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Measurement of moments for centroid estimation in Shack–Hartmann wavefront sensor—a wavelet-based approach and comparison with other methods

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Abstract

A wavelet-based method of moment calculation with a set of basis functions is presented for centroid estimation in a Shack-Hartmann Wavefront Sensor. The method has been compared with other algorithms such as statistical averaging, FFT and least-squares method. A comparative analysis shows that wavelet method has a high accuracy and processing speed, and better suited for wavefront reconstruction applications. Further, the wavelet method presented here has a variable accuracy and resolution, and can be optimized for a particular application under consideration.

Keywords: Wavelet; Moments; Scaling functions; Shack–Hartmann wavefront sensor; Centroiding methods

1. Introduction

Adaptive Optics [1,2] (AO) is a technique for measuring and correcting the aberrations of an optical wavefront. It is particularly useful for atmospheric turbulence compensation in real time. The Wavefront Sensor is the heart of an Adaptive Optics System (AOS). It is used to measure the specific characteristics of an object wavefront and atmospheric turbulence effects to be compensated, as also to determine the spatial and temporal requirements of AOS [3]. Wavefront sensing is most commonly done using Shack–Hartmann principle due to its accuracy and ease of implementation. In this method, the wavefront is spatially sampled and focused by micro-lenslet array on a suitable sensor such as a CCD camera. This provides an array of focus spots, each corresponding to a sub-aperture sample data. The centroid of each sub-aperture sampled image is displaced from its mean position by an amount proportional to the turbulence-induced tilt of the wavefront across the corresponding lenslet aperture. From the measurements of a local wavefront tilt, the wavefront reconstructor estimates the wavefront deformation.

The sensitivity and accuracy of the system mainly depends on the calculation of exact centroid positions of the sub-aperture focus spots with minimum error. The first step in measurement using the Shack–Hartmann Wavefront Sensor is to determine the location of diffraction-limited focus spots. There are several methods used for finding centroid positions of the sub-aperture data in Shack–Hartmann wavefront sensor. The most common method is the statistical averaging [4]. Other methods such as center of mass, thresholding,

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maximum-likelihood estimators, finding moments using FFT and least-squares fit can also be used for this purpose [5]. Each of these methods has its limitations with regard to the accuracy or the processing speed for real-time requirement. A method based on Wavelets for multi-resolution wavefront reconstruction has been reported [6]. The wavelet methods can be of different forms. In this work, we present a detailed methodology of one particular type of wavelet-based method for the measurement of moments in Shack–Hartmann sensor. The method has been practically evaluated by using data from a Shack–Hartmann wavefront sensor and relative evaluation of the other methods is also presented. The present method is both accurate as well as fast compared to other approaches.

2. Centroid estimation techniques

To find the locations of all the focused spots, we first divide the image into an array of small rectangular sections of pixels that are centered on the apertures with sides as long as the aperture spacing. The rectangular sections of pixels are defined to be integration areas, because we limit the centroid integration for each aperture to these areas. Centroid estimation can be done in many ways. Here we discuss statistical averaging, finding moments using FFT and least-squares fit along with the proposed wavelet-based measurement of moments in Shack–Hartmann wavefront sensor.

2.1. Statistical averaging technique

The centroid position formulae using statistical averaging are expressed as [4]

$$X_c(K) = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} x(i,j)s(i,j)}{\sum_{i=1}^{I}\sum_{j=1}^{J} s(i,j)}, \tag{1}$$

$$Y_c(K) = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J} y(i,j)s(i,j)}{\sum_{i=1}^{I}\sum_{j=1}^{J} s(i,j)}, \tag{2}$$

where $x\,(i, j)$ and $y(i, j)$ is the co-ordinate position of the (i, j)th pixel in the kth sub-aperture, $s\,(i, j)$ is the input wavefront intensity at the pixel (i, j) on the square sub-apertures having $I \times J$ pixels. With the formulae, the centroid position of the input wavefront at the Kth sub-aperture could be calculated as $(X_c(K),\ Y_c(K))$. It gives some reasonable pixel accuracy of centroid estimation in real-time calculation.

