Final Project - AERO 625

Platooning; An illustration.

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Outline I

Motivation

- Urban & suburban freeway congestion has been increasing rapidly in recent years, particularly in the largest and fastest growing metropolitan areas.
- The most authoritative projections at national level indicate that this trend will continue and become even more acute in years to come.
- The increased demand for travel cannot be met by simply constructing more lanes or limit the demand for travel.
- A more attractive alternative being, efficient usage of existing road and freeway infrastructure in order to avoid more construction.

Motivation

- A variety of methods have been tried to improve the utilization of our roads;
    - Traditional methods like adding separate turn lanes and arrows, adjusting signal timings, making signals traffic-responsive, installing signals on freeway entrance; and
    - Advanced electronic technologies have been employed to provide drivers with real-time information about congestion, navigation and route guidance.
- With these measures the utilization of the existing road facilities has improved by 15-20%, which is insufficient to meet the anticipated needs.
- To facilitate substantial increase in throughput, it is necessary to automate the control of vehicles.

Motivation

- The benefits that could be derived from vehicle automation are potentially extremely valuable. They include,
  - Providing a cost-effective roadway capacity increase without the addition of more lane-miles of road;
  - Improving safety by eliminating accidents caused by the frailties of human drivers;
  - Improving trip speed and reliability by permitting higher safe cruising speeds than at present; and
  - Increasing the convenience of road travel by eliminating the tedium.

Figure : Platooning.

Problem Description

Objective

To improve throughput on congested highways by allowing groups of vehicles (10 or 20) to travel together in tightly spaced platoons (1m intervals) at high speeds.

Approaches

Two fundamentally different approaches to platooning have been suggested:

- *Point - Following control*; Each vehicle is assigned a particular moving slot in the highway and maintains that position.
- *Vehicle - Following control*[1]; Each vehicle in the platoon regulates its position relative to the vehicle in front of it, based on information about the lead vehicle motion and locally measured variables.

[1] Approach adopted in this illustration.

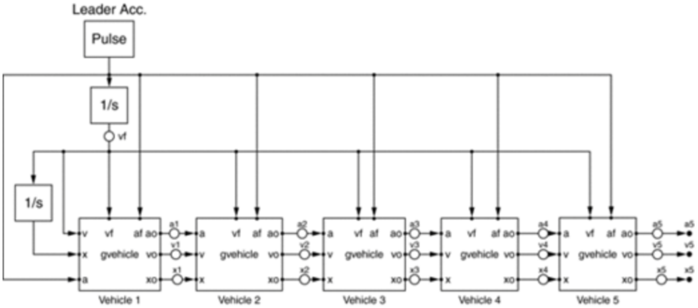


Figure : Model of a platoon with five vehicles.

Outline I

Equations of motion

- Each vehicle is represented by a third-order nonlinear model.
- This arises from the representation of each vehicle as a mass plus propulsion system (which has a first-order lag between the commanded and actual thrust).

$$\dot{a}_0 = -\frac{1}{\tau} a_0 + \frac{1}{\tau} u, ^2 \tag{1}$$

$$\dot{v}_0 = a_0, and \tag{2}$$

$$\dot{x}_0 = v_0. \tag{3}$$

- Here, the nonlinearities (saturation and quantization) and noise are ignored.

[2] For an electric power-train, the propulsion lag time constant τ is 0.1s, while for an internal combustion engine it is 0.5s.

- Each vehicle in the platoon is supplied with the following input data:
  - Acceleration, velocity, and position $(a, v, and\ d)$ of the preceding vehicle; and
  - acceleration and velocity of the lead vehicle (a_f, v_f).
- Each vehicle then generates as output its own acceleration, velocity, and position $(a_0, v_0, and\ d_0)$.

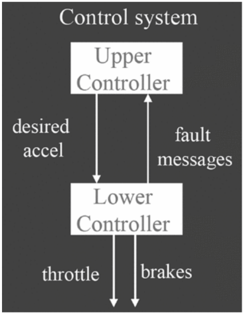


Figure : Control System

Assumptions

Some of the simplifying assumptions made for system analysis and subsequent controller design are as follows:

- The platoon has two vehicles - the leading and preceding vehicle.
- The vehicles are equipped with an electric power train.
- The propulsion time lag for an electric power train is 0.1s.
- The road grade hasnt been taken into account, instead the drag force alone accounts for the possible constant disturbance.

