Cooperative Guidance for Multi-missile Salvo Attack[1]

August 14, 2016

[1] Shiyu, Z., & Rui, Z. (2008). Cooperative guidance for multimissile salvo attack. Chinese Journal of Aeronautics, 21(6), 533-539.

Outline

Background

- Strategic and tactical targets such as airports, warfare ships, . . . etc are equipped with missile defense systems.
- Such systems serve as a barrier for missiles.
- To counter such systems, there are two alternatives.
  - *Single Missile Attack* - Maneuver in the terminal guidance phase to enhance survivability of the missile.
  - *Multiple Missiles Attack* - Many missiles are required to intercept the target simultaneously. Even if several missiles are intercepted, the remaining few can accomplish the mission.

Salvo attack - Many-to-one engagement scenario.

Background

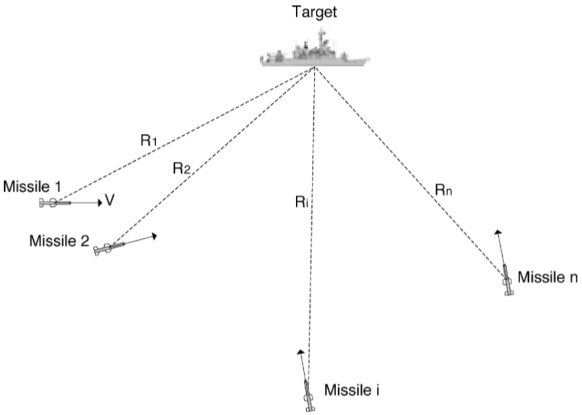

Figure : An Example Of A Salvo Attack[2]

[2] Jeon, I.S., Lee, J.I., &Tahk, M.J.(2006). Impact-time-control guidance for anti-ship missiles. Control Systems Technology, IEEE Transactions on, 14(2), 260-266.

Outline

Prevailing Approach[3]

- An *Impact-Time-Control Guidance*(ITCG) which controls the impact time of guidance of multiple missiles.
  - Requires the impact time to be manually pre-programmed.
  - No communication among missiles during guidance phase.
  - Salvo attack based on ITCG is simply *Open-loop*
  - It is a *Static Guidance Strategy*.
  - Not a *Genuine Multimissile Co-operative Attack*.

[3] Jeon, I.S., Lee, J.I., &Tahk, M.J.(2006). Impact-time-control guidance for anti-ship missiles. Control Systems Technology, IEEE Transactions on, 14(2), 260-266.

Outline

Proposed Method - Objectives:

1. Adopt a local guidance law, **Impact-Time Control Guidance**(ITCG) for each missile.
2. Achieve **rendezvous** of multiple missiles using co-ordination algorithms - centralized or distributed.
   - Use co-ordination variables to achieve information exchange.

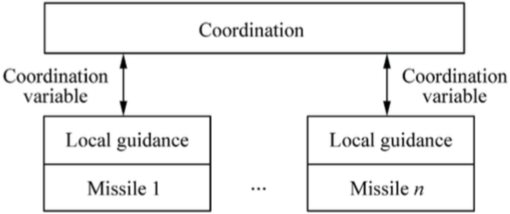

Figure : Two-level Heirarchical Co-operative Guidance Architecture.[4]

[4] Shiyu, Z., & Rui, Z. (2008). Cooperative guidance for multimissile salvo attack. Chinese Journal of Aeronautics, 21(6), 533-539.

Outline

Preliminaries - ITCG

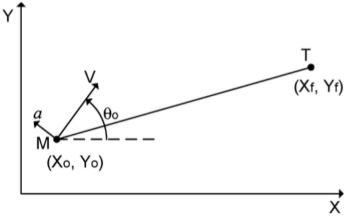

Figure : Engagement Geometry.[5]

- Planar Homing Guidance Problem
- Target(T) is **stationary**.
- Missile speed(V) is a **constant**.
- (X_0, Y_0) - Initial position of Missile(M)
- (X_f, Y_f) - Target position
- a - Acceleration command normal to V
- θ - Angle between the reference and the missile velocity vector(V)

[5] Jeon, I.S., Lee, J.I., &Tahk, M.J.(2006). Impact-time-control guidance for anti-ship missiles. Control Systems Technology, IEEE Transactions on, 14(2), 260-266.