2.2. First moment using FFT

The nth-order moment of a function is defined as

$$\begin{aligned} M_n &= \int_{-\infty}^{+\infty} t^n f(t)\,\mathrm{d}t, \\ M_1 &= \int_{-\infty}^{+\infty} t f(t)\,\mathrm{d}\,t \\ &= (-j)^{-1}\mathrm{d}/\mathrm{d}\omega \int_{-\infty}^{+\infty} f(t)\mathrm{e}^{-j\omega}\,\mathrm{d}\,t(\mathrm{at}\,I_{\omega=0}) \\ &= (-j)^{-1}\mathrm{d}f\hat{}(\omega)/\mathrm{d}\omega(at\,I_{\omega=0}). \end{aligned} \tag{3}$$

For the 2D spatial image data, this algorithm can be modified to estimate the centroid values with respect to Fourier plane information, and the peak positions can be identified with respect to x-and y-co-ordinates.

2.3. Least–squares method

For the 2D binary image with intensity function $f(x,y)$, the centroiding estimation is as follows:

Let

$$Z = f(x, y), \tag{4}$$

where Z is intensity level at spatial co-ordinate (x,y). By fitting the function Z in a polynomial using least-squares one can get the equation of the surface. For accuracy and convenience we have selected Zernike polynomial for fitting.

At the centroid co-ordinate, the gradient of the function $f(x,y)$ is zero.

$$\text{i.e.,}\quad \nabla \cdot f(x, y) = 0, \tag{5}$$

$\mathrm{d}f/\mathrm{d}x = 0$; $\mathrm{d}f/\mathrm{d}y = 0$; this gives x' and y', which represents the centroid co-ordinates. This process usually requires complex computing than other methods.

2.4. Wavelet techniques

Wavelet concept had been developed independently on several fronts. Different processing techniques have been developed for signal and image processing applications [7–9]. The wavelet transform has been successfully applied to non-stationary signals and images. Wavelet theory is based on analyzing signals to their components by using a set of basis functions. One important characteristic of the wavelet basis function is that they relate to each other by simple scaling and translation. A wavelet is a small wave with finite energy, which has its energy concentrated in time or space to give a tool for analysis of transient, non-stationary or time–varying phenomenon. The compactness and finite energy characteristic of wavelet function differentiate wavelet decompositions from other Fourier-like analysis in their applicability to different circumstances. In most wavelet transform applications, it is required that the original signal be synthesized from the wavelet coefficients. This condition is referred to as perfect reconstruction. In the perfect reconstruction, the set of wavelets for both analysis and synthesis compactly represent the signal and the wavelets should satisfy the orthogonality condition. By choosing two different sets of wavelets, one for analysis and other for synthesis, the two sets should satisfy the bio-orthogonality condition to achieve perfect reconstruction.

In this paper, we have computed the moments of a region represented by a scaling function on wavelet basis closely following the statistical formulation of Ma *et al.* [10]. The scaling function $\Phi(x)$ and wavelet $\Psi(x)$ are expressed as

$$\Phi(x) = \sum\nolimits_n h_n \sqrt{2}\Phi(2x-n), \tag{6}$$

$$\Psi(x) = \sum\nolimits_n g_n \sqrt{2}\Phi(2x-n). \tag{7}$$

The functions $\Phi^2(x)$ and $\Psi^2(x)$ are probability densities, which are non-negative with integral one. Further, the first moment (mean) of $\Psi^2(x)$ is the midpoint of the support of $\Psi(x)$, since Φ and Ψ are the scaling factors of the wavelet generated by an orthonormal analysis [10]. The scaling equation that recursively connect $\Phi(x)$ and $\Psi(x)$ with $\Phi(2x)$ give an algorithm for finding moments. The generalized moments $\xi_{k,t}$ for the wavelet $\Psi(x)$ is given by

$$\xi_{k,t} = \int x^k \Psi(x)\Psi(x-t)\,\mathrm{d}x \tag{8}$$

and the first moment is given by

$$\xi_{1,0} = \int x\Psi^2(x)\,\mathrm{d}x. \tag{9}$$

This holds good for the mean of any compactly supported wavelet associated with an orthogonal multi-resolution analysis.