Some further improvements that could be taken up to exact it with the real scenario :

- The acceleration and jerk limits is to be taken into account to serve as limits for the actuator.
- The road profile is to be taken into consideration.
- The distance of the follower is measured with respect to the leader, here a random source is assumed to exist for reference. This needs to be corrected.

State-Space Model - Discrete-time

State Equations:

$$X_{k+1} = AX_k + BU_k; \tag{4}$$

Output Equations:

$$Y_k = CX_k; \tag{5}$$

Where,

$$A = \begin{bmatrix} 1 & 0.0095 & 0 & 0 & 0 \\ 0 & 0.9048 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.01 & 0 \\ 0 & 0 & 0 & 1 & 0.0095 \\ 0 & 0 & 0 & 0 & 0.9048 \end{bmatrix}, \ B = \begin{bmatrix} 0.0005 & 0 \\ 0.0952 & 0 \\ 0 & 0 \\ 0 & 0.0005 \\ 0 & 0.0952 \end{bmatrix}, \ \&$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0. \end{bmatrix}.$$

While, $x = \begin{bmatrix} x_{1k} \\ x_{2k} \\ x_{3k} \\ x_{4k} \\ x_{5k} \end{bmatrix} = \begin{bmatrix} v_1 \\ a_1 \\ d_2 \\ v_2 \\ a_2 \end{bmatrix}$, $u = \begin{bmatrix} u_{1k} \\ u_{2k} \end{bmatrix}$, & $y = \begin{bmatrix} y_{1k} \\ y_{2k} \end{bmatrix}$.

Outline I

Open-loop Characteristics

Eigen values: $\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.9048 \\ 1 \\ 1 \\ 0.9048 \end{bmatrix}.$

It can be observed that the open-loop system is unstable with poles in the open Right-Half Plane (RHP).

Eigen vectors: v [3] $= \begin{bmatrix} 1 & -0.0995 & 0 & 0 & 0 \\ 0 & 0.9950 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0.099 \\ 0 & 0 & 0 & 0 & -0.0995 \\ 0 & 0 & 0 & 0 & 0.9950 \end{bmatrix}.$

[3] Columns v_i represent eigen vectors corresponding to eigen values λ_i, $i = 1, 2, \ldots, 6$.

Open-loop Characteristics

Pole	ζ	$\omega_n(rad/s)$	Time constant (s)
1	-1	1	-1
0.905	-1	0.905	-1.11
1	-1	1	-1
1	-1	1	-1
0.905	-1	0.905	-1.11

Table : Damping ratio (ζ) and Natural frequency (ω_n) of each pole.

Controllability

The controllability matrix,

$$[B, AB, A^2B, \ldots, A^4B] \tag{6}$$

$$\begin{matrix}
0 & 0 & 10 & 0 & -100 & 0 & 1000 & 0 & -10000 & 0 \\
10 & 0 & -100 & 0 & 1000 & 0 & -10000 & 0 & 100000 & 0 \\
0 & 0 & 0 & 0 & 0 & 10 & 0 & -100 & 0 & 1000 \\
0 & 0 & 0 & 10 & 0 & -100 & 0 & 1000 & 0 & -10000 \\
0 & 10 & 0 & -100 & 0 & 1000 & 0 & -10000 & 0 & 100000
\end{matrix}$$

is full rank $= 5$. Hence, the system (A,B) is completely controllable.

Observability

The observability matrix,

$$[C, CA, CA^2, \ldots, CA^4] \tag{7}$$

$$\begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 \\ 0 & -1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 \end{matrix}$$

is full rank = 5. Hence, the system (A,C) is completely observable.

Outline I

Sampled-Data Regulator (SDR)

Changing co-ordinates, we have

$$\tilde{x}_{k+1} = A\tilde{x}_k + B\tilde{u}_k; \tag{8}$$

Where, $\tilde{x}_{k+1} = x_{k+1} - x^*$, $\tilde{x}_k = x_k - x^*$, & $\tilde{u}_k = u_k - u^*$. Here, x^* and u^* are steady-state values.

