School of Mathematics and Statistics

Te Kura Mātai Tatauranga

STAT 292 **Assignment 5: Due Wednesday, 24 June 2020 at 12:00 PM**

Note: Solutions must be either typed or written neatly, and questions must be answered in order.

1. Table 1 presents a subset of data collected by Väisänen and Järvinen (1977) on bird species in the Krunnit Islands archipelago of Finland. In particular, they reported on the bird species found on each of the islands in 1949 and how many of those bird species were extinct by 1970. It is of interest to understand whether the area of the island (in km^2) is associated with species' survival. The data corresponding to Table 1 are available in the Excel file `Extinction.xlsx`.

		Extinct?	
Island	Area (X)	Yes	No
Ulkokrunni	185.80	5	70
Maakrunni	105.80	3	64
Ristikari	30.70	10	56
Isonkivenletto	8.50	6	45
Hietakraasukka	4.80	3	25
Kraasukka	4.50	4	16
Länsiletto	4.30	8	35

Table 1: Extinction of bird species from 1949 to 1970 on seven islands in the Krunnit Islands archipelago, Finland.

Fit the logistic regression model

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

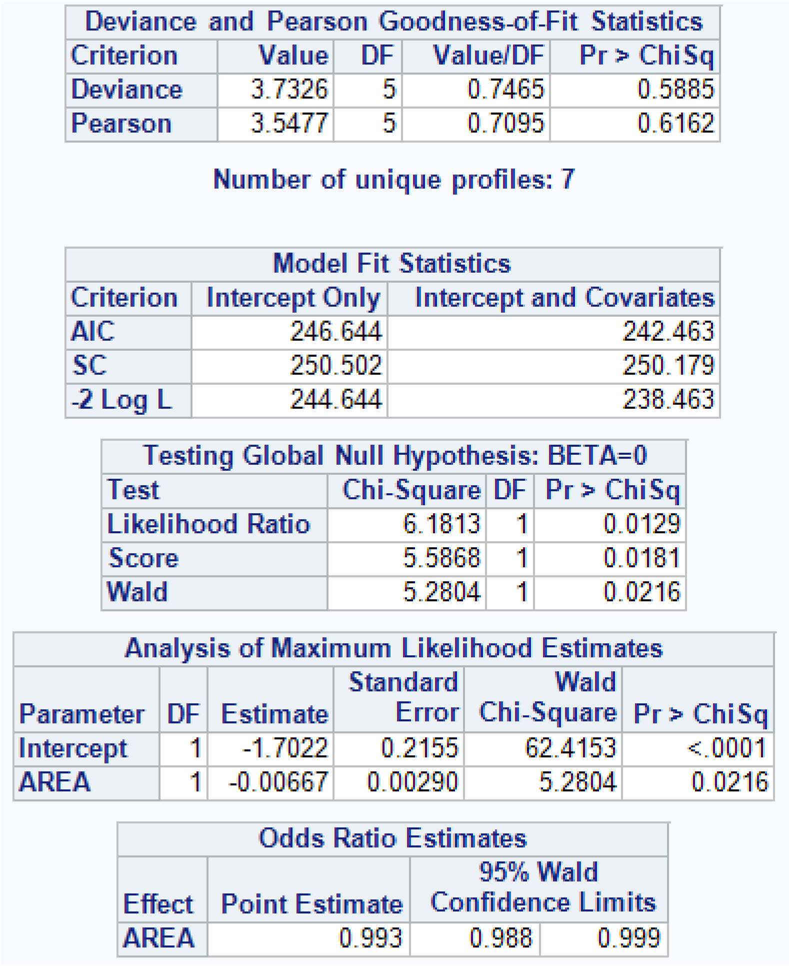
where X denotes island area and $p(X)$ denotes the probability of extinction.

Figure 1 shows relevant SAS output for the logistic regression model.

(a) Carry out an appropriate goodness-of-fit test to determine whether the model provides a good fit to the data. State the hypotheses, and give the test statistic and the p-value of the test. What do you conclude at the $\alpha = 0.05$ significance level?

(b) Give estimates of β_0 and β_1 (up to 5dp).

(c) Interpret the association between island area and extinction using the odds ratio. Demonstrate how the odds ratio is calculated from Figure 1. Additionally, provide a 95% confidence interval for the odds ratio.



Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	3.7326	5	0.7465	0.5885
Pearson	3.5477	5	0.7095	0.6162

Number of unique profiles: 7

Model Fit Statistics		
Criterion	Intercept Only	Intercept and Covariates
AIC	246.644	242.463
SC	250.502	250.179
-2 Log L	244.644	238.463

Testing Global Null Hypothesis: BETA=0			
Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	6.1813	1	0.0129
Score	5.5868	1	0.0181
Wald	5.2804	1	0.0216

Analysis of Maximum Likelihood Estimates					
			Standard	Wald	
Parameter	DF	Estimate	Error	Chi-Square	Pr > ChiSq
Intercept	1	-1.7022	0.2155	62.4153	<.0001
AREA	1	-0.00667	0.00290	5.2804	0.0216

Odds Ratio Estimates			
		95% Wald	
Effect	Point Estimate	Confidence Limits	
AREA	0.993	0.988	0.999

Figure 1: Summary output for the logistic regression model $\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$.