ITCG

Governing Equations:

$$\dot{X} = V\ cos\theta, X(0) = X_0, X(T_f) = X_f, \tag{1}$$

$$\dot{Y} = V\ sin\theta, Y(0) = Y_0, Y(T_f) = Y_f, \tag{2}$$

$$\dot{\theta} = \frac{a}{V}, \theta(0) = \theta_0 \tag{3}$$

Where,

- $X, Y \rightarrow$ Missile Position(m).
- $\theta \rightarrow$ Heading Angle($rads$).
- $T_f \rightarrow$ Terminal Time(s).
- $a \rightarrow$ Lateral Acceleration Command.

ITCG - Optimal Control Problem Formulation

Cost Function:

$$J = \frac{1}{2}\int_{t_0}^{T_f} a^2 dt \tag{4}$$

Boundary Conditions:

$$X(0) = X_0, X(T_f) = X_f, \tag{5}$$
$$Y(0) = Y_0, Y(T_f) = Y_f, \tag{6}$$
$$\theta(0) = \theta_0 \tag{7}$$

Path Constraints:

$$\int_{t_0}^{T_f} \sqrt{1+\theta^2} dt = VT_d,\ T_d \text{ is the designated impact-time.} \tag{8}$$

ITCG - Closed-Form Solution

ITCG Law:

$$a = a_p - \frac{60V^5}{a_p R_{go}^3}(T_d - \hat{T}_{go}) \quad (9)$$

Where,

- $a_p = NV\dot{\lambda}$, PN guidance law with $N = 3$.
- $\dot{\lambda}$, is LOS rate.
- $R_{go} = \sqrt{(X_t - X(t))^2 + (Y_t - Y(t))^2}$, is the current range between missile and target.
- $\hat{T}_{go} = \frac{(1+0.1(\theta-\lambda)^2)R_{go}}{V}$, is the estimated time-to-go.

Simulation

Scenario For Salvo Attack:

Missile	**Position(m)**	**Heading Angle($^\circ$)**	**Speed(m/s)**
1	(-6894,-5785)	70	280
2	(-3249,-8927)	95	320
3	(0,-8693)	135	260

Pre-programmed impact-time, $T_d = 38s$.