Outputs: $y_1 = \tilde{x}_1$, and $y_2 = \tilde{x}_3$.

Control Law:

$$u = K * (x_{ref} - \tilde{x}_k); \tag{9}$$

Where K $= \begin{bmatrix} 6.1650 & 0.5044 & 0 & 0 & 0 \\ 0 & 0 & 4.4054 & 3.3838 & 0.2985 \end{bmatrix}$, is obtained through LQR technique.

SDR Design

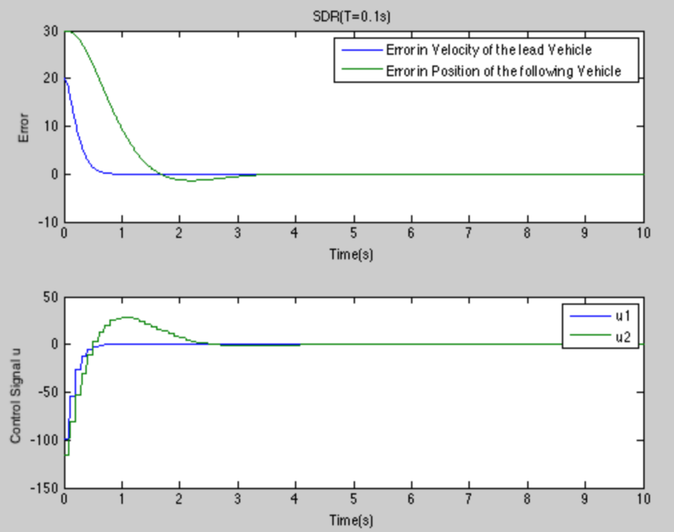
Continuous-time matrices,

$$Q = \begin{bmatrix} 2000 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2000 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}; R = \begin{bmatrix} 50 & 0 \\ 0 & 100 \end{bmatrix};$$

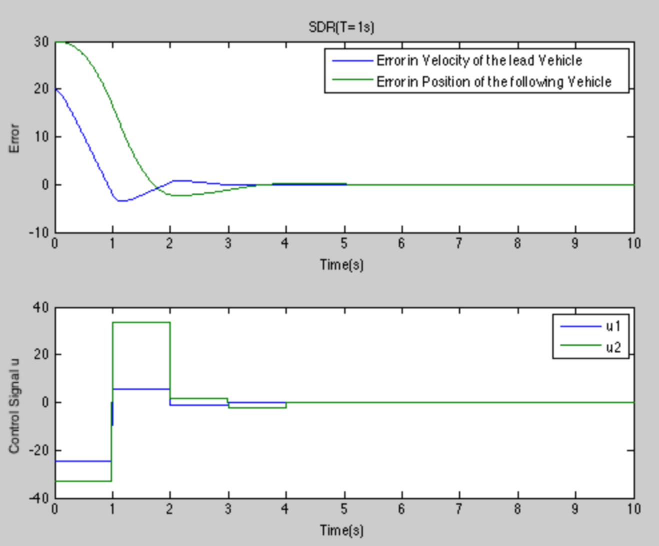
Discrete-time matrices,

$$\hat{Q} = \begin{bmatrix} 20 & 0.0967 & 0 & 0 & 0 \\ 0.0967 & 0.0097 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0.1 & 0.0003 \\ 0 & 0 & 0.100 & 0.0107 & 0.0001 \\ 0 & 0 & 0.0003 & 0.0001 & 0.0091 \end{bmatrix}; \text{ and } \hat{R} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1. \end{bmatrix}$$

