1)

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 2 & 5 & 7 & 6 & 8 & 9 & 3 & 1 \end{pmatrix}$$

$\sigma = (1\ 4\ 7\ 9)(3\ 5\ 6\ 8)$

$\operatorname{sgn} \sigma = (-1)^3 \cdot (-1)^3 = 1$

2)

$$(1\quad 2)\ \tau = (1\quad 2\quad 3).$$

2) $\tau = (1\ 2)(1\ 2\ 3) = (2\ 3)$

3)

$$A = \begin{bmatrix} 17 & 17 & 34 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| \overset{3.2.2}{=} 17 \cdot \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\overset{3.2.4}{=} 17 \cdot \begin{vmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & -2 & 0 \end{vmatrix}$$

② + ① × (−1)
③ + ① × (−1)

$$\overset{3.2.1}{=} 17 \cdot 1 \cdot \begin{vmatrix} -1 & -1 \\ -2 & 0 \end{vmatrix} = 17 \cdot (-2) = -34 .$$

4)

$$B = \begin{bmatrix} x-1 & x & x \\ x+1 & x & x-1 \\ x & x & x+1 \end{bmatrix}$$

$$B = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

3.2.2 & 3.2.3(2)

$$\Rightarrow |B| = \begin{vmatrix} x & x & x \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} -1 & 0 & 0 \\ x & x & x \\ 0 & 0 & 1 \end{vmatrix}$$

$$+ \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & -1 \\ x & x & x \end{vmatrix} + \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= x\cdot(-1) + x\cdot(-1) + x\cdot(-1) + 0$$

$$= -3x$$

5)

$$C = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \end{bmatrix}$$

$$\det C = \det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \det\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

$$= \det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot 0$$

$$= 0.$$

G)

$$D = \begin{bmatrix} 1 & 4 & x \\ -1 & 2 & y \\ 1 & 0 & z \end{bmatrix}$$

$$\det D = X \cdot \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} - y \begin{vmatrix} 1 & 4 \\ 1 & 0 \end{vmatrix} + z \begin{vmatrix} 1 & 4 \\ -1 & 2 \end{vmatrix}$$

(7)

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & a & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}} \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & a & -3 \end{bmatrix} = 2 \cdot (-3+1) = -4 \neq 0.$$

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$$x_1 = \frac{\begin{vmatrix} \boxed{\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}} & \begin{matrix} 1 \\ 2 \\ a \end{matrix} & \begin{matrix} -1 \\ 0 \\ -3 \end{matrix} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & a & -3 \end{vmatrix}} = \frac{1 \cdot (-6) - 1(-6) - 1(2a-2)}{-4}$$

$$= \frac{1}{2}a - \frac{1}{2}$$

$$x_2 = \frac{\begin{vmatrix} 1 & \boxed{1} & -1 \\ 0 & \boxed{2} & 0 \\ 1 & \boxed{1} & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & a & -3 \end{vmatrix}} = \frac{2\cdot(-3+1)}{-4} = 1$$

$$x_3 = \frac{\begin{vmatrix} 1 & 1 & \boxed{1} \\ 0 & 2 & \boxed{2} \\ 1 & a & \boxed{1} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & a & -3 \end{vmatrix}} = \frac{1\cdot(2-2a)}{-4} = \frac{1}{2}a - \frac{1}{2}$$

8)

$$F = \begin{bmatrix} 2 & -1 & 0 \\ 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\tilde{F} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\det F = 1$$

$$\Rightarrow F^{-1} = \frac{1}{\det F} \cdot \tilde{F} = \tilde{F} .$$

9)

$$G = \begin{bmatrix} 3 & 1 & 3^2 & 3^3 \\ 4 & 1 & 4^2 & 4^3 \\ 5 & 1 & 5^2 & 5^3 \\ 6 & 1 & 6^2 & 6^3 \end{bmatrix}$$

$$\det G = (-1) \prod_{1 \le i < j \le 4} (x_j - x_i)$$

$x_1 = 6$
$x_2 = 5$
$x_3 = 4$
$x_4 = 3$

12 13 14
23 24
34

$(-1)(-2)(-3)$
$(-1)(-2)$
(-1)

$$= (-1)\, 2^2 \cdot 3 = \underline{-12}$$

10) $H = \begin{bmatrix} h_{11} & \cdots & h_{14} \\ \vdots & & \\ h_{41} & \cdots & h_{44} \end{bmatrix}$

$$\det H = 2$$

$$H\tilde{H} = \det H \cdot E_4$$

$$\Rightarrow \det H \cdot \det \tilde{H} = (\det H)^4 \cdot \det E_4$$

$$\Rightarrow \quad \det \tilde{H} = (\det H)^3 = 2^3$$

$$= \underline{\underline{8}}$$