(1)

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \delta_{3-1,1} & \delta_{3-1,2} \\ \delta_{3-2,1} & \delta_{3-2,2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(2) $-A = {}^{t}A$

$$\iff \begin{bmatrix} 0 & -2 & -b \\ -c & -a & -b \\ -1 & -1 & -a \end{bmatrix} = \begin{bmatrix} 0 & c & 1 \\ 2 & a & 1 \\ b & b & a \end{bmatrix}$$

$$\iff \begin{aligned} a &= 0 \\ b &= -1 \\ c &= -2 \end{aligned}$$

(3) $BC = [d_{ij}]_{2\times 2}$

$$d_{ij} = \underline{b_{i1} \cdot c_{1j} + b_{i2} \cdot c_{2j}}$$

$$A(BC) = [e_{ij}]_{2\times 2}$$

$$e_{ij} = a_{i1} \cdot d_{1j} + a_{i2} \cdot d_{2j}$$

$$= a_{i1} \cdot [b_{11} \cdot c_{1j} + b_{12} c_{2j}]$$

$$+ a_{i2} \cdot [b_{21} c_{1j} + b_{22} c_{2j}]$$

$$= a_{i1} b_{11} c_{1j} + a_{i1} b_{12} c_{2j} + a_{i2} b_{21} c_{1j} + a_{i2} b_{22} c_{2j}$$

$$AB = [f_{ij}]$$

$$(AB)C = [g_{ij}]$$

$$g_{ij} = f_{i1} \cdot c_{1j} + f_{i2} \cdot c_{2j}$$

$$= (a_{i1} \cdot b_{11} + a_{i2} \cdot b_{21}) c_{1j}$$

$$+ (a_{i1} \cdot b_{12} + a_{i2} \cdot b_{22}) c_{2j}$$

$$= a_{i1} b_{11} c_{1j} + a_{i2} b_{21} c_{1j} + a_{i1} b_{12} c_{2j} + a_{i2} b_{22} c_{2j}.$$

$$\Rightarrow \quad e_{ij} = g_{ij}$$

$$\Rightarrow \quad A(BC) = (AB)C \qquad \square$$

(3') $(A(BC))_{ij} = \sum_{k=1}^{2} a_{ik}(BC)_{kj}$

$$= \sum_{k=1}^{2} a_{ik} \sum_{l=1}^{2} b_{kl} c_{lj}$$

$$= \sum_{k=1}^{2} \sum_{l=1}^{2} a_{ik} b_{kl} c_{lj}$$

$$= \sum_{l=1}^{2} \left(\sum_{k=1}^{2} a_{ik} b_{kl} \right) c_{lj}$$

$$= \sum_{l=1}^{2} (AB)_{i,l} \quad c_{lj}$$

$$= ((AB) \cdot C)_{i,j}$$

(4) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(5) $\begin{bmatrix} 1 & 3 & 2 \\ -1 & -3 & -2 \\ 2 & 6 & 4 \end{bmatrix}$

(6) $\begin{bmatrix} 2 & 1 \end{bmatrix}$

(7) $\begin{bmatrix} a_1 + 2a_2 \\ 3a_1 + 4a_2 \end{bmatrix}$

(8) $(s \in \mathbb{R},\ t = 1,\ u = 0)$

or

$(s = 0,\ t = 0,\ u = 0)$

(9) $\left[\begin{array}{ccc|c} -3 & 3 & 1 & 1 \\ 1 & -1 & 2 & 0 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & -\frac{1}{3} & -\frac{1}{3} \\ 1 & -1 & 2 & 0 \end{array}\right]$ ① $\times\left(-\frac{1}{3}\right)$

$\rightarrow \left[\begin{array}{cccc} 1 & -1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 2\frac{1}{3} & \frac{1}{3} \end{array}\right]$ ② $-$ ①

$$\rightarrow \begin{bmatrix} 1 & -1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{7} \end{bmatrix} \quad ② \times \frac{3}{7}$$

$$x_3 = \frac{1}{7}$$

$$x_2 = c \in \mathbb{R}$$

$$x_1 = -\frac{1}{3} + c + \frac{1}{21} = -\frac{2}{7} + c$$

解は

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} \\ 0 \\ \frac{1}{7} \end{bmatrix} + c \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad c \in \mathbb{R}$$

(10)

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 2 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 2 & 1 & 0 & -2 \\ 0 & -1 & 1 & 0 & 1 & -2 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & -2 & -1 & 0 & 2 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$$

$$\to A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$