Aero3000 Flight Dynamics Assignment 4 Pt. A

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30/5/2021

This report considers the lateral-directional equilibrium of the Pilatus PC-21. Section 1 considered the aircraft at cruise condition with a trainee pilot onboard while section 2 considers the aircraft in approach condition with a trainee pilot onboard. Both at an altitude of 500 feet.

Section 1

Table 1 Values for balanced & aileron-only turn at 300 kts with trainee-pilot.

Sub-section	Parameter	Value
Flight Conditions	V_T	300 kts
	$mass$	2760 kg
	$\frac{x_{cg}}{\bar{c}}$	29.22 %
	$Altitude$	500 ft
a. Balanced-Turn Trim States	C_L	0.426
	n_z	3.443
	ϕ	73.117 °
	$\dot{\psi}$	12.000 °/s
	p	0.000
	q	11.483 °/s
	r	3.485 °/s
b. Balanced-Turn Control Inputs	δ_T	26.233 %
	δ_e	1.464 °
	δ_a	-0.603 °
	δ_r	2.410 °
c. Aileron-Only Trim States & Control Inputs	β	-30.138 °
	δ_T	26.233 %
	δ_e	1.464 °
	δ_a	-0.538 °
	δ_r	0.000 °
d. Lateral Stability	$Lateral\ Stability$	Stable

a. Balanced Turn Trim States

For a steady turn, the yaw-rate is the angular rate the turn is completed in, which is specified as 360° within 30 seconds. The bank angle tilts the aircraft into the turn such that the generated lift force becomes the centripetal force for the turn. During a steady turn there is no roll-rate.

$$\dot{\psi} = \frac{2\pi}{30}$$

$$\phi = \operatorname{atan}\left(\frac{V\dot{\psi}}{g}\right)$$

$$n_z = \frac{1}{\cos\phi}$$

$$Q = \frac{1}{2}\rho V^2 \Rightarrow C_L = \frac{nW}{QS}$$

$$p = 0, \qquad q = \dot{\psi}\sin\phi\,, \qquad r = \dot{\psi}\cos\phi$$

b. Balanced-Turn Control Inputs

As the aircraft is in an equilibrium state, it is generating no forces or moments. The lateral stability equation is assessed for this special flight manoeuvre, then the longitudinal stability is assessed.

$$\delta_a = \frac{C_{l_r}}{C_{l_{\delta_a}}}\hat{r}, \qquad \delta_r = \frac{C_{n_r}}{C_{n_{\delta_a}}}\hat{r}$$

$$\begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_{m_\alpha} & C_{m_{\delta_e}} \\ C_{L_\alpha} & C_{L_{\delta_e}} \end{bmatrix}^{-1} \begin{bmatrix} 0 - C_{m_0} - C_{m_q}\hat{q} \\ C_L - C_{L_0} - C_{L_q}\hat{q} \end{bmatrix}$$

$$C_D = C_{D_0} + kC_L^2 \Rightarrow \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} = C_y(\alpha) \cdot \begin{bmatrix} -C_L \\ 0 \\ -C_D \end{bmatrix}$$

$$F_T = -QSC_X \Rightarrow \delta_T = \frac{F_T V}{P\sigma^{0.7}\eta}$$

c. Aileron-Only Trim States & Inputs

In an aileron-only turn, the rudder is not used to counter-act the sideslip.

$$\beta = \frac{\left(\frac{C_{n_{\delta_a}}}{C_{l_{\delta_a}}}\right) C_{l_r} - C_{n_r}}{C_{n_\beta} - \left(\frac{C_{n_{\delta_a}}}{C_{l_{\delta_a}}}\right) C_{l_\beta}}, \qquad \delta_a = \left(\frac{\left(\frac{C_{l_\beta}}{C_{n_\beta}}\right) C_{n_r} - C_{l_r}}{C_{l_{\delta_a}} - \left(\frac{C_{l_\beta}}{C_{n_\beta}}\right) C_{n_{\delta_a}}}\right)\hat{r}$$

d. Lateral Stability

An aircraft is laterally stable is the term below is larger than 1. It is indicative of whether an aircraft will continue to spiral after an aileron deflection. The PC-21 is laterally stable in this configuration.

$$Stability = \left(\frac{C_{l_\beta}}{C_{n_\beta}}\right)\left(\frac{C_{n_r}}{C_{l_r}}\right)$$

Section 2

Table 2 Values for steady-state flight for the PC-21 with damages at 100 kts with trainee pilot.

