Generative Adversarial Nets

2019年11月22日 13:12

$$V(G,D) = E_{x\sim p_{data}}[\log D(x)] + E_{z\sim p_z}[\log(1-(G(z)))]$$

$$= \int_x p_{data}(x)\log D(x)dx + \int_z p_z(z)\,(\log(1-D(G(z)))\,dz$$

$$= \int_x p_{data}(x)\log D(x) + \int_x p_g(x)\,(\log(1-(D(x)))\,dx$$

$$= \int_x p_{data}(x)\log D(x) + p_g(x)\log(1-D(x))\quad dx$$

$$\Leftrightarrow \max\ p_{data}(x)\log D(x) + p_g(x)\log(1-D(x))$$

$$f(x) = a\log x + b\log(1-x) \qquad x\in[0,1]$$

$$f'(x) = \frac{a}{x} - \frac{b}{1-x} = 0 \qquad \frac{a}{b} = \frac{x}{1-x}$$

$$a - \frac{bx}{1-x} = 0 \qquad \frac{a}{b} = \frac{1}{\frac{1}{x}-1} \qquad \frac{1}{x} - 1 = \frac{b}{a}$$

$$x = \frac{a}{a+b}$$

$$f''(x) = -\frac{a}{x^2} - \frac{b}{(1-x)^2} < 0$$

$\therefore\ x = \frac{a}{b+a}$ 为极大值.

即 $D(x) = \frac{p_{data}}{p_{data}+p_g} = D^*_G$

代入

$$C(G) = V(G, D^*_G) = \int_x p_{data}\log\frac{p_{data}}{p_{data}+p_g} + p_g\log\frac{p_g}{p_{data}+p_g}$$

$$= \int(\log 2 - \log 2)\,p_{data} + p_{data}\log\frac{p_{data}}{p_{data}+p_g} + (\log 2 - \log 2)\,p_g + p_g\log\frac{p_g}{p_{data}+p_g}$$

$$= -\log 2\int p_{data} + p_g\ dx + \int p_{data}\left(\log 2 + \log\frac{p_{data}}{p_{data}+p_g}\right) + p_g\left(\log 2 + \log\frac{p_g}{p_{data}+p_g}\right)$$

$$= -\log 4 + \int_x p_{data}\log\left(\frac{p_{data}}{\frac{p_{data}+p_g}{2}}\right)dx + \int_x p_g\log\left(\frac{p_g}{\frac{p_{data}+p_g}{2}}\right)dx$$

$$= -\log 4 + KL\left(p_{data}\,\Big\|\,\frac{p_{data}+p_g}{2}\right) + KL\left(p_g\,\Big\|\,\frac{p_{data}+p_g}{2}\right).$$

$$\left(KL = E_{x\sim p}\left[\log\frac{P(x)}{Q(x)}\right],\ JSD = \frac{1}{2}KL(P\|M) + \frac{1}{2}KL(Q\|M),\ M = \frac{P+Q}{2}\right)$$

$$= -\log 4 + 2JSD(p_{data}\,\|\,p_g)$$

JSD 非负 故 最小值为 $-\log 4$，此时 $p_g = p_{data}$