第十一回課題解説

教科書の問 11.1.

(1) $\displaystyle\int_0^2 x^3dx = \left[\frac{x^4}{4}\right]_0^2 = 4.$

(2) $\displaystyle\int_0^{\pi/6} \sin x dx = [-\cos x]_0^{\pi/6} = -\frac{\sqrt{3}}{2} + 1.$

(3) $\displaystyle\int_2^3 \frac{dx}{x^2} = \left[-x^{-1}\right]_2^3 = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}.$

(4) $\displaystyle\int_1^2 \frac{dx}{\sqrt{x}} = [2\sqrt{x}]_1^2 = 2(\sqrt{2}-1).$

教科書の問 11.2.

(1) $\displaystyle\lim_{n\to\infty} n\left\{\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(2n)^2}\right\} = \lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{(1+\frac{1}{n})^2} + \frac{1}{(1+\frac{2}{n})^2} + \cdots + \frac{1}{(1+\frac{n}{n})^2}\right\} = \int_0^1 \frac{dx}{(1+x)^2} = \left[-\frac{1}{1+x}\right]_0^1 = \frac{1}{2}.$

(2) $\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left\{\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{2n}}\right\} = \lim_{n\to\infty} \frac{1}{n}\left\{\frac{1}{\sqrt{1+\frac{1}{n}}} + \frac{1}{\sqrt{1+\frac{2}{n}}} + \cdots + \frac{1}{\sqrt{1+\frac{n}{n}}}\right\} = \int_0^1 \frac{dx}{\sqrt{1+x}} = [2\sqrt{1+x}]_0^1 = 2(\sqrt{2}-1).$

教科書の問 12.1. 以下の計算では積分定数を省略する.

(1) $\displaystyle\int_0^a x^2e^{-x}dx = \int_0^a x^2(-e^{-x})'dx = [x^2(-e^{-x})]_0^a - \int_0^a (2x)(-e^{-x})dx = -a^2e^{-a} + 2\int_0^a xe^{-x}dx$ である. $\displaystyle\int_0^a xe^{-x}dx = \int_0^a x(-e^{-x})'dx = [-xe^{-x}]_0^a + \int_0^a e^{-x}dx = -ae^{-a} + [-e^{-x}]_0^a = 1-(a+1)e^{-a}$ だから $\displaystyle\int_0^a x^2e^{-x}dx = -a^2e^{-a} + 2\{1-(a+1)e^{-a}\} = 2 - e^{-a}(a^2+2a+2).$

(2) $\displaystyle\int \frac{x}{(1+x^2)^2}dx$. $x^2 = t$ とおくと $2xdx = dt$ だから $\displaystyle\int \frac{x}{(1+x^2)^2}dx = \frac{1}{2}\int \frac{dt}{(1+t)^2} = -\frac{1}{2}(1+t)^{-1} = -\frac{1}{2}(1+x^2)^{-1}.$

(3) $\displaystyle\int \frac{dx}{1+e^x}$. $e^x = t$ とおくと $x = \log t$ だから $dx = \frac{dt}{t}$. よって $\displaystyle\int \frac{dx}{1+e^x} = \int \frac{dt}{t(1+t)} = \int\left(\frac{1}{t} - \frac{1}{1+t}\right)dt = \log\frac{t}{1+t} = \log\frac{e^x}{1+e^x}\left(= -\log(1+e^{-x})\right).$

(4) $\displaystyle\int \log x dx = \int (x)'\log x dx = x\log x - \int x\cdot\frac{1}{x}dx = x\log x - x.$

(5) $\displaystyle\int \tan x dx = \int \frac{\sin x}{\cos x}dx = \int \frac{(-\cos x)'}{\cos x}dx = -\log|\cos x|.$