<table>
<tr><th>Sub-section</th><th colspan="2">Parameter</th><th>Value</th></tr>
<tr><td rowspan="6">Flight Conditions</td><td colspan="2">V_T</td><td>100 kts</td></tr>
<tr><td colspan="2">W</td><td>2760 kg</td></tr>
<tr><td colspan="2">$\frac{x_{cg}}{\bar{c}}$</td><td>29.22 %</td></tr>
<tr><td colspan="2">$Altitude$</td><td>500 ft</td></tr>
<tr><td colspan="2">C_{l_0}</td><td>-0.02</td></tr>
<tr><td colspan="2">C_{l_0}</td><td>-0.05</td></tr>
<tr><td rowspan="3">a.
Correctional States and Controls I</td><td colspan="2">β</td><td>-0.845 °</td></tr>
<tr><td colspan="2">δ_a</td><td>-30.599 °</td></tr>
<tr><td colspan="2">δ_r</td><td>-15.0479 °</td></tr>
<tr><td rowspan="4">b.
Maximum Operational Damage</td><td rowspan="2">Controls:
$(25°, 25°)$</td><td>$(C_{l_0})_{max}$</td><td>0.0366</td></tr>
<tr><td>$(C_{n_0})_{max}$</td><td>0.0346</td></tr>
<tr><td rowspan="2">Controls:
$(-25°, 25°)$</td><td>$(C_{l_0})_{max}$</td><td>-0.0533</td></tr>
<tr><td>$(C_{n_0})_{max}$</td><td>0.0372</td></tr>
<tr><td rowspan="6">c.
Correctional States and Controls II</td><td colspan="2">C_L</td><td>1.114</td></tr>
<tr><td colspan="2">α</td><td>11.725 °</td></tr>
<tr><td colspan="2">ϕ</td><td>0.484 °</td></tr>
<tr><td colspan="2">δ_e</td><td>-0.329 °</td></tr>
<tr><td colspan="2">δ_a</td><td>-30.976 °</td></tr>
<tr><td colspan="2">δ_r</td><td>-13.471 °</td></tr>
</table>

a. <u>Correctional States and Controls I</u>

The indicative damage has been modelled as,

$$\overrightarrow{C_0} = \begin{bmatrix} C_{Y_0} \\ C_{l_0} \\ C_{n_0} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.02 \\ -0.05 \end{bmatrix}$$

This can be included into the lateral stability equations to find the appropriate sideslip angle, aileron, and rudder deflections. Where the rows to the matrix are side force, roll moment and yaw moment, respectively. Note that there is no bank angle in this flight condition.

$$\begin{bmatrix} \beta \\ \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} \overrightarrow{C_\beta} & \overrightarrow{C_{\delta_a}} & \overrightarrow{C_{\delta_r}} \end{bmatrix}^{-1} \cdot \left(-\overrightarrow{C_0}\right)$$

b. Maximum Operational Damage

Similar to how the correction controls were found in relation to offsets in the moments, the maximum amount of damage the aircraft can take before losing its ability to fly steady and level is to solve the lateral stability equations.

$$\begin{bmatrix} \beta \\ -C_{l_0} - C_{l_\beta}\beta \\ -C_{n_0} - C_{n_\beta}\beta \end{bmatrix} = \begin{bmatrix} \overrightarrow{C_{\delta_a}} & \overrightarrow{C_{\delta_r}} \end{bmatrix} \cdot \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$

As the equations are linearised, taking the most extreme values for the controls will determine the maximum operational damage. In this case, the aircraft can withstand more damage if one control is at its negative extreme while the other is at its positive extreme. Both cases are listed in *Table 2*.

c. Correctional States and Controls II

A similar approach will be taken as part a. However now there is no sideslip angle and there is a bank angle. The bank causes some of the lift force to act to the aircraft's side.

$$\begin{bmatrix} \phi \\ \delta_a \\ \delta_r \end{bmatrix} = \begin{bmatrix} C_L & C_{Y_{\delta_a}} & C_{Y_{\delta_r}} \\ 0 & C_{l_{\delta_a}} & C_{l_{\delta_r}} \\ 0 & C_{n_{\delta_a}} & C_{n_{\delta_r}} \end{bmatrix}^{-1} \cdot \left(-\overrightarrow{C_0}\right)$$

$$\begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} C_{m_\alpha} & C_{m_{\delta_e}} \\ C_{L_\alpha} & C_{L_{\delta_e}} \end{bmatrix}^{-1} \begin{bmatrix} 0 - C_{m_0} \\ C_L - C_{L_0} \end{bmatrix}$$