(d) Find the predicted probability of extinction for an island with an area of 50 km^2 (to 4dp).

(e) Find the fitted count of extinct bird species on the island of Ulkokrunni (to 2dp). Also find the fitted count of non-extinct bird species on Ulkokrunni (to 2dp).

(f) Test

$$\begin{aligned}\mathcal{H}_0 : \beta_1 &= 0\\ \mathcal{H}_1 : \beta_1 &\neq 0\end{aligned}$$

using the Wald statistic. Give the test statistic and the p-value of the test. What do you conclude at the $\alpha = 0.05$ significance level?

2. Consider data reported by Gilbert (1981) on the relationship between pre-marital sex (*i.e.*, sexual intercourse before marriage), extra-marital sex (*i.e.*, sexual intercourse with someone other than a spouse whilst married), and whether the person had been divorced for a random sample of heterosexual men and women who had been married at least once. These data are presented in Table 2 and are available in the Excel file `Divorce.xlsx`.

Gender	Pre-marital	Extra-marital	Divorced? (Z)	
(W)	Sex (X)	Sex (Y)	No	Yes
Woman	Yes	Yes	4	17
		No	25	54
	No	Yes	4	36
		No	322	214
Man	Yes	Yes	11	28
		No	42	60
	No	Yes	4	17
		No	130	68

Table 2: Data on reported pre-marital sex, extra-marital sex, and divorce for a random sample of heterosexual men and women.

First, use the backward model selection method to find the simplest model that provides a good fit to the data. Start from the following model, which we will denote by M_2,

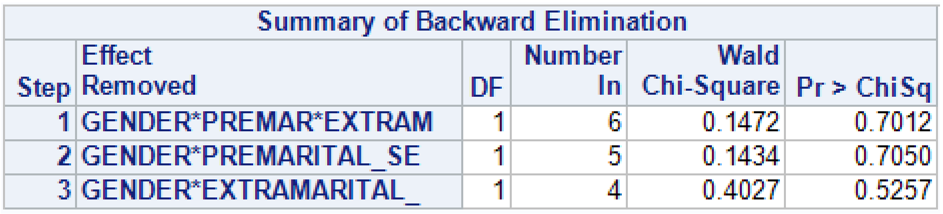
$$\log\left(\frac{p_{ijk}}{1-p_{ijk}}\right) = \beta_0 + \beta_i^W + \beta_j^X + \beta_k^Y + \beta_{ij}^{WX} + \beta_{ik}^{WY} + \beta_{jk}^{XY} + \beta_{ijk}^{WXY},$$

where p_{ijk} is the probability of divorce when the gender (W) is at level i, pre-marital sex status (X) is at level j, and extra-marital sex status (Y) is at level k.

Figure 2 shows relevant summary output from SAS.

(a) Is model M_2 a saturated model? Why or why not?

(b) What information does **Step 1** provide in the SAS output? Write down the test hypotheses. What do you conclude?



Summary of Backward Elimination					
Step	Effect Removed	DF	Number In	Wald Chi-Square	Pr > ChiSq
1	GENDER*PREMAR*EXTRAM	1	6	0.1472	0.7012
2	GENDER*PREMARITAL_SE	1	5	0.1434	0.7050
3	GENDER*EXTRAMARITAL_	1	4	0.4027	0.5257

Figure 2: Summary output for the backward selection method applied to the logit model $\log\left(\frac{p_{ijk}}{1-p_{ijk}}\right) = \beta_0 + \beta_i^W + \beta_j^X + \beta_k^Y + \beta_{ij}^{WX} + \beta_{ik}^{WY} + \beta_{jk}^{XY} + \beta_{ijk}^{WXY}$.

(c) What is the final model?