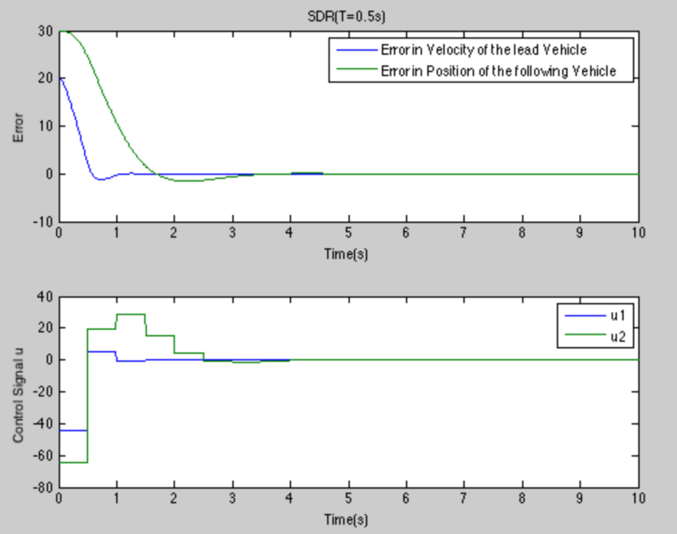
Time-domain response



Time-domain response



Time-domain response



Frequency-domain response

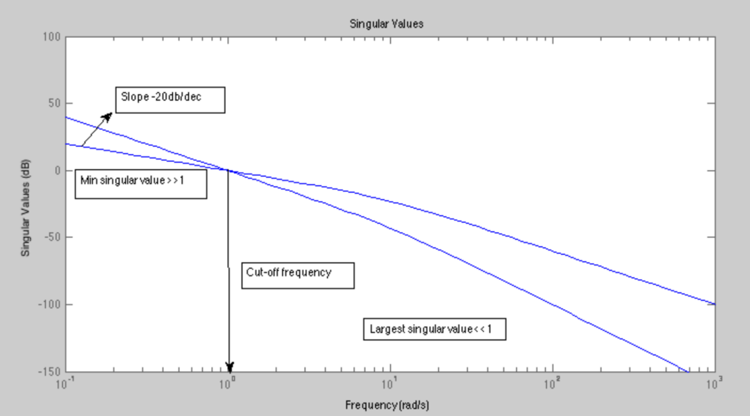


Figure : Sigma Bode Plot for SDR

PI-SDR

Consider (8). Also, let $y_{Ik} = [y_{1Ik}, y_{2Ik}]^T$
Augmenting this to the state-vector, we have,

$$\tilde{x}_{Ik} = \begin{bmatrix} \tilde{x}_{1k} \\ \tilde{x}_{2k} \\ \tilde{x}_{3k} \\ \tilde{x}_{4k} \\ \tilde{x}_{5k} \\ y_{1Ik} \\ y_{2Ik} \end{bmatrix}, \Phi = \begin{bmatrix} 1 & 0.0095 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9048 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0.0095 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9048 & 0 & 0 \\ 0.0100 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0.01 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Control Law: Same as (9). While,

$$K = \begin{bmatrix} 6.5625 & 0.5305 & 0 & 0 & 0 & 1.6861 & 0 \\ 0 & 0 & 5.6979 & 3.8543 & 0.3342 & 0 & 1.7031 \end{bmatrix}$$

PI-SDR design

Continuous-time matrices :

2000	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	2000	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	150	0
0	0	0	0	0	0	300

Figure : Q

While Discrete-time matrices,

20.0000	0.0967	0	0	0	0.0075	0
0.0967	0.0097	0	0	0	0.0000	0
0	0	20.0001	0.1000	0.0003	0	0.0150
0	0	0.1000	0.0107	0.0001	0	0.0001
0	0	0.0003	0.0001	0.0091	0	0.0000
0.0075	0.0000	0	0	0	1.5000	0
0	0	0.0150	0.0001	0.0000	0	3.0000

Figure : $\hat{Q}$

R & $\hat{R} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}$.

