Helmholtz Coil

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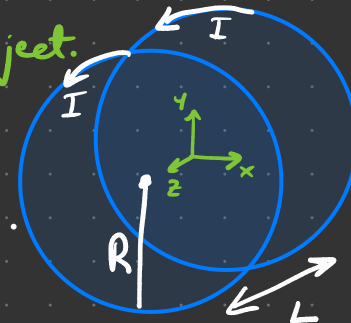
Prepared with accompanying Python code for my student Madhav Kopa as example for his Helmholtz Cage project.

The spacing that maximizes $B_z(z)$ field uniformity is $L = R$.



Proof.

Write $B_z(z)$ as superposition of contributions from each coil:

$$B_z(z) = \frac{\mu_0 I R^2}{2}\left[\frac{1}{\left(R^2 + \left(z - \frac{L}{2}\right)^2\right)^{3/2}} + \frac{1}{\left(R^2 + \left(z + \frac{L}{2}\right)^2\right)^{3/2}}\right]$$

The Taylor expansion of this function near $z = 0$ amounts to expanding the terms in the square bracket, each having form

$$f_{\pm}(z) = \left(R^2 + \left(z \pm \frac{L}{2}\right)^2\right)^{-3/2}.$$

The n^{th} derivative at $z=0$ for $f_+^{(n)}(0)$ and $f_-^{(n)}(0)$ match for n-even and cancel for n-odd. In particular, the 0^{th} order and 2^{nd} order (i.e. leading correction) terms go as

$$B_z^{(0)}(0) \propto \left(R^2 + \left(\frac{L}{2}\right)^2\right)^{-3/2}$$

and

$$B_z^{(2)}(0) \propto \left(L^2 - R^2\right)\left(R^2 + \left(\frac{L}{2}\right)^2\right)^{-7/2},$$

where we see, indeed, that $L=R$ makes $B_z^{(2)}(0)$ vanish, thus pushing back leading correction to 4^{th} order. Q.E.D.