Simulation Results

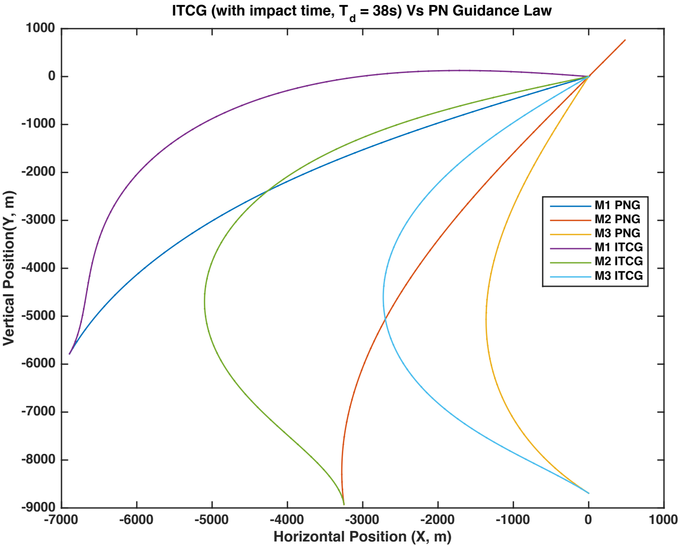

Figure : Missile Trajectories Using PN & ITCG

Simulation Results

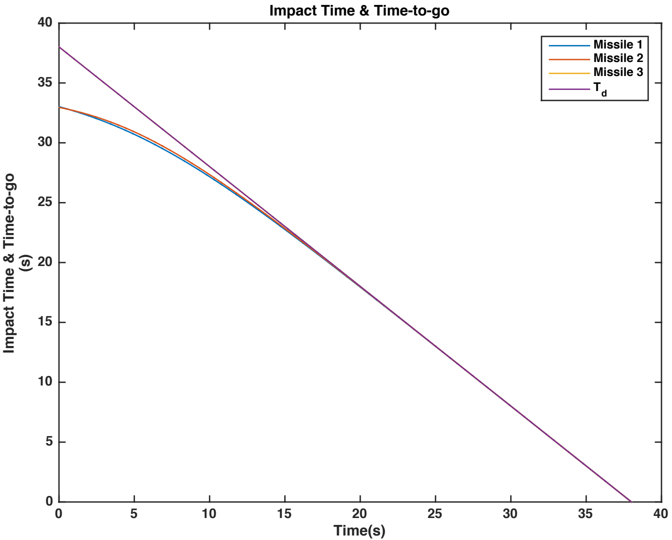

Figure : Histories Of Time-To-Go & Designated Impact Time

Outline

Centralized Co-Ordination Algorithm

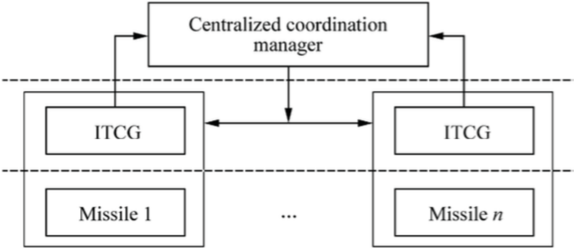

Figure : Co-Operative Guidance Architecture With Centralized Co-Ordination[4]

- CCM *collects* information from individual missiles.
- *Computes* the rendezvous time using the coordination algorithm.
- *Broadcasts* the rendezvous time to all missiles.

Centralized Co-Ordination Algorithm

Consider n missiles participating in a salvo attack. From (9), the control command of each missile is:

$$u_i = a_{pi} - \alpha_i(T_d - \hat{T}_{goi}), \; i = 1, 2, \ldots, n \tag{10}$$

$$\text{Where}, \alpha_i = \frac{60V_i^5}{a_{pi}R_{goi}^3} \tag{11}$$

The cost function is,

$$\bar{J} = \sum_{i=1}^{n} u_i^2 \tag{12}$$

Then, the optimal designated rendezvous time for all missiles is,

$$T_d^* = arg \min_{T_d} \bar{J} \tag{13}$$

Centralized Co-Ordination Algorithm

Using (10)-(13), T_d^* is obtained as below:

$$T_d^* = \frac{\sum_{i=1}^{n} \alpha_i^2 \hat{T}_{goi} + \sum_{i=1}^{n} \alpha_i a_{pi}}{\sum_{i=1}^{n} \alpha_i^2} \tag{14}$$

Let $\delta = \frac{\sum_{i=1}^{n} \alpha_i a_{pi}}{\sum_{i=1}^{n} \alpha_i^2}$, then (14) becomes

$$T_d^* = \frac{\sum_{i=1}^{n} \alpha_i^2 \hat{T}_{goi}}{\sum_{i=1}^{n} \alpha_i^2} + \delta \tag{15}$$

From (11), $\alpha_i^2 \to \infty$ as $R_{go} \to 0$ due to which δ is very small in comparison with the other part of (15). Therefore ignoring δ,

$$T_d^+ = \sum_{i=1}^{n} \frac{w_i \hat{T}_{goi}}{w_i} \tag{16}$$

is a sub-optimal designated impact time, with $w_i = \left[\frac{V_i^5}{a_{pi} R_{goi}^3}\right]^2$.