Time-domain Response

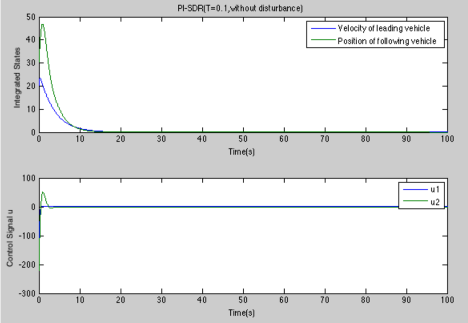


Figure : Response with T=0.1s without disturbance

Time-domain Response

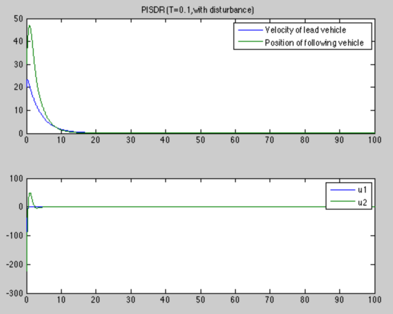


Figure : Response with T=0.1s with disturbance

Frequency-domain Response

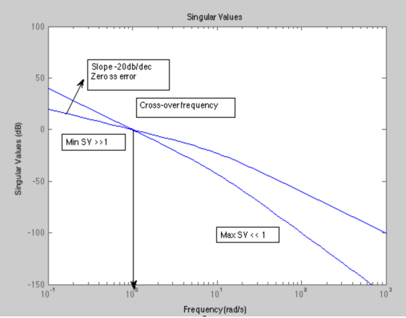


Figure : Sigma Bode

NZSP

Here the outputs y_1& y_2 are commanded to a non-zero value.
Control Law:

$$u = (\pi_{22} + K * \pi_{12}) * ym - K * x_k; \tag{10}$$

Where, K= $\begin{bmatrix} 4.3740 & 0.4374 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.9929 & 1.5088 & 0.1414 & \end{bmatrix}$;

$\pi_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, and $\pi 22 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Time-response characteristics

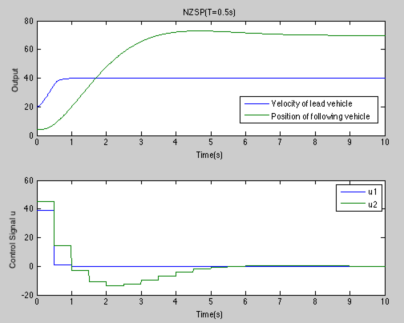


Figure : Response with T=0.5s

Time-response characteristics

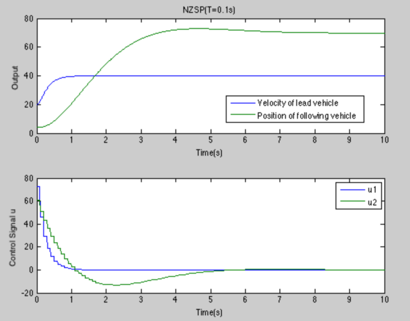


Figure : Response with T=0.1s

Time-response characteristics

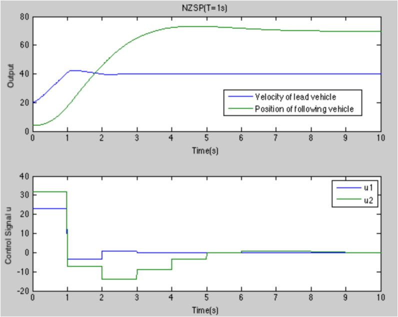


Figure : Response with T=1s

Time-response characteristics

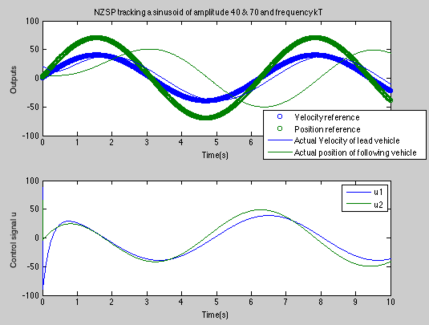


Figure : Response to sine input

PI-NZSP

Here the outputs y_1& y_2 are commanded to a non-zero value. The state vector is augmented with integrated states as in PI-SDR. **Control Law:**

$$u = (\pi_{22} + [K_1, K_2, K_3, K_4, K_5]\pi_{12})y_m - Kx_k \tag{11}$$

Where, K$= \begin{bmatrix} 9.1338 & 0.6894 & 0 & 0 & 13.6545 & 0 & \\ 0 & 0 & 13.9470 & 6.2414 & 0.5020 & 0 & 14.06 \end{bmatrix}$;