3. Experimental procedure

The experimental arrangement consisted of a 16×16 microlens array of 417 μm pitch and 45 mm focal length, illuminated by a collimated laser beam. The image spots were focused on a Sony 2/3 in monochrome CCD camera and a PC-SG framegrabber from Matrix-Vision was used for image capture. With a pixel size of the CCD being $11.6\,\mu m \times 11.2\,\mu m$ and the pitch of the microlens array as 417 μm, the sub-aperture allocation on the CCD was 36×36 pixels. The centroiding accuracy and processing time were studied for the various above methods. From 36×36 pixel sub aperture, we have chosen 26×26 pixels for centroid estimation, just to have a smaller matrix. For centroid estimation, the intensity distribution in a single sub-aperture containing one spot from the SHWFS was considered (see the window in Fig. 1 showing the intensity distribution). The Centroid is calculated using Statistical averaging, least squares, FFT and the suggested Wavelet method. Now, we are not sure which of these methods has given the most accurate co-ordinates of the Centroid. To check this, we theoretically translate the entire intensity distribution matrix, let us say by 1 pixel horizontally to the right and again calculate the centroids using all the methods. Now the X-co-ordinates of the Centroid should have a value $+1$ above the earlier values and the Y-co-ordinate should remain the same, because we have theoretically shifted the intensity matrix. Now referring to Table 1, and looking at the first two rows of X- and Y-co-ordinates of centroids obtained using the four methods, before and after the theoretically introduced shift, we find that there is either a small or large variation in the expected values of X- and Y-co-ordinates, which is only due to the centroiding method used.

The reasons for theoretically induced shifts are as follows. The shift mentioned above would correspond to a pure tilt in the x-direction of the light beam in the actual experimental case. When we give the tilt experimentally, we are not sure that we have given a tilt exactly corresponding to 1 pixel shift. Also, we are not sure that we have introduced only a tilt and not any other aberration, which would contribute to Centroid shift. Even if this is achieved, there is no guarantee that the intensity distribution in the sub-aperture considered is exactly the same before and after the tilt, except one pixel shift horizontally. If there is a change in the intensity distribution, definitely these methods would give a different Centroid value, because all these methods are based on statistical formulation. The idea

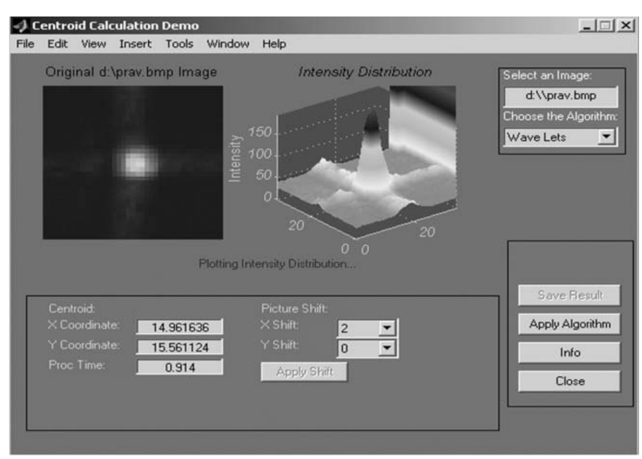

Fig. 1. The Centroid calculation window. A single spot from SHWFS was taken in a sub-aperture array of 26×26 pixels and centroid determination implementation using various algorithms in Mat-Lab environment.

Table 1

Applied shift	Statistiscal averaging method		FFT method		Least-squares method		Wavelet method	
	Calculated Centroid position (pixels)		Calculated Centroid position (pixels)		Calculated Centroid position (pixels)		Calculated Centroid position (pixels)	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
Initial condition	13.510024	15.295842	12.958865	15.567213	12.961127	15.569087	12.961038	15.561124
+1 HS	13.839728	15.308168	13.959436	15.567302	13.961127	15.549087	13.961036	15.551124
+1 VS	13.849728	15.912184	12.958844	16.566988	12.961129	16.561124	12.991039	16.531161
+2 HS	14.386728	15.312168	14.949456	15.567302	14.961127	15.559021	14.961636	15.561124
+2 VS	13.386728	16.848168	12.959456	17.567302	13.010234	17.549087	12.931236	17.551124
Processing time (s)	0.003		0.233		2.810		0.914	

HS – Horizontal shift; VS – Vertical shift.
Results obtained from various algorithms (Statistical Averaging, FFT, least-squares and Wavelet) for centroid estimation for the spot of in a sub-aperture of 26×26 pixels.

of the work was to check and compare the accuracy of the centroiding methods and that too without any contributions due to experimentally induced errors. Giving experimental shift might not give the accuracy of the method/algorithm used.

The centroid determination was carried out using all the above-mentioned methods using MatLab programming. The procedure mentioned above was repeated for 2 pixel horizontal shift, 1 and 2 pixel vertical shifts as well. Fig. 1 shows the Matlab window depicting the spot image, intensity plot, centroid values as well as the choice of algorithm. The results obtained and comparative analyses are summarized in Table 1. The centroid estimation algorithms were implemented in a Pentium III system with 733 MHz processor.