Simulation Results

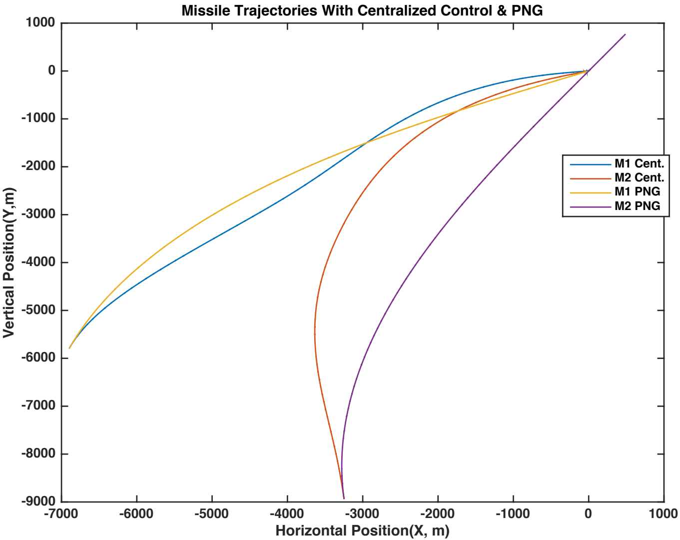

Figure : Missile Trajectories Using PN & Centralized Algorithm

Distributed Coordination Algorithm

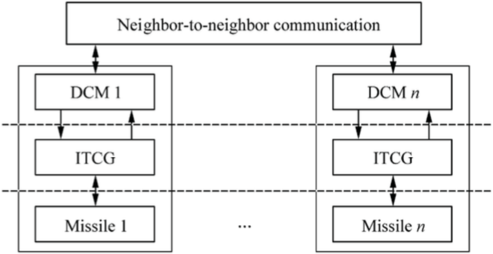

Figure : Co-Operative Guidance Architecture With Distributed Co-Ordination[4]

- CCM is replaced by DCM in each missile.
- Each DCM implements the distribution algorithm.
- According to which, multiple missiles will reach an agreement globally and asymptotically on the designated rendezvous time.
- Centralized algorithms produce the group decision value immediately, whereas distributed algorithms spend infinite time before $T_{di} \to T_d^+$.

Distributed Coordination Algorithm

Consider n missiles attacking a single target. Let T_{di} represent the impact time of missile i. Then, the agreement protocol assuming connectedness of the missiles is given by,

$$\dot{x}_i = c_i \sum_{j \in N_i} (x_j - x_i) \tag{17}$$

Here, $c_i = \frac{1}{w_i}$ and N_i represents the neighbors of missile i. Now, consider the following topology.

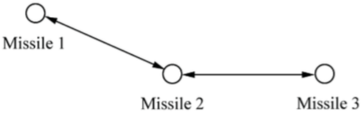

Figure : Communication Topology Of Missiles[4]

Distributed Coordination Algorithm

The distributed coordination algorithm is given by:

$$\dot{x_1} = c_1(x_2 - x_1) \tag{18}$$

$$\dot{x_2} = c_2[(x_2 - x_1) + (x_3 - x_2)] \tag{19}$$

$$\dot{x_3} = c_3(x_2 - x_3) \tag{20}$$

Lemma 1 - Assume that a network has a fixed topology $G(V, E, A)$, which is a strongly connected graph. If the node dynamics are

$$\gamma_i \dot{x}_i = \sum_{j=1}^{n} a_{ij}(x_j - x_i), (\gamma_i > 0, \ \forall i) \tag{21}$$

Where, x_i denotes the state of node i, a_{ij} is the entry of the adjacency matrix A and γ_i is a positive weight.

Distributed Coordination Algorithm

Lemma 1 . . . Continued

Subsequently, an agreement is globally and asymptotically reached, where the group decision value is

$$\alpha = \sum_i \frac{\gamma_i x_i(0)}{\gamma_i} \tag{22}$$

Conclusion

- The use of co-ordination algorithms overcomes the concerns of using ITCG alone.
- Centralized co-ordination algorithm in a leader-follower arrangement is ideal, since the impact time computation is immediate.
- Decentralized agorithm is time consuming since the nodes take time to agree upon a value for the designated impact time.