Now consider the logit model, which we will denote by M_1,

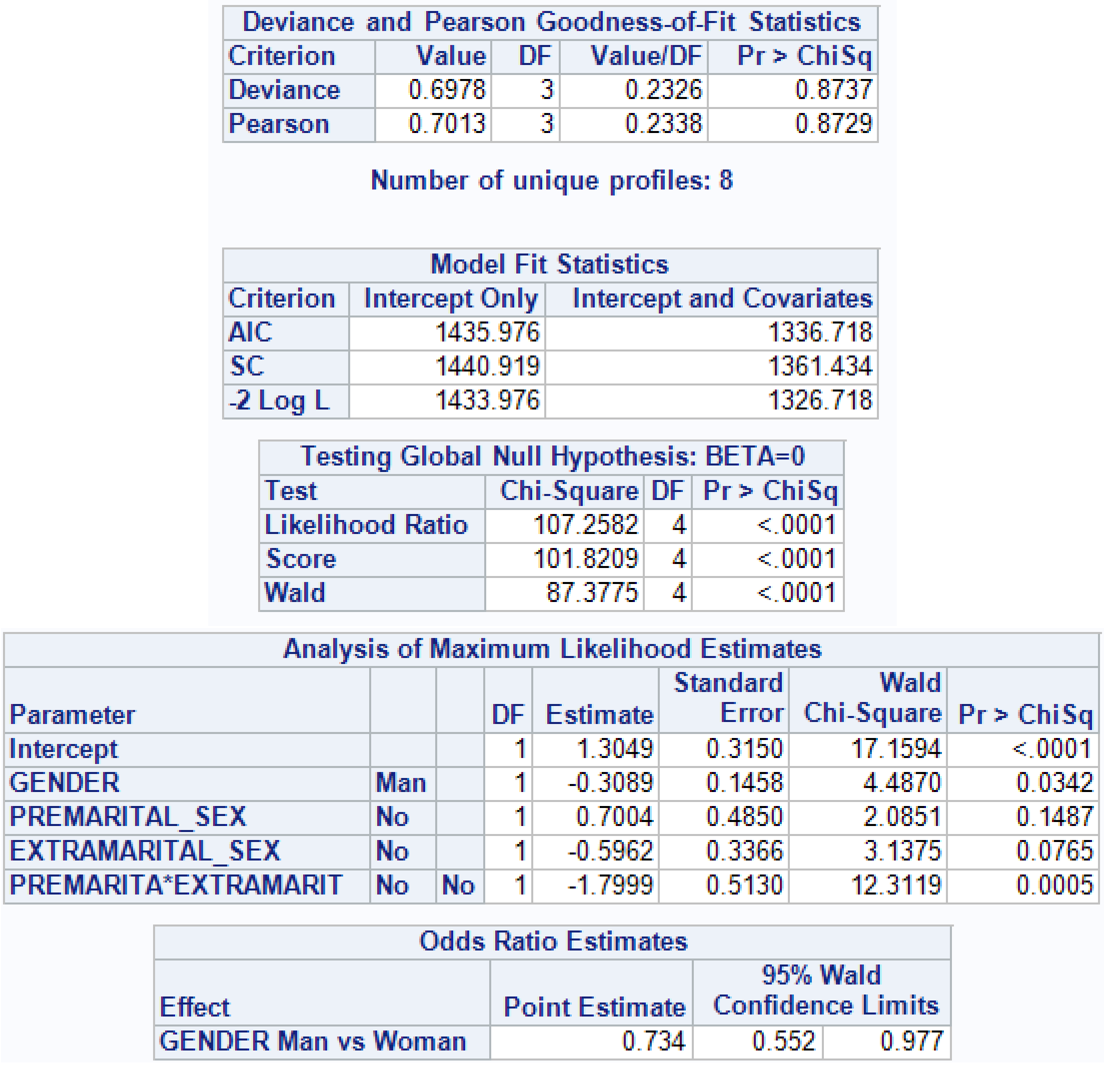
$$\log\left(\frac{p_{ijk}}{1-p_{ijk}}\right) = \beta_0 + \beta_i^W + \beta_j^X + \beta_k^Y + \beta_{jk}^{XY}.$$

which uses a reference level parametrisation for all factors.

Figure 3 shows relevant summary output from SAS.

(d) Carry out an appropriate goodness-of-fit test to determine whether model M_1 provides a good fit to the data. State the hypotheses, and give the test statistic and the p-value of the test. What do you conclude at the $\alpha = 0.05$ significance level?

(e) Compare the odds of divorce for men with the odds of divorce for women using an odds ratio, and interpret this odds ratio. Give a 95% confidence interval for the odds ratio.



Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	0.6978	3	0.2326	0.8737
Pearson	0.7013	3	0.2338	0.8729

Number of unique profiles: 8

Model Fit Statistics		
Criterion	Intercept Only	Intercept and Covariates
AIC	1435.976	1336.718
SC	1440.919	1361.434
-2 Log L	1433.976	1326.718

Testing Global Null Hypothesis: BETA=0			
Test	Chi-Square	DF	Pr > ChiSq
Likelihood Ratio	107.2582	4	<.0001
Score	101.8209	4	<.0001
Wald	87.3775	4	<.0001

Analysis of Maximum Likelihood Estimates							
					Standard	Wald	
Parameter			DF	Estimate	Error	Chi-Square	Pr > ChiSq
Intercept			1	1.3049	0.3150	17.1594	<.0001
GENDER	Man		1	-0.3089	0.1458	4.4870	0.0342
PREMARITAL_SEX	No		1	0.7004	0.4850	2.0851	0.1487
EXTRAMARITAL_SEX	No		1	-0.5962	0.3366	3.1375	0.0765
PREMARITA*EXTRAMARIT	No	No	1	-1.7999	0.5130	12.3119	0.0005

Odds Ratio Estimates			
		95% Wald	
Effect	Point Estimate	Confidence Limits	
GENDER Man vs Woman	0.734	0.552	0.977

Figure 3: Summary output for the logit model $\log\left(\frac{p_{ijk}}{1-p_{ijk}}\right) = \beta_0 + \beta_i^W + \beta_j^X + \beta_k^Y + \beta_{jk}^{XY}$.