Further, we have given the results of centroiding for one sub-aperture only and for a given set of 26×26 pixels, because we were also comparing the speed of each of these methods apart from accuracy. In a Shack–Hartmann wavefront sensor, the number of lenslets and hence the number of sub-apertures may vary from case to case. For instance, ours is 16×16 lenslet which corresponds to 256 sub-apertures. There are microlens arrays with much higher number of lenslets. Also, several lenslet arrays have different pitch (distance between two microlenses), which would mean varying number of pixels in the sub-aperture, depending on the pixel size of the CCD. So we decided to present the results for a single given sub-aperture, though we have checked these results for the intensity distributions in other sub-apertures.

As detailed above, a theoretically induced lateral shift of the entire intensity matrix, let us say 1 pixel horizontally, should change the *X*-co-ordinate by 1

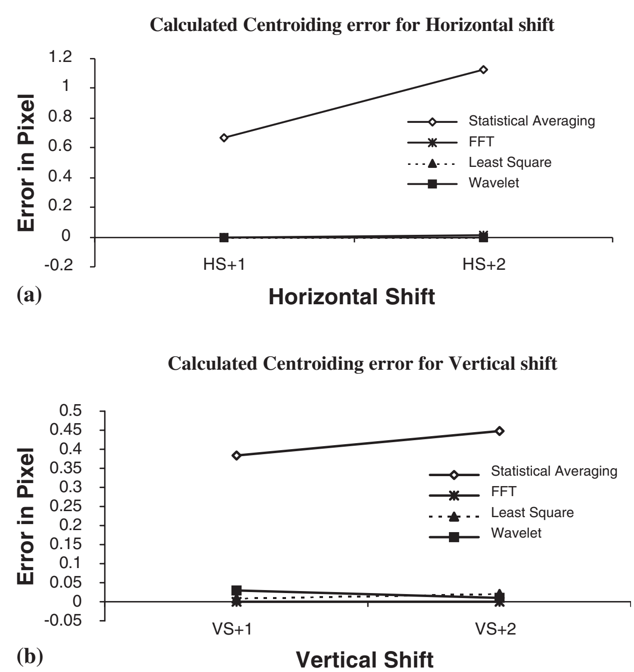

Fig. 2. Centroiding error for (a) horizontal shift and (b) vertical shift of the spot intensity.

and the *Y*-co-ordinate should remain unaltered from the reference Centroid values. But, as can be seen from Table 1, there is a small variation in the Centroid value of the *X*-co-ordinate from the expected value and *Y*-co-ordinate changes at the sub-pixel level, though in principle the *Y*-value should remain the same. This variation in the expected value of *X*-co-ordinate and the change of Y-co-ordinate of the Centroid due to a theoretically induced X-shift, and vice versa, arises only from the algorithm used for Centroid calculation. This proves the accuracy of the various algorithms. This shift is very large with the statistical averaging method meaning that it is the least accurate, and the least-squares method seems to be the most accurate. The centroiding errors obtained by using various algorithms for the x and y co-ordinates are plotted in Figs. 2 (a) and (b), respectively. From the processing time considerations, however, the statistical averaging method is the fastest, while the least-squares method is the most time consuming due to intensive computations.

4. Discussion

For real-time implementation of wavefront sensing and reconstruction, the accuracy as well as the speed of centroid estimation are both crucial. The error in the Centroid estimation will lead to undesired results and performance. From our observations both FFT and Wavelet methods are relatively quite accurate as well as reasonably fast. However, the wavelet method is better suited for Kolmogorov processes and Wavefront reconstruction with scale factors for AO applications [6]. The least squares method provides the best accuracy at the cost of time. Statistical averaging method provides faster processing with higher pixel error. The sub-pixel level of accuracy and processing time measured for the all the algorithms prove that high sub-pixel resolution can be achieved for real-time estimation of phase measurement in SHWFS using both FFT and Wavelet algorithm.

In statistical averaging, pixel level accuracy is not satisfied for our demand and also the noise factor of this estimator is high. Centroiding accuracy using this method can be somewhat improved by data-thresholding, where the noises are considered to be much lesser than the signal and the intensities below a particular value (threshold) are made zero. For higher thresholds or in cases where there is some signal information below the cut-off threshold, then that part of the signal is lost due to thresholding, especially when the wavefront aberration in sub-aperture is significant. This would introduce large errors in centroiding. What we need is an estimator, which does not discard any information in the sub-aperture and make the best use of all the available data.