$\pi_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, and $\pi 22 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Time-response characteristics

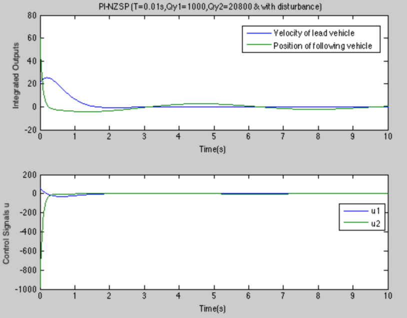


Figure : Response with T=0.01s with disturbance

Time-response characteristics

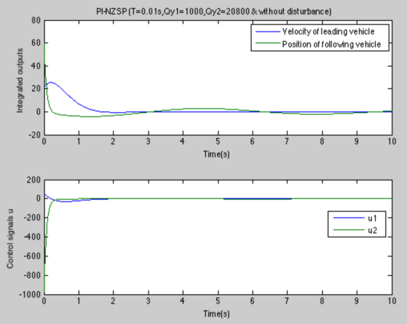


Figure : Response with T=0.01s with no disturabance

Time-response characteristics

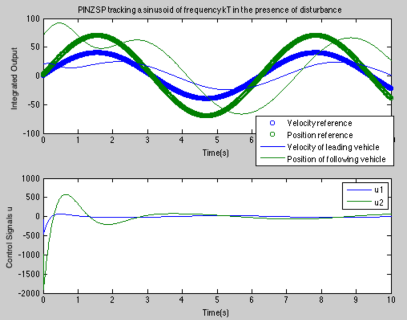


Figure : Response with sine input

Time-response characteristics

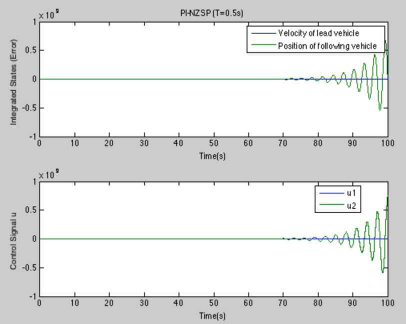


Figure : Response with T=0.5s

PIF-NZSP

The state vector is augmented with control inputs as in PI-SDR.

Control Law: same as in (10).

Where, K=

$$\begin{bmatrix} 4.4050 & 0.4404 & 0 & 0 & 0 & 3.0007 & 0 \\ 0 & 0 & 0.9892 & 2.1723 & 0.2111 & 0 & 2.1693 \end{bmatrix};$$

$$\pi_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \text{ and } \pi 22 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Time-response characteristics

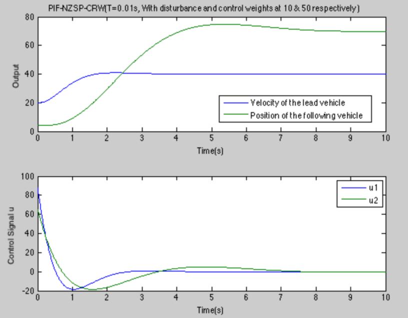


Figure : Response with T=0.01s with disturbance

Time-response characteristics

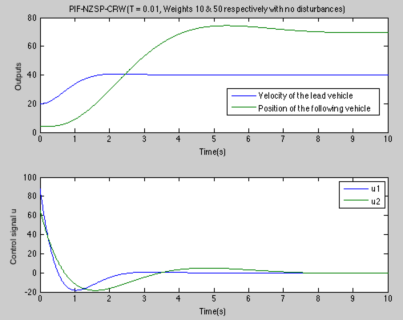


Figure : Response with T=0.01s with no disturabance

Time-response characteristics

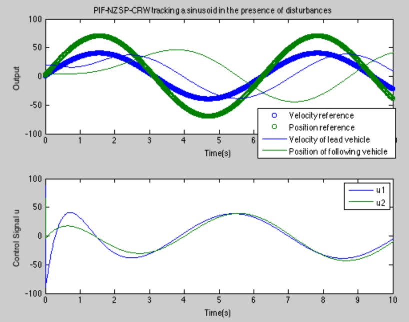


Figure : Response with sine input