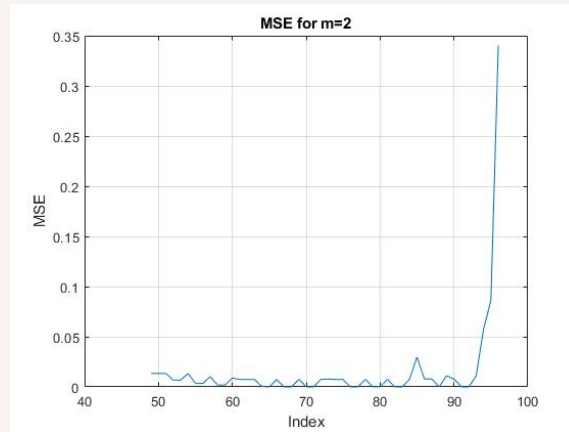
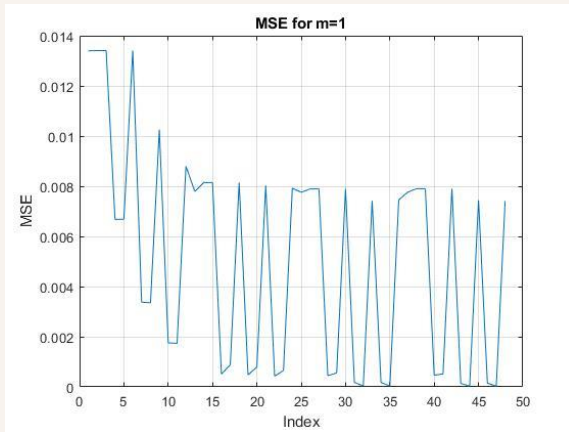
Creates 'combination' vector based on algorithm

# Tuning results

Results vary depending on input data

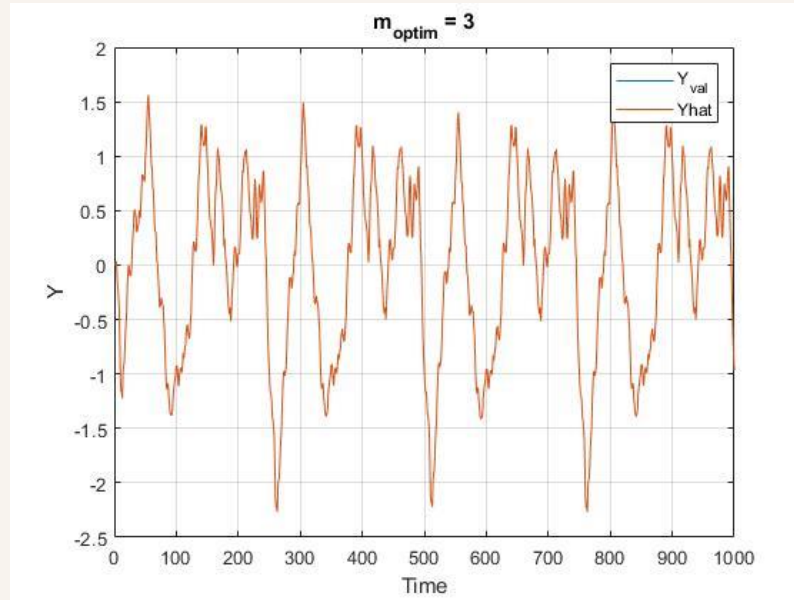
Depending on our data we calculated the mean-squared error

Best solution: for  $m = 3$ ,  $na = nb = 2$ ,  $nk = 2$

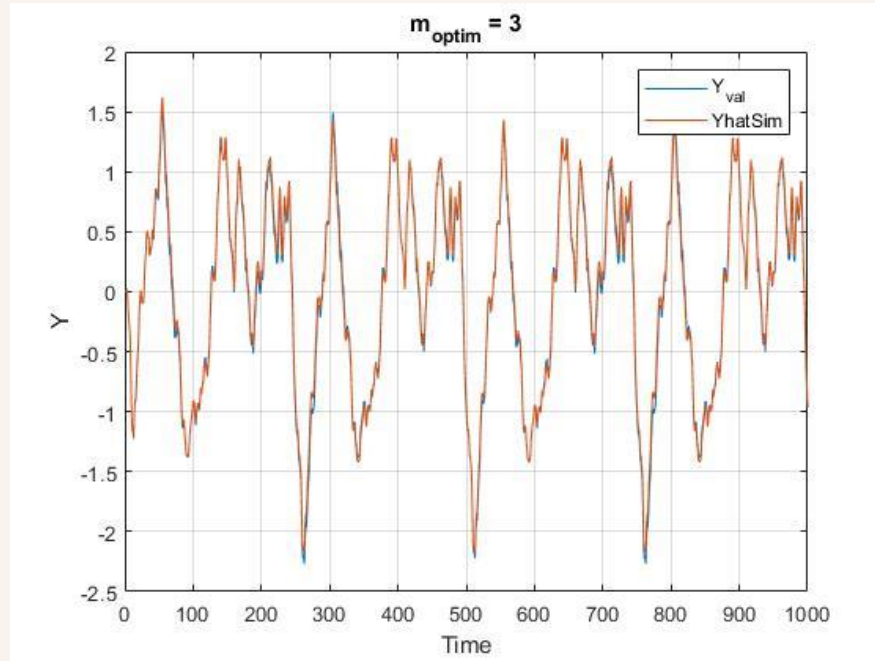




# Representative plot for Prediction



# Representative plot for Simulation



# Conclusion

Optimal results: case  $m = 3$  and  $n_a = n_b = n_k = 2$

For  $m = 1$  and  $n_a, n_k$  increasing, the model will remain stable.

Our model becomes more unstable, reaches extreme values, as  $m$  – also  $n_a, n_b, n_k$  – increase.

Prediction more accurate than simulation

# Thanks!

Do you have any questions?

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## Bibliography

- ~ [System Identification 2022 \(busoniu.net\)](http://busoniu.net)
- ~ L. Ljung, System Identification, Wiley Encyclopedia of Electrical and Electronics Engineering, 2007. Section 4, nonlinear models