Least–squares estimator for centroiding is computationally expensive. It generally needs lot of time for processing the given data, and it has inherent computational complexity than other methods for estimation of centroid. Fourier technique provides accuracy of the moment value with respect to spot image, but sometimes may filter out the high-frequency information in the sub-aperture region containing fine details of the image.

The power of the wavelet technique is the finer resolution of sub-pixel level accuracy of the centroid moments. Unlike Fourier-like transforms, which are generally based on a particular set of basis functions, there exist many different wavelet bases with different characteristics. We considered wavelets with certain properties needed for wavefront sensing and reconstruction application. The factors consider were compactly supported orthogonality, asymmetry, exact reconstruction, continuous transform and fast algorithm. This type of wavelet analysis is capable of revealing aspects of data that other analysis techniques miss, like discontinuities in higher derivatives and self-symmetry. Our proposed wavelet technique allows us for different views of data than those presented by traditional techniques. Also this method can often compress or denoise a signal without appreciable degradation. The inherent advantage of this wavelet analysis is that it has data thresholding in multi-resolution form for improved centroid measurement, and limits noise factors. Success of a given wavelet basis in a particular application does not necessarily mean that this set is efficient for other applications. Hence we have to choose only that wavelet function, which would be applicable for our centroiding purpose and we found such a function in Ref.[10]. Especially for multi-resolution wavefront sensing, wavelet methods outperform other wavefront reconstruction techniques, since there is a close statistical relationship between wavelet and Kolmogorov process. By multi-resolution sensing, we mean using sub-band coding wavelet decomposition and reconstruction for non-stationary signal analysis. The 2D wavelet decomposition is extended to non-standard form to get wavelet coefficient slopes from lowest resolution to highest resolution [6]. Also the wavelet decomposition compresses the wavefront reconstruction matrix, to improve the computational efficiency of the wavefront estimation.

The performance of centroid estimation also depends on read noise in the detector, photon noise, background subtraction, etc. In the present work, we have taken the intensity distribution in a particular sub-aperture, in a given experimental setup and carried out centroid estimation using various methods looking for their accuracy as well as speed. Hence, the above noise factors will be the same with the data under consideration for all the various Centroid estimation methods. Because the data is the same, the relative accuracy and speed should be dependent only on the centroiding method/algorithm. Even if the noise factors contribute to centroiding accuracy, it is all the more better that, from this study we can find out which method is least sensitive to noise factors.

5. Conclusion

A wavelet-based method of moment calculation with a set of basis functions has been explored for centroid estimation with the data from a Shack–Hartmann Wavefront Sensor. The proposed method has been compared with other centroiding algorithms such as statistical averaging, FFT and least-squares method. The comparative analysis shows that wavelet method has a high accuracy and processing speed and better suited for wavefront reconstruction applications. Further, the wavelet method presented here has a variable accuracy and resolution, is less sensitive to noise and could be optimized for a particular application under consideration.

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BOOK REVIEW

B. Javidi (Ed.), Image Recognition and Classification: Algorithms, Systems, and Applications, Marcel Dekker, New York, ISBN 0-8247-0783-4, 2002 (XII/506pp., numerous figs., US$175.00 Hardcover).

This book presents important recent advances in sensors, image processing algorithms, and systems for image recognition and classification with diverse applications in military, aerospace, security, image tracking, radar, biomedical, and intelligent transportation. The book includes contributions by some of the leading researchers in the field to present an overview of advances in image recognition and classification over the past decade. It provides both theoretical and practical information on advances in the field. The book illustrates some of the state-of-the-art approaches to the field of image recognition using image processing, nonlinear image filtering, statistical theory, Bayesian detection theory, neural networks, and 3D imaging. Currently there is now single winning technique that can solve all classes of recognition and classification problems. In most cases, the solutions appear to be application-dependant and may combine a number of these approaches to acquire the desired results. The book provides examples, tests, and experiments on real world applications to clarify theoretical concepts. A bibliography for each topic is included to aid the reader. It is a practical book, in which the systems and algorithms have commercial applications and can be implemented with commercially available computers, sensors, and processors. The book will be of great help for electrical or computer engineers with interest in signal/image processing, optical engineers, computer scientists, image scientist, biomedical engineers, applied physicists and others.

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